MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

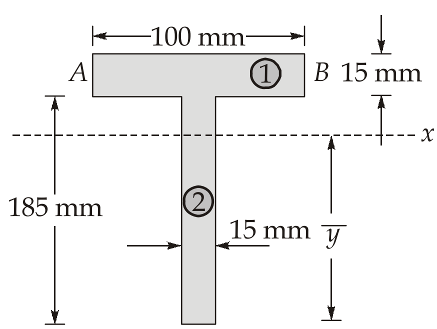
ESE-2024 Mains Test Series

Mechanical Engineering Test No : 15

Full Syllabus Test (Paper-2)

Section : A

1. (a)



Centroid of the cross-section is given by

$$\bar{y} = \frac{100 \times 15 \times 192.5 + 185 \times 15 \times 92.5}{100 \times 15 + 185 \times 15}$$

$$\bar{y} = 127.58 \text{ mm}$$

Now, from figure,

$$I_1 = \frac{100 \times 15^3}{12} + 100 \times 15 \times (64.92)^2 = 6350034.6 \text{ mm}^4$$

$$I_2 = \frac{15 \times 185^3}{12} + 15 \times 185 \times 35.08^2 = 11329464.01 \text{ mm}^4$$

$$\therefore \quad I_{xx} = I_1 + I_2$$
$$= 1767.95 \times 10^4 \text{ mm}^4$$

At the junction of the web,

$$A\bar{y} = 100 \times 15 \times 64.92 = 97380 \text{ mm}^3$$

$$\therefore \quad \tau_F = \frac{FA\bar{y}}{Ib} = \frac{250 \times 10^3 \times 97380}{1767.95 \times 10^4 \times 100}$$

$$\therefore \quad \tau_F = 13.77 \text{ N/mm}^2$$

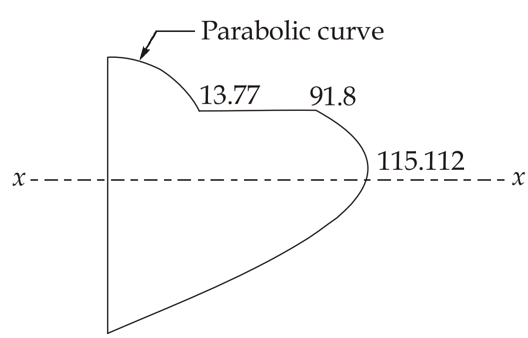
At the junction with the flange,

$$\tau_w = 13.77 \times \frac{100}{15} = 91.8 \text{ N/mm}^2$$

At the centroid, $$A\bar{y} = (A\bar{y})_F + (A\bar{y})_w$$
$$= (100 \times 15 \times 64.92) + (57.42 + 15 \times 28.71)$$
$$= 122107.923 \text{ mm}^3$$

$$\therefore \quad \tau_{max} = \tau_{NA} = \frac{250 \times 10^3 \times 122107.923}{1767.95 \times 10^4 \times 15} = 115.112 \text{ N/mm}^2$$ **Ans.**

The variation of shear stress is shown in figure below,



1. (b)

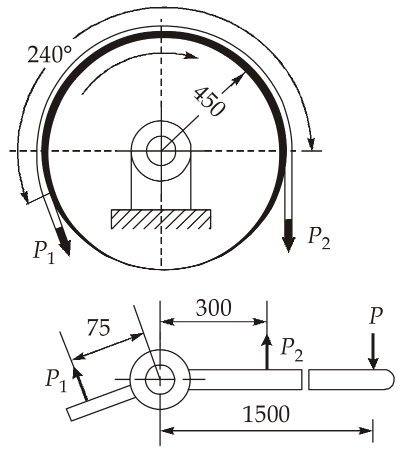
Given : $M_t = 3600$ Nm; $t = 3.5$ mm; $\mu = 0.3$; $\sigma_t = 60$ MPa; $R = 450$ mm; $\theta = 240°$

Width of band; $$M_t = (P_1 - P_2)R$$

$$\Rightarrow \quad 3600 = (P_1 - P_2) \times 0.45$$

$$\Rightarrow \quad P_1 - P_2 = 8000 \quad \text{...(i)}$$

Also, $$\frac{P_1}{P_2} = e^{\mu\theta}$$



$$\Rightarrow \quad \frac{P_1}{P_2} = e^{\left(0.3 \times 240° \times \frac{\pi}{180°}\right)}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = 3.5136 \quad \text{...(ii)}$$

On solving equation (i) and (ii), we get

$$P_1 = 11182.69 \text{ N and } P_2 = 3182.69 \text{ N}$$

The maximum tension in the band is P_1

$$P_1 = \sigma_t wt$$

$$\Rightarrow \quad 11182.69 = 60 \times w \times 3.5$$

$$\Rightarrow \quad w = 53.25 \text{ mm}$$ **Ans.**

Hence, width of the band is 53.25 mm.

Actuating force;

The free body diagram of forces acting on the band and the actuating lever is shown in the figure.

Taking moment of force about the fulcrum:

$$P_1 \times 75 + P \times 1500 - P_2 \times 300 = 0$$

$$\Rightarrow \quad 11182.69 \times 75 + P \times 1500 - 3182.69 \times 300 = 0$$

$$\Rightarrow \quad P = 77.40 \text{ N}$$ **Ans.**

Hence, the actuating force is 77.40 N.

Self-locking property;

Since $\frac{a}{b} = \frac{300}{75} = 4$ and $e^{\mu\theta} = 3.5136$

$$\therefore \quad \frac{a}{b} > e^{\mu\theta}$$

The brake is not self-locking. **Ans.**

1. (c)

$$m = 8 \text{ kg}; r = \frac{460}{2} = 230 \text{ mm}$$

$$F_1 = 1700 \text{ N at } r_1 = \frac{560}{2} = 280 \text{ mm}$$

$$F_2 = 800 \text{ N at } r_2 = \frac{320}{2} = 160 \text{ mm}$$

(i) The controlling force curve of a spring-controlling governor is a straight line and thus can be expressed as

$$F = ar - b$$

at (F_1, r_1): $\quad 1700 = a(0.280) - b \quad$ ...(i)

at (F_2, r_2): $\quad 800 = a(0.160) - b \quad$ ...(ii)

On solving equation (i) and (ii)

$$a = 7500 \text{ and } b = 400$$

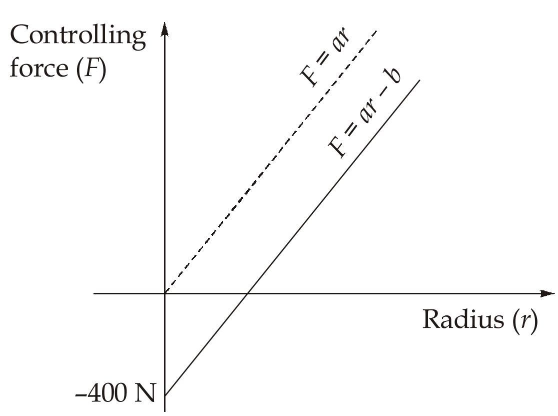
Now, $\quad F = mr\omega^2 = ar - b$

$$\Rightarrow \quad 8 \times (0.230) \times \omega^2 = 7500 \times (0.230) - 400$$

$$\Rightarrow \quad \omega = 26.8348 \text{ rad/s}$$

$$N = \omega \times \frac{60}{2\pi} = 26.8348 \times \frac{60}{2\pi} = 256.25 \text{ rpm}$$ **Ans.**

(ii) To make the governor isochronous, the controlling force line must pass through the origin, i.e., b is to be zero. This is possible only if the initial tension is increased by 400 N.



(iii) For isochronous governor, $b = 0$

$$F = mr\omega^2 = ar + b$$

$$8 \times r \times \omega^2 = 7500r + 0$$

$$\Rightarrow \quad 8\omega^2 = 7500$$

$$\Rightarrow \quad \omega = 30.6186 \text{ rad/s}$$

$$\Rightarrow \quad N = \omega \times \frac{60}{2\pi} = 30.6186 \times \frac{60}{2\pi} = 292.39 \text{ rpm}$$ **Ans.**

1. (d)

Given : $P = 60$ kN ; $\tau = 80$ N/mm^2

Primary shear stress:

The total area of two vertical welds is given by,

$$A = 2(300t) = (600t) \text{ mm}^2$$

The primary shear stress in the weld is given by,

$$\tau_1 = \frac{P}{A} = \frac{60 \times 10^3}{600t} = \frac{100}{t} \text{ N/mm}^2$$

Bending stress:

The moment of inertia of two welds about x-axis is given by,

$$I = 2\left[\frac{t(300)^3}{12}\right] = (4.5 \times 10^6 t) \text{ mm}^4$$

$$M_b = (60 \times 10^3) \times 250 = 1.5 \times 10^7 \text{ N-mm}$$

$$y = \frac{300}{2} = 150 \text{ mm}$$

$$\sigma_b = \frac{M_b y}{I} = \frac{(1.5 \times 10^7) \times 150}{4.5 \times 10^6 t} = \frac{500}{t} \text{N/mm}^2$$

Maximum shear stress:

The maximum principal shear stress in the weld is given by

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2}$$

$$= \sqrt{\left(\frac{500}{2t}\right)^2 + \left(\frac{100}{t}\right)^2} = \left(\frac{269.258}{t}\right) \text{ N/mm}^2$$

Size of weld:

The permissible shear stress in the weld is 80 N/mm². Therefore,

$$\left(\frac{269.258}{t}\right) = 80$$

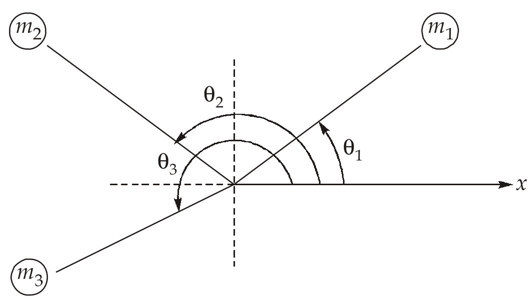
$$\Rightarrow \quad t = 3.366 \text{ mm}$$

$$h = t\sqrt{2} = 3.366 \times \sqrt{2}$$

$$= 4.76 \text{ mm}$$ **Ans.**

1. (e)

$m_1 = 5$ kg; $m_2 = 4$ kg; $m_3 = 2$ kg; $r_1 = 60$ mm; $r_2 = 90$ mm; $r_3 = 80$ mm; $\theta_1 = 45°$; $\theta_2 = 135°$; $\theta_3 = 225°$



$$r_b = 70 \text{ mm}$$

For static balance:

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3$$

$$= 5 \times 60 \times \cos(45°) + 4 \times 90 \times \cos(135°) + 2 \times 80 \times \cos(225°)$$

$$= -155.56 \text{ kg-mm}$$

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3$$

$$= 5 \times 60 \times \sin(45°) + 4 \times 90 \times \sin(135°) + 2 \times 80 \times \sin(225°)$$

$$= 353.55 \text{ kg-mm}$$

For static equilibrium,

$$m_c r_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$\Rightarrow \quad m_c \times 70 = \sqrt{(-155.56)^2 + (353.55)^2}$$

$$\Rightarrow \quad m_c = 5.518 \text{ kg}$$ **Ans.**

Angular position,

$$\tan\theta_c = \frac{-\Sigma V}{-\Sigma H} = \frac{-(353.55)}{-(-155.56)} = \frac{-353.55}{+155.56} = -2.2728$$

$$\Rightarrow \quad \theta_c = 293.75°$$ **Ans.**

θ_c is measured counter-clockwise from the reference line along the x-axis.

2. (a) (i)

$$F = 480 \text{ N}$$

$D(0, 500, 300)$, $B(600, 0, 700)$

$$\overrightarrow{DB} = (600-0)\hat{i} + (0-500)\hat{j} + (700-300)\hat{k}$$

$$= 600\hat{i} - 500\hat{j} + 400\hat{k}$$

$$\left|\overrightarrow{DB}\right| = \sqrt{(600)^2 + (-500)^2 + (400)^2} = 877.496 \text{ mm}$$

$$\vec{F}_D = F(\widehat{DB}) = F\frac{\overrightarrow{DB}}{\left|\overrightarrow{DB}\right|}$$

$$= \frac{480 \text{ N}}{877.496 \text{ mm}}\left[(600 \text{ mm})\hat{i} - (500 \text{ mm})\hat{j} + (400 \text{ mm})\hat{k}\right]$$

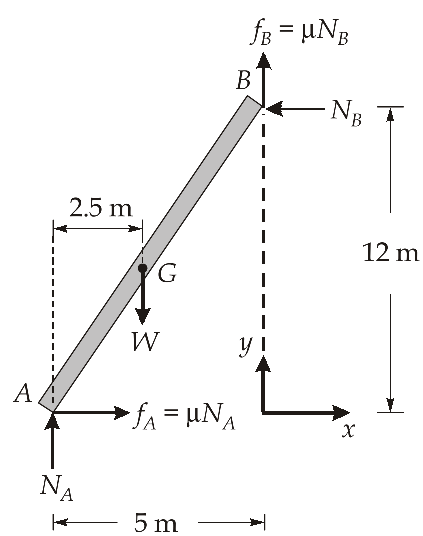
$$= (328.207 \text{ N})\hat{i} - (273.506 \text{ N})\hat{j} + (218.804 \text{ N})\hat{k}$$

Component of force exerted by cable on the support at D;

$$F_x = 328.207 \text{ N};\ F_y = -273.506 \text{ N};\ F_z = 218.804 \text{ N}$$ **Ans.**

2. (a) (ii)

Refer figure,



Frictional force,

at A; $f_A = \mu N_A$

at B; $f_B = \mu N_B$

Under static equilibrium;

$\Sigma M_A = 0$ (+ve clockwise)

$$W(2.5) - \mu N_B(5) - N_B(12) = 0$$

$$\Rightarrow \quad N_B = \frac{2.5W}{12+5\mu} \quad \text{...(i)}$$

$\Sigma F_y = 0;$

$$N_A + \mu N_B - W = 0$$

$$\Rightarrow \quad N_A = W - \mu N_B = W - \frac{2.5W\mu}{12+5\mu} \quad \text{...(ii)}$$

$\Sigma F_x = 0; \quad \mu N_A - N_B = 0$ ...(iii)

Substituting N_A and N_B from equation (ii) and (i) is equation (iii)

$$W\mu - \frac{2.5W\mu^2}{12+5\mu} - \frac{2.5W}{12+5\mu} = 0$$

$\Rightarrow 12W\mu + 5W\mu^2 - 2.5W\mu^2 - 2.5W = 0$

$$\Rightarrow \quad 2.5\mu^2 + 12\mu - 2.5 = 0$$

$$\Rightarrow \quad \mu = 0.2 \text{ and } \mu = -5 \text{ (Discarded)}$$

μ cannot be negative, so $\mu = 0.2$ **Ans.**

The minimum value of coefficient of static friction for which equilibrium is maintained is 0.2.

2. (b) (i)

1. **Rolling contact bearing :** There are two basic types of bearing failure breakage of parts like races or cage and the surface destruction.

   In general, the failure of antifriction bearing occurs not due to breakage of parts but due to damage of working surfaces of their parts. The principal types of surface wear are as follows:

   (i) Abrasive wear : Abrasive wear occurs when the bearing is made to operate in an environment contaminated with dust, foreign particles, rust, or spatter. Remedies against this type of wear are provision of oil seals, increasing surface hardness, and use of high viscosity oils. Thick lubricating film developed by these oils allows fine particles to pass without scratching.

   (ii) Corrosive wear : The corrosion of the surfaces of bearing parts is caused by the entry of water or moisture in the bearing. It is also caused due to corrosive elements present in the extreme pressure (EP) additives that are added in the lubricating oils. These elements attack the surfaces of the bearing, resulting in fine wear uniformly distributed over the entire surface. Remedies against this type of wear are, providing complete enclosure for the bearing free from external contamination, selecting proper additives, and replacing the lubricating oil at regular intervals.

(iii) Pitting : Pitting is the main cause of the failure of antifriction bearings. Pitting is a surface fatigue failure which occurs when the load on the bearing part exceeds the surface endurance strength of the material. This type of failure is characterized by pits, which continue to grow resulting in complete destruction of the bearing surfaces. Pitting depends upon the magnitude of Hertz' contact stress and the number of stress cycles. The surface endurance strength can be improved by increasing the surface hardness.

(iv) Scoring : Excessive surface pressure, high surface speed, and inadequate supply of lubricant result in breakdown of the lubricant film. This results in excessive frictional heat and overheating at the contacting surfaces. Scoring is a stick-slip phenomenon, in which alternate welding and shearing takes place rapidly at high spots. Here the rate of wear is faster. Scoring can be avoided by selecting the parameters, such as surface speed, surface pressure and the flow of lubricant in such a way that the resulting temperature at the contacting surfaces is within permissible limits.

2. Sliding contact bearing : Fatigue failures are not common in journal bearings unlike ball bearings. The failures in journal bearings are mainly associated with insufficient lubricant, contamination of lubricant, and faulty assembly. The principal types of bearing failure are as follows:

(i) Abrasive wear: Abrasive wear on the surface of the bearing is a common type of bearing failure. It is in the form of scratches in the direction of motion often with embedded particles. Abrasive wear occurs when the lubricating oil is contaminated with dust, foreign particles, rust, or spatter. Proper enclosures for the bearing and the housing, cleanliness of lubricating oil and use of high viscosity oil are some of the remedies against this type of wear.

(ii) Wiping of bearing surface: When the rotating journal touches the bearing, excessive rubbing occurs resulting in melting and smearing of the surface of the bearing. This type of failure is in the form of surface melting and flow of bearing material. The main causes for this type of wear are inadequate clearance, excessive transient load and insufficient oil supply. The remedy is to keep these factors under control.

(iii) Corrosion : The corrosion of bearing surface is caused by the chemical attack of reactive agents that are present in the lubricating oil. These oxidation products corrode materials such as lead, copper, cadmium, and zinc. Lead reacts rapidly with all oxidation agents. The remedy is to use oxidation inhibitors as additive in the lubricating oil.

(iv) Distortion : Misalignment and incorrect type of fit are the major sources of difficulties in journal bearings. When the fit is too tight, bore distortion occurs. When foreign particles are trapped between the bearing and the housing during the assembly, local bore distortion occurs. Correct selection of fit and proper assembly procedure is the remedy against this type of wear.

2. (b) (ii)

$$\text{Power} = 15 \text{ kW};\ N_1 = 1800 \text{ rpm}$$

For input shaft, $m_1 = 18$ kg; $k_1 = 80$ mm

For output shaft, $m_2 = 32$ kg; $k_2 = 140$ mm

$$I_1 = m_1 k_1^2 = 18 \times (0.080)^2 = 0.1152 \text{ kg-m}^2$$

$$I_2 = m_2 k_2^2 = 32 \times (0.140)^2 = 0.6272 \text{ kg-m}^2$$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 1800}{60} = 188.496 \text{ rad/s}$$

$$\omega_2 = 0$$

$$\text{Power} = \text{Torque} \times \text{Angular speed}$$

$$(15 \times 10^3) = M_t \times 188.496$$

$$\Rightarrow \quad M_t = 79.577 \text{ N-m}$$

The time required to bring the output shaft to the rated speed from rest is given as

$$t_1 = \frac{(\omega_1 - \omega_2) I_1 I_2}{(I_1 + I_2) M_t}$$

$$= \frac{(188.496 - 0) \times (0.1152) \times (0.6272)}{(0.1152 + 0.6272) \times (79.577)}$$

$$= 0.23053 \text{s}$$ **Ans.**

2. (c) (i)

Vibration Isolation and Transmissibility

Vibration isolation is a procedure by which the undesirable effects of vibration are reduced. Basically, it involves the insertion of a resilient member (or isolator) between the vibrating mass (or equipment or payload) and the source of vibration so that a reduction in the dynamic response of the system is achieved under specified conditions of vibration excitation.

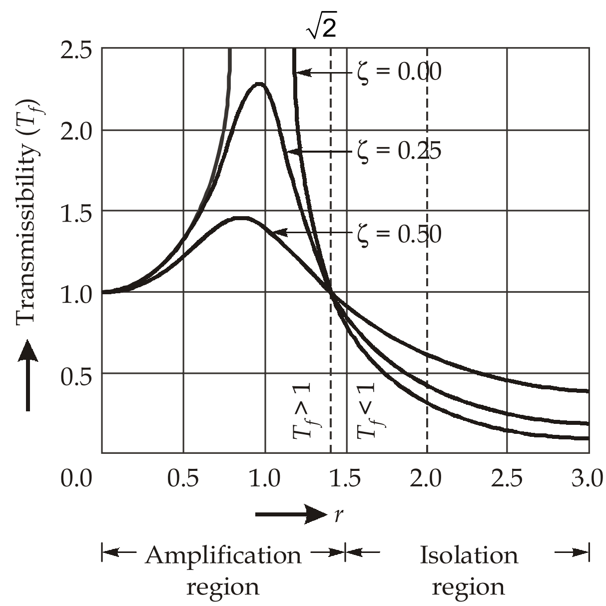


Fig. Variation of Transmissibility Ratio with frequency ratio

The transmissibility or transmission ratio of the isolator (TR or T_f) is defined as the ratio of the magnitude of the force transmitted to that of the exciting force. As the transmitted force is the vector sum of the spring force (kX) and the damping force ($c\omega X$) which are at perpendicular to each other.

$$F_T = \sqrt{(kX)^2 + (c\omega X)^2} = X\sqrt{k^2 + (c\omega)^2}$$

$$F_T = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}\sqrt{k^2 + (c\omega)^2}$$

$$F_T = \frac{F_0\sqrt{1 + \left(\frac{c}{k}\omega\right)^2}}{\sqrt{\left(1 - \frac{m}{k}\omega^2\right)^2 + \left(\frac{c\omega}{k}\right)^2}}$$

Transmissibility,
$$T_f = \frac{F_T}{F_0} = \left\{\frac{k^2 + \omega^2 c^2}{(k - m\omega^2)^2 + \omega^2 c^2}\right\}^{1/2}$$

$$= \left\{\frac{1 + (2\zeta r)^2}{\left[1 - r^2\right]^2 + (2\zeta r)^2}\right\}^{1/2} \quad ...(1)$$

The variation of T_f with the frequency ratio $r = \frac{\omega}{\omega_n}$ is shown in above figure. In order to achieve isolation, the force transmitted to the foundation needs to be less than the excitation force. It can be seen from figure, that the forcing frequency has to be greater than $\sqrt{2}$ times the natural frequency of the system in order to achieve isolation of vibration.

2. (c) (ii)

Given : $s = 7000$ N/m; $F = 600$ N; $\dot{x} = 5$ m/s; $m_0 = 0.6$ kg; m = 25 kg; $N = 450$ rpm; $e = 6$ cm

$$\text{Damping coefficient, } C = \frac{F}{\dot{x}} = \frac{600}{5} = 120 \text{ N-s/m}$$

$$\text{Angular speed, } \omega = \frac{2\pi N}{60} = \frac{2\pi \times 450}{60} = 47.124 \text{ rad/s}$$

Critical damping coefficient,

$$C_C = 2\sqrt{sm} = 2 \times \sqrt{7000 \times 25} = 836.66 \text{ N-s/m}$$

$$\text{Frequency at resonance, } \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{7000}{25}} = 16.733 \text{ rad/s}$$

1. $$\text{damping factor, } \xi = \frac{C}{C_C} = \frac{120}{836.66} = 0.1434$$ **Ans.**

2. $$\text{Amplitude, } A = \frac{\frac{m_0 e}{m}\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{\left(\frac{0.6 \times 0.06}{25}\right) \times \left(\frac{47.124}{16.733}\right)^2}{\sqrt{\left[1-\left(\frac{47.124}{16.733}\right)^2\right]^2 + \left[2 \times 0.1434 \times \frac{47.124}{16.733}\right]^2}}$$

$$= 1.6367 \times 10^{-3} \text{ m}$$

$$= 1.6367 \text{ mm} = 0.16367 \text{ cm}$$ **Ans.**

$$\text{Phase angle, } \tan\phi = \frac{2\left(\frac{\omega}{\omega_n}\right)\xi}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \frac{2 \times \left(\frac{47.124}{16.733}\right) \times 0.1434}{1-\left(\frac{47.124}{16.733}\right)^2} = -0.11653$$

$$\Rightarrow \quad \phi = 173.35°$$ **Ans.**

3. Resonant amplitude, $A_{reso} = \dfrac{m_0 e}{2\xi m} = \dfrac{0.6 \times 0.06}{2 \times 0.1434 \times 25}$

$= 5.0209 \times 10^{-3}$ m = 5.0209 mm **Ans.**

4. The force because of dashpot on the motor

$$F_c = c\omega A = 120 \times (47.124) \times (1.6367 \times 10^{-3}) = 9.255 \text{ N}$$

The force because of spring on the motor

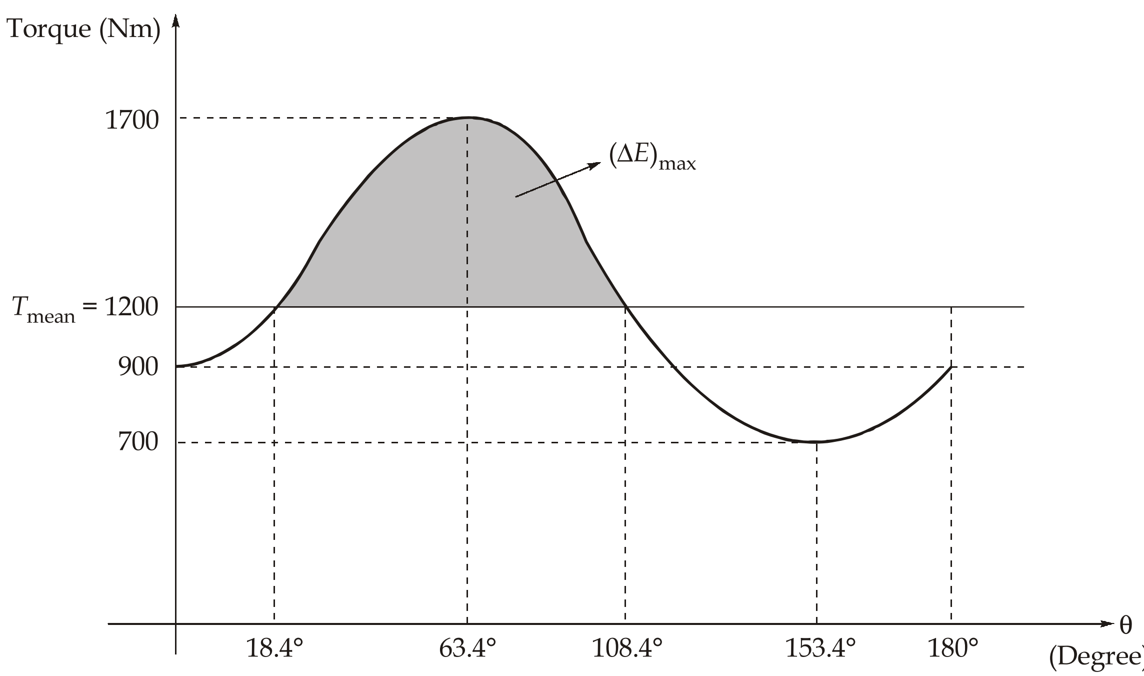
$$F_s = kA = 7000 \times (1.6367 \times 10^{-3}) = 11.457 \text{ N}$$

Resultant force on the motor, $F_R = \sqrt{F_c^2 + F_s^2} = \sqrt{(9.255)^2 + (11.457)^2}$

$= 14.728$ N **Ans.**

3. (a)

For the expression for torque being a function of 2θ, the cycle is repeated every 180° of the crank rotation.



(i) $$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta = \frac{1}{\pi}\int_0^{\pi} (1200 + 400\sin(2\theta) - 300\cos(2\theta)\, d\theta$$

$= 1200$ N-m

Power, $P = T\omega$

$$= 1200 \times \left(\frac{2\pi \times 270}{60}\right) = 33929.2 \text{ N} = 33.9292 \text{ kN}$$ **Ans.**

(ii) At any instant,

$$\Delta T = T - T_{mean}$$
$$= (1200 + 400 \sin(2\theta) - 300 \cos(2\theta)) - 1200$$
$$= 400 \sin(2\theta) - 300 \cos(2\theta)$$

For,

$$\Delta T = 0$$
$$400 \sin(2\theta) - 300 \cos(2\theta) = 0$$
$$\Rightarrow \tan(2\theta) = \frac{300}{400}$$
$$\Rightarrow 2\theta = 36.8° \quad \text{or} \quad 216.8°$$
$$\Rightarrow \theta = 18.4° \quad \text{or} \quad 108.4°$$
$$(\Delta E)_{max} = \int_{18.4°}^{108.4°} \Delta T \, d\theta = \int_{18.4°}^{108.4°} (400\sin(2\theta) - 300\cos(2\theta))\, d\theta$$
$$= [-200\cos(2\theta) - 150\sin(2\theta)]_{18.4°}^{108.4°}$$
$$= 499.9996 \text{ N-m} \simeq 500 \text{ Nm}$$

Also,

$$(\Delta E)_{max} = IC_s\omega^2$$
$$\Rightarrow C_s = \frac{(\Delta E)_{max}}{I\omega^2} = \frac{500}{\left(400 \times (0.350)^2\right) \times \left(\frac{2\pi \times 270}{60}\right)^2}$$
$$\Rightarrow C_s = 0.01276 \text{ or } 1.276\%$$

(iii) Acceleration or deceleration is produced by excess or deficit torque than the mean value at any instant.

$$\Delta T = 400 \sin(2\theta) - 300 \cos(2\theta)$$

when, $\theta = 60°$,

$$\Delta T = 400 \times \sin(120°) - 300 \times \cos(120°)$$
$$= 496.41 \text{ Nm}$$

Also,

$$\Delta T = I\alpha$$
$$\Rightarrow 496.41 = (400 \times (0.350)^2) \times \alpha$$
$$\Rightarrow \alpha = 10.13 \text{ rad/s}^2$$ **Ans.**

(iv) For $(\Delta T)_{max}$ and $(\Delta T)_{min}$

$$\frac{d}{d\theta}(\Delta T) = 0$$
$$\Rightarrow \frac{d}{d\theta}(400\sin(2\theta) - 300\cos(2\theta)) = 0$$
$$\Rightarrow 800\cos(2\theta) + 600\sin(2\theta) = 0$$

$$\Rightarrow \quad \tan(2\theta) = \frac{-800}{600}$$

$$\Rightarrow \quad 2\theta = 306.8° \text{ or } 126.8°$$

$$\Rightarrow \quad \theta = 153.4° \text{ or } 63.4°$$

Now, at $\theta = 153.4°$

$$(\Delta T)_{\theta = 153.4°} = 400 \times \sin(2 \times 153.4°) - 300 \times \cos(2 \times 153.4°)$$

$$= -499.999 \text{ Nm} \simeq -500 \text{ Nm}$$

At $\theta = 63.4°$

$$(\Delta T)_{\theta = 63.4°} = 400 \times \sin(2 \times 63.4°) - 300 \times \cos(2 \times 63.4°)$$

$$= 499.999 \text{ Nm} \simeq 500 \text{ Nm}$$

Hence, $(\Delta T)_{max} = (\Delta T)_{\theta = 63.4°} = 500$ N-m

$(\Delta T)_{min} = (\Delta T)_{\theta = 153.4°} = -500$ N-m

Maximum angular acceleration,

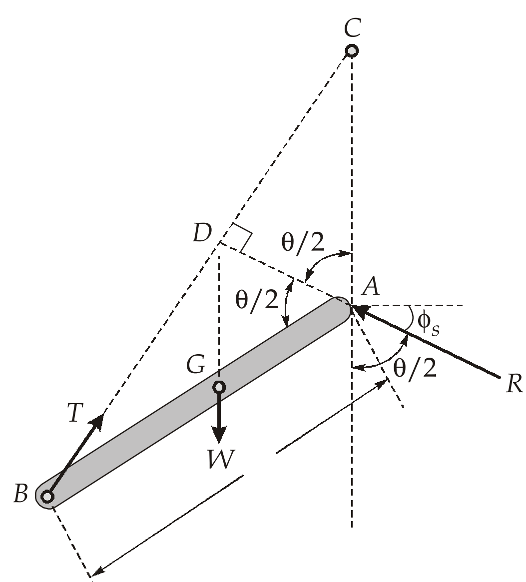
$$\alpha_{max} = \frac{(\Delta T)_{max}}{I} = \frac{500}{400 \times (0.350)^2} = 10.204 \text{ rad/s}^2$$

Minimum angular acceleration,

$$\alpha_{min} = \frac{(\Delta T)_{min}}{I} = \frac{-500}{400 \times (0.350)^2} = -10.204 \text{ rad/s}^2$$

Hence, both maximum angular acceleration and retardation is 10.204 rad/s^2.

3. (b) (i)



Free-body diagram

$$\mu_s = 0.3; \mu_k = 0.25$$

For a three force body under static equilibrium the line of action of three forces $\vec{T}, \vec{W}$ and $\vec{R}$ should intersect at a common point D as shown in the figure.

$$\text{Friction angle, } \tan\phi_s = \mu_s$$

$$\Rightarrow \quad \phi_s = \tan^{-1}(\mu_s) = \tan^{-1}(0.3) = 16.699°$$

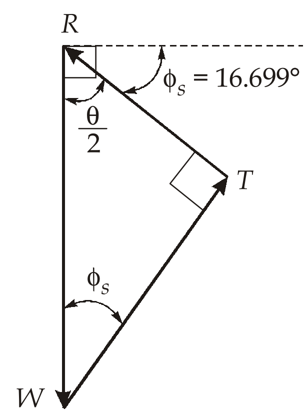
Since BG = GA, it follows that BD = DC and AD bisects ∠BAC

$$\frac{\theta}{2} + \phi_s = 90°$$

$$\Rightarrow \quad \frac{\theta}{2} + 16.699° = 90°$$

$$\Rightarrow \quad \theta = 146.602°$$ **Ans.**

From force triangle,



$$T = W\cos(\phi_s)$$

$$= 120 \times \cos(16.699°)$$

$$= 114.939 \text{ N}$$ **Ans.**

Alternatively,

From Lami's theorem,

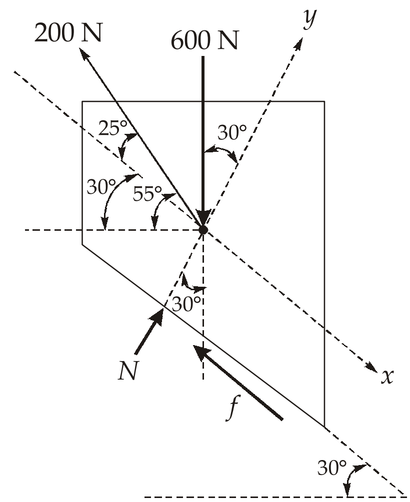
$$\frac{T}{\sin(90° + \phi_s)} = \frac{W}{\sin(90°)} = \frac{R}{\sin\left(90° + \frac{\theta}{2}\right)}$$

$$\Rightarrow \quad T = W\frac{\sin(90° + \phi_s)}{\sin(90°)} = \frac{120 \times \sin(90° + 16.699°)}{\sin(90°)}$$

$$= 114.939 \text{ N}$$ **Ans.**

Hence, the largest value of θ for which motion is impending is 146.602° and the corresponding value of tension in the string is 114.939 N.

3. (b) (ii)



Free-body diagram of the block

Assuming block to be in equilibrium,

$\Sigma F_x = 0;$

$$600 \sin(30°) - 200 \cos(25°) - f = 0$$

$$\Rightarrow \quad f = 118.738 \text{ N}$$

$\Sigma F_y = 0;$

$$-600 \cos(30°) + 200 \sin(25°) + N = 0$$

$$\Rightarrow \quad N = 435.092 \text{ N}$$

Maximum frictional force, $f_{max} = \mu_s N$

$$= 0.20 \times 484.886$$

$$= 96.977 \text{ N}$$

Since, $f > f_{max}$, block moves downwards. **Ans.**

Frictional force, $f = \mu_k N$

$$= 0.15 \times 484.886$$

$$= 72.733 \text{ N}$$ **Ans.**

3. (c)

Calculating support reactions:

Taking moment about A;

$\Sigma M_A = 0$ (+ve clockwise);

$$(120)(12)+\left(\frac{1}{2}\times30\times15\right)\left(\frac{2}{3}\times30\right)+\left(\frac{1}{2}\times12\times15\right)\left(30+\frac{12}{3}\right)-R_B\times30=0$$

$\Rightarrow \quad R_B = 300$ kN

Total force in vertical direction,

$\Sigma F_V = 0; \quad \uparrow +;$

$$R_A + R_B - 120 - \left(\frac{1}{2}\times30\times15\right)-\left(\frac{1}{2}\times12\times15\right)=0$$

$\Rightarrow \quad R_A + R_B = 435$ kN

$\Rightarrow \quad R_A = 435 \text{ kN} - R_B$

$= 435 - 300 = 135$ kN

Shear force diagram:

For portion AD : $\quad F_x = R_A - \frac{1}{2}\frac{wx}{l}x = 135 - \frac{1}{2}\times\frac{15\times x^2}{30}$

$= 135 - \frac{x^2}{4}$ (Parabolic)

$\Rightarrow \quad F_{x=0} = F_A = 135$ kN

$\Rightarrow \quad F_{x=12} = F_D = 99$ kN

For portion DB : $\quad F_x = 135 - \frac{x^2}{4} - 120 = 15 - \frac{x^2}{4}$ (Parabolic)

$\Rightarrow \quad F_{x=12} = F_D = -21$ kN

$\Rightarrow \quad F_{x=30} = F_B = -210$ kN

For Portion BC :

It is convenient to deal this portion by using variable x from the end C.

$$F_x = \frac{1}{2}\frac{wx}{l}x = \frac{1}{2}\times\frac{15\times x^2}{12} = \frac{5x^2}{8} \text{ (Parabolic)}$$

$\Rightarrow \quad F_x = F_c = 0$

$\Rightarrow \quad F_{x=12} = F_B = 90$ kN

Bending moment diagram:

For portion AD: $\quad M_x = 135x - \frac{x^2}{4}\cdot\frac{x}{3} = 135x - \frac{x^3}{12}$ (Cubic)

$M_{x=0} = M_A = 0$

$$M_{x=12} = M_D = 1476 \text{ kNm}$$

For portion DB: $$M_x = 135x - \frac{x^3}{12} - 120(x-12) \text{ (Cubic)}$$

$$M_{x=12} = M_D = 1476 \text{ kNm}$$

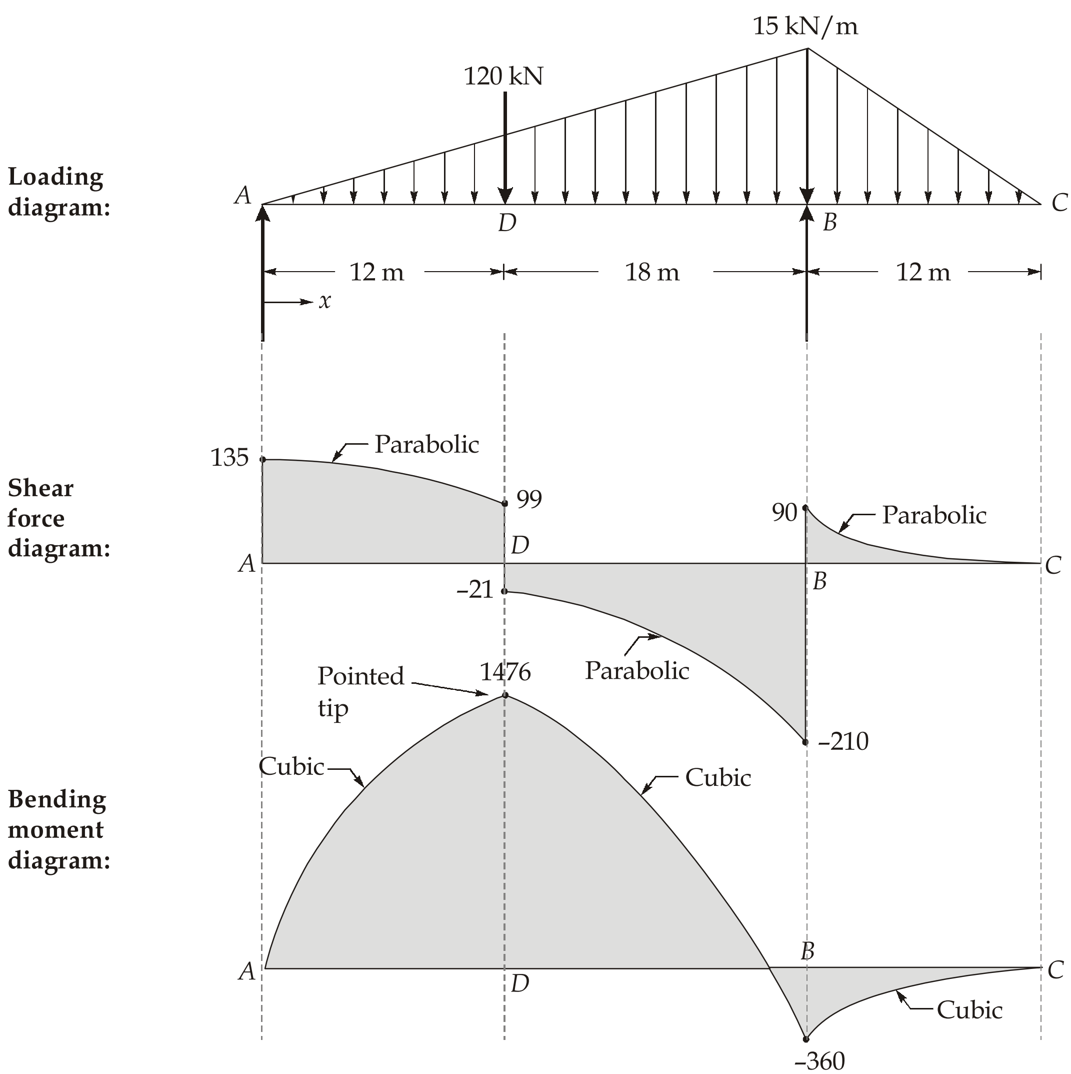
$$M_{x=30} = M_B = -360 \text{ kNm}$$

For portion BC: x from end C

$$M_x = \frac{5x^2}{8}\frac{x}{3} = \frac{-5x^3}{24} \text{ (Cubic)}$$

$$M_{x=0} = M_C = 0$$

$$M_{x=12} = M_B = -360 \text{ kNm}$$



4. (a)

Given: $(M_t)_{max}$ = 250 N-m, $(M_t)_{min}$ = 75 N-m, $(M_b)_{max}$ = 650 N-m, $(M_b)_{min}$ = 250 N-m, S_{ut} = 550 N/mm², S_{yt} = 400 N/mm², S_e = 220 N/mm², FOS = 2

Step-I : Mean and amplitude stress:

$$(M_b)_m = \frac{1}{2}\left[(M_b)_{\max} + (M_b)_{\min}\right] = \frac{1}{2}[650 + 250] = 450 \text{ N-m}$$

$$(M_b)_a = \frac{1}{2}\left[(M_b)_{\max} - (M_b)_{\min}\right] = \frac{1}{2}[650 - 250] = 200 \text{ N-m}$$

$$(M_t)_m = \frac{1}{2}\left[(M_t)_{\max} + (M_b)_{\min}\right] = \frac{1}{2}[250 + 75] = 162.5 \text{ N-m}$$

$$(M_t)_a = \frac{1}{2}\left[(M_t)_{\max} - (M_t)_{\min}\right] = \frac{1}{2}[250 - 75] = 87.5 \text{ N-m}$$

$$(\sigma_b)_m = \frac{32(M_b)_m}{\pi d^3} = \frac{32 \times 450 \times 10^3}{\pi d^3} = \frac{4583.66 \times 10^3}{d^3} \text{ N/mm}^2$$

$$(\sigma_b)_a = \frac{32(M_b)_a}{\pi d^3} = \frac{32 \times 200 \times 10^3}{\pi d^3} = \frac{2037.183 \times 10^3}{d^3} \text{ N/mm}^2$$

$$\tau_m = \frac{16(M_t)_m}{\pi d^3} = \frac{16 \times 162.5 \times 10^3}{\pi d^3} = \frac{827.605 \times 10^3}{d^3} \text{ N/mm}^2$$

$$\tau_a = \frac{16(M_t)_a}{\pi d^3} = \frac{16 \times 87.5 \times 10^3}{\pi d^3} = \frac{445.634 \times 10^3}{d^3} \text{ N/mm}^2$$

Using maximum distortion energy theory:

$$\sigma_m = \sqrt{(\sigma_b)_m^2 + 3\tau_m^2} = \sqrt{\left(\frac{4583.66 \times 10^3}{d^3}\right)^2 + 3 \times \left(\frac{827.605 \times 10^3}{d^3}\right)^2}$$

$$= \frac{4802.575 \times 10^3}{d^3} \text{ N/mm}^2$$

$$\sigma_a = \sqrt{(\sigma_b)_a^2 + 3\tau_a^2} = \sqrt{\left(\frac{2037.183 \times 10^3}{d^3}\right)^2 + 3 \times \left(\frac{445.634 \times 10^3}{d^3}\right)^2}$$

$$= \frac{2178.505 \times 10^3}{d^3} \text{ N/mm}^2$$

According to modified Goodman theory:

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} \le \frac{1}{N} \qquad \text{...(i)}$$

and $$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} \le \frac{1}{N} \quad \text{...(ii)}$$

From equation (i),

$$\frac{4802.575 \times 10^3}{d^3 \times 550} + \frac{2178.505 \times 10^3}{d^3 \times 220} \le \frac{1}{2}$$

$\therefore$ $d \ge 33.402$ mm

From equation (ii),

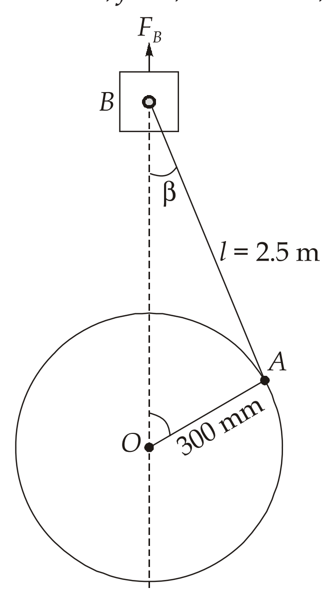
$$\frac{4802.575 \times 10^3}{d^3 \times 400} + \frac{2178.505 \times 10^3}{d^3 \times 400} \le \frac{1}{2}$$

$\therefore$ $d \ge 32.681$ mm

Hence, the diameter of the shaft should be 33.402 mm. **Ans.**

4. (b)

Given: $l = 2.5$ m, $m = 220$ kg, $a = 1000$ mm, $f = 8$, $T = 24$ sec, $r = 300$ mm, $N = 200$ rpm, $\theta = 40°$.



Mass at the crank pin, $m_a = 220 \times \left(\frac{2.5-1}{2.5}\right) = 132$ kg

Mass at the gudgeon pin, $m_b = 220 - 132 = 88$ kg

$$F_b = m_b r \omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= 88 \times 0.3 \times \left(\frac{2\pi \times 200}{60}\right)^2 \left(\cos 40° + \frac{\cos 80°}{2.5/0.3}\right)$$

$$= 9103.12 \text{ N}$$

As it is a vertical engine, the weight of the portion of the connecting rod at the piston pin also can be combined with this force i.e.

$$\text{Net force} = 9103.12 - 88 \times 9.81 = 8239.84 \text{ N}$$

$$\therefore \quad \text{Inertia torque, } T_b = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

$$T_b = 8239.84 \times 0.3\left(\sin 40^\circ + \frac{\sin 80^\circ}{2\sqrt{\left(\frac{2.5}{0.3}\right)^2 - \sin^2 40^\circ}}\right)$$

$$T_b = 8239.84 \times 0.3 \times 0.702$$

or $$T_b = 1735.44 \text{ Nm (Counter-clockwise)}$$

We have, $$b + \frac{k^2}{b} = L$$

Where, $b = 2.5 - 1 = 1.5$ m and 'L' can be found from,

$$t = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{24}{8} = 2\pi\sqrt{\frac{L}{9.81}} \text{ or } L = 2.238 \text{ m}$$

$$\therefore \quad 1.5 + \frac{k^2}{1.5} = 2.238$$

or $$k^2 = 1.108$$

or $$k = 1.052$$

or radius of gyration, $$k = 1052 \text{ mm}$$

$$\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2 - 1}{\left(n^2 - \sin^2\theta\right)^{3/2}}\right]$$

$$= -\left(\frac{2\pi \times 200}{60}\right)^2 \sin 40\left[\frac{8.33^2 - 1}{\left(8.33^2 - \sin^2 40\right)^{3/2}}\right]$$

$$= -33.62 \text{ rad/s}^2$$

$$\therefore \quad \text{Correction couple, } \Delta T = m\alpha_c b(l - L) = 220 \times (-33.62) \times 1.5 \times (2.5 - 2.238)$$

$$= -2906.78 \text{ Nm}$$

∴ Correction torque on the crankshaft,

$$T_c = \frac{\Delta T \cos\theta}{\sqrt{n^2 - \sin^2\theta}} = \frac{-2906.78\cos 40°}{\sqrt{8.33^2 - \sin^2 40}}$$

$$T_c = -268.11 \text{ Nm (Counter clockwise)}$$

Now, torque due to weight of mass at A,

$$T_a = m_a g r \sin\theta = 132 \times 9.81 \times 0.3 \times \sin 40°$$

$$= 249.7 \text{ Nm (clockwise)}$$

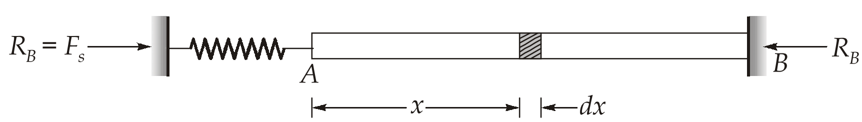
∴ Total inertia torque on crankshaft, i.e.

$$T_{net} = T_b - T_c + T_a = 1735.44 - (-268.11) - 249.7$$

$$T_{net} = 1753.85 \text{ Nm}$$

4. (c)

(i)



Consider an element dx at a distance x from end A.

$$(d\delta)_1 = \text{Elongation of element } dx \text{ due to temperature change}$$

$$\therefore \quad (d\delta)_1 = \alpha\Delta T(dx) = \alpha\Delta T_B\left(\frac{x^3}{L^3}\right)dx$$

$$\therefore \quad \delta_1 = \int_0^L (d\delta)_1 = \int_0^L \alpha\Delta T_B \frac{x^3}{L^3}dx = \frac{1}{4}\alpha(\Delta T_B)L$$

Axial deformation, $\delta_2 = \dfrac{R_B L}{AE} + \dfrac{F_S}{K}$

$$= \frac{R_B L}{AE} + \frac{R_B}{K} \qquad (\because F_S = R_B)$$

Using compatibility equation;

$$\delta_1 + \delta_2 = 0$$

$$\Rightarrow \quad \frac{1}{4}\alpha(\Delta T_B)L + \frac{R_B L}{AE} + \frac{R_B}{K} = 0$$

$$\Rightarrow \quad R_B\left(\frac{L}{AE} + \frac{1}{K}\right) = -\frac{\alpha(\Delta T_B)L}{4}$$

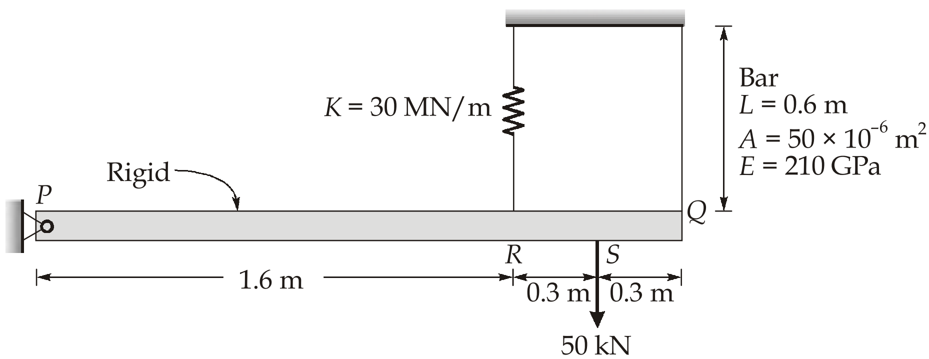
$$\Rightarrow \quad R_B = \frac{-\alpha(\Delta T_B)L}{4\left(\frac{L}{AE} + \frac{1}{K}\right)} = \frac{-\alpha(T_B)\times L \times AEK}{4(KL + AE)}$$

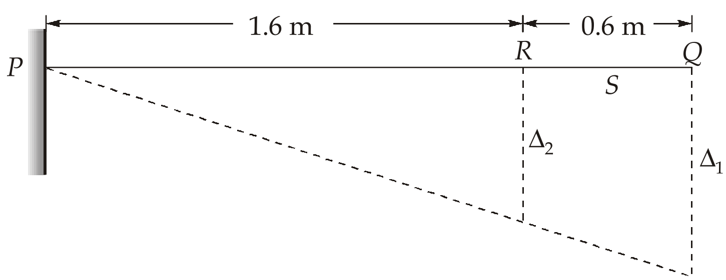
$$\Rightarrow \qquad \frac{R_B}{A} = \frac{-\alpha \Delta T_B \times L \times EK}{4(EA+KL)}$$

$$\Rightarrow \qquad \sigma_c = \frac{\alpha(\Delta T_B)LKE}{4KL\left(\frac{EA}{KL}+1\right)} = \frac{\alpha(\Delta T_B)E}{4\left(\frac{EA}{KL}+1\right)}$$

(ii)

Given : $K = 30$ MN/m^2; $A = 50 \times 10^{-6}$ m^2; $E = 210$ GPa; $P = 50$ kN; $L_{\text{steel rod}} = 0.6$ m





Taking moment about P:

$$P \times 1.9 = F_S \times 1.6 + T \times 2.2$$

$$\Rightarrow \qquad F_S \times 1.6 + T \times 2.2 = 50 \times 1.9 \times 1000$$

$$\Rightarrow \qquad F_S \times 1.6 + T \times 2.2 = 95000 \qquad \text{...(i)}$$

According to deformation equation;

$$\frac{\Delta_1}{2.2} = \frac{\Delta_2}{1.6}$$

$$\Rightarrow \qquad \frac{T \times L}{AE \times 2.2} = \frac{F_S}{K \times 1.6}$$

$$\Rightarrow \qquad \frac{T \times 0.6}{50 \times 10^{-6} \times 210 \times 10^{9} \times 2.2} = \frac{F_S}{30 \times 10^{6} \times 1.6}$$

$$\Rightarrow \qquad T = 0.802\, F_s \qquad \text{...(ii)}$$

From equation (i) and (ii),

$$\frac{T}{0.802} \times 1.6 + T \times 2.2 = 95000$$

$\Rightarrow$ $$T = 22645.939 \text{ N}$$

$\therefore$ Vertical displacement of point Q;

$$\frac{TL}{AE} = \frac{22645.939 \times 0.6}{50 \times 10^{-6} \times 210 \times 10^{9}} \times 1000$$

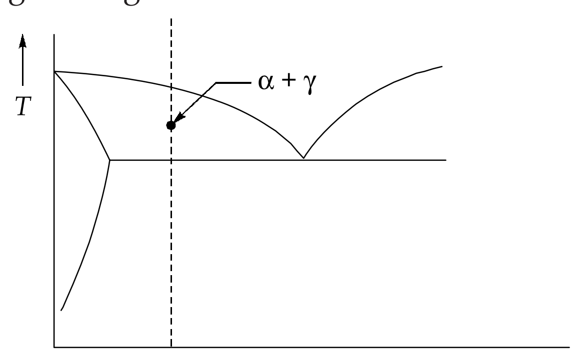
$$= 1.29 \text{ mm}$$

Section : B

5. (a)

Dual phase steels (DP steels):

- DP-steels are high strength steels that has a ferritic martensite microstructure.



- They are formed when very low carbon steels when they are heated in the range where two phase mixture α and γ are present and from this temperature material is cooled rapidly at a rate equal to or greater than critical cooling rate.
- Since, TTT-diagram is applicable only for austenite, so there will not be any change in α but austenite present will convert into micropockets of martensite.
- α-ferrite is already a very strong material and the micropockets of martensite creates further obstacles in the movement of dislocation, so strength of material increases exponentially.

Important characteristics are :

(a) Low yield strength

(b) Good uniform elongation

(c) High ultimate tensile strength

(d) A high strain rate sensitivity

(e) High initial strain hardening rates
(f) A good fatigue resistance

Applications:

- Automobile body panels
- Wheels
- Bumpers
- TMT-bars etc

Maraging steel:

- Since, Ti is very expensive material so the replacement is maraging steel.
- Maraging steel is having extensive application in defense and aerospace industries.

It contains:

- Ni – 17-19% : Increases ductility
- Co – 8-19% : Increases strength
- Mo – 3-35% : Increases strength
- Ti – 0.15% : Increases strength
- Al – 0.05-0.15% : as an impurity

5. (b)

(a) Vibration Monitoring: The noise and vibration are the most important parameters to monitor a machine, particularly in the moving parts such as shafts, rotors, cutting tools, gears, etc. The vibration level is recorded by attaching a transducer like velocity probe, accelerometer, or proximity probe to the machine. Special equipment is also available for using the output from the sensor to indicate the nature of vibration problem and even its precise cause. In some cases, it may become necessary to use the principles of Sonics and acoustics.

(b) Wear Debris Monitoring: This works on the principle that the working surfaces of a machine are washed by the lubricating oil, and any damage to them should be detectable from particles of wear debris in the oil. If the debris consists of relatively large ferrous lumps such as those generated by the fatigue of rolling element bearings and gears or the pitting of cams and taproots, these can be picked up by removable magnetic plugs inserted in the oil return lines. For small debris particle, spectrographic analysis or microscopic examination of oil samples after magnetic separation are commonly used techniques. Another popular technique is SOAP analysis for debris monitoring.

(c) Corrosion Monitoring: This is usually applied to fixed plant containing aggressive materials and is intended to monitor the rates of internal corrosion of the walls of the plant. This may be done by drilling sentinel holes part away, through the wall, which can be plugged when they leak or by inserting readily removable coupons of material of which the corrosion rate is assumed to relate to that of plant.

5. (c)

The various cold working processes are:

- Drawing
- Squeezing
- Bending
- Shearing
- Hobbing
- Shot peening
- Cold extruding, etc.

Advantages of cold working :

1. Good surface finish and better dimensional accuracy.
2. Energy saving since heating is not required.
3. Strength, fatigue and wear properties are improved.
4. Minimum contamination because of low working temperature, and no possibility of decarburisation of the surface.

Disadvantages:

1. Ductility of metal is reduced.
2. Deformation energy required is high, so rugged and more powerful equipment is required, thus equipment cost is high.
3. Severe stresses are set up, this requires stress relieving, which increases the cost.
4. Cold working, for large deformation, requires several stages with inter stage annealing, which increases the production cost.

The various hot working processes are:

- Rolling
- Forging
- Pipe welding
- Hot spinning
- Hot extruding, etc.

Advantages of hot working:

1. High production rate (since the process is faster) and very high reduction is possible without fear of fracture.
2. Metal is made tougher because pores get closed and impurities are segregated.
3. Deformation energy required is low, hence, less powerful equipments are required.
4. Since hot working promotes diffusion of constituents, segregation can be reduced or eliminated.

Disadvantages:

1. Handling of material is not so easy.
2. Heat resistant tools are required which are expensive.
3. Close tolerances cannot be held because of non-uniform cooling and thermal contraction.
4. Surface finish is poor because of scale formation.

5. (d)

(a) Percent set point, $SP(\%) = \dfrac{SP - \text{ Minimum operating value}}{\text{Operating range}}$

$$= \frac{1200 - 100}{2400 - 100} \times 100 = 47.83\%$$

(b) Percent measured value,

$$MV(\%) = \frac{\text{Current speed} - \text{Minimum speed}}{\text{Operating range}}$$

$$= \frac{1000 - 100}{2400 - 100} \times 100 = 39.13\%$$

(c) Percent error, $\text{Error}\,(\%) = \dfrac{\text{Error}}{\text{Operating range}} \times 100 = \dfrac{1200 - 1000}{2400 - 100} \times 100$

$$= 8.69\%$$

5. (e)

Probability Density Function (PDF)

The probability density function (PDF) is the basis for predicting the behaviour of any probabilistic situation such as reliability or availability. Given the PDF, one can compute reliability function and the instantaneous density function *F(t)*, usually referred to as failure density function.

Cumulative Distribution Function (CDF)

This function defines the probability that a random variable '*t*' lies between some lower limit (often zero) and upper limit '*t*'. For a continuous variable such as time to failure, the cumulative probability $F(t)$ is given by the integration of the density function between the limits required.

$$F(t) = f(x)\,dx \qquad \text{...(i)}$$

In reliability studies, this function is also known as unreliability function. The cumulative distribution function increases from zero to unity as 't' increases from its smallest to largest value.

Reliability Function

The reliability function $R(t)$, which is defined as the probability of a system or an item to perform or operate its required function without failure under given condition for an intended period of operation and it is mathematically given as

$$R(t) = 1 - F(t) = 1 - \int f(x)\,dx = \int R\,dx \quad ...(ii)$$

Hazard Function

The hazard function (t) or instantaneous failure rate function is a conditional expression that an item in service for time "t" will be in the next instant of time "dt" given that it has not previously failed, survived up to time 't'.

i.e., P [the unit will fail in $(t, t + dt)$] $\cong \lambda(t)$ dt ...(iii)

Where (t) is the hazard rate at time 't' and 'P' is the probability of occurrence. The mathematical relationship between failure density function $f(t)$ and hazard rate (t) is

$$\lambda(t) = \frac{f(t)}{[1 - F(t)]} \quad ...(iv)$$

The general equation showing the relationship between the reliability function $R(t)$ and hazard rate

$$R(t) = Exp\left[-\int \lambda(x)\,dx\right] \quad ...(v)$$

When λ is independent of time as in the case of constant failure rate, expression reduces to

$$R(t) = e^{-\lambda(t)} \quad ...(vi)$$

6. (a)

The data of the given problem can be summarized as follows:

Nutrient constituents	Nutrient content in the product		Minimum amount of nutrient
	A	B	
x	36	6	108
y	3	12	36
z	20	10	100
Cost	₹20	₹40	

Mathematical formulation of the linear programming is

Minimize $z = 20x_1 + 40x_2$

Subjected to: $36x_1 + 6x_2 \geq 108$

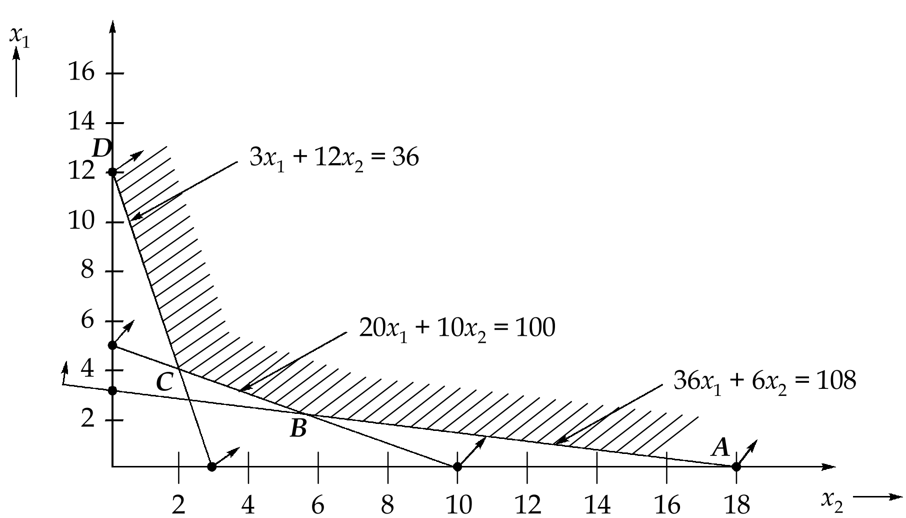
$$3x_1 + 12x_2 \geq 36$$
$$20x_1 + 10x_2 \geq 100$$
and $$x_1, x_2 \geq 0$$

where, x_1 = number of units of product *A*, x_2 = number of units of product *B*.

The constraints of the given problem are plotted by treating them as equation:

$$36x_1 + 6x_2 = 108$$
$$\Rightarrow \quad \frac{x_1}{3} + \frac{x_2}{18} = 1$$
$$3x_1 + 12x_2 = 36$$
$$\Rightarrow \quad \frac{x_1}{12} + \frac{x_2}{3} = 1$$
$$20x_1 + 10x_2 = 100$$
$$\Rightarrow \quad \frac{x_1}{5} + \frac{x_2}{10} = 1$$

The feasible region of the problem is shown below:



The coordinate of the extreme points of the feasible region are:

$$A = (0, 18), B = (2, 6), C = (4, 2) \text{ and } D = (12, 0)$$

The value of objective function at each of the extreme point can be evaluated as follows:

Extreme point	(x_1, x_2)	$z = 20x_1 + 40x_2$
A	(0, 18)	720
B	(2, 6)	280
C	(4, 2)	160 ← Minimum
D	(12, 0)	240

Hence, the optimum solution is to purchase 4 units of product *A* and 2 units of product *B* in order to maintain a minimum cost of ₹160/-.

6. (b)

$$^1T_2 = \left[\begin{array}{ccc|c} & {}^1R_2 & & {}^1D_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Because frame {2} is rotated relative to frame {1} about x-axis by 60°, equation, gives

$$^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60° & -\sin 60° \\ 0 & \sin 60° & \cos 60° \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500 & -0.866 \\ 0 & 0.866 & 0.500 \end{bmatrix} \quad \text{...(i)}$$

Substituting $^1R_2$ and $^1D_2$ in the above equation

$$^1T_2 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7.000 \\ 0 & 0.500 & -0.866 & 5.000 \\ 0 & 0.866 & 0.500 & 7.000 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad \text{...(ii)}$$

$$^2T_1 = \left[\begin{array}{ccc|c} & {}^2R_1 & & {}^2D_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[{}^1T_2\right]^{-1} \quad \text{...(iii)}$$

The inverse of $^1T_2$ is given by equation, that is,

$$^2T_1 = \left[\begin{array}{ccc|c} & {}^1R_2^T & & -{}^1R_2^T D_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

From $^1T_2$ in equation (ii),

$$^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500 & -0.866 \\ 0 & 0.866 & 0.500 \end{bmatrix}$$

and $$^1D_2 = [7.0 \quad 5.0 \quad 7.0]^T$$

Hence, $$^2R_1 = {^1R_2^T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500 & 0.866 \\ 0 & -0.866 & 0.500 \end{bmatrix} \quad \text{...(iv)}$$

and the position of the origin of frame {1} with respect to frame {2} is given by

$$^2D_1 = -{^1R_2} \cdot {^1D_2}$$

Substituting values

$$^2D_1 = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500 & 0.866 \\ 0 & -0.866 & 0.500 \end{bmatrix}\begin{bmatrix} 7.0 \\ 5.0 \\ 7.0 \end{bmatrix} = \begin{bmatrix} -7.000 \\ -8.562 \\ 0.830 \end{bmatrix} \quad \text{...(v)}$$

Therefore, $$^2T_1 = \begin{bmatrix} 1 & 0 & 0 & -7.000 \\ 0 & 0.500 & 0.866 & -8.562 \\ 0 & -0.866 & 0.500 & 0.830 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{...(vi)}$$

6. (c) (i)

Following are the assumptions with regard to this model:

(i) Work moves with a uniform velocity.

(ii) The surface where the shear occurs is a plane.

(iii) The tool is perfectly sharp and there is no contact along the clearance face.

(iv) The cutting edge is a straight line which extends perpendicular to the direction of motion and generates plane surface as the work moves past it.

(v) Width of the tool is greater than the width of work.

(vi) The stresses on the shear plane are uniformly distributed.

(vii) Uncut chip thickness is constant.

(viii) A continuous chip is produced without any built up edge.

(ix) The chip does not flow to either side, or there is no side spread.

(ii)

1. Cutting force, F_t :

Volume of chip before cut = Volume of chip after cut

$$l \times b \times t = l_c \times b_c \times t_c$$

$$\Rightarrow \qquad r = \frac{t}{t_c} = \frac{l_c}{l} \times \frac{b_c}{b}$$

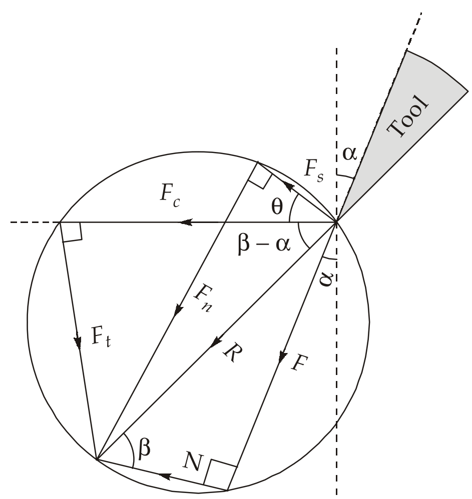
$$= \frac{48 \times 4.6}{170 \times 3.8} = 0.3418$$

Shear angle, $\phi = \tan^{-1}\left(\frac{r\cos\alpha}{1 - r\sin\alpha}\right) = \tan^{-1}\left(\frac{(0.3418)\times\cos(20°)}{1-(0.3418)\times\sin(20°)}\right)$

$$= \tan^{-1}(0.363705) = 19.9866°$$

Also, $\mu = \tan\beta$

$\Rightarrow \qquad \beta = \tan^{-1}(\mu) = \tan^{-1}(0.75) = 36.8699°$



Shear force, $F_S = \frac{\tau A}{\sin\phi} = \frac{\tau(b \cdot t)}{\sin\phi}$

$\Rightarrow \qquad F_S = \frac{248 \times 3.8 \times 0.2}{\sin(19.9866°)} = 551.43 \text{ N}$

From Merchant circle; $R = \frac{F_c}{\cos(\beta - \alpha)} = \frac{F_s}{\cos(\phi + \beta - \alpha)}$

$\Rightarrow \qquad F_c = F_s\left[\frac{\cos(\beta-\alpha)}{\cos(\phi+\beta-\alpha)}\right]$

$$= (551.43) \times \left[\frac{\cos(36.8699° - 20°)}{\cos(19.9866° + 36.8699° - 20°)}\right]$$

$= 659.51$ N **Ans.**

2. Thrust force, F_C :

From merchant circle,

$$F_t = F_c \cdot \tan(\beta - \alpha) = (659.51) \times \tan(36.8699° - 20°)$$
$$= 199.996 \text{ N}$$ **Ans.**

3. Power consumption:

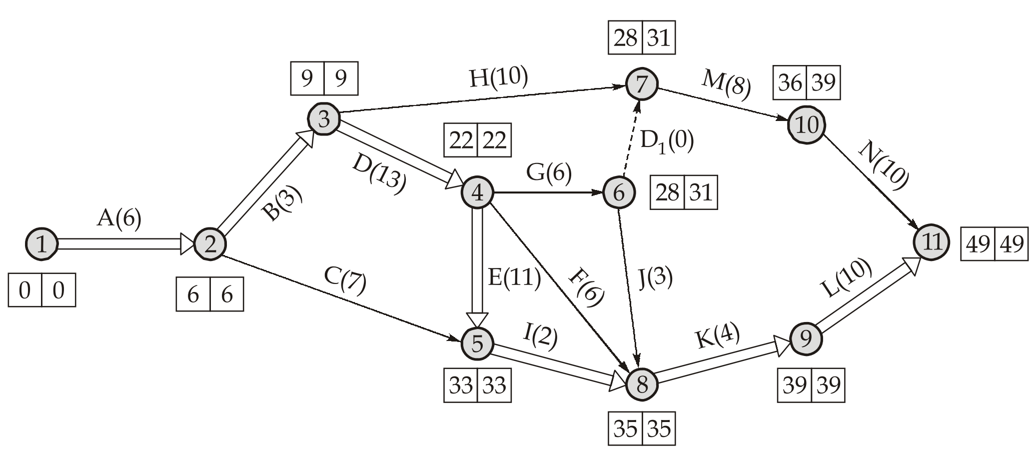
$$P = F_c \times v$$
$$= 659.51 \times \frac{36}{60} = 395.706 \text{ W}$$ **Ans.**

7. (a) (i)

PERT	CPM
1. PERT is used for non-repetitive jobs like planning the assembly of the space.	1. CPM is used for repetitive job like building a house.
2. It is a probabilistic model.	2. It is a deterministic model.
3. It is event-oriented as the results of analysis are expressed in terms of events or distinct points in time indicative of progress.	3. It is activity-oriented as the result or calculations are considered in terms of activities or operations of the project.
4. It is applied mainly for planning and scheduling research programmes.	4. It is applied mainly for construction and business problems.
5. PERT incorporates statistical analysis and thereby determines the probabilities concerning the time by which each activity or entire project would be completed.	5. CPM does not incorporate statistical analysis in determining time estimates, because time is precise and known.
6. PERT serves as useful control device as it assists management in controlling a project by calling attention to such delays.	6. It is difficult to use CPM as a control device for the simple reason that one must repeat the entire evaluation of the project each time the changes are introduced into the network.

7. (a) (ii)

1.



2.

Paths	Completion time (in days)
A - B - H - M - N	6 + 3 + 10 + 8 + 10 = 37
A - B - D - G - M - N	6 + 3 + 13 + 6 + 8 + 10 = 46
A - B - D - E - I - K - L	6 + 3 + 13 + 11 + 2 + 4 + 10 = 49 (maximum)
A - C - I - K - L	6 + 7 + 2 + 4 + 10 = 29
A - B - D - G - J - K - L	6 + 3 + 13 + 6 + 3 + 4 + 10 = 45
A - B - D - F - K - L	6 + 3 + 13 + 6 + 4 + 10 = 42

Critical path : A - B - D - E - I - K - L

Project completion time = 49 days

3.

Earliest start time : EST = E_i

Earliest finish time : EFT = EST + $t_E^{ij} = E_i + t_E^{ij}$

Latest finish time : LFT = L_j

Latest start time : LST = LFT – $t_E^{ij} = L_j - t_E^{ij}$

Total float : TF = LFT – EFT = $L_j - \left(E_i + t_E^{ij}\right)$

Activity	Duration (in days)	EST	EFT	LFT	LST	TF
A	6	0	6	6	0	0
B	3	6	9	9	6	0
C	7	6	13	33	26	20
D	13	9	22	22	9	0
E	11	22	33	33	22	0
F	6	22	28	35	29	7
G	6	22	28	31	25	3
H	10	9	19	31	21	12
I	2	33	35	35	33	0
J	3	28	31	35	32	4
K	4	35	39	39	35	0
L	10	39	49	49	39	0
M	8	28	36	39	31	3
N	10	36	46	49	39	3

7. (b)

Fixed angle representation

Let the fixed frame {1} frame {x, y, z} and moving frame {2} (frame {u v w}) be initially coincident. Consider the sequence of rotations about the three axes of fixed frame as shown in figure.

(i) First, moving frame {2} is rotated by an angle θ_1 about x-axis to frame {2′} as in figure (a). This rotation is described by the rotation matrix $R_x(\theta_1)$.

(ii) Next, the frame {2′} is rotated by an angle θ_2 about y-axis to give frame {2″} as in figure (b). This rotation is described by the rotation matrix $R_y(\theta_2)$.

(iii) Finally, it is rotated by an angle θ_3 about z-axis to frame {2} as in figure (c). This rotation is described by the rotation matrix $R_z(\theta_3)$.

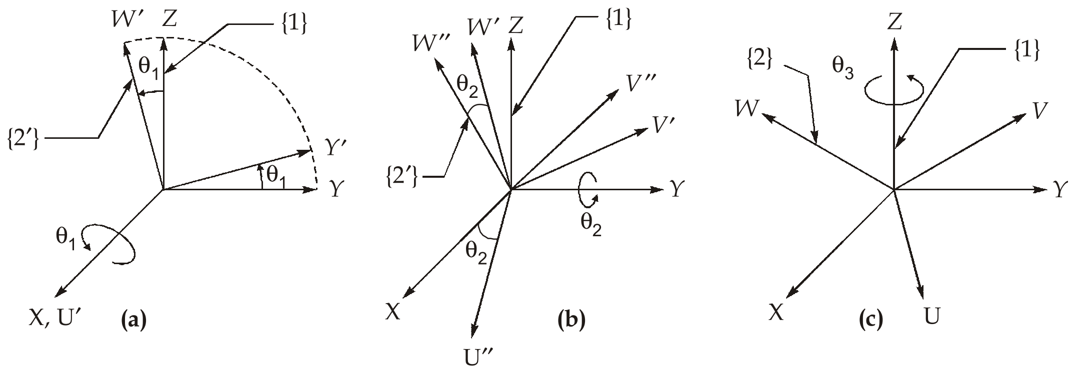


Figure: Three rotations of θ_1, θ_2, and θ_3 about fixed axes.

This convention for specifying orientation is known as fixed angle representation because each rotation is specified about an axis of fixed reference frame. The above three rotations are referred as XYZ-fixed angle rotations.

The final frame orientation is obtained by composition of rotations with respect to the fixed frame and the overall rotation matrix ${}^1R_2$ is computed by pre-multiplication of the matrices of elementary rotations, that is,

$$R_{xyz}(\theta_3\theta_2\theta_1) = {}^1R_2 = R_z(\theta_3)R_y(\theta_2)R_x(\theta_1)$$

(rotation ordering right to left)

Substituting the results of equation for fixed angle rotations, the final rotation matrix is

$$R_{xyz}(\theta_3\theta_2\theta_1) = \begin{bmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & C_1 & -S_1 \\ 0 & S_1 & C_1 \end{bmatrix}$$

or
$$R_{xyz}(\theta_3\theta_2\theta_1) = \begin{bmatrix} C_2C_3 & S_1S_2C_3 - C_1S_3 & C_1S_2C_3 + S_1S_3 \\ C_2S_3 & S_1S_2S_3 + C_1C_3 & C_1S_2S_3 - S_1C_3 \\ -S_2 & S_1C_2 & C_1C_2 \end{bmatrix}$$

The final frame orientation for any set of rotation performed about the axes of the fixed frame (e.g. ZYX, ZXZ, etc.) can be obtained by multiplying the rotation matrices in a consistent order as indicated in equation. In fixed angle representation, order of rotations XYZ or ZYX are equivalent, that is, $R_{xyz}(\theta_1\theta_2\theta_3) = R_{zyx}(\theta_1\theta_2\theta_3)$.

The three rotations about the three fixed principal axes in fixed angle rotation produce the motions, which are also known as roll, pitch, and yaw motions, as shown in figure below.

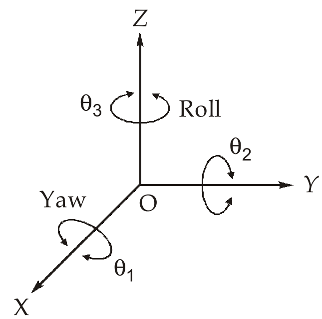


Figure: Representation of roll, pitch, and yaw (RPY) rotations

The three rotations about the three fixed principal axes in fixed angle rotation produce the motions, which are also known as roll, pitch, and yaw motions, as shown in figure. The XYZ-fixed angle transformation in equation is equivalent to roll-pitch-yaw (RPY) transformation.

Euler Angle Representations

The moving frame, instead of rotating about the principal axes of the fixed frame, can rotate about its own principal axes. Consider alternate rotations of frame {2} with respect to frame {1}, as shown in figure below, starting from the position when the two frames are coincident.

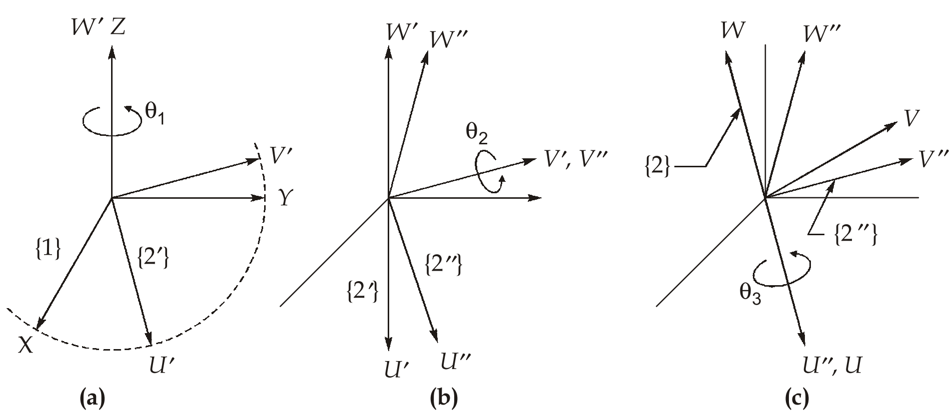


Figure: Eular angle representation for three rotations of θ_1, θ_2, and θ_3

(i) To begin with, frame {2} is rotated by an angle θ_1 about its w-axis coincident with z-axis of frame {1}. The rotated frame is now {2′} and the rotation is described by the rotation matrix $R_w(\theta_1)$.

(ii) Next, moving frame {2′} is rotated by an angle θ_2 about v'-axis, the rotated v-axis to frame {2″}. This rotation is described by the rotation matrix $R_{v'}(\theta_2)$.

(iii) Finally, frame {2″} is rotated by an angle θ_3 about u''-axis, the rotated u-axis to give frame {2}. This rotation is described by the rotation $R_{u''}(\theta_3)$.

This convention for specifying orientation is called WVU-Euler angle representation and is illustrated in figure (a), (b) and (c). Viewing each of these rotations as descriptions of frames relative to each other, the equivalent rotation matrix is computed by post multiplication of the matrices of the elementary rotations as

$$R_{wvu}(\theta_1\theta_2\theta_3) = {}^1R_2 = {}^1R_{2'}\,{}^{2'}R_{2''}\,{}^{2''}R_2$$

$$= R_w(\theta_1)R_{v'}(\theta_2)R_{u''}(\theta_3) \quad \text{(rotation ordering left to right)}$$

The rotations are performed about the current axes of the moving frame $\{uvw\}$.

Using the results of equation, the resulting frame orientation or the rotation matrix is

$$R_{wvu}(\theta_1\theta_2\theta_3) = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & C_3 & -S_3 \\ 0 & S_3 & C_3 \end{bmatrix}$$

or
$$R_{wvu}(\theta_1\theta_2\theta_3) = \begin{bmatrix} C_2C_3 & S_1S_2C_3 - C_1S_3 & C_1S_2C_3 + S_1S_3 \\ C_2S_3 & S_1S_2S_3 + C_1C_3 & C_1S_2S_3 - S_1C_3 \\ -S_2 & S_1C_2 & C_1C_2 \end{bmatrix}$$

It is observed that this result is exactly same as that obtained for fixed angle representation, but the rotations about the fixed axes were performed in opposite order. In general, three rotations performed about fixed axes give the same final orientation as obtained by the same three rotations performed in the opposite order about the moving axes. Hence,

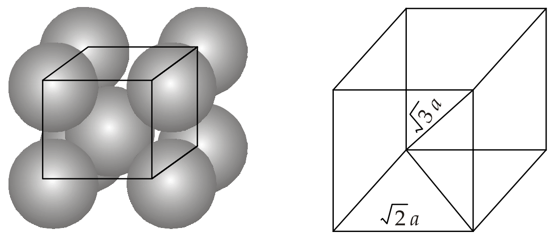
$$R_{xyz}(\theta_3\theta_2\theta_1) = R_{wvu}(\theta_1\theta_2\theta_3) = R_{zyx}(\theta_1\theta_2\theta_3)$$

7. (c) (i)

Atomic packing factor is defined as the percentage volume of unit cell covered by the atoms. It is given as;

$$\text{Atomic packing factor} = \frac{\text{Volume occupied by the atoms in a unit cell}}{\text{Volume of unit cell}}$$

In a BCC lattice each atom is bonded closely to the neighbouring atom along the body diagonal. There are 8 corners and hence number of corner atoms is 8.



(a) (b)

The number of atoms at the body center = 1

$\therefore$ The total number of atoms per unit cell is $\left(\frac{1}{8}\times 8\right)+1 = 2$

Volume occupied by atoms = No. of atoms × Volume of one atom = $2\times\frac{4}{3}\pi R^3$

$$\text{Volume of unit cell} = a^3$$

$$\text{From the figure, } \sqrt{3}a = 4R$$

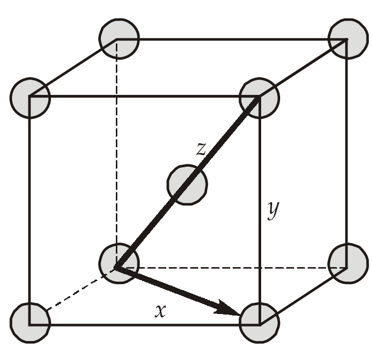
$$\therefore \quad \text{Volume of unit cell} = \left(\frac{4R}{\sqrt{3}}\right)^3$$

$$\text{Packing Factor} = \frac{\text{Volume occupied by atoms}}{\text{Volume of unit cells}}$$

$$= \frac{\frac{8}{3}\pi R^3}{\left(\frac{4R}{\sqrt{3}}\right)^3} = \frac{\sqrt{3}\pi}{8} = 0.68$$

7. (c) (ii)

In the figure below is shown a [1 1 0] direction within a BCC unit cell.



[110]

For this [1 1 0] direction there is one atom at each of the two unit cell corners, and, thus, there is the equivalence of 1 atom that is centered on the direction vector. The length of this direction vector is denoted by x in this figure, which is equal to

$$x = \sqrt{z^2 - y^2}$$

where y is the unit cell edge length, which, from equation is equal to $\frac{4R}{\sqrt{3}}$. Furthermore, z is the length of the unit cell diagonal, which is equal to 4R Thus, using the above equation, the length x may be calculated as follows:

$$x = \sqrt{(4R)^2 - \left(\frac{4R}{\sqrt{3}}\right)^2} = \sqrt{\frac{32R^2}{3}} = 4R\sqrt{\frac{2}{3}}$$

Therefore, the expression for the linear density of this direction is

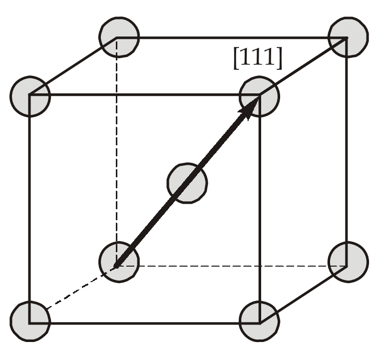
$$LD_{110} = \frac{\text{Number of atoms centered on [1 1 0] direction vector}}{\text{Length of [1 1 0] direction vector}}$$

$$= \frac{1 \text{ atom}}{4R\sqrt{\frac{2}{3}}} = \frac{\sqrt{3}}{4R\sqrt{2}}$$

From the table inside the front cover, the atomic radius for tungsten is 0.137 nm. Therefore, the linear density for the [1 1 0] direction is

$$LD_{110}(W) = \frac{\sqrt{3}}{4R\sqrt{2}} = \frac{\sqrt{3}}{(4)(0.137nm)\sqrt{2}} = 2.23 \text{ nm}^{-1} = 2.23 \times 10^9 \text{ m}^{-1}$$

A BCC unit cell within which is drawn a [1 1 1] direction is shown below.



For although the [1 1 1] direction vector shown passes through the centers of three atoms, there is an equivalence of only two atoms associated with this unit cell - one half of each of the two atoms at the end of vector, in addition to the center atom belongs entirely to the unit cell. Furthermore, the length of the vector shown is equal to 4R, since all of the atoms whose centers the vector passes through touch one another. Therefore, the linear density is equal to

$$LD_{111} = \frac{\text{Number of atoms centered on [1 1 1] direction vector}}{\text{Length of [1 1 1] direction vector}}$$

$$= \frac{2 \text{ atoms}}{4R} = \frac{1}{2R}$$

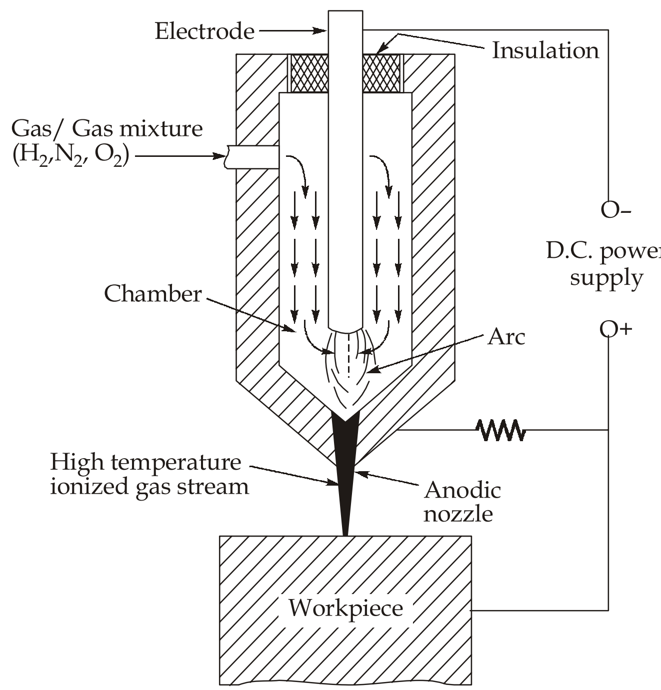
While for the [1 1 1] direction

$$LD_{111}(W) = \frac{1}{2R} = \frac{1}{2 \times (0.137 \text{ nm})} = 3.65 \text{ nm}^{-1} = 3.65 \times 10^9 \text{ m}^{-1}$$

8. (a) (i)

Plasma Arc Machining (PAM) : A plasma is a high temperature ionized gas. The plasma arc machining is done with a high speed jet by a high temperature plasma.

Principle and Working : A plasma is generated by subjecting a flowing gas to the electron bombardment of an arc. For this, the arc is set up between the electrode and the anodic nozzle; the gas is forced to flow through this arc as shown in figure. The high velocity electrons of the arc collide with the gas molecules, causing a dissociation of the diatomic molecules or atoms into ions and electrons resulting in a substantial increase in the conductivity of the gas which is now in plasma state. The free electrons subsequently accelerate and cause more ionization and heating.



Plasma Arc Machining (PAM).

Afterwards a further increase in temperature takes place when the ions and free electrons recombine into atoms or when the atoms recombine into molecules as these are exothermic processes. So a high temperature plasma is generated which is forced through the nozzle in the form of a jet. This jet (ionized stream of gas) is impinged on the workpiece which gets melted and eroded.

Characteristics of PAM :

- Tool: Plasma jet (maximum velocity 500 m/s).
- Workpiece materials: All conducting materials.

- Material removal: Melting (maximum material removal rate = 150 cm^3/min).
- Medium: Plasma.
- Maximum temperature: 16000°C.
- Power range: 2 to 200 kW.
- Critical parameter: Voltage (30 to 250 V); current (upto 600 A).

Advantages:

1. Excessively high temperatures are generated for use.
2. Can be used to cut any metal.
3. A faster process.

Disadvantages:

1. Initial cost of equipment is quite high.
2. Adequate safety precautions are always needed for the operator.
3. The work surface may undergo some metallurgical changes.

Applications:

1. Cutting of stainless steel and non-ferrous metals (such as aluminium alloys).
2. Turning and milling of 'hard to machine' materials.

8. (a) (ii)

$$\text{Total length, } L = 480 + 32 + 32 = 544 \text{ mm}$$

$$\text{Shaping width, } B = 158 + 6 + 6 = 170 \text{ mm}$$

$$V_C = 8 \text{ m/min}; V_r = 12 \text{ m/min}; f = 1.7 \text{ mm/tooth}$$

$$\text{Cutting stroke time, } t_1 = \frac{L}{V_c} = \frac{544}{8 \times 1000} = 0.0680 \text{ min}$$

$$\text{Return (idle) time, } t_2 = \frac{L}{V_r} = \frac{544}{12 \times 1000} = 0.0453 \text{ min}$$

$$\text{Total time per cycle, } t = t_1 + t_2 = 0.0680 + 0.0453 = 0.1133 \text{ min}$$

$$\text{Number of cycle required, } n = \frac{B}{f} = \frac{170}{1.7} = 100 \text{ cycles}$$

$$\therefore \quad \text{Machining time, } t_m = \text{Number of cycles} \times \text{Time per cycle}$$

$$= n \times t = 100 \times 0.1133 = 11.33 \text{ min}$$ **Ans.**

8. (b)

(i)

S.No.	Aspects	Microprocessor	Micro controller
1.	Components/parts	CPU, interrupt circuits and memory-addressing circuits.	Besides all parts of microprocessor, they also contain timers, parallel and serial I/O, internal RAM and ROM
2.	Access times for memory and I/O devices	More	Less
3.	Number of operational codes (for moving data from external memory to CPU)	Many	One or two
4.	Nature of deal with rapid movement of codes and data	From external address to chip.	Within the chip
5.	Operation as a digital computer	By adding external digital parts.	Without adding external digital parts
6.	Memory map for data and code.	Single	Separate
7.	Hardware used	More	Less
8.	Flexibility from design point of view	More	Less

(ii)

For an n-bit DAC,

$$V_0 = -\frac{2R_F V_R}{R}\left[\frac{b_1}{2}+\frac{b_2}{2^2}+.....\frac{b_n}{2^n}\right]$$

(a) For LSB, the binary input is 000000000001

Such that, $$V_{0,\text{LSB}} = \frac{-2(5)}{2^{12}} = -0.00244\,V = -2.44 \text{ mV}$$ **Ans.**

The resolution is also 2.44 mV.

(b) For MSB, the binary input is 100000000000 such that

$$V_{0,\text{MSB}} = \frac{-2(5)}{2^1} = -5 \text{ V}$$ **Ans.**

(c) The maximum value of digital input is 111111111111 such that

$$V_0 = -2(5)\left[\frac{1}{2^1}+\frac{1}{2^2}+....\frac{1}{2^{11}}+\frac{1}{2^{12}}\right]$$

$$V_0 = -9.99756 \text{ V}$$ **Ans.**

(d) For full-scale range voltage,

$$V_{FSR} = \frac{2R_F V_R}{R} = \frac{2\times5\times5}{5} = 10\text{V}$$ **Ans.**

It can be observed that

$$V_{FSR} - V_{0,MSB} = 10 - 9.99756 = 0.00244\text{V}$$

i.e. the same as the resolution.

8. (c)

(i)

Piezoelectric Accelerometer: A piezoelectric accelerator is probably the simplest and most commonly used transducer for measuring acceleration.

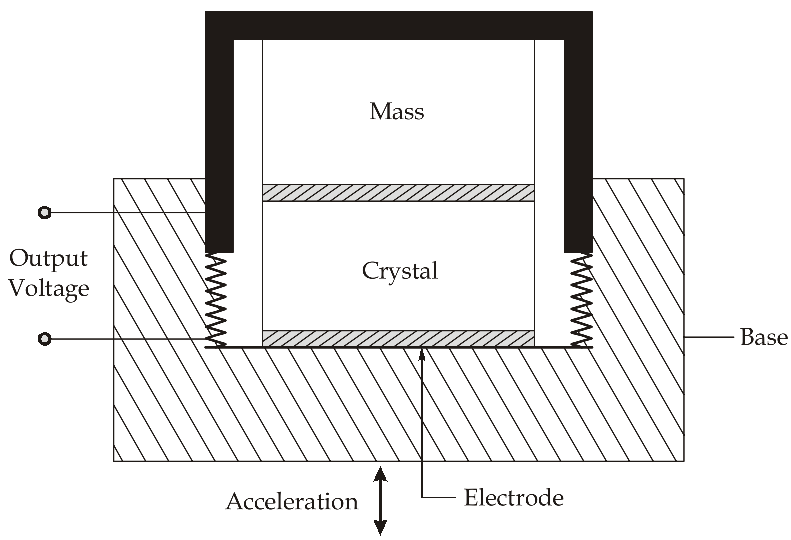


Figure: Piezoelectric accelerometer

Construction: It consists of a piezoelectric crystal sandwitched between two electrodes and has mass placed on it. The unit is fastened to the base whose acceleration characteristics are to be obtained. The can threaded to the base acts as a spring and squeezes the mass against the crystal. Mass exerts a force on the crystal and a certain voltage output is generated.

Working: When the base is accelerated downward inertial reaction force on the base acts upward against the top of the can. This relieves stress on the crystal. According to Newton's second law of motion, force = mass x acceleration, since the mass is a fixed quantity, the decrease in force is proportional to the acceleration. Similarly, an acceleration in the upward direction would increase the force on the crystal in proportion to the acceleration. The resulting change in the output voltage is recorded and correlated to the acceleration imposed on the base.

Advantages:

1. Small size and a small weight.
2. High output impedance.
3. Can measure acceleration from a fraction of g to thousands of g.
4. High sensitivity.
5. High frequency response (10 Hz to 50 kHz)

Disadvantages:

1. Unsuitable for applications where the input frequency is lower than 10 Hz.
2. Subject to hysteresis errors.
3. Sensitive to temperature changes.

(ii)

Given : $A = 6 \times 6 = 36\ \text{mm}^2$; $t = 1.5$ mm; $\in\ = 12 \times 10^{-9}$ F/m; F = 10 N; d = 120 pC/N; $E = 12 \times 10^6\ \text{N/m}^2$

$$\text{Pressure, } p = \frac{F}{A} = \frac{10}{36 \times 10^{-6}} = 0.277\ \text{MN/m}^2$$

$$\therefore \quad \text{Strain, } e = \frac{p}{E} = \frac{0.277 \times 10^6}{12 \times 10^6}$$

$$\therefore \quad e = 0.023$$ **Ans.(i)**

$$\text{Voltage sensitivity, } g = \frac{d}{\in_0 \in_r} = \frac{d}{\in}$$

$$g = \frac{120 \times 10^{-12}}{12 \times 10^{-9}} = 10 \times 10^{-3}\ \text{Vm/N or } 10^{-2}\ \text{Vm/N}$$

$$\therefore \quad \text{Voltage generated, } E = gtp$$

$$= 10^{-2} \times 1.5 \times 10^{-3} \times 0.277 \times 10^6$$

$$= 4.155\ \text{V}$$

Hence, $$\text{Charge, } Q = d \times F$$

$$= 120 \times 10^{-12} \times 10 = 1200\ \text{pC}$$ **Ans. (ii)**

and $$\text{capacitance, } C = \frac{1200 \times 10^{-12}}{4.155}$$

$$C = 288.8\ \text{pF}$$ **Ans.(ii)**

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