Detailed Solutions

ESE-2024 Mains Test Series

Electrical Engineering Test No : 15

Section-A

Q.1 (a) Solution:

The input power,

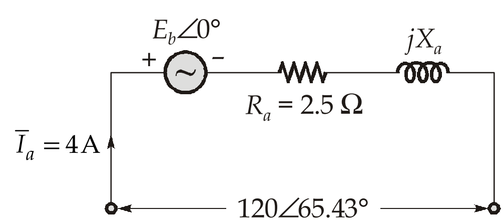
$$P_{in} = 200 \text{ W}$$

$$P_{in} = V_{in} \cdot I_a \cos\phi$$

$$200 = 120 \times 4 \times \cos\phi$$

$$\cos\phi = \frac{200}{120 \times 4} = \frac{5}{12}$$

$$\phi = 65.37°$$



$\bar{E}_b$ and $\bar{I}_a$ will be in phasor,

(i) From the circuit the following can be written

$$\bar{V}_{in} = \bar{E}_b + I_a[R_a + jX_a]$$

$$120\angle 65.37° = \bar{E}_b + 4\angle 0°[2.5 + jX_a]$$

$$50.01 + j109.08 = (E_b + 10) + j4X_a$$

On comparing,

$$50.01 = E_b + 10$$

$$E_b = 40.01 \text{ Volt}$$

Armature reactance, $$X_a = \frac{109.08}{4} = j27.27\ \Omega$$

(ii) We know, back emf

$$(E_b)_{\text{rms}} = \left(\frac{Z\phi NP}{60\ A}\right)\times\frac{1}{\sqrt{2}}$$

$$40.01 = \frac{840\times\phi_m\times 4000\times 2}{60\times 2\sqrt{2}}$$

$$\phi_m = 1.01 \text{ mWb}$$

Q.1 (b) Solution:

Let f be the output expression for different combinations of A, B, C, D, E.

(i) Either A or B or both must go,

$$f = 0, \text{ for } AB = 00$$

(ii) Either E or C, but not both must go,

$$f = 0, \text{ for } CE = 11$$

(iii) Either both A and C go or neither goes,

$$f = 0, \text{ for } AC = 01 \text{ and } AC = 10$$

(iv) If D goes, then E also must go,

$$f = 0, \text{ for } DE = 10$$

(v) If B goes, then A and D must also go,

$$f = 0, \text{ for } ABD = 010, 011, 110$$

Using the conditions (i) to (v). The truth table can be build as

***A B* = 0 0**

C	*D*	*E*	*f*	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	0	i
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	0	

***A B* = 0 1**

C	*D*	*E*	*f*	
0	0	0	0	(v)
0	0	1	0	(v)
0	1	0	0	(v)
0	1	1	0	(iv)
1	0	0	0	(iii)
1	0	1	0	(ii)
1	1	0	0	(iii)
1	1	1	0	(ii)

***A B* = 1 0**

C	*D*	*E*	*f*	
0	0	0	0	(iii)
0	0	1	0	(iii)
0	1	0	0	(iii)
0	1	1	0	(iii)
1	**0**	**0**	**1**	
1	0	1	0	(ii)
1	1	0	0	(iv)
1	1	1	0	(ii)

***A B* = 1 1**

C	*D*	*E*	*f*	
0	0	0	0	(iii)
0	0	1	0	(iii)
0	1	0	0	(iii)
0	1	1	0	(iii)
1	0	0	0	(v)
1	0	1	0	(ii)
1	1	0	0	(iv)
1	1	1	0	(ii)

Hence, the only possible combination for which $f = 1$ is

$$A = 1, B = 0, C = 1, D = 0, E = 0$$

$$f = A\bar{B}C\bar{D}\bar{E}$$

Q.1 (c) Solution:

$$H(Z) = \frac{1}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}} = \frac{z^2}{z^2 - 2zr\cos\theta + r^2}$$

It has a pair of poles at $z = r$.

$$z^2 - 2zr\cos\theta + r^2 = 0$$

at $z = r$, i.e., $$r^2 - 2r^2\cos\theta + r^2 = 0 \text{ at } z = r$$

$$2r^2 = 2r^2\cos\theta$$

$$1 - \cos\theta = 0$$

$$\cos\theta = 1$$

$$H(Z) = \frac{z^2}{z^2 - 2rz + r^2} = \frac{z^2}{(z-r)^2}$$

$$\frac{H(Z)}{z} = \frac{z}{(z-r)^2}$$

$$\frac{z}{(z-r)^2} = \frac{A}{z-r} + \frac{B}{(z-r)^2}$$

$$z = A(z - r) + B$$

$$z = Az + B - Ar$$

Comparing coefficient of z and constant term, we get

$$A = 1$$

and $$B - Ar = 0$$

$$B = Ar$$

$\therefore A = 1,$ $$B = Ar$$

$$B = r$$

$\therefore$ $$\frac{z}{(z-r)^2} = \frac{1}{z-r} + \frac{r}{(z-r)^2}$$

$$H(Z) = \frac{z^2}{(z-r)^2} = \frac{z}{z-r} + \frac{zr}{(z-r)^2}$$

Applying inverse Z-transform, we get

$$h(n) = r^n u(n) + n\, r^n u(n)$$

$$h(n) = (n + 1)\, r^n u(n)$$

Q.1 (d) Solution:

Given, differential equation,

$$\left[\frac{\partial}{\partial t^2} y(t)\right] + 16y(t) = 16u(t-3) - 16$$

$$Y(0) = 0; \quad \dot{y}(0) = 0$$

by taking Laplace transform,

$$s[sY(s) - Y(0)] - \dot{y}(0) + 16Y(s) = 16\left(\frac{e^{-3s}}{s}\right) - \frac{16}{s}$$

$$s^2Y(s) + 16Y(s) = 16\left[\frac{e^{-3s}}{s}\right] - \frac{16}{s}$$

$$Y(s)\left[s^2 + 16\right] = \frac{16e^{-3s} - 16}{s}$$

$$Y(s) = \frac{16e^{-3s} - 16}{s(s^2+16)}$$

$$\therefore \quad Y(s) = \frac{16e^{-3s}}{s(s^2+16)} - \frac{16}{s(s^2+16)}$$

Let

$$Y_1(s) = \frac{16}{s(s^2+16)}$$

$$Y_1(s) = \frac{1}{s} - \frac{s}{s^2+16}$$

$$\therefore \quad Y(s) = \frac{e^{-3s}}{s} - \frac{e^{-3s}s}{s^2+16} - \left[\frac{1}{s} - \frac{s}{s^2+16}\right]$$

$$= \frac{e^{-3s}}{s} - \frac{se^{-3s}}{s^2+16} - \frac{1}{s} + \frac{s}{s^2+16}$$

by taking Inverse Laplace transform

$$\therefore \quad y(t) = u(t-3) - \cos(4(t-3))u(t-3) - u(t) + \cos 4t$$

$$\therefore \quad y(t) = -u(t) + u(t-3) - \cos(4t - 12)\, u(t-3) + \cos 4t$$

Q.1 (e) Solution:

(i) From the circuit given:

$J_2 = \bar{Q}_1$; $\quad K_2 = Q_0$

$J_1 = Q_2$; $\quad K_1 = \bar{Q}_2$

$J_0 = Q_1$; $\quad K_0 = \bar{Q}_1$

Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
0	0	1	1	1	0	1	0	1
1	0	0	1	0	1	0	0	1
1	1	0	0	0	1	0	1	0
1	1	1	0	1	1	0	1	0
0	1	1	0	1	0	1	1	0
0	0	1	1	1	0	1	0	1

This is a divide by 5 counter, So $N = 5$.

(ii) $Q_2 = 1; Q_1 = 0; Q_0 = 1$

Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	1	1	1	1	0	0	1
0	1	0	0	0	0	1	1	0
0	0	1	1	1	0	1	0	1
1	0	0	1	0	1	0	0	1
1	1	0	0	0	1	0	1	0
1	1	1	0	1	1	0	1	0
0	1	1	0	1	0	1	1	0
0	0	1	1	1	0	1	0	1

This is also a divide by 5 counter, So, $N = 5$.

Q.2 (a) Solution:

An noninverting amplifier is shown in figure. The input resistance seen by the signal source is designated R_{if}. The equivalent circuit, including a finite open loop gain A_{OL}, Finite open-loop input differential resistance R_i and non-zero output resistance R_o, is shown in figure (b).

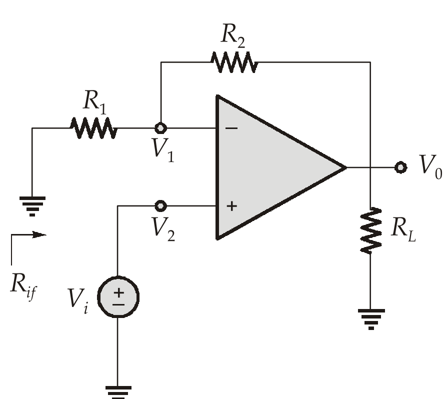


Figure (a)

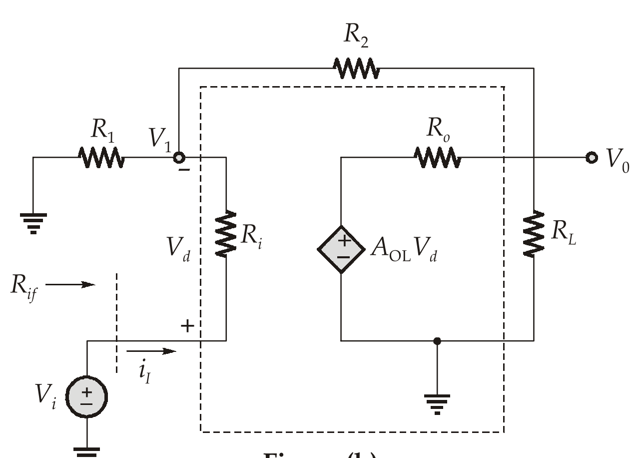


Figure (b)

Writing a KCL equation at the output node yields.

$$\frac{V_o}{R_L}+\frac{V_o-A_{oL}V_d}{R_o}+\frac{V_o-V_1}{R_2} = 0 \quad ...(i)$$

Solving for the output voltage, we have

$$V_o = \frac{\frac{V_1}{R_2}+\frac{A_{OL}V_d}{R_o}}{\frac{1}{R_L}+\frac{1}{R_o}+\frac{1}{R_2}} \quad ...(ii)$$

KCL equation at the V_1 node yields.

$$i_I = \frac{V_1}{R_1}+\frac{V_1-V_o}{R_2} \quad ...(iii)$$

$$V_o = V_1+\frac{V_1R_2}{R_1}-i_IR_2 \quad ...(iv)$$

Combining equations (ii) and (iv) and rearranging terms, we obtain,

$$i_I\left(1+\frac{R_o}{R_L}+\frac{R_o}{R_2}\right) = V_1\left\{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\left(1+\frac{R_o}{R_L}+\frac{R_o}{R_2}\right)-\frac{R_o}{R_2^2}\right\}-\frac{A_{OL}V_d}{R_2} \quad ...(v)$$

From figure (b), we see that,

$$V_d = i_1R_i \quad ...(vi)$$

and $$V_1 = V_i - i_IR_i \quad ...(vii)$$

Substituting equations (vi) and (vii) in (v) to we obtain an equation in i_I and v_I so that the input resistance R_{if} can be found as

$$R_{if} = \frac{V_1}{i_I}$$

Setting, $R_o = 0$ reduces equation (v) to

$$i_I = V_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{A_{OL}v_d}{R_2} \quad ...(viii)$$

Substituting, equation (vi) and (vii) into (viii), we find that the input resistance can be written in the form.

$$R_{if} = \frac{V_1}{i_I} = \frac{R_i(1+A_{OL})+R_2\left(1+\frac{R_i}{R_1}\right)}{1+\frac{R_2}{R_1}} \quad ...(ix)$$

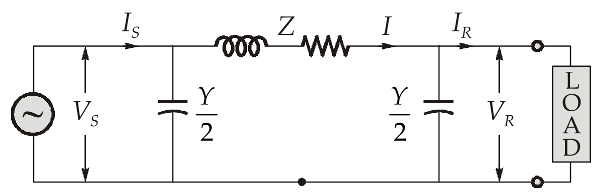
Equation (ix) describes the closed-loop input resistance of the noninverting amplifier with a finite open loop gain and a finite open-loop input resistance. In the limit as $A_{OL} \to \infty$, or as the open loop input resistance approaches infinity, we see that $R_{if} \to \infty$, which is a property of the ideal noninverting amplifier.

Q.2 (b) Solution:

Total series impedance per phase,

$$Z = (0.03 + j0.1) \times 100 = (3 + j10)\ \Omega$$

Total admittance per phase, $Y = j2\pi fC = j2\pi \times 50 \times 0.01 \times 10^{-6} \times 100 = j314 \times 10^{-6}\ S$



$$A = D = 1 + \frac{1}{2}YZ$$

$$= 1 + \frac{1}{2} j(314) \times 10^{-6} \times (3 + j10)$$

$$= 1 + j157\,(3 + j10) \times 10^{-6}$$

$$= 0.99843 + j0.000471 = 0.99843\angle 0.027°$$

$$B = Z = 3 + j10 = 10.44\angle 73.3°\ \Omega$$

$$C = Y + \frac{1}{4}Y^2Z = j314 \times 10^{-6} + \frac{1}{4}(j314 \times 10^{-6})^2(3 + j10)\,S$$

$$= j314 \times 10^{-6}\left[1 + \frac{(j314 \times 10^{-6})(3 + j10)}{4}\right]$$

$$= j314 \times 10^{-6}\,[0.9992 + j2.355 \times 10^{-4}]$$

$$= 313.75 \times 10^{-6}\angle 90°\ S$$

Taking receiving end phase voltage as reference

$$V_R = \frac{11000}{\sqrt{3}}\angle 0° = 6350.85\angle 0°\ V$$

So, load current, $I_L = I_L\angle -36.87° = \dfrac{2 \times 10^6 \angle -36.87°}{\sqrt{3} \times 11000 \times 0.8}$

$$= 131.216\angle -36.87°\ A$$

Capacitor current, $I_C = j\dfrac{\text{MVAR}}{\sqrt{3}V_C} = j\dfrac{10^6}{\sqrt{3}\times 11000} = j52.486$ A

Receiving end current, $I_R = I_L + I_C = 131.216\angle{-36.87°} + 52.486\angle 90°$

$= (104.976 - j26.242) = 108.206\angle{-14.03°}$ A

Phase voltage at sending end, $V_S = AV_R + B\,I_R$

$= (0.99843\angle 0.027° \times 6351\angle 0°)$

$+ (10.44\angle 73.3° \times 108.206\angle{-14.03°})$

$= 6341.028\angle 0.027° + 1129.71\angle 59.27°$

$= 6986.537\angle 8.014°$ V

Sending end line voltage $= \sqrt{3}V_S = \sqrt{3}\times 6986.537$

$= 12101.03\angle 8.014°$ V or 12.1 kV

Sending end current, $I_S = CV_R + DI_R$

$= (313.75 \times 10^{-6}\angle 90° \times 6350.85\angle 0°)$

$+ (0.99843\angle 0.027° \times 108.206\angle{-14.03°})$

$= (1.9925\angle 90°) + (108.036\angle{-14.003°})$

$= 107.571\angle{-12.973°}$ A

Q.2 (c) Solution:

(i) Here $V_{ml} = \sqrt{2}\times V_l = \sqrt{2}\times 230$ V

Average output voltage, $V_0 = \dfrac{3\,V_{ml}}{\pi} = \dfrac{3\sqrt{2}\times 230}{\pi} = 310.609$ V

But, $V_0 = E + I_0 R$

∴ Average value of battery charging current,

$$I_0 = \frac{V_o - E}{R} = \frac{310.56 - 240}{8} = 8.82 \text{ A or } 8.826 \text{ A}$$

Power delivered to battery $= EI_0 = 240 \times 8.826 = 2118.274$ W

Power delivered to load, $P_d = EI_0 + I_{or}^2 \cdot R$

Since, load current is ripple free,

$I_{or} = I_0 = 8.826$ A

∴ $P_d = 240 \times 8.826 + 8.826^2 \times 8 = 2741.46$ W

(ii) For ripple free load current, phase – a current i_a, or transformer secondary current i_s, would be constant at $I_0 = 8.826$ A from $\omega t = 30°$ to 150° and $-I_0$ from 210° to 330° and so on. As positive and negative half cycles are identical, average value of $i_s = 0$, i.e. $I_{dc} = 0$.

$$a_n = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_0 \cos n\omega t \cdot d(\omega t)$$

or,
$$a_1 = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_0 \cos \omega t \cdot d(\omega t) = \frac{2I_0}{\pi}[\sin 150° - \sin 30°] = 0$$

$$b_1 = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_0 \sin \omega t \cdot d(\omega t)$$

$$= \frac{2I_0}{\pi}[-\cos 150° + \cos 30°] = \frac{2\sqrt{3}}{\pi} I_0$$

Fundamental component of source current is given by

$$i_{s1} = \frac{2\sqrt{3}}{\pi} I_0 \sin \omega t \text{ and } \phi_1 = \tan^{-1}\left[\frac{0}{b_1}\right] = 0°$$

Input displacement factor, $DF = \cos\phi_1 = 1$

(iii) Rms value of fundamental component of source current,

$$i_{s1} = \frac{2\sqrt{3}}{\pi} \times \frac{I_0}{\sqrt{2}}$$

Rms value of source current,

$$I_s = \left[\frac{I_0^2 \times 2\pi}{\pi \times 3}\right]^{1/2} = \sqrt{\frac{2}{3}} \cdot I_0$$

Current distortion factor,

$$CDF = \frac{I_{s1}}{I_s} = \frac{2\sqrt{3} \times I_0}{\pi \cdot \sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{2} \cdot I_0} = \frac{3}{\pi} = 0.955$$

(iv) Input p.f. $= CDF \times DF$

$= 0.955 \times 1 = 0.955$ (lagging)

(v)
$$\text{HF} = \text{THD} = \left[\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right]^{1/2} = \left[\left(\frac{1}{0.955}\right)^2 - 1\right]^{1/2} = 0.3106$$

(vi)
$$\text{Transformer rating} = \sqrt{3} V_s \cdot I_s = \sqrt{3} \times 230 \times \sqrt{\frac{2}{3}} \times 8.826 = 2870.825 \text{ VA}$$

Also,
$$\text{transformer rating} = \frac{P_d}{\text{TUF}} = \frac{2741.46}{0.9541} = 2873.346 \text{ VA}$$

Q.3 (a) Solution:

(i)

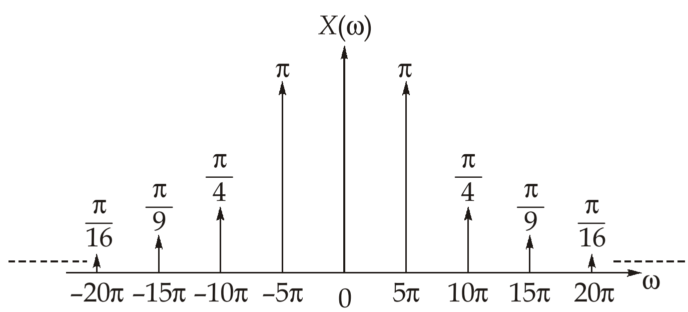
$$x(t) = \sum_{k=1}^{\infty} \frac{1}{k^2}\cos(5\pi kt)$$

$$\cos\omega_o t \xrightarrow{FT} \pi\delta(\omega-\omega_o)+\pi\delta(\omega+\omega_o)$$

$$\cos(5\pi kt) \xleftrightarrow{FT} \pi\delta(\omega-5\pi k)+\pi\delta(\omega+5\pi k)$$

$$\sum_{k=1}^{\infty} \frac{1}{k^2}\cos(5\pi kt) \xleftrightarrow{FT} \sum_{k=1}^{\infty}\left[\frac{\pi}{k^2}\delta(\omega-5\pi k)+\frac{\pi}{k^2}\delta(\omega+5\pi k)\right]$$

$$\therefore \quad X(\omega) = \sum_{k=1}^{\infty} \frac{1}{k^2}\left[\pi\delta(\omega-5\pi k)+\pi\delta(\omega+5\pi k)\right]$$



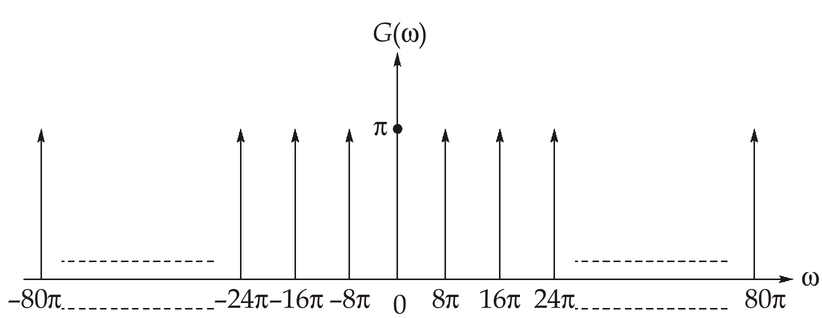
(ii)

$$g(t) = \sum_{k=1}^{10} \cos(8\pi kt)$$

$$\cos(8\pi kt) \xleftrightarrow{FT} \pi\delta(\omega-8\pi k)+\pi\delta(\omega+8\pi k)$$

$$\sum_{k=1}^{10} \cos(8\pi kt) \xleftrightarrow{FT} \sum_{k=1}^{10}\left[\pi\delta(\omega-8\pi k)+\pi\delta(\omega+8\pi k)\right]$$

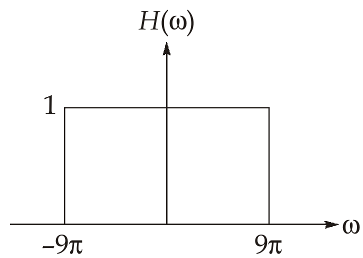
$$\therefore \quad G(\omega) = \sum_{k=1}^{10}\left[\pi\delta(\omega-8\pi k)+\pi\delta(\omega+8\pi k)\right]$$



(iii)

$$h(t) = \frac{\sin 9\pi t}{\pi t}$$

$H(\omega) \Rightarrow$



$H(\omega)$ is a low pass filter which allows only frequencies less than or equal to 9π rad/sec, when $x(t)$ is the input to $h(t)$, we get,

Output as $\pi\delta(\omega - 5\pi) + \pi\delta(\omega + 5\pi)$

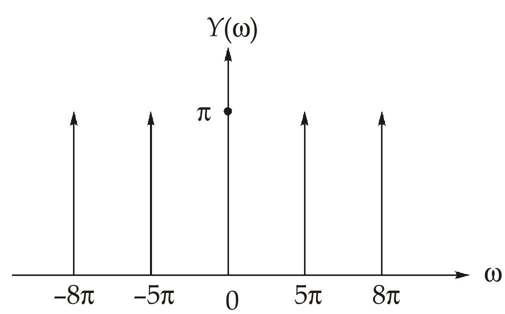
When $g(t)$ is the input to $h(t)$, we get output as $\pi\delta(\omega - 8\pi) + \pi\delta(\omega + 8\pi)$.

$$\therefore \quad y(t) = h(t) * [h(t) * x(t) + h(t) * g(t)]$$

$$Y(\omega) = H(\omega)\, [H(\omega)\, X(\omega) + H(\omega)\, G(\omega)]$$

$$Y(\omega) = H(\omega)\, X(\omega) + H(\omega)\, G(\omega)$$

$$Y(\omega) = \pi\delta(\omega - 5\pi) + \pi\delta(\omega + 5\pi) + \pi\delta(\omega - 8\pi) + \pi\delta(\omega + 8\pi)$$



(iv)

$$\pi\delta(\omega - 5\pi) + \pi\delta(\omega + 5\pi) \xleftrightarrow{IFT} \cos 5\pi t$$

$$\pi\delta(\omega - 8\pi) + \pi\delta(\omega + 8\pi) \xleftrightarrow{IFT} \cos 8\pi t$$

$$y(t) = \cos 5\pi t + \cos 8\pi t$$

Q.3 (b) Solution:

For truth table of seven segment display when inputs $b_3\, b_2\, b_1\, b_0$ = 0000, a decimal 0 will be displayed i.e., all segments lit except X_5.

$\therefore$ $$X_0 = X_1 = X_2 = X_3 = X_4 = X_6 = 1;\ X_5 = 0$$

and $E = 0$ because 0000 is a valid input combination.

The values at the outputs for all other input combinations can be derived in similar manner as below:

Decimal digit	b_3	b_2	b_1	b_0	E	X_6	X_5	X_4	X_3	X_2	X_1	X_0
0	0	0	0	0	0	1	0	1	1	1	1	1
1	0	0	0	1	0	0	0	1	1	0	0	0
2	0	0	1	0	0	1	1	0	1	1	0	1
3	0	0	1	1	0	1	1	1	1	1	0	0
4	0	1	0	0	0	0	1	1	1	0	1	0
5	0	1	0	1	0	1	1	1	0	1	1	0
6	0	1	1	0	0	1	1	1	0	1	1	1
7	0	1	1	1	0	0	0	1	1	1	0	0
8	1	0	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	0	1	1	1	1	1	1	0
Invalid	1	0	1	0	1	1	1	0	0	1	1	1
Invalid	1	0	1	1	1	1	1	0	0	1	1	1
Invalid	1	1	0	0	1	1	1	0	0	1	1	1
Invalid	1	1	0	1	1	1	1	0	0	1	1	1
Invalid	1	1	1	0	1	1	1	0	0	1	1	1
Invalid	1	1	1	1	1	1	1	0	0	1	1	1

From the above, we can write

$E = \Sigma m(10, 11, 12, 13, 14, 15)$

$X_6 = \Sigma m(0, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_5 = \Sigma m(2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_4 = \Sigma m(0, 1, 3, 4, 5, 6, 7, 8, 9)$

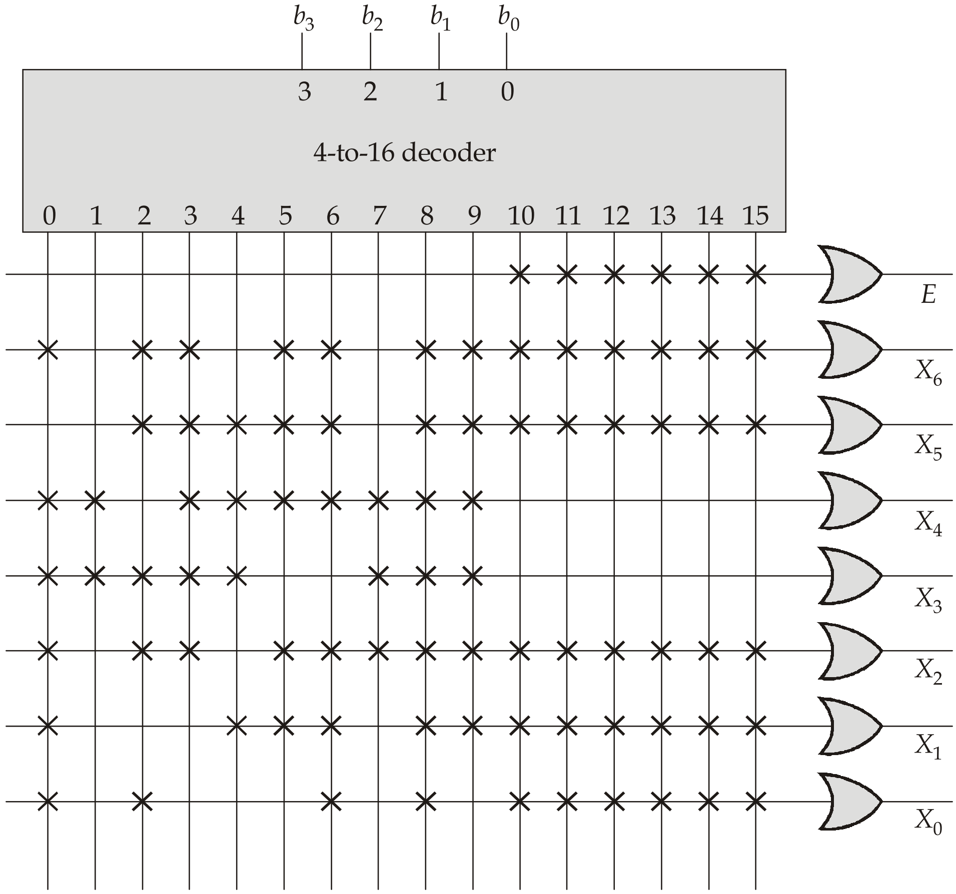
$X_3 = \Sigma m(0, 1, 2, 3, 4, 7, 8, 9)$

$X_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_1 = \Sigma m(0, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

$X_0 = \Sigma m(0, 2, 6, 8, 10, 11, 12, 13, 14, 15)$

The above can be implemented using a ROM as below:



Q.3 (c) Solution:

(i) When main thyristor T_1 is turned-on, an oscillatory current in the circuit C, T_1, L and D is set up and it is given by

$$i_c(t) = V_s \cdot \sqrt{\frac{C}{L}} \sin \omega_0 t$$

$\therefore$ Peak value of current through capacitor

$$I_p = V_s \cdot \sqrt{\frac{C}{L}} = 230\sqrt{\frac{40}{20}} = 325.269 \text{ A}$$

Peak value of current through main thyristor

$$T_1 = I_p + I_0 = 325.269 + 120 = 445.269 \text{ A}$$

Peak value of current through auxiliary thyristor $TA = I_0 = 120$ A

(ii)
$$I_0 = C\frac{V_s}{t_c}$$

∴ Circuit turn-off time for main thyristor,

$$t_c = C\frac{V_s}{I_0} = 40\times10^{-6}\frac{230}{120} = 76.67\ \mu s$$

An examination of figure reveals that when T_1 conducts and during the time upper plate of C is positive, $v_{TA} = -v_c$ i.e. auxiliary thyristors TA is reverse biased by v_c.

This gives circuit turn-off time t_{c1} for $TA = \dfrac{\pi}{2\omega_0}$.

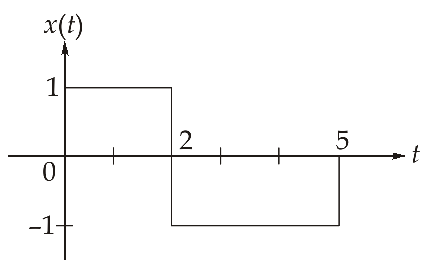
Here, $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{10^6}{\sqrt{20\times40}} = \frac{10^6}{\sqrt{800}}$$

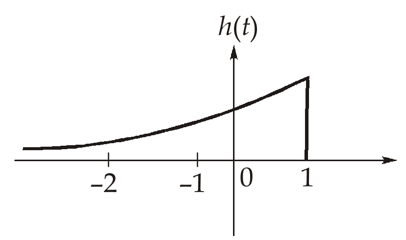
Circuit turn-off time for auxiliary thyristor,

$$t_{c1} = \frac{\pi}{2\omega_0} = \frac{\pi\sqrt{800}}{2\times10^6} = 44.43\ \mu s \text{ or } 44.428\ \mu s$$

Q.4 (a) Solution:

Given that, $h(t) = e^{2t}u(1-t)$ and $x(t) = u(t) - 2u(t-2) + u(t-5)$



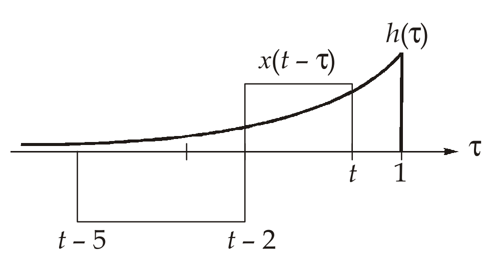


Using convolution integral,

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} e^{2\tau}u(1-\tau)[u(t-\tau) - 2u(t-2-\tau) + u(t-5-\tau)]\,d\tau$$

- **For $t \le 1$,**



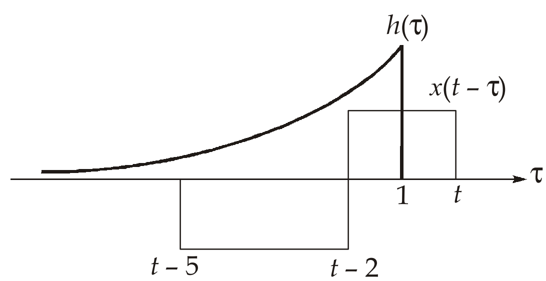
$$y(t) = \int_{t-5}^{t} e^{2\tau}[u(t-\tau)-2u(t-2-\tau)+u(t-5-\tau)]d\tau$$

$$= \int_{t-5}^{t-2} (-1)e^{2\tau}d\tau + \int_{t-2}^{t} e^{2\tau}d\tau$$

$$= -\frac{1}{2}\left[e^{2(t-2)}-e^{2(t-5)}\right]+\frac{1}{2}\left[e^{2t}-e^{2(t-2)}\right]$$

$$y(t) = \frac{1}{2}\left[1-2e^{-4}+e^{-10}\right]e^{2t}\ ;\ t \le 1$$

- **For $1 < t \le 3$ or $(t-2) \le 1 < t$,**



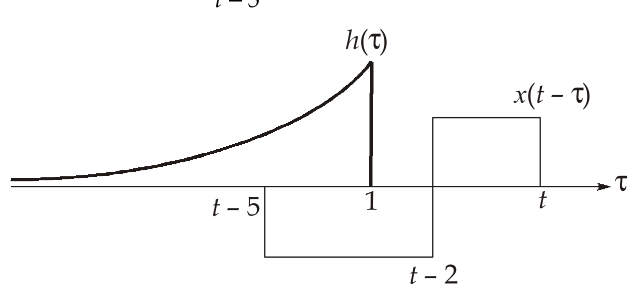
$$y(t) = \int_{t-5}^{t-2} -e^{2\tau}d\tau + \int_{t-2}^{1} e^{2\tau}d\tau$$

$$= -\frac{1}{2}\left[e^{2(t-2)}-e^{2(t-5)}\right]+\frac{1}{2}\left[e^{2}-e^{2(t-2)}\right]$$

$$y(t) = \frac{1}{2}\left[-2e^{-4}+e^{-10}\right]e^{2t}+\frac{1}{2}e^{2}\ ;\ 1 < t \le 3$$

- **For $3 < t \le 6$ or $(t-5) \le 1 < (t-2)$,**

$$y(t) = \int_{t-5}^{1} -e^{2\tau}d\tau = \frac{1}{2}\left[e^{2(t-5)}-e^{2}\right]\ ;\ 3 < t \le 6$$



- **For $t - 5 > 1$ or $t > 6$,**

$$y(t) = 0\,;\; t > 6$$

The overall solution is given by,

$$y(t) = \begin{cases} \frac{1}{2}\left[1-2e^{-4}+e^{-10}\right]e^{2t} & ; \quad t \le 1 \\ \frac{1}{2}\left[-2e^{-4}+e^{-10}\right]e^{2t}+\frac{1}{2}e^{2} & ; \quad 1 < t \le 3 \\ \frac{1}{2}\left[e^{2(t-5)}-e^{2}\right] & ; \quad 3 < t \le 6 \\ 0 & ; \quad t > 6 \end{cases}$$

Q.4 (b) Solution:

Given,

$$S_{G1} = P_{G1} + jQ_{G1}$$
$$S_{D1} = P_{D1} + jQ_{D1} = 2 + j1$$
$$S_{G2} = P_{G2} + jQ_{G2} = 0.5 + j1$$
$$S_{D2} = P_{D2} + jQ_{D2} = 0 + j0$$

Using nominal π-method for transmission line

$$y_{\text{series}} = \frac{1}{0.02 + j0.08} = 2.941 - j11.764$$
$$= 12.13\angle -75.96° \text{ p.u.}$$

$\therefore$ Each off diagonal term = $-2.941 + j11.764$ p.u.

$$\text{Each self term} = (2(2.941 - j11.764) + j0.01)$$
$$= 5.882 - j23.528$$
$$= 24.23\angle -75.95° \text{ p.u.}$$

$$Y_{\text{bus}} = \begin{bmatrix} 24.23\angle -75.95° & 12.13\angle 104.04° & 12.13\angle 104.04° \\ 12.13\angle 104.04° & 24.23\angle -75.95° & 12.13\angle 104.04° \\ 12.13\angle 104.04° & 12.13\angle 104.04° & 24.23\angle -75.95° \end{bmatrix} \mho$$

Also given, $V_2^0 = 1 + j0$ and $\delta_3^0 = 0$

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} + \delta_1 - \delta_2) + |V_2^2||y_{22}|\cos(\theta_{22})$$
$$+ |V_2||V_3||Y_{23}|\cos(\theta_{23} + \delta_3 - \delta_2)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} + \delta_1 - \delta_3)$$
$$+ |V_3||V_2||Y_{32}|\cos(\theta_{32} + \delta_2 - \delta_3) + V_3^2|Y_{33}|\cos(\theta_{33})$$

$$Q_2 = \left[-|V_2||V_1||Y_{21}|\sin(\theta_{21}+\delta_1-\delta_2) - V_2^2|Y_{22}|\sin(\theta_{22})\right.$$

$$\left.-|V_2|V_3||Y_{23}|\sin(\theta_{23}+\delta_3-\delta_2)\right]$$

$$P_2^0 = [(1.04)\ (1)\ (12.13)\cos(104.04+0-0) + (1)^2\ (24.23)\cos(-75.95°)$$

$$+ (1)\ (1.04)\ (12.13)\cos(104.04° + 0 - 0)]$$

$$= -3.064 + 5.8822 - 3.0604$$

$$= -0.2385 \text{ p.u.}$$

$$P_3^0 = [(1.04)\ (1.04)\ (12.13)\cos(104.04+0-0) + (1.04)\times 12.13\ (\cos 104.04)$$

$$+ (1.04)^2\ (24.23)\cos(-75.95)]$$

$$= -3.18285 - 3.0604 + 6.3622$$

$$= 0.119 \text{ p.u.}$$

Similarly, $Q_2^0 = -0.9715$ p.u.

Checking for Q_3 range

$$Q_3 = -|V_3|V_1Y_{31}\sin(\theta_{31}+\delta_1^{(0)}-\delta_3^{(0)}) - |V_3|^2 Y_{33}\sin(\theta_{33})$$

$$-V_3V_2Y_{32}\sin(\theta_{32}+\delta_2^{(0)}-\delta_3^{(0)})$$

$$= -12.7278 + 25.423 - 12.238 = 0.2686$$

$$Q_{G3}^{(0)} = Q_{D3} + \Delta Q_3^{(0)} = 0.6 + 0.2686 = 0.8686 \text{ p.u.}$$

Power residual,

$$\Delta P_2^0 = P_2 \text{ (specified)} - \Delta P_2^0 \text{(Calculated)} = 0.5 - (-0.235)$$

$$= 0.7385 \text{ p.u.}$$

$$\Delta P_3^0 = -1.5 - (0.119) = -1.619 \text{ p.u.}$$

$$\Delta Q_2^0 = 1 - (-0.9715) = 1.9715 \text{ p.u.}$$

So matrix equations for the solution of load flow by FDLF method,

$$\begin{bmatrix} \dfrac{\Delta P_2^{(0)}}{|V_2^{(0)}|} \\ \dfrac{\Delta P_3^{(0)}}{|V_3^{(0)}|} \end{bmatrix} = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix}$$

and $$\left[\frac{\Delta Q_2^{(0)}}{\left|V_2^{(0)}\right|}\right] = [-B_{22}]\left[\Delta\left|V_2^{(0)}\right|\right]$$

Here B_{22} and B_{33} are the imaginary parts of Y_{22} and Y_{33}

$$\begin{bmatrix} \frac{0.7385}{1} = 0.7385 \\ \frac{-1.619}{1.04} = -1.5567 \end{bmatrix} = \begin{bmatrix} 23.528 & -11.764 \\ -11.764 & 23.528 \end{bmatrix}\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix}$$

Solving above equation,

$$\Delta\delta_2^{(0)} = -0.0023 \text{ rad}$$

$$\Delta\delta_3^{(0)} = -0.0673 \text{ rad}$$

$$\delta_2^{(1)} = 0 - 0.0023 = -0.0023 \text{ rad or } -0.13178°$$

$$\delta_3^{(1)} = 0 - 0.0673 = -0.0673 \text{ rad or } -3.856°$$

$$Q_2^{(0)} = \left(-1.04 \times 1 \times 12.13 \sin\left(104.04° + 0 + 0.0023 \times \frac{180}{\pi}\right) - (24.23)\sin(-75.95°)\right.$$

$$\left.-(1.04)(12.13 \sin\left(104.04 + 0.0023 \times \frac{180}{\pi} - 0.0673 \times \frac{180}{\pi}\right)\right)$$

$$= -12.231 + 23.50 - 12.411 = -1.14227 \text{ p.u.}$$

$$\Delta Q_2^{(0)} = 1 - (-1.14227) = -2.14227 \text{ p.u.}$$

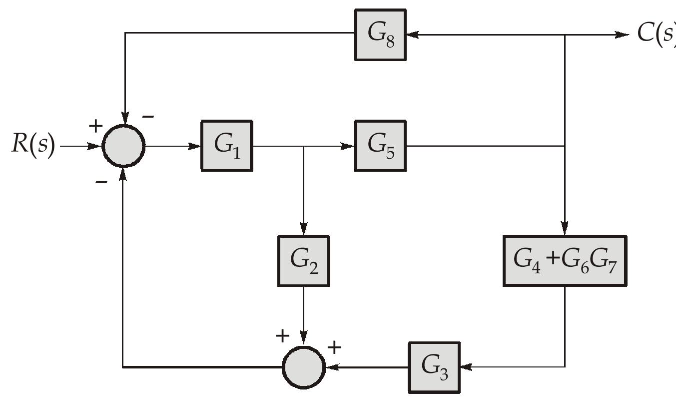
$$-2.14227 = 23.528\left(\Delta V_2^{(0)}\right)$$

$$\Delta\left|V_2^0\right| = 0.091$$

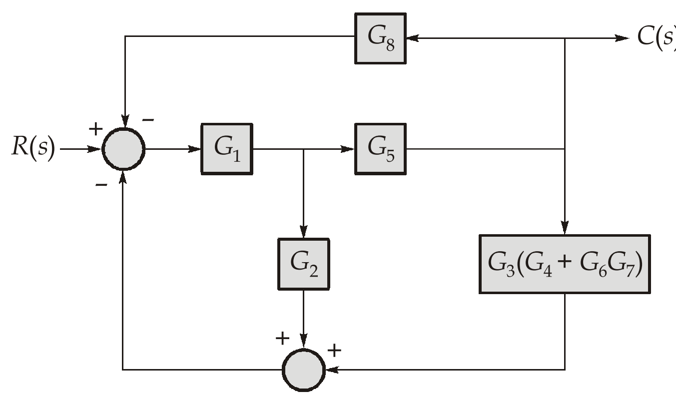
$$\left|V_2^{(1)}\right| = \left|V_2^{(0)}\right| + \Delta\left|V_2^{(0)}\right| = 1 + 0.091 = 1.091 \text{ p.u.}$$

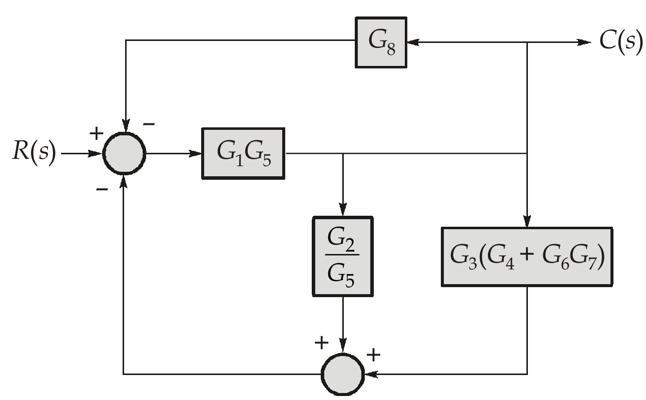
Q.4 (c) Solution:

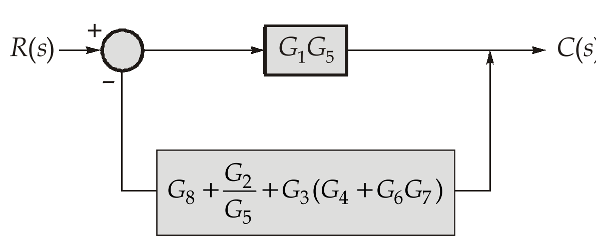
Combining blocks G_6 and G_7 and adding the forward path block G_4 , gives



Combining G_3 and $(G_4 + G_6G_7)$ gives







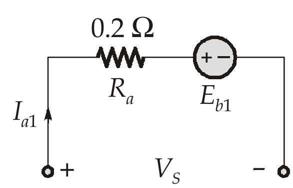
$$\frac{C(s)}{R(s)} = \frac{G_1G_5}{1 + G_1G_5\left[G_8 + \frac{G_2}{G_5} + G_3(G_4 + G_6G_7)\right]}$$

$$= \frac{G_1G_5}{1 + G_1G_5G_8 + G_1G_2 + G_1G_3G_5(G_4 + G_6G_7)}$$

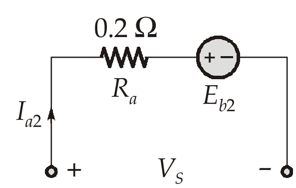
$$= \frac{G_1G_5}{1 + G_1(G_2 + G_5G_8 + G_3G_4G_5 + G_3G_5G_6G_7}$$

Section-B

Q.5 (a) Solution:



Motor-1



Motor-2

For motor-1,

The speed $N_1 = 600$ rpm

$I_{a1} = I_{a2} = 50$ A

Back emf, $E_{b1} = V_s - I_{a1} \cdot R_a = 400 - 50 \times 0.2 = 390$ volt

It will also be the same for motor-2,

$E_{b2} = E_{b1} = 390$ volt

Since current is contant in all cases

$E_b \propto N$

$\Rightarrow$ $E_b = kN$

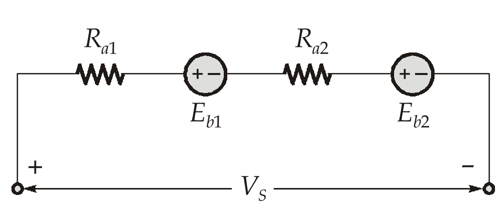
$E_{b1} = K_1N_1$ and $E_{b2} = K_2N_2$

$390 = K_1(600)$; $390 = K_2(650)$

$K_1 = 0.65$ V/rpm; $K_2 = 0.6$ V/rpm

Now motors are mechanically couled,

So,



$$E_{b1} + E_{b2} = V_S - I_a (R_{a1} + R_{a2})$$
$$= 500 - 50 (2 \times 0.2) = 480 \text{ V}$$

Let speed is N,

So,
$$E_{b1} = K_1 \cdot N = 0.65 \text{ N}$$
$$E_{b2} = K_2 \cdot N = 0.6 \text{ N}$$
$$E_{b1} + E_{b2} = 0.65 \text{ N} + 0.6 \text{ N} = 480 \text{ V}$$
$$1.25 \text{ N} = 480$$
$$\text{N} = 384 \text{ rpm}$$

Q.5 (b) Solution:

Given, variable capital cost of cable = 20000 a per km

Annual charges on account of interest and depreciation

$$P_2 a = \text{Rs } 20000\, a \times \frac{12}{100} = 2400\, a \text{ per km}$$

Resistance of each conductor of 'a' cm^2 cross-sectional area,

$$R = \frac{0.173}{a} \text{ per km}$$

Current in each conductor,
$$I = \frac{\text{Power supplied (in kW)}}{\sqrt{3} \times \text{line voltage in kV} \times \text{p.f.}}$$
$$= \frac{1500}{\sqrt{3} \times 11 \times 0.8} = 98.412 \text{ A}$$

Total power loss in all three conductors for 1 km length

$$W = 3I^2R = 3 \times (98.412)^2 \times \frac{0.173}{a} = \frac{5026.47}{a} \text{Watt}$$

$$\text{Energy loss per annum} = \text{Loss (kW)} \times \text{working hour/annum}$$
$$= \frac{5026.47}{a \times 1000} \times 300 \times 8 = \frac{12063.528}{a} \text{ kWh}$$

Annual cost of energy loss,

$$\frac{P_3}{a} = 0.02 \times \frac{12063.528}{a} = \frac{241.27}{a}$$

For most economical cross section of conductor

$$2400\, a = \frac{241.27}{a}$$

$$a^2 = \frac{241.27}{2400} \text{ or } a = 0.317 \text{ cm}^2$$

Diameter of conductor, $$d = \sqrt{\frac{4a}{\pi}} = \sqrt{\frac{4 \times 0.317}{\pi}} = 0.635 \text{ cm}$$

So, current density, $$D = \frac{I}{a} = \frac{98.412}{0.317} = 310.4 \text{ A/cm}^2$$

Q.5 (c) Solution:

XRA	11H	(To reset the *CY* flag)
MVI	C, FFH	(Load FFH to register C immediately)
MOV	A, C	(Transfer FFH to accumulator from register C)
ADI	01H	(Add 01H to FFH which gives 100H as result)
MOV	C, A	(Move contents of accumulator to register C)
OUT	Display	(Display the content of register C to LED location)
HLT		(Stop the operation)

(Display is the location where LED's are connected for display of content.)

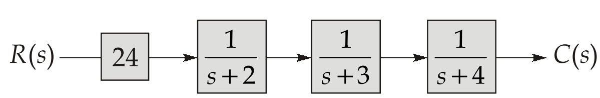
Q.5 (d) Solution:

Given transfer function,

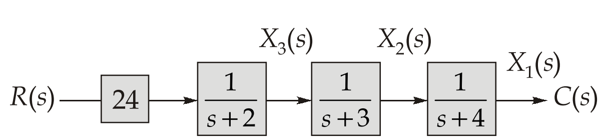
$$\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

$$\frac{C(s)}{R(s)} = \frac{24}{(s+2)(s+3)(s+4)}$$

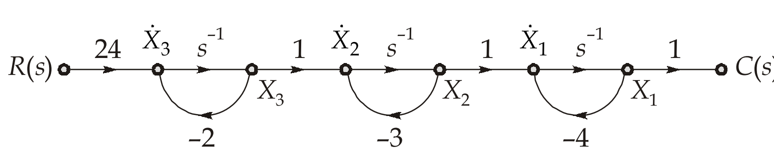
Block diagram model,



State model:



The above block diagram can be draw as



$$\frac{X_1(s)}{X_2(s)} = \frac{1}{s+4} = \frac{s^{-1}}{1+4s^{-1}} = \frac{s^{-1}}{1-(-4s^{-1})}$$

$$\frac{X_2(s)}{X_3(s)} = \frac{1}{s+3} = \frac{s^{-1}}{1+3s^{-1}} = \frac{s^{-1}}{1-(-3s^{-1})}$$

$$\frac{X_3(s)}{R(s)} = \frac{24}{s+2} = \frac{24s^{-1}}{1+2s^{-1}} = \frac{24s^{-1}}{1-(-2s^{-1})}$$

∴

$$\dot{X}_1 = X_2 - 4X_1$$

$$\dot{X}_2 = X_3 - 3X_2$$

$$\dot{X}_3 = -2X_3 + 24R$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} R$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Q.5 (e) Solution:

Given, characteristic equation of the control system,

$$s^5 + 2s^4 + 4s^3 + 8s^2 + 3s + 1 = 0$$

RH-criteria,

s^5	1	4	3
s^4	2	8	1
s^3	0	2.5	
s^2	∞		
s^1			
s^0			

Here, the RH-criteria fails. To remove the above difficulty,

1. Replace 0 by $\in$ (very small number) and complete the array with $\in$
2. Examine the sign change by taking $\in \to 0$.

Now, Routh array becomes,

$$\begin{array}{c|ccc} s^5 & 1 & 4 & 3 \\ s^4 & 2 & 8 & 1 \\ s^3 & \in & 2.5 & 0 \\ s^2 & \dfrac{5-8\in}{\in} & 1 & 0 \\ s^1 & \dfrac{2.5\dfrac{5-8\in}{\in}-\in}{\dfrac{5-8\in}{\in}} & & \\ s^0 & 1 & & \end{array}$$

Now putting $\in \rightarrow 0$, Routh's array becomes,

$$\begin{array}{c|ccc} s^5 & 1 & 4 & 3 \\ s^4 & 2 & 8 & 1 \\ s^3 & \in & 2.5 & 0 \\ s^2 & \dfrac{5-8\in}{\in} = -\infty & 1 & 0 \\ s^1 & 2.5 & & \\ s^0 & 1 & & \end{array}$$

There are two sign changes in first column elements of this array. Therefore the system is unstable.

Q.6 (a) Solution:

(i) Using table, the amplitude of the 60 Hz fundamental frequency is

$$V_1 = m_a V_{dc} = (0.8)(100) = 80 \text{ V}$$

The current amplitudes are determined using phasor analysis,

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega_0 L)^2}}$$

For the fundamental frequency,

$$I_1 = \frac{80}{\sqrt{10^2 + [(1)(2\pi 60)(0.02)]^2}}$$

$$= 6.39 \text{ A}$$

(ii) With $m_f = 21$, the first harmonics are at $n = 21$, 19 and 23. Using table,

n	$f_{n(\text{Hz})}$	V_n(V)	$Z_n(\Omega)$	$I_{n,\text{rms}}$(A)	P_n(W)
1	60	80.0	12.5	4.52	204.0
19	1140	22.0	143.6	0.11	0.1
21	1260	81.8	158.7	0.36	1.3
23	1380	22.0	173.7	0.09	0.1

$$V_{21} = (0.82)\,(100) = 82\text{ V}$$

$$V_{19} = V_{23} = (0.22)\,(100) = 22\text{ V}$$

Current at each of the harmonics is determined.

Power at each frequency is determined,

$$P_n = (I_{n,\text{rms}})^2 R = \left(\frac{I_n}{\sqrt{2}}\right)^2 R$$

The resulting voltage amplitudes, currents, and powers at these frequencies are summarized in table.

Power absorbed by the load resistor is,

$$P = \sum P_n \approx 204.0 + 0.1 + 1.3 + 0.1 = 205.5\text{ W}$$

Higher order harmonics contribute little power and can be neglected.

(c) The THD of the load current is determined with the rms current of the harmonics approximated by the first few terms indicated in table.

$$\text{THD}_1 = \frac{\sqrt{\sum_{n=2}^{\infty} I_{n,\text{rms}}^2}}{I_{1,\text{rms}}} \approx \frac{\sqrt{(0.11)^2 + (0.36)^2 + (0.09)^2}}{4.52}$$

$$= 0.087 = 8.7\%$$

By using the truncated Fourier series in table, the THD will be underestimated. However, since the impedance of the load increases and the amplitudes of the harmonics generally decrease as n increases, the above approximation should be acceptable. (Including currents through $n = 100$ gives a THD of 9.1 percent).

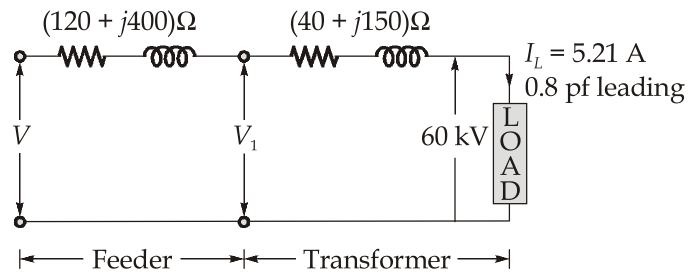
Q.6 (b) (i) Solution:

1. Impedance of 66 kV feeder = $(120 + j400)\ \Omega$

Equivalent impedance of transformer

referred to LV side = $(0.4 + j1.5)\Omega$

referred to HV side = $(40 + j150)\Omega$



Load of 250 kW at 0.8 pf leading at 6 kV

$$I_L = \frac{250}{6 \times 0.8} = 52.08\angle 36.86° \text{ A}$$

$$I_L' \text{ (HV side)} = \frac{52.08}{10} = 5.21\angle 36.86 \text{ A}$$

Sending end voltage, $V = 60 \times 10^3 + 5.21\,(160 \times 0.8 - 550 \times 0.6)$

$= 58.95$ kV

2. Primary voltage of transformer,

$$V_1 = 60 \times 10^3 + 5.21\,(40 \times 0.8 - j150 \times 0.6) = 59.7 \text{ kV}$$

$$\text{Active power loss} = (5.21)^2 \times 160 \times 10^{-3} = 4.34 \text{ kW}$$

$$\text{Reactive power loss} = (5.21)^2 \times 550 \times 10^{-3} = 14.92 \text{ kVAR}$$

$$\text{Power received} = (250 - j250 \tan \cos^{-1} 0.8)$$

$$= (250 - j187.5) \text{ kVA}$$

3. Complex power input $= \vec{S}$

$$\vec{S} = (250 + 4.34) + j(-187.5 + 14.92)$$

$$\vec{S}_{input} = (254.34 - j172.60) \text{ kVA}$$

Q.6 (b) (ii) Solution:

1. $P_C = 95$ Watt $\rightarrow$ Core loss

$$I_2 \text{ (H.V. side)} = \frac{20 \times 10^3}{2000} = 10 \text{ A}$$

$$\text{Full load } P_C = 10^2 \times 2.65 = 265 \text{ watt}$$

$$P_{out} = 20 \times 0.8 = 16 \text{ kW}$$

$$\text{Percentage, } \eta = \frac{x \times \text{kVA} \times \cos\phi}{x \times \text{kVA} \times \cos\phi + P_C + P_{CU}(f_L)} \times 100$$

$$\%\eta = \frac{16}{16 + 0.95 + 0.265} \times 100 = 97.80\%$$

2. $$X_{eq}\,(HV) = \sqrt{Z_{eq}^2 - R_{eq}^2} = \sqrt{(4.23)^2 - (2.65)^2}$$

$$= 3.2970\ \Omega \approx 3.30\ \Omega$$

Voltage drop across impedance = $I_2(R_{eq} \cos \phi + X_{pu} \sin \phi)$

$$\text{Voltage drop} = 10(2.65 \times 0.8 + 3.3 \times 0.6) = 41 \text{ Volt}$$

$$\text{HV side applied voltage} = 2000 + 41 = 2041 \text{ Volt}$$

3. $$\%\text{ voltage regulation} = \frac{2041 - 2000}{2000} \times 100 = 2.05\%$$

Q.6 (c) Solution:

(i) The transfer function of the given system is $\dfrac{C(s)}{R(s)} = \dfrac{10}{s^2 + 2s + 10}$

Here $$H(s) = 1 \text{ and } G(s) = \frac{10}{s(s+2)}$$

Therefore, $$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

The characteristic equation is

$$s^2 + 2s + 10 = 0$$

Comparing with characteristic equation of second-order unity feedback system,

We get $$\omega_n = \sqrt{10} = 3.16 \text{ rad/sec}$$

and $$2\xi\omega_n = 2$$

or $$\xi = \frac{1}{\omega_n} = \frac{1}{3.16} = 0.32$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.16\sqrt{1-0.32} = 3 \text{ rad/sec}$$

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-0.32^2}}{0.32} = 71.34°$$

Now the output response is given by

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \beta)$$

Substituting the values in the expression, we get

$$c(t) = 1 - \frac{e^{-0.32\times 3.16t}}{\sqrt{1-0.32^2}}\sin(3t + 71.34°)$$

or $$c(t) = 1 - 1.05e^{-t}\sin(3t + 71.34°)$$

(ii) The natural frequency (ω_n) and damping ratio (ξ) have been calculated in part (i) above as

$$\omega_n = 3.16 \text{ rad/sec}$$

(iii) Peak overshoot

$$\%M_p = 100\times e^{-\frac{0.32}{\sqrt{1-\xi^2}}\pi} = 100\times e^{-\frac{0.32}{\sqrt{1-0.32^2}}} = 34.63\%$$

Peak time, $$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{3.16\sqrt{1-0.32^2}} = 1.04 \text{ sec}$$

(iv) Steady-state error

The input is = 1 + 4t. In Laplace transform for

$$R(s) = \frac{1}{s}+\frac{4}{s^2} = \frac{s+4}{s^2}$$

Now $$E(s) = \frac{R(s)}{1+G(s)H(s)} = \left[\frac{1}{1+\frac{10}{s(s+2)}\times 1}\right]\times\frac{s+4}{s^2} = \frac{s(s+4)(s+2)}{s^2(s^2+2s+10)}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \frac{s\times s(s+4)(s+2)}{s^2(s^2+2s+10)} = \frac{8}{10} = 0.8$$

(v) Now $$G(s) = \frac{10}{s(s+2)}$$

It is given that poles are added at $\pm j\sqrt{3}$. This mean $G(s)$ will have another denominator term equal to (s^2 + 3). Therefore

$$G(s) = \frac{10}{s(s+2)(s^2+3)}$$

The characteristic equation is 1 + GH = 0 ;

i.e $$1+\frac{10}{s(s+2)(s^2+3)} = 0$$

or $$s(s+2)(s^2+3)+10 = 0$$

or $$s^4+2s^2+3s^2+6s+10 = 0$$

Now, we have to find the absolute stability. This we will ascertain by Routh Hurwitz criterion.

s^4	1	3	10
s^3	2	6	0
s^2	0	10	0

A zero has appeared in the first column s^2 row.

Therefore, putting $s = \frac{1}{z}$ in the characteristic equation, we get

$$10z^4 + 6z^3 + 3z^2 + 2z + 1 = 0$$

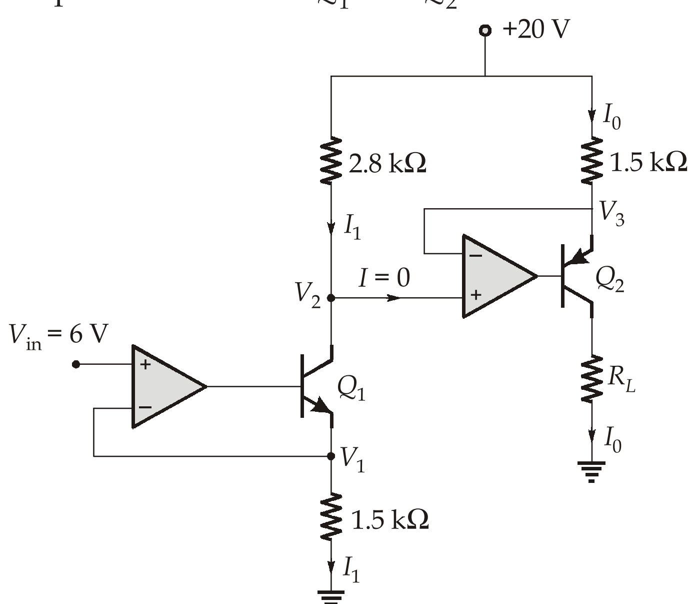
Developing the Rough's array, we get

	z^4	1	3	1
Sign change	z^3	2	6	0
	z^2	$-1/3$	10	0
Sign change	z^1	20	0	0
	z^0	1	0	0

There are two sign changes in the first column of the Routh's array. Hence, there are two roots lying on the right hand side of s-plane. Hence, the closed-loop system unstable.

Q.7 (a) (i) Solution:

Assuming: both op-amps to be ideal and Q_1 and Q_2 to be in saturation.



Here, $V_1 = V_{in} = 6$ V (voltage follower)

So, $I_1 = \frac{V_1}{1.5} = \frac{6}{1.5} = 4$ mA

By KVL, $V_2 = V_{CC} - (I_1)2.8$

$= 20 - 4 \times 2.8 = 8.8$ V

Using the virtual-short concept,

$$V_3 = V_2 = 8.8 \text{ V}$$

So, $I_0 = \frac{V_{CC} - V_3}{1.5} = \frac{20 - 8.8}{1.5} = 7.467$ mA

So, output current, $I_0 = 7.467$ mA

For maximum value of R_L, the transistor Q_2 should be at boundary of active and saturation.

So, $(V_{EC})_{sat} = 0.2$ V

Applying KVL in transistor Q_2,

$$V_{CC} = I_0(1.5 + R_L) + (V_{EC})_{sat}$$

$$20 = 7.467(1.5 + R_L) + 0.2$$

$$R_L + 1.5 = \frac{20 - 0.2}{7.467}$$

$$R_L = \frac{19.8}{7.467} - 1.5 = 1.151 \text{ k}\Omega$$

$$R_L = 1.151 \text{ k}\Omega$$

So, maximum value of $R_L = 1.15$ kΩ

Q.7 (a) (ii) Solution:

Given,

$$m(t) = 5 \sin(2\pi \times 1000t) + 2 \sin(2\pi \times 2000t)$$

$$f_s = 56000 \text{ Hz}$$

To avoid slope overload distortion, we have

$$\frac{\Delta}{T_s} \geq \left.\frac{dm(t)}{dt}\right|_{max}$$

or

$$\Delta f_s \geq \left.\frac{dm(t)}{dt}\right|_{max}$$

$$\Delta \times 56000 = 10000\pi\cos(2\pi\times 1000t) + 8000\pi\cos(2\pi\times 2000t)\big|_{max}$$

$$= 10000\pi + 8000\pi$$

$$\Delta \times 56000 = 18000\pi$$

$$\therefore \quad D = \frac{18000\pi}{56000} = 1 \text{ V}$$

Q.7 (b) Solution:

Step-1: Establishing bus (1) with its impedance to the reference bus.

$$Z_{bus,\ 1} = [j\,1.2]\ 1 \times 1 \text{ bus impedance matrix.}$$

Step-2: Establishing bus (2) with its impedance to bus.

$$Z_{bus,\ 2} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.4 \end{bmatrix}$$

The term $j\,1.4$ above is sum of $j\,1.2$ and $j\,0.2$. The element $j\,1.4$ in the new row and column are the repetition of the element of row 1 and column 1 of the matrix being modfied.

Step-3: Bus (3) with the impedance connecting to bus (2),

$$Z_{bus,\ 3} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix}$$

Step-4: Add the impedance $Z_s = j\,1.5$ between the reference bus and bus (3).

Using below equation - Type 3 modification

$$Z_{bus,\ new} = Z_{bus\,old} - \frac{1}{Z_{kk} + Z_s}\begin{bmatrix} Z_{1k} \\ Z_{2k} \\ ,, \\ ,, \\ Z_{nk} \end{bmatrix}[Z_{k_1} Z_{k_2} \,.....Z_{k_n}] \qquad ...(i)$$

$$Z_{bus,\ 4} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix} - \frac{1}{j1.55 + j1.5}\begin{bmatrix} j1.2 \\ j1.4 \\ j1.55 \end{bmatrix}[j1.2\; j1.4\; j1.55]$$

$$= \begin{bmatrix} j0.7279 & j0.6492 & j0.5302 \\ j0.6492 & j0.7574 & j0.6885 \\ j0.5902 & j0.6885 & j0.7623 \end{bmatrix}$$

Step-5: Add the impedance $Z_s = j\,0.3$ between thee bus (1) and bus (3).

Use type 4 modification:

$$Z_{\text{bus, new}} = Z_{\text{Bus,old}} - \frac{1}{Z_{kk} + Z_{ii} + Z_s - 2Z_{ik}} \begin{bmatrix} Z_{1i} & - & Z_{1k} \\ & \vdots & \\ Z_{ni} & - & Z_{nk} \end{bmatrix} \times [Z_{i1} - Z_{k1} \ldots\ldots Z_{in} - Z_{kn}]$$

$$Z_{\text{bus, 5}} = \begin{bmatrix} j0.7279 & j0.6492 & j0.5902 \\ j0.6492 & j0.7574 & j0.6885 \\ j0.5902 & j0.6885 & j0.7623 \end{bmatrix} - \frac{1}{j0.6098} \begin{bmatrix} j0.3177 \\ j0.0393 \\ -j0.1721 \end{bmatrix} \times [j0.3177 - j0.0393 - j0.1721]$$

$$Z_{\text{bus, 5}} = \begin{bmatrix} j0.7279 & j0.6492 & j0.5902 \\ j0.6492 & j0.7574 & j0.6885 \\ j0.5902 & j0.6885 & j0.7623 \end{bmatrix} - \begin{bmatrix} j0.031 & -j0.0087 & -j0.03886 \\ -j0.00887 & j0.002533 & j0.01109 \\ -j0.03886 & j0.01109 & j0.04857 \end{bmatrix}$$

$$= j \begin{bmatrix} 0.6969 & 0.65807 & 0.62906 \\ 0.65807 & 0.75487 & 0.67741 \\ 0.62906 & 0.67741 & 0.71373 \end{bmatrix}$$

Step-6: Add the impedance $Z_s = j\,0.25$ between thee bus (3) and bus (4).

i.e. repeat the step. (2) or similar as step (2),

$$Z_{\text{bus, 6}} = j \begin{bmatrix} 0.6969 & 0.65807 & 0.62906 & 0.62906 \\ 0.65807 & 0.75487 & 0.67741 & 0.67741 \\ 0.62906 & 0.67741 & 0.71373 & 0.71373 \\ 0.62906 & 0.67741 & 0.71373 & 0.96373 \end{bmatrix}$$

Step-7: Add the impedance $Z_s = j\,0.12$ between thee bus (2) and bus (4).

Similar as step (5),

$$Z_{\text{bus, 7}} = j\left[Z_{\text{Bus,6}}\right] - \frac{j}{0.48377} \begin{bmatrix} 0.02901 \\ 0.077457 \\ -0.03632 \\ -0.28632 \end{bmatrix} [0.02901, 0.077457 - 0.03632 - 0.28632]$$

$$Z_{\text{bus, 7}} = j\left[Z_{\text{Bus,6}}\right] - j \begin{bmatrix} 0.00174 & 0.004645 & -0.002178 & -0.01717 \\ 0.004645 & 0.0124 & -0.005815 & -0.04584 \\ -0.002178 & -0.005815 & 0.002727 & 0.21496 \\ -0.01717 & -0.04584 & 0.021496 & 0.169456 \end{bmatrix}$$

$$Z_{bus,\,7} = j\begin{bmatrix} 0.69516 & 0.653425 & 0.631238 & 0.64623 \\ 0.653425 & 0.742467 & 0.683225 & 0.72325 \\ 0.631238 & 0.683225 & 0.711003 & 0.692234 \\ 0.64623 & 0.72325 & 0.692234 & 0.794274 \end{bmatrix}$$

Note: If any element removing from the system then connected those element with - Z_b (Z_b with negative sign) in respective buses.

Q.7 (c) Solution:

(i) Given, message signal $m(t)$

$$\therefore \quad f_m = 10 \text{ kHz}$$

Since, the VCO output is an FM signal

$\therefore$ The bandwidth of FM signal is according to Carson's formula,

$$\text{Bandwidth} = 2\Delta f + 2f_m$$

where, frequency deviation,

$$\Delta f = K_f(\text{maximum } m(t))$$

$$\therefore \quad \Delta f = 25 \text{ kHz/mV} \times 2 \text{ mV} = 50 \text{ kHz}$$

$$\therefore \quad \text{Bandwidth} = 2 \times 50 \text{ k} + 2 \times 10 \text{ k} = 120 \text{ kHz}$$

(ii) The output of VCO is the FM signal,

i.e, $$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\, dt\right]$$

or, $$= A_c \cos[2\pi f_c t + \theta_c(t)]$$

The output of the filter, $f_c \pm n f_m$

Given, $f_c = 5$ MHz and $f_m = 10$ kHz

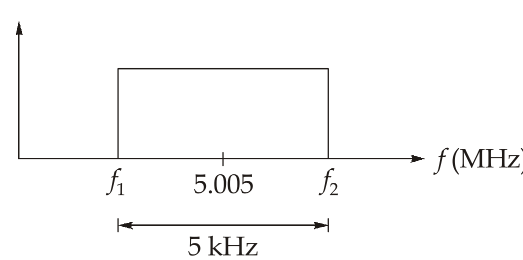
For $n = 1$, $5000 \text{ K} + 1 \times 10 \text{ K} = 5010 \text{ K}$

$n = 2$, $5000 \text{ K} + 2 \times 10 \text{ K} = 5020 \text{ K}$

$n = 3$, $5000 \text{ K} + 3 \times 10 \text{ K} = 5030 \text{ K}$

⋮ ⋮ ⋮

Given, bandpass filter,



$$\therefore \qquad f_1 = 5005\text{ K} - 2.5\text{ K} = 5.0025\text{ MHz}$$
$$f_2 = 5005\text{ K} + 2.5\text{ K} = 5.0075\text{ MHz}$$

∴ The given bandpass filter does not capture any of the frequency components of FM.

Hence, the output of BPF is zero.

Q.8 (a) Solution:

The process of conversion of an analog signal to digital signal is referred to as analog to digital conversion, whereas the process of conversion of a digital signal to analog signal is referred to as digital to analog conversion.

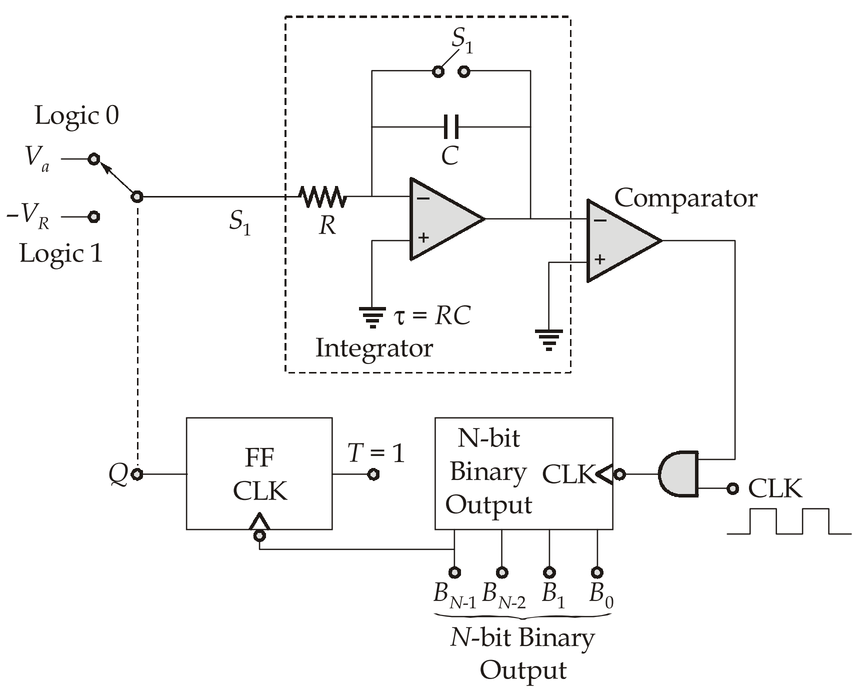
Digital to Analog Converters :

1. R-2R ladder digital to analog converter
2. Weighted-resistor type analog converter

Analog to Digital Converters :

1. Counter type ADC
2. Successive approximation type ADC
3. Flash type ADC
4. Ramp type ADC
5. Dual slope type ADC

Circuit Diagram of Dual Slope ADC :



The conversion process begins at $t = 0$ with the switch S_1 in position 0 thereby connecting the analog voltage V_a to the input of the integrator.

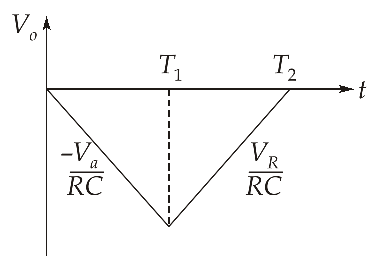
The integrator output

$$V_o = -\frac{1}{\tau}\int_0^t V_a dt = -\left(\frac{V_a}{\tau}\right)t$$

This results in high V_c, thus enabling the AND gate and the clock. The counter counts from 00 --- 00 to 111 --- 11. When $2^N - 1$ clock pulses are applied. At the next clock pulse 2^N, the counter is cleared and Q becomes 1. This controls the state of S_1 which now moves to position 1 at T_1, thereby connecting $-V_R$ to the input of the integrator.

The output of the integrator now starts to move in the positive direction. The counter continues to count until $V_o < 0$. As soon as V_o goes positive at T_2, V_c goes low disabling the AND gate.

The counter will stop counting in the absence of the clock pulses. The waveforms of voltage V_o and V_c are shown in figure below.



The time T_1 is given by :

$$T_1 = 2^N T_c$$

where T_c is the time period of the clock pulses. When the switch S_1 is in position 1,

$$V_o = -\frac{V_a}{\tau}T_1 + \frac{V_R}{\tau}(t - T_1) \qquad (V_o = 0 \text{ at } t = T_2)$$

Therefore, $$T_2 - T_1 = \frac{V_a}{V_R}T_1 = \frac{V_a}{V_R}2^N T_c$$

Let the count recorded in the counter be n at T_2.

Therefore, $$T_2 - T_1 = n.T_c = \frac{V_a}{V_R}2^N T_c$$

$\therefore$ $$n = \frac{V_a}{V_R}2^N$$

Q.8 (b) Solution:

$$G(s)H(s) = \frac{1.25(s+1)}{(s+0.5)(s-2)} = \frac{1.25(j\omega+1)}{(j\omega+0.5)(j\omega-2)}$$

At $\omega = 0$;
$$G(j\omega)\,H(j\omega) = \frac{1.25\times 1}{0.5\times 2} = -1.25$$

Gives,
$$M = \frac{1.25\sqrt{1+\omega^2}}{\sqrt{0.5^2+\omega^2}\sqrt{(-2)^2+\omega^2}}$$

and
$$\phi = \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{0.5}\right) - \tan^{-1}\left(\frac{\omega}{-2}\right)$$

When, $\omega = 0$;
$$M = \frac{1.25}{\sqrt{0.5^2\times 4}} = 1.25\angle 0 - 0 - (-180°) = -1.25\angle 180°$$

$\omega = \infty$;
$$M = 0\angle 90° - 90° - (-270°) = 0\angle 270°$$

Intersection with negative real axis

$$G(j\omega)H(j\omega) = \frac{1.25(1+j\omega)}{(0.5+j\omega)(j\omega-2)}$$

$$= \frac{1.25(1+j\omega)(0.5-j\omega)(-2-j\omega)}{(0.5+j\omega)(0.5-j\omega)(-2+j\omega)(-2-j\omega)}$$

$$= \frac{1.25(1+j\omega)(-1+1.5j\omega-\omega^2)}{(0.25+\omega^2)(4+\omega^2)}$$

$$= \frac{-1.25(1+2.5\omega^2)}{(0.25+\omega^2)(4+\omega^2)} + \frac{1.25j\omega(0.5-\omega^2)}{(0.25+\omega^2)(4+\omega^2)}$$

Equating imaginary part to zero gives

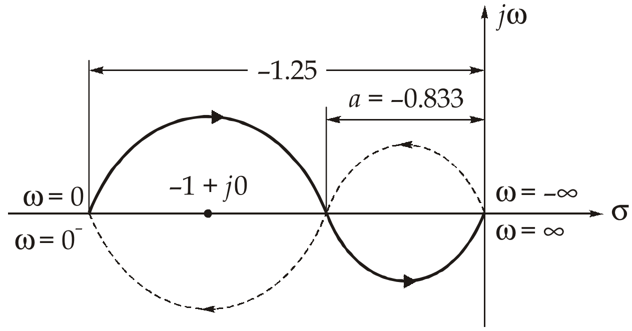
$$0.5 - \omega^2 = 0$$

or
$$\omega = 0.707$$

Putting the value of ω in the real part gives

$$a = \frac{-1.25(1+2.5\times 0.5)}{(0.25+0.5)(4+0.5)} = -0.833$$

Nyquist plot is shown in figure below,



It can be seen that the Nyquist plot makes one clockwise encirclement of point –1 + $j0$ and hence N = –1. Also, from $G(s)H(s)$, we get P = 1, substituting the values in

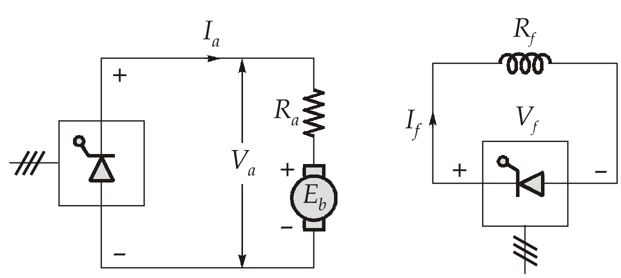
$$N = P - Z$$

we get, $$-1 = 1 - Z$$

or $$Z = 2$$

Which means two roots of characteristics equation lying on the right hand side of the s-plane. Thefore, the closed-loop system is unstable. The open-loop system is also unstable as P = 1.

Q.8 (c) Solution:



For continuous conduction, output voltage of three-phase full converter is given by,

$$V_0 = \frac{3V_{mL}}{\pi}\cos\alpha$$

For maximum possible field current α = 0°

$$V_f = \frac{3 \times 230 \times \sqrt{2}}{\pi} = 310.61 \text{ V}$$

Field current, $$I_f = \frac{V_f}{R_f} = \frac{310.61}{200} = 1.553 \text{ A}$$

(i) Rated shaft power P_{shaft} = 15000 W

$$P_{dev} = P_{shaft} + P_{rot}$$

$$P_{dev} = 15000 + 500$$
$$= 15500 \text{ W}$$

$$\text{Back emf } E_b = K I_f \omega$$
$$= 1.2 \times 1.553 \times \frac{2\pi \times 1200}{60} = 234.19 \text{ V}$$

$$\text{Rated developed power} = E_b I_a$$
$$\frac{15500}{234.19} = I_a$$
$$\text{Rated armature current} = 66.19 \text{ A}$$

Armature supply voltage,

$$V_a = E_b + I_a R_a$$
$$= 234.19 + 66.19 \times 0.25$$
$$V_a = 250.74 \text{ V}$$

V_a is output voltage of three phase full converter

$$V_a = V_0$$
$$= \frac{3V_{mL}}{\pi} \cos\alpha$$
$$250.74 = \frac{3 \times 230\sqrt{2}}{\pi} \cos\alpha$$
$$\alpha = 36.17°$$

(ii) For armature current of 10% of rated value

$$(I_a)_{\text{no load}} = 66.19 \times \frac{10}{100} = 6.619 \text{ A}$$

Delay angle is same as part (i) i.e,

$$\alpha = 36.17°$$

Back emf at no load,

$$(E_b)_{\text{no load}} = V_a - (I_a) \text{ no-load } R_a$$
$$(E_b)_{\text{no load}} = 250.74 - 6.619 \times 0.25$$
$$= 249.085 \text{ V}$$

No load speed can be calculated as,

$$E_b = K I_f \omega$$
$$(\omega)_{\text{no load}} = \frac{(E_b)_{\text{no-load}}}{K I_f}$$
$$\frac{2\pi (N)_{\text{no-load}}}{60} = \frac{(E_b)_{\text{no-load}}}{K I_f}$$

$$(N)_{\text{no load}} = \frac{60 \times (E_b)_{\text{no-load}}}{2\pi K I_f} = \frac{60 \times 249.085}{2\pi \times 1.2 \times 1.553}$$

$$\text{No load speed } (N)_{\text{no load}} = 1276.34 \text{ rpm}$$

(iii)

$$\text{Speed regulation} = \frac{\text{No load speed} - \text{Full load speed}}{\text{Full load speed}}$$

$$= \frac{1276.34 - 1200}{1200}$$

$$\% \text{ speed regulation} = 6.362\%$$

❍❍❍❍