MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 14

Section A

Q.1 (a) Solution:

(i) Given, temperature, T = 300 K

$$V_1 = 0.4 \text{ V}$$
$$V_2 = 0.42 \text{ V}$$
$$I_1 = 10 \text{ mA}$$
$$I_2 = 30 \text{ mA} \quad \text{(given)}$$
$$V_T = 26 \text{ mV}$$

The expression for diode current is given by,

$$I = I_0\left[e^{V/\eta V_T} - 1\right]$$

For $V_1 = 0.4 \text{ V}, \quad I_1 = 10 \text{ mA}, \quad V_T = 26 \text{ mV}$

$$10 \times 10^{-3} = I_0\left[e^{\frac{0.4}{\eta \times 26 \times 10^{-3}}} - 1\right] = I_0\left[e^{\frac{400}{26\eta}} - 1\right]$$

$\because \quad e^{\frac{400}{26\eta}} >> 1$

$$\therefore \quad 10 \times 10^{-3} = I_0\left[e^{\frac{400}{26\eta}}\right] \quad \text{...(i)}$$

For $V_2 = 0.42 \text{ V}, \quad I_2 = 30 \text{ mA}, \quad V_T = 26 \text{ mV}$

$$30 \times 10^{-3} = I_0\left[e^{\frac{0.42}{26\times 10^{-3}\eta}} - 1\right] = \left[e^{\frac{420}{26\eta}} - 1\right]$$

$\because$ $$e^{\frac{420}{26\times\eta}} >> 1$$

$\therefore$ $$30 \times 10^{-3} = I_0\left[e^{\frac{420}{26\eta}}\right] \quad ...(ii)$$

Dividing equation (ii) with equation (i),

$$\frac{30\times 10^{-3}}{10\times 10^{-3}} = \frac{I_0\left[e^{\frac{420}{26\eta}}\right]}{I_0\left[e^{\frac{400}{26\eta}}\right]}$$

$$3 = e^{\frac{420-400}{26\eta}} = e^{\frac{20}{26\eta}}$$

$$\ln 3 = \frac{20}{26\eta}$$

$$\frac{10}{13\eta} = 1.099$$

$\therefore$ $$\eta = \frac{10}{13\times 1.099} = 0.7$$

Substituting η = 0.7 in equation (i),

$$10 \times 10^{-3} = I_0\left[e^{\frac{400}{26\times 0.7}}\right]$$

$\therefore$ $$I_0 = \frac{10\times 10^{-3}}{e^{\frac{400}{26\times 0.7}}}$$

$\therefore$ $$I_0 = 2.85 \times 10^{-12}\text{ A} \quad \text{(or)} \quad 2.85\text{ pA}$$

(ii) **Given:** $I_0 = 30\ \mu A$; T = 125°C = 125 + 273 = 398 K

Forward bias, $V_f = 0.2$ V

Reverse bias, $V_r = -0.2$ V

We know that,

$$\text{Thermal voltage, } V_T = \frac{T}{11{,}600}$$

$$V_T = \frac{398}{11{,}600} = 34.3\times 10^{-3}\text{ V}$$

Forward dynamic resistance,

$$r_f = \frac{\eta V_T}{I_0 \, e^{V_f / \eta V_T}}$$

$$r_f = \frac{2 \times 34.3 \times 10^{-3}}{30 \times 10^{-6} e^{\left(\frac{0.2}{2 \times 34.3 \times 10^{-3}}\right)}} = 123.9 \ \Omega$$

Reverse dynamic resistance,

$$r_r = \frac{\eta V_T}{I_0 \, e^{V_r / \eta V_T}}$$

$$r_r = \frac{2 \times 34.3 \times 10^{-3}}{30 \times 10^{-6} \times e^{(-0.2 / 2 \times 34.3 \times 10^{-3})}} = 42.2 \ \text{k}\Omega$$

Q.1 (b) Solution:

We have,

$$r_{Na^+} = 0.102 \text{ nm} = 0.102 \times 10^{7} \text{ cm}$$

$$r_{Cl^-} = 0.181 \text{ nm} = 0.181 \times 10^{-7} \text{ cm}$$

$$A_{Na} = 23 \text{ g/mol}$$

$$A_{Cl} = 35.45 \text{ g/mol}$$

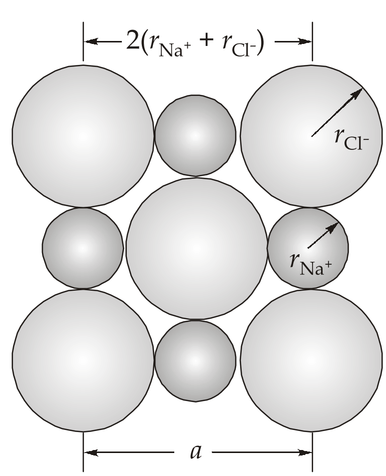
We know that,

Theoretical density is given by

$$\rho = \frac{\text{Atomic volume}}{\text{Volume of unit cell } (a^3)}$$

$$= \frac{n'(A_{Na} + A_{Cl})}{(2r_{Na^+} + 2r_{Cl^-})^3 N_A}$$

Here, n' = the number of NaCl units per unit cell and lattice parameter $a = 2(r_{Na^+} + r_{Cl^-})$



NaCl has FCC lattice. The Na^+ occupies the edge centres and body centred positions of cube while Cl^- occupies the corners and face centres of the cube. Hence, $n' = 4$

On putting all the values, we get

$$\rho = \frac{4(23+35.45)}{\left[(2\times0.102\times10^{-7})+(2\times0.181\times10^{-7})\right]^3\times6.023\times10^{23}}$$

$$\rho = 2.14 \text{ g/cm}^3$$

We have, $$\rho_{experimental} = 2.16 \text{ g/cm}^3$$

$$\text{Error} = \rho_{experimental} - \rho_{theoretical}$$

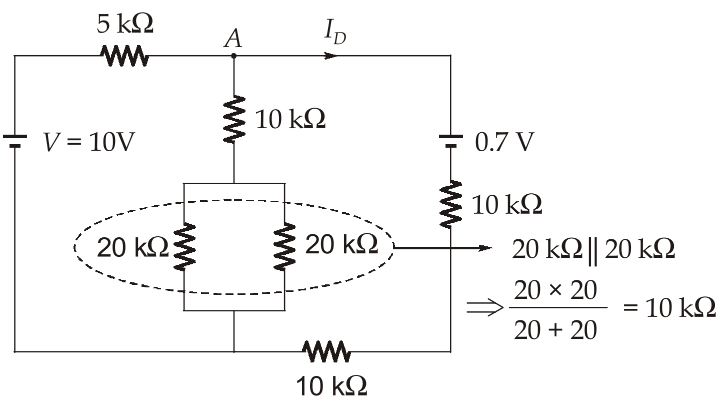
$$= 2.16 - 2.14$$

$$\text{Error} = 0.02 \text{ g/cm}^3$$

Q.1 (c) Solution:

Case-I: Considering DC source only

⇒ Since the diode is forward biased, so we assume $V_D = 0.7$ V and redraw the circuit as



On applying KCL at node A, we get,

$$\frac{V_A - 10}{5\text{ k}\Omega} + \frac{V_A}{20\text{ k}\Omega} + \frac{V_A - 0.7}{20\text{ k}\Omega} = 0$$

$$\Rightarrow \quad 4V_A - 40 + V_A + V_A - 0.7 = 0$$

$$\Rightarrow \quad 6V_A = 40.7$$

$$V_A = 6.78 \text{ Volt}$$

$$I_D = \frac{V_A - 0.7}{20\text{ k}\Omega} = 0.304 \text{ mA}$$

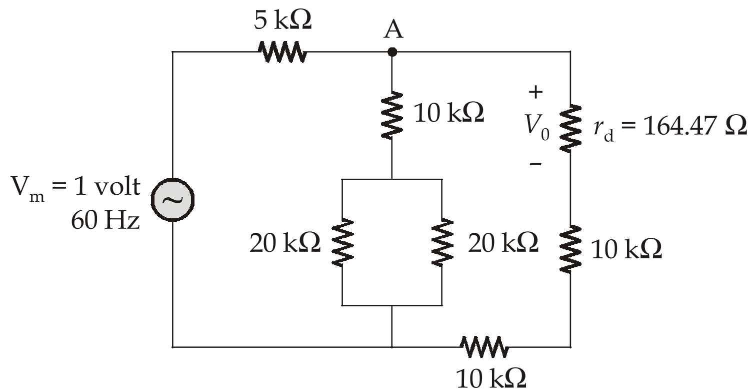
At this operating point, the diode incremental resistance r_d is

$$r_d = \frac{\eta V_T}{I_D} = \frac{2\times25\times10^{-3}}{0.304\times10^{-3}} = 164.47\ \Omega$$

Case-II: Considering AC source only

(Replacing diode with its small signal resistance, r_d)

We redraw the circuit as,



Again apply KCL at node A, we get,

$$\frac{V_A - V_{in}(t)}{5\text{ k}\Omega} + \frac{V_A}{20\text{ k}\Omega} + \frac{V_A}{20.16\text{ k}\Omega} = 0$$

$$403.2V_A - 403.2\ V_{in}(t) + 100.8V_A + 100V_A = 0$$

$$604V_A = 403.2\ V_{in}(t)$$

$$V_A = 0.67\ V_{in}(t)$$

Using voltage division rule to calculate $V_0(t)$,

$$V_o(t) = \frac{0.67 \times 164.47}{20.17\text{ k}\Omega}$$

$$= 5.46 \times 10^{-3}\ V_{in}(t) = 5.46 \sin(2\pi \times 60t) \text{ mV}$$

We note that, since this value is quite small, our use of the small-signal model of the diode is justified.

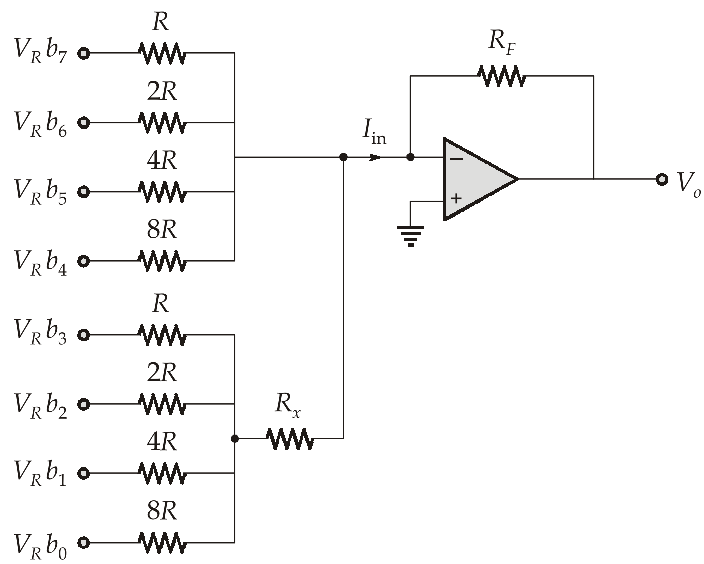
Q.1 (d) Solution:

(i) Let us assume that the binary logic values (1 or 0) are represented as $b_7, b_6, b_5, \ldots b_1, b_0$.

So,
$$d_i = \begin{cases} V_R & ; \quad \text{for } b_i = 1 \\ 0 & ; \quad \text{for } b_i = 0 \end{cases}$$

$$d_i = V_R\, b_i$$

Considering the given circuit with $d_i = V_R\, b_i$ as shown below:



$$V_o = -I_{in} R_F$$

For a digital to analog converter,

$$V_0 \propto \text{[Decimal equivalent of input binary code]}$$

$$V_0 = -I_{in} R_F$$

So, $I_{in} \propto$ [Decimal equivalent of input binary code]

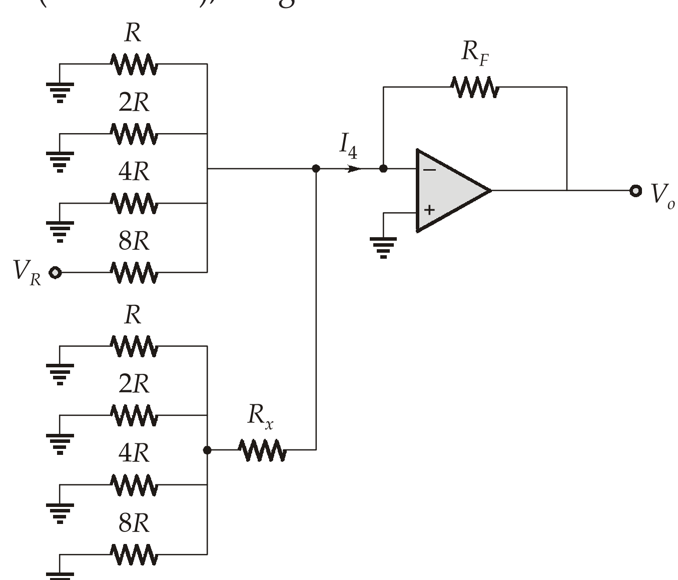
Let, $I_{in} = I_0$ when $(b_7 \ldots b_0) = 0000\ 0001 = (1)_{10}$

and $I_{in} = I_4$ when $(b_7 \ldots b_0) = 0001\ 0000 = (16)_{10}$

So, $I_4 = 16I_0$ ...(i)

To calculate I_4:

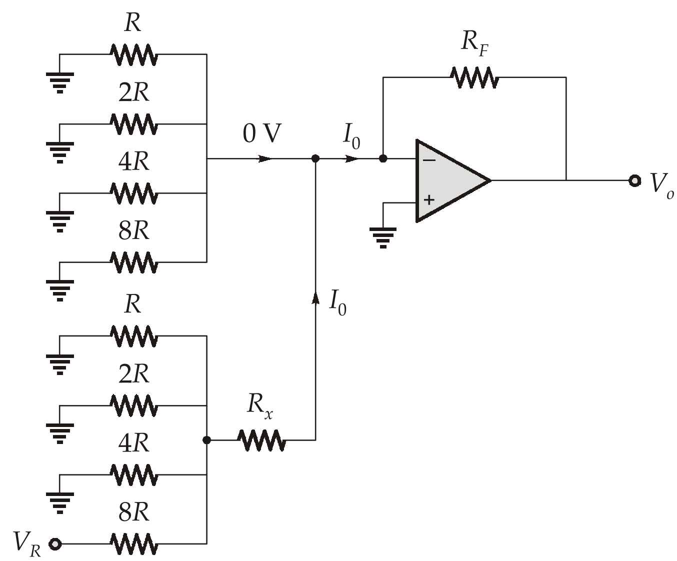
When $(b_7 \ldots b_0)$ = (00001 0000), the given circuit can be drawn as,



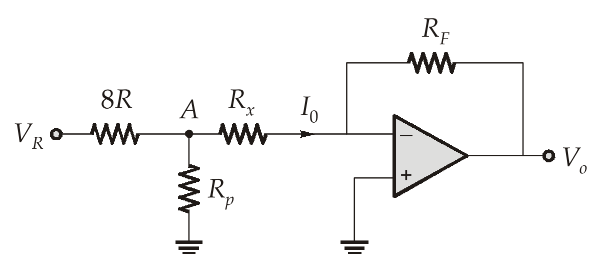
So, $$I_4 = \frac{V_R}{8R} \quad \text{...(ii)}$$

To calculate I_0:

When $(b_7 \ldots b_0)$ = (0000 0001), the given circuit can be drawn as,



The above circuit can be further reduced as,



By applying KCL at node 'A', we get

$$\frac{V_R - V_A}{8R} = \frac{V_A}{R_p} + \frac{V_A - 0}{R_x}$$

where $$R_p = R \,||\, 2R \,||\, 4R = \frac{4R}{7}$$

$$V_A\left(\frac{1}{8R} + \frac{7}{4R} + \frac{1}{R_x}\right) = \frac{V_R}{8R}$$

$$V_A(R_x + 14R_x + 8R) = V_R R_x$$

$$V_A = \frac{V_R R_x}{8R + 15R_x}$$

and $$I_0 = \frac{V_A - 0}{R_x} = \frac{V_A}{R_x} = \frac{V_R R_x}{R_x(8R + 15R_x)} \quad \text{...(iii)}$$

From equations (i), (ii) and (iii),

$$\frac{V_R}{8R} = 16\left[\frac{V_R R_x}{R_x(8R + 15R_x)}\right]$$

$$15R_x^2 + 8RR_x = 128R\,R_x$$

$$15R_x = 120R$$

$$R_x = 8R$$

So, when $R_x = 8R$, the given circuit can be used as an 8-bit (binary) digital to analog converter.

(ii) When the given circuit is used as a 2-decade *BCD* digital to analog converter,

$$I_4 = 10I_0 \quad \text{...(iv)}$$

From equations, (ii), (iii) and (iv), we get

$$\frac{V_R}{8R} = 10\left[\frac{V_R R_x}{R_x[8R + 15R_x]}\right]$$

$$15R_x^2 + 8R\,R_x = 80R\,R_x$$

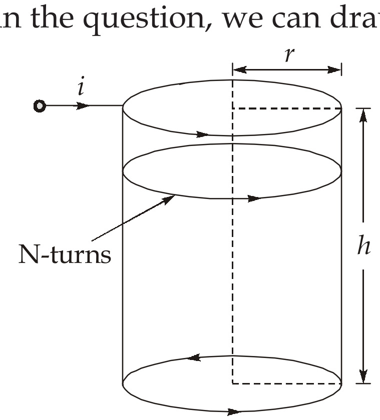
$$15R_x = 72R$$

$$R_x = \frac{72}{15}R = 4.8R$$

So, when $R_x = 4.8R$, the given circuit can be used as a 2-decade *BCD* digital to analog converter.

Q.1 (e) Solution:

As per the description given in the question, we can draw the solenoid as



(i) For the direction of current i shown in figure above, the magnetic field inside the solenoid is directed axially upward.

$\therefore$ Magnetic field intensity inside the solenoid,

$$H = nI = \frac{iN}{h}\text{AT/m},$$

where n is the number of turns per length = N/h

$$\text{flux density, } B = \mu_0 H = \frac{\mu_0 iN}{h}\ \text{Wb/m}^2$$

The field energy density inside the solenoid,

$$\rho_{fld} = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\mu_0\left(\frac{iN}{h}\right)^2\ \text{J/m}^3$$

The field energy stored inside the solenoid,

$$W_{fld} = \rho_{fld} \times \text{Volume of solenoid}$$

$$= \frac{1}{2}\mu_0\left(\frac{iN}{h}\right)^2 \times \pi r^2 h$$

$$= \frac{1}{2}\frac{\mu_0 (Ni)^2}{h}\pi r^2\ \text{Joule}$$

(ii) Radial magnetic force,

$$F_r = \frac{\partial W_{fld}(i,r)}{\partial r} = \frac{1}{2}\frac{\mu_0 (iN)^2}{h} \times 2r\pi$$

$$F_r = \frac{\mu_0 (iN)^2}{h}\pi r\ \text{Newton}$$

(iii) Radial pressure on the sides of solenoid

$$= \frac{f_r}{\text{surface area of solenoid}}$$

$$= \frac{\mu_0 (iN)^2}{h}\pi r \times \frac{1}{2\pi r h}$$

$$= \frac{\mu_0}{2}\frac{(iN)^2}{h^2}\ \text{N/m}^2$$

(iv) Solenoid inductance, $L = \dfrac{N^2 \mu_0 A}{l}$

$$L = \frac{N^2 \mu_0 \pi r^2}{h}\ \text{Henry}$$

Q.2 (a) Solution:

(i) The resolution of a $3\frac{1}{2}$ digit voltmeter on 10 V range is $\frac{V_{FS}}{10^3} = 10^{-3} \times 10 = 10^{-2}$ = 0.01 V. So, the error due to ±1 digit on 10 V range is ±0.01 V.

1. When the instrument is reading 5 V on the 10 V range,

$$\text{error} = (\pm 0.5\% \text{ of } 5 \text{ V}) \pm 0.01 \text{ V}$$

$$= \pm\left(\frac{0.5}{100}\times 5\right) \pm 0.01 \text{ V} = \pm 0.035 \text{ V}$$

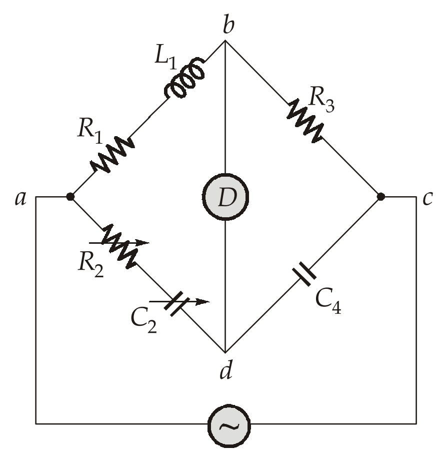
2. When the instrument is reading 0.1 V on 10 V range,

$$\text{error} = (\pm 0.5\% \text{ of } 0.1 \text{ V}) \pm 0.01 \text{ V}$$

$$= \pm\left(\frac{0.5}{100}\times 0.1\right) \pm 0.01 \text{ V} = \pm 0.0105 \text{ V}$$

$$\%\text{error} = \frac{0.0105}{0.1}\times 100 = 10.5\%$$

(ii) From the given data, the circuit diagram of Owen's Bridge can be drawn as below:



We have,

$$Z_1 = R_1 + j\omega L_1, \; Z_3 = R_3$$

$$Z_2 = R_2 - \frac{j}{\omega C_2}, \; Z_4 = \frac{-j}{\omega C_4}$$

At balance condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\left(\frac{-j}{\omega C_4}\right) = R_3\left(R_2 - \frac{j}{\omega C_2}\right)$$

$$\frac{-jR_1}{\omega C_4}+\frac{L_1}{C_4} = R_2R_3-\frac{jR_3}{\omega C_2}$$

On equating Real and Imaginary parts,

$$\frac{L_1}{C_4} = R_2R_3$$

$$L_1 = R_2R_3C_4 \qquad \text{...(i)}$$

and

$$\frac{R_1}{\omega C_4} = \frac{R_3}{\omega C_2}$$

$$R_1 = R_3\left(\frac{C_4}{C_2}\right) \qquad \text{...(ii)}$$

Given: $R_2 = 834\ \Omega,\ R_3 = 100\ \Omega$

$C_2 = 0.124\ \mu\text{F},\ C_4 = 0.1\ \mu\text{F},\ f = 2\text{ kHz}$

From equation (i), $L_1 = R_2R_3C_4 = 834 \times 100 \times 0.1 \times 10^{-6}$

$L_1 = 8.34\text{ mH}$

From equation (ii), $R_1 = R_3\left(\frac{C_4}{C_2}\right) = 100\left(\frac{0.1}{0.124}\right) = 80.65\ \Omega$

$R_1 = 80.65\ \Omega$

Impedance of specimen,

$$Z_1 = R_1 + j\omega L_1 = 80.65 + j2\pi \times 2 \times 10^3 \times 8.34 \times 10^{-3}$$

$$= 80.65 + j104.8\ \Omega$$

$$Z_1 = \sqrt{(80.65)^2+(104.8)^2}$$

$$Z_1 = 132.24\ \Omega$$

Q.2 (b) Solution:

For given circuit, carrying out DC analysis to obtain small signal parameters,

The dc or quiescent gate to source voltage,

$$V_{GSQ} = V_{DD}\times\frac{R_2}{R_1+R_2} = \frac{12\times 29.1}{70.9+29.1}$$

$$V_{GSQ} = 3.492\text{ V}$$

Let the transistor is in saturation region,

∴ The quiescent drain current,

$$I_{DQ} = \frac{K_n}{2}(V_{GSQ} - V_t)^2(1 + V_{DSQ}\lambda)$$

$$= \frac{0.5 \times 10^{-3}}{2}(3.492 - 1.5)^2(1 + V_{DSQ}\lambda)$$

$$I_{DQ} = 0.992(1 + \lambda V_{DSQ})\text{mA} \quad ...(1)$$

Also, $V_{DSQ} = V_{DD} - I_{DQ} R_D$

$$V_{DSQ} = 12 - 5\, I_{DQ}$$

Put in equation (1)

$$I_{DQ} = 0.992[1 + 0.01(12 - 5I_{DQ})]$$

$$I_{DQ} = 0.992 + 0.01 \times 0.992 \times 12 - 5 \times 0.992 \times 0.01 I_{DQ}$$

$$1.0496\, I_{DQ} = 0.992 + 0.11904$$

$$1.0496\, I_{DQ} = 1.11104$$

$$I_{DQ} = 1.059 \text{ mA}$$

$\therefore$ $V_{DSQ} = V_{DD} - I_{DQ}R_D$

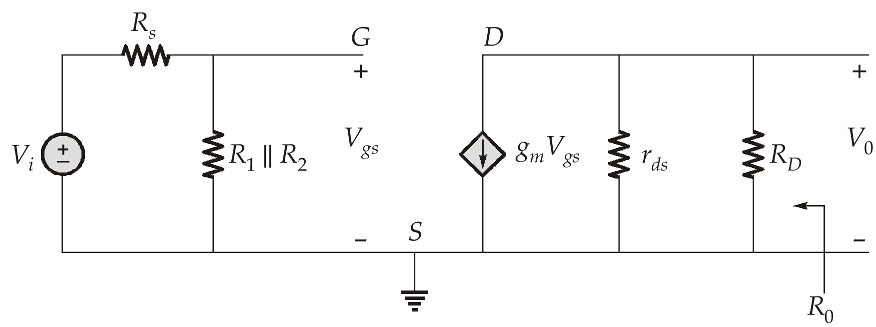
$$= 12 - 5 \times 1.059$$

$$V_{DSQ} = 6.705 \text{ volt}$$

Hence, $V_{DSQ} > V_{GSQ} - V_t$

Hence, our assumption is correct. Transistor is operating in saturation region.

Drawing small signal equivalent circuit,



$$r_{ds} = \frac{1}{\lambda I_{DQ}} = \frac{1}{0.01 \times 1.059 \times 10^{-3}}$$

$$r_{ds} = 94.43 \text{ k}\Omega$$

Amplifier input resistance, $R_i = R_1 \parallel R_2 = 29.1 \parallel 70.9 = 20.63 \text{ k}\Omega$

Amplifier output resistance, $R_0 = r_{ds} \parallel R_D = 94.43 \parallel 5$

$$R_0 = 4.7486 \text{ k}\Omega$$

Small signal transconductance

$$g_m = \sqrt{2I_{DQ} \times K_n(1+\lambda V_{DSQ})}$$

$$= \sqrt{2 \times 1.059 \times 10^{-3} \times 0.5 \times 10^{-3}(1+0.01 \times 6.705)}$$

$$g_m = 1.063 \text{ m℧}$$

Now, $$V_0 = -g_m V_{gs} R_0$$

and $$V_{gs} = \frac{V_i R_i}{R_i + R_s}$$

$\therefore$ $$A_V = \frac{V_0}{V_i} = \frac{-g_m V_{gs} R_0 \times R_i}{V_{gs}(R_i + R_s)}$$

$$A_V = \frac{-g_m R_0 R_i}{(R_i + R_s)} = \frac{-1.063 \times 4.7486 \times 20.63}{(4 + 20.63)}$$

$$A_V = -4.23 \text{ V/V}$$

Q.2 (c) Solution:

(i) Given, $$\Delta E_g = 18.7 \ln\left(\frac{N}{7 \times 10^{17}}\right) \text{ meV}$$

$$T = 300 \text{ K} \qquad \text{(for } kT = 26 \text{ meV)}$$

$$N_E = 10^{19} \text{ cm}^{-3}$$

$$N_B = 10^{18} \text{ cm}^{-3}$$

The emitter injection efficiency,

$$\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{p_{E0}}{n_{B0}}\frac{x_B}{x_E}} = \frac{1}{1 + \frac{p_{E0}}{n_{B0}}} = \frac{1}{1 + \frac{N_B}{N_E}\frac{n_{Ei}^2}{n_{Bi}^2}}$$

$(\because D_E = D_B \text{ and } X_B = X_E)$

Here, n_{Ei} and n_{Bi} are the intrinsic concentration of free carriers in emitter and base respectively.

We know that,

Intrinsic carrier concentration, $n_i^2 \propto \exp(-E_g / kT)$

$\therefore$ $$n_{Ei}^2 \propto \exp(-\Delta E_{gE} / kT)$$

and $$n_{Bi}^2 \propto \exp(-\Delta E_{gB} / kT)$$

$\therefore$ $$\frac{n_{Ei}^2}{n_{Bi}^2} = \frac{\exp\left(-\Delta E_{gE} / kT\right)}{\exp(-\Delta E_{gB} / kT)}$$

$$\frac{n_{Ei}^2}{n_{Bi}^2} = \exp\left(\frac{-\Delta E_{gE} + \Delta E_{gB}}{kT}\right)$$

For, $N = N_{E1} = 10^{19}$ cm^{-3},

$$\Delta E_{gE_1} = 18.7 \ln\left(\frac{10^{19}}{7\times 10^{17}}\right) \text{ meV}$$

$$\therefore \quad \Delta E_{gE_1} = 49.73 \text{ meV}$$

For, $N = N_{E2} = 10^{20}$ cm^{-3}

$$\Delta E_{gE_2} = 18.7 \ln\left(\frac{10^{20}}{7\times 10^{17}}\right) \text{ meV}$$

$$\therefore \quad \Delta E_{gE_2} = 92.78 \text{ meV}$$

For $N = N_B = 10^{18}$ cm^{-3},

$$\Delta E_{gB} = 18.7 \ln\left(\frac{10^{18}}{7\times 10^{17}}\right) \text{ meV} = 6.67 \text{ meV}$$

$\therefore$ For $N = N_{E_1} = 10^{19}$ cm^{-3},

The emitter injection efficiency,

$$\gamma = \frac{1}{1 + \frac{N_B}{N_{E_1}} \cdot \frac{n_{Ei_1}^2}{n_{Bi_1}^2}}$$

where

$$\frac{n_{Ei_1}^2}{n_{Bi_1}^2} = \exp\left(-\frac{\Delta E_{gE_1} + \Delta E_{gB}}{kT}\right)$$

$$= \exp\left(-\frac{49.73 + 6.67}{26}\right) = 0.19$$

$$\therefore \quad \gamma_1 = \frac{1}{1 + \frac{10^{18}}{10^{19}}(0.19)} = 0.98$$

For $N = N_{E_2} = 10^{20}$ cm^{-3},

$$\frac{n_{Ei_2}^2}{n_{Bi}^2} = \exp\left(-\frac{92.78 + 6.67}{26}\right) = 0.036$$

$$\gamma_2 = \frac{1}{1+\frac{10^{18}}{10^{20}}\times 0.036} = 0.99$$

(ii) **Given:** $D_B = 50\ \text{cm}^2/\text{s}$; $L_B = 3.5\ \mu\text{m}$; $x_B = 0.5\ \mu\text{m}$

For common-emitter configuration, the frequency is determined by the lifetime of the minority carriers in the base region

i.e., $$f_E \propto \frac{1}{\tau_B} \Rightarrow f_E = \frac{1}{2\pi\left(\frac{L_B^2}{D_B}\right)}$$

where, $$f_E = \frac{1}{2\pi}\frac{D_B}{L_B^2} = \frac{50}{2\pi(3.5\times 10^{-4})^2} = 65\ \text{MHz}$$

For common-base configuration, the frequency depends on the diffusion rate through base region,

i.e., $$f_B = \frac{1}{2\pi\tau_{diff}}$$ where, $\tau_{diff} = \frac{x_B^2}{2D_B}$

$$f_B = \frac{1}{2\pi}\left(\frac{2D_B}{x_B^2}\right) = \frac{1}{2\pi}\left(\frac{2\times 50}{(0.5\times 10^{-4})^2}\right)$$

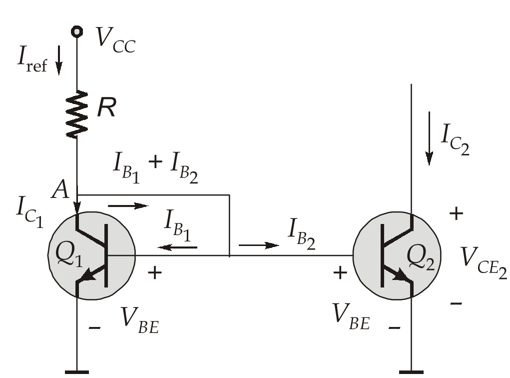
∴ $$f_B = 6.37\times 10^9\ \text{Hz} = 6.37\ \text{GHz}$$

Q.3 (a) Solution:

(i) A current mirror is a circuit designed to copy a current through one active device by controlling the current in another active device of a circuit, keeping the output current constant regardless of loading. Current mirrors are used to provide bias currents and active loads to circuits.

Basic Current Mirror:

The circuit diagram of basic current mirror is drawn below:



Here, we assume both the transistors are identical and having infinite early voltage.

On applying KCL at node A, we get,

$$I_{ref} = I_{C_1} + I_{B_1} + I_{B_2}$$

Since, both the transistors are identical and $V_{BE1} = V_{BE2}$, hence $I_{B_1} = I_{B_2}; I_{C_1} = I_{C_2}$

$$I_{ref} = I_{C_1} + 2I_{B_1} = I_{C_1} + \frac{2I_{C_1}}{\beta} = I_{C_1}\left[1+\frac{2}{\beta}\right]$$

$$I_{C_1} = \frac{I_{ref}}{\left[1+\frac{2}{\beta}\right]}$$

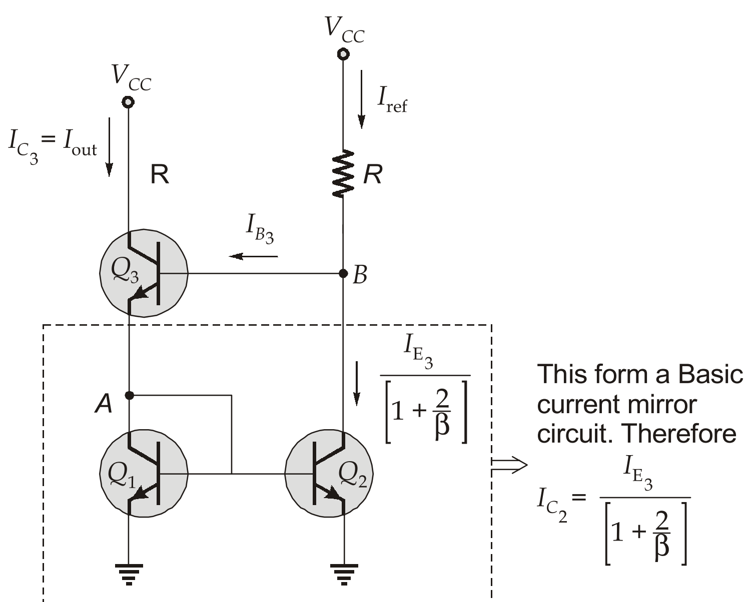
$$I_{C_2} = I_{C_1} = \frac{I_{ref}}{\left[1+\frac{2}{\beta}\right]}$$

For large β transistor, $I_{C_2} = I_{ref}$

Hence, basic current mirror is preferred for high 'β' transistors.

Wilson Current Mirror

The circuit diagram of Wilson current mirror is drawn below:



Now, by applying KCL at node B, we get,

$$I_{ref} = I_{B_3} + \frac{I_{E_3}}{\left[1+\frac{2}{\beta}\right]} \qquad \therefore\ I_{B_3} = \frac{I_{C_3}}{\beta}$$

$$I_{ref} = \frac{I_{C_3}}{\beta} + \frac{I_{E_3}}{\left[1+\frac{2}{\beta}\right]}$$

Since, $$I_{E_3} = I_{C_3} + I_{B_3}$$

$$I_{E_3} = I_{C_3} + \frac{I_{C_3}}{\beta} = I_{C_3}\left[1+\frac{1}{\beta}\right]$$

$$I_{ref} = \frac{I_{C_3}}{\beta} + \frac{I_{C_3}\left(1+\frac{1}{\beta}\right)}{\left(1+\frac{2}{\beta}\right)}$$

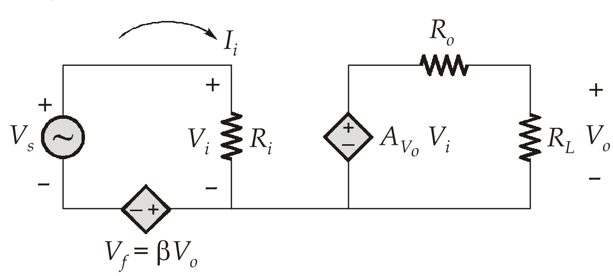
We have, $$I_{C_3} = I_{out}$$

$\therefore$ $$I_{ref} = \frac{I_{out}}{\beta} + \frac{I_{out}\left(1+\frac{1}{\beta}\right)}{\left(1+\frac{2}{\beta}\right)} = \left[\frac{1}{\beta} + \frac{(\beta+1)}{(\beta+2)}\right]I_{out}$$

$$I_{out} = \frac{I_{ref}}{\left(1+\frac{2}{\beta(2+\beta)}\right)}$$

Wilson current mirror provide output current same as input current, even if transistor have smaller 'β'.

(ii) The circuit for voltage series feedback is shown below:



where, R_i = Input impedance without feedback

R_o = Output impedance without feedback

A_V = Voltage gain for finite 'R_L'

A_{V_o} = Voltage gain for infinite 'R_L'

Input resistance with feedback is given as $R_{if} = \dfrac{V_s}{I_i}$

On applying KVL in input loop, we get,

$$-V_s + V_i + V_f = 0$$

$$V_s = V_i + V_f$$

$$V_s = V_i + \beta V_o \qquad [\because\ A_V V_i = V_o]$$

$$V_s = V_i + \beta A_V V_i = (1 + \beta A_V)V_i$$

$$R_{if} = \frac{V_s}{I_i} = \frac{V_i(1+\beta A_V)}{I_i} = R_i(1+\beta A_V)$$

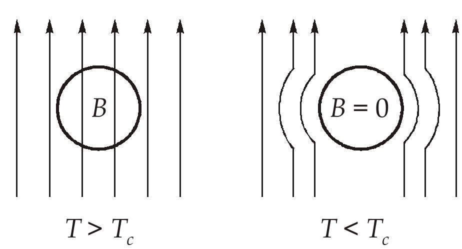
Hence, from above equation, it is proved that the input impedance is increased by the factor of $(1 + \beta A_V)$.

Q.3 (b) Solution:

(i) 1. Meissner Effect:

Superconductors not only exhibits zero resistance but also spontaneously expel all magnetic flux when cooled through the superconducting transition, that is they are also perfect diamagnets. This is called as Meissner effect.

Meissner effect is not a consequence of zero resistance and Lenz's law. The flux is expelled as the superconductor is cooled in constant magnetic field. There is no time rate of change of the magnetic induction. Lenz's law does not apply. Perfect diamagnetism is an independent property of superconductors and shows that superconductivity involves a change of thermodynamic state, not just a spectacular change in electrical resistance. Figure below shows the illustration of Meissner effect that is one of the popular symbols of superconductivity.



It has been observed that when a long superconductor is cooled in a longitudinal magnetic field from above the transition temperature, the lines of induction are pushed out. Then inside the specimen, $B = 0$. We know from magnetic properties of materials that $B = \mu_0(H + M)$, for $B = 0$, $H = -M$, consequently since $\chi_m = \frac{M}{H}$, we may state that magnetic susceptibility of superconductor is negative, this is referred to as perfect diamagnetism. This phenomenon is called Meissner effect. One of the Maxwell's equation is

$$\nabla \times \vec{E} = \frac{-\partial B}{\partial t}$$

and ohm's law, $J = \sigma E$ (or) $E = \rho J$

with $\rho = 0$, $E = 0$, so $\frac{\partial B}{\partial t} = 0$, but this is not so because the flux exclusion from normal to superconducting state takes place. A perfect diamagnetism and zero resistivity are two independent essential properties of superconducting state.

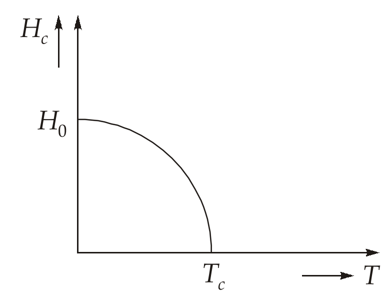
2. **Silsbee's Rule:**

The critical value of magnetic field for destruction of superconductivity, H_c is a function of temperature, at $T = T_c$, $H_c = 0$.

With only small deviations, the critical field H_c varies with temperature according to parabolic law

$$H_c = H_0\left(1-\left(\frac{T}{T_C}\right)^2\right)$$

H_0 is the critical field at absolute zero and T_c is the transition temperature. For any particular superconductor, the shape of variation of H_c with temperature is shown in figure below.



The magnetic field which causes a superconductor to become normal from a superconducting state is not necessarily an external applied field it may arise as a result of electric current flow in the conductor.

In a long superconductor wire of radius r, the superconductivity may be destroyed when a current I exceeds the critical current value I_C, which at the surface of wire will produce a critical field H_C, given by $I_C = 2\pi r$Hc called silsbee's rule.

3. **Frequency effect:**

Superconductivity is observed for d.c. and upto radio frequencies. It is not observed for higher frequencies. For a superconductor, the resistance is zero only when the current is steady or varies slowly. When the current fluctuates or alternates, small absorption of energy roughly proportional to rate of alternation occurs. When the frequency of alternation rises above 10 MHz, appreciable resistance arises, and at infrared frequencies (10^{13} Hz) the resistivity is same in the normal and superconducting states, and is independent of temperature.

(ii) Alpha Ray: (α-ray)

- Definition: Alpha rays or Alpha particle are positively charged particles consisting of two protons and two neutrons.

  (They are like helium nucleus having zero electron)

- Example: ${}_{92}U^{238} \longrightarrow {}_{90}Th^{234}$ + Alpha particle
- Penetration power: They have least penetration power because of high mass. Therefore, they can't penetrate the skin.
- Ionization power: They have highest ionization power because of high positive charge and heavy mass.

Beta Ray: (β-ray)

- Definition: Beta rays or Beta particle are extremely energetic electrons that are released from nucleus of heavy atoms. When a neutron in the nucleus is divided into a proton and electron, then negative charged electron having negligible mass is released from nucleus with high energy.
- Example: ${}_{90}Th^{234} \longrightarrow {}_{91}Pa^{234} + {}_{-1}e^{0}$ (β- particle)
- Penetration power: β-rays have higher penetration power as compare to α-rays.
- Ionization power: β-rays have lower ionization power than the α-rays.

Gamma Ray: (γ-ray)

- Definition: Gamma rays are photons (energy) having very high frequency and very high energy but they have no mass and no charge.
- Example: ${}_{90}Th^{230} \longrightarrow {}_{90}Th^{230}$ + γ-rays
- Penetration power: γ-rays has highest penetration power among the three. Because of no mass, γ-rays can penetrate through skin and enter into human bones.
- Ionization power: γ-rays have least ionization power because there is no charge.

Q.3 (c) Solution:

(i) Given: $N_a = N_d = 10^{18}$ cm^{-3} ; Peak electric field, $E_{max} = 10^6$ V/cm

We know that,
$$E_{max} = \frac{qN_D W_n}{\in} = \frac{1.6\times10^{-19}\times10^{18}\times W_n}{11.7\times8.85\times10^{-14}}$$
$$10^6 = \frac{1.6\times10^{-19}\times10^{18}\times W_n}{11.7\times8.85\times10^{-14}}$$
$$W_n = 6.47\times10^{-6}\text{ cm}$$

We know that, Width of depletion region on n-side,
$$W_n = \left[\frac{2\in(V_{bi}+V_R)}{q}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$
$$(6.47\times10^{-6})^2 = \left[\frac{2\times11.7\times8.85\times10^{-14}}{1.6\times10^{-19}}\left(\frac{10^{18}}{10^{18}}\right)\left(\frac{1}{10^{18}+10^{18}}\right)\right]\times(V_{bi}+V_R)$$
$$\therefore \quad V_{bi}+V_R = 6.468\text{ V}$$

But built-in potential,
$$V_{bi} = V_T\ln\left(\frac{N_aN_d}{n_i^2}\right)$$
$$= 0.0259\ln\left(\frac{10^{18}\times10^{18}}{(1.5\times10^{10})^2}\right) = 0.933\text{ V}$$

We have, $V_{bi}+V_R = 6.468$ V

$\Rightarrow$ $0.933 + V_R = 6.468$ V

$\therefore$ $V_R = 5.54$ V

(ii) Applying nodal analysis at V_1, we get
$$\frac{V_1}{3}+\frac{V_1-V_2}{-j5}+\frac{V_1-V_2}{j3}+5\angle90^\circ = 0$$
$$V_1\left[\frac{1}{3}+\frac{1}{-j5}+\frac{1}{j3}\right]-V_2\left[\frac{1}{-j5}+\frac{1}{j3}\right] = -j5$$
$$(0.33-j0.133)V_1 + V_2(j0.133) = -j5$$
$$V_1 = \frac{-j5-j0.133V_2}{0.33-j0.133} = 5.26-j13.02-V_2[-0.139+j0.346]$$
$$V_1 = 5.26-j13.02+(0.139-j0.346)V_2 \qquad ...(i)$$

Applying KCL at node (2)

$$\frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j3} + \frac{V_2}{6} = 10\angle 0°$$

$$-V_1\left[\frac{1}{-j5} + \frac{1}{j3}\right] + V_2\left[\frac{1}{-j5} + \frac{1}{j3} + \frac{1}{6}\right] = 10$$

$$j0.133V_1 + (0.166 - j0.133)V_2 = 10$$

Using equation (i),

$$j0.133[5.26 - j13.02 + (0.139 - j0.346)\ V_2] + (0.166 - j0.133)V_2 = 10$$

$$j0.7 + 1.73 + j0.0184V_2 + 0.046V_2 + 0.166V_2 - j0.133V_2 = 10$$

$$(0.212 - j0.1146)V_2 = -j0.7 - 1.73 + 10$$

$$V_2 = 31.569 + j13.76$$

$$V_2 = 34.44\angle 23.56°\ \text{V}$$

Q.4 (a) Solution:

The state table of the given state diagram is

Present State	Next State		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	c	b	1	1
b	d	c	0	0
c	g	d	0	1
d	e	f	1	0
e	a	f	1	0
f	g	f	1	0
g	a	f	1	0

For states "g"' and "e", next state and output are same for an applied input. Hence, 'g' and 'e' are said to be equivalent states. So, remove "g" and replace it with "e".

The reduced state table is

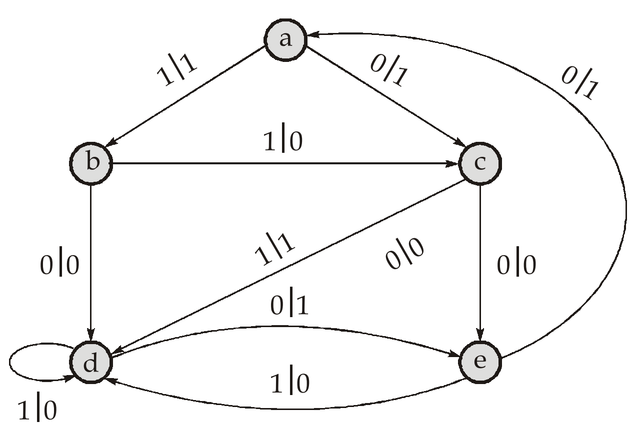
Present State	Next State		Output	
	X = 0	X = 1	X = 0	X = 1
a	*c*	*b*	1	1
b	*d*	*c*	0	0
c	*e*	*d*	0	1
d	*e*	*f*	1	0
e	*a*	*f*	1	0
f	*e*	*f*	1	0

"*d*" and "*f*" are said to be equivalent states. So, remove "*f*" and replace it with "*d*".

The reduced state table is

Present State	Next State		Output	
	X = 0	X = 1	X = 0	X = 1
a	*c*	*b*	1	1
b	*d*	*c*	0	0
c	*e*	*d*	0	1
d	*e*	*d*	1	0
e	*a*	*d*	1	0

No further reduction is possible, hence, the reduced state diagram is as shown:



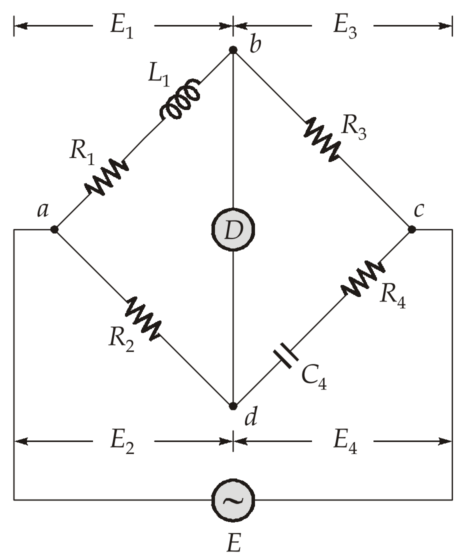
Q.4 (b) Solution:

(i) The Hay's bridge is a modification of Maxwell's bridge. This bridge uses a resistance in series with the standard capacitor as shown in the circuit below.

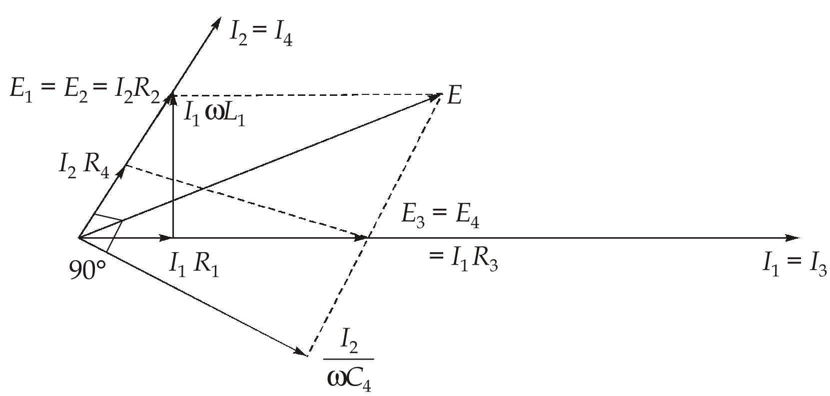
Let, L_1 = Unknown inductance having a resistance R_1

R_2, R_3, R_4 = Known non-inductive resistance,

and C_4 = Standard capacitor



Phasor Diagram:



At balance, $(R_1 + j\omega L_1)\left(R_4 - \dfrac{j}{\omega C_4}\right) = R_2 R_3$

or

$$R_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = R_2 R_3$$

Separating the real and imaginary terms, we obtain,

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \quad \text{and} \quad L_1 = \frac{R_1}{\omega^2 R_4 C_4}$$

Solving the above two equations, we have

$$L_1 = \frac{R_2 R_3 C_4}{1+\omega^2 C_4^2 R_4^2}$$

and, $$R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1+\omega^2 C_4^2 R_4^2}$$

The Q-factor of the coil is, $$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

Hence, $$L_1 = \frac{R_2 R_3 C_4}{1+(1/Q)^2}$$

When Q is greater than 10 then $1/Q^2$ is very small and hence, it can be neglected and we get, $L_1 = R_2R_3C_4$

The expressions for the unknown inductance and resistance contain the frequency term. Therefore, it appears that the frequency of the source of supply to the bridge must be accurately known.

Advantages:

1. This bridge gives very simple expression for unknown inductance for high Q coils, and is suitable for coils having $Q > 10$.
2. This bridge also gives a simple expression for Q factor.
3. If we examine the expression for Q factor: $Q = \dfrac{1}{\omega C_4 R_4}$, we find that the resistance

   R_4 appears in the denominator and hence for high Q coils, its value should be small. Thus, this bridge requires only a low value resistor for R_4, whereas the Maxwell's bridge requires a parallel resistor, R_4, of a very high value.

Disadvantages: The Hay's bridge is suited for the measurement of high Q inductors, especially Q >10. For those inductors having Q values smaller than 10, the term $\left(\dfrac{1}{Q}\right)^2$ in the expression for inductance L_1 becomes rather important and thus, cannot be neglected.

Hence, this bridge is not suited for measurement of coils having Q less than 10 and for these applications, a Maxwell's bridge is more suited.

(ii) The resistance at ice point (T = 0°C = 273 K), R_o = 3980 Ω.

Using the resistance temperature relationship,

$$R_T = aR_0 \exp(b/T)$$

$$\therefore \quad 3980 = a \times 3980 \times \exp\left(\frac{b}{273}\right)$$

or $$1 = a\exp\left(\frac{b}{273}\right) \quad ...(i)$$

Resistance at 50°C is R_T = 794 Ω.

Absolute temperature corresponding to 50°C

$$T = 273 + 50° = 323 \text{ K}$$

Hence, $$794 = a \times 3980 \exp\left(\frac{b}{323}\right)$$

$$= 3980a\exp\left(\frac{b}{323}\right) \quad ...(ii)$$

Solving equations (i) and (ii), we have

$$a = 30 \times 10^{-6} \text{ and } b = 2843$$

Absolute temperature at 40°C = 273 + 40 = 313 K

$\therefore$ $$\text{Resistance at 40°C} = 30 \times 10^{-6} \times 3980 \times \exp\left(\frac{2843}{313}\right) = 1051.308\ .$$

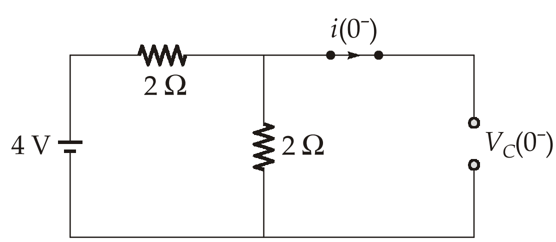
Absolute temperature at 100°C = 273 + 100 = 373 K

$\therefore$ $$\text{Resistance at 100°C} = 30 \times 10^{-6} \times 3980 \times \exp\left(\frac{2843}{373}\right) = 244\ \Omega$$

Thus, the range of resistance is 244 Ω to 1051.308 Ω.

Q.4 (c) Solution:

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit and the capacitor acts as an open circuit.



$$v_c(0^-) = 4 \times \frac{2}{2+2} = 2 \text{ V}$$

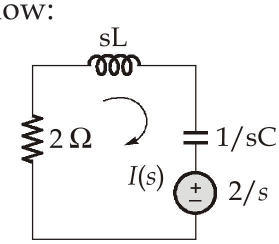
$$i(0^-) = 0$$

Since current through the inductor and voltage across the capacitor cannot change instantaneously,

$$v_c(0^+) = 2 \text{ V}$$

$$i(0^+) = 0$$

When switch is closed, the circuit is s-domain can be drawn as below:



The characteristic equatio of RLC circuit is given by

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\Rightarrow \quad s = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

- If $\alpha > \omega_0$, we have, $s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = s_1, s_2$ i.e., over-damped system. Thus,

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

- If $\alpha = \omega_0$, we have, $s_1 = s_2 = -\alpha$ i.e., critical damping. Thus,

$$i(t) = (A_1 + A_2 t)e^{-\alpha t}$$

- If $\alpha < \omega_0$, we have, $s = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2} = \alpha \pm j\omega_d$ i.e., under damped system. Thus,

$$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

Case I: When $R = 2\ \Omega$, $L = \frac{1}{2}$H, $C = 1$ F

$$\alpha = \frac{R}{2L} = \frac{2}{2 \times \frac{1}{2}} = 2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \times 1}} = \frac{1}{\sqrt{0.5}} = 1.414$$

Since, $\alpha > \omega_0$

This indicates an overdamped case.

$$\therefore \quad i(t) = A_1 e^{s_1 t} - A_2 e^{s_2 t}$$

where $s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$$= -2 - \sqrt{4 - 2} = -2 - \sqrt{2} = -3.414$$

and $s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2 + \sqrt{2} = -0.586$

$$\therefore \quad i(t) = A_1 e^{-3.414t} + A_2 e^{-0.586t}$$

At $t = 0$, $i(0) = 0$

$$A_1 + A_2 = 0 \quad \text{...(i)}$$

Also, $$v_L(0^+) + v_C(0^+) + v_R(0^+) = 0$$

$$v_L(0^+) = -v_R(0^+) - v_C(0^+)$$
$$= -2i(0^+) - v_C(0^+)$$
$$= -2 \text{ V} \quad \text{...(ii)}$$

We have, $$v_L(0^+) = L\frac{di}{dt}(0^+)$$

$$\frac{di}{dt}(0^+) = \frac{v_L(0^+)}{L} = -\frac{2}{0.5} = -4 \text{ A/s}$$

Differentiating the equation of $i(t)$ and putting the condition at $t = 0$, we get

$$-3.414A_1 - 0.586A_2 = -4 \quad \text{...(iii)}$$

Solving Eqs (i) and (iii), we get

$A_1 = 1.414$ and $A_2 = -1.414$

$$i(t) = 1.414(e^{-3.414t} - e^{-0.586t})\text{A} \quad \text{for } t > 0$$

Case II: When $R = 2\ \Omega$, $L = 1$ H, $C = 1$ F

$$\alpha = \frac{R}{2L} = \frac{2}{2\times 1} = \frac{2}{2} = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1}} = 1$$

Since, $$\alpha = \omega_0$$

This indicates a critically damped case.

$\therefore$ $$i(t) = e^{-\alpha t}(A_1 + A_2 t)$$
$$= e^{-t}(A_1 + A_2 t)$$

At $t = 0$, $i(0) = 0$

$$A_1 = 0$$

Also, $$v_L(0^+) = L\frac{di}{dt}(0^+)$$

$$\frac{di}{dt}(0^+) = \frac{v_L(0^+)}{L} = -\frac{2}{1} = -2 \text{ A/s}$$

Differentiating the equation of $i(t)$ and putting the condition at $t = 0$, we get

$$\left.\frac{di(t)}{dt}\right|_{t=0} = -A_1 + A_2 = -2$$

$$A_2 = -2$$

$$i(t) = -2te^{-t} \text{A} \qquad \text{for } t > 0$$

Case III: When $R = 2\ \Omega$, $L = 5$ H, $C = 1$ F

$$\alpha = \frac{R}{2L} = \frac{2}{10} = 0.2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5}} = 0.447$$

Since, $\alpha < \omega_0$

This indicates an underdamped case.

$$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

where, $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(0.447)^2 - (0.2)^2} = 0.4$

$$s_1, s_2 = -\alpha \pm j\,\omega_d$$
$$= -0.2 \pm j\,0.4$$

$\therefore$ $i(t) = e^{-0.2t}\,(B_1 \cos 0.4\,t + B_2 \sin 0.4t)$

Applying the initial conditions,

$$i(0^+) = 0$$

and $\frac{di}{dt}(0^+) = -\frac{v_L(0^+)}{L} = -\frac{2}{5}\text{A/s}$

$$B_1 = i(0) = 0$$

We have, $\left.\frac{di(t)}{dt}\right|_{t=0} = -0.2B_1 + 0.4B_2 = -0.4$

$\Rightarrow$ $B_2 = -1$

$\therefore$ $i(t) = -e^{-0.2t} \sin 0.4t$ A $\qquad$ for $t > 0$

Section B

Q.5 (a) Solution:

(i) Heisenberg uncertainty Principle:

Heisenberg's uncertainty principle states that for particles exhibiting both particle and wave nature, it will not be possible to accurately determine both the position and velocity at the same time. This principle is based on the wave-particle duality of matter. It states that the product of uncertainty in position and uncertainty in momentum must be greater than or equal to $\frac{h}{4\pi}$.

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi};$$

where, Δx – Uncertainty in position
Δp – Uncertainty in momentum
$h = 6.6256 \times 10^{-34}$ Joule sec

(ii) De-Broglie Wave equation:

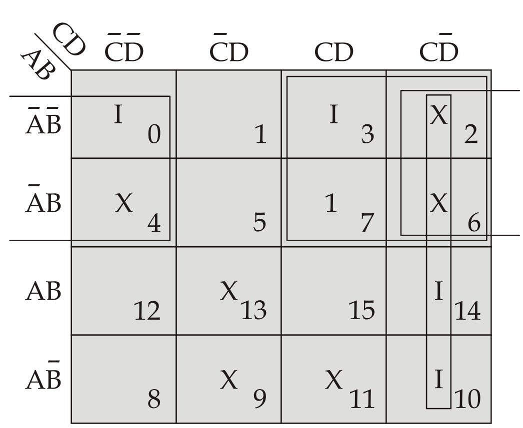
Quantum mechanics assume matter to be both like a wave as well as a particle at the sub-atomic level. The De Broglie equation states that every particle that moves can sometimes act as a wave, and sometimes as particle. The wave which is associated with the particles that are moving are known as the matter-wave and also as the De Broglie wave. The wavelength of the matter wave is known as the de Broglie wavelength given as:

$\lambda = \frac{h}{mv}$ $h \rightarrow$ Planck's constant; λ = De Broglie wavelength; $m \rightarrow$ mass of particle
$V \rightarrow$ Velocity of particle

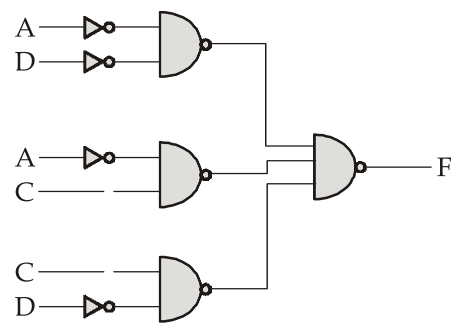
Q.5 (b) Solution:

(i)

Decimal Number	5211 BCD				
	A	*B*	*C*	*D*	*F*
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	1	1
3	0	1	0	1	0
4	0	1	1	1	1
5	1	0	0	0	0
6	1	0	1	0	1
7	1	1	0	0	0
8	1	1	1	0	1
9	1	1	1	1	0



$$F = \bar{A}C + \bar{A}\bar{D} + C\bar{D}$$



(ii) Let us assume that the current drawn by the entire circuit from V_{CC} is I_{CC}.

When $Y = 1$.

- Y will be '1', when all the five driving gates are having output at logic –1 (or logic HIGH)

So,
$$I_{CC} \geq 5\,(I_{OH}) + +6(I_{IH})$$

$$\frac{V_{CC} - V_{OH}}{R_C} \geq 5(250\,\mu\text{A}) + 6(40\,\mu\text{A})$$

$$R_C \leq \frac{(5-2.4)\times 1000}{5\times 250 + 6\times 40}\,\text{k}\Omega$$

$$R_C \leq 1.745\ \text{k}\Omega \qquad ...(1)$$

When $Y = 0$;

- Y will be at logic-0, when at least one driving gate has output at logic-0 (or logic-low)

So,
$$I_{CC} \leq (I_{OL}) + 6(I_{IL})$$

$$\frac{V_{CC} - V_{OL}}{R_C} \leq 16\ \text{mA} + 6 \times (-1.6)\ \text{mA}$$

$$R_C \geq \frac{5-0.4}{16 - 6\times 1.6} = 0.718\ \text{k}\Omega$$

$$R_C \geq 718.75\ \Omega \qquad ...(2)$$

From (1) and (2), we get, the required range of the value of R_C is,

$$718.75\ \Omega \leq R_C \leq 1.745\ \text{k}\Omega$$

Q.5 (c) Solution:

Given:

$$\text{Generation rate, } g_{op} = 10^{19}\ \text{cm}^{-3}/\text{sec}$$

$$\text{Donor concentration, } n_0 = 10^{15}\ \text{cm}^{-3}$$

$$\text{Carrier life time } \tau = 10 \times 10^{-6} = 10^{-5}\text{s}$$

$$D_p = 12\ \text{cm}^2/\text{s},\ \mu_n = 1300\ \text{cm}^2/\text{V-sec}$$

Electron-hole concentration due to light induced

$$= \delta n = \delta p = g_{op}\,\tau$$

$$\delta n = \delta p = 10^{19} \times 10^{-5} = 10^{14}\ \text{cm}^{-3}$$

Here, $\delta n <<$ dopant concentration of n_0. So low level injection

$$n = n_0 + \delta n = 10^{15} + 10^{14} = 1.1 \times 10^{15}\ \text{cm}^{-3}$$

$$p = p_0 + \delta p = \frac{n_i^2}{n_0} + \delta p = \frac{(1.5\times 10^{10})^2}{10^{15}} + 10^{14} \cong 10^{14}\,\text{cm}^{-3}$$

We have,

$$\mu_p = \frac{D_p}{V_T} = \frac{12}{0.0259} = 463.32\ \text{cm}^2/\text{Vs}$$

Quasi-fermi level separation

$$= F_n - F_p = KT\ln\left(\frac{n\cdot p}{n_i^2}\right)$$

$$F_n - F_p = 0.0259\ln\left(\frac{1.1\times 10^{15}\times 10^{14}}{(1.5\times 10^{10})^2}\right) = 0.5182\ \text{eV}$$

Change in conductivity,

$$\Delta\sigma = q(\mu_n\delta n + \mu_p\delta p)$$

$$= 1.6 \times 10^{-19}\,(1300 \times 10^{14} + 463.32 \times 10^{14})$$

$$\Delta\sigma = 0.0282/\Omega\ \text{cm}$$

Q.5 (d) Solution:

Apply KCL at node (1)

$$\frac{V_1}{4} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{2} + 3 = 5$$

$$2V_1 + V_1 - V_2 + 4V_1 - 4V_3 = 16$$

$$7V_1 - V_2 - 4V_3 = 16 \qquad \text{...(1)}$$

Apply KCL at node (2)

$$\frac{V_2 - V_1}{8} + \frac{V_2}{2} + \frac{V_2 - V_3}{4} = 0$$

$$V_2 - V_1 + 4V_2 + 2V_2 - 2V_3 = 0$$

$$-V_1 + 7V_2 - 2V_3 = 0 \qquad ...(2)$$

Apply KCL at node (3)

$$\frac{V_3 - 12}{8} + \frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{2} = 3$$

$$V_3 - 12 + 2V_3 - 2V_2 + 4V_3 - 4V_1 = 24$$

$$-4V_1 - 2V_2 + 7V_3 = 36 \qquad ...(3)$$

Writing equations (1), (2) and (3) in matrix form,

$$\begin{bmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 36 \end{bmatrix}$$

Determinant of conductance matrix,

$$\Delta = \begin{vmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{vmatrix} \Rightarrow \Delta = 7(49-4) + 1(-7-8) - 4(2+28)$$

$$\Delta = 7 \times 45 + 1 \times (-15) - 4 \times 30$$

$$\Delta = 180$$

$$\Delta_1 = \begin{vmatrix} 16 & -1 & -4 \\ 0 & 7 & -2 \\ 36 & -2 & 7 \end{vmatrix} \Rightarrow \Delta_1 = 16(49-4) + 1(0+72) - 4(-252)$$

$$\Delta_1 = 1800$$

$$\Delta_2 = \begin{vmatrix} 7 & 16 & -4 \\ -1 & 0 & -2 \\ -4 & 36 & 7 \end{vmatrix} \Rightarrow \Delta_2 = 7(72) - 16(-7-8) - 4(-36)$$

$$\Delta_2 = 888$$

$$\Delta_3 = \begin{vmatrix} 7 & -1 & 16 \\ -1 & 7 & 0 \\ -4 & -2 & 36 \end{vmatrix} \Rightarrow \Delta_3 = 7(252-0) + 1(-36) + 16(30)$$

$$\Delta_3 = 2208$$

Using Cramer's rule,

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{1800}{180} = 10\text{ V}; V_2 = \frac{\Delta_2}{\Delta} = \frac{888}{180} = 4.933\text{ V};$$

$$V_3 = \frac{\Delta_3}{\Delta} = 12.267\text{ V}$$

Q.5 (e) Solution:

The magnetization for a paramagnetic spin system is given by curie law as

$$M = \frac{NP_B^2\mu_0 H}{KT} = N_p$$

where N = Number of atoms per unit volume and p = Magnetic moment

The average magnetic moment per spin in Bohr magneton becomes

$$M' = \frac{M}{NP_B} = \frac{P_B\mu_0 H}{KT}$$

where P_B = Bohr magneton = 9.27×10^{-24} Am^2

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$H = 10^6$ A/m

Boltzmann's constant, $K = 1.38 \times 10^{-23}$ J/K

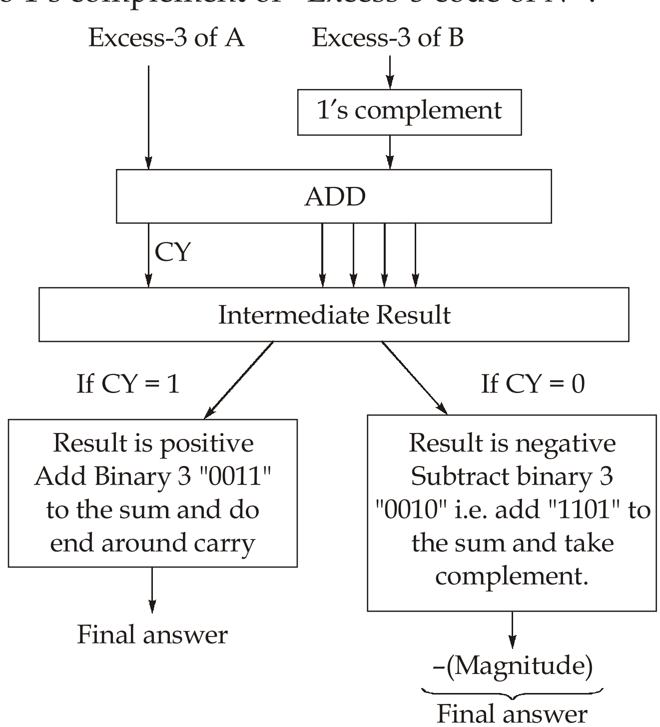
Temperature, T = 300 K

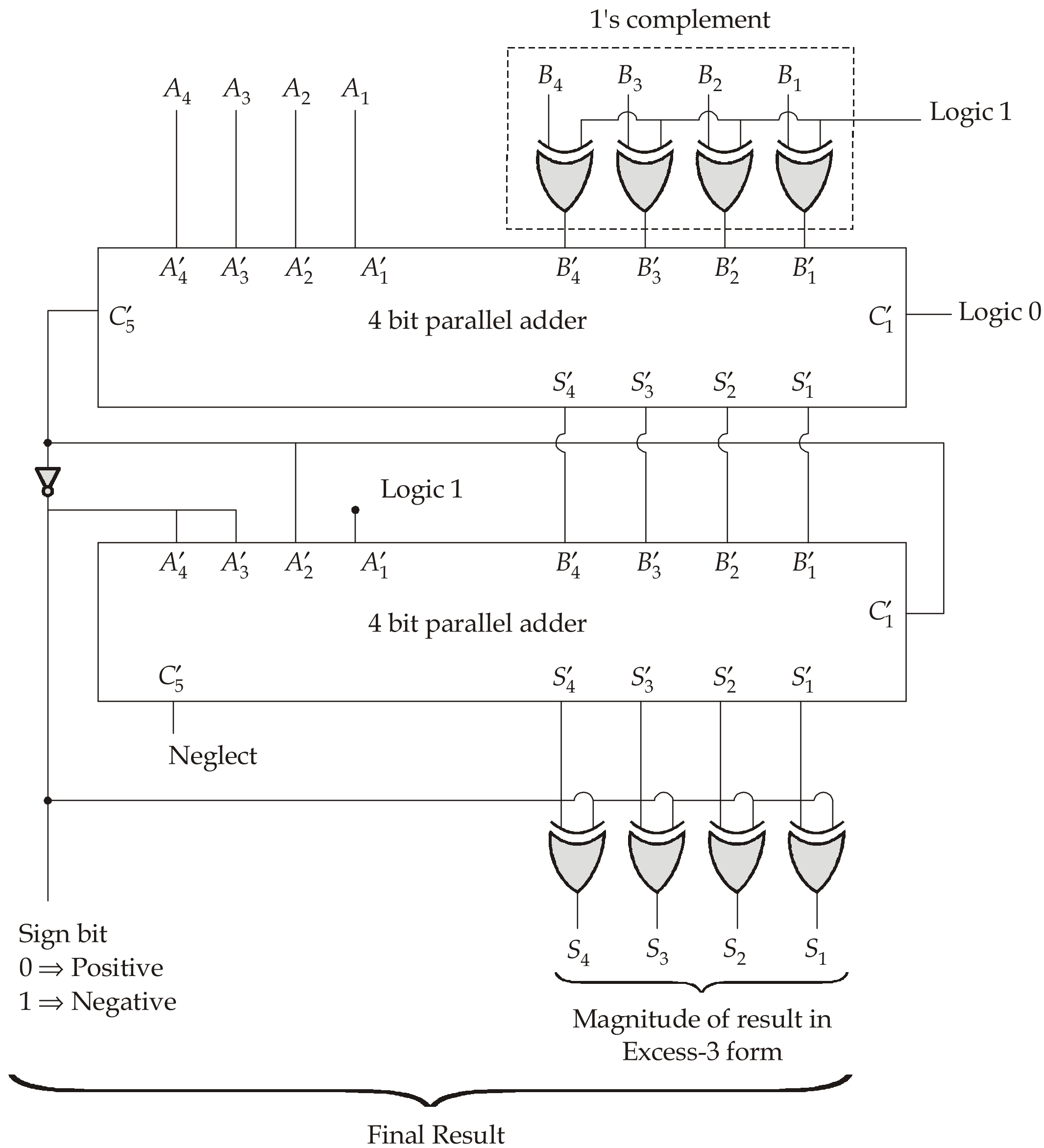
So,
$$M = \frac{9.27\times10^{-24}\times4\pi\times10^{-7}\times10^{6}}{1.38\times10^{-23}\times300}$$

$M = 2.81 \times 10^{-3}$ Bohr magneton

Q.6 (a) Solution:

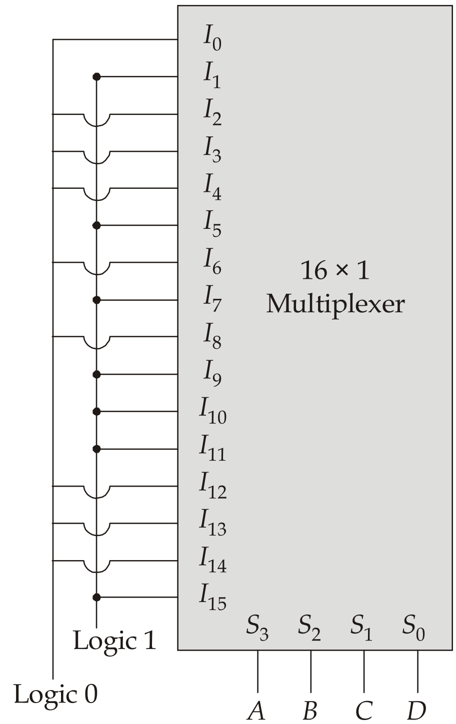
(i) Excess-3 code is self complementing code. So, 9s complement of excess-3 code i.e. (9-N) is equal to 1's complement of "Excess-3 code of N".





(ii) 1. Implementation using 16 × 1 multiplexer: The given function can be implemented using a 16 × 1 multiplexer by connecting A, B, C and D to the select lines as shown below:

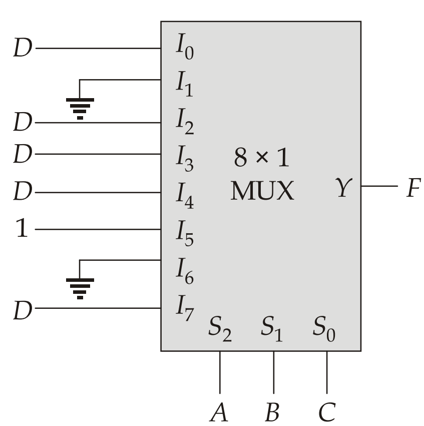
$$F(A, B, C, D) = \Sigma m(1, 5, 7, 9, 10, 11, 15)$$



2. Implementation using 8 × 1 multiplexer:

When the variables A, B and C are connected to the select lines S_2, S_1 and S_0 of the 8 × 1 multiplexer respectively, the signals required at various input lines of the multiplexer can be determined by using the following table:

D \ ABC	000	001	010	011	100	101	110	111
0	0	2	4	6	8	(10)	12	14
1	(1)	3	(5)	(7)	(9)	(11)	13	(15)
	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓
	$I_0 = D$	$I_1 = 0$	$I_2 = D$	$I_3 = D$	$I_4 = D$	$I_5 = 1$	$I_6 = 0$	$I_7 = D$



3. Implementation using 4 × 1 multiplexer: When the variables A and B are connected to the select lines S_1 and S_0 of the 4 × 1 multiplexer respectively, the signals at various input lines to the multiplexer can be determined by using the following table.

CD \ AB	00	01	10	11
00	0	4	8	12
01	①	⑤	⑨	13
10	2	6	⑩	14
11	3	⑦	⑪	⑮
	⇓ I_0	⇓ I_1	⇓ I_2	⇓ I_3

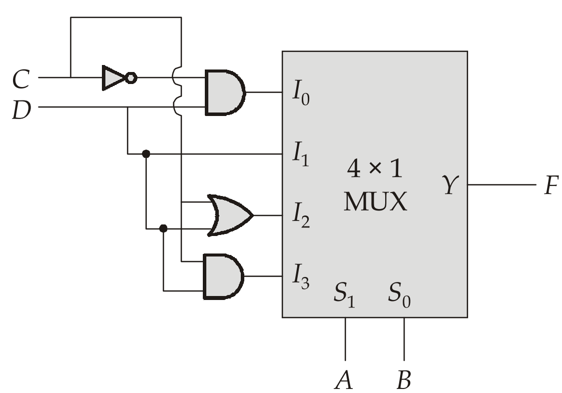
$$I_0 = \bar{C}D$$

$$I_1 = \bar{C}D + CD = D$$

$$I_2 = \bar{C}D + C\bar{D} + CD = C + D$$

$$I_3 = CD$$

Logic circuit:



Q.6 (b) Solution:

(i) The value of the interplanar spacing d_{hkl} is determined with a = 0.2866 nm and h = 2, k = 2 and l = 0 as we are considering the (220) planes. Therefore,

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d_{hkl} = \frac{0.2866}{\sqrt{(2)^2 + (2)^2 + (0)^2}} = 0.1013 \text{ nm}$$

Using Bragg's law, the value of θ may now be computed with $n = 1$ for a first order reflection as:

$$\sin\theta = \frac{n\lambda}{2d_{hkl}} = \frac{(1)(0.1790)}{(2)(0.1013)} = 0.8835$$

$$\theta = \sin^{-1}(0.8835) = 62.06°$$

The diffraction angle is 2θ or

$$2\theta = (2)(62.06) = 124.13°$$

(ii) Given: $V = 1200$ Volt ; Order, $n = 2$; Glancing angle = 60°

We know that, De Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m(qV)}} \qquad (\because\ E = qV)$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times1200}}$$

$$\lambda = 3.547\times10^{-11}\ \text{m}$$

Using Bragg's law, $n\lambda = 2d\sin\theta$

$$d = \frac{n\lambda}{2\sin\theta} = \frac{2\times3.547\times10^{-11}}{2\times\sin60°}$$

Interplanar spacing, $d = 4.09\times10^{-11}$ m

Q.6 (c) Solution:

(i) We have, number of poles = 4

Generated EMF, E = 250 V

Speed, N = 600 rpm

Diameter of pole shoe = 0.35 m

$$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7$$

Length of shoe = 0.2 m

We know that,

Pole pitch = Distance between two adjacent poles

$$= \frac{\text{Periphery of armature}}{\text{Number of poles of the generator}}$$

$$= \frac{\pi D}{P} = \frac{\pi\times0.35}{4}\ \text{m}$$

$$\text{Pole pitch} = 0.275 \text{ m}$$

Since, $$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7 \quad \Rightarrow \quad \text{Pole arc} = 0.7 \times \text{Pole pitch}$$

$$\text{Pole arc} = 0.7 \times 0.275$$
$$= 0.1925 \text{ m}$$

or, $$\text{Area of pole shoe} = \text{Pole arc} \times \text{axial length}$$
$$= 0.1925 \times 0.2$$
$$= 0.0385 \text{ m}^2$$

As,EMF generated is given by,

$$E = \frac{NP\phi Z}{60A}$$

where N = running speed of the rotor (in rpm); P = number of poles; ϕ = flux per pole; Z = number of conductors

$$250 = \frac{600 \times 4 \times \phi \times 1200}{60 \times 4}$$

$$\phi = \frac{250 \times 60 \times 4}{600 \times 4 \times 1200}$$

$$\phi = 0.021 \text{ Wb}$$

Flux density in the air gap,

$$B = \frac{\text{Flux per pole}}{\text{area of pole shoe}}$$

$$B = \frac{0.021}{0.0385} = 0.545 \text{ T}$$

(ii) **Given:** $N_D = 10^{18}$ cm^{-3} ; $N_A = 10^{16}$ cm^{-3}, $\tau_p = \tau_n = 10^{-6}$ s; $n_i = 9.65 \times 10^9$ cm^{-3}

Device area, $A = 1.2 \times 10^{-5}$ cm^2.

1. The saturation current, $I_s = J_s \times A$

where, $$J_s = qn_i^2\left[\frac{1}{N_D}\sqrt{\frac{D_p}{\tau_p}} + \frac{1}{N_A}\sqrt{\frac{D_n}{\tau_n}}\right]$$

$$J_s = 1.6 \times 10^{-19} \times (9.65 \times 10^9)^2 \left(\frac{1}{10^{18}}\sqrt{\frac{10}{10^{-6}}} + \frac{1}{10^{16}}\sqrt{\frac{21}{10^{-6}}}\right)$$

$$J_s = 6.87 \times 10^{-12} \text{ A/cm}^2$$

$\therefore \qquad I_s = A \times J_s = 1.2 \times 10^{-5} \times 6.87 \times 10^{-12} = 8.244 \times 10^{-17}$ A

2. Total current density,

$$J = J_s\left[e^{\frac{qV}{kT}} - 1\right]$$

(or)
$$I = I_s\left[e^{\frac{qV}{kT}} - 1\right]$$

At $V = +0.7$ V,

$$I = 8.244 \times 10^{-17}\left[e^{\frac{0.7}{0.0259}} - 1\right]$$

$\therefore \qquad I = 4.51 \times 10^{-5}$ A

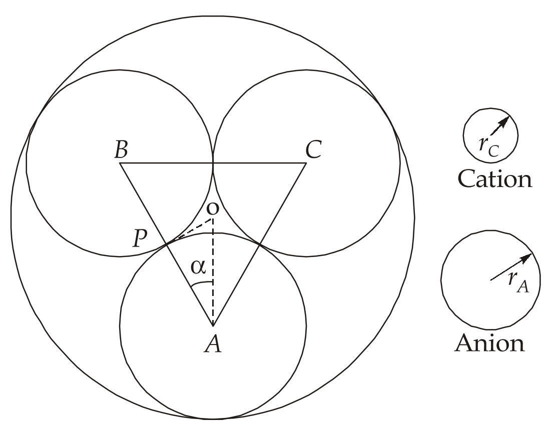
At $V = -0.7$ V,

$$I = I_s\left[e^{\frac{qV}{kT}} - 1\right] = 8.244 \times 10^{-17}\left[e^{-\frac{0.7}{0.0259}} - 1\right]$$

$\therefore \qquad I = -8.244 \times 10^{-17}$ A (from n-side to p-side)

Q.7 (a) Solution:

(i) For the coordination number 3, the small cation is surrounded by three anions to form an equilateral triangle as shown in the figure below as, triangle ABC, with the centers of all four ions coplanar.



This boils down to a relatively simple plane trigonometry problem. Consideration of the right triangle APO makes it clear that the side lengths are related to the anion and cation radii r_A and r_C as

$$\overline{AP} = r_A$$

and
$$\overline{AO} = r_A + r_C$$

Furthermore, the side length ratio $\overline{AP}/\overline{AO}$ is a function of the angle α as

$$\frac{\overline{AP}}{\overline{AO}} = \cos\alpha$$

The value of α is 30°, since line $\overline{AO}$ bisects the 60° angle BAC of equilateral triangle. Thus,

$$\frac{\overline{AP}}{\overline{AO}} = \frac{r_A}{r_A + r_C} = \cos 30° = \frac{\sqrt{3}}{2}$$

Solving for the cation-anion radius ratio,

$$\frac{r_C}{r_A} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = 0.155$$

(ii) A ceramic is a material that is neither metallic nor organic. It may be crystalline, glassy or both crystalline and glassy. Ceramics are typically hard and chemically non-reactive and can be formed or densified with heat. Typically, they demonstrate excellent strength and hardness properties; however they are often brittle in nature. Ceramics are more than pottery and dishes: clay, bricks, tiles, glass and cement are probably the best-known examples. Advanced ceramics consist of carbides (SiC), pure oxides (Al_2O_3), nitrides (Si_3N_4), non-silicate glasses and many others. Ceramic materials are used in electronics because depending on their composition, they may be semiconducting, superconducting, ferroelectric or an insulator. Ceramics are also used to make objects as diverse as plugs, fiber optics, artificial joints, space shuttle tiles, cooktops, race car brakes, micropositioners, chemical sensors, body armor etc.

Some of the applications of ceramics are listed below:

1. Aerospace: space shuttle tiles, thermal barriers, high temperature glass windows, fuel cells.
2. Medical: Orthopedic joint replacement, dental restoration, bone implants.
3. Defence: Structural components for ground, air and naval vehicles, missiles, sensors.
4. Communication: Fiber optic/laser Communication, TV and radio components, microphones.
5. Other industries: Bricks, cement, resistors, insulators, ferroelectric sensor etc.

Q.7 (b) Solution:

(i) **1.** Given that the CsCl structure has one cation (Cs^+) and one anion (Cl^-) in the unit cell and the lattice parameter $a = 0.412 \times 10^{-9}$ m.

The number of ion pairs (N_i) per unit volume is,

$$N_i = \frac{1}{a^3} = \frac{1}{(0.412 \times 10^{-9})^3}$$

$$N_i = 1.43 \times 10^{28} \text{ m}^{-3}$$

where N_i is also the concentration of cations and anions individually.

At low frequency, both electronic and ionic polarization contributes to '$\in_r$'. From the Clausius-Mossotti equation,

$$\frac{\in_r - 1}{\in_r + 2} = \frac{1}{3\in_0}\left[N_i\alpha_e(Cs^+) + N_i\alpha_i + N_i\alpha_e(Cl^-)\right]$$

where, α_e and α_i are electronic and ionic polarizabilities respectively.

$$\frac{\in_r - 1}{\in_r + 2} = \frac{1}{3\in_0}\left[(1.43 \times 10^{28})(4 \times 10^{-40} + 3 \times 10^{-40} + 6 \times 10^{-40})\right]$$

$$\frac{\in_r - 1}{\in_r + 2} = 0.7$$

$$\in_r - 1 = 0.7\in_r + 1.4$$

$$0.3\in_r = 2.4$$

$$\in_r = 8$$

2. At optical frequencies (high frequencies), the ionic polarization is too sluggish to allow it to contribute to $\in_r$.

Thus, $\in_{r\,opt}$, the relative permittivity at optical frequencies is given by,

$$\frac{\in_{r\,opt} - 1}{\in_{r\,opt} + 2} = \frac{1}{3\in_0}\left[N_i\alpha_e(Cs^+) + N_i\alpha_e(Cl^-)\right]$$

$$\frac{\in_{r\,opt} - 1}{\in_{r\,opt} + 2} = \frac{1}{3 \times 8.85 \times 10^{-12}}\left[(1.43 \times 10^{28})(4 \times 10^{-40} + 3 \times 10^{-40})\right]$$

$$\frac{\in_{r\,opt} - 1}{\in_{r\,opt} + 2} = 0.38$$

$$\in_{r\,opt} - 1 = 0.38\in_{r\,opt} + 0.76$$

$$0.62\in_{r\,opt} = 1.76$$

$$\in_{r\,opt} = 2.84$$

(ii) Given:

$T_C = 7.18°K$; $H_0 = 6.5 \times 10^{-4}$ A/m, $R = 0.2$ cm, $T = 5°K$

then critical current density, $J_C = ?$

The critical magnetic field at temperature T is given by

$$H_C = H_0\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

At $T = 5°K$,
$$H_C = 6.5 \times 10^{-4}\left[1 - \left(\frac{5}{7.18}\right)^2\right]$$

$$H_C = 3.34 \times 10^{-4} \text{ A/m}$$

We have,
$$\oint H \cdot dl = I$$

$\therefore$
$$H \cdot (2\pi R) = J \times \pi R^2$$

$$J = \frac{2H}{R}$$

Critical current density,
$$J_C = \frac{2H_C}{R}$$

$$J_C = \frac{2 \times 3.34 \times 10^{-4}}{0.2 \times 10^{-2}}$$

$$J_C = 0.334 \text{ A/m}^2$$

Q.7 (c) Solution:

(i) Applying Kirchhoff's voltage law, we get the time domain equation as

$$Ri(t) + L\frac{di(t)}{dt} = v(t)$$

Taking Laplace transform, we have

$$RI(s) + LsI(s) - Li(0^+) = V(s)$$

$$[R + Ls]I(s) = V(s) \qquad [\text{since } i(0^+) = i(0) = 0]$$

Substituting $R = 2\ \Omega$ and $L = 2$ H, we get

$$(2 + 2s)I(s) = V(s)$$

$$I(s) = \frac{V(s)}{2 + 2s} = \frac{V(s)}{2(s+1)}$$

The Laplace transform of the given triangular wave is obtained as,

$$v(t) = r(t) - 2r(t-1) + r(t-2)$$

$$\Rightarrow \quad V(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2}$$

Therefore, $$I(s) = \frac{1 - 2e^{-s} + e^{-2s}}{2s^2(s+1)}$$

By partial fraction expansion,

$$F(s) = \frac{1}{s^2(s+1)} = \frac{A_0}{s^2} + \frac{A_1}{s} + \frac{A_2}{(s+1)}$$

where, $$A_0 = F(s)s^2\Big|_{s=0} = \frac{1}{s+1}\Big|_{s=0} = 1$$

$$A_1 = \frac{d}{ds}\frac{1}{s+1}\Big|_{s=0} = -\frac{1}{(s+1)^2}\Big|_{s=0} = -1$$

$$A_2 = (s+1)F(s)\Big|_{s=-1} = \frac{1}{s^2}\Big|_{s=-1} = 1$$

Therefore, $$\frac{1}{s^2(s+1)} = \frac{1}{s^2} - \frac{1}{s} + \frac{1}{(s+1)}$$

$$L^{-1}\left\{\frac{1}{s^2(s+1)}\right\} = t - 1 + e^{-t}$$

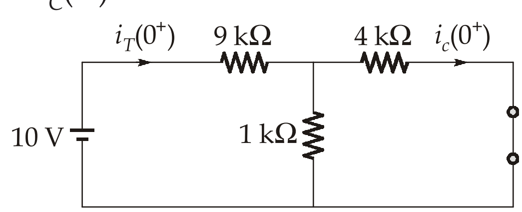
Therefore, $$i(t) = L^{-1}[I(s)] = L^{-1}\left\{\frac{1 - 2e^{-s} + e^{-2s}}{2s^2(s+1)}\right\}$$

$$= \frac{1}{2}\left[t - 1 + e^{-t}\right]\left[u(t) - 2u(t-1) + u(t-2)\right]$$

(ii) At $t = 0^-$, the capacitor is uncharged. Hence,

$$v_C(0^-) = 0$$

$$i_C(0^-) = 0$$



Since voltage across the capacitor cannot change instantaneously,

$$v_C(0^+) = 0$$

At $t = 0^+$, from the circuit drawn above, we have,

$$i_T(0^+) = \left[\frac{10}{9\,\text{k} + (4\,\text{k} \,\|\, 1\,\text{k})}\right] = \frac{10}{9.8\,\text{k}} = 1.02\text{ mA}$$

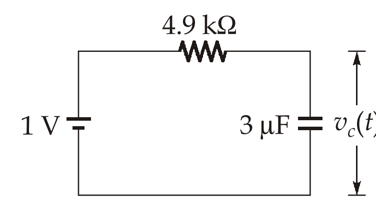
$$i_C(0^+) = 1.02\text{ mA} \times \frac{1\,\text{k}}{4\,\text{k} + 1\,\text{k}} = 0.204\text{ mA}$$

For $t > 0$, representing the network to the left of the capacitor by Thevenin's equivalent network,

$$V_{\text{Th}} = 10 \times \frac{1\,\text{k}}{9\,\text{k} + 1\,\text{k}} = 1\text{ V}$$

$$R_{\text{Th}} = (9\text{ k} \,\|\, 1\text{ k}) + 4\text{ k} = 4.9\text{ k}\Omega$$

Using above, the Thevenin's equivalent circuit is obtained as shown below:



Writing KCL equation for $t > 0$,

$$3 \times 10^{-6} \frac{dv_C}{dt} + \frac{v_C - 1}{4.9 \times 10^3} = 0$$

$$\frac{dv_C}{dt} + 68.02\, v_C = 68.02$$

The solution of this linear differential equation is given by

$$v_C(t) = \frac{Q}{P} + Ke^{-Pt}, \quad \text{were } P = Q = 68.02$$

$$= 1 + Ke^{-68.02\,t}$$

At $t = 0$, $v_C(0) = 0$

$$0 = 1 + K$$

$$K = -1$$

Hence, $$v_C(t) = (1 - e^{-68.02t})\text{ V} \qquad \text{for } t > 0$$

We have, $$i_C(t) = C\frac{dv_c(t)}{dt}$$

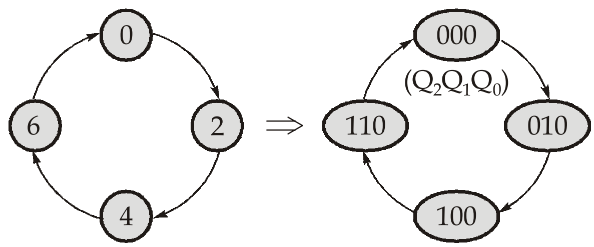
$$= 3 \times 10^{-6} \frac{d}{dt}(1 - e^{-68.02t})$$

$$= 3 \times 10^{-6} \times 68.02\, e^{-68.02t}$$

$$= 204.06 \times 10^{-6}\, e^{-68.02t} = 204.06e^{-68.02t}\ \mu\text{A} \qquad \text{for } t > 0$$

Q.8 (a) Solution:

Assuming the state of the counter is represented by the output of D flip-flops, $Q_2\,Q_1\,Q_0$, the sequence diagram of the counter to be designed is,



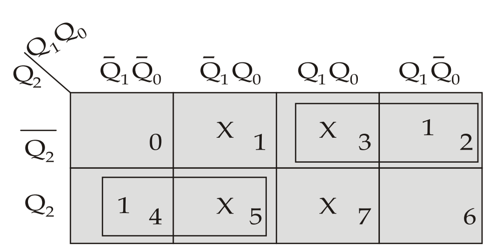
Excitation Table:

Present State			Next State			Excitations		
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	D_2	D_1	D_0
0	0	0	0	1	0	0	1	0
0	0	1	X	X	X	X	X	X
0	1	0	1	0	0	1	0	0
0	1	1	X	X	X	X	X	X
1	0	0	1	1	0	1	1	0
1	0	1	X	X	X	X	X	X
1	1	0	0	0	0	0	0	0
1	1	1	X	X	X	X	X	X

"X" indicates don't care corresponding to unused states of the counter.

Minimization:

K-Map for D_2



$$D_2 = \overline{Q_2}Q_1 + Q_2\overline{Q_1}$$

$$D_2 = Q_2 \oplus Q_1$$

K-Map for D_1

$Q_2 \backslash Q_1 Q_0$	$\bar{Q}_1 \bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1 \bar{Q}_0$
$\bar{Q}_2$	1 (0)	X (1)	X (3)	(2)
Q_2	1 (4)	X (5)	X (7)	(6)

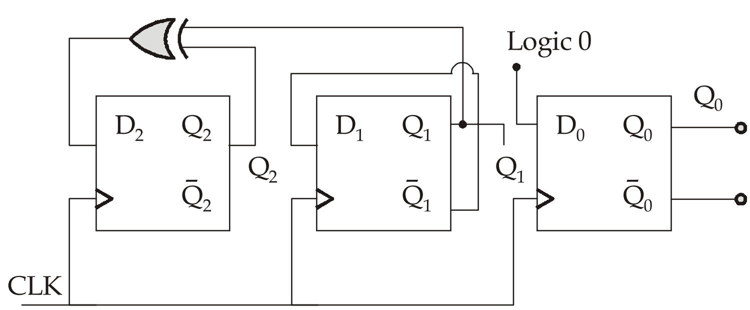
$$D_1 = \overline{Q_1}$$

K-Map for D_0

$Q_2 \backslash Q_1 Q_0$	$\bar{Q}_1 \bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1 \bar{Q}_0$
$\bar{Q}_2$	(0)	X (1)	X (3)	(2)
Q_2	(4)	X (5)	X (7)	(6)

$$D_0 = 0$$

Logic circuit:



Checking for self starting:

- A counter is said to be self starting when it enters into a used or valid state from an unused state within finite number of clock cycles.
- In the above designed counter, there are four unused states (1, 3, 5, 7). In order to determine the self starting capability of the counter, the next states of the unused states are to be examined, which can be done as shown below:

$$D_2 = Q_2 \oplus Q_1$$

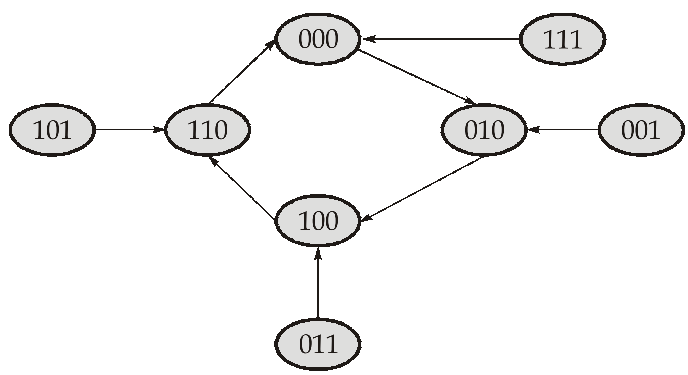
$$D_1 = \overline{Q_1}$$

$$D_0 = 0$$

Present State			Excitations			Next State		
Q_2	Q_1	Q_0	D_2	D_1	D_0	Q_2^+	Q_1^+	Q_0^+
0	0	1	0	1	0	0	1	0
0	1	1	1	0	0	1	0	0
1	0	1	1	1	0	1	1	0
1	1	1	0	0	0	0	0	0

From the above sequence table, it is clear that, from all the unused states, the counter will enter into a valid state within finite number of clock cycles. So, the designed counter is said to be self starting.

Complete sequence diagram:



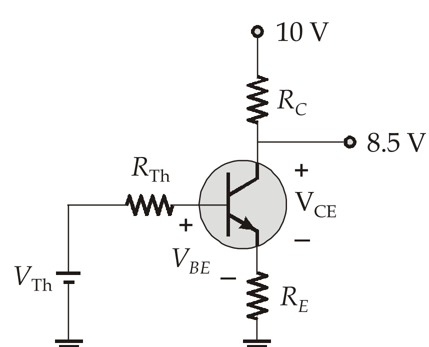
Q.8 (b) Solution:

We have, $I_C = 5$ mA ; $V_{CE} = 7.5$ Volt

Stability factor, $S = 12$

$\beta = 100$, $V_C = 8.5$ V

Now, redraw the Thevenin's equivalent of the given circuit:



where, $R_{Th} = R_1 \parallel R_2 = \dfrac{10R_2}{R_1 + R_2}$

On applying KVL in collector to emitter loop, we get,

$$-10 + I_C R_C + 8.5 = 0$$

$$I_C R_C = 1.5$$

$$R_C = \frac{1.5}{5 \times 10^{-3}} = 0.3 \times 1000 = 300\ \Omega \qquad \text{...(i)}$$

Since, we have, $V_{CE} = 7.5$ V and $V_C = 8.5$ V

$$V_{CE} = V_C - V_E$$

$$7.5 = 8.5 - V_E$$

$$V_E = 1 \text{ Volt}$$

Now, from circuit diagram, we have

$$V_E = I_E R_E$$

$$R_E = \frac{V_E}{I_E}$$

$\because$ $$I_E = I_C + I_B = I_C + \frac{I_C}{\beta} = 5 + \frac{5}{100} = 5.05 \text{ mA}$$

Hence, $$R_E = \frac{1}{5.05 \times 10^{-3}} = 198.02\ \Omega \qquad \text{...(ii)}$$

We know that, for voltage divider bias circuit, stability factor 'S' is given as,

$$S = \frac{1+\beta}{1 + \frac{\beta R_E}{R_E + R_{Th}}} = \frac{1+100}{1 + \left(\frac{100 \times 198.02}{198.02 + R_{Th}}\right)}$$

$$12 = \frac{101}{1 + \frac{19802}{198.02 + R_{Th}}}$$

$$1 + \frac{19802}{198.02 + R_{Th}} = \frac{101}{12}$$

$$\frac{19802}{198.02 + R_{Th}} = \frac{89}{12}$$

$$\frac{19802 \times 12}{89} = 198.02 + R_{Th}$$

$$R_{Th} = 2471.91\ \Omega \qquad \text{...(iii)}$$

Now on again applying the KVL in base loop, we get,

$$-V_{Th} + I_B R_{Th} + V_{BE} + I_E R_E = 0$$

$$V_{Th} = I_B R_{Th} + V_{BE} + I_E R_E$$

$$V_{Th} = \left(\frac{5\times10^{-3}}{100}\times 2471.91\right) + 0.7 + (5.05\times10^{-3})(198.02)$$

$$V_{Th} = 1.82 \text{ Volt} \quad \text{...(iv)}$$

Since, $$V_{Th} = \frac{10R_2}{R_1+R_2} = 1.82$$

$$\Rightarrow \quad \frac{R_2}{R_1+R_2} = 0.182 \quad \text{...(v)}$$

From equation (v) and (iii), we get

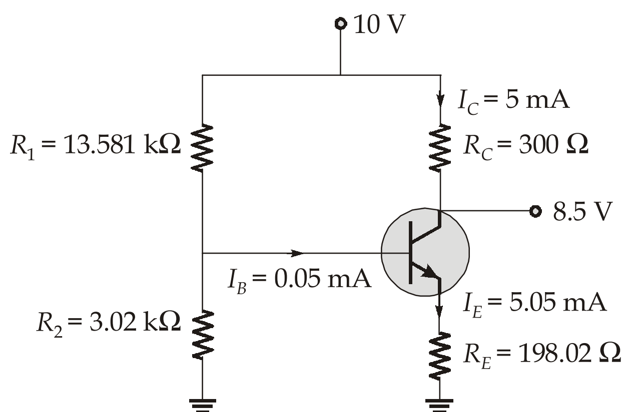
$$R_{Th} = 2471.91 = \frac{R_1 R_2}{R_1+R_2} = 0.182R_1$$

$$R_1 = 13.581 \text{ k}\Omega \quad \text{...(vi)}$$

and $$\frac{R_2}{13.581+R_2} = 0.182$$

On solving, we get, $$R_2 = 3.02 \text{ k}\Omega \quad \text{...(vii)}$$

Now, we get all the component values. We redraw the circuit as



Q.8 (c) Solution:

(i) For MOS transistor to be in saturation region

$$\Rightarrow \quad V_{DS} \geq V_{GS} - V_{Th}$$

$$V_D \geq V_G - V_{Th}$$

So, for maximum value of gate voltage, we have,

$$V_D = V_G - V_{Th}$$

Now, $$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{Th})^2 \quad \text{...(i)}$$

and $$1.8 - I_D(R_D) - V_D = 0 \quad \text{...(ii)}$$

Substituting equation (i) in equation (ii), we get,

$$1.8 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_G - V_{Th})^2 R_D - V_G + V_{Th} = 0$$

$$1.8 - \frac{1}{2}\times 10\times 10^{-6}\times 11.11(V_G - 0.4)^2\times 10^3 - V_G + 0.4 = 0$$

$$2.2 - 0.055\,(V_G - 0.4)^2 - V_G = 0$$

$$0.055V_G^2 + 0.956V_G - 2.1912 = 0$$

∴ $V_G = 2.05\text{ V},\quad -19.43\text{ V}$

Thus, $V_G = 2.05$ V (Since $V_{GS} > G_{Th}$)

(ii) Given, diffusion resistance,

$$r_d < 48\ \Omega$$

Diffusion conductance,

$$g_d = \frac{1}{r_d} = \frac{1}{48} = 0.0208\text{ S}$$

But, $$g_d = \frac{I_D}{\eta V_T}$$

Diode current, $I_D = g_d V_T$ (Assume $\eta = 1$)

∴ $I_D < 0.0208 \times 0.0259 = 0.539$ mA

We know that, $$I_D = I_s \exp\left(\frac{V_a}{V_T}\right)$$

where V_a = applied forward bias voltage.

$$\frac{I_D}{I_s} = \exp\left(\frac{V_a}{V_T}\right)$$

$$\frac{V_a}{V_T} = \ln\left(\frac{I_D}{I_s}\right)$$

∴ $$V_a = V_T \ln\left(\frac{I_D}{I_s}\right)$$

Hence, $$V_a < 0.0259\ln\left(\frac{0.539\times 10^{-3}}{2\times 10^{-11}}\right) = 0.443\text{ V}$$

Therefore, the maximum forward-bias voltage that can be applied to meet the given specification is 0.443V.

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