MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

Mechanical Engineering Test No : 14

Full Syllabus Test (Paper-1)

Section : A

1. (a)

Differentiating above equation with respect to x, we get:

$$\frac{\partial u}{\partial x} = \frac{k\left(x^2+y^2+z^2\right)^{3/2} - kx\left(x^2+y^2+z^2\right)^{1/2} \times 2x \times \frac{3}{2}}{\left(x^2+y^2+z^2\right)^3}$$

$$= \frac{k\left(x^2+y^2+z^2\right)^{3/2} - 3kx^2\left(x^2+y^2+z^2\right)^{1/2}}{\left(x^2+y^2+z^2\right)^3}$$

$$\frac{\partial u}{\partial x} = \frac{k\left(z^2+y^2-2x^2\right)}{\left(x^2+y^2+z^2\right)^{5/2}} \quad \text{...(i)}$$

Similarly, differentiating v with respect to y, we get,

$$\frac{\partial v}{\partial y} = \frac{k\left(x^2-2y^2+z^2\right)}{\left(x^2+y^2+z^2\right)^{5/2}} \quad \text{...(ii)}$$

For a steady, three dimensional incompressible fluid flow, the continuity equation is,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \frac{k\left(y^2+z^2-2x^2\right)}{\left(x^2+y^2+z^2\right)^{5/2}}+\frac{k\left(x^2+z^2-2y^2\right)}{\left(x^2+y^2+z^2\right)^{5/2}}+\frac{\partial\omega}{\partial z}=0$$

$$\Rightarrow \qquad \frac{\partial\omega}{\partial z} = \frac{k\left(x^2+y^2-2z^2\right)}{\left(x^2+y^2+z^2\right)^{5/2}}$$

Integrating above equation with respect to z,

$$\omega = k\int\frac{\left(x^2+y^2-2z^2\right)}{\left(x^2+y^2+z^2\right)^{5/2}}dz$$

Putting $x^2 + y^2 = a^2$

$$\omega = k\int\frac{\left(a^2-2z^2\right)}{\left(a^2+z^2\right)^{5/2}}dz = k\int\frac{a^2}{\left(a^2+z^2\right)^{5/2}}dz - 2k\int\frac{z^2}{\left(a^2+z^2\right)^{5/2}}dz$$

Putting $\qquad z = a\tan\theta \Rightarrow dz = a\sec^2\theta d\theta$

$$\omega = k\int\frac{a^2\left(a\sec^2\theta\right)d\theta}{\left(a^2+a^2\tan^2\theta\right)^{5/2}}dz - 2k\int\frac{a^2\tan^2\theta\times a\sec^2\theta d\theta}{\left(a^2+a^2\tan^2\theta\right)^{5/2}}$$

$$= \frac{k}{a^2}\int\cos^3\theta d\theta - \frac{2k}{a^2}\int\sin^2\theta\cos\theta d\theta = \frac{k}{a^2}\int\left(\cos\theta-\sin^2\theta\cos\theta\right)d\theta - \frac{2k}{a^2}\int\sin^2\theta\cos\theta d\theta$$

$$= \frac{k}{a^2}\int\cos\theta d\theta - \frac{3k}{a^2}\int\sin^2\theta\cos\theta d\theta = \frac{k}{a^2}\sin\theta - \frac{k}{a^2}\sin^3\theta = \frac{k}{a^2}\sin\theta\left(1-\sin^2\theta\right)$$

$$= \frac{k}{a^2}\frac{z}{\left(a^2+z^2\right)^{1/2}}\left(1-\frac{z^2}{a^2+z^2}\right) \qquad \left[\because z = a\tan\theta, \sin\theta = \frac{z}{\left(a^2+z^2\right)^{1/2}}\right]$$

$$= \frac{kz}{\left(a^2+z^2\right)^{3/2}} = \frac{kz}{\left(x^2+y^2+z^2\right)^{3/2}}$$

Thus the component of velocity in z-direction is

$$\omega = \frac{kz}{\left(x^2+y^2+z^2\right)^{3/2}}+C$$

The constant C should be independent of z but may a function of x and y.

$$\therefore \qquad \omega = \frac{kz}{\left(x^2+y^2+z^2\right)^{3/2}}+f(x,y)$$

1. (b)

Given: $V = 1.67\ m^3$, $W_{out} = 106$ kJ/kg, $P_1 = 20$ bar $= 2000$ kPa, $T_1 = T_o = 300$ K, $P_2 = 700$ kPa, $P_o = 101.325$ kPa.

The initial mass of compressed air in the bottle,

$$m_1 = \frac{P_1 V}{RT_1} = \frac{2000 \times 1.67}{0.287 \times 300} = 38.79 \text{ kg}$$

The mass of air remaining in bottle, when its pressure reaches to 7 bar,

$$m_2 = \frac{P_2 V}{RT_2} = \frac{700 \times 1.67}{0.287 \times 300} = 13.58 \text{ kg}$$

The mass of air supplied to air motor,

$$m = m_1 - m_2 = 38.79 - 13.58 = 25.21 \text{ kg}$$

Total work delivered by air motor, $W = mW_{out} = 25.21 \times 106 = 2672.26$ kJ

(a) Maximum work possible in air motor = Change in availability of compressed air

$$= \phi_1 - \phi_o = (u_1 - u_o) + P_o (V_1 - V_o) - T_o (s_1 - s_o)$$

where, $u_1 - u_o = 0$

$$P_o (V_1 - V_o) = P_o\left[\frac{RT_1}{P_1} - \frac{RT_0}{P_0}\right] = 101.325 \times 0.287 \times 300 \times \left[\frac{1}{2000} - \frac{1}{101.325}\right]$$

$$= -81.74 \text{ kJ/kg}$$

$$T_o (s_1 - s_o) = T_o\left[C_V \ln\left(\frac{T_1}{T_o}\right) - R\ln\left(\frac{P_1}{P_o}\right)\right]$$

$$= 300 \times \left[0 - 0.287 \times \ln\left(\frac{2000}{101.325}\right)\right] = -256.80 \text{ kJ/kg}$$

$\therefore$ $\phi_1 - \phi_o = 0 + (-81.74) - (-256.80) = 175.06$ kJ/kg

Total availability of compressed air, $W_{max} = m(\phi_1 - \phi_o)$

$$= 38.79 \times 175.06 = 6790.58 \text{ kJ}$$

(b) Second law efficiency, $\eta_{II} = \dfrac{\text{Actual work delivered}}{\text{Maximum possible work}} = \dfrac{2672.26}{6790.58}$

$$= 0.3935 \text{ or } 39.35\%$$

1. (c)

Given: r = 10 cm, $q_g = 4 \times 10^6$ W/m^3, T_a = 150°C, h = 750 W/m^{2}K, k = 50 W/mK

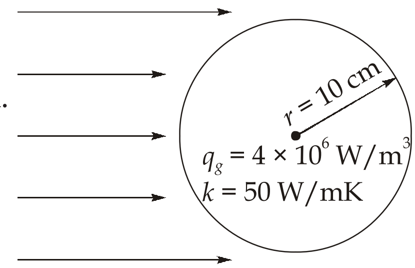
Assumption:

(i) Steady state heat transfer condition.

(ii) Heat conduction takes place only in radial direction.

(iii) Constant thermal conductivity.

(iv) Negligible radiation heat transfer.



$$h = 750 \text{ W/m}^2\text{K}$$
$$T_\infty = 150°\text{C}$$

Now, heat generated in the sphere is convected over the surface.

i.e. $$q_g \times \frac{4}{3}\pi r^3 = h \times 4\pi r^2 \times (T_w - T_a)$$

or, $$q_g \times \frac{r}{3} = h \times (T_w - T_a)$$

$\therefore$ $$T_w = \frac{q_g r}{3h} + T_a = \frac{4 \times 10^6 \times 0.1}{3 \times 750} + 150$$
$$= 327.777°\text{C}$$

Maximum temperature occurs at centre i.e. at $r = 0$; its value is prescribed by the relation.

$$T_{\max} = T_w + \frac{q_g R^2}{6k}$$

$$T_{\max} = 327.327.777 + \frac{4 \times 10^6 \times 0.1^2}{6 \times 50}$$

$\therefore$ $$T_{\max} = 461.110°\text{C}$$ **...Ans.(i)**

Temperature at any radius 'r' can be worked out from the relation,

$$\frac{T - T_w}{T_{\max} - T_w} = 1 - \left(\frac{r}{R}\right)^2$$

$$\frac{T - 327.777}{461.11 - 327.777} = 1 - \left(\frac{0.08}{0.1}\right)^2$$

$$T = 375.776°\text{C}$$ **...Ans.(ii)**

1. (d)

Heat developed by pump (H) = Static lift + Head lost in pipe friction

$$= 25.8 + \frac{fL}{2gd}\left[\frac{4Q}{\pi d^2}\right]^2 = 25.8 + \frac{0.02 \times 10^3}{2 \times 9.81 \times 0.4}\left[\frac{4Q}{\pi \times 0.4^2}\right]^2$$

$= 25.8 + 161.38\ Q^2$

Also, $H = 43.8 + 251Q - 2764Q^2$

From the two expressions for head, we obtain,

$2925.38Q^2 - 251Q - 18 = 0$

On solving, we get $Q = 0.1068\ m^3/s$

∴ Operating head, $H = 25.8 + 161.38 \times 0.1068^2$

$= 27.64$ m

Power output of the pump, $P_o = \rho g Q H = 10^3 \times 9.81 \times 0.1068 \times 27.64$

$= 28.96$ kW

Power required to drive the pump

$$P_{act} = \frac{P_o}{\eta_o} = \frac{28.96}{0.75} = 38.61 \text{ kW}$$

1. (e)

(i) Scavenging is the process of pushing of punching exhausted gas charge out of the cylinder and drawing in fresh charge of air or fuel/air mixture for the next cycle. This process is essential in having a smooth running engines. If scavenging is incomplete, the following stroke will begin with a mix of exhaust fumes rather than clean air. This may be inadequate for proper combustion, leading of poor running conditions.

In the ideal scavenging process, the fresh mixture would push the residual gases without mixing or exchanging heat with them, and this process would continue until all the burned gases had been replaced with fresh mixture, at which point the flow would cease.

Limitations:

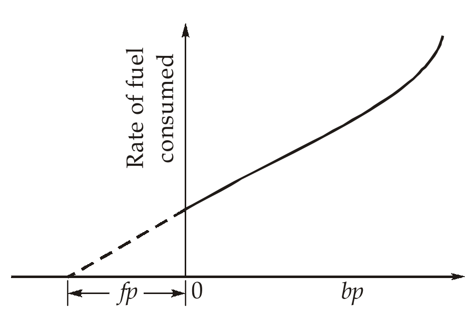
1. In ideal scavenging process, not only is the cylinder completely filled with fresh mixture, but also no fresh mixture escapes from the exhaust ports, and thus all of the mixture supplied remains to take part in the subsequent combustion and expansion. But, in actual engines, the fresh mixture mixes and exchanges heat with residual gases during the actual scavenging process and some portion of the fresh mixture is usually lost through the exhaust ports. Thus two stroke engines never realize the ideal scavenging process.
2. In ideal scavenging, it is assumed that the scavenging process stops suddenly, which is not possible in actual process.

(ii) Mechanism of smoke formation in IC engine is discussed as follows:

1. Smoke formation (solid particles) are usually formed by dehydrogenation, polymerization and agglomeration.
2. In the combustion process of different hydrocarbons, acetylene (C_2H_2) is formed as an intermediate product. These acetylene molecules after simultaneous polymerization and dehydration produce carbon particles, which are the main constituents of the particulates.
3. Smoke emissions increase with the increase in engine load due to overall richer fuel-air mixture ratios and hence the rated engine power was specified based on the maximum permitted smoke density to curb black smoke emissions during engine operation. The rated power is also known as 'smoke limited power'.
4. Poor control of the fuel injection rate during acceleration also increase smoke.

2. (a)

Willan's Line Method: This is a method of determining the friction power and hence the indicated power (ip = bp + fp) of an unthrottled compression ignition engine. This method is not suitable for use with petrol engines. It is based on the fact that at light loads a relatively small amount of fuel is pumped into the air charge. Hence, there is plenty of air available for complete combustion within the engine cylinder. Therefore, at a given engine speed in the light load region, a straight line law exists between the rate at which fuel is consumed and the engine load or brake power. This straight line is Willan's line and is shown in figure. By extrapolation, the fuel flow rate to give zero brake power can be determined. This is the fuel flow rate necessary to overcome friction, and consequently, the amount of negative brake power at zero rate of fuel consumption represents the friction power. From this, the indicated power and mechanical efficiency can be evaluated.

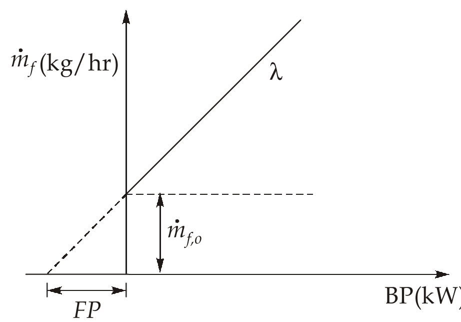


Willan's line

The rapid increase in slope of the line at the high load end denotes a progressive reduction in combustion efficiency as more and more fuel is pumped into the given volume of air. It is therefore important that the extrapolation of Willan's line is carried out as accurately as possible, and that sufficient readings at light load are taken to define the line.

Since a petrol engine is throttled to maintain a high fuel/air ratio with load, combustion is not complete within the cylinder and a plot of brake power versus the rate of fuel consumption does not yield a straight line. Hence extrapolation is virtually impossible.

Refer figure,



Fuel flow rate, $\dot{m}_f = \lambda \times BP + \dot{m}_{f,o}$

$$10 = \lambda \times 100 + \dot{m}_{f,o} \quad \text{... (i)}$$

$$20 = \lambda \times 220 + \dot{m}_{f,o} \quad \text{... (ii)}$$

From equation (i) and (ii), we get

$$\Rightarrow \quad \lambda = \frac{1}{12} \text{ and } \dot{m}_{f,o} = 1.67 \text{ kg/hr}$$

$$\dot{m}_{f,o} = 1.67 \text{ kg/hr}$$ **Ans.**

$$\Rightarrow \quad \dot{m}_f = \frac{1}{12} \times BP + 1.67 \quad \text{... (iii)}$$

At $\dot{m}_f = 0 \Rightarrow$ –BP = FP

So, from equation (iii), we get

$$0 = -\frac{BP}{12} + 1.67$$

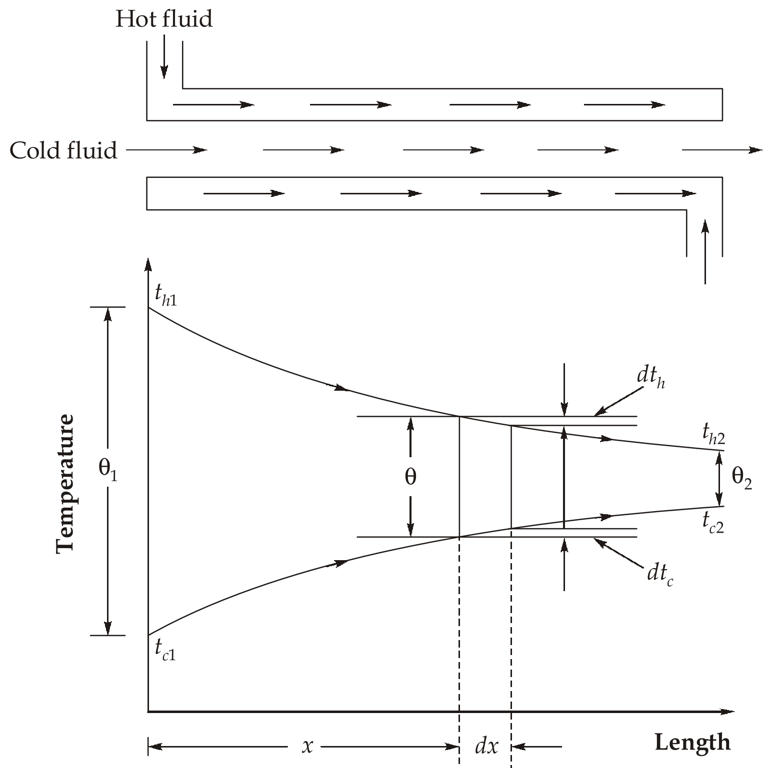
$\Rightarrow$ FP = 1.67 × 12 = 20.04 kW **Ans.**

$$\dot{m}_f = 15 \text{ kg/hr} = \frac{1}{12} \times BP + 1.67$$

$\Rightarrow$ BP = 159.96 kW **Ans.**

$$\therefore \quad \text{bsfc} = \frac{\dot{m}_f}{BP} = \frac{15}{159.96} = 0.09377 \text{ kg/kWh}$$ **Ans.**

2. (b)



Temperature changes of mediums (fluids) during parallel flow arrangement

For a parallel flow heat exchanger, as given in above figure the heat flow dQ through an elementary strip of area dA is given by:

$$dQ = -m_h c_h dt_h = m_c c_c dt_c = -C_h dt_h = C_c dt_c$$

$$dt_h = -\frac{dQ}{C_h} \text{ and } dt_c = \frac{dQ}{C_c}$$

$$d(t_h - t_c) = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$dQ = \frac{-d(t_h - t_c)}{\frac{1}{C_h} + \frac{1}{C_c}} = \frac{-d\theta}{\frac{1}{C_h} + \frac{1}{C_c}} \quad \text{... (i)}$$

Also, $$dQ = UdA\theta = (a + b\theta)\, dA\theta \quad \text{... (ii)}$$

Through integration of expression (i) from θ_1 to θ_2, we get

$$Q = \frac{\theta_1 - \theta_2}{\frac{1}{C_h} + \frac{1}{C_c}} \Rightarrow \left(\frac{1}{C_h} + \frac{1}{C_c}\right) = \frac{\theta_1 - \theta_2}{Q} \quad \text{... (iii)}$$

From expression (i) and (ii)

$$(a + b\theta)dA\theta = \frac{-d\theta}{\frac{1}{C_h} + \frac{1}{C_c}} = \frac{-d\theta}{\frac{(\theta_1 - \theta_2)}{Q}} = \frac{Qd\theta}{\theta_1 - \theta_2}$$

Separating the variable and integrating between the inlet and outlet sections of the heat exchanger:

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta(a + b\theta)} = \frac{\theta_1 - \theta_2}{Q} \int_o^A dA$$

$$\frac{\theta_1 - \theta_2}{Q} A = \frac{1}{a}\log_e\left[\frac{\theta}{a + b\theta}\right]_{\theta_1}^{\theta_2} = \frac{1}{a}\log_e\left[\frac{\theta_2}{a + b\theta_2} - \frac{\theta_1}{a + b\theta_1}\right]$$

$$= \frac{1}{a}\log_e\left[\frac{\theta_2(a + b\theta_1)}{\theta_1(a + b\theta_2)}\right] = \frac{1}{a}\log_e\left(\frac{\theta_2 U_1}{\theta_1 U_2}\right) \quad \text{... (iv)}$$

From the identities, $U_1 = a + b\theta_1$ and $U_2 = a + b\theta_2$, the constant a is found to be

$$a = \frac{U_1\theta_2 - U_2\theta_1}{\theta_2 - \theta_1}$$

Upon substitution of this value in expression (iv),

$$A \cdot \frac{\theta_1 - \theta_2}{Q} = \frac{\theta_2 - \theta_1}{U_1\theta_2 - U_2\theta_1} \times \frac{1}{a}\log_e\left(\frac{\theta_2 U_1}{\theta_1 U_2}\right)$$

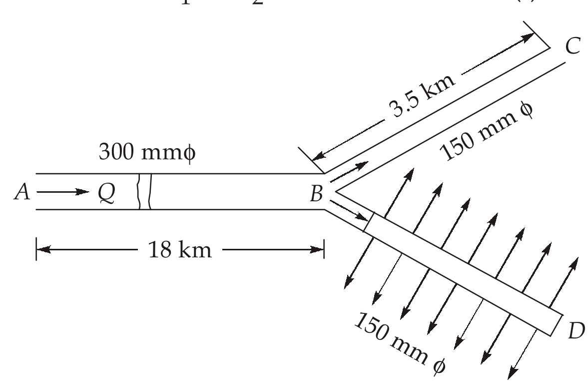
Therefore, the heat exchange rate is

$$Q = \frac{U_1\theta_2 - U_2\theta_1}{\log_e(\theta_2 U_1 / \theta_1 U_2)} A$$

2. (c)

Let Q be the flow rate through the pipe AB and be divided at B into Q_1 and Q_2 for pipes BC and BD, respectively. Then from continuity,

$$Q = Q_1 + Q_2 \quad \text{... (i)}$$



Flow through a branched pipe with side tappings

Since the entire flow at inlet to the pipe BD is drained off through side tapping at a constant rate of 0.01 litre/s per meter length, $Q_2 = 0.01 \times 3500 = 35$ litre/s $= 0.035\ m^3/s$.

Hence, average velocity at the inlet to pipe BD $= \dfrac{4 \times 0.035}{\pi \times 0.15^2} = 1.98$ m/s

$$\text{Loss of lead BD, } (h_f)_{BD} = \frac{1}{3}\frac{fLV^2}{2gD} = \frac{1}{3} \times \frac{0.012 \times 3500 \times 1.98^2}{2 \times 9.81 \times 0.15} = 18.65 \text{ m}$$

Since, B is a common point and C and D are at the same horizontal level and have the same pressure which is equal to that of the atmosphere, the loss of head in the parallel pipe BC and BD are equal.

$\therefore \qquad (h_f)_{BC} = 18.65$ m

$\therefore$ Average flow velocity in pipe BC $= \dfrac{4Q_1}{\pi \times D_1^2} = \dfrac{4 \times Q_1}{\pi \times 0.15^2} = 56.59Q_1$

Equating $(h_f)_{BC}$ with different losses taking place in pipe BC,

$$(h_f)_{BC} = 18.65 = \frac{fL_1V_1^2}{2gD_1} + \frac{V_1^2}{2g}$$

$$= \frac{0.012 \times 3500 \times (56.59Q_1)^2}{2 \times 9.81 \times 0.15} + \frac{(56.59Q_1)^2}{2 \times 9.81}$$

$$18.65 = 45865.6 \times Q_1^2$$

$\Rightarrow \qquad Q_1 = 0.02\ m^3/s$

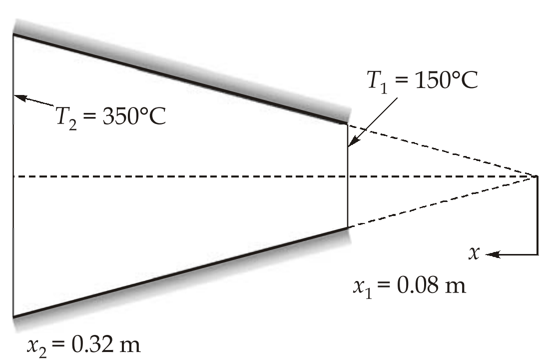
From equation (i), we have

$$Q = Q_1 + Q_2 = 0.035 + 0.02 = 0.055\ m^3/s \qquad \textbf{Ans.}$$

Velocity in the main pipe, AB $= \dfrac{4 \times 0.055}{\pi \times 0.3^2} = 0.78$ m/s

$\therefore$ Head loss in the main pipe Ab $= \dfrac{fLV^2}{2gD} = \dfrac{0.012 \times 18000 \times 0.78^2}{2 \times 9.81 \times 0.3} = 22.33$ m **Ans.**

3. (a)



From Fourier's law of heat conduction,

$$Q = -kA_c \times \frac{dT}{dx} \quad \text{... (i)}$$

Cross -sectional area, $A = \frac{\pi D^2}{4} = \frac{\pi a^2 x^2}{4}$ ... (ii)

From equation (i) and (ii), we get

$$\Rightarrow \quad Q = -k\frac{\pi a^2 x^2}{4} \times \frac{dT}{dx}$$

$$\Rightarrow \quad \frac{4Q}{\pi a^2 k} \times \frac{dx}{x^2} = -dT$$

Integrating above equation from x_1 to x,

$$\Rightarrow \quad \frac{4Q}{\pi a^2 k} \times \int_{x_1}^{x} \frac{dx}{x^2} = -\int_{T_1}^{T} dT$$

$$\Rightarrow \quad \frac{4Q}{\pi a^2 k} \times \left[\frac{1}{x}\right]_{x}^{x_1} = [T]_{T}^{T_1}$$

$$\Rightarrow \quad \frac{4Q}{\pi a^2 k} \times \left[\frac{1}{x_1} - \frac{1}{x}\right] = T_1 - T$$

$$\Rightarrow \quad T = T_1 - \left[\frac{4Q}{\pi a^2 k} \times \left(\frac{1}{x_1} - \frac{1}{x}\right)\right]$$

A $x = x_2$, $T = T_2$

$$\Rightarrow \quad T_2 = T_1 - \left[\frac{4Q}{\pi a^2 k} \times \left(\frac{1}{x_1} - \frac{1}{x_2}\right)\right]$$

$$\Rightarrow \quad \frac{4Q}{\pi a^2 k} \times \left(\frac{x_2 - x_1}{x_1 x_2}\right) = T_1 - T_2$$

$$\Rightarrow \quad Q = \frac{\pi a^2 k (T_1 - T_2)}{4\left(\frac{x_2 - x_1}{x_1 x_2}\right)} = \frac{\pi \times 0.3^2 \times 3.4 \times (150 - 350)}{4\left(\frac{0.32 - 0.08}{0.32 \times 0.08}\right)}$$

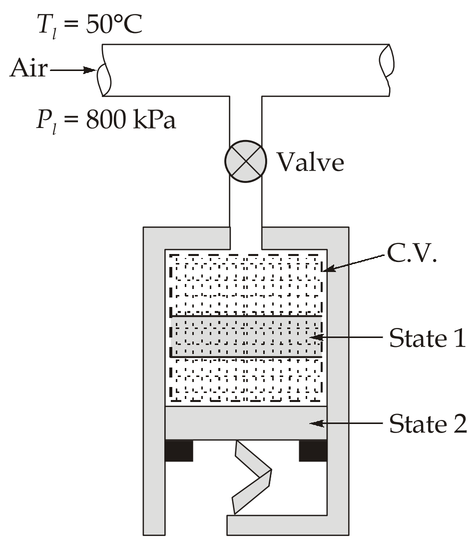
$$\Rightarrow \quad Q = -5.13 \text{ W}$$ **Ans.**

3. (b)

Given: T_∞ = 10°C = 283 K, A_P = 0.1 m^2, V_{stop} = 50L = 50 × 10^{-3} m^3, $C_{P\ air}$ = 1.005 kJ/kgK, R = 0.287 kJ/kgK, k = 100 kN/m, V_1 = 20L = 20 × 10^{-3} m^3, P_1 = 200 kPa, T_1 = 10°C = 283 K, P_l = 800 kPa, T_l = 50°C = 323 K, P_2 = 800 kPa, T_2 = 80°C = 353 K

Assumption: Air as an ideal gas

So, $$m_1 = \frac{P_1V_1}{RT_1} = \frac{200 \times 20 \times 10^{-3}}{0.287 \times 283} = 0.0492 \text{ kg}$$



At state (1), $$P_1A_P = F_S + P_{atm}A_P$$

$$P_1A_P = kx_1 + P_{atm}A_P \quad \text{... (i)}$$

When piston reaches the stoppers,

$$P_{stop}A_P = kx_2 + P_{atm}A_P \quad \text{... (ii)}$$

From equation (i) and (ii),

$$\Rightarrow \quad P_1A_P - P_{stop}A_P = k(x_1 - x_2)$$

$$\Rightarrow \quad (P_{stop})A_P = P_1A_P + k(x_2 - x_1)$$

$$(P_{stop})A_P = P_1A_P + k\left(\frac{V_{stop}}{A_P} - \frac{V_1}{A_P}\right)$$

$$P_{stop} = P_1 + \frac{k}{A_P^2}\left(V_{stop} - V_1\right)$$

$$= 200 + \frac{100}{(0.1)^2}\left(50 \times 10^{-3} - 20 \times 10^{-3}\right)$$

$$= 500 \text{ kPa}$$

Since, $P_2 > P_{stop}$, piston will hit the stops.

Now, $$V_2 = 50 \times 10^{-3} = V_S$$

So, $$m_2 = \frac{P_2V_2}{RT_2} = \frac{800 \times 50 \times 10^{-3}}{0.287 \times 353} = 0.3948 \text{ kg}$$

Now, from mass conservation,

$$\left(\frac{dm}{dt}\right)_{CV} = \dot{m}_i - \cancel{\dot{m}_e}^{\,o} = \dot{m}_i$$

and, $$m_i = m_2 - m_1 \quad \text{... (iii)}$$

From energy conservation,

$$\left(\frac{dU}{dt}\right)_{CV} = \dot{m}_i h_i + \dot{Q} - \cancel{\dot{m}_e}^{\,o} h_e - \dot{W}_{CV}$$

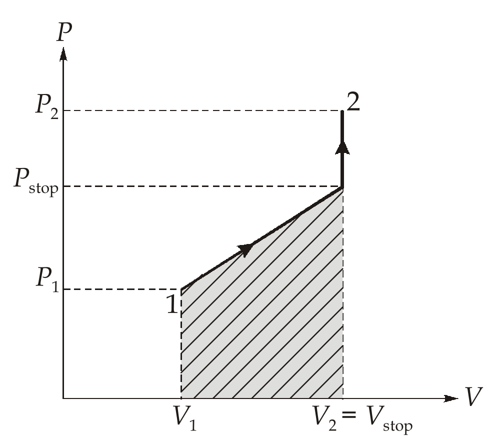
$$\Rightarrow \quad m_2u_2 - m_1u_1 = m_ih_i + Q - W_{CV}$$

$$Q = m_2u_2 - m_1u_1 - m_ih_i + W_{CV}$$

From equation (iii), we get,

$$Q = m_2C_VT_2 - m_1C_VT_1 - (m_2 - m_1)C_PT_i + W_{CV} \quad \text{... (iv)}$$

Now, $$W_{CV} = \int PdV$$



$$W_{CV} = \frac{(P_1 + P_{stop})}{2}(V_{stop} - V_1)$$

$$= \left(\frac{200 + 500}{2}\right)(50 \times 10^{-3} - 20 \times 10^{-3})$$

$$W_{CV} = 10.5 \text{ kJ}$$

From equation (iv), we have

$Q = 0.3948\,(1.005 - 0.287)(353) - 0.0492(1.005 - 0.287)\,(283) - (0.3948 - 0.0492)(1.005)(323) + 10.5$

$Q = 100.0636 - 9.9971 - 112.1869 + 10.50$

$Q = -11.62$ kJ

Now, entropy change associated with process,

$$(\Delta S)_{net} = m_2 s_2 - m_1 s_1 - m_i s_i - \frac{Q_{CV}}{T_\infty}$$

$$= m_2 s_2 - m_1 s_1 - (m_2 - m_1) s_i - \frac{Q_{CV}}{T_\infty}$$

$$= m_2 (s_2 - s_i) - m_1 (s_1 - s_i) - \frac{Q_{CV}}{T_\infty} \qquad \text{... (v)}$$

and

$$s_2 - s_i = C_P \ln\left(\frac{T_2}{T_l}\right) - R\ln\left(\frac{P_2}{P_l}\right)$$

$$= 1.005\ln\left(\frac{353}{323}\right) - 0.287\ln\left(\frac{800}{800}\right) = 0.08925 \text{ kJ/kgK}$$

Also,

$$s_1 - s_i = C_P \ln\left(\frac{T_1}{T_l}\right) - R\ln\left(\frac{P_1}{P_l}\right)$$

$$= 1.005\ln\left(\frac{283}{323}\right) - 0.287\ln\left(\frac{200}{800}\right) = 0.265 \text{ kJ/kgK}$$

Now, from equation (v) we get,

$$(\Delta S)_{net} = 0.3948(0.8925) - 0.0492(0.265) - \frac{(-11.62)}{283}$$

$$= 0.35235 - 0.01304 + 0.04113 = 0.380 \text{ kJ/K}$$

3. (c)

(i) Total black body emissive power: The total emissive power of a black body is defined as the total radiant energy emitted by the surface in all directions over the entire wavelength range per unit surface area per unit time. The basic rate equation for radiation transfer is based on Stefan-Boltzmann law which states that the amount of radiant energy emitted per unit time from unit area of black surface is proportional to the fourth power of its absolute temperature.

$E_b = \sigma_b T^4$, where σ_b = Radiation coefficient of a black body or Stefan-Boltzmann coefficients.

Spectral black body emissive power: The spectral or monochromatic emissive power of a blackbody is defined as the energy emitted by the black surface (in all directions) at a given wavelength λ per unit wavelength interval around λ.

Relation: At any temperature, the area under the monochromatic emissive power versus wavelength gives the rate of radiant energy emitted within the wavelength interval $d\lambda$.

Thus, $$dE_b = (E_\lambda)_b \, d\lambda$$

Upon integration over the entire range of wavelength.

$$E_b = \int_{\lambda=0}^{\lambda=\infty} (E_\lambda)_b \, d\lambda$$

The integral measures the total area under the monochromatic emissive power versus wavelength curve for the blackbody, and it represents the total emissive power per unit area (radiant energy flux density) radiated from a blackbody.

(ii) Given: A_1 = 0.0015 m^2, $A_2 = A_4$ = 0.001 m^2, A_3 = 0.00125 m^2, I_n = 6500 W/m^2-sr.

$r_{21} = r_{31} = r_{41}$ = 0.8 m.

1. for a diffused emitter, the intensity of the emitted radiation is independent of direction. Therefore,

$$I = 6500 \text{ W/m}^2\text{-sr for each of the three directions.}$$

2. In terms of differential surface and the distance radiation travel, the solid angle is given by:

$$dw = \frac{dA_n}{r^2} = \frac{dA\cos\theta}{r^2}$$

where θ is the angle between the surface normal and the direction of radiation. Therefore solid angle subtended by surface A_2 with respect to surface A_1 is

$$w_{2\text{-}1} = \frac{0.001 \times \cos(90^\circ - 55^\circ)}{0.8^2} = 1.28 \times 10^{-3} \text{ sr}$$

Similarly, $$w_{3\text{-}1} = \frac{0.00125 \times \cos 0^\circ}{0.8^2} = 1.95 \times 10^{-3} \text{ sr}$$

$$w_{4\text{-}1} = \frac{0.001 \times \cos 0^\circ}{0.8^2} = 1.56 \times 10^{-3} \text{ sr}$$

3. The rate at which radiation is intercepted by each of three surfaces A_2, A_3 and A_4 can be obtained from the expression, $Q_{1\text{-j}} = I A_1 \cos\theta_1 \, w_{j\text{-}1}$

where,. θ_1 is the angle between normal to the emitting surface A_1 and the direction of propagation of radiation. Therefore

$$Q_{1\text{-}2} = 6500 \times 0.0015 \times \cos 55^\circ \times 1.28 \times 10^{-3}$$
$$= 7.16 \times 10^{-3} \text{ W}$$

$$Q_{1\text{-}3} = 6500 \times 0.0015 \times \cos 0° \times 1.95 \times 10^{-3}$$
$$= 19.01 \times 10^{-3} \text{ W}$$

and
$$Q_{1\text{-}4} = 6500 \times 0.0015 \times \cos 40° \times 1.56 \times 10^{-3}$$
$$= 11.65 \times 10^{-3} \text{ W}$$

4. (a)

Displacement thickness, $\delta^* = \int_o^\delta \left(1 - \frac{U}{U_o}\right) dy = \int_o^\delta \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy$

$$= \left[y - \frac{7}{8} \times \frac{y^{8/7}}{\delta^{1/7}}\right]_o^\delta = \frac{\delta}{8}$$

Momentum thickness, $\theta = \int_o^\delta \frac{u}{U_o}\left(1 - \frac{U}{U_o}\right) dy = \int_o^\delta \left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy$

$$= \int_o^\delta \left[\left(\frac{y}{\delta}\right)^{1/7} - \left(\frac{y}{\delta}\right)^{2/7}\right] dy = \left[\frac{7}{8} \times \frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{9} \times \frac{y^{9/7}}{\delta^{2/7}}\right] = \frac{7\delta}{72}$$

Shape factor, $H = \frac{\delta^*}{\theta} = \frac{\delta/8}{7\delta/72} = 1.285$

Energy thickness, $\delta^E = \int_o^\delta \frac{U}{U_o}\left[1 - \left(\frac{U}{U_o}\right)^2\right] dy$

$$= \int_o^\delta \left[\left(\frac{y}{\delta}\right)^{1/7} - \left(\frac{y}{\delta}\right)^{3/7}\right] dy$$

$$= \left[\frac{7}{8} \times \frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{10} \times \frac{y^{10/7}}{\delta^{3/7}}\right]_o^\delta = \frac{7\delta}{40}$$

For the given boundary layer thickness,

$$\delta^E = \frac{7 \times 3.2}{40} = 0.56 \text{ cm}$$

Energy loss per unit width, due to boundary layer $= \frac{1}{2}\rho U_o^3 \delta^E = \frac{1}{2} \times 1.2 \times 12^3 \times 56 \times 10^{-4}$

$$= 5.806 \text{ Nm}$$

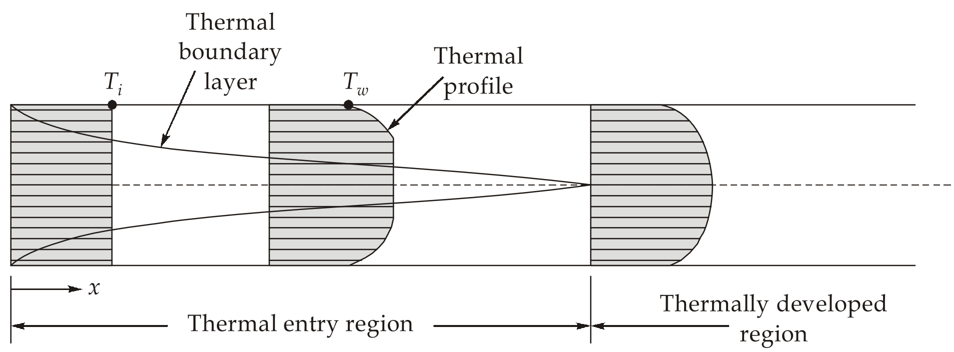
If 'h_L' represent the energy loss in terms of metre of head, then.

$$\rho Q h_L = 5.806$$

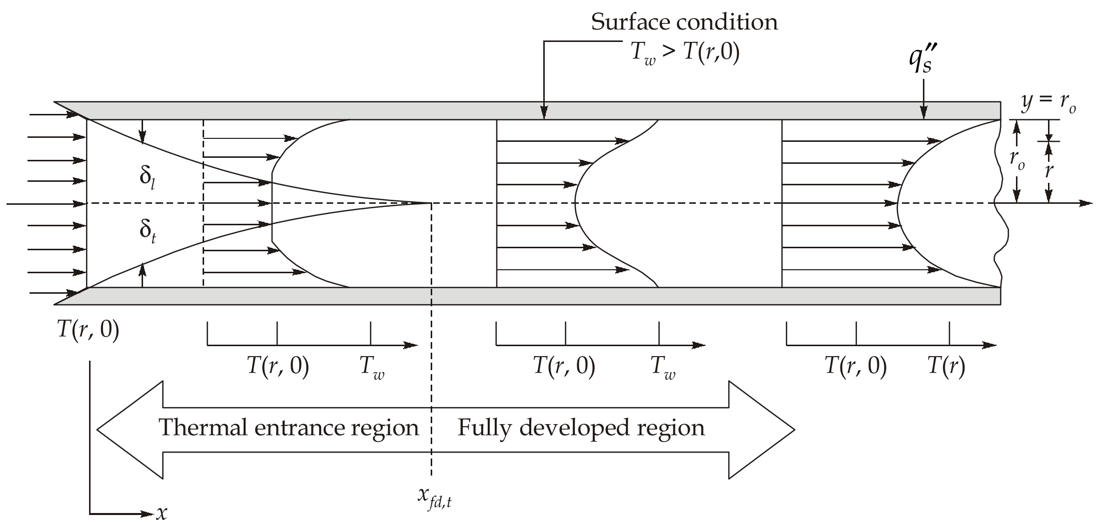
$$\Rightarrow \qquad h_L = \frac{5.806}{1.2 \times 5} = 0.967 \text{ m}$$

4. (b)

(i)



(a)



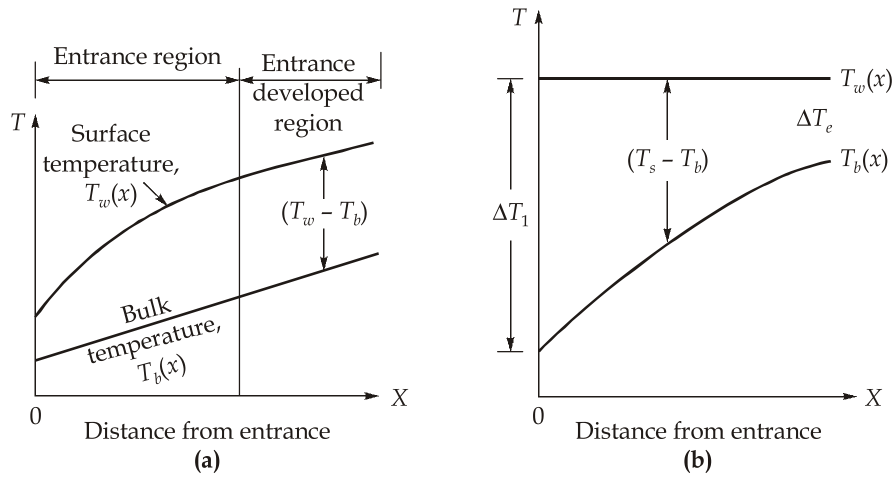
(b)

Let us consider that a fluid at a uniform temperature enters a circular tube with its wall at a different temperature. The fluid particles in the layer in contact with the surface of the tube will assume the tube surface or wall temperature T_w.

This will initiate convection heat transfer in the tube followed by the development of the thermal boundary layer along the tube. The thickness of this thermal boundary layer reaches the tube center and thus fills the entire tube.

The region of flow over which the thermal boundary layer develops and reaches the tube centre is called the thermal entry region. The region beyond the thermal entry region in which the temperature profile remains unchanged is called the thermally developed region/zone.

The dimensionless temperature profile $\left(\dfrac{T - T_w}{T_c - T_w}\right)$ does not also change upstream of thermal entry length. The cone in which the flow is both hydrodynamically and thermally developed is called the fully developed region. The shape of the fully developed temperature profile $T(t, x)$ a uniform heat flux is maintained. For both surface conditions, however, the amount by which fluid temperature exceed the entrance temperature increases with increasing x.



Variation of average bulk temperature of a fluid in a pipe for (a) Constant heat flux and (b) Constant wall temperature

Nusselt number of fully developed laminar flow in a tube is given as,

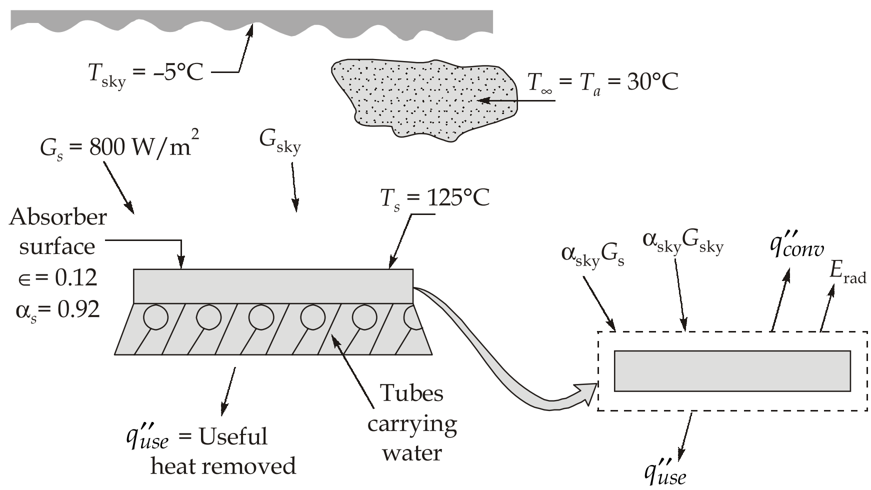
$$\text{Nu}_d = \frac{hD}{k} = \frac{48}{11} = 4.364 \text{ (for constant heat flux)}$$

$$= \frac{hD}{k} = 3.66 \text{ (for } T_w = \text{Constant)}$$

(ii) Referring to figure, using the energy balance on the absorber, $E_{\text{in}} = E_{\text{out}}$

$$\Rightarrow \quad \alpha_s G_s + \alpha_{\text{sky}} G_{\text{sky}} = q''_{conv} + E_{rad} + q''_{use}$$

where E_{rad} = Energy lost due to radiation, W/m^2



(a) Solar collector (b) Control volume

q''_{use} = Useful heat removal rate, W/m^2

q_{conv} = Energy lost due to convection, W/m^2 = $h(T_s - T_\infty)$

$= 0.22(T_s - T_\infty)^{4/3}$

G_{sky} = Earth's irradiation, W/m^2

G_s = Solar irradiation, W/m^2

α = Absorptivity

Since the sky radiation is concentrated in approximately the same spectral region as that of the surface emission, we may reasonably assume

$$\alpha_{sky} = \in = 0.12$$

It is incorrect to assume $\alpha_{sky} = \alpha$ as spectral range of both is much different.
Writing the expressions for various energy terms, we get

$$\alpha_a G_s + \in \sigma T_{sky}^4 = 0.22(T_s - T_\infty)^{4/3} + \in \sigma T_s^4 + q''_{use}$$

$$\Rightarrow q''_{use} = 0.92 \times 800 + 0.12 \times 5.68 \times 10^{-8} \times (268)^4 - 0.22(125 - 30)^{4/3} - 0.12 \times 5.67 \times 10^{-8} \times 398^4$$

$$= 505.07 \text{ W/m}^2$$

$$\text{The efficiency of collector} = \frac{q''_{use}}{G_s} = \frac{505.07}{800} = 0.6313 \text{ or } 63.13\%$$

Comments: If the heat losses due to convection and radiation increase, the useful energy extracted and the collector efficiency will significantly decrease. Use of cover plate will reduce these losses.

4. (c)

Indicated power, IP = imep × LANK

$$= 7.2 \times 100 \times 0.36 \times \frac{\pi}{4} \times 0.18^2 \times \frac{300}{60 \times 2} = 16.49 \text{ kW}$$

$$\text{Brake power, BP} = \frac{2\pi NT}{60} = \frac{2\pi \times 300 \times 420}{60 \times 1000} = 13.19 \text{ kW}$$

Heat equivalent to BP = 13.19 × 60 = 791.4 kJ/min

$$\eta_{ith} = \frac{IP}{m_f \times CV} = \frac{16.49}{\left(\frac{3.5}{3600}\right) \times 46000} = 0.3687 \textit{ For } 36.87\%$$ **Ans.**

$$\text{Air flow rate, } \dot{m}_a = 25 \times \frac{3.5}{3600} = 0.0243 \text{ kg/s}$$

$$\rho_a = \frac{P_a}{RT} = \frac{1.013 \times 100}{0.287 \times 294} = 1.2 \text{ kg/m}^3$$

$$\text{Volume of air consumed, } \dot{V}_a = \frac{\dot{m}_a}{\rho_a} = \frac{0.0243}{1.2} = 0.02025 \text{ m}^3/s$$

$$\dot{V}_s = \frac{\pi}{4} \times D^2 \times L \times \frac{N}{2 \times 60} = \frac{\pi}{4} \times 0.18^2 \times 0.36 \times \frac{300}{2 \times 60}$$

$$= 0.0229 \text{ m}^3/\text{s}$$

$$\therefore \quad \text{Volumetric efficiency, } \eta_v = \frac{\dot{V}_a}{\dot{V}_s} = \frac{0.02025}{0.0229} = 0.8843 \text{ or } 88.43\%$$ **Ans.**

$$\text{Heat input per min} = \dot{m}_f \times CV = \frac{3.5}{60} \times 46000 = 2683.33 \text{ kJ/min}$$

$$\text{Heat lost in cooling water} = m_{co}\, C_{pw}\, \Delta T_{cv}$$

$$= 4.5 \times 4.18 \times 36 = 677.16 \text{ kJ/min}$$

As one kg of H_2 in the fuel will be converted to 9 kg of H_2O during combustion.

$$H_2O = 9 \times \%H_2 \times \dot{m}_f$$

$$= 9 \times 0.15 \times \frac{3.5}{60} = 0.07875 \text{ kg/min}$$

$$\text{Mass of wet exhaust gas/min} = \dot{m}_a + \dot{m}_f = (0.0243 \times 60) + \left(\frac{3.5}{60}\right)$$

$$= 1.5163 \text{ kg/min}$$

∴ Mass of dry exhaust gas/min = 1.5163 – 0.07875 = 1.43755 kg/min

Heat lost to dry exhaust gas/min = 1.43755 × 1.005 × (415 – 21)

$$= 569.22 \text{ kJ/min}$$

Enthalpy of steam h_{steam} = 4.18 × (100 – 21) + 2250 + 2.05 × (415 – 100)

$$= 3225.97 \text{ kJ/kg}$$

∴ Heat carried by steam = 0.07875 × 3225.97

= 254.04 kJ/min

Unaccounted losses = 2683.33 – (791.40 + 677.16 + 569.22 + 254.04)

= 391.51 kJ/min

Heat input per min	(kJ)	S.No.	Heat expenditure per min	(kJ)
Heat supplied by fuel	2683.33	1	BP equivalent	791.4
		2	Heat lost to cooling water	677.16
		3	Heat lost to dry exhaust gas	569.22
		4	Heat carried by steam	254.04
		5	Unaccounted heat losses	391.51

Section : B

5. (a)

In a fuel cell, electrochemical reactions take place whereby reactants are converted to products in a steady flow process. If the temperature and pressure of the flow stream from entrance to exit (during reaction) remain unchanged, from the first law of thermodynamics:

$$\Delta Q - \Delta W = \Delta H + \Delta(\text{KE}) + \Delta(\text{PE}) \quad \text{...(i)}$$

where,

ΔQ = Heat transferred to the steady flow stream from the surrounding.

ΔW = Work done by the flow stream on the surrounding.

ΔH = Change in enthalpy of the flow stream from entrance to exit (of the cell)

The change in KE and PE of the stream are usually negligible. Thus,

$$\Delta W = \Delta Q - \Delta H \quad \text{...(ii)}$$

For ΔW to be the maximum, the process must be reversible. From the second law of thermodynamics, for a reversible process

$$\Delta Q = T\Delta S \quad \text{...(iii)}$$

where T is the temperature of the process and it remains constant.

Thus from equation (ii),

$$\Delta W_{max} = -(\Delta H - T\Delta S) \quad \text{...(iv)}$$

Gibbs free energy is given by,

$$G = H - TS$$

or, $$\Delta G = \Delta H - (T\Delta S - S\Delta T)$$

As there is no change in temperature, $\Delta T = 0$, and thus

$$\Delta G = \Delta H - T\Delta S$$

Therefore, from equation (iv),

$$\Delta W_{max} = -\Delta G \qquad \text{...(v)}$$

Combining equations (iii) and (v), we can write,

$$\Delta G = \Delta H - \Delta Q$$

or,

$$\Delta Q = \Delta H - \Delta G$$

The efficiency of energy conversion of a fuel cell:

$$\eta = \frac{\Delta W}{-\Delta H}$$

$$\text{Maximum efficiency, } \eta_{max} = \frac{\Delta W_{max}}{-\Delta H} = \frac{\Delta G}{\Delta H}$$

5. (b)

The duration of the ignition lag depends on the following factors:

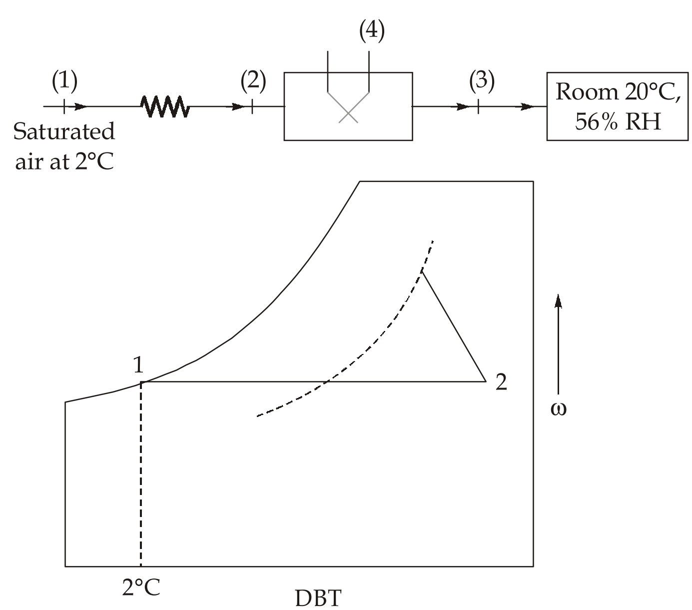
1. **Fuel:** The ignition lag depends on the chemical nature of the fuel. The higher the self-ignition temperature of the fuel, the longer the ignition lag.
2. **Initial temperature and pressure:** The ignition lag, therefore, decreases with an increase in the temperature and pressure of the gas at the time of the spark. Thus, increasing the intake temperature and pressure, increasing the compression ratio and retarding the spark, all reduce the ignition lag.
3. **Electrode gap:** The electrode gap is important from the point of view of establishment of the nucleus of flame. If the gap is too small, quenching of the flame nucleus may occur and the range of fuel-air ratio for the development of a flame nucleus is reduced. The lower the compression ratio, the higher is the electrode gap required. For a compression ratio of 7 or more a gap of 0.625 mm is satisfactory. The voltage required at the spark plug electrode to produce the spark is found to increase with decrease in fuel-air ratio and with increase in compression ratio and engine load.

The ill effect of detonation are as follows:

1. **Noise and Roughness:** Mild knock is seldom audible and is not harmful. When the intensity of the knock increases a loud pulsating noise is produced due to the development of a pressure wave which vibrates back and forth across the cylinder. The presence of vibratory motion causes crankshaft vibrations and the engine runs rough.

2. **Mechanical Damage:** In most cases of knocking a local and very rapid pressure rise is observed with subsequent waves of large amplitudes. This gives rise to increased rate of wear. Erosion of piston crown, in a manner similar to that of marine propeller blades by cavitation, occurs. The cylinder head and valves may also be pitted.
3. **Carbon deposits:** Detonation results in increased carbon deposits.
4. **Increase in heat transfer:** Knocking is accompanied by an increase in the rate of heat transfer to the combustion chamber walls. The increase in heat transfer is due to two reasons. The minor reason is that the maximum temperature in a detonating engine is about 150°C higher than in a non-detonating engine, due to rapid completion of combustion. The major reason for increased heat transfer is the scouring away of protective layer of inactive stagnant gas on the cylinder walls due to pressure waves. The inactive layer of gas normally reduces the heat transfer by protecting the combustion chamber walls and piston crown from direct contact with flame.
5. **Decrease in power output and efficiency:** Due to increase in the rate of heat transfer the power output as well as efficiency of a detonating engine decreases.
6. **Pre-ignition:** The increase in the rate of heat transfer to the walls has yet another effect. It may cause local overheating, especially of the sparking plug, which may reach a temperature high enough to ignite the charge before the passage of spark, thus causing pre-ignition. An engine detonating for a long period would most probably lead to pre-ignition and this is the real danger of detonation.

5. (c)



At 20°C: $P_{vs3} = 2.339$ kPa

$$\phi_3 = \frac{P_{v3}}{P_{vs3}}$$

$\Rightarrow$ $$P_{v3} = \phi_3 \times P_{vs3} = 0.5 \times 2.339 = 1.1695 \text{ kPa}$$

$$\omega_3 = \frac{0.622 \times P_{v3}}{P_t - P_{v3}} = \frac{0.622 \times 1.1695}{101.3 - 1.1695}$$

$$= 7.265 \times 10^{-3} \text{ kg/kg of d.a.}$$

At 2°C: $$P_{vs1} = 0.7156 \text{ kPa}$$

$$\phi_1 = \frac{P_{v1}}{P_{vs1}} = 1$$

$\Rightarrow$ $$P_{v1} = P_{vs1} = 0.7156 \text{ kPa}$$

$$\omega_1 = \frac{0.622 \times P_{v1}}{P_t - P_{v1}} = \frac{0.622 \times 0.7156}{101.3 - 0.7156}$$

$$= 4.425 \times 10^{-3} \text{ kg/kg of d.a.}$$

$$\omega_3 - \omega_1 = 7.265 \times 10^{-3} - 4.425 \times 10^{-3} = 2.84 \times 10^{-3} \text{ kg/kg of d.a.}$$

$$v_{a3} = \frac{R_a T_3}{P_{a3}} = \frac{0.287 \times 293}{(101.3 - 1.1695)} = 0.8398 \text{ m}^3\text{/kg of d.a.}$$

$$\text{Mass of spray water required} = \frac{2.84 \times 10^{-3}}{0.8398} = 3.382 \times 10^{-3} \text{ kg moisture/m}^3.$$ **Ans.**

From energy balance,

$$h_2 + (\omega_3 - \omega_2)h_4 = h_3$$

$$\Rightarrow [C_P\, t_{db2} + \omega_2\, h_{vapour,2}] + (\omega_3 - \omega_2) \times h_4 = C_P\, t_{db3} + \omega_3\, h_{vapour,3}$$

$$\Rightarrow C_P\,(t_{db3} - t_{db2}) + \omega_3\, h_{vapour,3} - \omega_2\, h_{vapour,2} - (\omega_3 - \omega_2) \times h_4 = 0$$

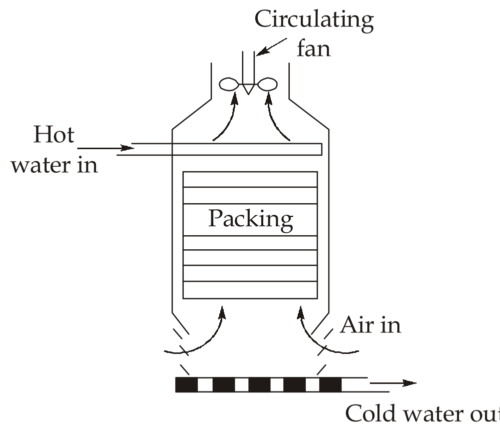
$$\Rightarrow 1.005 \times (20 - t_{db2}) + 7.265 \times 10^{-3} \times [2518 + 1.884 \times (20 - 9.65)] - 4.425 \times 10^{-3} \times [2518 + 1.884 \times (t_{db2} - 9.65)] - (2.84 \times 10^{-3} \times 4.187 \times 10) = 0$$

$$\Rightarrow 20.1 - 1.005 \times t_{db2} + 18.435 - 11.142 - 8.337 \times 10^{-3}\,(t_{db2} - 9.65) - 0.1189 = 0$$

$\Rightarrow$ $$t_{db2} = 26.99°\text{C}$$

5. (d)

Cooling towers in thermal power plants serve the purpose of removing excess heat from the plant. They do this by transferring the heat to the atmosphere. The hot water from the plant is circulated through the cooling tower, where some of it evaporates, taking away heat and cooling the remaining water, which is then recirculated back into the plant. This process helps maintain the efficiency and proper functioning of the power plant by preventing overheating.



Air entering the tower is unsaturated and as it comes in contact with the water spray, water continues to evaporates till the air becomes saturated. So, the minimum temperature to which water can be cooled is the adiabatic saturation or wet bulb temperature of the ambient air.

At this temperature (WBT), air is 100% saturated and cannot absorb any more water vapour. Hence, there will be no further evaporation and cooling. The humid air while moving up comes in contact with warm water spray and so the air temperature rises.

A cooling tower is specified by (i) approach, (ii) range, and (iii) cooling efficiency. The approach (A) is defined at the difference between the exit temperature of cooling water and the wet bulb temperature of the ambient air, or

$$A = t_{C2} - t_{wb}$$

Warm water from the condenser enters the cooling tower at temperature t_{C1} and is cooled to temperature T_{C2}, higher than the minimum value, the wet bulb temperature, t_{wb}, and this unattainable temperature difference is the approach. The approach varies from 6°C to 8°C.

The cooling range or simply range (R) is the defined as the difference in temperature of the incoming warm water (t_{C1}) and the exiting cooled water (t_{C2}), or

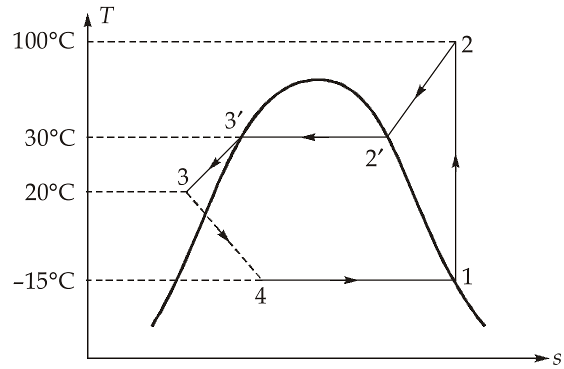
$$R = t_{C1} - t_{C2}$$

It is the range by which warm water from condenser is cooled. The range varies from 6°C to 10°C. The cooling efficiency is defined as the ratio of the actual cooling of water to the maximum cooling possible, or

$$\eta_{cooling} = \frac{\text{Actual cooling}}{\text{Maximum cooling possible}} = \frac{t_{C1} - t_{C2}}{t_{C1} - t_{wb}}$$

5. (e)

Given: $\dot{V}_P = 2\ m^3/min$, $P_C = 12$ bar, $P_e = 2.5$ bar, $Q_{rej,\ cond} = 5040$ kJ/hr $= 1.4$ kW, $\eta_V = 0.82$.



$h_1 = 1426.58$ kJ/kg; $h_{2'} = 1468.87$ kJ/kg

$h_{3'} = 323.08$ kJ/kg, $v_1 = 0.5098\ m^3/kg$

$h_2 = h_{2'} + C_{PV} \times (T_2 - T_{2'}) = 1468.87 + 2.763 \times (100 - 30)$

$= 1662.28$ kJ/kg

$h_3 = h_{3'} - C_{Pl}\,(T_{3'} - T_3) = 323.08 - 4.606 \times (30 - 20)$

$= 277.02$ kJ/kg $= h_4$

$$\eta_V = \frac{\dot{m}_R v_1}{\dot{V}_P} \Rightarrow \dot{m}_R = \frac{\eta_V \times \dot{V}_P}{v_1} = \frac{0.82 \times 2}{0.5098 \times 60}$$

$\Rightarrow$ $\dot{m}_R = 0.0536$ kg/s

Refrigeration capacity, $RC = \dot{m} \times (h_1 - h_4)$

$= 0.0536 \times (1426.58 - 277.02) = 61.61$ kW

Work done during compression of the refrigerant

$$\dot{W}_C = \dot{m}_R \times (h_2 - h_1) = 0.0536 \times (1662.28 - 1426.58)$$

$= 12.63$ kW

Indicated power of the system= Total work done by the system

$= \dot{W}_C + Q_{rej,cond}$

$= 12.63 + 1.4 = 14.03$ kW

$$\text{COP of the system} = \frac{\text{Refrigeration capacity}}{\text{Total work done}} = \frac{61.61}{14.03} \simeq 4.39$$

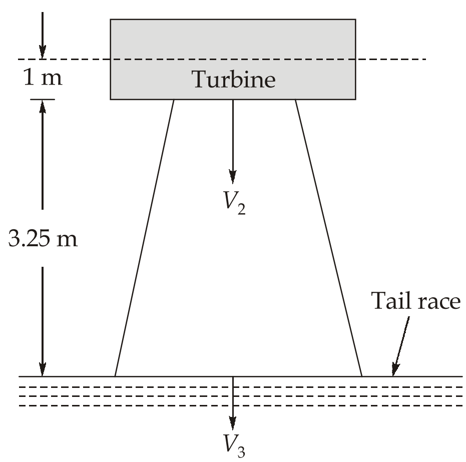
6. (a)

(a) Total head across the turbine (above the tail race level)

= Total head in the spiral casing measured above the tail race

or
$$H = \frac{P}{\gamma} + \frac{V_2^2}{2g} + (1 + 3.25)$$

$$= 50 + \frac{5^2}{2 \times 9.81} + 4.25 = 55.52 \text{ m}$$ **Ans.**



(b) Shaft power, $P = \rho g Q H.\eta_o = 9.81 \times 10^3 \times 2.5 \times 55.52 \times 0.8$

$P = 1089.3$ kW **Ans.**

At draft tube exit, $$V_3 = \frac{4Q}{\pi d^2} = \frac{4 \times 2.5}{\pi \times (1.5)^2} = 1.414 \text{ m/s}$$

(c) Head utilized by the turbine $= H - h_{ft} - h_{fd} - \frac{V_3^2}{2g} = 55.52 - \left(h_{ft} + h_{fd}\right) - \frac{1.414^2}{2 \times 9.81}$

$$H_{net} = 55.418 - (h_{ft} + h_{fd})$$

Hydraulic efficiency, $$\eta_h = \frac{\text{Head utilized by the turbine}}{\text{Head supplied to the turbine}}$$

$\therefore$
$$0.83 = \frac{55.418 - \left(h_{ft} + h_{fd}\right)}{55.52}$$

or
$$h_{ft} + h_{fd} = 9.336 \text{ m}$$

$\therefore$ Combined Head lost due to friction in turbine and draft tube = 9.336 m

Ans.

(d) Power delivered to the shaft = shaft power + Power lost in mechanical friction

$$= P_s + P_f$$

Mechanical efficiency,

$$\eta_{mech} = \frac{\text{Shaft power}}{\text{Power delivered to the shaft}} = \frac{P_s}{P_s + P_f}$$

or, $$\frac{1}{\eta_{mech}} = \frac{P_s + P_f}{P_s} = 1 + \frac{P_f}{P_s}$$

or, $$P_f = P_s \times \left(\frac{1}{\eta_{mech}} - 1\right)$$

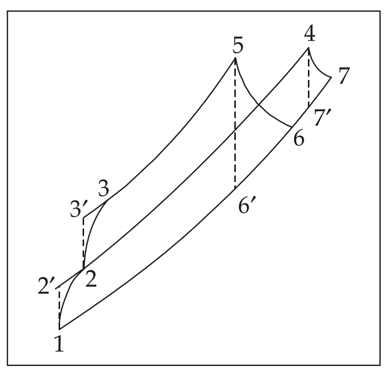
$\because$ $$\eta_{mech} = \frac{\eta_o}{\eta_h} = \frac{0.8}{0.83} = 0.9638$$

$$P_f = 1089.3\left(\frac{1}{0.9638} - 1\right)$$

$\therefore$ $$P_f = 40.91 \text{ kW}$$ **Ans.**

6. (b)

Refer to figure,



$$T_2 = T_1\left[1 + \frac{1}{\eta_{isen,c}}\left(r_{LP}^{(\gamma-1)/\gamma}\right)\right] = 288\left[1 + \frac{1}{0.85}\left(3^{0.4/1.4} - 1\right)\right]$$

$$= 412.94 \text{ K}$$

Similarly $$T_3 = T_2\left[1 + \frac{1}{\eta_{isen,c}}\left(r_{HP}^{(\gamma-1)/\gamma}\right)\right] = 412.94\left[1 + \frac{1}{0.85}\left(3^{0.4/1.4} - 1\right)\right]$$

$$= 592.07 \text{ K}$$

Now, $$T_{6'} = \frac{T_5}{(9)^{(1.33-1)/1.33}} = \frac{853}{(9)^{0.33/1.33}} = 494.51 \text{ K}$$

and $$T_6 = T_5 - \eta_{isen,T}(T_5 - T_{6'}) = 853 - 0.88\,(853 - 494.51)$$

$$= 537.53 \text{ K}$$

Let x be the mass of air passing through the power turbine, then

$$\dot{m}_a C_{pa}\left[(T_2 - T_1) + (1-x)(T_3 - T_2)\right] = \dot{m}_a C_{pg}(1-x)(T_5 - T_6)$$

or,
$$(T_2 - T_1) = (1-x)\left[\frac{C_{pg}}{C_{pa}}(T_5 - T_6) - (T_3 - T_2)\right]$$

or,
$$x = 1 - \frac{T_2 - T_1}{\frac{C_{pg}}{C_{pa}}(T_5 - T_6) - (T_3 - T_2)}$$

$$= 1 - \frac{412.94 - 288}{\frac{1.147}{1.005}(853 - 537.53) - (592.07 - 412.94)}$$

$$= 0.309$$ **Ans.**

$\therefore$ Percentage of the total air intake that passes to the power turbine = 30.9%.

Now,
$$T_7 = T_4\left[1 - \eta_T\left(1 - \frac{1}{3^{\gamma_g - 1/\gamma_g}}\right)\right]$$

$$= 973\left[1 - 0.88\left(1 - \frac{1}{3^{0.33/1.33}}\right)\right] = 768.7 \text{ K}$$

$\therefore$
$$\eta_{th} = \frac{xC_{pg}(T_4 - T_7)}{(1-x)C_{pg}(T_5 - T_3) + xC_{pg}(T_4 - T_2)}$$

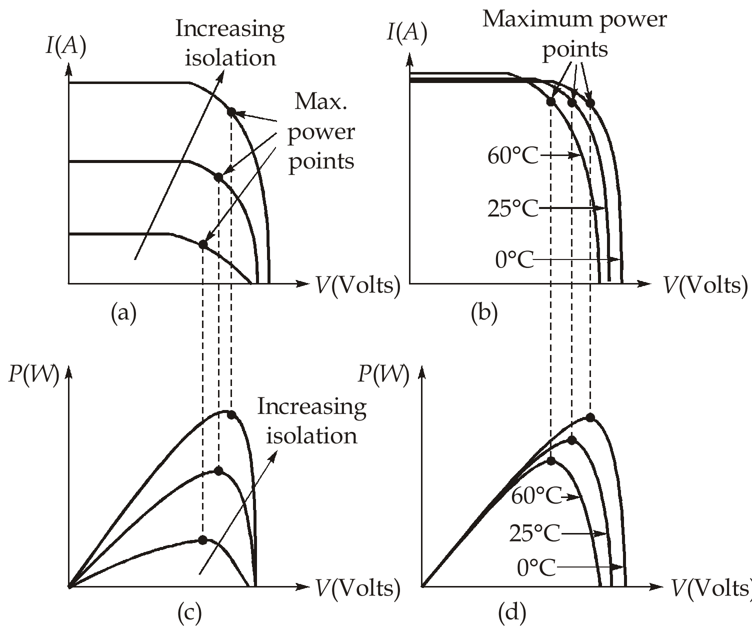
$$= \frac{0.309 \times 1.147(973 - 768.7)}{(1 - 0.309) \times 1.147(853 - 592.07) + 0.309 \times 1.147(973 - 412.94)}$$

$$\eta_{th} = 0.1786 \text{ or } 17.86\%$$ **Ans.**

6. (c)

(i)

As the insolation keeps on varying throughout the day, it is important to observe its effects on PV characteristics. If the spectral content of the radiation remains unaltered, and temperature and all other factors remain same, both I_{sc} and V_{oc} increase with increasing the intensity of radiation. The photo-generated current depends directly on insolation. Therefore, the short-circuit current depends linearly while the open-circuit voltage depends logarithmically on the insolation. This is is shown in figure (a, c).



Effect of variation of (a) and (c) insolation and (b) and (d) Temperature on the characteristic of solar cell.

An illuminated PV cell converts only a small fraction (approx. less than 20%) of irradiance into electrical energy. The balance is converted into heat, resulting into heating of the cell. As a result, the cell can be expected to operate above the ambient temperature. Keeping insolation level as constant, if the temperature is increased, there is a marginal increase in the cell current but a marked reduction in the cell voltage. An increase in temperature causes reduction in the band gap. This, in turn, causes some increase in photo-generation rate and thus a marginal increase in current. However, the reverse saturation current increase rapidly with temperature. Due to this, the cell voltage decreases by approximately 2.2 mV per °C rise in its operating temperature, depending on the resistivity of the silicon used-higher the silicon resistivity, more marked is the temperature effect. Also, the fill factor decreases slightly with temperature. This is shown in figure (b, d).

(ii)

Given : V_{oc} = 0.8V, I_{sc} = 220 A/m^2, T = 50 + 273 = 323 K

$$\text{Load current, } I = I_{sc} - I_0\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

where I_0 = Reverse saturation current; V = Voltage; k = Boltzmann constant; T = Absolute temperature, I_{sc} = Short circuit current

For an open circuit, $I = 0$, $V = V_{oc}$,

$$\therefore \quad \frac{I_{sc}}{I_o} = \exp\left(\frac{eV_{oc}}{kT}\right) - 1$$

or

$$I_0 = \frac{I_{sc}}{\exp\left(\frac{eV_{oc}}{kT}\right) - 1}$$

Now,

$$\frac{eV_{oc}}{kT} = \frac{1.602 \times 10^{-19} \times 0.8}{1.381 \times 10^{-23} \times 323} = 28.73$$

$$\therefore \quad I_0 = \frac{220}{\exp(28.73) - 1} = 7.33 \times 10^{-11}\,\text{A/m}^2$$

Power obtained from photovoltaic cell is

$$P = \left[I_{sc} - I_0\left(\exp\left(\frac{eV}{kT}\right) - 1\right)\right] \times V$$

Maximum power is obtained if,

$$\frac{dP}{dV} = 0, \text{ we get}$$

$$\left(1 + \frac{eV_m}{kT}\right)\exp\left(\frac{eV_m}{kT}\right) = 1 + \frac{I_{sc}}{I_o}$$

$$\frac{e}{kT} = \frac{1.602 \times 10^{-19}}{1.381 \times 10^{-23} \times 323} = 35.91$$

$$\therefore \quad (1 + 35.91\,V_m)\exp(35.91\,V_m) = 1 + \frac{220}{7.33 \times 10^{-11}}$$

or $(1 + 35.91 V_m)\exp(35.91 V_m) = 3 \times 10^{12}$

On solving, we get

$$V_m = 0.7088\text{V}$$ Ans.

Maximum current density, $I_m = I_{sc} - I_0\left(\exp\left(\frac{eV_m}{kT}\right) - 1\right)$

$$= 220 - 7.33 \times 10^{-11}(\exp(35.91 \times 0.7088) - 1)$$

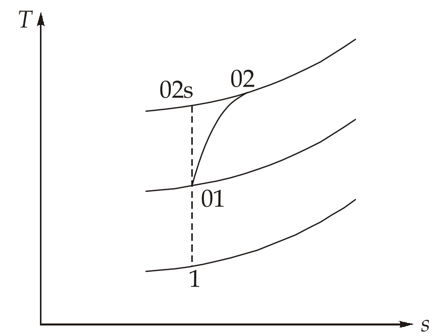
$$I_m = 211.7\ \text{A/m}^2$$ Ans.

$\therefore$ Maximum power, $P_m = I_m \times V_m = 211.7 \times 0.7088$

$$P_m = 150.05\ \text{W/m}^2$$ Ans.

7. (a)

Refer to figure.



$$T_{02s} = T_{01}\,(r_1)^{\gamma-1/\gamma} = 293 \times 5^{(0.4/1.4)} = 464.059 \text{ K}$$

Also,
$$T_{02} = T_{01} + \frac{T_{02s} - T_{01}}{\eta_{isen,C}} = 293 + \frac{464.059 - 293}{0.82} = 501.6 \text{ K}$$

$$U_2^2 = C_p\Delta T = C_P\,(T_{02} - T_{01})$$
$$= 1.005 \times (501.6 - 293) \times 10^3$$

or,
$$U_2 = 457.867 \text{ m/s}$$

$\therefore$
$$W_C = U_2^2 = 457.867^2 = 209.64 \text{ kW/kg/s}$$

Now,
$$U_2 = \frac{\pi D_2 N}{60}$$

$\therefore$
$$D_2 = \frac{60 \times 457.867}{\pi \times 12000} = 0.728 \text{ m}$$ **Ans.**

and
$$D_1 = \frac{D_2}{2} = \frac{0.728}{2} = 0.364 \text{ m}$$ **Ans.**

Now, Static temperature and pressure,

$$T_1 = T_{01} - \frac{V_1^2}{2C_P} = 293 - \frac{60^2}{2 \times 1005} \qquad [\because V_1 = V_f]$$
$$= 291.2 \text{ K}$$

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 1 \times \left(\frac{291.2}{293}\right)^{\frac{1.4}{0.4}} = 0.978 \text{ bar}$$

$\therefore$ Density of air, $\rho_1 = \dfrac{P_1}{RT_1} = \dfrac{0.978 \times 10^2}{0.287 \times 291.2} = 1.17 \text{ kg/m}^3$

Power input to the compressor,

$$p_c = \rho Q \times W_C = 1.17 \times \frac{500}{60} = 209.64$$

$= 2045.90 \text{ kW}$ **Ans.**

Flow rate, $\dot{Q} = A \times V_F = \pi D_1 b_1 \times V_F$

$\therefore \quad \frac{500}{60} = \pi \times 0.364 \times b_1 \times 60$

$\therefore \quad b_1 = 0.12 \text{ m}$ **Ans.**

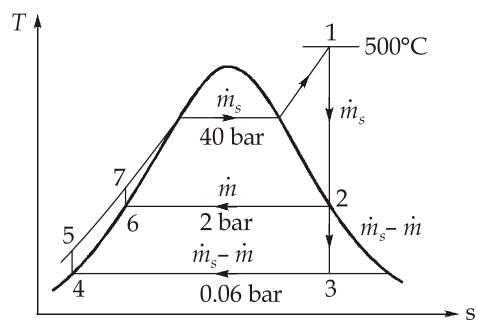
Now, $U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.364 \times 12000}{60} = 228.7 \text{ m/s}$

$\therefore \quad \alpha_1 = \tan^{-1}\left(\frac{V_F}{U_1}\right) = \tan^{-1}\left(\frac{60}{228.7}\right) = 14.7°$ **Ans.**

$\therefore \quad \alpha_2 = \tan^{-1}\left(\frac{V_F}{U_2}\right) = \tan^{-1}\left(\frac{60}{457.867}\right) = 7.46°$ **Ans.**

7. (b)

With reference to figure:



From steam table:

$h_1 = 3446.0 \text{ kJ/kg}$

$s_1 = 7.0922 \text{ kJ/kgK} = s_2 = s_3$

$h_2 = h_g = 2706.2 \text{ kJ/kg}$

$h_2 - h_6 = h_{fg} = 2201.5 \text{ kJ/kg}$

$h_{f3} = 151.48 \text{ kJ/kg}, h_{fg3} = 2415.2 \text{ kJ/kg},$

$s_{f3} = 0.52082 \text{ kJ/kgK}$

$s_{fg3} = 7.8082 \text{ kJ/kgK}$

If $\dot{m}$ is the rate of the extraction for process heating,

$\dot{m}(h_2 - h_6) = 1.2 \times 10^3$

$\dot{m} = \frac{1.2 \times 10^3}{2201.5} = 0.545 \text{ kg/s}$

Now, $s_1 = s_3 = s_{f3} + x_3 s_{fg3}$

$$7.0922 = 0.52082 + x_3 \times 7.8082$$

$\therefore$ $$x_3 = 0.8416$$

$\therefore$ $$h_3 = h_{f3} + x_3 \times h_{fg3} = 151.48 + 0.8416 \times 2415.2$$
$$= 2184.11 \text{ kJ/kg}$$

Total work output, $$W_T = \dot{m}_s(h_1 - h_2) + (\dot{m}_s - m)(h_2 - h_3)$$

$$6.5 \times 10^3 = \dot{m}_s(3446 - 2706.2) + (\dot{m}_s - 0.545)(2706.2 - 2184.11)$$

$\therefore$ $$6.5 \times 10^3 = 739.8\dot{m}_s + 522.09\dot{m}_s - 284.539$$

$\therefore$ $$\dot{m}_s = 5.37 \text{ kg/s or } 19.35 \text{ t/h}$$ **Ans. (a)**

Now, $$h_7 = h_6 + v_6\Delta p = 504.7 + 0.00106052 \times (40 - 2) \times 10^2$$
$$h_7 = 508.73 \text{ kJ/kg}$$
$$h_5 = h_4 + v_1\Delta p = 151.48 + 0.00100645 \times 100\ (40 - 0.06)$$
$$= 155.5 \text{ kJ/kg}$$

$\therefore$ $$Q_1 = (\dot{m}_s - m)(h_1 - h_5) + m(h_1 - h_7)$$
$$= (5.37 - 0.545)(3446 - 155.5) + 0.545(3446 - 508.73)$$
$$= 17.477 \text{ MW}$$ **Ans. (b)**

$$\eta_{boiler} = \frac{Q_1}{\dot{m}_f \times CV}$$

$$0.88 = \frac{17.477}{\dot{m}_f \times 30} \Rightarrow \dot{m}_f = 0.66 \text{ kg/s}$$

or, $$\dot{m}_f = 2.38 \text{ t/h}$$ **Ans. (c)**

$$Q_2 = (m_s - m)(h_3 - h_4) = (5.37 - 0.545)(2184.11 - 151.48)$$
$$= 9807.44 \text{ kW}$$
$$Q_2 = 9.807 \text{ MW}$$ **Ans. (d)**

7. (c)

Given: $U_H = 10$ m/s, $H = 15$ m, $Z = 120$ m, $\rho = 1.23$ kg/m^3, $D = 60$ m,

$A_1 = \frac{\pi}{4} \times 60^2 = 2827.43 \text{ m}^2$, $U_1 = 0.8U_o$, $\eta_{gen} = 0.85$

Now, $$U_Z = U_H\left(\frac{Z}{H}\right)^{\alpha} = 10\left(\frac{120}{15}\right)^{0.14} = 13.38 \text{ m/s} = U_o$$

and $U_1 = 0.8U_o = 0.8 \times 13.38 = 10.704$ m/s

Total power available in wind,

$$P_o = \frac{1}{2}\rho A U_o^3 = \frac{1}{2} \times 1.23 \times 2827.43 \times 13.38^3$$

$\therefore$ $P_o = 4165.2$ kW **Ans. (i)**

Interference factor, $a = \dfrac{U_o - U_1}{U_o} = \dfrac{13.38 - 10.704}{13.38} = 0.2$

Power coefficient, $C_p = 4a(1-a)^2 = 4 \times 0.2(1-0.2)^2 = 0.512$

$\therefore$ Power extracted by the turbine,

$P_T = C_P P_o = 0.512 \times 4165.2 = 2132.58$ kW **Ans. (ii)**

Electrical power generated, $P_E = \eta_{gen} \times P_T = 0.85 \times 2132.58 = 1812.7$ kW **Ans. (iii)**

Axial thrust on the turbine, $F_A = 4a(1-a) \times \dfrac{\rho U_o^2}{2} \times A$

$$= 4 \times 0.2 \times 0.8 \times \frac{1.23 \times 13.38^2}{2} \times 2827.43$$

$= 199.23$ kN **Ans. (iv)**

Maximum axial thrust on the turbine,

when, $a = 0.5, C_F = 1$

$$F_{A\,max} = \frac{\rho U_o^2}{2} \times A_1 = \frac{1.23 \times 13.38^2}{2} 2827.43$$

$= 311.3$ kN **Ans. (v)**

8. (a)

(i) The factors affecting the amount of draught to be produced in a boiler are:

1. The average gas temperature (T_g). Higher is the T_g, higher is the draught produced.
2. The ambient temperature (T_a).
3. The mass flow of flue gases through the chimney in a plant.
4. Velocity of flue gases flowing through the chimney.

The merits of induced draught over the forced draught are:

1. These are normally located at the foot of the stack and handle hot combustion gases.
2. It uses a more compact ground area than a forced draught.

3. Fan equipment in warm exhaust gases is less liable to icing up in winter operation.

Demerits

1. The power requirements are greater than forced draught fans.
2. They must cope with corrosive combustion products and fly ash. Therefore they face high maintenance problems.
3. Their capital and operating costs are higher.
4. There are chances of air leakage as compared to forced draught due to negative pressure at system upstream.

(ii) Density of air, $\rho_a = \frac{P}{RT} = \frac{101.3}{0.287 \times 300} = 1.176 \text{ kg/m}^3$

$$\text{Velocity head}, = \frac{\rho V^2}{2} = \frac{1.176 \times 12^2}{2} = 84.67 \text{ N/m}^2$$

$$\text{Static pressure head} = 25 \text{ mm of water} = 0.025 \times 10^3 \times 9.81$$
$$= 245.25 \text{ N/m}^2$$

Total pressure head to be produced by the fan,

$$\Delta P = 245.25 + 84.67 = 329.92 \text{ N/m}^2$$

Volume of air to be handled,

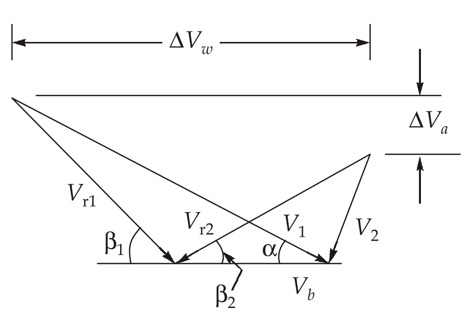
$$\dot{V} = \frac{2500}{3600} \times \frac{15}{1.176} = 8.857 \text{ m}^3/s$$

∴ Power required by the FD fan,

$$\dot{P} = \frac{\Delta P \cdot \dot{V}}{1000} = \frac{329.92 \times 8.857}{1000} = 2.92 \text{ kW} \qquad \textbf{Ans.}$$

8. (b)

Given: $\alpha = 20°$, $\beta_2 = 30°$, $U = 120$ m/s, $V_1 = 350$ m/s



Refer to figure, $\tan\beta_1 = \frac{V_1 \sin\alpha}{V_1 \cos\alpha - U} = \frac{350 \sin 20°}{350 \cos 20° - 120}$

$\therefore \quad \beta_1 = 29.81°$

$$V_{r1} = \frac{V_1 \sin\alpha}{\sin\beta_1} = \frac{350 \sin 20°}{\sin 29.81°} = 239.41 \text{ m/s}$$

$$V_{r2} = 0.8 \times 239.41 = 191.53 \text{ m/s}$$

$$\Delta V_w = V_{r1} \cos\beta_1 + V_{r2} \cos\beta_2$$

$$= 239.41 \cos 29.81 + 191.53 \cos 30 = 373.6 \text{ m/s}$$

$$W_D = \dot{m}_s \Delta V_w \cdot u = 1 \times 373.6 \times 120 = 44.83 \text{ kJ/kg}$$

Diagram efficiency, $\eta_D = \frac{2 \times \Delta V_w \times u}{V_1^2} = \frac{2 \times 373.6 \times 120}{350^2} = 0.7319$ or 73.19%

$\therefore$ Stage efficiency, $\eta_{stage} = \eta_{nozzle} \times \eta_{blade} = 0.85 \times 0.7319 = 0.6221$ or 62.21%

$$\eta_{internal} = \eta_{stage} \times \text{reheat factor}$$

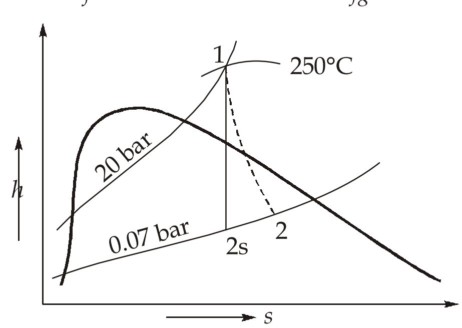
$$= 0.6221 \times 1.06 = 0.6594 \text{ or } 65.94\%$$

Now, from steam table

At 20 bar, 250°C, $h_1 = 2903.2$ kJ/kg, $s_1 = 6.5475$ kJ/kgK

At 0.07 bar, $h_f = 163.35$ kJ/kg, $h_{fg} = 2408.4$ kJ/kg

$s_f = 0.55903$ kJ/kg, $s_{fg} = 7.7154$ kJ/kg



Now, $s_1 = s_{2s}$

or, $6.5475 = 0.55903 + x_{2s} \times 7.7154$

$\therefore \quad x_{2s} = 0.7761$

$$h_{2s} = 163.35 + 0.7761 \times 2408.4 = 2032.509 \text{ kJ/kg}$$

$$h_1 - h_2 = \eta_{int} \times (h_1 - h_{2s}) = 0.6594 \times (2903.2 - 2032.509)$$

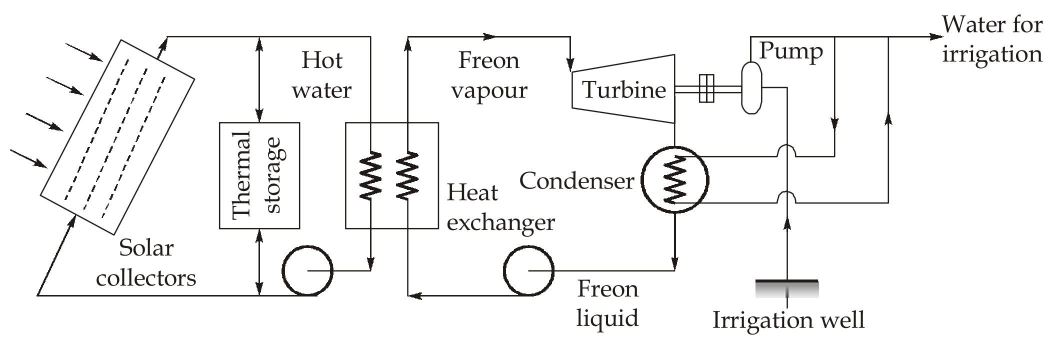
$$= 574.13 \text{ kJ/kg}$$

$\therefore$ Number of stages, $\eta = \frac{(\Delta h)_{total}}{(\Delta h)_{stage}} = \frac{574.13}{44.83} = 12.8 \simeq 13$ stages

8. (c)

(i)

A schematic diagram of a typical Rankine cycle, solar thermal water pumps is shown in figure. A solar-collector system may consist of flat plate-collectors, non-focusing type (stationary) collectors or sun-tracking concentrators. Water is used as a heat-transport fluid, and yields its heat to a low-boiling point organic working fluid (such as Freon R113, R12, isobutane etc.) in a heat exchanger. Surplus heat is stored in the thermal storage to be used later when the sun is not available. The high-pressure vapours of the working fluid expand in the turbine, condense in the condenser and return in the heat exchanger (boiler). A part of the irrigation-pumped water is diverted through the condenser for cooling purposes.



Solar thermal water pump

(ii) Given: $\gamma = 0°$ and $n = 309$ for 5 November

From Cooper's equation,

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + 309)\right] = -16.54°$$

At 9.00 a.m.

$$\omega = 15\,(\text{Solar time} - 12:00) = 15\,(9:00 - 12:00) = -45°$$

Now,

$$\cos\theta = \sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega \cos(\phi - \beta)$$

$$= \sin(-16.54°)\sin(28.58° - 35°) + \cos(-16.54°) \times \cos(-45°)\cos(28.58° - 35°)$$

$$= 0.705$$

$\therefore$ $\theta = 45.13°$ **Ans.**

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