Detailed Solutions

# ESE-2024 Mains Test Series

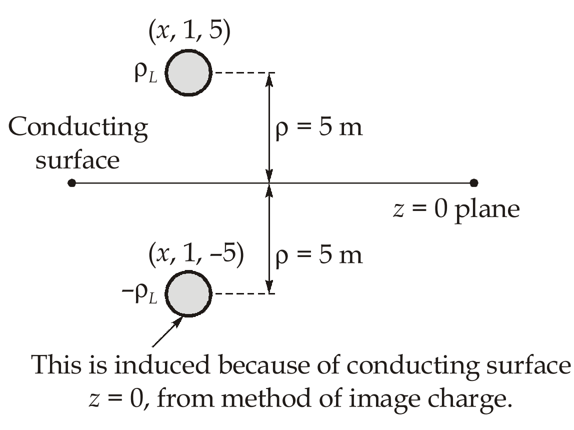
# Electrical Engineering Test No : 14

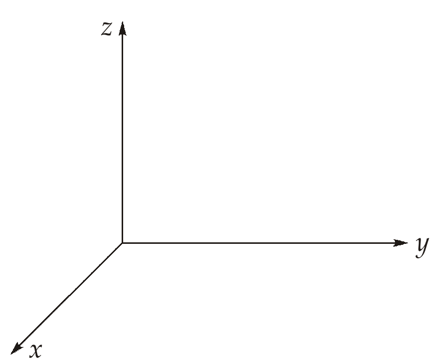
## Section-A

**Q.1 (a) Solution:**

Image theory states that a given charge configuration above an infinite grounded perfect conducting plane may be replaced by the charge configuration itself, its image and an equipotential surface in the plane of conducting plane.

Given that, $\rho_L$ = 20 nC/m is present at $y = 1$, $z = 5$ plane.




General co-ordinate for $\rho_L$ line charge is $(x, 1, 5)$

General co-ordinate for $-\rho_L$ line charge is $(x, 1, -5)$

**(i)**
$$\vec{E}_{(1,2,3)} = \vec{E}_{(x,1,5)} + \vec{E}_{(x,1,-5)}$$

$$\vec{R} = (1, 2, 3) - (1, 1, 5) = \hat{a}_y - 2\hat{a}_z$$

$$\Rightarrow \qquad \rho = \sqrt{5}, \quad \hat{a}_\rho = \frac{\hat{a}_y}{\sqrt{5}} - \frac{2}{\sqrt{5}}\hat{a}_z$$

Due to $\rho_L$ line charge,

$$\vec{E}_{(x,1,5)} = \frac{\rho_L}{2\pi \in_o \rho}\hat{a}_\rho = \frac{20\times10^{-9}\times18\times10^{9}}{5}\left[\hat{a}_y - 2\hat{a}_z\right]$$

where, $$\in_o = \frac{1}{36\pi}\times10^{-9}\ \text{F/m}$$

$$\vec{E}_{(x,1,5)} = 72\hat{a}_y - 144\hat{a}_z\ \text{V/m}$$

Due to $-\rho_L$ line charge,

$$\vec{E}_{(x,1,-5)} = \frac{-\rho_L}{2\pi \in_o \rho}\hat{a}_\rho$$

$$\vec{R} = (1, 2, 3) - (1, 1, -5)$$

$$\vec{R} = \hat{a}_y + 8\hat{a}_z$$

$$\rho = \sqrt{1^2 + 8^2} = \sqrt{65}\,\text{m}$$

$$\hat{a}_\rho = \frac{\hat{a}_y}{\sqrt{65}} + \frac{8}{\sqrt{65}}\hat{a}_z$$

$$\vec{E}_{(x,1,-5)} = \frac{-20\times10^{-9}\times18\times10^{9}}{65}\left[\hat{a}_y + 8\hat{a}_z\right]\text{V/m}$$

$$\vec{E}_{(x,1,-5)} = -5.54\hat{a}_y - 44.30\hat{a}_z$$

So, $$\vec{E}_{(1,2,3)} = \vec{E}_{(x,1,5)} + \vec{E}_{(x,1,-5)} = 66.46\hat{a}_y - 188.30\hat{a}_z\ \text{V/m}$$

**(ii)** $$\vec{E}_{(2,4,-8)} = \vec{E}_{(x,1,5)} + \vec{E}_{(x,1,-5)}$$

Due to $\rho_L$ line charge,

$$\vec{E}_{(x,1,5)} = \frac{\rho_L}{2\pi \in_o \rho}\hat{a}_\rho$$

$$\vec{R} = (2, 4, -8) - (2, 1, 5)$$

$$\vec{R} = 3\hat{a}_y - 13\hat{a}_z$$

$$\rho = \sqrt{3^2 + 13^2} = \sqrt{178}\ \text{m}$$

$$\hat{a}_\rho = \frac{\vec{R}}{\rho} = \frac{3\hat{a}_y}{\sqrt{178}} - \frac{13}{\sqrt{178}}\hat{a}_z$$

$$\vec{E}_{(x,1,5)} = \frac{20\times10^{-9}\times18\times10^{9}}{178}\left[3\hat{a}_y - 13\hat{a}_z\right]$$

$$\vec{E}_{(x,1,5)} = 6.067\hat{a}_y - 26.30\hat{a}_z \text{ V/m}$$

Due to $-\rho_L$ line charge,

$$\vec{E}_{(x,1,-5)} = \frac{\rho_L}{2\pi\epsilon_o\rho}\hat{a}_\rho$$

$$\vec{R} = (2, 4, -8) - (2, 1, -5)$$

$$\vec{R} = 3\hat{a}_y - 3\hat{a}_z$$

$$\rho = 3\sqrt{2}\text{ m},\ \hat{a}_\rho = \frac{\hat{a}_y}{\sqrt{2}} - \frac{\hat{a}_z}{\sqrt{2}}$$

$$\vec{E}_{(x,1,-5)} = \frac{-20\times10^{-9}\times18\times10^{9}}{6}\left[\hat{a}_y - \hat{a}_z\right]\text{V/m}$$

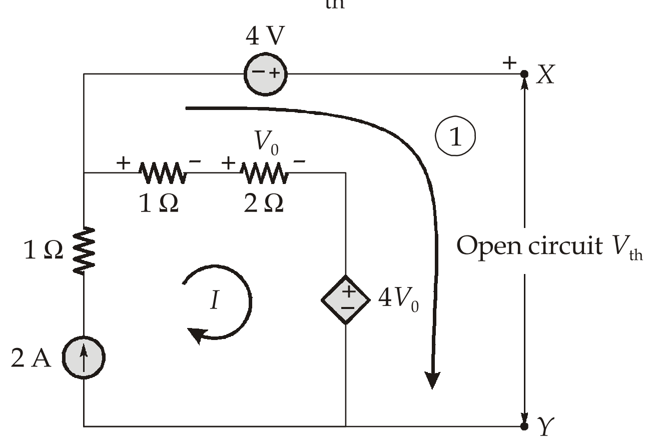
$$\vec{E}_{(x,1,-5)} = -60\hat{a}_y + 60\hat{a}_z$$

$$\vec{E}_{(2,4,-8)} = \vec{E}_{(x,1,5)} + \vec{E}_{(x,1,-5)}$$

$$= -53.93\hat{a}_y + 33.7\hat{a}_z \text{ V/m}$$

**Q.1 (b) Solution:**

Open circuit the load resistance to obtain $V_{th}$.




Hence when the load is removed, current from the 2 A source flows in the closed path.

So, Voltage, $V_o = 2 \times 2 = 4$ V

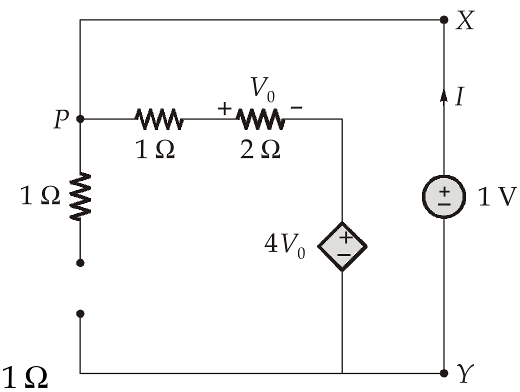
KVL in the loop-1:

$$-V_{th} + 4 + (2 \times 1) + (2 \times 2) + 4V_o = 0$$

$$V_{th} = 26 \text{ V}$$

$R_{th}$ can be obtained by short circuiting the all independent voltage sources and open circuiting the independent current source. Take 1 V voltage source across load and imagine '$I$' current is flowing from source.

Applying KCL at node $P$,




We get, $$\frac{1-5V_o}{1} = I$$

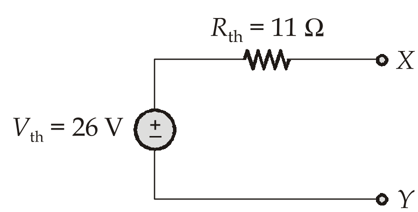
Since, $V_o = 2I$, therefore

$$\frac{1-5(2I)}{1} = I$$

$$I = \frac{1}{11}\text{A}$$

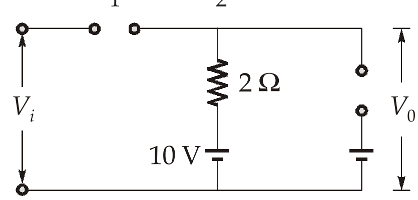
$\therefore$ $$R_{th} = \frac{1}{I} = \frac{1}{(1/11)} = 11\ \Omega$$

Equivalent Thevenin's circuit is



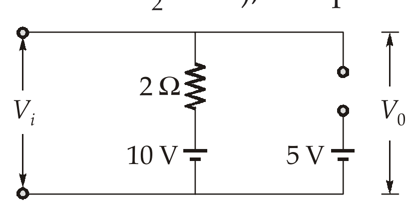

**Q.1 (c) (i) Solution:**

In the circuit when $V_i < 10$ V, both $D_1$ and $D_2$ are off



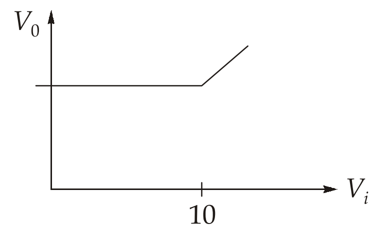

Output, $V_0 = 10$ V

When $V_i > 10$ ($D_1$ is forward bias and $D_2$ is off), so equivalent circuit is

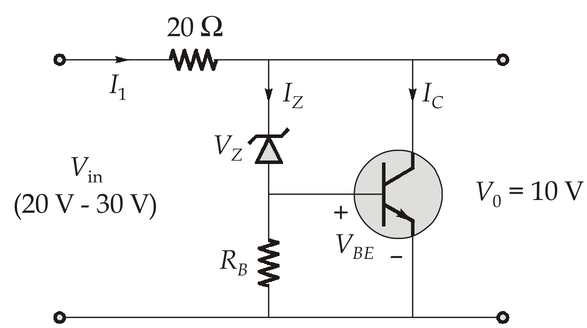



Output, $V_0 = V_i$

Transfer characteristics of the circuit,




**Q.1 (c) (ii) Solution:**




$$I_{1\,max} = \frac{V_{in\,max} - V_0}{20}$$

$$I_1 = \frac{30-10}{20} = 1\text{ A } (I_Z = 0)$$

$$I_E = I_C + I_Z$$

$$I_B = I_Z \text{ (as no current flows in } R_B)$$

$$\beta = \frac{I_C}{I_B} = \frac{I_C}{I_Z}$$

$$I_C = \beta I_Z$$

$$I_E = \beta I_Z + I_Z = (99+1)I_Z$$

$$I_E = 100\, I_Z$$

$$I_1 = I_E = 100 I_Z$$

$$I_Z = \frac{I_1}{100} = 0.01\text{ A}$$

$$P_Z = V_Z I_Z = 9.5 \times 0.01 = 95\text{ mW}$$

$$I_C = 99 I_Z = 99 \times 0.01 = 0.99\text{ A}$$

$$P_T = V_C I_C = 10 \times 0.99 = 9.9\text{ W}$$

**Q.1 (d) Solution:**

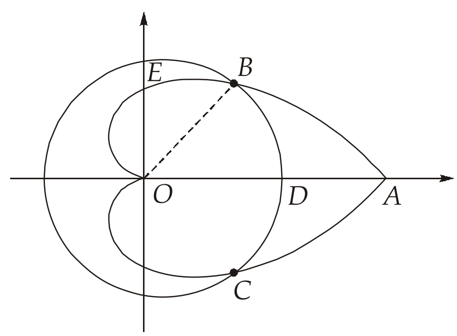
The points of intersection of the two curves are:

$$\frac{3}{2}a = a(1+\cos\theta)$$

i.e. $$\theta = \pm\frac{\pi}{3} \text{ or } B\left(\frac{3}{2}a, \frac{\pi}{3}\right), C\left(\frac{3}{2}a, \frac{-\pi}{3}\right)$$

Here $D\left(\frac{3}{2}a, 0\right)$ and $A(2a, 0)$, $E\left(a, \frac{\pi}{2}\right)$

Common area included between the circle and the cardioid is twice the area of the shaded region ODBEO because both the curves circle and cardioid are symmetric about the initial line ($x$-axis). Again area of ODBEO = Area of ODB + Area of OBEO common area.




$$= 2\left[\int_0^{\pi/3}\frac{1}{2}r^2 d\theta + \int_{\pi/3}^{\pi}\frac{1}{2}r^2 d\theta\right]$$

$$= \int_0^{\pi/3}\frac{9}{4}a^2\, d\theta + \int_{\pi/3}^{\pi} a^2(1+\cos\theta)^2 d\theta$$

$$= \frac{9}{4}a^2\theta\Big|_0^{\pi/3} + a^2\int_{\pi/3}^{\pi}\left[1+2\cos\theta+\frac{1+\cos 2\theta}{2}\right]d\theta$$

$$= \frac{3\pi a^2}{4} + a^2\left[\frac{3}{2}\theta + 2\sin\theta + \frac{\sin 2\theta}{4}\right]_{\pi/3}^{\pi}$$

$$= \frac{3\pi a^2}{4} + a^2\left[\frac{3}{2}\times\frac{2\pi}{3} + 2\left(0-\frac{\sqrt{3}}{2}\right) + \frac{1}{4}\left(0-\frac{\sqrt{3}}{2}\right)\right]$$

$$A = \frac{7\pi a^2}{4} - \frac{9}{8}\sqrt{3}\,a^2 = \left(\frac{7\pi}{4} - \frac{9\sqrt{3}}{8}\right)a^2$$

Now, area of the cardiode = twice of area of $OABEO$

$$= 2\times\frac{1}{2}\int_0^{\pi} r^2 d\theta = \frac{1}{2}\int_0^{\pi} a^2(1+\cos\theta)^2 d\theta$$

$$= a^2\int_0^{\pi}\left[1+2\cos\theta+\frac{1+\cos 2\theta}{2}\right]d\theta$$

$$= a^2\left[\frac{3}{2}\theta+2\sin\theta+\frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$= a^2\left[\frac{3}{2}\pi+0+0\right]=\frac{3\pi a^2}{2}$$

Now area outside the circle but inside the cardioid is

$$= \frac{3\pi a^2}{2}-\left(\frac{7\pi}{4}-\frac{9\sqrt{3}}{8}\right)a^2 = \left(\frac{9\sqrt{3}}{8}-\frac{\pi}{4}\right)a^2$$

**Q.1 (e) Solution:**

Here, $d\vec{s} = r\, d\phi\, dz\, \hat{a}_r$

$$\therefore \quad \oint \vec{A}\cdot d\vec{s} = \left[\int_{\phi=0}^{2\pi}\int_{z=0}^{10}\left(\frac{5r^3}{4}\hat{a}_r\right)(r\, d\phi\, dz\, \hat{a}_r)\right]_{r=2} - \left[\int_{\phi=0}^{2\pi}\int_{z=0}^{10}\left(\frac{5r^3}{4}\hat{a}_r\right)(r\, d\phi\, dz\, \hat{a}_r)\right]_{r=1}$$

$$= \left[\int_{\phi=0}^{2\pi}\int_{z=0}^{10}\frac{5r^4}{4}\hat{a}_r\, d\phi\, dz\right]_{r=2} - \left[\int_{\phi=0}^{2\pi}\int_{z=0}^{10}\frac{5r^4}{4}\hat{a}_r\, d\phi\, dz\right]_{r=1}$$

$$= \int_{\phi=0}^{2\pi}\int_{z=0}^{10}\frac{75}{4}d\phi\, dz = \frac{75}{4}\times 2\pi\int_{z=0}^{10} dz = 375\pi$$

Also, $$\nabla\cdot\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{5r^3}{4}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{5r^4}{4}\right)$$

$$= \frac{5}{4}\times\frac{1}{r}\times 4r^3 = 5r^2$$

$$\therefore \quad \int_V (\nabla \cdot \vec{A}) dV = \int_V 5r^2 \, r \, dr \, d\phi \, dz$$

$$= \int_{r=1}^{2} \int_{\phi=0}^{2\pi} \int_{z=0}^{10} 5r^3 dr \, d\phi \, dz$$

$$= 2\pi \times 10 \int_{r=1}^{2} 5r^3 dr = 100\pi \left(\frac{r^4}{4}\right)_1^2 = 375\pi$$

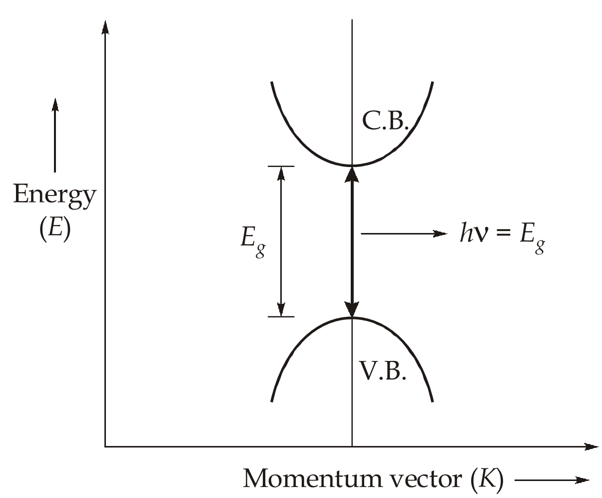
Since, $\int_V (\nabla \cdot \vec{A}) dV = \oint_S \vec{A} \cdot d\vec{s}$ divergence theorem is verified.

**Q.2 (a) Solution:**

**Direct Band Gap Material:**

In a direct band gap semiconductor such as GaAs, an electron in the conduction band can fall to an empty state in the valance band, giving off the energy difference $E_g$ as a photon of light.

- This property of direct band gap material can be used in designing devices requiring light output. E-K diagram for direct band gap material is as.



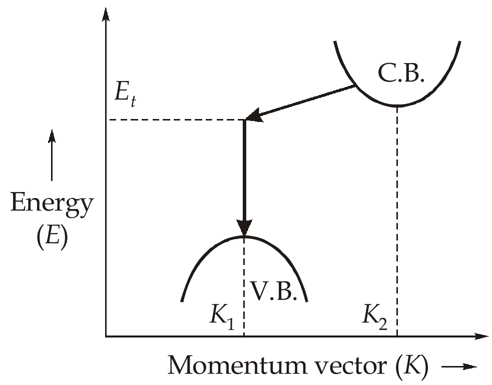

- These are the materials for which lowest energy state of conduction band and higher energy state of valance band occurs for **"same value of momentum"**.
- In these type of materials, the recombination occurs without the help of an external agent.

  No second particle is emitted.

**Indirect Band Gap Material:**

An electron in lowest energy conduction band of an indirect semiconductor cannot fall directly to the valance band of maximum energy but also **"undergo a momentum change"** as well as **changing its energy**.

E-K diagram for indirect band gap material is




- These are the materials in which higher-most energy state of valance band and lower-most energy state of conduction band occurs for different values of momentum.
- The probability of recombination is very less.
- So an external agent (Au) is required.
- In these materials recombination occurs in following steps:
  1. Gold creates trap levels (energy state which has momentum value $K_1$).
  2. As soon as any electron jumps from C.B. to trap level, its momentum value becomes $K_1$.
  3. Now it falls and recombine with hole having same value of momentum $K_1$ in opposite direction.
- Energy is released mainly in the form of heat.
- A second particle phonon is emitted when electron falls from C.B. to trap energy state $E_t$ and this phonon collides with lattice crystal and losses its energy in the form of heat.
- In GaAs → Ga is from Group III and As is from Group V.
- So in order to make GaAs n-type, we will increase the concentration of As (V group) in GaAs compared to Ga.
- In order to make GaAs p-type, we will increase the concentration (doping) of Ga compared to As in GaAs.
- In GaAs → bonding is mixed bonding having covalent as well as ionic character.
- Ionic bonding is due to difference in placement of Ga and As atoms in the periodic table.

**Q.2 (b) Solution:**

C.F: Here A.E is $m^2 - 4m + 1 = 0$ with real distinct roots $m_1 = 2+\sqrt{3}$ and $m_2 = 2-\sqrt{3}$ so that the C.F. is

$$y_c = C_1 e^{(2+\sqrt{3})x} + C_2 e^{(2-\sqrt{3})x}$$

P.I. note that:

$$\cos x \cdot \cos 2x = \frac{1}{2}(\cos 3x + \cos x)$$

and $$\sin^2 x = \frac{1}{2}[1-\cos 2x]$$

So that,

$$\cos x \cdot \cos 2x + \sin^2 x = \frac{1}{2}[\cos 3x + \cos x + 1 - \cos 2x]$$

$$y_P = \frac{1}{D^2 - 4D + 1}[\cos x \cdot \cos 4x + \sin^2 x]$$

$$= \frac{1}{2}\frac{1}{D^2 - 4D + 1}[\cos 3x + \cos x + 1 - \cos 2x]$$

$$= \frac{1}{2}[I_1 + I_2 + I_3 + I_4]$$

Here, $$I_1 = \frac{1}{D^2 - 4D + 1}\cdot \cos 3x$$

Replacing, $D^2$ by $-3^2$

$$I_1 = \frac{-1}{4}\frac{D-2}{(D+2)(D-2)}\cdot \cos 3x$$

$$= \frac{-1}{4}\frac{(D-2)}{(D^2 - 2^2)}\cdot \cos 3x$$

Replacing $D^2$ by $-3^2$, we get

$$I_1 = \frac{-1}{4}\frac{(D-2)}{-3^2 - 2^2}\cdot \cos 3x = \frac{1}{52}(D-2)\cos 3x$$

$$= \frac{1}{52}[-3\sin 3x - 2\cos 3x]$$

Similarly, $I_2 = \frac{1}{D^2 - 4D + 1} \cdot \cos x$

$$= \frac{1}{-1^2 - 4D + 1} \cdot \cos x$$

$\because$ $D^2 = -1^2$

$$= -\frac{1}{4}\frac{1}{D}\cos x = -\frac{1}{4}\int \cos x \, dx$$

$$= -\frac{1}{4}\sin x$$

Also, $I_3 = \frac{1}{D^2 - 4D + 1} \cdot 1 = \frac{1}{D^2 - 4D + 1} e^{0x}$

$$= \frac{1}{0 - 4.0 + 1} = 1$$

Where $D$ is replaced by $a = 0$,

Finally, $I_4 = \frac{1}{D^2 - 4D + 1}\cos 2x = \frac{1}{-2^2 - 4D + 1}\cos 2x$

Where $D^2$ is replaced by $-2^2$

$$I_4 = -\frac{1}{(4D + 3)}\cos 2x$$

Rewriting this to get $D^2$ terms we have,

$$I_4 = -\frac{(4D - 3)}{(4D + 3)(4D - 3)} \cdot \cos 2x = \frac{-(4D - 3)}{16D^2 - 3^2} \cdot \cos 2x$$

Now replacing $D^2$ by $-2^2$, we get

$$I_4 = \frac{-(4D - 3)}{(4D + 3)(4D - 3)} \cdot \cos 2x = \frac{(4D - 3)}{73}\cos 2x$$

$$= \frac{(4D - 3)}{73}\cos 2x$$

$$= \frac{1}{73}[4(-2)\sin 2x - 3\cos 2x]$$

$$I_4 = -\frac{1}{73}(8\sin 2x + 3\cos 2x)$$

Thus the P.I.

$$y_p = \frac{1}{2}[I_1 + I_2 + I_3 - I_4]$$

$$y_p = \frac{1}{2}\left[-\frac{1}{52}(3\sin 3x + 2\cos 3x) - \frac{1}{4}\sin x + 1 - \frac{1}{73}(8\sin 2x + 3\cos x)\right]$$

G.S:

$$y = y_c + y_p$$

$$y = C_1 e^{(2+\sqrt{3})x} + C_2 e^{(2-\sqrt{3})x}$$

$$+ \left[1 - \frac{1}{4}\sin x - \frac{1}{73}(3\cos 2x + 8\sin 2x) - \frac{1}{52}(3\cos 3x + 2\cos 3x)\right] \times \frac{1}{2}$$

**Q.2 (c) Solution:**

Disk Scheduling algorithms:

1. FCFS
2. SSTF
3. Elevator

**1. FCFS disk scheduling algorithms:**

- First come first served, scheduled the request as they arrived.
- Simple, easy to implement
- No chance of starvation.
- May cause more seek time and more number of times head changing its direction.

**2. SSTF disk scheduling algorithms:**

- Shortest seek time first (Nearest cylinder next)
- Schedule the request next which is nearest from current head position (shortest seek time)
- Produces minimum average seek time.
- Chance of starvation.
- May cause more number of times head change its direction.

**3. Elevator disk scheduling algorithms:**

- Head move from one end to other and service request in that order.
- No chance of starvation.
- Minimum number of times, head change its direction (only at end)
- Types of elevator disk scheduling algorithm are:

  (a) Scan (b) C-scan (c) Look (d) C-Look

**(ii) 1.** Increasing the associativity will reduce the number of conflict misses (misses that occur because addresses compete for a limited number of spaces in the cache) but will not affect the number of compulsory misses (misses that occur the first time a memory address is accessed) or the number of capacity misses (misses that occur because a program references more data than will fit in the cache). Therefore, the best that is achieved with increasing the associativity is the elimination of all the conflict misses.

Since, the miss rate is 7 percent and the program makes 10,00,000 memory references, therefore, the total number of misses is 70,000.

$$\text{Number of conflict misses} = \frac{1}{2}\times 70,000 = 35,000$$

Hence, the maximum number of misses that can be eliminated by increasing the associativity of the cache is 35,000.

**2.** Increasing both the size and associativity of the cache will eliminate both capacity and conflict misses.

$$\text{Number of capacity and conflict misses} = \frac{3}{4}\times 70,000 = 52,500$$

Hence, the maximum number of misses that can be eliminated by increasing the size and associativity of the cache is 52,500.

**Q.3 (a) (i) Solution:**

$$\text{Power factor} = \cos\phi = 0.5 \text{ lagging}$$

$$\text{Phase angle, } \phi = 60°$$

The phase angle between applied voltage and load current is 60°. Under ideal condition, the voltage magnet flux $\phi_p$ should lag behind the applied voltage $V$ by 90° and the current magnet flux $\phi_s$, should be in phase with load current $I$. This is shown in figure (a),

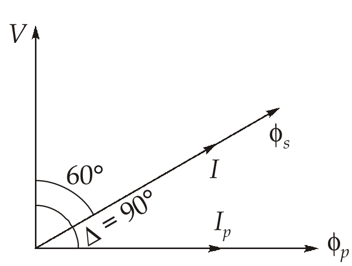


Figure (a)

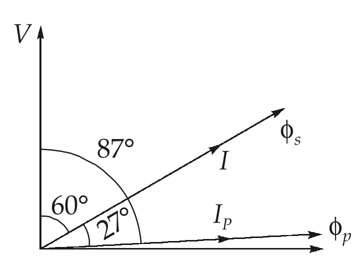


Figure (b)

$$\text{Energy registered} = K\int VI\sin(\Delta-\phi)dt$$

$$\propto \sin(\Delta-\phi)$$

Now under ideal conditions, $\Delta = 90°$

$\therefore$ Energy registered under ideal conditions $\propto \sin(90° - \phi)$

$$\propto \cos\phi$$

$$\propto \cos 60°$$

The phasor diagram for actual working conditions in shown in figure (b)

Now,

$\because$ $\Delta - \phi = 27°$

$\therefore$ $\Delta = 27° + 60° = 87°$

$\therefore$ Energy registered under actual working conditions

$$\propto \sin(\Delta - \phi) \propto \sin(87° - 60°) \propto \sin 27°$$

It is clear from the above conclusion that the energy registered under actual working condition is not the same as under ideal condition. Therefore, the meter does not read correctly

$$\text{Error} = \frac{(\sin 27° - \cos 60°)}{\cos 60°} \times 100$$

$$= \frac{0.454 - 0.5}{0.5} \times 100 = -9.2\%$$

**Q.3 (a) (ii) Solution:**

Angle of shear, $\theta = \dfrac{2T}{\pi G r^3}$

Where, $G$ is the shaft shear modulus

$r$ is the radius of the shaft

$T$ is the applied torque

An area of the shaft surface, originally square with the sides of unit length, is deformed by stain to a parallelogram. The original length of the diagonal is $\sqrt{2}$. If the angle of shear, $\theta$ is small, the length of the diagonal of the parallelogram is longer than the diagonal of the square. The difference in lengths is $\dfrac{\theta}{\sqrt{2}}$. Therefore, the longitudinal strain is:

$$\varepsilon = \frac{\Delta L}{L} = \frac{\theta/\sqrt{2}}{\sqrt{2}} = \frac{\theta}{2}$$

But $\dfrac{\Delta R}{R} = G_f \varepsilon = 2 \times \dfrac{\theta}{2} = \theta$

or $$\theta = \frac{\Delta R}{R} = \frac{0.24}{120} = 2 \times 10^{-3} \text{ rad}$$

$\therefore$ Torque, $$T = \frac{\pi G r^3}{2}\theta = \frac{\pi \times 80 \times 10^9 \times (15 \times 10^{-3})^3}{2} \times 2 \times 10^{-3} = 848.23 \text{ Nm}$$

**Q.3 (b) Solution:**

**(i)** $P_1 = 7500$ W and $P_2 = -1500$ W

Total power, $P = P_1 + P_2 = 7500 - 1500 = 6000$ W

$$\phi = \tan^{-1}\sqrt{3}\cdot\frac{P_1 - P_2}{P_1 + P_2}$$

$$= \tan^{-1}\sqrt{3}\cdot\frac{7500-(-1500)}{7500+(-1500)} = 69°$$

Power factor $\cos\phi = \cos 69° = 0.358$

**(ii)** Power consumed by each phase

$$= \frac{6000}{3} = 2000 \text{ W}$$

Current of each phase $= \dfrac{2000}{231 \times 0.358} = 24.1$ A

Impedance of each phase $= \dfrac{231}{24.1} = 9.55\ \Omega$

Resistance of each phase $= \dfrac{2000}{(24.1)^2} = 3.44\ \Omega$

Reactance of each phase $= \sqrt{(9.55)^2 - (3.44)^2} = 8.9\ \Omega$

In order that one of the wattmeters should read zero, the power factor should be 0.5

$\therefore$ $\cos\phi = 0.5$ and $\tan\phi = 1.73$

Now, $$\tan\phi = \frac{X}{R}$$

$\therefore$ Reactance of circuit, $X = R\tan\phi = 3.44 \times 1.73 = 5.9\ \Omega$

$\therefore$ Capacitive reactance required

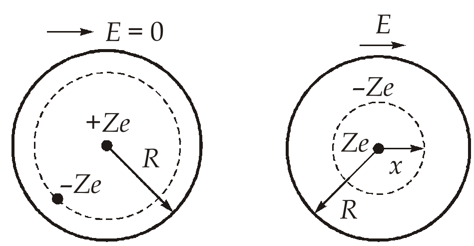
$$= 8.9 - 5.9 = 3.0\ \Omega$$

and capacitance, $$C = \frac{1}{2\pi \times 50 \times 3} F = 1060\ \mu\text{F}$$

**Q.3 (c) Solution:**

**Electronic Polarizability:**

- Electronic polarization is observed in inert gases in which it is assumed that interaction among the atoms is negligible.
- A simple model of an atom is shown below in which a positive nucleus of charge $Ze$ ($Z$ is the atomic number of the atom and e is the charge of an electron) is surrounded by a spherical negative cloud of charge having a magnitude $-Ze$, having atomic radius equals to '$R$'.
- When external field ($E$) is applied then, under equilibrium, the positive charge remains at a distance x from the centre due to forces: one is the coulombic attraction between the charges and the other is the force on the nucleus due to the field $E$, equal to $ZeE$.




i.e. $$F_e = ZeE$$

The charge enclosed in the sphere of radius $x$ is given by $\Delta q$,

- $$\Delta q = \frac{-Ze}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi x^3 = \frac{-Zex^3}{R^3} \quad \text{...(i)}$$

The magnitude of coulombic attraction force between this charge treated concentrated at a point, and the nucleus will be

$$F = \frac{\left(\frac{-Zex^3}{R^3}\right)(Ze)}{(4\pi\varepsilon_0 x^2)} = \frac{-(Ze)^2 x}{4\pi\varepsilon_0 R^3} \quad \text{...(ii)}$$

The total force on the nucleus must be zero in equilibrium, so we obtain (equating two forces)

i.e. $$|F_{att.}| = |F_e|$$

$\Rightarrow$ $$ZeE = \frac{(Ze)^2 x}{4\pi\varepsilon_0 R^3}$$

or $$x = (4\pi\varepsilon_0 R^3/Ze)E \quad \text{...(iii)}$$

The dipole moment induced by the field, will be given by

$$P_{ind} = Ze\,x = 4\pi\varepsilon_0 R^3 E = \alpha_e E \quad \text{...(iv)}$$

This dipole is induced by the field and never existed in the absence of the field. The induced dipole moment is proportional to the field strength and proportionality factor $\alpha_e$ is called the electronic polarizability.

We see that $\alpha_e$ is proportional to $R^3$, i.e., to the volume of electron cloud.

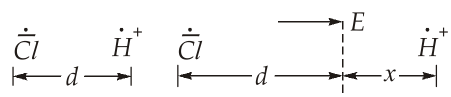
Because the electronic structure of an atom is relatively temperature independent, the variation of $\alpha_e$ with the temperature is expected to be zero.

**Note:** Since, $R_{He} < R_{Ne} < R_{Ar} < R_{Kr} < R_{Xe}$ (where $R$ = radius of an atom)

then, $$\alpha_{e_{(He)}} < \alpha_{e_{(Ne)}} < \alpha_{e_{(Ar)}} < \alpha_{e_{(kr)}} < \alpha_{e_{(Xe)}}$$

**Ionic Polarizability:**

- Ionic polarization occurs in the materials having ionic bonds. E.g. NaCl, HCl etc. Even in the absence of an applied field, these molecules have a permanent dipole moment $(e \times d)$ where $d$ is the distance of separation of ions.
- The field produces force on the two charges $\pm e$, as well as a torque on the dipole. The distance between ions increases from $d$ to $d + x$,




- The field has induced an additional dipole moment, $p_{\text{ind}} = e \cdot x$ in the molecule. The induced dipole moment is proportional to the applied electric field, and the proportionality constant is the ionic polarizability. So, we have,

$$p_{\text{ind}} = \sigma_i E$$

where $\alpha_i$ is the ionic polarizability.

**Orientational Polarizability:**

- Orientational polarization is found in the material having covalent bond with partly ionic bonds, i.e. polar type of covalent bond.

  For moderate fields and all but very low temperatures, the orientational polarization $P_0$ may be written

$$P_0 = \frac{Np_p^2 E}{3\,kT}$$

But $$P_0 = N\,\alpha_0\,E$$

where $\alpha_0$ = Orientational polarizability

Therefore, $$\alpha_0 = \frac{p_p^2}{3\,kT}$$

where, $p_p$ = Permanent dipole moment

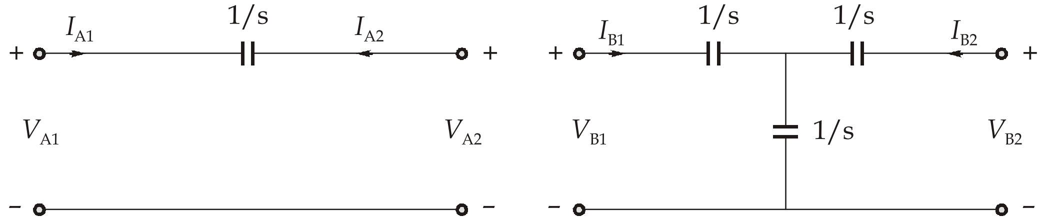
$T$ = Temperature in Kelvin

$k$ = Boltzmann's constant = $1.38 \times 10^{-23}$ J/K

$N$ = Number of dipoles/m$^3$.

**Q.4 (a) (i) Solution:**

It can be considered as two networks in parallel,



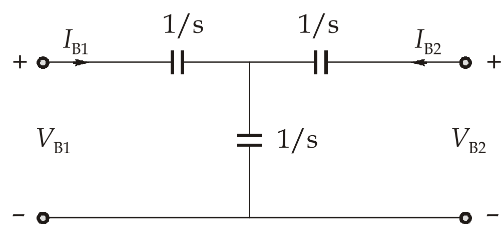

**Network A** **Network B**

$Y$-parameter of network A,

$$y_{A11} = \left.\frac{I_{A1}}{V_{A1}}\right|_{V_{A2}=0} = s = y_{A22}$$

and
$$y_{A12} = \left.\frac{I_{A1}}{V_{A2}}\right|_{V_{A1}=0} = -s = y_{A21}$$

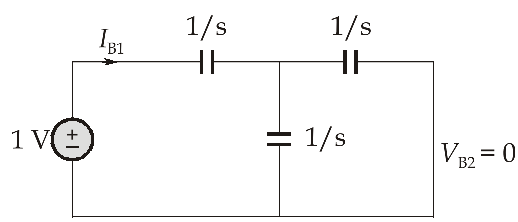
$Y$-parameter of network $B$,




**Network B**

$$Y_{B11} = \left.\frac{I_{B1}}{V_{B1}}\right|_{V_{B2}=0} \text{ and } V_{B1} = 1\text{ V}$$

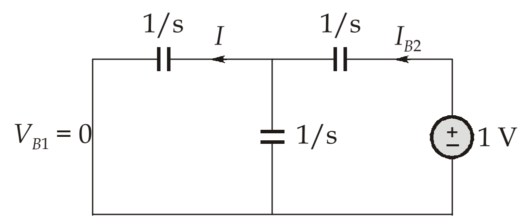
Taking $V_{B1} = 1$ V,




$$I_{B1} = \frac{1}{\frac{1}{s} + \frac{\frac{1}{s} \times \frac{1}{s}}{\frac{1}{s} + \frac{1}{s}}} = \frac{2s}{3}$$

$$y_{B11} = \frac{2s}{3} = y_{B22}$$

$$y_{B12} = \left.\frac{I_{B1}}{V_{B2}}\right|_{V_{B1}=0}$$




$$I_{B2} = \frac{2s}{3}$$

$$I = \frac{\frac{2s}{3} \times \frac{1}{s}}{\frac{1}{s} + \frac{1}{s}} = \frac{s}{3}$$

$$I_{B1} = -I$$

$$y_{B12} = -\frac{s}{3} = y_{B21}$$

$$[Y_B] = \begin{bmatrix} \frac{2s}{3} & -\frac{s}{3} \\ -\frac{s}{3} & \frac{2s}{3} \end{bmatrix}$$

$$Y_{\text{(for combined circuit)}} = Y_A + Y_B = \begin{bmatrix} \frac{5s}{3} & -\frac{4s}{3} \\ -\frac{4s}{3} & \frac{5s}{3} \end{bmatrix}$$

**Q.4 (a) (ii) Solution:**

Here $I_1 = 6$ A, $L_1 = 4$ H, $L_2 = 9$ H and $M = 3$ H

So energy stored,

$$W = 0.5\, L_1 I_1^2 + 0.5 L_2 I_2^2 - M I_1 I_2$$
$$= 0.5 \times 4 \times 36 + 0.5 \times 9 \times I_2^2 - 3 \times 6 \times I_2$$
$$= 4.5\, I_2^2 - 18\, I_2 + 72 = f(I_2)$$

For finding the minimum value of stored energy. $\frac{df(I_2)}{dI_2}$ should be equal to zero and solve for $I_2$:

$$\frac{dW}{dI_2} = \frac{df(I_2)}{dI_2} = 9I_2 - 18 = 0$$

This yields $I_2 = 2A$, and the corresponding minimum stored energy,

$$W_{min} = 4.5 \times 2^2 - 18 \times 2 + 72$$
$$= 18 - 36 + 72 = 54 \text{ J}$$

**Q.4 (b) (i) Solution:**

The expression for deflection for a square low response voltmeter is

$$\theta = \frac{V^2}{KZ^2}\frac{dM}{dQ}$$

For d.c. impedance, $Z$ = resistance $R$

and therefore, $$\theta = \frac{V^2}{KR^2}\frac{dM}{d\theta}$$

But $$\frac{V}{R} = 0.05 \text{ A},$$

$$K = 0.5 \times 10^{-6} \text{ and } \theta = 90°$$

Thus, we have $$90 = \frac{(0.05)^2}{0.5\times10^{-6}}\frac{dM}{d\theta}$$

or $$\frac{dM}{d\theta} = 18 \times 10^{-3} \text{ H/rad}$$

$$\text{Angle of deflection} = 90° = \frac{\pi}{2} \text{ rad}$$

Total change in mutual inductance

$$= 18\times10^{-3}\times\frac{\pi}{2} = 28.3 \times 10^{-3} \text{ H}$$

Mutual inductance at $\theta$ = 90°

= Initial mutual inductance + change in mutual inductance

$$= 0.25 + 28.3 \times 10^{-3}$$

$$= 0.2783 \text{ H}$$

Reactance of instrument at 50 Hz

$$= 2\pi \times 50 \times 0.2783 = 87.43\ \Omega$$

$$\text{Resistance of instrument} = \frac{50}{0.05} = 1000\,\Omega$$

Impedance of instrument at 50 Hz

$$= \sqrt{(1000)^2 + (87.43)^2} = 1003.8\ \Omega$$

Current through the instrument when reading 50 V is 0.05 A

∴ Actual voltage across the instrument

$$= 0.05 \times 1003.8 = 50.19 \text{ V}$$

**Q.4 (b) (ii) Solution:**

Let $V$ and $I$ be the phase voltage and phase current repectively. cos ϕ is the power factor of the load,

$$\text{Phase voltage, } V = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

and phase current, $I = 30$ A

In the first case, the wattmeter measures power in one phase,

In the first case, the wattmeter measures the power in one phase,

$$\therefore \quad \text{VI} \cos\phi = 5.54 \times 10^3$$

or
$$\cos\phi = \frac{5.54 \times 10^3}{231 \times 30} = 0.8$$

$$\sin\phi = 0.6$$

When the current coil is connected in the red phase and the pressure coil circuit is connected across the yellow and blue phases, the reading of wattmeter is,

$$= V_{YB} I_R \cos(90° - \phi)$$

$$= \sqrt{3}\, VI \sin\phi$$

But $VI \sin\phi$ is the reactive power of each phase and therefore the wattmeter indicates $\sqrt{3}$ × reactive power of each phase.

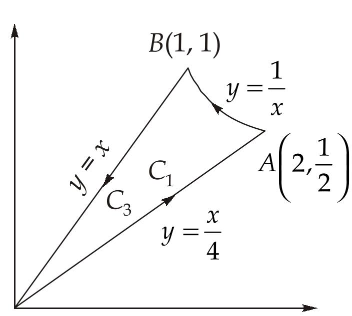
$$\text{Magnitude of reading} = \sqrt{3} \times 231 \times 30 \times 0.6 \times 10^{-3}$$

$$= 7.2 \text{ KVAr}$$

**Q.4 (c) (i) Solution:**

By Green's theorem area $A$ of the region bounded by a closed curve $C$ is given by

$$A = \frac{1}{2}\oint x\,dy - y\,dx$$



Here $C$ consists of the curves $C_1$ :

$$y = \frac{x}{4}, \quad C_2 : y = \frac{1}{x} \text{ and } C_3 : y = x$$

So,
$$A = \frac{1}{2}\left[\int_{C_1} + \int_{C_2} + \int_{C_3}\right] = \frac{1}{2}[I_1 + I_2 + I_3]$$

Along $C_1$ :
$$y = \frac{x}{4}, \quad dy = \frac{1}{4}dx, \quad x : 0 \text{ to } 2$$

$$I_1 = \int_{C_1} x\,dy - y\,dx = \int_{C_1} x\frac{1}{4}dx - \frac{x}{4}dx = 0$$

Along $C_2$ :
$$y = \frac{1}{x}, \quad dy = -\frac{1}{x^2}dx, \; x : 2 \text{ to } 1$$

$$I_2 = \int_{C_2} x\,dy - y\,dx = \int_2^1 x\left(-\frac{1}{x^2}\right)dx - \frac{1}{x}dx$$

$$= -2\ln|_2^1 = 2\ln 2$$

Along $C_3$ :
$$y = x, \quad dy = dx, \quad x : 1 \text{ to } 0$$

$$I_3 = \int_{C_3} x\,dy - y\,dx = \int x\,dx - x\,dx = 0$$

$$A = \frac{1}{2}(I_1 + I_2 + I_3) = \frac{1}{2}(0 + 2\ln 2 + 0) = \ln 2$$

**Q.4 (c) (ii) Solution:**

Characteristics equation of $\begin{bmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{bmatrix}$ is:

$(1 - \lambda)(2 - \lambda)(3 - \lambda) - (1 - \lambda) \times 2 - 1(2 - 2(2 - \lambda)) = 0$

$-\lambda^3 + 5\lambda^2 - 6\lambda + \lambda^2 - 5\lambda + 6 - 2 + 2\lambda - 2 + 4 - 2\lambda = 0$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$$

So, $\lambda = 1, 2, 3$ are three distinct eigen values of $A$

For $\lambda = 1$,

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_3 = 0$$

$$x_1 + x_2 = 0$$

$$X_1 = C_1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$,

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 + x_3 = 0$$

$$2x_1 + 2x_2 + x_3 = 0$$

$$x_1 = -x_3 \, , \, x_2 = \frac{1}{2}x_3 \, , \, x_2 = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

For $\lambda = 3$,

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 = -x_2$$

$$x_1 = -\frac{1}{2}x_3$$

$$x_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Thus these are three linearly independent eigen vectors $x_1$ , $x_2$ , $x_3$ corresponding to the three distinct eigen values

Since, $X_1^T X_2 = 3 \neq 0,$

$X_2^T X_3 = 5 \neq 0,$

$X_3^T X_1 = 0$

Therefore only $X_1$ and $X_3$ are orthogonal.

## Section-B

**Q.5 (a) Solution:**

**(i)** The Hall voltage can be given as,

$$V_H = E_H W = -(16.5 \times 10^{-3})\,(5 \times 10^{-2})$$

$$V_H = -0.825 \text{ mV}$$

**(ii)** Since the value of Hall voltage is negative, the type of semiconductor has electrons as their majority carriers. Hence, the semiconductor is *n*-type.

**(iii)** The majority carrier concentration can be calculated as

$$n = \frac{-I_x B_z}{qdV_H} = \frac{-(0.5\times10^{-3})(6.5\times10^{-2})}{(1.6\times10^{-19})(5\times10^{-5})(-0.825\times10^{-3})}$$

$$n = 4.92 \times 10^{21} \text{ m}^{-3} = 4.92 \times 10^{15} \text{ cm}^{-3}$$

**Q.5 (b) Solution:**

Note that Meshes 1 and 2 form a supermesh since they have an independent current source in common. Also meshes 2 and 3 form another supermesh because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh.

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or $$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \quad \text{...(i)}$$

For the independent current source, we apply KCL to node $P$:

$$i_2 = i_1 + 5$$

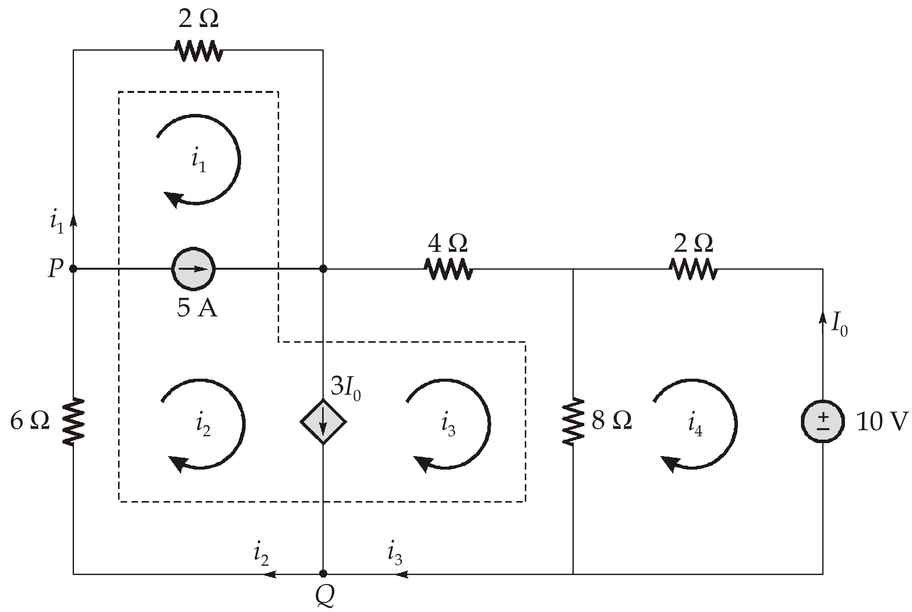
$$i_1 = i_2 - 5 \quad \text{...(ii)}$$

Put equation (ii) in equation (i)

$$4i_2 + 6i_3 - 4i_4 = 5 \quad \text{...(iii)}$$

For the dependent current source, we apply KCL to node $Q$:

$$i_2 = i_3 + 3I_o$$




But, $I_o = -i_4$

$$i_2 = i_3 - 3i_4$$

$$i_2 - i_3 + 3i_4 = 0 \quad \text{...(iv)}$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

$$5i_4 - 4i_3 = -5 \quad \text{... (v)}$$

From equation (iii), (iv), (v)

$$i_2 = -2.5 \text{ A}, \qquad i_3 = 3.93 \text{ A}$$

and $i_4 = 2.143$ A

Putting $i_2 = -2.5$ A in equation (ii),

$$i_1 = -7.5 \text{ A}$$

**Q.5 (c) Solution:**

$$\text{Power consumed by load} = 25 \times 10 \times 0.174$$

$$= 43.5 \text{ W}$$

In figure (a), the current coil is on the load side,

Power factor, $\cos\phi = 0.174$

$\therefore$ $\phi = 80°$ lagging

$$\text{Current, } I = 10(\cos 80° - j\sin 80°)$$

$$= 10(0.1736 - j0.985)$$

$$= 1.736 - j9.85 \text{ A}$$

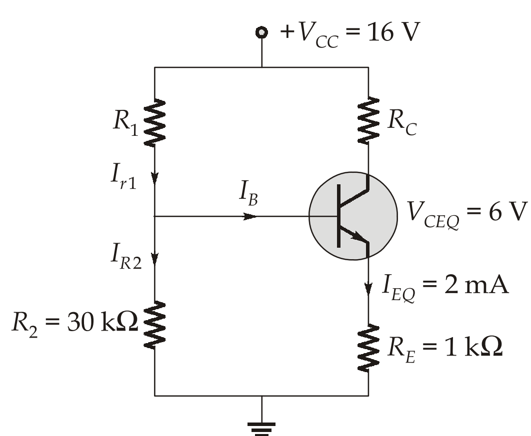
$$\text{Voltage drop across current coil} = (1.736 - j9.85)(0.06 + j0.02)$$
$$= 0.301 - j0.5569$$
$$= 0.632\angle{-61.54°}\text{ V}$$
$$\text{Voltage across pressure coil, } V_p = 25 + 0.301 - j0.5569$$
$$= 25.316\angle{-1.26°}\text{ V}$$
$$\text{Power indicated by wattmeter} = 25.316 \times 10 \times \cos(80° - 1.26°)$$
$$= 49.5\text{ W}$$
$$\text{Reading of wattmeter} = \text{Power consumed in load + Power consumed in CC}$$
$$= 43.5 + (10)^2 \times 0.06 = 49.6\text{ W}$$
$$\text{Error} = \frac{49.5 - 43.5}{43.5} \times 100 = 13.8\%$$

When the pressure coil is on the load side as in figure '*b*'

$$\text{Power indicated by wattmeter} = 43.5 + \frac{(25)^2}{6250} = 43.5 + 0.1 = 43.6\text{ W}$$
$$\text{Error} = \frac{43.6 - 43.5}{43.5} \times 100 = 0.23\%$$

**Q.5 (d) Solution:**

The circuit diagram will be as shown below:




Let us first calculate the value of β using the following relationship

$$\beta = \frac{\alpha}{1 - \alpha}$$

Substituting the given values of α, we get

$$\beta = \frac{0.985}{1-0.985} = 65.65$$

and $$I_{CQ} = \alpha I_E = 0.985 \times 2 = 1.97 \text{ mA}$$

**(i)** Now, applying KVL to the collector circuit in figure,

We have,

$$V_{CC} = I_{CQ}R_C + V_{CE\,Q} + I_{EQ}R_E$$

$$R_C = \frac{V_{CC} - V_{CEQ} - I_{EQ}R_E}{I_{CQ}}$$

Substituting the given values, we get

$$R_C = \frac{16-6-(2\times 1)}{1.97\times 10^{-3}} = 4.06 \text{ k}\Omega$$

Applying KVL to the base circuit, we write

$$V_{CC} = I_{R1}R_1 + V_{BE} + I_{EQ}R_E$$

$$R_1 = \frac{V_{CC} - V_{BE} - I_{EQ}R_E}{I_{R1}}$$

$$I_{R1} = I_{R2} + I_B = \frac{V_B}{R_2} + \frac{I_{CQ}}{\beta}$$

$$= \frac{2+0.2}{30\times 10^3} + \frac{1.97\times 10^{-3}}{65.65} = 103.33 \ \mu\text{A}$$

Substituting this values of $I_{R1}$

$$R_1 = \frac{16-0.2-(2\times 1)}{103.33\times 10^{-6}} = 133.54 \text{ k}\Omega$$

**Q.5 (e) Solution:**

Superconductivity or zero resistivity (electrical) depends on three parameters.

1. Critical temperature ($T_c$)
2. Critical magnetic field ($H_c$)
3. Critical current density ($J_c$)

The process of superconductivity is reversible like liquid-gas phase.

According to Silsbee's rule if a current greater than critical current is applied then superconductivity of the material is lost. At critical values of parameter there is a transition in state of material.

In a long superconducting wire of radius $r$, the superconductivity may be destroyed when a current $I$ exceeds the critical current value $I_c$, which at the surface of wire will produce a critical field $H_c$.

$$I_c = 2\pi r H_c$$

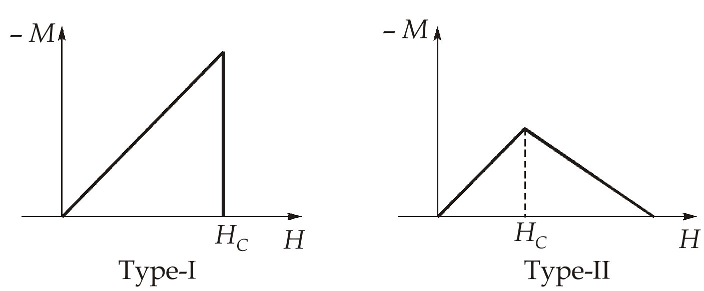
$$\text{Critical current density} = \frac{I_c}{A} = \frac{2\pi r H_c}{\pi r^2} = \frac{2H_c}{r}$$

On the basis of their magnetic properties superconductors are categorized in two types :

(1) Type I (2) Type II

Type I : These superconductor shows perfect diamagnetism upto critical field $H_c$ and shows normal state after that. Soft metals belongs to this group. e.g. lead, Indium. These materials are not suitable for high field applications. These materials follows Silsbee's rule and shows Meissner's effect.

Type II : These superconductors have different magnetization properties. Hard metals and alloys belongs to this group. These have high transition temperature, high critical field, incomplete Meissner's effect, breakdown of Silsbee's rule and broad transition region.




Applications

1. In magnetic levitation trains
2. Making electricity generation more efficient.
3. Very fast computing
4. In detecting even the weakest magnetic field in superconducting quantum interference devices (SQUID's)
5. In particle accelerator.

**Q.6 (a) (i) Solution:**

$$\text{Characteristic equation is } |A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(2-\lambda)^2 - 1] + 1[-1(2-\lambda) + 1] + 1[1 - (2-\lambda)] = 0$$

$$(2-\lambda)^3 - (2-\lambda) - 2 + \lambda + 1 + 1 - 2 + \lambda = 0$$

$$-\lambda^3 + 8 - 12\lambda + 6\lambda^2 - 2 + 2\lambda - 2 + \lambda = 0$$

$$-\lambda^3 + 6\lambda^2 - 9\lambda + 4I = 0$$

$$\lambda^3 - 6\lambda^2 + 9\lambda - 4I = 0$$

By Cayley-Hamilton theorem,

$$A^3 - 6A^2 + 9A - 4I = 0$$

Multiiply by $A^{-1}$, we get

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

or $$4A^{-1} = A^2 - 6A + 9I$$

$$4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Also, $$A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$$

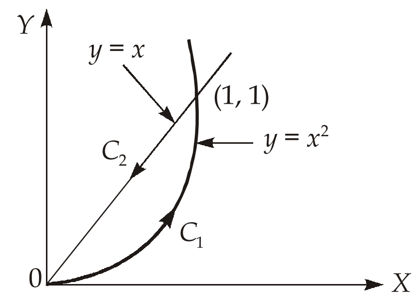
$$= A^3(A^3 - 6A^2 + 9A - 4I) + 2(A^3 - 6A^2 + 9A - 4I) + 5A - I$$

$$= 5A - I$$

**Q.6 (a) (ii) Solution:**

Here, $\phi = xy + y^2$ and $\Psi = x^2$

$$\therefore \quad \int_c (\phi\, dx + \Psi dy) = \int_{c_1} + \int_{c_2}$$




Along $c_1$, $y = x^2$ and $x$ varies from 0 to 1

$$\therefore \quad \int_{c_1} = \int_0^1 [\{x(x)^2 + (x^2)^2\}\, dx + x^2 d(x^2)]$$

$$= \int_0^1 (3x^3 + x^4)\, dx$$

$$= \left[\frac{3x^4}{4} + \frac{x^5}{5}\right]_0^1 = \frac{3}{4} + \frac{1}{5} = \frac{15+4}{20} = \frac{19}{20}$$

Along $c_2$, $y = x$ and $x$ varies from 1 to 0

$$\therefore \quad \int_{c_2} = \int_1^0 [\{x(x) + (x)^2\}\, dx + x^2 d(x)]$$

$$= \int_1^0 3x^2 dx = \left[\frac{3x^3}{3}\right]_1^0 = -1$$

Thus,

$$\int_c (\phi\, dx + \Psi dy) = \frac{19}{20} - 1 = -\frac{1}{20} \quad \text{...(i)}$$

$$\text{RHS} = \iint_S \left(\frac{\partial \Psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\, dy$$

$$= \iint_S \left[\frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial y}(xy + y^2)\right] dx\, dy$$

$$= \int_0^1 \int_{x^2}^{x} (2x - x - 2y)\, dy\, dx$$

$$= \int_0^1 [xy - y^2]_{x^2}^{x}\, dx = \int_0^1 (x^4 - x^3)\, dx$$

$$= \frac{x^5}{5} - \frac{x^4}{4} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20} \quad \text{...(ii)}$$

Hence, Green theorem is verified from the equality of (i) and (ii).

**Q.6 (b) Solution:**

**(i)** Given :

$$H = 10^5 \rho^2 \hat{a}_\phi \ \text{A/m}$$

$$\rho = 5 \text{ mm}, V = 0.1 \text{ V}, L = 20 \text{ m}$$

From Maxwell's equation,

$$J = \nabla \times H$$

$$J = \frac{1}{\rho}\begin{bmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho 10^5 \rho^2 & 0 \end{bmatrix}$$

$$= \frac{1}{\rho}\left[0 + 0 + a_z\left(\frac{\partial}{\partial \rho}(10^5 \rho^3)\right) - 0\right]$$

$$= \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}(10^5 \rho^3) a_z$$

$$J = \frac{3 \times 10^5 \rho^2}{\rho}\hat{a}_z = 3 \times 10^5 \rho \hat{a}_z$$

Electric field,

$$\vec{E} = \frac{V}{L} = \frac{0.1}{20} = 5 \times 10^{-3} \text{ V/m}$$

Also,

$$\vec{J} = \sigma \vec{E}$$

$$\sigma = \frac{\vec{J}}{\vec{E}}$$

$$\sigma = \frac{3 \times 10^5 \rho}{5 \times 10^{-3}} = \frac{3 \times 10^5 (5 \times 10^{-3})}{5 \times 10^{-3}} = 3 \times 10^5 \text{ S/m}$$

**(ii)** The current in the wire,

$$I = \int_s \vec{J} \cdot ds$$

$$ds = \rho d\rho d\phi \hat{a}_z$$

$$I = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{5\text{mm}} 3\times10^5 \rho \cdot \rho d\rho d\phi$$

$$= 3\times10^5 \left[\frac{\rho^3}{3}\right]_0^{5\text{mm}} \times [\phi]_0^{2\pi}$$

$$I = 1\times10^5 \left[\rho^3\right]_0^{5\text{mm}} \times [\phi]_0^{2\pi}$$

$$= 1 \times 10^5[(5 \times 10^{-3})^3 - 0] \times 2\pi$$

$$I = 0.0785 \text{ A}$$

Resistance,
$$R = \frac{V}{I} = \frac{0.1}{0.0785} = 1.273\ \Omega$$

$$R = 1.273\ \Omega$$

**Q.6 (c) Solution:**

**(i)** Capacity of main memory = 256 MB
Capacity of cache memory = 1 MB
Block size = 128 bytes
A set contain 8 blocks.
Since the address space of processor is 256 MB.
The processor will generate address of 28-bits to access a byte (word).

The number of blocks contained by main memory = $\frac{256 \text{ MB}}{128 \text{ B}} = 2^{21}$

Therefore, number of bits required to specify one block in main memory = 21.
Since the block size is 128 bytes.
The number of bits required access each word (byte) = 7.
For associative, the address format :

| Tag | Word |
|---|---|
| 21 | 7 |

The number of blocks contained by cache memory = $\frac{1 \text{ MB}}{128 \text{ B}} = 2^{13}$.

Therefore, the number of bits required to specify one block in cache memory = 13.
For direct cache, the address format is

| Tag | Block | Word |
|---|---|---|
| 8 | 13 | 7 |

(Block + Word: Index)

**(ii)** Cache access time, $t_c$ = 50 nsec

Main memory access time, $t_m$ = 500 nsec

Probability of read, $p_r$ = 0.8

Hit ratio for read access, $h_r$ = 0.9

Hit ratio for write access, $h_w$ = 1 ...(by default 1)

Writing scheme : Write through

1. Considering only memory read cycle

   The average access time

$$t_{av,r} = h_r \times t_c + (1 - h_r) \times t_m$$
$$= 0.9 \times 50 + (1 - 0.9) \times 500$$
$$= 95 \text{ nsec}$$

2. For both read and write cycle,

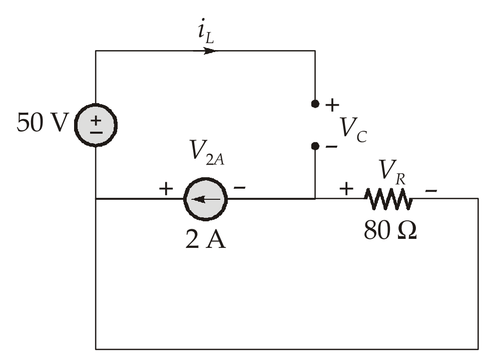
$$\text{Average access time} = p_r \times t_{av,r} + (1 - p_r) \times t_m$$

   In write-through method, access time for write cycle will be main memory access time.

$$= 0.8 \times 95 + (1 - 0.8) \times 500$$
$$= 176 \text{ nsec}$$

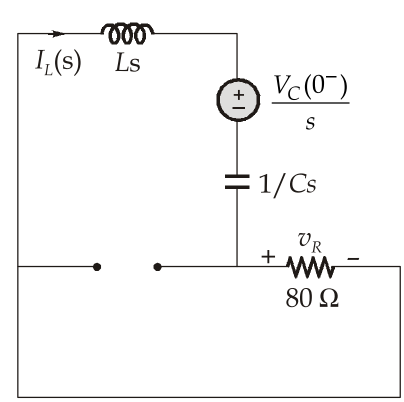
**Q.7 (a) (i) Solution:**

Fot $t < 0$;




$$i_L(0^-) = i_L(0^+) = 0 \text{ A}$$
$$V_R(0^-) = -2 \times 80$$
$$= -160 \text{ V}$$
$$V_C(0^-) = 50 - V_R = 210 \text{ V}$$

Applying laplace transform




$$I_L(s) = \frac{-210/s}{2s + \frac{10^3}{s} + 80} = \frac{-105}{s^2 + 40s + 500}$$

$$i_L(t) = -10.5\, e^{-20t} \sin 10t \text{ A}$$
$$v_L(t) = 210e^{-20t}\,[2 \sin 10t - \cos 10t] \text{ V}$$
$$v_R(t) = Ri_L(t) = 80[-10.5\, e^{-20t} \times \sin 10t]$$
$$= -840e^{-20t} \sin 10t \text{ V}$$
$$v_C(t) = -[V_L(t) + V_R(t)]$$
$$= 210e^{-20t}\,[2 \sin 10t + \cos 10t] \text{ V}$$

**Q.7 (a) (ii) Solution:**

Before closing the switch,

$$\text{Energy stored} = \frac{1}{2}C_1V^2 = \frac{1}{2}\times 4\times 12^2 = 288 \text{ J}$$

and $\text{Charge, } Q = CV = 4 \times 12 = 48 \text{ C}$

After closing the switch,

$$\text{Charge, } Q = (C_1 + C_2)v$$

Where, $v$ is the common terminal voltage.

$$48 = (4 + 2)v$$

$$v = 8 \text{ V}$$

and the energy stored after closing the switch

$$= \frac{1}{2}CV^2 = \frac{1}{2}\times(4+2)8^2 = 192 \text{ J}$$

By charge conservation,

$$\text{Charge, } Q_1 + Q_2 = 48\ C$$

Also, $$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 = 2Q_2$$

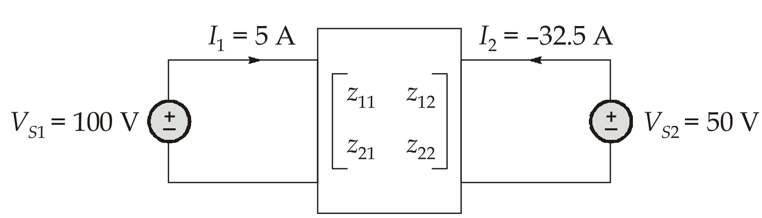
Solving $$Q_1 = 32\ C$$

$$Q_2 = 16\ C$$

**Q.7 (b) (i) Solution:**

For $z$-parameter $$[Z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

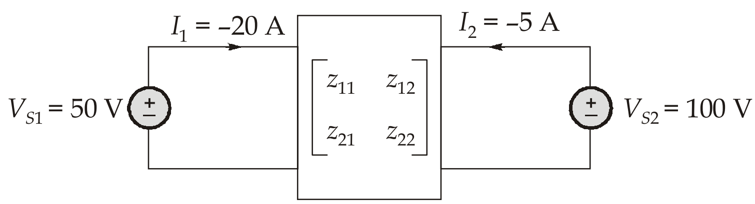
From experimeter-1




$$V_{s1} = z_{11}I_1 + z_{12}I_2$$

$$100 = z_{11} \times 5 + z_{12}(-32.5) \quad \text{...(i)}$$

From experiment-2




$$V_{s1} = z_{11}I_1 + z_{12}I_2$$

$$50 = z_{11}(-20) + z_{12}(-5) \quad \text{...(ii)}$$

On solving equation (i) and (ii)

$$z_{11} = -1.66\ \Omega$$

$$z_{12} = -3.33\ \Omega$$

Similarly, from experiment 1

$$V_{s2} = z_{21}I_1 + z_{22}I_2$$

$$50 = z_{21}(5) + z_{22}(-32.5) \quad \text{...(iii)}$$

From experiment 2

$$V_{s2} = z_{21}I_1 + z_{22}I_2$$

$$100 = z_{21}(-20) + z_{22}(-5) \quad \text{...(iv)}$$

On solving equation (iii) and (iv),

$$z_{21} = -4.44\Omega$$

$$z_{22} = -2.22\Omega$$

Hence, $Z$-parameter matrix

$$[Z] = \begin{bmatrix} -1.66 & -3.33 \\ -4.44 & -2.22 \end{bmatrix}\Omega$$

$Y$-parameter matrix, $[Y] = [Z]^{-1}$

$$[Y] = \begin{bmatrix} -1.66 & -3.33 \\ -4.44 & -2.22 \end{bmatrix}^{-1}$$

$$= \frac{\begin{bmatrix} -2.22 & 3.33 \\ 4.44 & -1.66 \end{bmatrix}}{[(1.66 \times 2.22) - (3.33 \times 4.44)]} = \frac{1}{11.133}\begin{bmatrix} -2.22 & 3.33 \\ 4.44 & -1.66 \end{bmatrix}$$

$$\therefore \quad [Y] = \begin{bmatrix} 0.2 & -0.3 \\ -0.4 & 0.15 \end{bmatrix}℧$$

**Q.7 (b) (ii) Solution:**

Given, $V_0(t) = 2e^{-t} - e^{-3t};\ t > 0$

by taking Laplace transform

$$V_0(s) = \frac{2}{s+1} - \frac{1}{s+3}$$

also given, $i_s(t) = \delta(t) \Rightarrow I_s(s) = 1$

$$V_0(s) = Z(s)I_s(s) \quad \text{...(i)}$$

$$Z(s) = \frac{V_0(s)}{I_s(s)} = \frac{2}{s+1} - \frac{1}{s+3}$$

when $i_s(t)$ is a pulse of 1A height and a duration of 2 seconds then it may written as,

$$i_s(t) = u(t) - u(t-2)$$

$$I_s(s) = \frac{1}{s} - \frac{1}{s}e^{-2s}$$

From equation (i),

$$V_0(s) = Z(s)I_s(s) = \left[\frac{2}{s+1} - \frac{1}{s+3}\right]\left[\frac{1}{s} - \frac{e^{-2s}}{s}\right]$$

$$= \frac{2}{s(s+1)} - \frac{1}{s(s+3)} - 2e^{-2s}\left[\frac{1}{s(s+1)}\right] + \frac{e^{-2s}}{s(s+3)}$$

$$V_0(s) = 2\left[\frac{1}{s} - \frac{1}{s+1}\right] - \frac{1}{3}\left[\frac{1}{s} - \frac{1}{s+3}\right] - 2e^{-2s}\left[\frac{1}{s} - \frac{1}{s+1}\right] + \frac{e^{-2s}}{3}\left[\frac{1}{s} - \frac{1}{s+3}\right]$$

by taking inverse Laplace transform,

$$V_0(t) = 2[u(t) - e^{-t}u(t)] - \frac{1}{3}[u(t) - e^{-3t}u(t)] - 2[u(t-2) - e^{-(t-2)}u(t-2)] + \frac{1}{3}[u(t-2) - e^{-3(t-2)}u(t-2)]$$

for $t > 0$.

**Q.7 (c) (i) Solution:**

We find out the force on charge at $A$ due to the other two charges situated at point $B$ and point $C$. Force on charge at $A$ due charge at $B$ is:

$$\vec{F}_{BA} = \frac{Q_1Q_2}{4\pi\varepsilon r^2} = \frac{q^2}{4\pi\varepsilon d^2}$$

Force on charge at $A$ due to charge at $C$ is

$$\vec{F}_{CA} = \frac{q^2}{4\pi\varepsilon d^2}$$

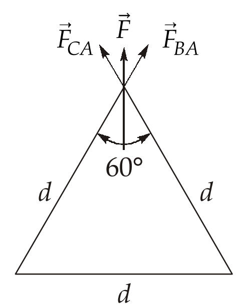
It is observed that the horizontal components of the total force cancel out. Hence, the total force acts along the upward direction and its magnitude is given as

$$F = F_{BA} \cos 30° + F_{CA} \cos 30°$$

$$= 2\times\frac{q^2}{4\pi\varepsilon d^2}\times\cos 30°$$

$$= 2\times\frac{q^2}{4\pi\varepsilon d^2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}q^2}{4\pi\varepsilon d^2}$$

$$F = \frac{\sqrt{3}q^2}{4\pi\varepsilon d^2}$$




**Q.7 (c) (ii) Solution:**

$$\rho_V = \rho_0 e^{-(\sigma/\varepsilon)t} = \rho_0 e^{-t/\lambda}$$

At $t = \frac{\varepsilon}{\sigma}$, $\rho_V = \rho_0 e^{-1} = \frac{1}{e}\rho_0$

This shows that if at any instant a charge density ρ existed within a conductor, it would decrease to $\frac{1}{e}$ times this value in a time $\frac{\varepsilon}{\sigma}$ second

For copper, $\sigma = 5.8 \times 10^7$ mho/m and $\varepsilon_r = 1$

the value of this time is $\tau = \frac{\varepsilon}{\sigma} = \frac{\varepsilon_0\varepsilon_r}{\sigma} = \frac{8.854\times 10^{-12}\times 1}{5.8\times 10^7}$

$$= 1.53 \times 10^{-19} \text{ second}$$

**Q.8 (a) (i) Solution:**

Full scale deflection current, $i = \frac{100}{20}\times 10^{-3} = 5\times 10^{-3}\,\text{A}$

Total deflecting torque exerted on the coil.

$$T_d = BilNb \text{ (N-m)}$$

where, $i = 5 \times 10^{-3}$ A, $l$ = 30 mm, $b$ = 25 mm, $N$ = 100

$$T_d = B \times 5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100$$

The control torque of the spring is

$$T_C = k_s \times \theta$$

$$= 0.375 \times 10^{-6} \times 120$$

At equilibrium, $T_d = T_C$

$$B \times 5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100 = 0.375 \times 10^{-6} \times 120$$

$$B = \frac{0.375 \times 10^{-6} \times 120}{5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100} = 0.12 \text{ wb/m}^2$$

Coil winding resistance = 20 × 0.3 = 6 Ω

If the copper wire has a cross-sectional area of '$a$' m then

$$R = N\frac{\rho l}{A}$$

$$6 = 100 \times 1.7 \times 10^{-8} \times \frac{2 \times (30+25) \times 10^{-3}}{A}$$

$$A = 31.16 \times 10^{-3} \text{ mm}^2$$

If $d$ be the diameter of the copper wire then area, $A = \frac{\pi d^2}{4}$,

$$d = \sqrt{\frac{4 \times A}{\pi}} = \sqrt{\frac{4 \times 31.16 \times 10^{-3}}{\pi}} = 0.199 \text{ mm}$$

**Q.8 (a) (ii) Solution:**

The voltage drop per cm length of potentiometer wire,

$$V = \frac{1.0186}{55} = 0.01852$$

**1.** The emf of a cell balanced at 70 cm,

$$= V.l = 0.01852 \times 70 = 1.2964 \text{ volt.}$$

**2.** The potential difference which is balanced at 60 cm,

$$= V.l = 0.01852 \times 60 = 1.1112 \text{ volt}$$

Magnitude of the standard resistor, $R_s$ = 2 Ω.

Therefore, current flowing through 2 Ω resistance

$$I = \frac{V}{R_s} = \frac{1.1112}{2} = 0.5556 \text{ A}$$

**3.** The potential difference which balances at 85 cm.

$$V' = V.l = 0.01852 \times 85 = 1.5742 \text{ V}$$

$$\text{Voltage of supply main} = V' \times \text{ratio of volt-ratio box}$$

$$= 1.5742 \times 100 = 157.42 \text{ V}$$

**4.** The potential difference which balances at 80 cm

$$V' = V.l = 0.01852 \times 80 = 1.4816 \text{ volt}$$

$$\text{Voltmeter reading} = 1.40 \text{ volt}$$

percentage error in voltmeter reading,

$$= \frac{1.4 - 1.4816}{1.4816} \times 100 = -5.507\%$$

**5.** The potential difference which balances at 45 cm,

$$V' = V.l = 0.01852 \times 45 = 0.8334 \text{ volt}$$

Current flowing through 2.5 Ω resistance, $V' = 0.8334$ V is

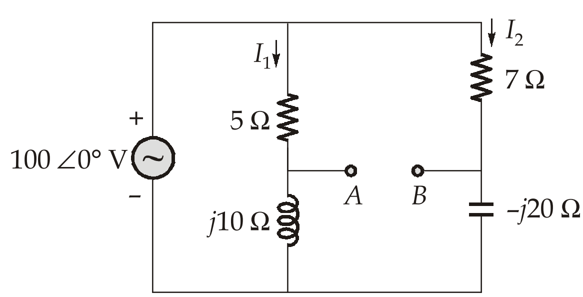
$$I' = \frac{V'}{R_s} = \frac{0.8334}{2.5} = 0.33336 \text{ A}$$

Percentage error in ammeter reading

$$= \frac{0.35 - 0.33336}{0.33336} \times 100 = 4.991\%$$

**Q.8 (b) (i) Solution:**

For calculation of Thevenin's voltage ($V_{th}$), remove load impedance '$Z_L$' across terminals $A$ and $B$.




$$I_1 = \frac{100\angle 0°}{5 + j10} = \frac{100\angle 0°}{11.18\angle 63.43°} = 8.94\angle -63.43° A$$

$$I_2 = \frac{100\angle 0°}{7 - j20} = \frac{100\angle 0°}{21.19\angle -70.7°} = 4.72\angle 70.7° A$$

$$\text{Thevenin's voltage, } V_{th} = V_A - V_B = [(8.94\angle -63.43°)(j10)] - [(4.72\angle 70.7°)(-j20)]$$

$$= 89.4\angle 26.57° - 94.4\angle -19.3°$$

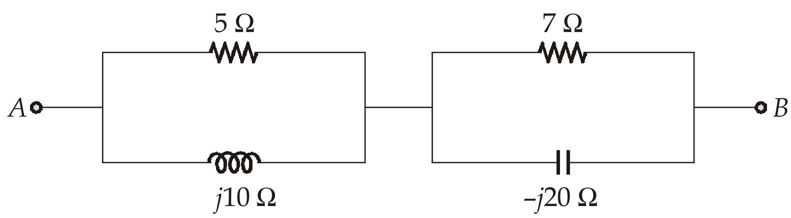
$$= 79.96 + j39.98 - 89.09 + j31.2$$

$$= -9.13 + j71.18$$

$$\therefore \quad V_{th} = 71.76\angle 97.3° \text{ V}$$

for calculation of $Z_{th}$:

The impedance seen from the terminals $A$ and $B$ with short-circuiting voltage source is shown below:




$$Z_{th} = \frac{5(j10)}{5+j10} + \frac{7(-j20)}{7-j20}$$

$$= \frac{50\angle 90°}{11.18\angle 63.43°} + \frac{140\angle -90°}{21.19\angle -70.7°}$$

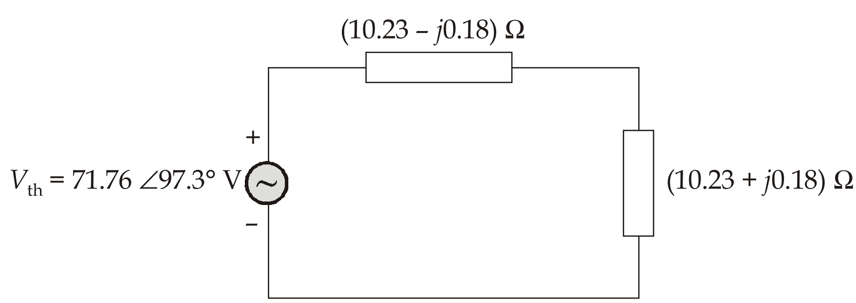
$$= 4.47\angle 26.57° + 6.6\angle -19.3°$$

$$= 4 + j2 + 6.23 - j2.18$$

$$Z_{th} = 10.23 - j0.18\ \Omega$$

For maximum power transfer, the load impedance should be complex conjugate of the source impedance.

$$\therefore \quad Z_L = Z_{th}^* = 10.23 + j0.18\ \Omega$$




∴ maximum power transferred to load:

$$P_{max} = I_L^2 R_L$$

where,

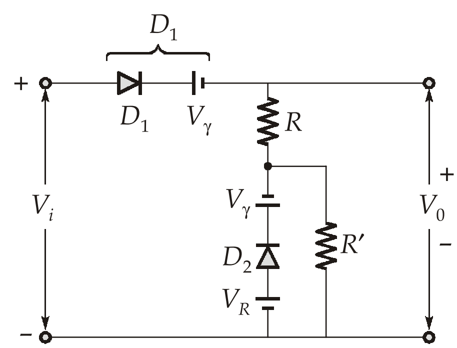
$$I_L = \frac{71.76\angle 97.3°}{10.23 - j0.18 + 10.23 + j0.18} = \frac{71.76\angle 97.3°}{20.46}$$

$$I_L = 3.51 \angle 97.3° \text{A}$$

$$P_{max} = I_L^2 R_L = (3.51)^2 \times 10.23 \simeq 126 \text{ W}$$

**Q.8 (b) (ii) Solution:**

**1.**




If $V_i \leq V_R$ then $D_1$ OFF and $D_2$ ON hence diode $D_1$ is open and diode $D_2$ is short circuit.

$$\therefore \quad V_0 = V_R$$

If $V_i > V_R$ then $D_1$ ON and $D_2$ is:

**Case-1:**

$$V_x = V_i \times \frac{R'}{R + R'} < V_R ; D_2 = 01$$

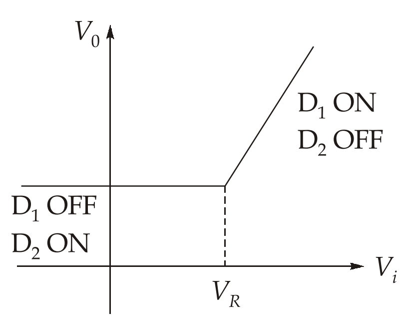
**Case-2:**

$$V_X > V_R$$

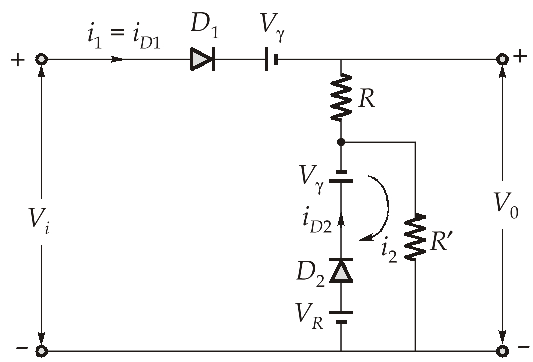
$$D_2 = \text{OFF}$$

In both cases, $V_0 = V_i$

Therefore, the transfer curve,



**2.** Assume both diodes are in ON state i.e., conducting.




$$V_i = Ri_1 + V_R + V_\gamma - V_\gamma \quad \text{...(i)}$$

and $$V_R - V_\gamma = R'i_2$$

But also, $$i_1 = i_{D1} = \frac{V_i - V_R}{R} \text{ from equation (i)}$$

$$i_{D2} = i_2 - i_1$$

$$i_{D2} = \frac{V_R - V_\gamma}{R'} - \frac{V_i - V_R}{R}$$

The current $i_{D2}$ becomes (–ve), when,

$$V_i \geq V_R + \frac{R}{R'}(V_R - V_\gamma)$$

Then $D_1$ is ON and $D_2$ is OFF.

Therefore, the maximum value of the input voltage $V_i$, so that the current in $D_2$ is always in the forward direction,

i.e., $$V_{i,\,\max} = V_R + \frac{R}{R'}\left(V_R - V_\gamma\right)$$

**Q.8 (c) (i) Solution:**

Given that series resistance,

$$R_S = 5\text{ k}\Omega = 5000\ \Omega$$

Series inductance, $$L_S = 0.8\text{ H}$$

Series capacitance, $$C_S = 0.08\text{ pF} = 8 \times 10^{-14}\text{ F}$$

Parallel capacitance, $$C_P = 1.0\text{ pF} = 1 \times 10^{-12}\text{ F}$$

We know that,

Series resonant frequency, $$f_s = \frac{L}{2\pi\sqrt{L_S C_S}} = \frac{1}{2\pi\sqrt{0.8 \times 8 \times 10^{-14}}} = 629\text{ kHz}$$

Also, parallel resonant frequency,

$$f_p = \frac{1}{2\pi}\sqrt{\frac{1+\frac{C_S}{C_P}}{L_S C_S}} = \frac{1}{2\pi}\sqrt{\frac{1+\frac{8\times10^{-14}}{1\times10^{-12}}}{0.8\times8\times10^{-14}}} = 654 \text{ kHz}$$

**Q.8 (c) (ii) Solution:**

Given,

$$R_C = 4.7 \text{ k}\Omega, \qquad R_E = 3.3 \text{ k}\Omega$$

$$V_{CC} = 12 \text{ V}, \qquad V_{EE} = -12 \text{ V}$$

We have,

$$I_E = \frac{V_{EE} - V_{BE}}{2R_E} = \frac{12 - 0.7}{2\times3.3\times10^3} = 1.712 \text{ mA}$$

Now, $I_C = I_E = 1.712$ mA

Therefore, $I_{CQ} = 1.712$ mA

Also we write,

$$V_{CEQ} = V_{CC} + V_{BE} - I_{CQ}R_C$$

$$= 12 + 0.7 - 1.712 \times 10^{-3} = 4.653 \text{ V}$$

Therefore $Q$ points is (1.712 mA, 4.653 V)

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