



MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

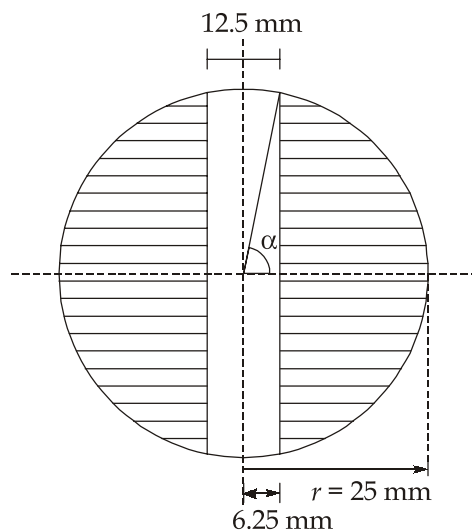
**ESE-2024
Mains Test Series**

**Civil Engineering
Test No : 14**

Section-A

Q.1 (a) (i) Solution:

For the circular cross-section of the bar,



Now,

$$\cos \alpha = \frac{6.25}{25}$$

$$\alpha = 75.52^\circ \simeq 1.318 \text{ rad}$$

$$\text{Area of cross-section} = 4 \times \left[\frac{\alpha r^2}{2} - \frac{1}{2} \times 6.25 \times \sqrt{25^2 - 6.25^2} \right]$$

$$= 4 \times \left[\frac{1.318 \times 25^2}{2} - 75.64 \right]$$

$$= 1344.94 \text{ mm}^2$$

∴ Allowable load in tension,

$$P = \sigma_{\text{allow}} A$$

$$= 80 \times 1344.94 \text{ N}$$

$$= 107.6 \text{ kN}$$

(ii) $L = 5000 \text{ mm}$

Diameter of bar = 50 mm

Tensile test:

Tensile stress, $\sigma_T = \frac{P}{A} = \frac{75 \times 10^3}{\left(\frac{\pi \times 50^2}{4} \right)} = 38.197 \text{ N/mm}^2 \simeq 38.2 \text{ N/mm}^2$

Tensile strain, $\epsilon_T = \frac{\Delta L}{L} = \frac{4.2}{5000} = 8.4 \times 10^{-4}$

∴ Young's modulus, $E = \frac{\sigma}{\epsilon_T} = \frac{38.2}{8.4 \times 10^{-4}} = 4.545 \times 10^4 \text{ N/mm}^2$

Effective length of column, $L_e = \frac{L}{2} = \frac{5000}{2} = 2500 \text{ mm}$

[Both ends are fixed]

Euler's buckling load is calculated as

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 4.55 \times 10^4 \times \pi \times 50^4}{64 \times (2500)^2} \text{ N} = 22 \text{ kN}$$

∴ Safe load, $P_s = \frac{P_{cr}}{FOS} = \frac{22}{3} = 7.33 \text{ kN}$

Q.1 (b) Solution:

Management is the art of getting things done through and with people. So, the overall job of a manager is to create within the organization and to develop an atmosphere which will facilitate the accomplishment of its predetermined objectives. For this purpose, it has to perform a series of functions. The most primary and basic functions of the management are:

- | | | |
|--------------|------------------|----------------|
| 1. Planning | 2. Organizing | 3. Staffing |
| 4. Directing | 5. Co-ordinating | 6. Controlling |

The following are some of the important benefits of planning:

1. Planning helps in determining the objectives of an enterprise.
2. Planning helps in better coordination because the well defined objectives, well publicised policies, well developed programmes and procedures help in coordination.
3. Planning helps in control by distributing the responsibilities of different persons and jobs.
4. Planning makes possible rational and realistic forecasting of the business activities of the enterprise.
5. Planning minimises the cost by utilising the available resources in the best way.
6. Planning improves motivation and morale of managers as well as workers.
7. Planning imparts competitive strength to the enterprise.

Q.1 (c) Solution:

Given:

$$b_w = 150 \text{ mm}$$

$$f_{ck} = 40 \text{ MPa}$$

$$D = 300 \text{ mm}$$

$$f_{cp} = 5 \text{ N/mm}^2$$

$$d = 300 - 50 = 250 \text{ mm}$$

$$f_y = 415 \text{ N/mm}^2$$

$$V = 130 \text{ kN}$$

$$f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40}$$

$$= 1.518 \text{ N/mm}^2$$

Now, ultimate shear strength of section

$$V_{cw} = 0.67b_wD\sqrt{f_t^2 + 0.8f_{cp}f_t}$$

$$= 0.67 \times 150 \times 300 \sqrt{1.518^2 + 0.8 \times 5 \times 1.518}$$

$$\therefore V_{cw} = 87259.76 \simeq 87260 \text{ N} = 87.26 \text{ kN}$$

Hence, balance of shear force to be resisted by stirrups

$$= V - V_{cw}$$

$$= 130 - 87.26$$

$$= 42.74 \text{ kN}$$

Using 8 mm diameter two legged stirrups, spacing of stirrups is,

$$S_v = \frac{0.87 f_y A_{sv} d}{V - V_{cw}}$$

$$= \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 250}{42.74 \times 10^3} = 212.31 \text{ mm}$$

Maximum permissible spacing of vertical stirrups

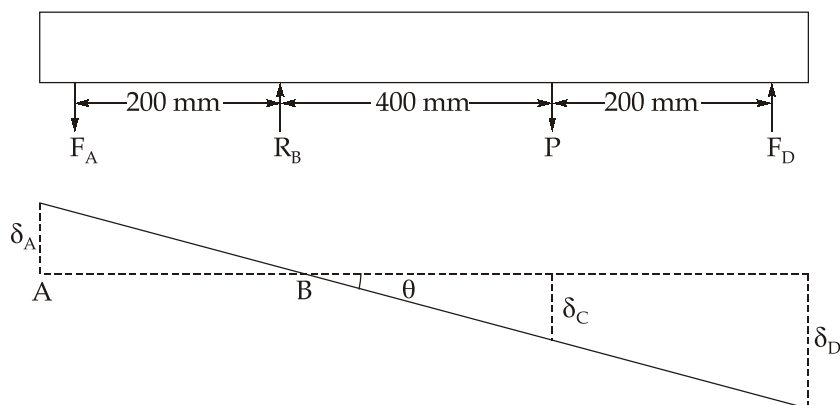
$$= 0.75 d = 0.75 \times 250$$

$$= 187.5 \text{ mm} < 212.31 \text{ mm}$$

So, provide 8 mm diameter two legged stirrups @ 180 mm c/c.

Q.1 (d) Solution:

Free body diagram and displacement diagram,



Equations of equilibrium

$$\sum M_B = 0,$$

$$\Rightarrow -F_A(200) - F_D(600) + P(400) = 0$$

$$\Rightarrow F_A + 3F_D = 2P \quad \dots(1)$$

As per equation of compatibility,

$$\frac{\delta_A}{200} = \frac{\delta_D}{600}$$

$$\Rightarrow \frac{F_A}{K_1} = \frac{F_D}{3K_2} \quad [\because F = Kx]$$

$$\Rightarrow \frac{F_A}{15} = \frac{F_D}{3 \times 25}$$

$$\Rightarrow F_D = 5F_A \quad \dots(2)$$

From (1) and (2),

$$F_A = \frac{P}{8} \quad \text{and} \quad F_D = \frac{5P}{8}$$

(i) Now,

$$\text{Permissible angle of rotation, } \theta = \frac{\delta_D}{600} = \frac{F_D}{K_2 \times 600}$$

$$\Rightarrow 4^\circ \times \frac{\pi}{180^\circ} = \frac{5P}{8 \times 25 \times 600}$$

$$\Rightarrow P = 1675.52 \text{ N}$$

(ii) Now,
$$\Delta_A = \frac{F_A}{K_A} = \frac{P}{8 \times 15} = \frac{1675.52}{120} = 13.96 \text{ mm}$$

$$\Delta_D = \frac{F_D}{K_D} = \frac{5P}{8 \times 25} = \frac{1675.52}{40} = 41.89 \text{ mm}$$

From the deflection profile it can be seen that spring at A will elongate and spring at D will contract by 13.96 mm and 41.89 mm respectively.

Q.1 (e) Solution:

Beams of large depth reduce the head room or ceiling height available. Hence, flat slab is a RCC slab built monolithically with the supporting column and reinforced in two or more directions. Beams are not provided in flat slabs to support it. The load carried by the slab is directly transmitted to the column. It provides greater clear ceiling heights.

Generally, in flat slabs, the columns are provided with enlarged heads called column heads or capitals. To support heavy loads, the thickness of slab over the column is increased. This thickened portion of the slab is known as drop panel. In light loading, column heads are omitted. Such a flat slab of constant thickness supported on column is called flat slab plate.

Advantages of flat slab

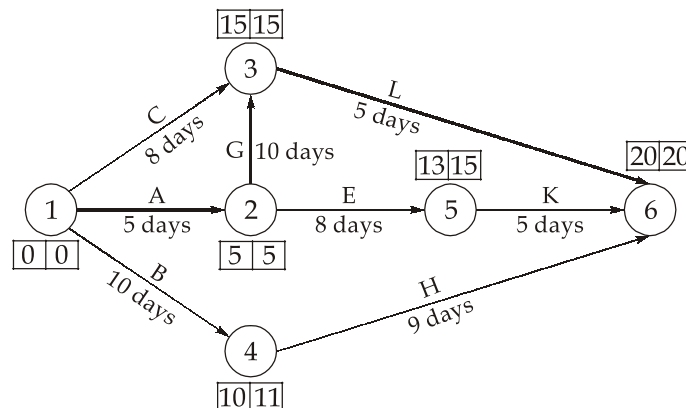
1. Concrete is more economically used in flat slab construction.
2. Floor systems require lesser depth and hence reduction in storey height.
3. There is reduction of dead load and foundation loads because weight of structure gets reduced because of reduction in height.
4. Improved fire resistance and better illumination in flat slab construction.
5. Easier to provide a acoustical treatment to the underside of the slab.
6. Flat slab can better withstand point loads.

Q.2 (a) Solution:

Calculation of EST, EFT, LST, LFT and F_T are done in table below.

Activity	t_{ij}	EST	EFT	LST	LFT	F_T	Remarks
D_0	0	0	0	0	0	0	–
A	5	0	5	0	5	0	Critical
B	10	0	10	1	11	1	
C	8	0	8	7	15	7	
E	8	5	13	7	15	2	
G	10	5	15	5	15	0	Critical
H	9	10	19	11	20	1	
K	5	13	18	15	20	2	
L	5	15	20	15	20	0	Critical
F_0	0	20	20	20	20	0	–

A-O-A network diagram:



Critical path is A-G-L and project duration is 20 days.

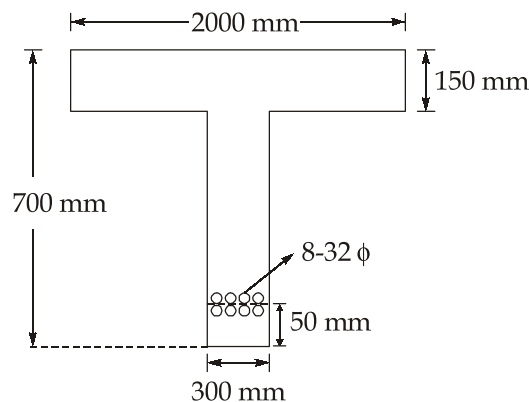
Q.2 (b) Solution:

- (i) 1. The use of high strength concrete and steel in prestressed members results in lighter and slender members than those with reinforced concrete. The dead load moments are neutralized by the prestressing moment. The economy of prestressed concrete is well established for long span structures. Standard precast bridge beams between 10 m to 30 m long span and precast prestressed piles have proved to be more economical than steel and RCC. Thus, in the long span range prestressed concrete is generally more economical than RCC, Also the high strength concrete is very cheap to prepare as its cost does not increase in the same proportion as its strength does.
2. Prestress concrete members possess improved resistance to shearing force, due to the effect of compressive prestress which reduces the principal tensile stress.

The use of curved cables particularly in long span members, help to reduce the shear force developed at the support sections.

In a prestressed concrete member, the shear stress is generally accompanied by a direct stress in the axial direction of the member and if transverse vertical prestressing is adopted, compressive stresses in the direction perpendicular to the axis of the member will be prestress. The direct stresses being compressive, the magnitude of the principal tensile stress is considerably reduced and in some cases even eliminated, so that under working loads both major and minor principal stresses are compressive thereby eliminating the risk of diagonal tension cracks in concrete.

(ii)



Effective width of flange, b_{eff} for an isolated T-beam is given by,

$$b_{\text{eff}} = \frac{l_0}{\frac{l_0}{b} + 4} + b_w$$

$$= \frac{10,000}{\frac{10,000}{2,000} + 4} + 300$$

$$= 1411.11 \text{ mm} < b = 2000 \text{ mm}$$

$$\text{Area of steel, } A_{\text{st}} = 8 \times \frac{\pi}{4} \times 32^2 = 6434 \text{ mm}^2$$

Case I.

Assuming that the neutral axis, x_u lies in the flange-

$$x_u = \frac{0.87 \cdot f_y \cdot A_{\text{st}}}{0.36 f_{\text{ck}} \cdot b_{\text{eff}}} = \frac{0.87 \times 415 \times 6434}{0.36 \times 25 \times 1411.11}$$

$$= 182.91 \text{ mm} > d_f = 150 \text{ mm} \quad (\text{Not OK})$$

Case II.

Assume $d_f < x_u < \frac{7}{3}d_f (= 350 \text{ mm})$

$$\begin{aligned} x_u &= \frac{0.87 f_y A_{st} - 0.45 f_{ck} (b_{eff} - b_w) (0.15 x_u + 0.65 d_f)}{0.36 f_{ck} b_w} \\ &= \frac{0.87 \times 415 \times 6434 - 0.45 \times 25 (1411.11 - 300) \times (0.15 x_u + 0.65 \times 150)}{0.36 \times 25 \times 300} \\ &= 860.37 - 0.6944 x_u - 451.39 \end{aligned}$$

$$\therefore x_u = 241.37 \text{ mm} < \frac{7}{3} \times 150 = 350 \text{ mm}$$

Hence our assumptions is correct,

Now ultimate moment of resistance is given by,

$$M_{uR} = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_{eff} - b_w) y_f \left(d - \frac{y_f}{2} \right)$$

where,

$$\begin{aligned} y_f &= 0.15 x_u + 0.65 d_f \\ &= 0.15 \times 241.37 + 0.65 \times 150 \\ &= 133.706 \text{ mm} \end{aligned}$$

And

$$d = D - E.C = 700 - 50 = 650 \text{ mm}$$

$$\begin{aligned} \therefore M_{uR} &= 0.36 \times 25 \times 300 \times 241.37 (650 - 0.42 \times 241.37) + 0.45 \times \\ &\quad 25 \times (1411.11 - 300) (133.706) \left(650 - \frac{133.706}{2} \right) \\ &= 1322.14 \times 10^6 \text{ N-mm} \\ &= 1322.14 \text{ kN-m} \end{aligned}$$

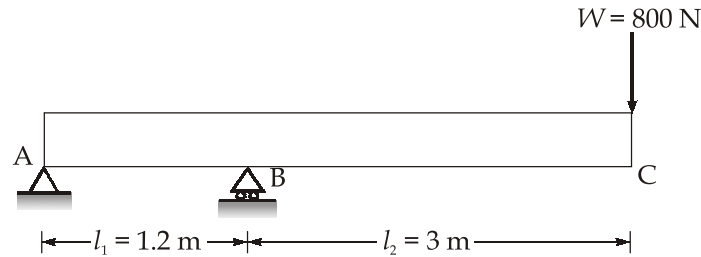
Q.2 (c) Solution:

(i) For the rectangular cross-section,

$$\begin{aligned} I &= \frac{bd^3}{12} = \frac{450 \times 50^3}{12} \\ &= 4.6875 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} \text{Section modulus, } Z &= \frac{I}{\frac{d}{2}} = \frac{4.6875 \times 10^6}{25} \\ &= 1.875 \times 10^5 \text{ mm}^3 \end{aligned}$$

For static load W at overhanging end C.



$$\begin{aligned}\text{Deflection at end C, } \delta_{st} &= \frac{Wl_2^3}{3EI} + \frac{Wl_2 \times l_1}{3EI} \times l_2 \\ &= \frac{1}{3 \times 12 \times 10^3 \times 4.6875 \times 10^6} \left[800 \times 3000^3 + 800 \times 3000^2 \times 1200 \right] \\ &= 179.2 \text{ mm}\end{aligned}$$

$$\text{Now maximum static bending stress, } \sigma_{st} = \frac{M_{\max}}{Z} = \frac{Wl_2}{Z} = \frac{800 \times 3000}{1.875 \times 10^5} = 12.8 \text{ N/mm}^2$$

$$\text{Now, } \frac{\delta_{\max}}{\delta_{st}} = 1 + \sqrt{1 + \frac{2h_{\max}}{\delta_{st}}}$$

For a linearly elastic beam, the bending stress σ is proportional to the deflection δ .

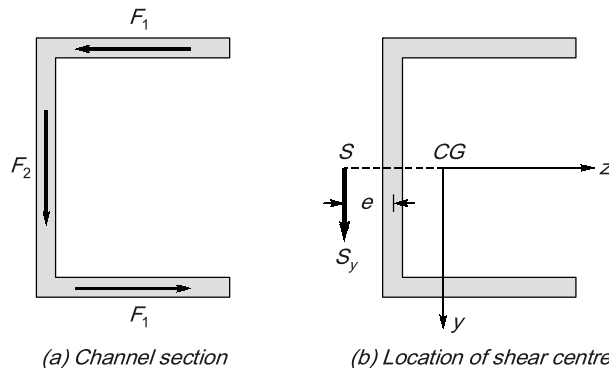
$$\therefore \frac{\sigma_{\max}}{\sigma_{st}} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$

$$\Rightarrow \frac{45}{12.8} = 1 + \sqrt{1 + \frac{2h_{\max}}{179.2}}$$

$$\Rightarrow h_{\max} = 477.42 \text{ mm} \simeq 0.48 \text{ m}$$

(ii)

Consider a channel section, in which bending is about z-axis. Assuming section is subjected to vertical shear force S_y parallel to y-axis.



We know shear stress in beams of thin walled open cross-section is given by,

$$\tau = \frac{S_y A \bar{y}}{I t}$$

where, S_y = vertical shear force parallel to y-axis

Hence, maximum shear stress in flange τ_1 is

$$\tau_1 = \frac{S_y A_f \bar{y}}{I_z t_f}$$

where,

$$A_f \bar{y} = \frac{b t_f h}{2}$$

$$\therefore \tau_1 = \frac{S_y b h}{2 I_z} \quad \dots(i)$$

$$\text{Similarly, } \tau_2 \text{ (just within the web at junction)} = \frac{S_y b t_f h}{2 I_z t_w} \quad \dots(ii)$$

Also, at the neutral axis, shear stress will be maximum and is given by

$$\tau_{\max} = \left(\frac{b t_f}{t_w} + \frac{h}{4} \right) \frac{h S_y}{2 I_z} \quad \dots(iii)$$

The total shear force in flange either top or bottom is given as

$$F_1 = \left(\frac{1}{2} \times b \times t_f \right) \times \tau_1 = \frac{h b^2 t_f S_y}{4 I_z}$$

The vertical shear force F_2 in web will be equal to S_y

$$\therefore F_2 = (h t_w) \tau_2 + \frac{2}{3} (\tau_{\max} - \tau_2) h t_w$$

From eq. (ii) and (iii) we get,

$$F_2 = \left(\frac{t_w h^3}{12} + \frac{b h^2 t_f}{2} \right) \frac{S_y}{I_z}$$

where,

$$I_z = \frac{t_w h^3}{12} + \frac{b h^2 t_f}{2}$$

Let shear centre is at a distance 'e' from the centre line of the web of section, then as per the definition of shear centre

$$\begin{aligned} F_1 \times h - F_2 \times e &= S_y \times 0 \\ \Rightarrow \left(\frac{h b^2 t_f S_y}{4 I_z} \right) \times h - \left(\frac{t_w h^3}{12} + \frac{b h^2 t_f}{2} \right) \frac{S_y}{I_z} \times e &= 0 \end{aligned}$$

$$\Rightarrow \frac{b^2 h^2 t_f}{4} - I_z \times e = 0$$

$$\Rightarrow e = \frac{b^2 h^2 t_f}{4 I_z}$$

Q.3 (a) Solution:

(i) Given, $d_2 = 45 \text{ cm}$, $d_1 = 30 \text{ cm}$, $\tau_{\text{allow}} = 3.1 \times 10^7 \text{ N/m}^2$

$$\begin{aligned} \text{Polar moment of inertia, } I_p &= \frac{\pi}{32} (d_2^4 - d_1^4) \\ &= \frac{\pi}{32} (0.45^4 - 0.30^4) \\ &= 3.23 \times 10^{-3} \text{ m}^4 \end{aligned}$$

Now, maximum torque that can be applied,

$$\begin{aligned} T_{\text{max}} &= \frac{2\tau_{\text{allow}} I_p}{d_2} \\ &= \frac{2 \times 3.1 \times 10^7 \times 3.23 \times 10^{-3}}{0.45} = 445022.22 \text{ N-m} \end{aligned}$$

$$\begin{aligned} 1. \quad \text{Power transmitted, } P &= \frac{2\pi NT}{60} \\ &= \frac{2 \times \pi \times 120 \times 445022.22}{60 \times 745.7} \text{ HP} = 7499.42 \text{ HP} \end{aligned}$$

$$2. \quad \text{As} \quad P = \frac{2\pi NT}{60}$$

If N is doubled, but P remains same, then T is halved, so shear stress is also halved.

(ii) Let H_B and V_B be the horizontal and vertical reactions at B and V_D be the vertical reaction at D.

Using equations of equilibrium,

$$\sum F_x = 0$$

$$\Rightarrow H_B = 0$$

$$\text{Also, } \sum F_y = 0$$

$$\Rightarrow V_B + V_D = 15$$

Also, $\sum M_B = 0$

$$\Rightarrow -8 + 2 + 6 + 5 \times 3 \times \frac{3}{2} - V_D (3) = 0$$

$$\Rightarrow V_D = 7.5 \text{ kN}$$

$$\therefore V_B = 7.5 \text{ kN}$$

Shear force calculations:

$$(\text{S.F.})_A = 0$$

Just left of B, $(\text{S.F.})_{B^-} = 0$

Just right of B, $(\text{S.F.})_{B^+} = V_B = 7.5 \text{ kN}$

In span BD, shear force at x from B,

$$\text{SF}_x = 7.5 - 5x \quad [0 \leq x < 3]$$

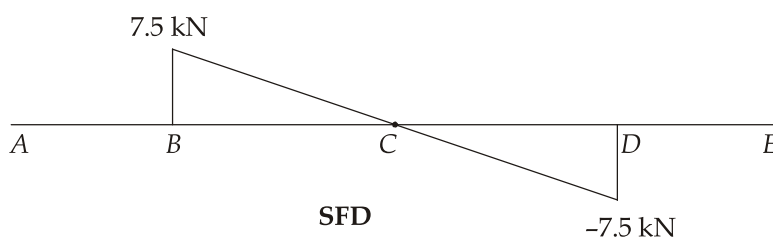
At $x = 0$, $\text{SF}_{B^+} = 7.5 \text{ kN}$

At $x = 3 \text{ m}$, $\text{SF}_D^- = 7.5 - 5(3)$
 $= -7.5 \text{ kN}$

At just right of D,

$$\text{S.F.}_D^+ = \text{S.F.}_D^- + V_D = -7.5 + 7.5 = 0$$

Also, $\text{S.F.}_E = 0$



Bending moment calculations

In span AB, At x distance from A,

$$M_x = -8 \text{ kN-m} \quad [0 \leq x < 1 \text{ m}]$$

At $x = 0$, $M = -8 \text{ kNm}$

At $x = 1 \text{ m}$, $M = -8 \text{ kN-m}$

In span BC, at x distance from B,

$$M_x = -8 + 7.5x - \frac{5x^2}{2} \quad [0 \leq x < 1.5 \text{ m}]$$

At $x = 0$, $M_B = -8 \text{ kN-m}$

At $x = 1.5 \text{ m}$, $M_C = -2.375 \text{ kN-m}$

In span CD, At x distance from B $M_x = -8 + 7.5x - \frac{5x^2}{2} + 2$

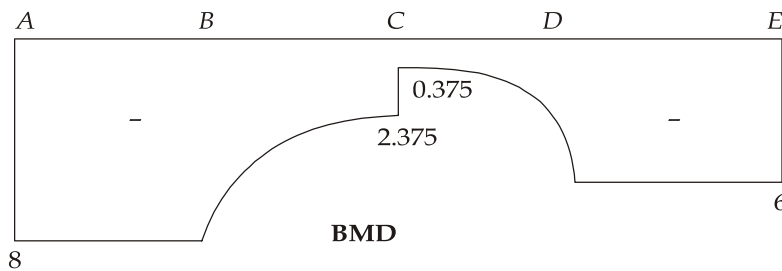
$$= -6 + 7.5x - \frac{5x^2}{2}$$

At $x = 1.5 \text{ m}$, $M_C^+ = -0.375 \text{ kN-m}$

At $x = 3 \text{ m}$, $M_D = -6 \text{ kN-m}$

In span DE, $M_D = -6 \text{ kN-m}$

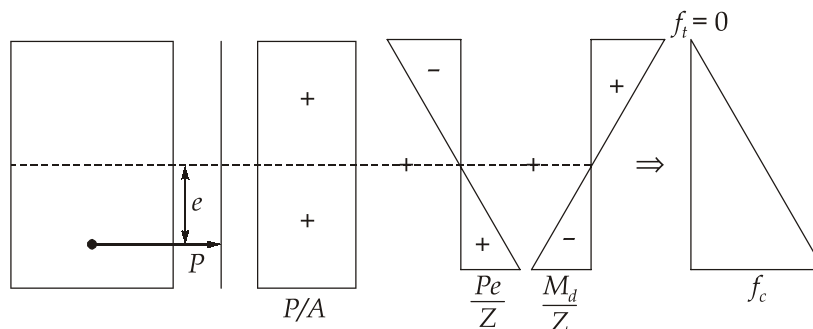
$M_E = -6 \text{ kN-m}$



(Values in KN-m)

Q.3 (b) Solution:

At transfer stage:



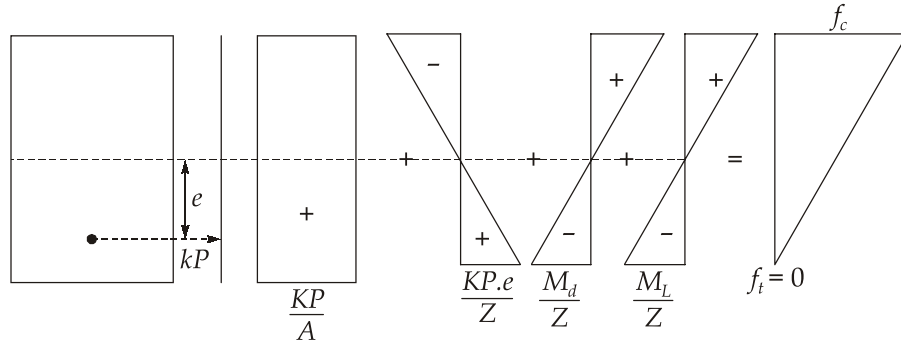
Stresses

At top, $0 = \frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z} \quad \dots(i)$

At bottom,

$$f_c = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z} \quad \dots(ii)$$

At final stage



Stresses

At top,

$$f_c = \frac{KP}{A} - \frac{KPe}{Z} + \frac{M_d}{Z} + \frac{M_l}{Z} \quad \dots(iii)$$

At bottom,

$$0 = \frac{KP}{A} + \frac{KPe}{Z} - \frac{M_d}{Z} - \frac{M_l}{Z} \quad \dots(iv)$$

Now, by solving equations (i), (ii), (iii) and (iv) we get,

$$P = \frac{A \cdot f_c}{2} \quad \dots(A)$$

And,

$$Z = \frac{(1-K)M_d + M_l}{K \cdot f_c} \quad \dots(B)$$

And,

$$e = \frac{(1+K)M_d + M_l}{2 \cdot K \cdot P} \quad \dots(C)$$

Now,

$$M_l = \frac{wl^2}{8} = \frac{80 \times 15^2}{8} = 2250 \text{ kN-m}$$

$$f_c = 25 \text{ MPa}$$

$$K = 1 - \frac{\text{Loss}}{100} = \left(1 - \frac{15}{100}\right) = 0.85$$

Assume, width, B = 500 mm

And, overall depth, D = 800 mm

$$\text{Dead load, } w_d = 0.5 \times 0.8 \times 1 \times 25 = 10 \text{ kN/m}$$

$$M_d = \frac{w_d \cdot l^2}{8} = \frac{10 \times 15^2}{8} = 281.25 \text{ kN-m}$$

Now, section modulus from equation (B) is given by,

$$Z = \frac{(1-0.85) \times 281.25 \times 10^6 + 2250 \times 10^6}{0.85 \times 25}$$

$$= 107.87 \times 10^6 \text{ mm}^3$$

Now,

$$Z = \frac{BD^2}{6}$$

$$\Rightarrow D = \sqrt{\frac{6Z}{B}} = \sqrt{\frac{6 \times 107.87 \times 10^6}{500}} = 1137.73 \text{ mm}$$

\therefore Adopt overall depth as

$$D = 1200 \text{ mm}$$

Now,

$$w_d = 0.5 \times 1.2 \times 1 \times 25 = 15 \text{ kN/m}$$

$$M_d = \frac{15 \times 15^2}{8} = 421.875 \text{ kNm}$$

Section modulus,

$$Z = \frac{(1-K)M_d + M_l}{K.f_c}$$

$$= \frac{(1-0.85) \times 421.875 \times 10^6 + 2250 \times 10^6}{0.85 \times 25}$$

$$= 108.86 \times 10^6 \text{ mm}^3$$

Now,

$$Z = \frac{BD^2}{6} = 108.86 \times 10^6$$

$$\Rightarrow D = \sqrt{\frac{6 \times 108.86 \times 10^6}{500}}$$

$$= 1143 \text{ mm} < 1200 \text{ mm} \quad (\text{OK})$$

Hence,

$$B = 500 \text{ mm}$$

and D

$$= 1200 \text{ mm}$$

Now P force is given force equation (A)

$$P = \frac{Af_c}{2}$$

$$= \frac{500 \times 1200 \times 25 \times 10^{-3}}{2} \text{ kN}$$

$$= 7500 \text{ kN}$$

$$\begin{aligned}\text{Area of steel, } A_s &= \frac{P}{f_s} = \frac{7500 \times 10^3}{1500} \\ &= 5000 \text{ mm}^2\end{aligned}$$

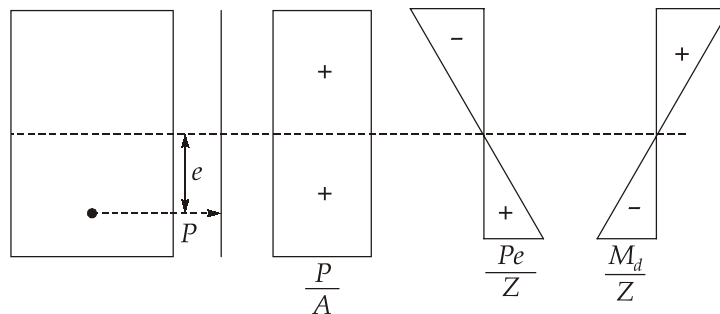
$$\text{No. of strands required} = \frac{5000}{8 \times \frac{\pi}{4} \times 8^2} = 12.43 \simeq 13 \text{ (say)}$$

Now eccentricity e is given from equation (C)

$$\begin{aligned}e &= \frac{(1+K)M_d + M_l}{2KP} \\ &= \frac{(1+0.85) \times 421.875 \times 10^6 + 2250 \times 10^6}{2 \times 0.85 \times 7500 \times 10^3} \\ &= 237.68 \text{ mm} \\ &\text{say } 238 \text{ mm ; } 238 \text{ mm (say)}\end{aligned}$$

Check for stresses :

(a) At transfer stage



$$\frac{P}{A} = \frac{7500 \times 10^3}{500 \times 1200} = 12.5 \text{ MPa}$$

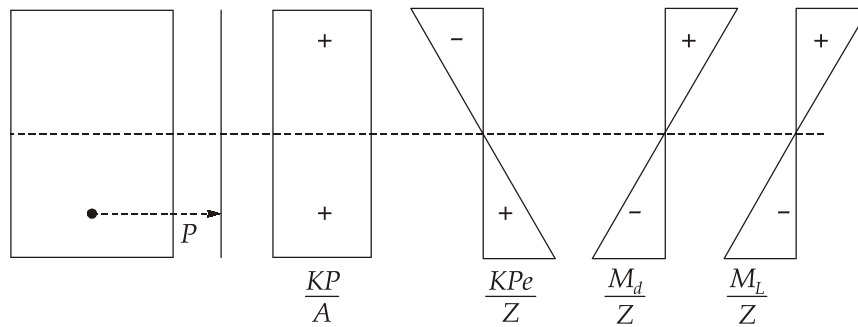
$$\frac{Pe}{Z} = \frac{7500 \times 10^3 \times 238}{500 \times \frac{1200^2}{6}} = 14.875 \text{ MPa}$$

$$\frac{M_d}{Z} = \frac{421.875 \times 10^6}{500 \times \frac{1200^2}{6}} = 3.52 \text{ MPa}$$

$$\therefore \text{ Stress at top, } = 12.5 - 14.875 + 3.52 = 1.145 > 0$$

$$\text{Stress at bottom} = 12.5 + 14.875 - 3.52 = 23.855 < 25 \text{ MPa} \quad (\text{OK})$$

(b) At final stage



$$\frac{KP}{A} = 0.85 \times 12.5 = 10.625 \text{ MPa}$$

$$\frac{KPe}{Z} = 0.85 \times 14.875 = 12.644 \text{ MPa}$$

$$\frac{M_d}{Z} = 3.52 \text{ MPa}$$

$$\frac{M_L}{Z} = \frac{2250 \times 10^6}{500 \times 1200^2} = 18.75 \text{ MPa}$$

$$\therefore \text{Stress at top} = 10.625 - 12.644 + 3.52 + 18.75 = 20.251 < 25 \text{ MPa} \quad (\text{OK})$$

$$\text{Stress at bottom} = 10.625 + 12.644 - 3.52 - 18.75 = 0.999 \text{ MPa} > 0 \quad (\text{OK})$$

Q.3 (c) Solution:

(i)

Factored shear force, $V_u = 15 \text{ kN}$ Factored torsional moment, $T_u = 5 \text{ kN-m}$

$$\begin{aligned} \text{Equivalent shear force, } V_e &= V_u + \frac{1.6T_u}{B} \\ &= 15 + \frac{1.6 \times 5}{0.3} \\ &= 41.67 \text{ kN} \end{aligned}$$

$$\text{Equivalent shear stress, } \tau_e = \frac{V_e}{B.d} = \frac{41.67 \times 10^3}{300 \times 500} = 0.2778 \simeq 0.28 \text{ MPa}$$

∴ Shear strength of M20 concrete for 0.25% tension steel, $\tau_c = 0.36 \text{ MPa}$

∴ $\tau_c (0.36 \text{ MPa}) > \tau_e (0.28 \text{ MPa})$

Hence no torsional reinforcement is required.

Minimum shear reinforcement should be provided, such that,

$$\frac{A_{sv}}{S_v} \geq \frac{0.4B}{0.87f_y}$$

Use 8 mm – 2 legged stirrups of Fe415 grade steel.

$$\therefore A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$\text{Now, } S_v \leq \frac{0.87f_y \cdot A_{sv}}{0.4B} \leq \frac{0.87 \times 415 \times 100.53}{0.4 \times 300} \leq 302.47 \text{ mm}$$

$$\text{But } S_v \nless 300 \text{ mm}$$

Hence provide 2 legged - 8 mm ϕ @ 300 mm c/c.

(ii)

Vibrators: These are mechanical devices which are used to compact concrete in the framework.

Following are the four types of vibrators:

1. **Internal or immersion vibrators:** These vibrators consist of a steel tube which is inserted in fresh concrete. This steel tube is called the poker and it is connected to an electric motor or a petrol engine through a flexible tube. They are available in sizes varying from 40 mm to 100 mm diameters and the size is decided by keeping in mind the spacing between reinforcing bars in concrete. The frequency of vibration is about 300 to 600 rpm.
2. **Surface vibrators:** These vibrators are mounted on platform or screeds. They are used to finish concrete surfaces such as bridge floors, roads slabs, station platform etc. These vibrators are found to be more effective for compacting very dense concrete mixes because the vibration acts in the same direction of gravity and the concrete is compacted in confined zone.
3. **Form or shutter vibrators:** These vibrators are attached to the framework and external centering of walls, column etc. The vibrating action is conveyed to the concrete through the framework during transmission of vibrations. Hence they are not generally used. But they are very much helpful for concrete sections which are too thin for the use of internal vibrators.

4. **Vibrating tables:** These are in the form of a rigidly built steel platform mounted on flexible springs and they are operated by electromagnetic action or electric motors. They are found to be very effective in compacting stiff and harsh concrete mixes and hence they are invariably used in the preparation of pre-cast structural products in factories and test specimens in laboratories.

Q.4 (a) Solution:

- (i) Given data,

$$B = 250 \text{ mm}$$

$$D = 550 \text{ mm}$$

$$d = D - E.C = 550 - 50 = 500 \text{ mm}$$

$$l_{\text{eff}} = 5 \text{ m}$$

M20 and Fe415

$$\text{Area of tension steel, } A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942.5 \text{ mm}^2$$

Dead load of beam, is given by,

$$\begin{aligned} w_d &= B \times D \times 1 \times \gamma_{\text{RCC}} \\ &= 0.25 \times 0.55 \times 1 \times 25 \\ &= 3.4375 \text{ kN/m} \end{aligned}$$

Ultimate depth of neutral axis, x_u is given as -

$$\begin{aligned} x_u &= \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot B} \\ &= \frac{0.87 \times 415 \times 942.5}{0.36 \times 20 \times 250} \\ &= 189.1 \text{ mm} \end{aligned}$$

Limiting depth of neutral axis, $x_{u \text{ lim}}$ is given by,

$$\begin{aligned} x_{u \text{ lim}} &= K_0 d \\ &= 0.48 \times 500 \quad (\text{Fa Fe415}) \\ &= 240 \text{ mm} \end{aligned}$$

Hence $x_{u \text{ lim}} > x_u$, and thus section is under reinforced

Now ultimate moment of resistance is given by,

$$\begin{aligned} M_{uR} &= 0.36 f_{ck} \cdot B x_u (d - 0.42 x_u) \\ &= 0.36 \times 20 \times 250 \times 189.1 (500 - 0.42 \times 189.1) \end{aligned}$$

$$= 143.16 \times 10^6 \text{ N-mm}$$

$$= 143.16 \text{ kN-m}$$

Now working moment of resistance is given by,

$$\begin{aligned} M_R &= \frac{M_{uR}}{1.5} \\ &= 95.44 \text{ kN-m} \end{aligned}$$

If w is the net super imposed load including self weight of the beam, then the bending moment for the beam will be,

$$(BM)_{\max} = \frac{w.l_{\text{eff}}^2}{8} = M_R$$

$$\text{Now,} \quad \frac{w.l_{\text{eff}}^2}{8} = 95.44$$

$$\Rightarrow \quad \frac{w \times (5)^2}{8} = 95.44$$

$$\Rightarrow \quad w = 30.54 \text{ kN/m}$$

But self weight of the beam, is

$$w_d = 3.4375 \text{ kN/m}$$

Hence safe superimposed load = $30.54 - 3.4375$

$$= 27.1 \text{ kN/m}$$

(ii) Retaining walls are classified based on how they resist the back fill pressure. Retaining walls may be gravity retaining walls or reinforced concrete retaining walls. The R.C.C. retaining walls may be cantilever retaining walls or counterfort retaining walls or buttressed retaining walls.

1. **Gravity retaining walls:** These walls resist the backfill pressure and remain stable by their own weight. These walls are of masonry or plain concrete. These walls have to be massive to avoid tensile stresses at any section and also for their stability.
2. **Cantilever retaining walls:** This is a commonly provided retaining wall which is in the shape of an inverted T-section. Its three components are the stem, the heel slab and the toe slab. All the components act as cantilevers.
3. **Counterfort retaining walls:** These walls are meant to retain soil for greater heights. The vertical slab and the heel slab act as continuous slabs supported by vertical cantilevering counterforts, provided at intervals of about 3 meters. The toe slab however acts as a cantilever.

4. **Buttressed retaining walls:** These are similar to counterfort retaining walls. In this case the buttresses are provided on the side opposite to the back fill.

Q.4 (b) Solution:

(i) Without repair:

Present worth of annual payment of Rs. 5000 for 15 years,

$$\begin{aligned} P_{15} &= 5000 \left[\frac{P}{A}, 6\%, 15 \right] \\ &= 5000 \left[\frac{(1+0.06)^{15} - 1}{(0.06)(1+0.06)^{15}} \right] \\ &= \text{Rs. } 48561.24 \end{aligned}$$

(ii) With repair:

Present worth of annual payment of Rs. 5000 for $15 + 10 = 25$ years.

$$\begin{aligned} P_{25} &= 5000 \left[\frac{(1+0.06)^{25} - 1}{(0.06)(1+0.06)^{25}} \right] \\ &= \text{Rs. } 63916.78 \end{aligned}$$

So by doing the repair, present worth of gain

$$\begin{aligned} &= 63916.78 - 48561.24 \\ &= \text{Rs. } 15355.54 \end{aligned}$$

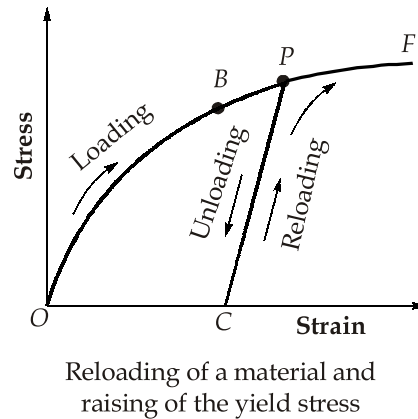
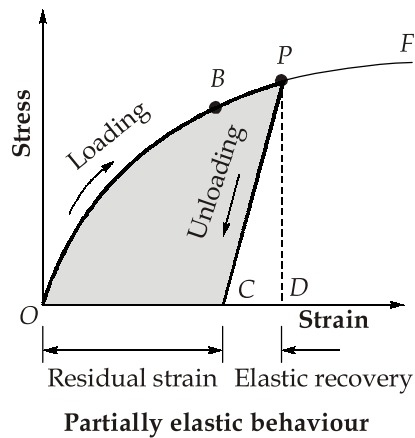
And the present investment for this gain = Rs. 20,000

Therefore it will be uneconomical to do the major repair.

(ii)

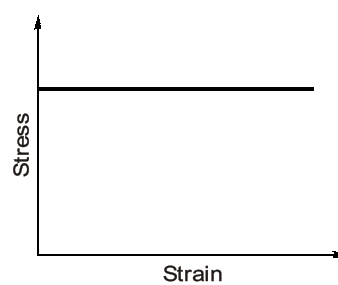
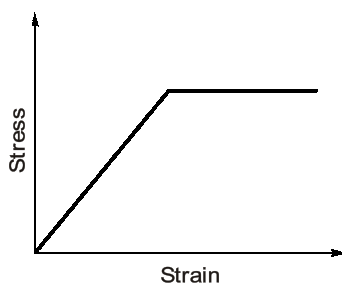
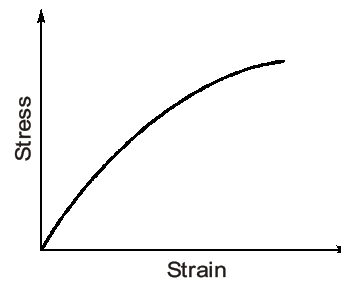
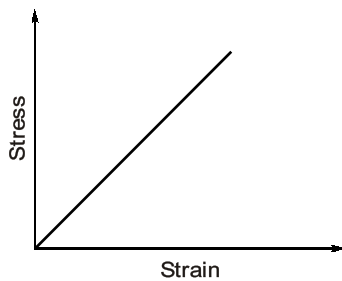
Let a material is loaded to a much higher level than elastic limit (B), such that point P is reached on stress-strain diagram. When unloading occurs, the material follows path PC as shown in the figure, which is parallel to the initial portion of original stress-strain curve. When point C is reached, the load has been entirely removed but a permanent strain or residual strain OC remains in material. The corresponding residual elongation of the specimen is called **permanent set**.

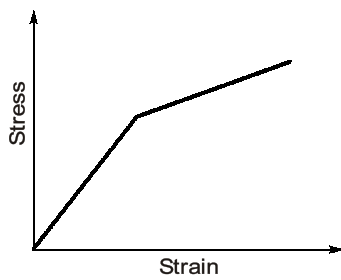
During unloading, only CPD part of strain energy is recovered and is called as elastic strain energy, whereas a large part OPC is lost in permanent deformation and is called **inelastic strain energy**.



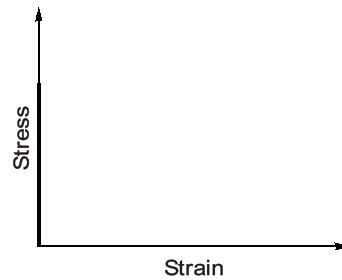
If material undergoes continuous cyclic loading and unloading beyond elastic limit, then yield limit of material continuously increases. This concept is used in cold working of mild steel bar to avoid yield plateau.

Types of material behaviour

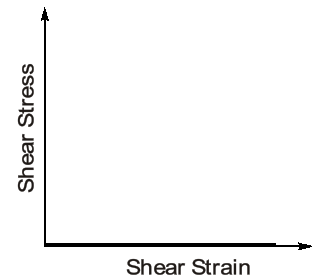




(v) Elasto-plastic with strain hardening

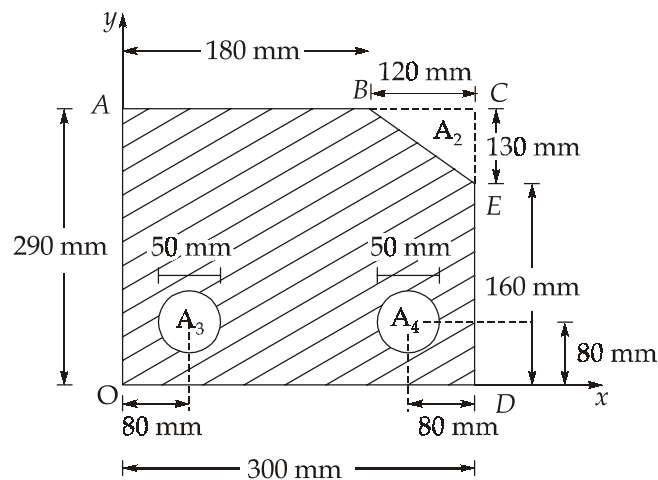


(vi) Ideal rigid



(vii) Ideal fluid

Q.4 (c) (i) Solution:

 $A_1 = \text{Large rectangle } OACD$ $A_2 = \text{Triangular cutout } BCE$ $A_4 = A_3 = \text{Circular holes}$

Now,

$$A_1 = 290 \times 300 = 87000 \text{ mm}^2$$

$$\bar{x}_1 = 150 \text{ mm}, \quad \bar{y}_1 = 145 \text{ mm}$$

$$A_2 = \frac{1}{2} \times (290 - 160) \times (300 - 180) \\ = 7800 \text{ mm}^2$$

$$\bar{x}_2 = 300 - \frac{120}{3} = 260 \text{ mm}$$

$$\bar{y}_2 = 290 - \frac{130}{3} = 246.67 \text{ mm}$$

$$A_3 = \frac{1}{4} \times \pi \times 50^2 = 1963.5 \text{ mm}^2$$

$$\bar{x}_3 = 80 \text{ mm}, \quad \bar{y}_3 = 80 \text{ mm}$$

$$A_4 = \frac{1}{4} \times \pi \times 50^2 = 1963.5 \text{ mm}^2$$

$$\bar{x}_4 = 220 \text{ mm}, \quad \bar{y}_4 = 80 \text{ mm}$$

$$\begin{aligned} \text{Total area} &= A_1 - A_2 - A_3 - A_4 \\ &= 87000 - 7800 - 1963.5 - 1963.5 \\ &= 75273 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} Q_x &= \sum x_i A_i \\ &= A_1 \bar{x}_1 - A_2 \bar{x}_2 - A_3 \bar{x}_3 - A_4 \bar{x}_4 \\ &= 87000 \times 150 - 7800 \times 260 - 1963.5 \times 80 - 1963.5 \times 220 \\ &= 10432950 \text{ mm}^3 \end{aligned}$$

$$\therefore \bar{x} = \frac{Q_x}{A} = \frac{10432950}{75273} = 138.60 \text{ mm}$$

$$\begin{aligned} Q_y &= \sum \bar{y}_i A_i \\ &= A_1 \bar{y}_1 - A_2 \bar{y}_2 - A_3 \bar{y}_3 - A_4 \bar{y}_4 \\ &= 87000 \times 145 - 7800 \times 246.67 - 1963.5 \times 80 - 1963.5 \times 80 \\ &= 10376814 \text{ mm}^3 \end{aligned}$$

$$\therefore \bar{y} = \frac{Q_y}{A} = \frac{10376814}{75273} = 137.86 \text{ mm}$$

(ii)

Actual depth of neutral axis, x_a is given by,

$$\frac{B}{2} x_a^2 + m.A_{st}.x_a - m.A_{st}.d = 0$$

$$\Rightarrow \frac{400}{2} x_a^2 + 13.33 \times 800 x_a - 13.33 \times 800 \times 500 = 0$$

$$\therefore x_a = 138.78 \text{ mm}$$

Check critical depth of neutral axis :

$$\begin{aligned} x_c &= 0.2886 d \\ &= 0.2886 \times 500 \\ &= 144.3 \text{ mm} \end{aligned}$$

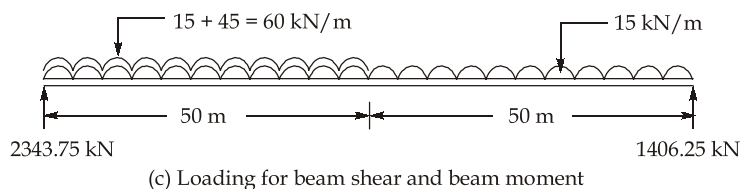
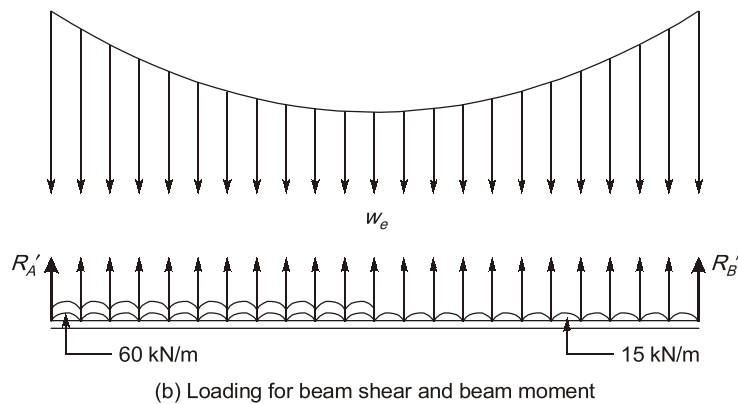
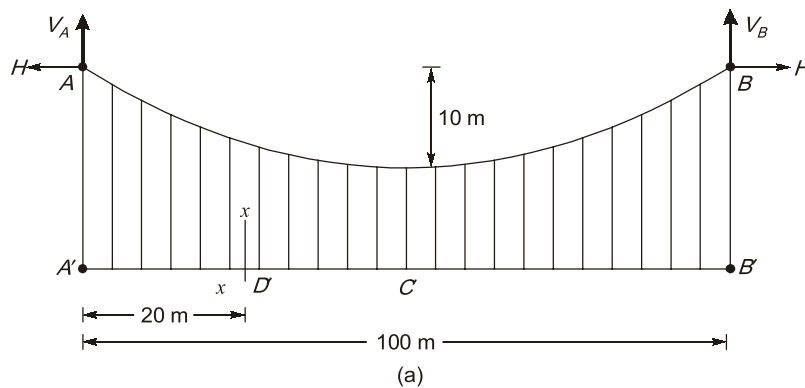
 \therefore As $x_a < x_c$ and thus section is under reinforced.

Moment of resistance is given by,

$$\begin{aligned}
 M_R &= \sigma_{st} \cdot A_{st} \cdot \left(d - \frac{x_a}{3} \right) \\
 &= 230 \times 800 \left(500 - \frac{138.73}{3} \right) \times 10^{-6} \text{ kN-m} \\
 &= 83.49 \text{ kN-m}
 \end{aligned}$$

Section-B

Q.5 (a) Solution



(Because there are two girders so load will be equally distributed to each of them)

Now,

$$l = 100 \text{ m}, h = 10 \text{ m}$$

$$\text{Dead load per girder} = 5 \times \frac{6}{2} = 15 \text{ kN/m}$$

$$\text{Live load per girder} = 15 \times \frac{6}{2} = 45 \text{ kN/m on left half portion}$$

$$\therefore \text{Total load on girder} = 60 \times 50 + 15 \times 50 = 3750 \text{ kN}$$

Considering cable:

\therefore Equivalent UDL transmitted to cables is

$$w_e = \frac{\text{Total load}}{\text{Span}} = \frac{3750}{100} = 37.5 \text{ kN/m}$$

$$H = \frac{w_e l^2}{8h} = \frac{37.5 \times 100^2}{8 \times 10} = 4687.5 \text{ kN}$$

$$V = w_e \frac{l}{2} = 37.5 \times \frac{100}{2} = 1875 \text{ kN}$$

$$\therefore T_{\max} = \sqrt{V^2 + H^2} = \sqrt{1875^2 + 4687.5^2} = 5048.6 \text{ kN}$$

Considering girder:

$$\text{Shear at } D'(V'_D) = \text{Beam shear} + \text{Shear due to } w_e$$

$$\Rightarrow R'_A = \frac{M_B}{100} = \frac{15 \times 50 \times 25 + 60 \times 50 \times 75}{100}$$

$$\Rightarrow R'_A = 2437.5 \text{ kN}$$

$$\therefore \text{Beam shear at } D' = R'_A - 60 \times 20 \\ = 2437.5 - 1200 = 1237.5 \text{ kN}$$

$$\therefore V'_D = 1237.5 - w_e \left(\frac{l}{2} - x \right) \\ = 1237.5 - 37.5 (50 - 20) = 112.5 \text{ kN}$$

$$\begin{aligned} \text{Now, } M_D &= \text{Beam moment} + \text{Moment due to } w_e \\ &= \text{Beam moment} - w_e \frac{x(l-x)}{2} \\ &= [2437.5 \times 20 - 60 \times 20 \times 10] - 37.5 \times 20 \times \frac{(100-20)}{2} \\ &= 6750 \text{ kNm} \end{aligned}$$

Q.5 (b) Solution:

1. **Compressive strength test:** This test is carried out to determine the compressive strength of cement. Following procedure is adopted:

- (i) The mortar of cement and sand is prepared. The proportion is 1:3 which means that x gm of cement is mixed with 3 x gm of sand.

- (ii) The water is added to the mortar. The water-cement ratio is kept as 0.4 which means that $0.4x$ gm of water is added to dry mortar.
 - (iii) The mortar is placed in moulds. The test specimens are in the form of cubes with side as 70.6 mm or 76 mm. The moulds are of metal and they are constructed in such a way that the specimens can be easily taken out without being damaged. For 70.6 mm and 76 mm cubes, the cement required is 185 gm and 235 gm respectively.
 - (iv) The mortar, after being placed in the moulds, is compacted in vibrating machine for 2 minutes.
 - (v) The moulds are placed in a damp cabin for 24 hours.
 - (vi) The specimens are removed from the moulds and they are submerged in clean water for curing.
 - (vii) The cubes are then tested in compression testing machine at the end of 3 days and 7 days. The testing of cubes is carried out on their three sides without packing. Thus, three cubes are tested each time to find out the compressive strength at the end of 3 days and 7 days. The average value is then worked out.
 - (viii) The compressive strength at the end of 3 days should not be less than 115 kg/cm^2 or 11.5 N/mm^2 and that at the end of 7 days should not be less than 175 kg/cm^2 or 17.5 N/mm^2 .
2. **Tensile strength:** This test was formerly used to have an indirect indication of compressive strength of cement. It is at present generally used for the rapid hardening cement. Following procedure is adopted:
- (i) The mortar of cement and sand is prepared. The proportion is 1 : 3 which means that x gm of cement is mixed with $3x$ gm of sand.
 - (ii) The water is added to the mortar. The quantity of water is 8 per cent by weight of cement and sand.
 - (iii) The mortar is placed in briquette moulds. The mould is filled with mortar and then a small heap of mortar is formed at its top. It is beaten down by a standard spatula till water appears on the surface. Same procedure is repeated for the other face of briquette. Such twelve standard briquettes are prepared. The quantity of cement may be 600 gm for 12 briquettes.
 - (iv) The briquettes are kept in a damp cabin for 24 hours.
 - (v) The briquettes are carefully removed from the moulds and they are submerged in clean water for curing.
 - (vi) The briquettes are tested in testing machine at the end of 3 days and 7 days. Six briquettes are tested in each test and average is found out. During the test, the load is to be applied uniformly at the rate of 35 kg/cm^2 or 3.5 N/mm^2

(vii) It may be noted that cross-sectional area of briquette at its least section is 6.45 cm². Hence, the ultimate tensile stress of cement paste is obtained from the following relation:

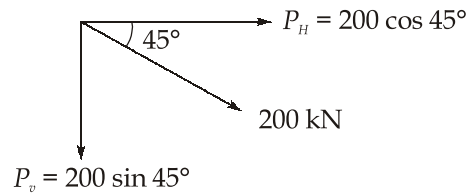
$$\text{Ultimate tensile stress} = \text{Failing load} / 6.45$$

(viii) The tensile stress at the end of 3 days should not be less than 20 kg/cm² or 2 N/mm² and that at the end of 7 days should not be less than 25 kg/cm² or 2.5 N/mm².

Q.5 (c) Solution:

Given : 16 mm diameter bolt of 4.6 grade

∴ Diameter of bolt hole (d_0) = 16 + 2 = 18 mm; $f_{ub} = 400$ N/mm²,
 $f_{yb} = 240$ N/mm², $f_u = 410$ N/mm², $f_y = 250$ N/mm²



Factored tension force in one bolt,

$$T_b = \frac{200 \times \cos 45^\circ}{8} = 17.68 \text{ kN}$$

Factored shear force in one bolt, $V_{sb} = \frac{P_v}{n} = \frac{200 \times \sin 45^\circ}{8} = 17.68 \text{ kN}$

Design strength of the bolt :

Here bolts are in single shear.

Design shear strength of the bolts,

$$\begin{aligned} V_{dsb} &= \frac{f_{ub}}{\sqrt{3} \times \gamma_{mb}} \times (0.78 \times A_{sb}) \\ &= \frac{400}{\sqrt{3} \times 1.25} \times 0.78 \times \pi \times \frac{16^2}{4} \times 10^{-3} \text{ kN} = 28.97 \text{ kN} \end{aligned}$$

Design bearing strength of the bolt,

$$\begin{aligned} V_{dpb} &= 2.5 \times K_b \times \frac{f_u}{1.25} \times d \times t \\ \text{where } K_b &= \text{minimum} \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}; 1 \right\} \end{aligned}$$

Adopting

$$e = e_{\min} = 1.5 d_0 = 1.5 \times 18 = 27 \text{ mm}$$

$$p = p_{\min} = 2.5 d = 2.5 \times 16 = 40 \text{ mm}$$

$$\therefore K_b = \text{minimum} \left\{ \frac{27}{3 \times 18}, \frac{40}{3 \times 18} - 0.25, \frac{400}{410}, 1 \right\} = 0.49$$

$$V_{dpb} = \left(2.5 \times 0.49 \times 16 \times 10 \times \frac{410}{1.25} \times 10^{-3} \right) \text{ kN} = 64.3 \text{ kN}$$

$$\therefore \text{Strength of the bolt, } V_{db} = \text{minimum} \left\{ \begin{array}{l} V_{dsb} = 28.97 \text{ kN,} \\ V_{dpb} = 64.3 \text{ kN} \end{array} \right\} = 28.97 \text{ kN}$$

$$\begin{aligned} \text{Tensile strength of bolt, } T_b &= 0.9 \times \frac{f_{ub}}{1.25} \times A_{nb} \leq \left(f_{yb} \times \frac{\gamma_{mb}}{\gamma_{mo}} \times A_{sb} \right) \\ &= 0.9 \times \frac{400}{1.25} \times 0.78 \times \pi \times \frac{16^2}{4} \leq 240 \times \frac{1.25}{1.1} \times \pi \times \frac{16^2}{4} \\ &= 45.17 \text{ kN} \leq 54.84 \text{ kN} \quad (\text{OK}) \end{aligned}$$

\therefore For safety, in combined shear and tension

$$\begin{aligned} &\left(\frac{V_{sb}}{V_{db}} \right)^2 + \left(\frac{T_b}{T_{db}} \right)^2 \leq 1.0 \\ \Rightarrow &\left(\frac{17.68}{28.97} \right)^2 + \left(\frac{17.68}{45.17} \right)^2 = 0.53 < 1 \quad (\text{OK}) \end{aligned}$$

Hence, joint is safe in combined shear and tension.

Q.5 (d) Solution:

(1) Internal or immersion vibrators: These vibrators consist of a steel tube which is inserted in fresh concrete. This steel tube is called the poker and it is connected to an electric motor or a petrol engine through a flexible tube. They are available in sizes varying from 40 mm to 100 mm diameters and the size is decided by keeping in mind the spacing between reinforcing bars in concrete. The frequency of vibration is about 3000 to 6000 r.p.m.

The poker vibrates while it is being inserted. The internal vibrators should be inserted and withdrawn slowly and they should be operated continuously while they are being withdrawn. Otherwise holes will be formed inside the concrete. The vibrator can be placed vertically or at a slight inclination not exceeding 10° to the vertical

with a view to avoid flow of concrete due to vibration into the mould and consequent scope of segregation. Hence, skilled and experienced personnel should handle internal vibrators. These vibrators are more efficient than other types of vibrators and hence they are most commonly used.

- (2) **Surface vibrators:** These vibrators are mounted on platform or screeds. They are used to finish concrete surfaces such as bridge floors, road slabs, station platform, etc. These vibrators are found to be more effective for compacting very dry concrete mixes because the vibration acts in the same direction of gravity and the concrete is compacted in a confined zone. These vibrators also cause movement of fine material to the top and it aids in finishing operations. However the movement of excess fine material at top will not be desirable for plastic mixes as the wearing resistance of such fine material is very low.
- (3) **Form or shutter vibrators:** These vibrators are attached to the formwork and external centering of walls, columns, etc. The vibrating action is conveyed to the concrete through the formwork during transmission of vibrations. Hence, they are not generally used. But they are very much helpful for sections which are too thin for the use of internal vibrators.

These vibrators require more power because of loss of some power in vibrating the rigid shutters. They are also heavy and hence they cannot be clamped at as many points as possible for uniform compaction of concrete. The compaction by these vibrators is found to be effective only upto a distance of about 450 mm from the face of the formwork.

- (4) **Vibrating tables:** These are in the form of a rigidly built steel platform mounted on flexible springs and they are operated by electromagnetic action or electric motors. They are found to be very effective in compacting stiff and harsh concrete mixes and hence they are invariably used in the preparation of pre-cast structural products in factories and test specimens in laboratories.

The tables are vibrated either mechanically or by placing the springs under the supports of tables. The frequency of vibration varies from 3000 to 7200 vibrations per minute. The two parameters of vibrations are frequency and time and they are related as below :

$$\text{Frequency } \alpha = 1/\text{Time}$$

It means that the frequency is inversely proportional to the time of vibration. In other words, if frequency is more, the consolidation of concrete will be achieved in less time and vice versa.

Q.5 (e) Solution:

Static indeterminacy, D_s is,

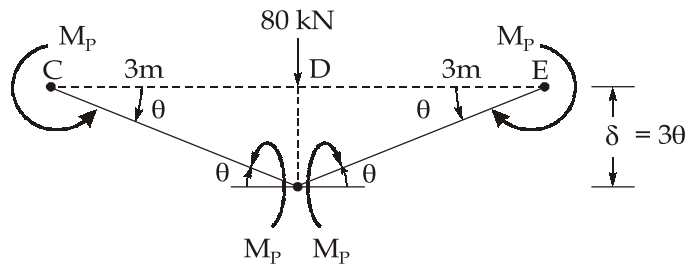
$D_s = \text{Number of reactions} - \text{Number of equilibrium equations}$

$$\Rightarrow D_s = 4 - 3 = 1$$

Number of possible plastic hinges = 4 (at B, C, D & E.)

\therefore Number of independent mechanisms = $N - D_s = 4 - 1 = 3$

i.e. 2 beam mechanisms and 1 sway mechanism.

1. Beam mechanism in span CE

$$80 \times \delta = 4 M_p \theta$$

$$\Rightarrow 4 M_p \times \frac{\delta}{3} = 80 \times \frac{\delta}{1} \quad \left[\because \theta = \frac{\delta}{3} \right]$$

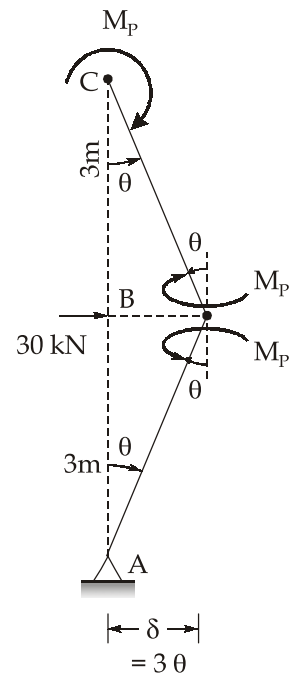
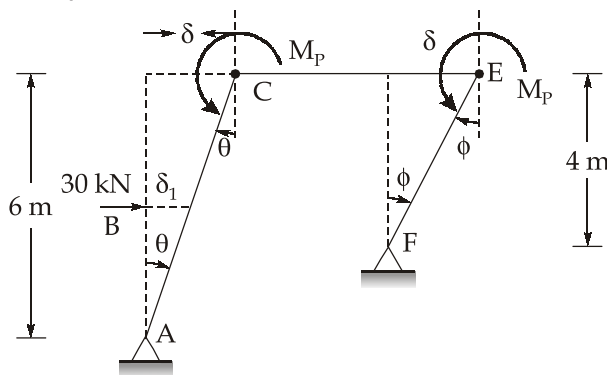
$$\Rightarrow M_p = 80 \times \frac{3}{4} = 60 \text{ kN-m}$$

2. Beam mechanism in span AC

$$3 M_p \theta = 30 \times \delta$$

$$\Rightarrow 3 M_p \times \frac{\delta}{3} = 30 \times \delta \quad \left[\because \theta = \frac{\delta}{3} \right]$$

$$\Rightarrow M_p = 30 \text{ kN-m}$$

3. Sway mechanism

$$M_P \times \theta + M_P \phi = 30 \times \delta_1$$

Now, $\theta = \frac{\delta}{6}, \phi = \frac{\delta}{4}$

Also $\delta_1 = \delta/2$

$$\Rightarrow M_P \times \frac{\delta}{6} + M_P \times \frac{\delta}{4} = 30 \times \frac{\delta}{2}$$

$$\Rightarrow 2M_P + 3M_P = 30 \times 6$$

$$\Rightarrow M_P = 36 \text{ kNm}$$

Now there are two possible combined mechanisms as below.

1. Combined Mechanism- 1

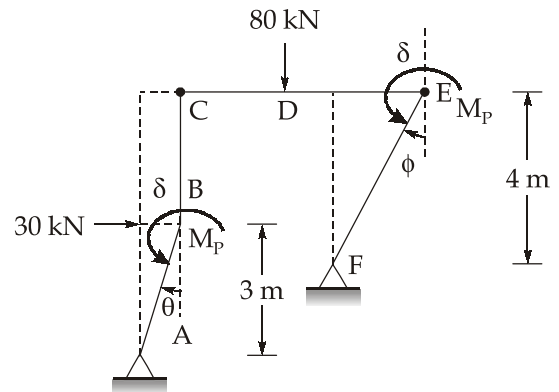
$$M_P \theta + M_P \phi = 30 \times \delta$$

$$\theta = \frac{\delta}{3}, \phi = \frac{\delta}{4}$$

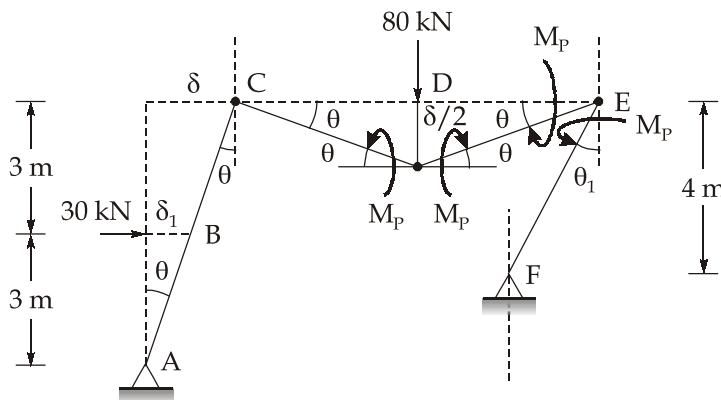
$$\therefore M_P \times \frac{\delta}{3} + M_P \times \frac{\delta}{4} = 30 \times \delta$$

$$\Rightarrow 4M_P + 3M_P = 360 \text{ kN-m}$$

$$\Rightarrow M_P = 51.43 \text{ kNm}$$



2. Combined mechanism - 2



$$30 \times \delta_1 + 80 \times \frac{\delta}{2} = 3M_P \theta + M_P \theta_1$$

$$\delta_1 = \frac{\delta}{2}, \theta = \frac{\delta}{6}, \theta_1 = \frac{\delta}{4}$$

$$\therefore 30 \times \frac{\delta}{2} + 80 \times \frac{\delta}{2} = 3M_P \times \frac{\delta}{6} + M_P \times \frac{\delta}{4}$$

$$\Rightarrow 110 = M_p + \frac{M_p}{2}$$

$$\Rightarrow 1.5 M_p = 110 \text{ kN-m}$$

$$\Rightarrow M_p = 73.33 \text{ kN-m}$$

Moment capacity of the frame will be equal to maximum of 60 kN-m, 30 kN-m, 36 kN-m, 51.43 kN-m and 73.33 kN-m.

$$\therefore M_p = 73.33 \text{ kN-m}$$

Q.6 (a) Solution:

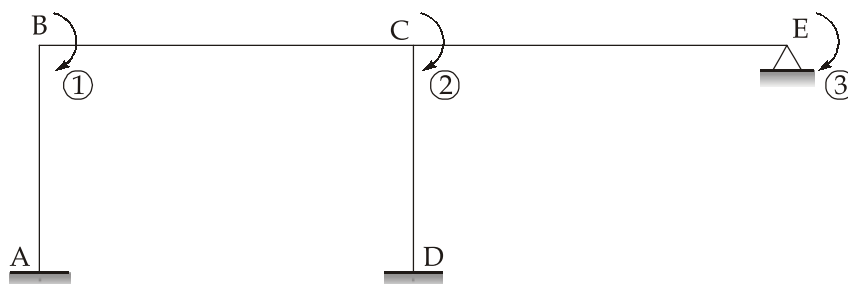
$$D_K = 3j - r_e - m$$

Here, $j = 5, r_e = 8, m = 4$

$$\therefore D_K = 3 \times 5 - 8 - 4 = 3$$

Take θ_B, θ_C and θ_E as unknown displacements.

Assign coordinate (1) in the direction of θ_B , coordinate (2) in the direction of θ_C and coordinate (3) in the direction of θ_E .

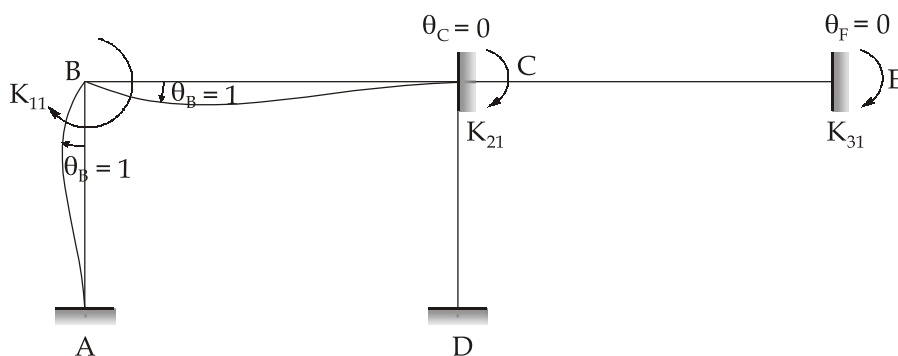


Stiffness matrix

1st column

Give unit displacement in the direction of co-ordinate ①,

$$\therefore \theta_B = 1 \text{ and ensuring } \theta_C = \theta_E = 0$$



$$K_{11} = \frac{4EI}{6} + \frac{4EI}{6} = \frac{4EI}{3}$$

$$K_{21} = \frac{2EI}{6} = \frac{EI}{3}$$

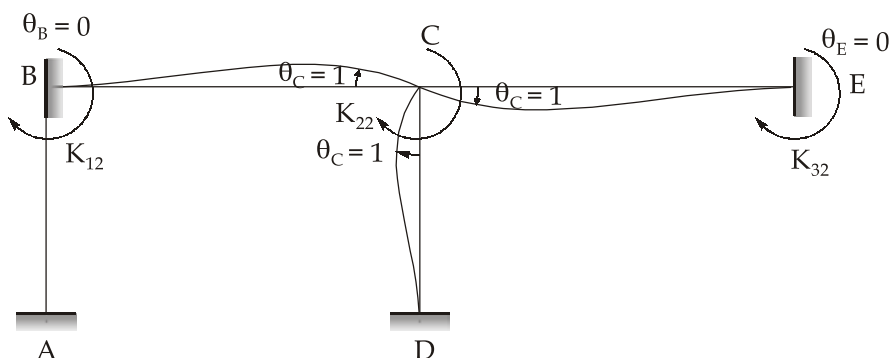
$$K_{31} = 0$$

2nd column

Give unit displacement in the direction of co-ordinate ②.

∴

$$\theta_C = 1 \text{ and } \theta_B = \theta_E = 0$$

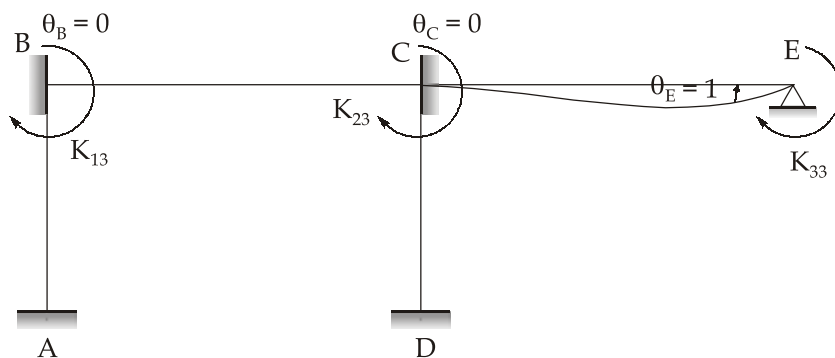


$$K_{12} = K_{21} = \frac{EI}{3}$$

$$K_{22} = \frac{4EI}{6} + \frac{4EI}{6} + \frac{4EI}{6} = 2EI$$

$$K_{32} = \frac{2EI}{6} = \frac{EI}{3}$$

3rd column



Give unit displacement in the direction of co-ordinate ③,

∴

$$\theta_E = 1 \text{ and } \theta_B = \theta_C = 0$$

$$K_{13} = K_{31} = 0$$

$$K_{33} = \frac{4EI}{6} = \frac{2EI}{3}$$

$$K_{23} = K_{32} = \frac{EI}{3}$$

Now, the stiffness matrix for selected co-ordinate system is given below.

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} = \begin{bmatrix} \frac{4EI}{3} & \frac{EI}{3} & 0 \\ \frac{EI}{3} & 2EI & \frac{EI}{3} \\ 0 & \frac{EI}{3} & \frac{2EI}{3} \end{bmatrix}$$

$$\Rightarrow [K] = \frac{EI}{3} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 6 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

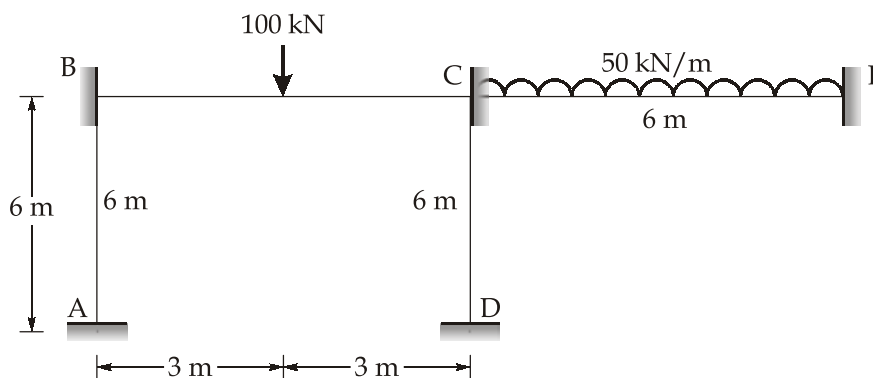
Inverse of stiffness matrix is

$$adj(K) = \frac{3}{EI} \begin{bmatrix} 11 & -2 & 1 \\ -2 & 8 & -4 \\ 1 & -4 & 23 \end{bmatrix}$$

$$|K| = 4(11) - 1(2) + 0 = 42$$

$$\therefore [K]^{-1} = \frac{adj(K)}{|K|} = \frac{3}{42EI} \begin{bmatrix} 11 & -2 & 1 \\ -2 & 8 & -4 \\ 1 & -4 & 23 \end{bmatrix}$$

Let P_{1L} , P_{2L} , and P_{3L} are forces developed in co-ordinate direction due to the given load in locked structure.



$$P_{1L} = \bar{M}_{BC} = -\frac{100 \times 6}{8}$$

$$= -75 \text{ kN-m}$$

$$\begin{aligned} P_{2L} &= \bar{M}_{BC} - \bar{M}_{CE} \\ &= \frac{100 \times 6}{8} - \frac{50 \times 6^2}{12} \\ &= -75 \text{ kN-m} \end{aligned}$$

$$P_{3L} = \frac{50 \times 6^2}{12} = 150 \text{ kN-m}$$

Now,

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_E \end{bmatrix} = \frac{3}{42EI} \begin{bmatrix} 11 & -2 & 1 \\ -2 & 8 & -4 \\ 1 & -4 & 23 \end{bmatrix} \begin{bmatrix} -(-75) \\ -(-75) \\ -150 \end{bmatrix}$$

$$\therefore \theta_B = \frac{3}{42EI} [11(75) - 2(75) - 150] = \frac{37.5}{EI}$$

$$\theta_C = \frac{3}{42EI} [-2(75) + 8(75) + 4(150)] = \frac{75}{EI}$$

$$\theta_E = \frac{3}{42EI} [1(75) - 4(75) - 23(150)] = \frac{-262.5}{EI}$$

Now final end moments using slope deflection equations are as below.

$$\begin{aligned} \Rightarrow M_{AB} &= \bar{M}_{AB} + \frac{2EI}{6} [2\theta_A + \theta_B] \\ &= \frac{EI\theta_B}{3} = 12.5 \text{ kN-m} \quad \left[\because \bar{M}_{AB} = \theta_A = 0 \right] \end{aligned}$$

$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{6} [2\theta_B] = 25 \text{ kN-m}$$

$$\begin{aligned} M_{BC} &= \bar{M}_{BC} + \frac{2EI}{6} [2\theta_B + \theta_C] \\ &= -75 + \frac{EI}{3} \left[\frac{150}{EI} \right] = -25 \text{ kN-m} \end{aligned}$$

$$\begin{aligned} M_{CB} &= \bar{M}_{CD} + \frac{2EI}{6} [2\theta_C] \\ &= 75 + \frac{EI}{3} \left[\frac{187.5}{EI} \right] = 137.5 \text{ kN-m} \end{aligned}$$

$$M_{CD} = \bar{M}_{CD} + \frac{2EI}{6} [2\theta_C]$$

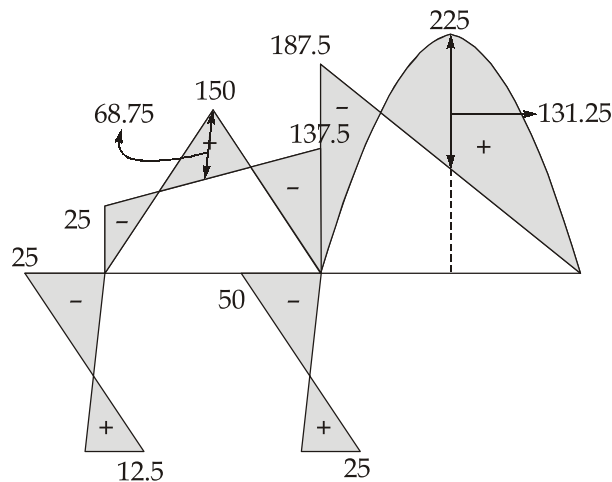
$$= 0 + \frac{EI}{3} \left[\frac{150}{EI} \right] = 50 \text{ kN-m}$$

$$M_{DC} = \bar{M}_{DC} + \frac{2EI}{6} [\theta_C] = 25 \text{ kN-m}$$

$$M_{CE} = \bar{M}_{CE} + \frac{2EI}{6} [2\theta_C + \theta_E]$$

$$= -150 + \frac{EI}{3} \left[\frac{-112.5}{EI} \right] = -187.5 \text{ kN-m}$$

$$M_{EC} = \bar{M}_{EC} + \frac{2EI}{6} [2\theta_E + \theta_C] = 150 - \frac{450}{3} = 0$$



(Values in kN-m)

Bending moment diagram (BMD)

Q.6 (b) Solution:

Defects due to natural forces

1. Knots

- These are the bases of branches or limbs which are broken or cut, encased by the wood of the free trunk.
- Knots are formed in timber when tree lose its branch. When the tree grows further and puts on more wood, the stumps of these branches which have fallen off are covered and appear as knots in the sawn pieces of timber.
- As continuity of wood fibres is broken by knots, they form a source of weakness and reduces the workability of timber.

2. Shakes

- It is longitudinal separations (cracks) in the wood between the annual rings.
- This lengthwise separations reduce the allowable shear strength without much effect on compressive and tensile strength.
- Wood appearance becomes undesirable.

(i) Heart shake:

- It occurs due to shrinkage of heartwood, (interior of a tree) when tree is overmatured.
- Cracks start from pith and run towards sapwood.
- These are wider at centre and diminish outwards.

(ii) Cup shake:

- It appears as curved split which partly or wholly separates annual rings from one another.
- It is caused due to excessive frost action or non-uniform growth.

(iii) Star shake:

- It is radial splits or cracks wide at circumference (bark) and diminishing towards the centre of the tree. It is confined usually to sapwood thus giving star appearance at the end of a piece.
- This may arise from severe frost and fierce heat of sun. Star shakes appear as the wood dries below the fibre saturation point. It is fault leading to separation of log into number of pieces when sawn.

(iv) Ring shakes:

- When cup shakes cover the entire ring, they are known as the ring shakes.

(v) Radial shakes:

- These are similar to star shakes. But they are fine, irregular and numerous.
- This split starts from bark and sapwood and extends to the heartwood and pitch.
- These defect occurs when outer tissues dry at faster rate than inner ones. This defect can also occur during seasoning process due to excessive heat of sun or cold or frost.

3. Foxiness

- It is indicated by red or yellow tinge stain in wood; or reddish or brown stains or spots round the pith of tree.

It is caused either due to poor ventilation during storage or by commencement of decay due to over-maturity or due to growth of tree in a marshy soil.

4. Druxiness

- It is indicated by white decayed spots which are formed by fungi

5. Burls (Rind Galis)

- Also known as the excrescences (distinct outgrowth resulting from abnormality) and are formed when tree receives injury in its young age or due to unsuccessful attempts at the formation of branches.

6. Coarse grain

- If a tree grows rapidly, the annual rings are widened. This results in the coarse grained timber. Such timber possesses lesser strength.

7. Twisted fibres (Wondering heart) caused by twisting of young trees constantly in one direction by fast blowing wind.

- Timber with twisted fibres is unsuitable for sawing. It can however be used for posts and poles in an unsawn condition.

8. Upsets (Ruptures)

- They indicate the wood fibres which are injured by crushing or compression.
- Upsets are mainly due to improper felling of tree and exposure of tree in its young age to fast blowing wind.

9. Wind cracks:

- If wood is exposed to atmospheric agencies, its exterior surface shrinks. Such a shrinkage results in cracks. These cracks are not very deep.

Q.6 (c) Solution:

Let's first find the spacing of the channel sections

Let the spacing be S , then for economical design

$$I_{zz} = I_{yy}$$

$$\Rightarrow 310.8 \times 10^4 + 4564 \left(23.6 + \frac{S}{2} \right)^2 = 6362.6 \times 10^4$$

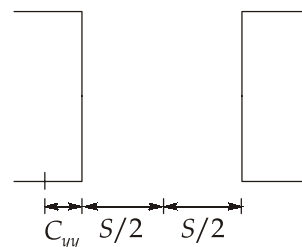
$$\Rightarrow S = 183.1 \text{ mm}$$

In the design of built up columns it is desirable to keep $I_{yy} \geq I_{zz}$.

Therefore, let's assume a spacing of 200 mm.

Check for design compressive strength of the column.

$$\text{Effective slenderness ratio} = 1.1 \times \left(\frac{KL}{r_z} \right) = 1.1 \times \frac{10 \times 10^3}{118.1} = 93.14$$



Using table given in question,

$$\frac{100 - 90}{107 - 121} = \frac{93.14 - 90}{f_{cd} - 121}$$

$$\Rightarrow f_{cd} = 116.6 \text{ N/mm}^2$$

$$\begin{aligned} \therefore \text{Design compressive strength} &= 2 \times 4564 \times 116.6 \times 10^{-3} \text{ kN} \\ &= 1064.3 \text{ kN} > 1000 \text{ kN} \end{aligned}$$

[OK]

Design of battens

Let spacing of battens be C

$$\therefore \frac{C}{r_y} \nlessgtr 0.7 \text{ times slenderness ratio of column as a whole and also } C \nlessgtr 50$$

$$\therefore \frac{C}{26.1} \nlessgtr 0.7 \times \frac{1.1 \times 10000}{118.1}$$

$$\Rightarrow C \nlessgtr 1701.69 \text{ mm} \quad \dots(i)$$

$$\text{Also } \frac{C}{26.1} \nlessgtr 50$$

$$\Rightarrow C \nlessgtr 1305 \text{ mm} \quad \dots(ii)$$

From (i) and (ii), provide battens at a spacing of 1300 mm.

Size of end battens:

Provide 20 mm diameter bolts of grade 4.6 for connection.

$$\therefore \text{Diameter of bolt hole, } d_O = 20 + 2 = 22 \text{ mm}$$

$$\text{Minimum edge distance, } 1.5 d_O = 22 \times 1.5 = 33 \text{ mm} \simeq 35 \text{ mm}$$

$$\text{Effective depth of batten } d' = 200 + 2 \times 23.6 = 247.2 \text{ mm} > 2 \times 90 \text{ mm}$$

$$\text{Overall depth, } d = 247.2 + 2 \times 35 = 317.2 \text{ mm} \simeq 320 \text{ mm}$$

$$\text{Length of the battens} = 200 + 2 \times 90 = 380 \text{ mm}$$

$$\text{Thickness of the batten} = \frac{1}{50} \times (200 + 2 \times 50) = 6 \text{ mm}$$

Provide 380 × 320 × 6 mm end batten plates

Size of intermediate battens:

Effective depth of intermediate batten plate,

$$d'_1 = \frac{3}{4} \times 247.2 = 185.4 \text{ mm} > 2 \times 90 \text{ mm}$$

$$\text{Overall depth of batten, } d' = 185.4 + 2 \times 35 = 255.4 \text{ mm} \simeq 260 \text{ mm}$$

$$\text{Thickness of batten} = \frac{1}{50} \times (200 + 2 \times 50) = 6 \text{ mm}$$

Provide $380 \times 260 \times 6$ mm intermediate batten plates.

Design forces

Transverse shear, $V_t = 2.5\%$ of axial load.

$$= \frac{2.5}{100} \times 1000 = 25 \text{ kN}$$

$$\text{Longitudinal shear, } V_b = \frac{25 \times 1300}{2 \times (200 + 2 \times 50)} = 54.17 \text{ kN}$$

$$\text{Moment, } M = \frac{25 \times 1300}{2 \times 2 \times 10^3} = 8.125 \text{ kN}$$

Check for end battens:

$$\text{Shear stress} = \frac{54.17 \times 10^3}{320 \times 6} = 28.21 \text{ N/mm}^2 < \frac{f_y}{\sqrt{3}\gamma_0} = \frac{250}{\sqrt{3} \times 1.1} = 131.2 \text{ N/mm}^2$$

$$\begin{aligned} \text{Bending stress} &= \frac{6M}{td^2} = \frac{6 \times 8.125 \times 10^6}{6 \times 320^2} = 79.35 \text{ N/mm}^2 < \frac{f_y}{\gamma_0} \\ &= \frac{250}{1.1} = 227.2 \text{ N/mm}^2 \end{aligned} \quad (\text{OK})$$

Check for intermediate battens:

$$\text{Shear stress} = \frac{54.17 \times 10^3}{260 \times 6} = 34.72 \text{ N/mm}^2 < 131.2 \text{ N/mm}^2 \quad (\text{OK})$$

$$\text{Bending stress} = \frac{6M}{td^2} = \frac{6 \times 8.125 \times 10^6}{6 \times 260^2} = 120.19 \text{ N/mm}^2 < \frac{250}{1.1} \text{ N/mm}^2 \quad (\text{OK})$$

Q.7 (a) Solution:

(i) Fixed end moments:

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = \frac{-WL}{8} = \frac{-48 \times 4}{8} = -24 \text{ kN-m}$$

$$M_{FCB} = \frac{WL}{8} = \frac{48 \times 4}{8} = 24 \text{ kN-m}$$

(ii) Slope deflection equations:**Member AB**

$$M_{AB} = M_{FAB} + \frac{2E(2I)}{l} \left(2\theta_A + \theta_B - \frac{3\delta}{6} \right)$$

where, δ is the horizontal displacement of joint B

$$\begin{aligned} \therefore M_{AB} &= 0 + \frac{2EI}{3} \left(2\theta_A + \theta_B - \frac{\delta}{2} \right) \\ &= \frac{4}{3}EI\theta_A + \frac{2}{3}EI\theta_B - \frac{EI\delta}{3} \end{aligned} \quad \dots(i)$$

$$\begin{aligned} M_{BA} &= M_{FBA} + \frac{2E(2I)}{l} \left(2\theta_B + \theta_A - \frac{3\delta}{6} \right) \\ &= 0 + \frac{2E(2I)}{6} \left(2\theta_B + \theta_A - \frac{3\delta}{6} \right) \\ &= \frac{2}{3}EI\theta_A + \frac{4}{3}EI\theta_B - \frac{EI\delta}{3} \end{aligned} \quad \dots(ii)$$

Member BC

$$\begin{aligned} M_{BC} &= M_{FBC} + \frac{2E(2I)}{l} (2\theta_B + \theta_C) \\ &= -24 + \frac{2E(2I)}{4} (2\theta_B + \theta_C) \\ &= -24 + 2EI\theta_B + EI\theta_C \end{aligned} \quad \dots(iii)$$

$$\begin{aligned} M_{CB} &= M_{FCB} + \frac{2E(2I)}{l} (2\theta_C + \theta_B) \\ &= 24 + \frac{2E(2I)}{4} (2\theta_C + \theta_B) \\ &= 24 + EI\theta_B + 2EI\theta_C \end{aligned} \quad \dots(iv)$$

Member CD

$$\begin{aligned} M_{CD} &= M_{FCD} + \frac{2EI}{l} \left(2\theta_C - \frac{3\delta}{l} \right) \quad [\because \theta_D = 0 \text{ as D is fixed}] \\ &= 0 + \frac{2EI}{4} \left(2\theta_C - \frac{3\delta}{4} \right) = EI\theta_C - \frac{3}{8}EI\delta \end{aligned} \quad \dots(v)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l} \left(\theta_C - \frac{3\delta}{l} \right)$$

$$= 0 + \frac{2EI}{4} \left(\theta_C - \frac{3\delta}{4} \right) = \frac{EI\theta_C}{2} - \frac{3}{8}EI\delta \quad \dots(vi)$$

(iii) Joint equilibrium conditions

$$\Rightarrow \Sigma M_B = 0$$

$$\Rightarrow M_{BA} + M_{BC} = 0$$

$$\Rightarrow \frac{2}{3}EI\theta_A + \frac{4}{3}EI\theta_B - \frac{EI\delta}{3} - 24 + 2EI\theta_B + EI\theta_C = 0$$

$$\Rightarrow \frac{2}{3}EI\theta_A + \frac{10}{3}EI\theta_B + EI\theta_C - \frac{EI\delta}{3} = 24 \quad \dots(vii)$$

$$\Sigma M_C = 0$$

$$\Rightarrow M_{CB} + M_{CD} = 0$$

$$\Rightarrow 24 + EI\theta_B + 2EI\theta_C + EI\theta_C - \frac{3}{8}EI\delta = 0$$

$$\Rightarrow EI\theta_B + 3EI\theta_C - \frac{3}{8}EI\delta = -24 \quad \dots(viii)$$

Also, $M_{AB} = 0$ [\because End A is hinged]

$$\Rightarrow \frac{4}{3}EI\theta_A + \frac{2}{3}EI\theta_B - \frac{EI\delta}{3} = 0 \quad \dots(ix)$$

Shear equation :

$$\frac{M_{AB} + M_{BA}}{6} + \frac{M_{CD} + M_{DC}}{4} = 0$$

$$\Rightarrow \frac{\frac{4}{3}EI\theta_A + \frac{2}{3}EI\theta_B - \frac{EI\delta}{3} + \frac{2}{3}EI\theta_A + \frac{4}{3}EI\theta_B - \frac{EI\delta}{3}}{6} + \frac{EI\theta_C - \frac{3}{8}EI\delta + \frac{EI\theta_C}{2} - \frac{3}{8}EI\delta}{4} = 0$$

$$\Rightarrow \frac{EI\theta_A}{3} + \frac{EI\theta_B}{3} - \frac{EI\delta}{9} + \frac{3}{8}EI\theta_C - \frac{3}{16}EI\delta = 0$$

$$\Rightarrow \frac{EI\theta_A}{3} + \frac{EI\theta_B}{3} + \frac{3}{8}EI\theta_C - \frac{43}{144}EI\delta = 0 \quad \dots(x)$$

Solving (vii), (viii), (ix) and (x), we get

$$EI\theta_A = -9.6 \quad \square$$

$$EI\theta_B = 11.77$$

$$EI\theta_C = -13.78$$

$$EI\delta = -14.9$$

So,

$$M_{AB} = \frac{4}{3}(-9.6) + \frac{2}{3} \times 11.77 - \frac{1}{3} \times (-14.9) = 0$$

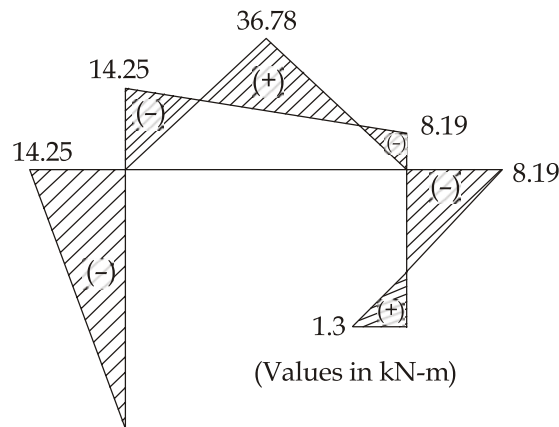
$$M_{BA} = \frac{2}{3}(-9.6) + \frac{4}{3} \times 11.77 - \frac{1}{3} \times (-14.9) = 14.25 \text{ kN-m}$$

$$M_{BC} = -24 + 2 \times 11.77 + (-13.78) = -14.25 \text{ kN-m}$$

$$M_{CB} = 24 + 11.77 + 2 \times (-13.78) = 8.21 \text{ kN-m}$$

$$M_{CD} = (-13.78) - \frac{3}{8} \times (-14.9) = -8.19 \text{ kN-m}$$

$$M_{DC} = \frac{1}{2} \times (-13.78) - \frac{3}{8} \times (-14.9) = -1.3 \text{ kN-m}$$



Q.7 (b) (i) Solution:

Determination of required stiffener.

Using,
$$\frac{d}{t_w} = \frac{1500}{15} = 100$$

For no stiffeners,
$$\frac{c}{d} = \infty$$

From table,
$$\tau_b = 95.7 \text{ N/mm}^2$$

Hence shear strength corresponding to web buckling.

$$V_{cr} = dt_w \tau_b = 1500 \times 15 \times 95.7 \times 10^{-3} \text{ kN} = 2153.25 \text{ kN} < 2800 \text{ kN}$$

Hence stiffeners are required.

$$\text{Required shear buckling strength } \tau_b \text{ for no stiffeners} = \frac{2800 \times 1000}{1500 \times 15} = 124.44 \text{ N/mm}^2$$

From second table,

For
$$\tau_b = 124.44 \text{ N/mm}^2,$$

c/d ratio can be found out as

$$\frac{130 - 122.3}{1 - 1.2} = \frac{124.44 - 122.3}{(c/d) - 1.2}$$

$$\Rightarrow \frac{c}{d} = 1.14$$

∴ Provide stiffeners at a spacing of $(c) = 1.14 \times d$
 $= 1.14 \times 1500 = 1710 \text{ mm}$

(b) Clearly, thickness of the web required will be more than 15 mm for no stiffeners.

Let the thickness be 18 mm, then

$$\frac{d}{t_w} = \frac{1500}{18} = 83.33$$

$$\text{For } \frac{c}{t} = \infty, \frac{d}{t_w} = 83.33$$

Using first table,

$$\frac{85 - 80}{116.9 - 123.9} = \frac{85 - 83.33}{116.9 - \tau_b}$$

$$\Rightarrow \tau_b = 119.238 \text{ N/mm}^2$$

$$\therefore V_{cr} = 1500 \times 18 \times 119.238 \times 10^{-3} \text{ Kn} = 3219.43 \text{ kN} > 2800 \text{ kN}$$

Hence adopt 18 mm web thickness.

(ii)

Table: Comparison between Clamp Burning and Kiln Burning

S.No.	Property	Clamp-burning	Kiln burning
1.	Capacity	About 20000-100000 bricks.	Avg. 25000 bricks.
2.	Cost of fuel	Low as grass, cow dung, litter may be used.	High because coal dust is to be used.
3.	Initial cost	Very low as no structures are to be built.	More as permanent structures are to be constructed.
4.	Quality of bricks	The percentage of good quality bricks is small about 60%.	Percentage of good quality bricks is high about 90%.
5.	Regulation of fire	It is not possible to control or regulate fire during the process of burning.	The fire is under control throughout the process of burning.
6.	Skilled supervision	Not necessary through out the process of burning.	The Continuous skilled supervision is necessary.
7.	Structure	Temporary structure.	Permanent structure.
8.	Suitability	For small scale.	For large scale.
9.	Time of burning and cooling	It requires about 2-6 months.	Actual burning time is 24 hours and 12 days are required for cooling of bricks.

Q.7 (c) Solution:

(i)

For Fe410 (E250) grade of steel,

$$f_u = 410 \text{ N/mm}^2, f_y = 250 \text{ N/mm}^2$$

Nominal diameter of bolt (d) = 16 mmDiameter of bolt hole, $d_o = d + 2 = 16 + 2 = 18 \text{ mm}$

Tensile strength of section based on gross-section yielding

$$T_{dg} = \frac{f_y}{\gamma_{m0}} \times A_g = \left(\frac{250}{1.1} \times 2490 \right) \times 10^{-3} \text{ kN} = 565.91 \text{ kN}$$

Tensile strength of section based on net section rupture,

$$T_{dn} = 0.9 \frac{f_u}{\gamma_{m1}} \times A_{nc} + \beta \frac{f_y}{\gamma_{m0}} A_{g0}$$

where β is shear lag factor and is given by

$$\beta = 1.4 - 0.076 \times \left(\frac{w}{t} \right) \left(\frac{f_y}{f_u} \right) \left(\frac{b_s}{L_c} \right)$$

Also,

$$0.7 \leq \beta \leq \frac{0.9 f_u}{f_y} \times \frac{\gamma_{m0}}{\gamma_{m1}}$$

 w = width of outstand leg = 75 mmThickness of the web, (t) = 10.2 mm

Shear lag width,
$$b_s = \left(75 + 50 - \frac{6 + 10.2}{2} \right) = 116.9 \text{ mm}$$

Length of connection in the direction of load transfer = 100 mm

$$\begin{aligned} \therefore \beta &= 1.4 - 0.076 \frac{b_s}{L_c} \times \frac{w}{t} \times \frac{f_y}{f_u} \\ &= 1.4 - 0.076 \times \frac{116.9}{100} \times \frac{75}{10.2} \times \frac{250}{410} \\ &= 1.00167 \simeq 1 > 0.7 \end{aligned} \quad (\text{OK})$$

$$0.9 \times \frac{f_u}{f_y} \times \frac{\gamma_{m0}}{\gamma_{m1}} = \frac{0.9 \times 410}{250} \times \frac{1.1}{1.25} = 1.299$$

$$\therefore 0.7 < 1 < 1.299 \quad (\text{Hence OK})$$

$$A_{nc} = (175 - 2 \times 18 - 10.2) \times 6 = 772.8 \text{ mm}^2$$

$$A_{go} = (2490 - 772.8) = 1717.2 \text{ mm}^2$$

$$\begin{aligned} \text{Now, } T_{dn} &= \left(0.9 \times \frac{410}{1.25} \times 772.8 + 1 \times \frac{250 \times 1717.2}{1.1} \right) \times 10^{-3} \text{ kN} \\ &= 618.4 \text{ kN} \end{aligned}$$

Check for block shear strength :

$$T_{db} = \text{minimum} \begin{cases} (1) \text{ Shear yielding and tension rupture} \\ (2) \text{ Tension yielding and shear rupture} \end{cases}$$

(1) Shear yielding and tension rupture

$$A_{vg} = 130 \times 6 \times 2 = 1560 \text{ mm}^2$$

$$A_{tn} = (75 - 18) \times 6 = 342 \text{ mm}^2$$

$$\begin{aligned} \therefore T_{db1} &= \left(\frac{250 \times 1560}{\sqrt{3} \times 1.1} + \frac{0.9 \times 410 \times 342}{1.25} \right) \times 10^{-3} \text{ kN} \\ &= 305.66 \text{ kN} \end{aligned}$$

(2) Tension yielding and shear rupture

$$A_{vn} = (130 - 45) \times 6 \times 2 = 1020 \text{ mm}^2$$

$$A_{tg} = 450 \text{ mm}^2$$

$$\begin{aligned} \therefore T_{db2} &= \left(\frac{250 \times 450}{1.1} + 0.9 \times \frac{1020 \times 410}{\sqrt{3} \times 1.25} \right) \times 10^{-3} \\ &= 276.12 \text{ kN} \end{aligned}$$

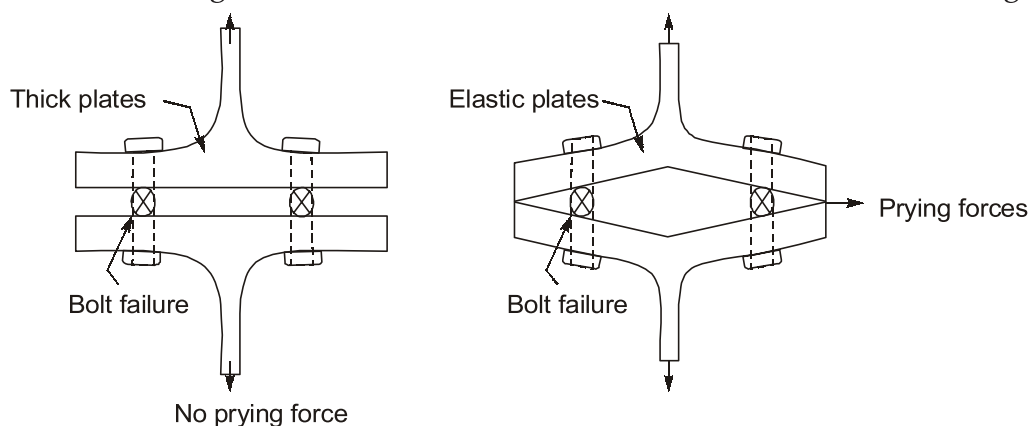
\therefore Tensile strength of section,

$$\begin{aligned} T_d &= \text{minimum} \{565.91, 276.12, 618.4\} \\ &= 276.12 \text{ kN} \end{aligned}$$

(ii) Prying Force: The connections where bolts are required to transfer load by direct tension an additional force is generated in the bolts due to thin jointed plates. These are called as prying forces. These forces can develop only when ends of connected plates are in contact due to external loads. The design force will be sum of the tensile force and additional prying force on bolt. Prying force is negligible if the plates to be jointed are thick and stiff.

Moment resisting beam to column connections often contain regions in which the bolts will be required to transfer load by direct tension such as the upper bolts in the end plate connection. In the design of such connections we should consider the additional forces as a result of prying action. These additional prying forces induced in the bolts are mainly due to the flexibility of connected plates. Thus in a simple

T-stub connection as shown in figure, the prying force will develop only when the ends of the flanges are in contact due to the external load as shown in figure.



Q.8 (a) Solution:

For the given truss,

Number of members, $m = 5$

Number of external reactions, $r_e = 4$

Number of joints, $j = 4$

So, degree of static indeterminacy,

$$\begin{aligned} D_s &= m + r_e - 2j \\ &= 5 + 4 - 2 \times 4 = 1 \end{aligned}$$

P system of forces :

Hence, the given truss is externally indeterminate by 1 degree. Let the support at C is removed as shown below.

Let, the length $AE = EC = L$

Now,

$$\sum M_C = 0$$

$$\Rightarrow V_A \times 2L + V_D \times L = 0$$

$$\Rightarrow V_A = -V_D/2$$

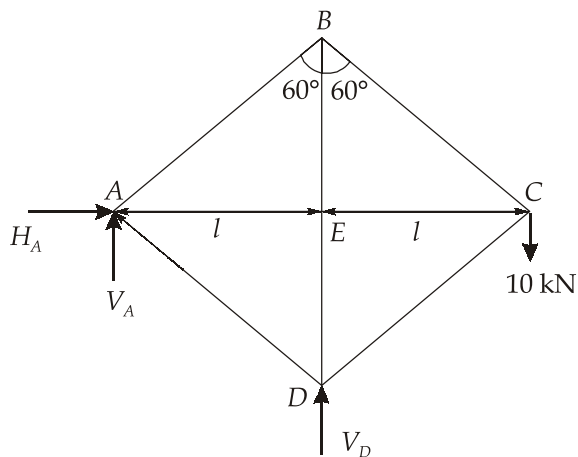
Also,

$$\sum F_y = 0$$

$$\Rightarrow V_A + V_D - 10 = 0$$

$$\Rightarrow \frac{-V_D}{2} + V_D - 10 = 0$$

$$\Rightarrow V_D = 20 \text{ kN}$$

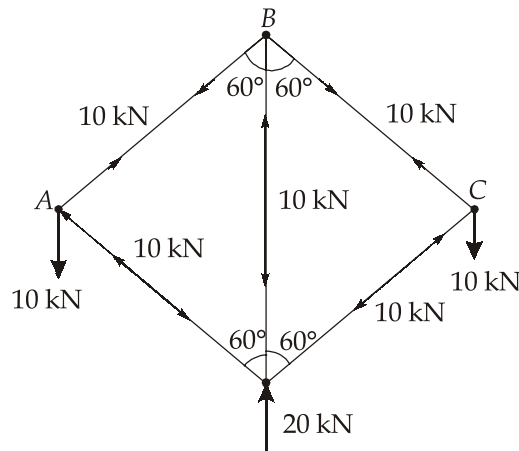


$$\therefore V_A = \frac{-V_D}{2} = \frac{-20}{2} = -10 \text{ kN} = 10 \text{ kN}(\downarrow)$$

$$\text{Also, } \sum F_x = 0$$

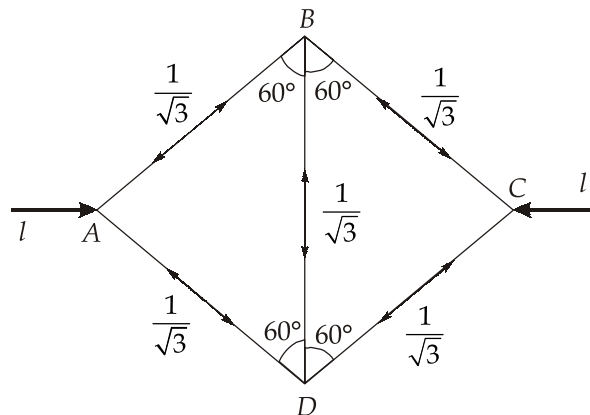
$$\Rightarrow H_A = 0$$

The forces in members are shown below.



K system of forces :

Remove the external load and apply a unit load in the direction of horizontal reaction at roller support C. The forces due to this unit load are shown below.



Now, the horizontal reaction at support C is,

$$X = \frac{-\sum PKL}{\sum \frac{K^2 L}{AE}} = \frac{-\sum \frac{PK}{A}}{\sum \frac{K^2}{A}} \quad \left[\begin{array}{l} \text{Length and 'E' value are} \\ \text{same for all the members} \end{array} \right]$$

Now the calculations are tabulated below.

Member	P (kN)	K	A (mm ²)	$\frac{PK}{A}$	$\frac{K^2}{A}$
AB	-10	$\frac{1}{\sqrt{3}}$	500	$\frac{-1}{50\sqrt{3}}$	$\frac{1}{1500}$
BC	-10	$\frac{1}{\sqrt{3}}$	500	$\frac{-1}{50\sqrt{3}}$	$\frac{1}{1500}$
CD	10	$\frac{1}{\sqrt{3}}$	1000	$\frac{1}{100\sqrt{3}}$	$\frac{1}{3000}$
DA	10	$\frac{1}{\sqrt{3}}$	1000	$\frac{1}{100\sqrt{3}}$	$\frac{1}{3000}$
BD	10	$\frac{-1}{\sqrt{3}}$	1000	$\frac{-1}{100\sqrt{3}}$	$\frac{1}{3000}$
			$\Sigma =$	$\frac{-3}{100\sqrt{3}}$	$\frac{7}{3000}$

Now,

$$X = \left(\frac{\frac{-3}{100\sqrt{3}}}{\frac{7}{3000}} \right) = 7.42 \text{ kN}$$

S system of forces :

Now, final forces in members can be calculated using the formula

$$S = P + KX$$

So,

$$S_{AB} = -10 + \frac{1}{\sqrt{3}} \times 7.42 = -5.72 \text{ kN}$$

$$S_{BC} = -10 + \frac{1}{\sqrt{3}} \times 7.42 = -5.72 \text{ kN}$$

$$S_{CD} = 10 + \frac{1}{\sqrt{3}} \times 7.42 = 14.28 \text{ kN}$$

$$S_{DA} = 10 + \frac{1}{\sqrt{3}} \times 7.42 = 14.28 \text{ kN}$$

$$S_{BD} = 10 - \frac{1}{\sqrt{3}} \times 7.42 = 5.72 \text{ kN}$$

Q.8 (b) Solution:

(i)

1. **Class 1 (Plastic compression member)** : Class 1 cross-sections are those which can form a plastic hinge and have (inelastic) rotation mechanism. In other words, the cross-sections have sufficient plastic hinge rotation capacity to allow redistribution of moments within the structure. The stress distribution for these sections is rectangular.
2. **Class 2 (Compact compression member)** : Class 2 cross-sections are those which are capable of developing a fully plastic stress distribution i.e. the plastic moment capacity, but local buckling may prevent the development of plastic hinge with sufficient rotation capacity at the section. In other words, these sections have limited plastic hinge (inelastic) rotation capacity for formation of plastic mechanism. The stress distribution for these sections is rectangular.
3. **Class 3 (Semi compact compression member)** : These are the cross-sections in which the stress in the extreme compression member/fibre of the section (assuming elastic distribution of stresses) can reach the yield strength, but the section is not capable of developing the plastic moment of resistance due to local buckling effects. The width-to-thickness $\left(\frac{w}{t}\right)$ ratio of plate elements should be equal or less than that specified under class 3 but greater than that specified under class 2 (compact). The stress distribution for these sections is triangular.
4. **Class 4 (Slender compression member)** : These are the cross-sections in which one or more parts of the cross-section buckle locally even before reaching yield stress. The reduction in design stress is severe. In such cases, to account for the post-local buckling strength of the cross-section, the width of the compression plate element in excess of the semi-compact section limit is deducted.

As a result, it is usually more economically to thicken the members to take them out of slender range.

(ii)

Properties and uses of different types of glass:

1. **Soda-lime glass or commercial glass**: This is also known as the soda-glass or soft-glass. It is mainly a mixture of sodium silicate and calcium silicate.

Properties: Following are the properties of soda-lime glass:

- (i) It is available in clean and clear state.
- (ii) It is cheap.
- (iii) It is easily fusible at comparatively low temperatures.

- (iv) It is possible to blow or to weld articles made from this glass with the help of simple sources of heat.

Uses: It is used in the manufacture of glass tubes and other laboratory apparatus, plate glass, window glass, etc.

2. **Potash-lime glass:** This is also known as the Bohemian-glass or hard-glass. It is mainly a mixture of potassium silicate and calcium silicate.

Properties: Following are the properties of potash-lime glass:

- (i) It fuses at high temperatures.
- (ii) It is not easily affected by water and other solvents.
- (iii) It does not melt so easily.

Uses: This glass is used in the manufacture of glass articles which have to withstand high temperatures such as combustion tubes, etc.

3. **Potash-lead glass:** This is also known as the flint glass. It is mainly a mixture of potassium silicate and lead silicate.

Properties: Following are the properties of potash-lead glass:

- (i) It fuses very easily.
- (ii) It is easily attacked by aqueous solutions.
- (iii) It possesses bright lustre and great refractive power.
- (iv) Its specific gravity is about 3 to 3.30.
- (v) It turns black and opaque, if it comes into contact with reducing gases of the furnace during heating.

Uses: It is used in the manufacture of artificial gems, electric bulbs, lenses, prisms, etc.

4. **Common glass:** This is also known as the bottle glass. It is prepared from cheap raw materials. It is mainly a mixture of sodium silicate, calcium silicate and iron silicate.

Properties: Following are the properties of common glass:

- (i) It fuses with difficulty.
- (ii) It is brown, green or yellow in colour.
- (iii) It is easily attacked by acids.

Uses: It is mainly used in the manufacture of medicine bottles.

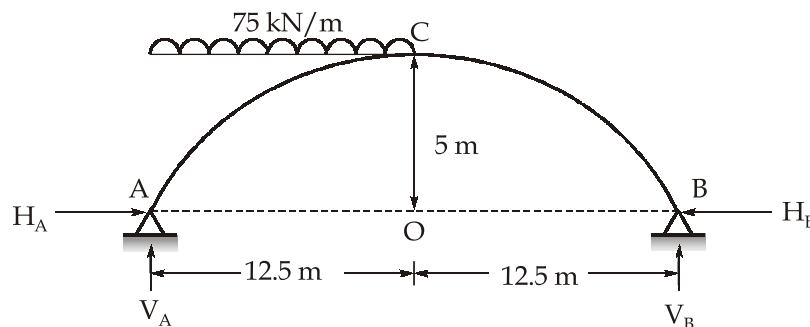
5. **Borosilicate glass:** Most of us are more familiar with this type of glass in the form of ovenware and other heat-resisting ware, better known under the trade name Pyrex. Borosilicate glass is made mainly of 70% to 80% silica and 7% to 13% boric oxide with smaller amounts of the alkalis (sodium and potassium oxides) and aluminium oxide.

Properties: Following are the properties of borosilicate glass:

- (i) It has a relatively low alkali content and consequently has good chemical durability and thermal shock resistance.
- (ii) It has high softening point.
- (iii) It does not break when temperature changes quickly.

Uses: This glass is widely used in the chemical industry, for laboratory apparatus, for ampoules and other pharmaceutical containers, for various high intensity lighting applications and as glass fibres used in the reinforced plastics to make protective helmets, boats, piping, car chassis, ropes, car exhausts and many other items and also in textile industry.

Q.8 (c) (i) Solution:



$$\sum M_B = 0$$

$$\Rightarrow V_A \times 25 - 75 \times 12.5 \times \left(12.5 + \frac{12.5}{2} \right) = 0$$

$$\Rightarrow V_A = 703.125 \text{ kN}$$

$$\sum F_y = 0$$

$$\Rightarrow V_A + V_B - 75 \times 25 = 0$$

$$\Rightarrow V_B = 75 \times 25 - 703.125 = 234.375 \text{ kN}$$

$$\text{Now, Horizontal thrust, } H = \frac{wl^2}{16h} = \frac{75 \times 25^2}{16 \times 5} = 585.94 \text{ kN}$$

Maximum positive bending moment will occur at a section in AC at 'x' distance from A.

Now, B.M. at x distance from A,

$$\begin{aligned} M_x &= V_A x - H \times y - 75 \times \frac{x^2}{2} \\ &= 703.12x - \frac{585.94 \times 4 \times 5 \times x}{25^2} \times (25 - x) - 37.5x^2 \\ &= 703.12x - 18.75x(25 - x) - 37.5x^2 \end{aligned}$$

$$= 703.125x - 468.75x + 18.75x^2 - 37.5x^2$$

$$= 234.375x - 18.75x^2$$

For maximum bending moment,

$$\frac{dM_x}{dx} = 0$$

$$\Rightarrow 234.375 - 2 \times 18.75x = 0$$

$$\Rightarrow x = 6.25 \text{ m}$$

$$\text{So, } M_{\max} = 234.375 \times 6.25 - 18.75 \times 6.25^2$$

$$= 732.42 \text{ kN-m}$$

(ii)

Proportion by Volume = 1 : 2 : 4

Assume volume of cement in 1 m³ concrete = $x \text{ m}^3$

\therefore Volume of fine aggregate = $2x \text{ m}^3$

Volume of coarse aggregate = $4x \text{ m}^3$

Weight of cement in concrete = $1500 x \text{ kg}$

Weight of fine aggregate in concrete = $1800 \times 2x \text{ kg}$

Weight of coarse aggregate in concrete = $2100 \times 4x \text{ kg}$

Weight of water in concrete = $(1500x) 0.55 \text{ kg}$

$$\text{Solid density of cement} = \frac{\text{Weight of cement solids}}{\text{Volume of cement solids}}$$

$$\gamma_{mc} = \frac{M_C}{V_S}$$

$$\text{Also, } \frac{V_V}{V_S} = e$$

$$\gamma_C = \frac{M_C}{V_V + V_S} = \frac{\gamma_{mc}}{1 + e}$$

$$\Rightarrow \gamma_{mc} = \gamma_c [1 + e] = 1500 \times 1.55 = 2325 \text{ kg/m}^3$$

$$\gamma_{MFA} = \gamma_{FA} [1 + e] = 1800 \times 1.35 = 2430 \text{ kg/m}^3$$

$$\gamma_{MCA} = \gamma_{CA} [1 + e] = 2100 \times 1.45 = 3045 \text{ kg/m}^3$$

\therefore Volume of concrete = Volume of air + Volume of water + Volume of solids

$$\Rightarrow 1 \text{ m}^3 = 0.025 + \frac{0.55 \times 1500x}{1000} + V_s$$

Here, $V_s = \frac{1500x}{2325} + \frac{1800 \times 2x}{2430} + \frac{2100 \times 4x}{3045}$

$\therefore 1 = 0.025 + \frac{0.55 \times 1500x}{1000} + \frac{1500x}{2325} + \frac{1800 \times 2x}{2430} + \frac{2100 \times 4x}{3045}$

$\Rightarrow x = 0.17075 \text{ m}^3$

$\therefore \text{Mass of cement} = 1500 (0.17075) = 256.1 \text{ kg}$

$\text{Mass of fine aggregate} = 1800 (2) (0.17075) = 614.7 \text{ kg}$

$\text{Mass of coarse aggregate} = 2100 (4) (0.17075) = 1434.3 \text{ kg}$

$\text{Mass of water} = 0.55 (256.1) = 140.8 \text{ kg}$

