MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 13

Section A

Q.1 (a) Solution:

(i) The pre envelope of a real valued signal $x(t)$ can be given by

$$x_{pre}(t) = x(t) + j\hat{x}(t)$$

where, $\hat{x}(t)$ is the Hilbert transform of $x(t)$.

$$x(t) \xleftrightarrow{\text{Fourier transform}} X(f)$$

$$\hat{x}(t) \xleftrightarrow{\text{Fourier transform}} (-j\,\text{sgn}(f))\,X(f)$$

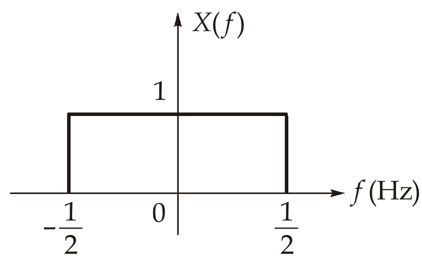
$$x_{pre}(t) \xleftrightarrow{\text{Fourier transform}} X_{pre}(f)$$

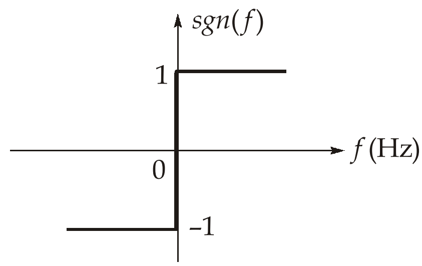
So,
$$X_{pre}(f) = X(f) + j(-j\text{sgn}(f))\,X(f)$$

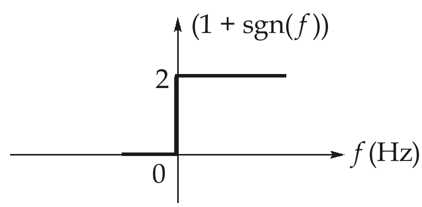
$$X_{pre}(f) = X(f)[1 + \text{sgn}(f)]$$

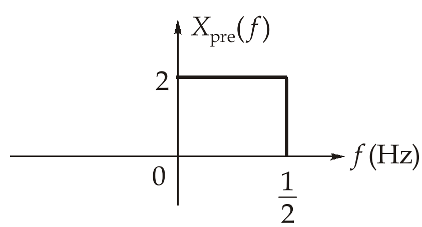
Now,
$$x(t) = \frac{\sin \pi t}{\pi t} = \text{sinc}(t) \xleftrightarrow{\text{F.T.}} X(f)$$

where, $X(f) = \text{rect}(f)$









The Fourier transform of $x_{\text{pre}}(t)$ can be expressed in terms of rect(f) as,

$$X_{\text{pre}}(f) = 2\text{rect}\left(2f - \frac{1}{2}\right)$$

We have, $\quad \sin c(t) \xleftrightarrow{\text{F.T}} \text{rect}(f)$

$$\sin c(t) e^{j2\pi\left(\frac{1}{2}\right)t} \xleftrightarrow{\text{F.T}} \text{rect}\left(f - \frac{1}{2}\right)$$

Using the scaling property of Fourier Transform,

$$\sin c\left(\frac{t}{2}\right) e^{j\pi\left(\frac{1}{2}\right)t} \xleftrightarrow{\text{F.T}} 2\text{rect}\left(2f - \frac{1}{2}\right)$$

So, the pre-envelope of the given signal $x(t)$ is,

$$x_{\text{pre}}(t) = \sin c\left(\frac{t}{2}\right) e^{j\frac{\pi t}{2}} = \frac{2\sin\left(\frac{\pi t}{2}\right)}{\pi t} e^{j\frac{\pi t}{2}}$$

(ii) For the received message r_0, we have,

$$p\left(\frac{r_0}{m_0}\right)p(m_0) > p\left(\frac{r_0}{m_1}\right)p(m_1) > p\left(\frac{r_0}{m_2}\right)p(m_2)$$

$$\Rightarrow \quad (0.6)\,(0.3) > (0.1)\,(0.5) > (0.1)\,(0.2)$$

Hence, we select m_0 wherever r_0 is received

For the received message r_1, we have,

$$p\left(\frac{r_1}{m_1}\right)p(m_1) > p\left(\frac{r_1}{m_0}\right)p(m_0) > p\left(\frac{r_1}{m_2}\right)p(m_2)$$

$$(0.5)\,(0.5) > (0.3)\,(0.3) > (0.1)\,(0.2)$$

Hence, we select m_1 wherever r_1 is received.

For the received message r_2, we have,

$$p\left(\frac{r_2}{m_1}\right)p(m_1) > p\left(\frac{r_2}{m_2}\right)p(m_2) > p\left(\frac{r_2}{m_0}\right)p(m_0)$$

$$(0.4)\,(0.5) > (0.8)\,(0.2) > (0.1)\,(0.3)$$

Hence, we select m_1 whenever r_2 is received.

Using the above decision rule, the probability of being correct is

$$p(c) = p(m_0)\,p\left(\frac{r_0}{m_0}\right) + p(m_1)\cdot p\left(\frac{r_1}{m_1}\right) + p(m_1)\,p\left(\frac{r_2}{m_1}\right)$$

$$= (0.6)\,(0.3) + (0.5)\,(0.5) + (0.5)\,(0.4) = 0.63$$

Hence, probability of error, $p(e) = 1 - p(c)$

$$p(e) = 1 - 0.63 = 0.37$$

Q.1 (b) Solution:

Error detection and correction are crucial functions of the data link layer in ensuring reliable data transmission across networks.

Error Detection: Error detection involves identifying errors in the transmitted data.

Common techniques include:

1. **Parity check:**
   - **Single Parity Bit:** A single bit is added to the data to make the number of 1's either even (even parity) or odd (odd parity). This detects simple-bit error but not multiple-bit errors.
   - **Two Dimensional Parity Check:** Adds parity bits both to rows and columns of a data block, improving error detection capability.

2. **Cyclic Redundancy Check (CRC):**
   - **Process:** The sender appends a CRC code (also known as a checksum) to the end of the data frame. This CRC code is generated by dividing the data by a pre-determined polynomial and taking the remainder.
   - **Verification:** At the receiver's end, the incoming frame is divided by the same polynomial. If the remainder is zero, the data is considered error-free; otherwise, an error is detected.
   - **Strengths:** CRC is highly effective for detecting burst errors and is widely used in networks like ethernet.
3. **Checksum**
   - **Usage:** Commonly used in higher-layer protocols (such as TCP/IP), but can also be implemented at the data link layer.
   - **Process:** The data is divided into equal segments, and their binary values are added together. The sum is then complemented to produce the checksum, which is appended to the data frame.
   - **Verification:** The receiver performs the same process on the received data, including the checkum and generates a checksum. This checksum is compared with the one sent along with the data. If the two checksums match, it indicates that the data was transmitted without errors.

II. **Error Correction:** Error correction involves not only detecting errors but also correcting them without the need for retransmission. Techniques include:

1. **Hamming Code**

   Process: Adds redundancy bits to the original data at specific positions, allowing detection and correction of single-bit errors.

   Parity Bits: Inserted at positions that are powers of two (1, 2, 4, 8 etc.). The positions of these bits help in identifying and correcting errors.

   Detection and correction: The receiver checks the parity bits. If any parity bit is incorrect, it identifies the bit position where the error occurred and correct it.

2. **Forward Error Correction (FEC)**

   Process: Uses algorithms to add redundant data that allows the receiver to detect and correct errors without needing retransmission.

   Techniques: Common FEC codes include read-solomon, convolutional codes, and low-density parity-check (LDPC) codes.

Usage: Widely used in applications where retransmission is costly or impossible, such as satellite and deep-space communications.

3. **Automatic Repeat Request (ARQ)**

Types

Stop and wait ARQ: The sender waits for an acknowledgment (ACK) for each frame before sending the next one. If a negative acknowledgment (NAK) or timeout occurs, the frame is retransmitted.

Go-Back-NARQ: The sender can send several frames before needing an acknowledgment but must retransmit all frames starting from the erroneous frame upon detecting an error.

Selective Repeat ARQ: Only the erroneous frames are retransmitted, allowing more efficient use of the network bandwidth.

Usage: ARQ relies on acknowledgments and timeouts for error control and is commonly used in reliable data communication protocols.

III. Implementation in Protocols

(a) **Ethernet:** Uses CRC for error detection but does not have built-in error correction through retransmission.

(b) **Wi-fi:** Uses CRC for error detection and ARQ for error correction, specifically employing protocols like stop-and-wait ARQ and selective repeat ARQ for reliable data transfer.

(c) **Point-to-point protocol (PPP):** Uses frame check sequence (FCS) which is essentially a CRC for error detection.

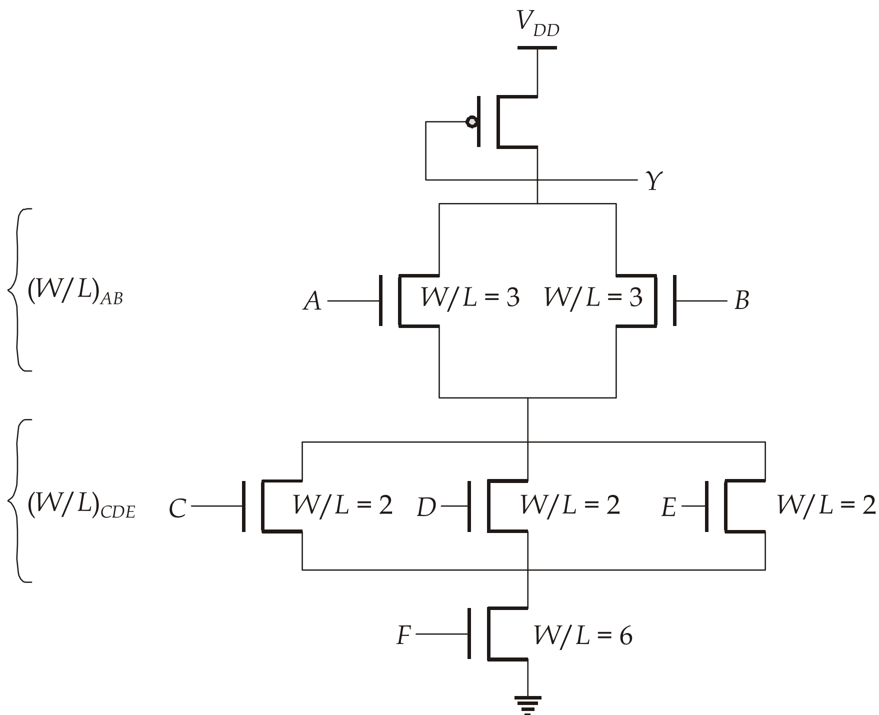
Q.1 (c) Solution:

(i) The function, $Y = \overline{(A+B)(C+D+E)F}$ is implemented as shown in figure.

The output resistance of a MOS is inversely proportional to (W/L).

For the output resistance of the circuit to be same as that of an inverter,

$$\left(\frac{W}{L}\right)_{eq} = \left(\frac{W}{L}\right)_{inv} = 2$$



For MOSFETs in series,

$$\frac{1}{(W/L)_{eq}} = \frac{1}{(W/L)_1} + \frac{1}{(W/L)_2} + \ldots$$

For MOSFETs in parallel,

$$(W/L)_{eq} = (W/L)_1 + (W/L)_2 + \ldots$$

Thus, we have,

$$\frac{1}{W_{eq}} = \frac{1}{\left(\frac{W}{L}\right)_{AB}} + \frac{1}{\left(\frac{W}{L}\right)_{CDE}} + \frac{1}{\left(\frac{W}{L}\right)_f} = \frac{1}{\left(\frac{W}{L}\right)_{inv}} \quad \ldots(i)$$

where,

$$\left(\frac{W}{L}\right)_{AB} = \left(\frac{W}{L}\right)_A + \left(\frac{W}{L}\right)_B$$

and

$$\left(\frac{W}{L}\right)_{CDE} = \left(\frac{W}{L}\right)_C + \left(\frac{W}{L}\right)_D + \left(\frac{W}{L}\right)_E$$

From equation (i), we can write,

$$\left(\frac{W}{L}\right)_{AB} = 3(W/L)_{inv} = 6 = \left(\frac{W}{L}\right)_A + \left(\frac{W}{L}\right)_B$$

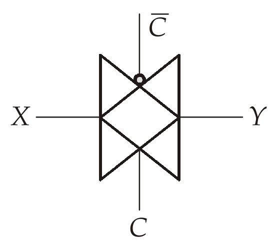
$$\Rightarrow \quad \left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 3$$

Similarly,
$$\left(\frac{W}{L}\right)_{CDE} = 3\left(\frac{W}{L}\right)_{\text{inv}} = 6 = \left(\frac{W}{L}\right)_C + \left(\frac{W}{L}\right)_D + \left(\frac{W}{L}\right)_E$$

$$\Rightarrow \quad \left(\frac{W}{L}\right)_C = \left(\frac{W}{L}\right)_D = \left(\frac{W}{L}\right)_E = 2$$

and
$$\left(\frac{W}{L}\right)_F = 3\left(\frac{W}{L}\right)_{\text{inv}} = 6$$

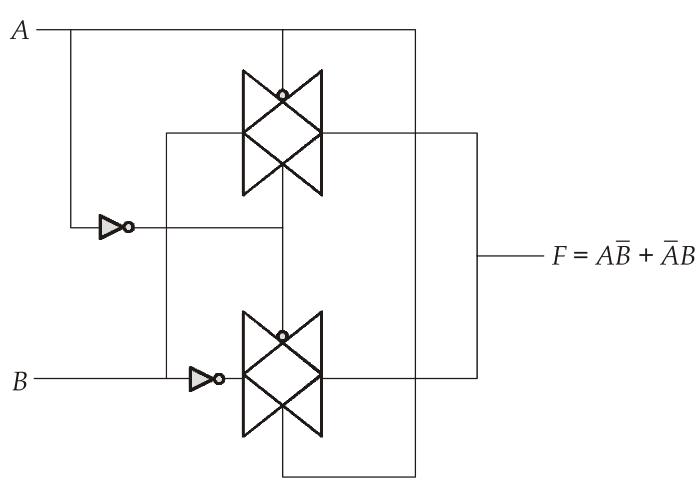
(ii) **1.** In a CMOS Transmission Gate, when the control signal "C" is high, the transmission gate passes the input to the output . When "C" is low, the input and output are isolated from each other. For the below CMOS transmission gate,



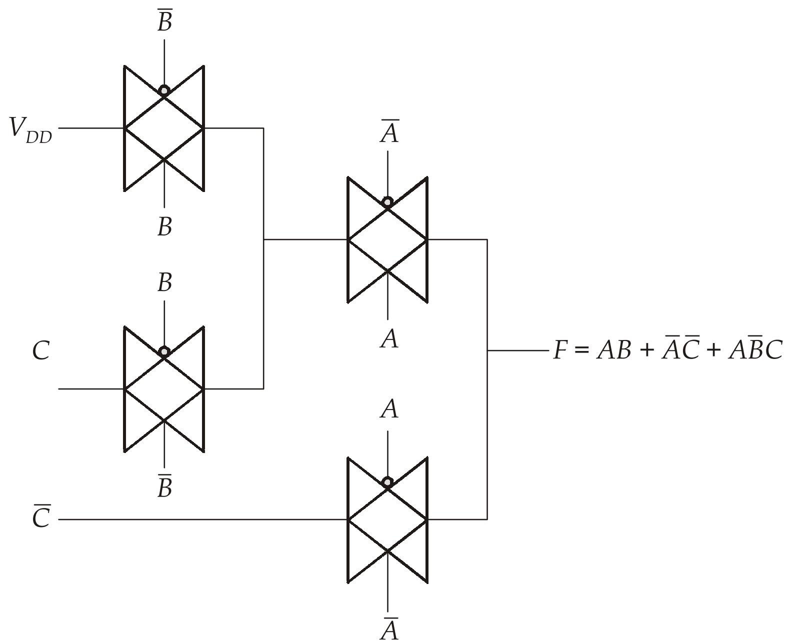
$Y = XC$

The given boolean functions can be implemented using CMOS Transmission Gate as below:

$F = A\bar{B} + \bar{A}B$



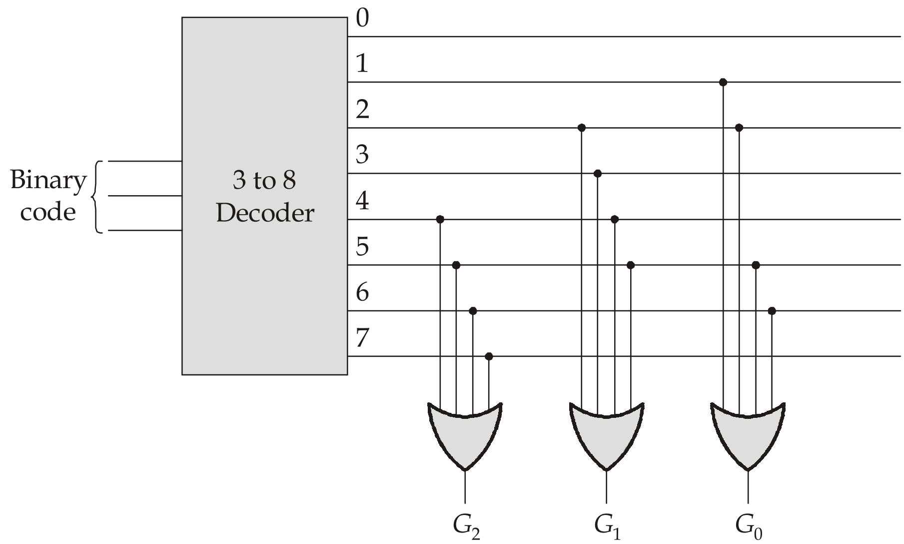
2. $F = AB + \bar{A}\bar{C} + A\bar{B}C$



(iii) The truth table for the given combinational logic circuit is obtained as below:

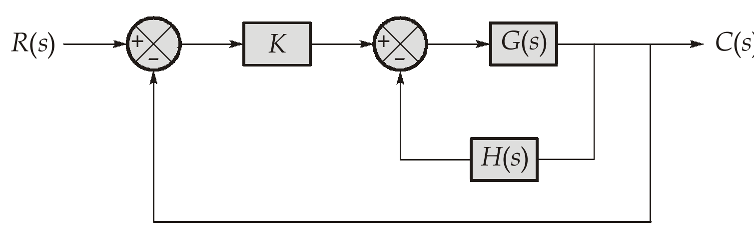
S. No.	Binary code			Gray code		
	A	B	C	G_2	G_1	G_0
0	0	0	0	0	0	0
1	0	0	1	0	0	1
2	0	1	0	0	1	1
3	0	1	1	0	1	0
4	1	0	0	1	1	0
5	1	0	1	1	1	1
6	1	1	0	1	0	1
7	1	1	1	1	0	0

Circuit diagram:



Q.1 (d) Solution:

(i) The given system can be represented by the block diagram shown below:



Given:

$$K = 30$$

$$G(s) = \frac{1}{s(s+8)}, H(s) = sK_a$$

The closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K\,G(s)}{1+G(s)H(s)+KG(s)}$$

$$\frac{C(s)}{R(s)} = \frac{30\cdot\frac{1}{s(s+8)}}{1+\frac{1}{s(s+8)}s\cdot K_a+\frac{30}{s(s+8)}}$$

$$= \frac{30}{s^2+8s+sK_a+30} = \frac{30}{s^2+(8+K_a)s+30}$$

Comparing with transfer function of standard second order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

gives $\omega_n^2 = 30$

$\therefore$ $\omega_n = \sqrt{30} = 5.477 \text{ rad/sec}$

and $2\xi\omega_n = 8 + k_a$

For $\xi = 0.8$ (given), $\omega_n = 5.477$

$\Rightarrow$ $2 \times 0.8 \times 5.477 = 8 + k_a$

$$K_a = 8.763 - 8 = 0.763$$

So, derivative feedback constant, $K_a = 0.763$.

(ii) **1.** For $r(t) = 12t$

$$R(s) = \frac{12}{s^2}$$

Steady state error for unity feedback system is

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{s\left(\frac{12}{s^2}\right)}{1+\frac{K}{s(s+2)(0.5s+1)}}$$

$$= \lim_{s\to 0} \frac{s\left(\frac{12}{s^2}\right)s(s+2)(0.5s+1)}{s(s+2)(0.5s+1)+K}$$

If $K = 6$,

$$e_{ss} = \lim_{s\to 0} \frac{12(s+2)(0.5s+1)}{s(s+2)(0.5s+1)+6}$$

$$= \frac{12\times 2}{6} = 4$$

2. For unit ramp input r(t),

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} < 0.06$$

$\therefore$ We have,
$$e_{ss} = \lim_{s\to 0} \frac{s\times \frac{1}{s^2}}{1+\frac{K}{s(s+2)(0.5s+1)}} < 0.06$$

$$\Rightarrow \quad \lim_{s\to 0} \frac{(s+2)(0.5s+1)}{s(s+2)(0.5s+1)+K} < 0.06$$

$$\frac{2}{K} < 0.06$$

$$K > \frac{200}{6}$$

Hence, Minimum value of $K = 33.334$.

Q.1 (e) Solution:

(i) 1. For a lossless line, characteristic impedance,

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\text{Phase constant, } \beta = \omega\sqrt{LC}$$

$$L = 0.75 \ \mu\text{H/m}$$

$$C = 80 \text{ pF/m}$$

$$\omega = 2\pi f$$

$$f = 150 \text{ kHz (given)}$$

$$\omega = (2\pi)(150 \times 10^3) \text{ rad/sec}$$

$$\therefore \quad Z_0 = \sqrt{\frac{0.75\times 10^{-6}}{80\times 10^{-12}}} = 96.824\ \Omega$$

and

$$\beta = 2\pi\times(150\times 10^3)\sqrt{(0.75)\times(80\times 10^{-18})}$$

$$\beta = 7.3 \times 10^{-3} \text{ rad/m}$$

2. Velocity of propagation,

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.75\times 10^{-6})(80\times 10^{-12})}}$$

$$v_p = 1.29 \times 10^8 \text{ m/s}$$

Also,

$$v_p = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_0 \in_0 \in_r}}$$

$$\Rightarrow \quad \in_r = \frac{1}{\in_0 \mu_0 v_p^2} \quad \text{as } \mu = \mu_0 \text{ (given)}$$

$$\in_r = \frac{1}{(8.85\times10^{-12})(1.29\times10^8)^2\times4\pi\times10^{-7}}$$

$$= 5.403$$

3. Delay generated by transmission line,

$$t_d = \frac{l}{v_p}$$

We have, $l = 60$ m

$$\therefore \quad t_d = \frac{60}{1.29\times10^8} = 4.65116\times10^{-7}\text{ s}$$

$$t_d = 465.116 \text{ ns}$$

(ii) Given: $\beta = 0.8$ rad/m, $\omega = 2\pi \times 10^7$ rad/sec, $\mu = \mu_0$ (non-magnetic)

1. Since $\alpha = 0$ and $\beta \neq \omega/c$, the medium is not free space but a lossless medium.

Considering, $\in = \in_0 \in_r$, we have

$$\beta = \omega\sqrt{\mu\in} = \omega\sqrt{\mu_0\in_0\in_r} = \frac{\omega}{c}\sqrt{\in_r}$$

$$\sqrt{\in_r} = \frac{\beta c}{\omega} = \frac{0.8\times3\times10^8}{2\pi\times10^7} = \frac{12}{\pi}$$

$$\in_r = 14.59$$

Intrinsic impedance, $\eta = \sqrt{\frac{\mu}{\in}} = \sqrt{\frac{\mu_0}{\in_0\in_r}} = \frac{120\pi}{\sqrt{\in_r}} = 120\pi\frac{\pi}{12} = 10\pi^2 = 98.7\ \Omega$

2. The Poynting vector gives the instantaneous power density as

$$\vec{P} = \vec{E}\times\vec{H} = \frac{E_0^2}{\eta}\sin^2(\omega t - \beta x)\hat{a}_x$$

The time-average power carried by the wave is obtained as

$$\vec{P}_{\text{avg}} = \frac{1}{T}\int_0^T P\,dt = \frac{E_0^2}{2\eta}\hat{a}_x = \frac{16}{2\times10\pi^2}\hat{a}_x$$

$$= 81\hat{a}_x \text{ mW/m}^2$$

3. For plane $2x + y = 5$

$$\hat{a}_n = \frac{2\hat{a}_x + \hat{a}_y}{\sqrt{5}}$$

Hence the total power crossing 100 cm^2 of the plane is

$$P = \int \overrightarrow{P_{avg}} \cdot \overrightarrow{ds} = \overrightarrow{P_{avg}} \cdot S\hat{a}_n, \quad \text{where } S = 100 \text{ cm}^2$$

$$= (81 \times 10^{-3} a_x) \cdot (100 \times 10^{-4}) \left[\frac{2\hat{a}_x + \hat{a}_y}{\sqrt{5}} \right]$$

$$= \frac{162 \times 10^{-5}}{\sqrt{5}} = 724.5 \ \mu\text{W}$$

Q.2 (a) Solution:

(i) Given: Wavelength, $\lambda = 1.5$ µm ; Maximum bit error rate = 2×10^{-9}

1. The probability $P(z)$ of detecting z photons in time period τ when it is expected on average to detect z_m photons obeys the Poisson distribution as

$$P(z) = \frac{z_m^z \, e^{-z_m}}{z!} \quad \text{...(i)}$$

When z_m is the average number of electron-hole pair generated in time τ.

From equation (i), the probability of no pairs being generated when a light pulse is present is obtained by

$$P(0/1) = e^{-z_m}$$

Assuming the absence of electron-hole pairs on the absence of illumination, we can write

$$\text{BER} = e^{-z_m} = 2 \times 10^{-9}$$

Thus, $\qquad z_m = -\ln(2 \times 10^{-9}) = 20.03$

z_m corresponds to an average number of photons detected in a time period τ for a BER of 2×10^{-9}.

If P_0 is the incident optical power and hf is the energy of a photon, then we can write

$$z_m = \frac{\eta P_0 \tau}{hf} = 20.03$$

Hence, the minimum pulse energy or quantum limit:

$$E_{\min} = P_0 \tau = \frac{z_m hf}{\eta} = \frac{20.03 hf}{\eta}$$

Thus, the quantum limit at the receiver to maintain a maximum BER of 2×10^{-9} is $\dfrac{20.03 hf}{\eta}$.

2. From part (i) the minimum pulse energy:

$$P_0\tau = \frac{20.03hf}{\eta}$$

Therefore, the average received optical power required to provide the minimum pulse energy is

$$P_0 = \frac{20.03hf}{\tau\eta}$$

However, for ideal binary signalling there are an equal number of ones and zeros (50% in the on state and 50% in the off state). Thus the average received optical power may be considered to arrive over two bit periods and

$$P_0(\text{binary}) = \frac{20.03hf}{2\tau\eta} = \frac{20.03hf\,B_T}{2\eta}$$

where B_T is the bit rate. At a wavelength of 1.5 μm,

$$f = 2 \times 10^{14}\text{ Hz}$$

Hence,
$$P_0(\text{binary}) = \frac{20.03 \times 6.626 \times 10^{-34} \times 2 \times 10^{14} \times 12 \times 10^{6}}{2 \times 1}$$
$$= 15.92\text{ pW}$$

In decibels (dB):

$$P_0(\text{dB}) = 10\log(15.92 \times 10^{-12}) = -107.98\text{ dB}$$

(ii) The modal birefringence is given by

$$B_F = \frac{\lambda}{L_B}$$

where,
$$L_B = \text{Beat length} = 9\text{ cm}\ ;\ \lambda = 0.9\ \mu\text{m}$$

$$\therefore \quad B_F = \frac{\lambda}{L_B} = \frac{0.9 \times 10^{-6}}{9 \times 10^{-2}} = 10^{-5}$$

Birefringent coherence is maintained over a length of fiber L_{bc} (i.e. coherence length) when:

$$L_{bc} \cong \frac{c}{B_F\,\delta f} = \frac{\lambda^2}{B_F\,\delta\lambda}$$

where c is the velocity of light and $\delta\lambda$ is the source linewidth.

$$L_{bc} \cong \frac{(0.9)^2 \times 10^{-12}}{10^{-5} \times 10^{-9}} = 81\text{ m}$$

The difference between the propagation constant for the two orthogonal modes.

$$\beta_x - \beta_y = \frac{2\pi}{L_B} = \frac{2\pi}{0.09} = 69.8 \text{ rad/m}$$

or

$$B_F = \frac{\beta_x - \beta_y}{\left(\frac{2\pi}{\lambda}\right)}$$

$$\beta_x - \beta_y = B_F \times \left(\frac{2\pi}{\lambda}\right) = 10^{-5} \times \frac{2\pi}{0.9 \times 10^{-6}} = 69.8 \text{ rad/m}$$

Q.2 (b) Solution:

An embedded system is a computing device designed to perform specific tasks within a larger system, with real time constraints and limited resources.

Architecture of the embedded system for a smart thermostat:

(i) Hardware Components:

1. **Microcontroller:** Select a microcontroller with sufficient processing power and I/O capabilities, such as an ARM cortex-M series or an AVR microcontroller.
2. **Temperature Sensor:** Integrate a temperature sensor like the LM35 or the DS18B20 to measure ambient temperature accurately.
3. **Display:** Choose an appropriate display module, such as an LCD or an OLED display, to show temperature readings and user interface elements.
4. **Input Interface:** It includes buttons or a touchscreen interface for user interaction and adjustment of settings.
5. **Relay:** Use a relay module to control the heating or cooling system based on temperature readings.

(ii) Software Modules:

1. **Sensor Interface:** Develop a software module to interface with the temperature sensor, read temperature values, and convert them into a usable format.
2. **Control Algorithm:** Implement a control algorithm to regulate the heating or cooling system based on the desired temperature set by the user.
3. **User Interface:** Design software for display module to show temperature reading, settings and user prompts and implement touch or button handling routines for user interaction.
4. **Communication Protocol:** Develop communication protocols for exchanging data between the microcontroller and external devices, such as a central heating system or a mobile app.

5. **Error handling:** Implement error handling routines to detect and handle sensor malfunctions communication failures, or abnormal temperature readings.

(iii) Communication Interface:

1. **Serial communication:** Use UART or SPI communication for data exchange with external devices or for debugging purposes.
2. **Wireless connectivity:** Optionally, integrate Wi-fi or Bluetooth module to enable remote monitoring and control of the thermostat via a mobile app or a web interface.
3. **I2C interface (Inter Integrated Circuit):** Utilize the I2C interface for communication between the microcontroller and peripherals like the display module and the temperature sensor.

(iv) Interaction Flow:

1. The temperature sensor continuously measure ambient temperature.
2. The microcontroller reads temperature values from the sensor and compare them to the desired temperature set by the user.
3. Based on the temperature data, the control algorithm activates the heating or cooling system through the relay module.
4. The display module shows the current temperature readings and allow the user to adjust using buttons or touchscreen controls.
5. User input is processed by the micro controller, updating the control algorithm parameters or triggering other actions as necessary.

The embedded system architecture provides a robust framework for building a smart thermostat with temperature sensing, control display, and user interaction capabilities. The module design allows for easy scalability and customization to meet specific requirements or add additional features in the future.

Q.2 (c) Solution:

$$G(s)H(s) = \frac{K}{(s+5)(s+7)(s^2+4s+6)}$$

Open loop poles $s = -5, -7, s = -2 \pm 1.414j$

Hence, $P = 4$

and $Z = 0$

Thus, there are four branches of the root locus, which terminate at infinity. Hence there are four asymptotes.

Centroid of asymptotes

$$\sigma_0 = \frac{\Sigma\text{open loop poles} - \Sigma\text{open loop zeros}}{P-Z}$$

$$= \frac{-5-7-2+1.414j-2-1.414j-0}{4} = -4$$

The angle of asymptotes:

$$\phi = \frac{(2K+1)180°}{P-Z}; K = 0,1,2,3$$

$$\phi = \frac{(2K+1)180°}{4} = (2K+1)45°; K = 0,1,2,3$$

$$= 45°, 135°, 225°, 315°$$

(i) Angle of departure from $s = -2 + 1.414j$,

$$\theta_{d1} = 180° + \text{Angle}\left[\frac{K}{(s+5)(s+7)(s+2+1.414j)}\right]\Bigg|_{s=-2+1.414j}$$

$$= 180° + \text{Angle}\left[\frac{K}{(-2+1.414j+5)(-2+1.414j+7)(-2+1.414j+2+1.414j)}\right]$$

$$= 180° - 25.236° - 15.79° - 90°$$

$$= 48.974°$$

Angle of departure from $s = -2 - 1.414j$

$$\theta_{d2} = 180° + \text{Angle}\left[\frac{K}{(s+5)(s+7)(s+2-1.414j)}\right]\Bigg|_{s=-2-1.414j}$$

$$= 180° + \text{Angle}\left[\frac{K}{(-2-1.414j+5)(-2-1.414j+7)(-2-1.414j+2-1.414j)}\right]$$

$$= 180° + \tan^{-1}\frac{1.414}{3} + \tan^{-1}\frac{1.414}{5} + 90°$$

$$= 311.026° = -48.974°$$

(ii) The characteristic equation is given by

$$1 + G(s)H(s) = 0$$

$$\Rightarrow \quad 1 + \frac{K}{(s+5)(s+7)(s^2+4s+6)} = 0$$

$$\Rightarrow \quad K = -(s+5)(s+7)(s^2+4s+6)$$
$$= -(s^2+12s+35)(s^2+4s+6)$$
$$= -(s^4+16s^3+89s^2+212s+210)$$

At breakaway points,

$$\frac{dK}{ds} = -(4s^3+48s^2+178s+212) = 0$$

On solving

$$s = -6.204, -2.897 + 0.3815j, -2.89 - 0.3815j$$

$s = (-2.89 \pm 0.3815j)$ doesn't represent a breakaway point. Hence, the break away point $s = -6.204$

(iii) Intersection with the imaginary axis:

The characteristic equation is

$$q(s) = s^4 + 16s^3 + 89s^2 + 212s + 210 + K$$

Forming the Routh array,

$$\begin{array}{c|ccc} s^4 & 1 & 89 & 210+K \\ s^3 & 16 & 212 & \\ s^2 & 75.75 & 210+K & \\ s^1 & \dfrac{16059-3360-16K}{75.75} & & \\ s^0 & 210+K & & \end{array}$$

For intersection with imaginary axis, odd row of routh array must be zero, i.e.

$16K - 12699 \quad \Rightarrow \quad K = 793.68$

To find the imaginary axis crossing point, we use the auxiliary equation given by

$$75.75s^2 + 210 + K = 0$$
$$75.75s^2 + 210 + 793.68 = 0$$
$$75.75s^2 + 1003.68 = 0$$

So, $$\omega_c = \sqrt{\frac{1003.68}{75.75}} = 3.64 \text{ rad/sec}$$

Hence, the root locus intersects the imaginary axis at $s = \pm j3, 64$.

(iv) For the system to be stable, there should be no sign change in the first column of the Routh Array. Hence, we have, closed loop system will be stable for $0 < K < 793.68$.

$$12699 - 16K > 0 \quad \Rightarrow K < 793.68$$

and $210 + K > 0 \Rightarrow K > -210$

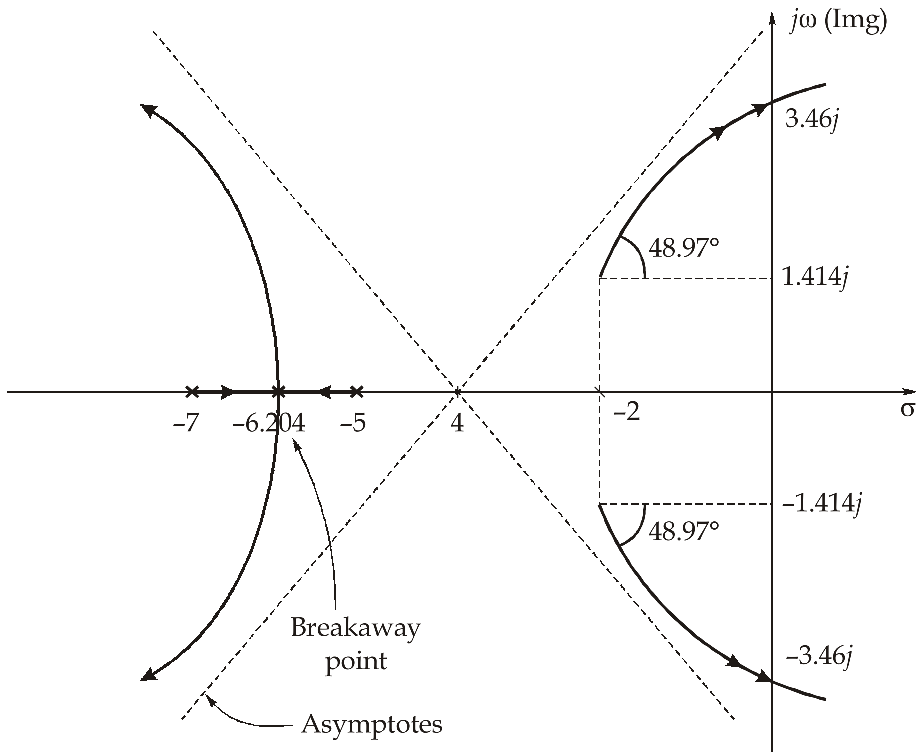
Since, $K > 0$, the

(v) For $K = 20$

$$\text{Gain margin} = \frac{\text{Marginal value of } K}{\text{Specified value of } K} = \frac{793.68}{20} = 39.684$$

$$\text{In decibels, GM} = 20\log(39.684) = 31.972 \text{ dB}$$

Root locus is shown below:



Q.3 (a) Solution:

We know, $R_X(\tau) = R_X(-\tau) = E[X(t)\,X(t+\tau)]$

$R_Y(\tau) = R_Y(-\tau) = E[Y(t)\,Y(t+\tau)]$

For LTI system, $Y(t) = X(t) * h(t)$

Laplace transform of $h(t)$, $H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}\,dt$

$$\therefore \quad H(0) = \int_{-\infty}^{\infty} h(t)\,dt \quad \text{...(i)}$$

Convolution of two signals,

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\alpha)\,x_2(t-\alpha)\,d\alpha = \int_{-\infty}^{\infty} x_1(t-\alpha)\,x_2(\alpha)\,d\alpha$$

Convolution of three signals,

$$x_1(t) * x_2(t) * x_3(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1(\alpha)x_2(\beta)x_3(t-\alpha-\beta)\,d\alpha\,d\beta \quad \text{...(ii)}$$

(i) To prove $E[Y(t)] = H(0)\,E[X(t)]$:

For an LTI system, when the input process is WSS, the output will also be a WSS process.

For the given LTI system,

$$Y(t) = X(t) * h(t)$$

$$E[Y(t)] = E[X(t) * h(t)]$$

$$= E\left[\int_{-\infty}^{\infty} X(t-\alpha)h(\alpha)\,d\alpha\right]$$

$$= \int_{-\infty}^{\infty} E\left[X(t-\alpha)\,h(\alpha)\right]d\alpha = \int_{-\infty}^{\infty} E\left[X(t-\alpha)\right]h(\alpha)\,d\alpha$$

Since $X(t)$ is a WSS process,

$$E[X(t-\alpha)] = E[X(t)]$$

So,
$$E[Y(t)] = E[X(t)]\int_{-\infty}^{\infty} h(\alpha)\,d\alpha$$

From equation (i), $\int_{-\infty}^{\infty} h(\alpha)\,d\alpha = H(0)$

So,
$$E[Y(t)] = H(0)E[X(t)]$$

(ii) To prove $R_{XY}(\tau) = R_X(\tau) * h(\tau)$:

$$R_{XY}(\tau) = E[X(t)\,Y(t+\tau)] = E\left[X(t)\left(X(t+\tau) * h(t)\right)\right]$$

$$= E\left[X(t)\int_{-\infty}^{\infty} X(t+\tau-\alpha)h(\alpha)\,d\alpha\right] = \int_{-\infty}^{\infty} E[X(t)X(t+\tau-\alpha)]\,h(\alpha)\,d\alpha$$

$$= \int_{-\infty}^{\infty} R_X(\tau-\alpha)\,h(\alpha)\,d\alpha = R_X(\tau) * h(\tau)$$

(iii) To prove $R_{YX}(\tau) = R_X(\tau) * h(-\tau)$:

$$R_{YX}(\tau) = E[Y(t)\,X(t+\tau)]$$

$$= E\left[X(t+\tau)\left(X(t) * h(t)\right)\right] = E\left[X(t+\tau)\int_{-\infty}^{\infty} X(t-\alpha)h(\alpha)\,d\alpha\right]$$

$$= \int_{-\infty}^{\infty} E[X(t+\tau)X(t-\alpha)]\, h(\alpha)\, d\alpha \;=\; \int_{-\infty}^{\infty} R_X(\tau+\alpha)\, h(\alpha)\, d\alpha$$

Let $\beta = -\alpha \Rightarrow d\beta = -d\alpha$

So,
$$R_{YX}(\tau) = -\int_{\infty}^{-\infty} R_X(\tau-\beta)h(-\beta)\,d\beta = \int_{-\infty}^{\infty} R_X(\tau-\beta)h(-\beta)\,d\beta$$
$$= R_X(\tau) * h(-\tau)$$

(iv) To prove $R_Y(\tau) = R_X(\tau) * h(\tau) * h(-\tau)$:

$$R_Y(\tau) = E[Y(t)\,Y(t+\tau)]$$
$$= E\Big[\big(X(t) * h(t)\big)\big(X(t+\tau) * h(t)\big)\Big]$$
$$= E\left[\int_{-\infty}^{\infty} X(t-\alpha)h(\alpha)\,d\alpha \int_{-\infty}^{\infty} X(t+\tau-\beta)h(\beta)\,d\beta\right]$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[X(t-\alpha)X(t+\tau-\beta)]\, h(\alpha)h(\beta)\,d\alpha\, d\beta$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_X(\tau-\beta+\alpha)\, h(\alpha)h(\beta)\,d\alpha\, d\beta$$

Let $\gamma = -\alpha \Rightarrow d\gamma = -d\alpha$

$$R_Y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_X(\tau-\beta-\gamma)\, h(\beta)h(-\gamma)\,d\beta\, d\gamma$$

By comparing the above equation with the equation (ii), we get,

$$R_Y(\tau) = R_X(\tau) * h(\tau) * h(-\tau)$$

Q.3 (b) Solution:

(i) According to given information, instruction format can be drawn as

← 32 bit →

Op-Code 8-bits	Address 8-bits	Address 8-bits	Address 8-bits

⇓

No. of instructions
= 248
$\Rightarrow n \geq \log_2 248$
$n \geq 8$

- Total number of instructions set possible with 8 bit opcode = 2^8 = 256
- Out of which 248 are used as three-address instructions. Thus, remaining opcodes are 256 – 248 = 8 that can be used for single address instructions.

The format of one address instruction is as below:

one address instruction

opcode (24-bits)	Address (8-bits)

Maximum number of one address instructions

$$= 8 \times 2^{16} = 5{,}24{,}288$$

(ii) As per the given data, we can have instruction format as below:

← 42 →

opcode	Register operand	Memory operand	immediate operand
7 bit	8 bit	19 bit	

- Word size = 32 bit
- Instruction size = 42 bit
- Memory size = 512 kB

  ⇒ number of bits required for memory operand,

  $$n \geq \log_2(512 \text{ kB})$$
  $$n \geq \log_2(2^9 \times 2^{10})$$
  $$n \geq 19$$

- Register file size = 200

  ⇒ number of bits required for register operand,

  $$n \geq \log_2 200$$
  $$n \geq 8$$

- Number of instructions = 100

  ⇒ number of bits required for opcode,

  $$n \geq \log_2 100$$
  $$n \geq 7$$

- Immediate constant field size = 42 – (7 + 8 + 19)

  $$= 8 \text{ bit}$$

1. Range of unsigned data possible in the instruction

$$= (0 \text{ to } 2^n - 1)$$
$$= 0 \text{ to } 2^8 - 1$$

$= 0 \text{ to } 255$

Hence, the largest unsigned constant possible in the instruction = 255.

2. Number of constants possible = $2^{\text{immediate field size}}$

$= 2^8$

$= 256$

3. Range of signed constant (By default 2's complement)

$= -(2^{n-1}) \text{ to } (2^{n-1} - 1)$

$= -(2^{8-1}) \text{ to } (2^{8-1} - 1)$

$= -128 \text{ to } 127$

4. Range of signed magnitude constant possible in the instruction

$= -(2^{n-1} - 1) \text{ to } (2^{n-1} - 1)$

$= -(2^7 - 1) \text{ to } (2^7 - 1)$

$= -127 \text{ to } 127$

Q.3 (c) Solution:

(i) Using partial fraction expansion, $H(s)$ can be written as

$$H(s) = \frac{1}{(s+0.5)(s^2+0.5s+2)} = \frac{A_1}{s+0.5} + \frac{A_2 s + A_3}{s^2+0.5s+2}$$

Therefore, $A_1(s^2 + 0.5s + 2) + (A_2 s + A_3)(s + 0.5) = 1$

Comparing the coefficients of s^2, s and the constants on either side of the above expression, we get

$$A_1 + A_2 = 0$$

$$0.5A_1 + 0.5A_2 + A_3 = 0$$

$$2A_1 + 0.5A_3 = 1$$

Solving the above simultaneous equations, we get $A_1 = 0.5$, $A_2 = -0.5$ and $A_3 = 0$.

The system response can be written as,

$$H(s) = \frac{0.5}{s+0.5} - \frac{0.5s}{s^2+0.5s+2}$$

$$= \frac{0.5}{s+0.5} - 0.5\left(\frac{s}{(s+0.25)^2+(1.3919)^2}\right)$$

$$= \frac{0.5}{s+0.5} - 0.5\left(\frac{s+0.25}{(s+0.25)^2+(1.3919)^2} - \frac{0.25}{(s+0.25)^2+(1.3919)^2}\right)$$

Using impulse invariant transformation,

$$\frac{s+a}{(s+a)^2+b^2} \xrightarrow[\text{(is transformed to)}]{} \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \xrightarrow[\text{(is transformed to)}]{} \frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{A_i}{s+p_i} \xrightarrow[\text{(is transformed to)}]{} \frac{A_i}{1-e^{-p_i t}z^{-1}}$$

$$= \frac{0.5}{s+0.5} - 0.5\left(\frac{s+0.25}{(s+0.25)^2+(1.3919)^2}\right) + 0.0898\left(\frac{1.3919}{(s+0.25)^2+(1.3919)^2}\right)$$

We get, $H(z) = \dfrac{0.5}{1-e^{-0.5T}z^{-1}} - 0.5\left[\dfrac{1-e^{-0.25T}(\cos 1.3919T)z^{-1}}{1-2e^{-0.25T}(\cos 1.3919T)z^{-1}+e^{-0.5T}z^{-2}}\right]$

$$+ 0.0898\left[\frac{e^{-0.25T}(\sin 1.3919T)z^{-1}}{1-2e^{-0.25T}(\cos 1.39197T)z^{-1}+e^{-0.5T}z^{-2}}\right]$$

Assume $T = 1$ s,

$$H(z) = \frac{0.5}{1-0.6065z^{-1}} - 0.5\left(\frac{1-0.1385z^{-1}}{1+0.277z^{-1}+0.6065z^{-2}}\right)$$

$$+ 0.0898\left[\frac{0.7663z^{-1}}{1-0.277z^{-1}+0.6065z^{-2}}\right]$$

(ii) From the fact 2,

$$F^{-1}[(1+j\omega)X(\omega)] = Ae^{-2t}u(t)$$

By taking Fourier transform,

$$(1+j\omega)X(\omega) = \frac{A}{2+j\omega}$$

$$X(\omega) = \frac{A}{(1+j\omega)(2+j\omega)}$$

$$= A\left[\frac{A'}{1+j\omega} + \frac{B'}{2+j\omega}\right]$$

$$A' = \left.\frac{1}{2+j\omega}\right|_{j\omega=-1} = 1$$

$$B' = \left.\frac{1}{1+j\omega}\right|_{j\omega=-2} = -1$$

$$\therefore \quad X(\omega) = A\left(\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right)$$

By taking inverse Fourier transform of the above $X(\omega)$, we get

$$x(t) = Ae^{-t}u(t) - Ae^{-2t}u(t)$$

Using Parseval's theorem,

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

$$\therefore \quad \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \times 2\pi = 1$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} \left|Ae^{-t}u(t) - Ae^{-2t}u(t)\right|^2 dt = 1$$

$$\int_{-\infty}^{\infty} \left[A^2e^{-2t} - 2A^2e^{-3t} + A^2e^{-4t}\right]u(t)dt = 1$$

$$\int_{0}^{\infty} \left[A^2e^{-2t} - 2A^2e^{-3t} + A^2e^{-4t}\right]dt = 1$$

$$\frac{A^2}{-2}\left[e^{-2t}\right]_0^{\infty} - \frac{2A^2}{(-3)}\left[e^{-3t}\right]_0^{\infty} + \frac{A^2}{(-4)}\left[e^{-4t}\right]_0^{\infty} = 1$$

$$\frac{A^2}{2} - \frac{2}{3}A^2 + \frac{A^2}{4} = 1$$

$$6A^2 - 8A^2 + 3A^2 = 12$$

$$A^2 = 12$$

$$\therefore \quad A = \pm\sqrt{12}$$

Since from the fact 1, $x(t)$ is non-negative

$$\therefore \quad A = \sqrt{12}$$

$\therefore$ Closed form of $x(t) = \sqrt{12}\left[e^{-t} - e^{-2t}\right]u(t)$

Q.4 (a) Solution:

(i)

$$\vec{E} = \text{Re}[E_s e^{j\omega t}]$$

$$= \left(6\vec{a}_x + 15\vec{a}_y\right)e^{-0.4z}\cos(\omega t - 3.6z)$$

$(\because \alpha + j\beta = 0.4 + j3.6/\text{m})$

At $z = 5$ m and $t = \dfrac{T}{12}$,

$$\omega t = \frac{2\pi}{T}\cdot\frac{T}{12} = \frac{\pi}{6}\text{ rad}$$

Hence,

$$\vec{E} = (6\vec{a}_x + 15\vec{a}_y)e^{-2}\cos\left(\frac{\pi}{6} - 3.6\times 5\right)$$

$$= (6\vec{a}_x + 15\vec{a}_y)e^{-2}\cos\left(\frac{\pi}{6} - 18\right)$$

$$|\vec{E}| = \sqrt{6^2 + 15^2}\,e^{-2}\left|\cos\left(\frac{\pi}{6} - 18\right)\right| = 3\sqrt{29}e^{-2}\left|\cos\left(\frac{\pi}{6} - 18\right)\right|$$

$$|\vec{E}| = 0.429\text{ V/m}$$

(ii) For a attenuation constant α (in dB), the loss over a distance Δz is given by

$$\text{Loss (in dB)} = \alpha\Delta z$$

Given: $\Delta z = 4$ m

$\alpha = 0.4$ Np/m

As $1\text{ Np} = 8.686\text{ dB} \Rightarrow \alpha = 0.4 \times 8.686 = 3.4744\text{ dB}$

$\therefore$ Loss $= 4 \times 3.744$

$= 13.8976$ dB

(iii) We have,

$$\alpha = \omega\sqrt{\frac{\mu\in}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\in}\right)^2} - 1\right)}$$

Let,

$$x = \sqrt{1 + \left(\frac{\sigma}{\omega\in}\right)^2}$$

$\therefore$

$$\alpha = \omega\sqrt{\frac{\mu\in}{2}(x - 1)}$$

Similarly, $\beta = \omega\sqrt{\frac{\mu\in}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}+1\right)} = \omega\sqrt{\frac{\mu\in}{2}(x+1)}$

$\therefore$ $\frac{\alpha}{\beta} = \sqrt{\frac{x-1}{x+1}} = \frac{0.4}{3.6} = \frac{1}{9}$

$$\frac{x-1}{x+1} = \frac{1}{81} \quad \Rightarrow \quad x = \frac{82}{80} = 1.025$$

Thus, $\alpha = \omega\sqrt{\frac{\mu\in}{2}}\sqrt{1.025-1} = \frac{\omega}{c}\sqrt{\frac{\in_r}{2}}\sqrt{x-1}$

$\Rightarrow$ $\sqrt{\frac{\in_r}{2}} = \frac{\alpha c}{\omega\sqrt{x-1}} = \frac{0.4\times 3\times 10^8}{10^9\sqrt{0.025}} = 0.7589$

$$\in_r = 1.152$$

We have, intrinsic impedance,

$$\eta = \sqrt{\frac{j\mu\omega}{\sigma + j\omega\in}} = \sqrt{\frac{j\mu}{\in\left[\frac{\sigma}{\omega\in}+j\right]}}$$

$\Rightarrow$ $|\eta| = \frac{\sqrt{\frac{\mu_0}{\in_0}}\frac{1}{\sqrt{\in_r}}}{x} = \frac{120\pi}{\sqrt{1.152\times 1.025}} = 342.67$

$\therefore$ $\tan 2\theta_\eta = \frac{\sigma}{\omega\in} = \sqrt{x^2-1} = 0.225$

$$\theta_\eta = \frac{1}{2}\tan^{-1}(0.225) = 6.34°$$

Thus, $\eta = 342.67\angle 6.34°\ \Omega$

$$\vec{H}_s = \hat{a}_k \times \frac{\vec{E}_s}{\eta} = \frac{\vec{a}_z}{\eta}\times\left(6\vec{a}_x + 15\vec{a}_y\right)e^{-\gamma z}, \text{ where } |\eta| = 342.67\ \Omega$$

$$= \frac{-15\vec{a}_x + 6\vec{a}_y}{|\eta|}e^{-j6.34° e^{-\gamma z}}$$

$$|\eta| = 342.67$$

$$\vec{H} = (-43.776\vec{a}_x + 17.514\vec{a}_y)e^{-0.4z}\cos(\omega t - 3.6z - 6.34°)\text{mA/m}$$

Poynting vector, $\vec{P} = \vec{E}\times\vec{H}$

$$= (6\vec{a}_x + 15\vec{a}_y)\times(-43.776\vec{a}_x + 17.514\vec{a}_y)\times 10^{-3} e^{-0.8z} \times \cos(\omega t - 3.6z)\cos(\omega t - 3.6z - 6.34°)$$

$$\vec{P} = 0.761 e^{-0.8z}\cos(\omega t - 3.6z)\cos(\omega t - 3.6z - 6.34°)\hat{a}_z$$

At $z = 5, t = \frac{T}{12}$

$$\omega t = \frac{\pi}{6}$$

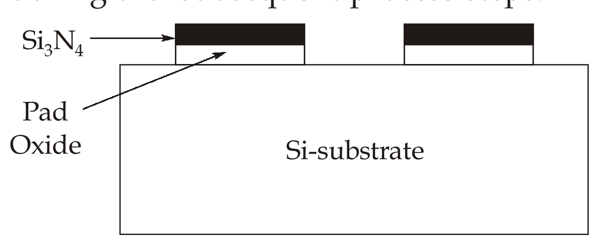
$$\therefore \quad \vec{P} = 0.761 e^{-0.8\times 5}\cos\left(\frac{\pi}{6} - 3.6\times 5\right)\cos\left(\frac{\pi}{6} - 3.6\times 5 - 6.34°\right)\hat{a}_z$$

$$\vec{P} = 0.830\hat{a}_z \text{ mW/m}^2$$

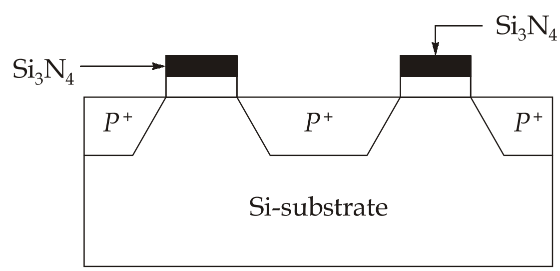
Q.4 (b) Solution:

(i) The local oxidation of silicon (LOCOS) technique is based on the principle of selectively growing the field oxide in certain regions. It is achieved by shielding the active areas with silicon nitride (Si_3N_4) during oxidation which effectively inhibits the oxide growth. The basic steps of the LOCOS method are shown below:

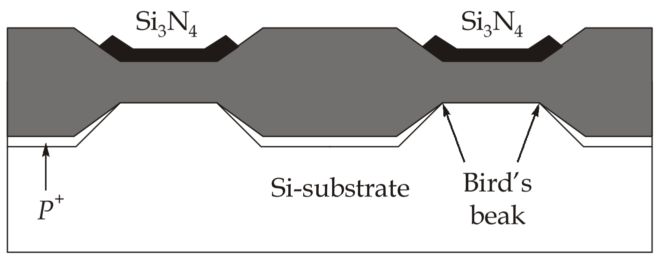
Step-1: A thin pad oxide is grown on the silicon surface, followed by the deposition and patterning of a silicon nitride layer to mask the active areas. The thin pad oxide underneath the silicon nitride layer is used to protect the silicon surface from stress caused by nitride during the subsequent process steps.



Step-2: The exposed areas of the silicon surface, which will eventually form the isolation regions, are doped with a P-type impurity to create the channel stop implants that surround the transistor.

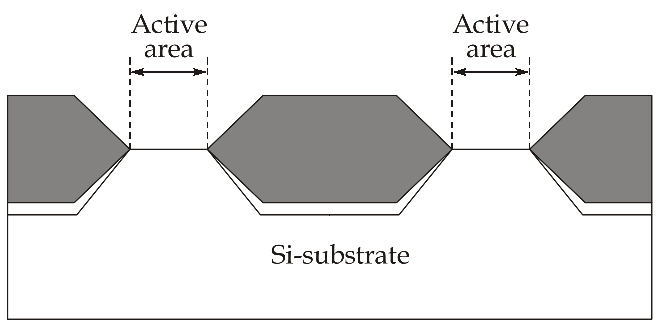


Step-3: A thick field oxide is grown in the area not covered with Si_3N_4 as shown:



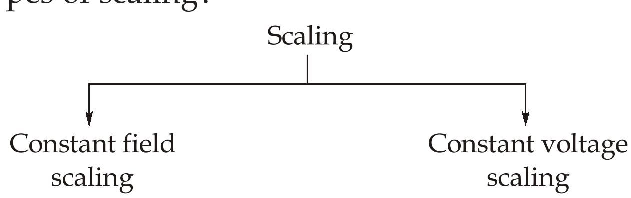
The field oxide is partially recessed into the surface since the thermal oxidation process also consumes some of the silicon. Also, the field oxide forms a lateral extension under the nitride layer, called the bird's beak region.

Step-4: The Si_3N_4 layer and thin pad oxide layer are etched in the final step, resulting in active areas surrounded by the partially recessed field oxide.



(ii) The reduction of the size i.e. the dimensions of MOSFET is commonly referred to as scaling.

There are two types of scaling:



1. **Constant Field Scaling:** It does not alter the magnitude of the internal fields in the MOSFET and dimensions are scaled down by a factor of S. The effect of constant field scaling on device parameter are given

Before Scaling	After Scaling
Channel length L	$L' = L/S$
Channel width W	$W' = W/S$
Junction depth x_j	$x_j' = x_j / S$
Threshold voltage V_{Th}	$V_{\text{Th}}' = V_{\text{Th}}/S$
Gate oxide thickness t_{ox}	$t_{ox}' = t_{ox} / S$
Power supply voltage V_{DD}	$V_{DD}' = V_{DD}/S$
Doping densities N_A	$N_A' = SN_A$
Doping densities N_D	$N_D' = SN_D$
Tranconductance paramter	$K_n' = SK_n$

Effect of Constant Field scaling on Key Device Characteristics:

Oxide capacitance: The gate oxide capacitance per unit area is changed as follows:

$$C_{ox}' = \frac{\epsilon_{ox}}{t_{ox}'} = S \cdot \frac{\epsilon_{ox}}{t_{ox}} = S \cdot C_{ox}$$

The aspect ratio W/L of the MOSFET will remain unchanged under scaling.

Drain current: Since all terminal voltages are scaled down by the factor S as well, the linear mode drain current of the scaled MOSFET can be found as

$$I_D' = \frac{K_n'}{2}\left[2(V_{GS}' - V_{\text{Th}}')V_{DS}' - V'^2_{DS}\right]$$

$$I_D' = \frac{S}{2} \cdot K_n \cdot \frac{1}{S^2}\left[2(V_{GS} - V_{\text{Th}})V_{DS} - V_{DS}\right] = \frac{I_D}{S}$$

Here, $$K_n = \mu_n C_{ox} \frac{W}{L}$$

Power Dissipation:

The instantaneous power dissipation of the MOSFET before scaling can be found as

$$P = I_D \cdot V_{DS}$$

Now, full scaling reduces both the drain current I_D and drain to source voltage V_{DS} by a factor of S. So, the power dissipation of the transistor will be reduced by the factor of S^2.

$$P' = I_D' V_{DS}' = \frac{1}{S^2} \cdot I_D \cdot V_{DS} = \frac{P}{S^2}$$

2. **Constant Voltage Scaling:** All dimensions of the MOSFET are reduced by a factor of S. In constant voltage scaling the power supply voltage and terminal voltages remains unchanged.

The effect of constant voltage scaling on key dimensions, voltage and doping densities is shown below:

Quantity	**Before Scaling**	**After Scaling**
Dimensions	W, L, t_{ox}, x_j	reduced by S $\left(W' = \frac{W}{S}, L' = \frac{L}{S}, t_{ox}' = \frac{t_{ox}}{S}, x_j' = \frac{x_j}{S}\right)$
Voltage	V_{DD}, V_{Th}	Remains unchanged
Doping densities	N_A, N_D	Increased by s^2 $\left(N_A' = S^2 \cdot N_A, \text{ and } N_D' = S^2 N_D\right)$

Effect of Constant Voltage scaling on Key Device Characteristics:

Oxide capacitance: The gate oxide capacitance per unit area is changed as follows:

$$C_{ox}' = \frac{\in_{ox}}{t_{ox}'} = S \cdot \frac{\in_{ox}}{t_{ox}} = S \cdot C_{ox}$$

Thus, C_{ox} is increased by a factor of S.

Drain current: Since all terminal voltages remain unchanged, the linear mode drain current of the scaled MOSFET can be written as

$$I_D' = \frac{K_n'}{2}\left[2(V_{GS}' - V_{Th}')V_{DS}' - V'^2_{DS}\right]$$

$$I_D' = \frac{S \cdot K_n}{2}\left[2(V_{GS} - V_{Th}) \cdot V_{DS} - V_{DS}^2\right] = S \cdot I_D$$

Power dissipation: Since the drain current I_D is increased by a factor of S while the drain to source voltage V_{DS} remain unchanged. The power dissipation of the MOSFET increases by a factor of S.

$$P' = I_D' V_{DS}' = (S \cdot I_D) \cdot V_{DS} = S \cdot P$$

Effect of constant voltage scaling upon key device characteristics:

Before Scaling	After Scaling
Oxide capacitance C_{ox}	$C'_{ox} = S \cdot C_{ox}$
Drain current I_D	$I'_D = S \cdot I_D$
Power dissipation P_D	$P' = S \cdot P$
Power density P/Area	$\frac{P'}{\text{Area}} = S^3 \cdot \frac{P}{\text{Area}}$

Q.4 (c) Solution:

(i) Given data for geosynchronous satellite:

The gravitational coefficient, g_o = GM = 3.986 × 10^5 km^3/s^2

Radius of the orbit, $r_e + h$ = 42000 km.

We know, velocity in the orbit,

$$V_s = \sqrt{\frac{g_o}{r_e+h}} = \sqrt{\frac{3.986\times10^5}{42000}} = 3.08 \text{ km/s}$$

Orbit period,

$$T_s = \frac{2\pi(r_e+h)^{3/2}}{\sqrt{g_o}} = \frac{2\pi(r_e+h)}{V_s}$$

$$= \frac{2\pi\times(42000)^{3/2}}{\sqrt{3.986\times10^5}} = 85661.397 \text{ sec}$$

If the radius of the orbit is reduced to 36,500 Km,

$$r_e + h = 36{,}500 \text{ km}$$

$$V_s = \sqrt{\frac{g_o}{r_e+h}} = \sqrt{\frac{3.986\times10^5}{36500}} = 3.3046 \text{ km/s}$$

Change in velocity = 3.304 – 3.08 = 0.224 km/sec.

(ii) 1. A channel kept busy for one hour is defined as having a traffic of one Erlang. Thus,

$$\text{Average traffic} = T = 10000\times\frac{1\text{ hr}}{24\text{ hr}}$$

$$T = 416.667 \text{ Erlangs}$$

Peak traffic,

$$T_p = 10000\times\frac{30\text{ min}}{60\text{ min}} = 5000 \text{ Erlangs}$$

$$T_p = 5000 \text{ Erlangs}$$

2. Average traffic in cell $(T_c) = \dfrac{416.66}{120}$

$$T_c = 3.472 \text{ Erlangs}$$

$$\text{Peak traffic in cell} = \frac{\text{Peak traffice in cellular system}}{\text{Number of cells}} = \frac{T_p}{120}$$

$$\text{Peak traffic in cell} = \frac{5000}{120} = 41.66 \text{ Erlangs}$$

Section B

Q.5 (a) Solution:

(i) Deadlock exist among a set of processes if every process in the set is waiting for an event that can be caused only by another process in the set.

Deadlock can exist if and only if given four conditions hold simultaneously:

1. Mutual Exclusion:

- This condition ensures that each resource can be accessed by only one process at a time.
- When a process acquires a resource, it has exclusive use of that resource until it releases it.
- This exclusivity can lead to resource contention if other processes also require the same resource, causing them to wait.

2. Hold and Wait:

- Hold and wait refers to processes holding onto resources while waiting for additional resources to be allocated.
- This means a process may hold some resources and is waiting for others, leading to potential resource starvation as processes might wait indefinitely for the resources held by others.

3. No Preemption:

- Preemption involves forcibly taking away a resource from a process that's currently using it.
- In the context of deadlock, preemption is not allowed, meaning resources can only be released voluntarily by the process holding them.
- This condition contributes to the deadlock as it allows processes to hold onto resources, even if they are not actively using them, preventing other processes from accessing them.

4. **Circular Wait:**
   - Circular wait exists when there's circular chain of processes, where each process is waiting for a resource held by the next process in the chain.
   - This means process A is waiting for a resource held by process B, process B is waiting for a resource held by Process C, and so on until some process is waiting for a resource held by Process A.
   - This cyclic dependency of processes waiting for resources from each other is a key characteristic of deadlock.

When these four conditions are met simultaneously, they create a situation where processes are stuck in a waiting state indefinitely, unable to proceed due to resource allocation conflicts, leading to deadlock.

(ii) A cyclic dependency does not always lead to deadlock, and here's why;

1. **Nature of Resources:**
   - Not all resources are essential for a process to complete its execution.
   - Some resources can be released and reacquired, allowing processes to break out of a cyclic dependency by releasing non-essential resources.
2. **Resources Allocation Strategy:**
   - If a resource allocation strategy ensure that atleast one resource in the cycle can be preempted or has a limited hold and wait time, deadlock can be avoided.
   - For instance, a time limit can be set for a process to wait for a resource, after which it releases its held resources to break the cycle.
3. **Resource Requirements:**
   - Processes may not always require all the resources they request to execute.
   - Some resources might be optional or can be substituted by other resources, allowing processes to release the resources they hold and continue execute without waiting for all requested resources.
4. **External Intervention:**
   - External interventions like process termination or resource reallocation can break the cyclic dependency.
   - If the system detect potential deadlock situations and takes corrective actions, such as terminating one of the processes or reallocating resources, the cyclic dependancy can be broken, preventing deadlock.

Q.5 (b) Solution:

(i) Comparison of memory mapping and IO mapping of IO devices in an 8085 based system:

Memory mapping of an IO device:

1. 16 bit addresses are provided for IO devices.
2. The devices are accessed by memory read or memory write cycles.
3. The IO ports or peripheral can be treated like memory locations and so all instructions related to memory can be used for data transfer between the IO device and the processor.
4. In memory-mapped ports, the data can be moved from any register to the ports and vice versa.
5. When memory mapping is used for IO devices the full memory address space cannot be used for addressing memory. Hence memory mapping is useful only for small systems, where the memory requirement is less.
6. In memory-mapped IO devices, a large number of IO ports can be interfaced.
7. For accessing memory-mapped devices, the processor executes the memory read or write cycle with $IO/\overline{M}$ asserted low $(IO/\overline{M} = 0)$.

IO mapping of an IO device:

1. 8 bit addresses are provided for IO devices.
2. The devices are accessed by IO read or IO write cycle. During these cycles, the 8 bit port address is placed both in the lower and higher order address bus.
3. Only IN and OUT instructions can be used for data transfer between the IO device and the processor.
4. In IO-mapped ports the data transfer can take place only between the accumulator and the ports.
5. When IO mapping is used for IO devices, then the full memory address space can be used for addressing the memory. Hence, it is suitable for systems which requires a large memory capacity.
6. In IO mapping, only 256 ports ($2^8 = 256$) can be interfaced.
7. For accessing the IO mapped devices, the processor executes the IO read or write cycle. During this cycle, $IO/\overline{M}$ is asserted high $(IO/\overline{M} = 1)$.

(ii) Memory mapping of IO device in 8086 based system:

1. 20 bit addresses are provided for IO devices.
2. The ID ports or peripherals can be treated like memory locations and so all instructions related to memory can be used for data transfer between the IO device and the processor.
3. In memory-mapped ports, the data can be moved from any register to the ports and vice-versa.
4. When memory mapping is used for IO devices, the full memory address space cannot be used for addressing memory where the memory requirement is less.
5. For accessing memory mapped devices, the processor executes the memory read or write cycle. During this cycle, $\mathrm{M}/\overline{\mathrm{IO}}$ is asserted high.

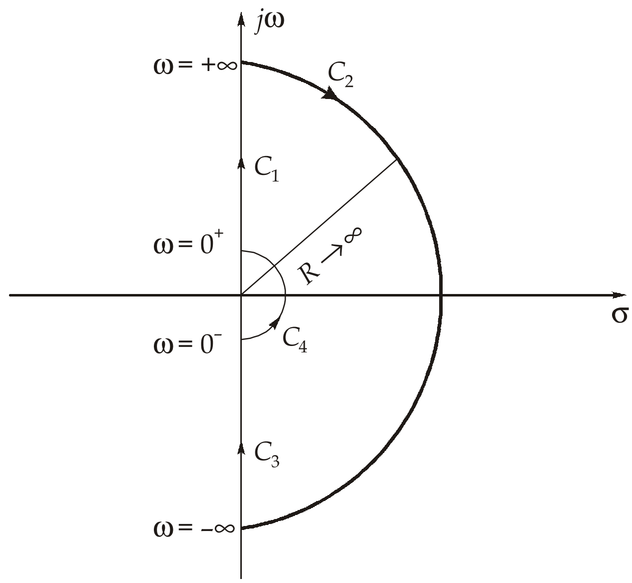
IO mapping of IO device in 8086 based system:

1. 8 bit or 16 bit addresses are provided for IO devices.
2. Only IN and OUT instructions can be used for data transfer between the IO device and the processor.
3. In IO mapped ports, the data transfer can take place only between the accumulator and the ports.
4. When IO mapping is used for IO devices, then the full memory address space can be used for addressing memory. Hence it is suitable for systems which require large memory capacity.
5. For accessing IO mapping devices, the processor executes the IO read or write cycle. During this cycle, $\mathrm{M}/\overline{\mathrm{IO}}$ is asserted low.

Q.5 (c) Solution:

Given that, $$G(s)H(s) = \frac{1+6s}{s^2(1+2s)(1+3s)}$$

The given open-loop system has a double pole at the origin. The Nyquist contour is, therefore, indented to bypass the origin. The Nyquist contour on s-plane is shown in figure.



Mapping of section C_1:

In section C_1, ω varies from 0 to ∞.

Substituting $s = j\omega$ in $G(s)H(s)$, we get

$$G(j\omega)H(j\omega) = \frac{(1+6j\omega)}{(j\omega)^2(1+2j\omega)(1+3j\omega)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{\sqrt{1+36\omega^2}}{\omega^2\sqrt{1+4\omega^2}\sqrt{1+9\omega^2}}$$

$$\phi = \angle G(j\omega)H(j\omega) = \tan^{-1}6\omega - 180° - \tan^{-1}2\omega - \tan^{-1}3\omega$$

When the $G(j\omega)H(j\omega)$ locus cross real axis, the phase will be –180°.

So, at $\omega = \omega_{pc}$

$$\angle G(j\omega_{pc})H(j\omega_{pc}) = -180°$$

$$\therefore \quad \tan^{-1}(6\omega_{pc}) - 180° - \tan^{-1}2\omega_{pc} - \tan^{-1}3\omega_{pc} = -180°$$

$$\tan^{-1}(6\omega_{pc}) - \tan^{-1}\frac{5\omega_{pc}}{1-6\omega_{pc}^2} = 0$$

$$6\omega_{pc} = \frac{5\omega_{pc}}{1-6\omega_{pc}^2}$$

$$6\omega_{pc} - 36\omega_{pc}^3 = 5\omega_{pc}$$

$$\omega_{pc} = 36\omega_{pc}^3$$

$$\omega_{pc}(36\omega_{pc}^2 - 1) = 0$$

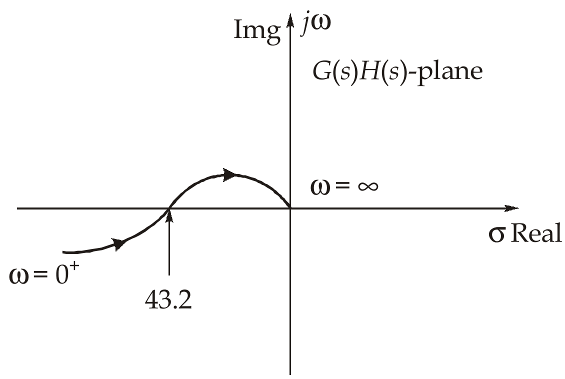
$$\omega_{pc} = 0,\ \omega_{pc} = \frac{1}{\sqrt{36}} = \frac{1}{6} = 0.167 \text{ rad/sec}$$

Hence,
$$|G(j\omega_{pc})H(j\omega_{pc})| = \frac{\sqrt{1+36\times(0.167)^2}}{(0.167)^2\sqrt{1+4(0.167)^2}\sqrt{1+9\times(0.167)^2}}$$
$$= 43.2$$

At $\omega = 0$, $\quad G(j\omega)H(j\omega) = \infty\angle{-180°}$

At $\omega = \infty$, $\quad G(j\omega)H(j\omega) = 0\angle{-270°}$

The mapping of section C_1 in $G(s)H(s)$-plane is shown below:



Mapping of section C_2:

$$s = \lim_{R\to\infty} Re^{j\theta}$$

where, $\theta \in \left[\frac{\pi}{2}, -\frac{\pi}{2}\right]$ and $R \to \infty$

As,
$$G(s)H(s) = \frac{1+6s}{s^2(1+2s)(1+3s)}$$

Put $s = Re^{j\theta}$ where $R \to \infty$ and $\theta \in \left[\frac{\pi}{2}, \frac{-\pi}{2}\right]$

$$G(Re^{j\theta})H(Re^{j\theta}) = \frac{1+6Re^{j\theta}}{(Re^{j\theta})^2(1+2Re^{j\theta})(1+3Re^{j\theta})}$$

For R $\to \infty$; $1 << 6Re^{j\theta}$; $1 << 2Re^{j\theta}$; $1 << 3Re^{j\theta}$

Therfore we can write,

$$G(Re^{j\theta})H(Re^{j\theta}) = \frac{6Re^{j\theta}}{(Re^{j\theta})^2(2Re^{j\theta})(3Re^{j\theta})} = \frac{1}{(Re^{j\theta})^3}$$

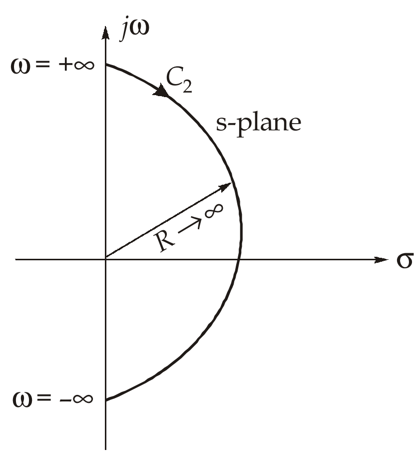
For R $\to \infty$, we get

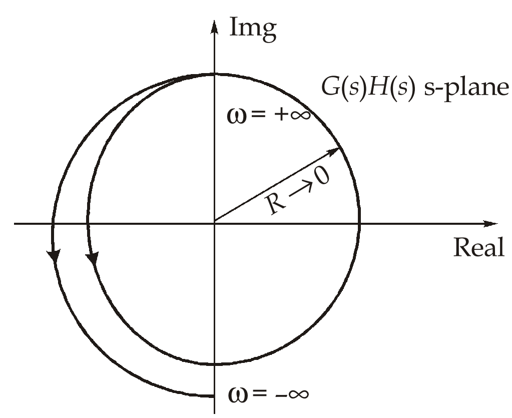
$$G(Re^{j\theta})H(Re^{j\theta}) = \lim_{R\to\infty} \frac{1}{(Re^{j\theta})^3}$$

$$\therefore \quad G(s)H(s)\Big|_{s = \lim\limits_{R\to\infty} Re^{j\theta}} = \frac{1}{\lim\limits_{R\to\infty} (Re^{j\theta})^3} = 0e^{-j3\theta}$$

when $\theta = \dfrac{\pi}{2}$; $G(s)H(s) = 0e^{-j3\pi/2}$

$\theta = \dfrac{-\pi}{2}$; $G(s)H(s) = 0e^{j3\pi/2}$





Mapping of section C_3: This locus is the inverse polar plot obtained for section C_1.

Mapping of section C_4:

$$s = \lim_{R\to 0} Re^{j\theta}; \quad \theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$G(s)H(s) = \frac{1+6s}{s^2(1+2s)(1+3s)},$$

$$G(s)H(s)\Big|_{s=Re^{j\theta}} = \frac{1+6Re^{j\theta}}{(Re^{j\theta})^2(1+2Re^{j\theta})(1+3Re^{j\theta})}$$

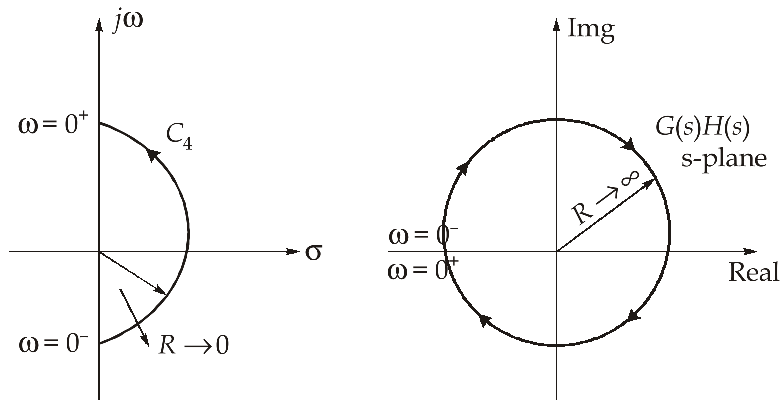
For $\lim\limits_{R\to 0} \dfrac{1+6Re^{j\theta}}{(Re^{j\theta})^2(1+2Re^{j\theta})(1+3Re^{j\theta})} = \dfrac{1}{(Re^{j\theta})^2}$

Thus, for section C_4:

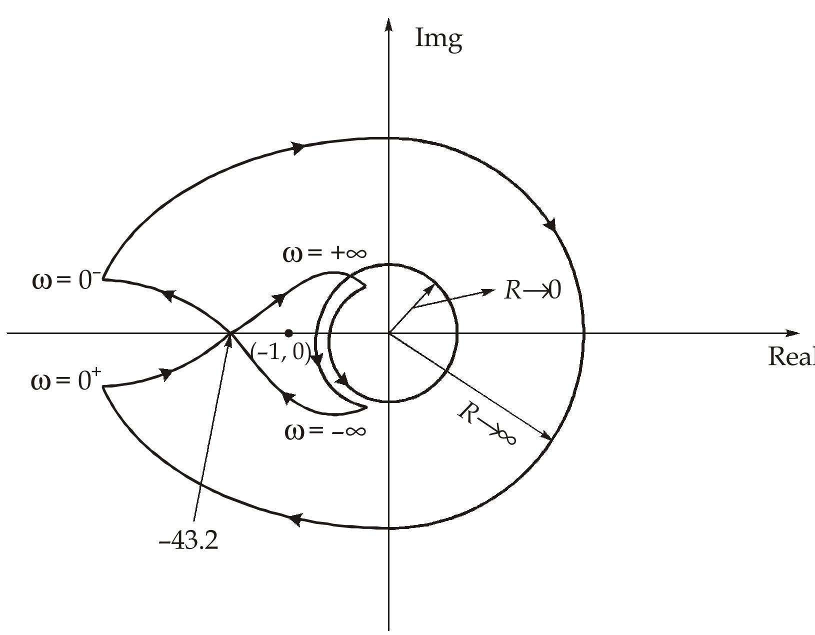
$$G(s)H(s)\Big|_{s=\lim\limits_{R\to 0} Re^{j\theta}} = \infty e^{-2j\theta}$$

$$\text{For } \theta = -\pi/2;\ G(s)H(s) = \infty e^{+j\pi}$$

$$\text{For } \theta = \pi/2;\ G(s)H(s) = \infty e^{-j\pi}$$



Complete Nyquist plot:



Here, encirclement around (–1, 0) by the Nyquist plot,

$$N = -2$$

From Nyquist Criterion, $N = P - Z$

where, P = Open loop poles in right side of s-plane.

Z = Closed loop poles in right side of s-plane

Here, $P = 0$

So, $N = -2 = 0 - Z$

$\Rightarrow$ $Z = 2$

- Closed loop system is unstable.
- Two poles of closed loop system are lying on the right half of s-plane.

Q.5 (d) Solution:

$$F[e^{jat^2}] = \int_{-\infty}^{\infty} e^{jat^2} e^{-j\omega t} dt = \int^{\infty} e^{j(at^2-\omega t)} dt$$

Here, $$at^2 - \omega t = a\left(t^2 - \frac{\omega}{a}t\right)$$

$$= a\left[t^2 - \frac{2\omega}{2a}t + \frac{\omega^2}{4a^2}\right] - \frac{\omega^2}{4a}$$

$$= a\left[t - \frac{\omega}{2a}\right]^2 - \frac{\omega^2}{4a} = \left[\sqrt{a}\,t - \frac{\omega}{2\sqrt{a}}\right]^2 - \frac{\omega^2}{4a} = y^2 - \frac{\omega^2}{4a}$$

where $$y = \sqrt{a}\,t - \frac{\omega}{2\sqrt{a}}$$

Therefore, $$dy = \sqrt{a}\,dt \text{ and } dt = \frac{dy}{\sqrt{a}}$$

$$F[e^{jat^2}] = \int_{-\infty}^{\infty} e^{j\left[y^2 - \frac{\omega^2}{4a}\right]} \frac{dy}{\sqrt{a}}$$

$$= \frac{1}{\sqrt{a}} e^{-j\left(\frac{\omega^2}{4a}\right)} \int_{-\infty}^{\infty} e^{jy^2} dy = \frac{2}{\sqrt{a}} e^{-j\left(\frac{\omega^2}{4a}\right)} \int_{0}^{\infty} e^{jy^2 dy} \quad \text{...(1)}$$

Considering the Gamma function,

$$\Gamma(a) = \int_0^{\infty} x^{(a-1)} e^{-x} dx$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^{\infty} x^{-1/2} e^{-x} dx$$

Putting $x = y^2$ in the above equation, we get

$$\Gamma\left(\frac{1}{2}\right) = \int_0^{\infty} y^{-1} e^{-y^2} 2y dy = 2\int_0^{\infty} e^{-y^2} dy = \sqrt{\pi}$$

Replacing y by $\sqrt{-j}x$ in the above integral, we get

$$2\int_0^\infty e^{jx^2}\sqrt{(-j)}dx = \sqrt{\pi}$$

$$2\int_0^\infty e^{jx^2}dx = \sqrt{\frac{\pi}{-j}} = \sqrt{j\pi}$$

From equation (1), $$F[e^{jat^2}] = \sqrt{\frac{j\pi}{a}}\, e^{\frac{-j\omega^2}{4a}} \quad ...(2)$$

But $$j = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = e^{j\frac{\pi}{2}} \text{ and } \sqrt{j} = e^{j\frac{\pi}{4}}$$

$\therefore$ $$F[e^{jat^2}] = \sqrt{\frac{\pi}{a}}\, e^{j\left[\frac{\pi}{4}-\frac{\omega^2}{4a}\right]}$$

From equation (2), we can derive Fourier's transform of some trigonometric functions.

$$F[e^{jat^2}] = \int_{-\infty}^{\infty}(\cos at^2 + j\sin at^2)e^{-j\omega t}dt$$

$$= \sqrt{\frac{\pi}{a}}\left[\cos\left(\frac{\pi}{4}-\frac{\omega^2}{4a}\right) + j\sin\left(\frac{\pi}{4}-\frac{\omega^2}{4a}\right)\right]$$

Equating real and imaginary parts from both the sides,

$$F[\cos at^2] = \int_{-\infty}^{\infty}\cos at^2 e^{-j\omega t}dt = \sqrt{\frac{\pi}{a}}\cos\left(\frac{\pi}{4}-\frac{\omega^2}{4a}\right)$$

$$F[\sin at^2] = \int_{-\infty}^{\infty}\sin at^2 e^{-j\omega t}dt = \sqrt{\frac{\pi}{a}}\sin\left(\frac{\pi}{4}-\frac{\omega^2}{4a}\right)$$

Putting $a = jb^2$ in equation (2), we get

$$F[e^{-b^2t^2}] = \sqrt{\frac{j\pi}{jb^2}}\,e^{-j\frac{\omega^2}{4jb^2}} = \frac{\sqrt{\pi}}{b}e^{-\left(\frac{\omega}{2b}\right)^2}$$

Therefore, $$F[e^{-b^2t^2}] = \frac{\sqrt{\pi}}{b}e^{-\left(\frac{\omega}{2b}\right)^2}$$

Putting $a = jb$ in Equation (2), we get

$$F[e^{-bt^2}] = \sqrt{\frac{j\pi}{jb}}e^{-\omega^2/4b}$$

Therefore,
$$F[e^{-bt^2}] = \sqrt{\frac{\pi}{b}}e^{-\omega^2/4b}$$

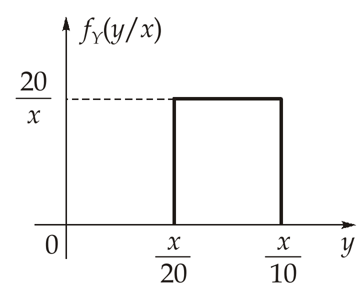
Q.5 (e) Solution:

The joint probability density function of the random variables X and Y can be written as,

$$f_{XY}(x,y) = f_X(x)f_Y(y/x)$$

From the given plots,
$$f_X(x) = \begin{cases} \frac{1}{2\times 10^4}x & ; \text{ for } 0 \le x \le 200 \\ 0 & ; \text{ otherwise} \end{cases}$$

$$f_Y(y|x) = \frac{20}{x}\left[u\left(y-\frac{x}{20}\right)-u\left(y-\frac{x}{10}\right)\right] = \frac{f_{XY}(x,y)}{f_X(x)}$$



The marginal probability density function $f_Y(y)$ can be defined as,

$$f_Y(y) = \int_{x=-\infty}^{\infty} f_{XY}(x,y)\,dx = \int_{-\infty}^{\infty} f(x)\cdot f(y/x)dx$$

$$= \int_{x=0}^{200}\left(\frac{x}{2\times 10^4}\right)\left(\frac{20}{x}\right)\left[u\left(y-\frac{x}{20}\right)-u\left(y-\frac{x}{10}\right)\right]dx$$

$$= \frac{1}{1000}\int_{x=0}^{200}\left[u\left(y-\frac{x}{20}\right)-u\left(y-\frac{x}{10}\right)\right]dx \quad \text{...(i)}$$

To solve $\int_{x=0}^{200} u\left(y-\frac{x}{20}\right)dx$:

Put
$$y-\frac{x}{20} = \tau$$

$$dx = -20d\tau$$

when $x = 0, \tau = y$

$x = 200, \tau = y - 10$

So,
$$\int_{x=0}^{200} u\left(y - \frac{x}{20}\right)dx = -20\int_{\tau=y}^{y-10} u(\tau)\,d\tau = -20\left[\tau u(\tau)\right]_y^{y-10}$$

$$= -20\left[(y-10)\,u(y-10) - yu(y)\right] \quad \text{...(ii)}$$

To solve, $\displaystyle\int_{x=0}^{200} u\left(y - \frac{x}{10}\right)dx$:

Put,
$$y - \frac{x}{10} = \tau$$

$$dx = -10d\tau$$

when, $x = 0, \tau = y$

$x = 200, \tau = y - 20$

So,
$$\int_{x=0}^{200} u\left(y - \frac{x}{10}\right)dx = -10\int_{\tau=y}^{y-20} u(\tau)\,d\tau$$

$$= -10\left[\tau u(\tau)\right]_y^{y-20}$$

$$= -10\left[(y-20)\,u(y-20) - yu(y)\right] \quad \text{...(iii)}$$

From equations (i), (ii) and (iii), we get,

$$f_Y(y) = \frac{1}{1000}\left[20yu(y) - 20(y-10)\,u(y-10) + 10(y-20)\,u(y-20) - 10yu(y)\right]$$

$$f_Y(y) = \frac{y}{100}u(y) - \frac{1}{50}(y-10)\,u(y-10) + \frac{1}{100}(y-20)\,u(y-20)$$

The above expression of $f_Y(y)$ can also be expressed as

$$f_Y(y) = \begin{cases} \dfrac{y}{100} & ; \quad 0 < y \le 10 \\ \dfrac{1}{5} - \dfrac{y}{100} & ; \quad 10 < y \le 20 \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

Q.6 (a) Solution:

(i) IEEE formats for representation of floating point data is of two types:

(a) Single precision format

(b) Double precision format

The format for the IEEE floating point representation is as below:

Sign	Bias Exponent	Mantissa

1 bit

- **Sign bit:** Sign bit 0 implies the number is positive whereas sign bit 1 implies the number is negative.
- **Bias Exponent:** In floating-point representation, a biased exponent is the result of adding some constant (Bias) to the actual exponent to make the range of the exponent non-negative.

  or

  In IEEE floating-point numbers, the exponent is biased in the engineering sense of the word-the value stored is offset from the actual value by the exponent bias, also called a biased exponent. Biasing is done because exponents have to be signed values in order to be able to represent both tiny and huge values, but two's complement, the usual representation for signed values, would make comparison harder. To solve this problem the exponent is stored as an unsigned value which is suitable for comparison, and when being interpreted it is converted into an exponent within a signed range by subtracting the bias.

  Bias Exponent: Actual Exponent + Bias

  Where, Bias $= (2^{n-1} - 1)$.... for normal bias

  $= 2^{n-1}$ ... for excess bias

  By default, we take normal bias.
- **Mantissa:**

  As in floating point representaion, number is arranged as $(-1)^{\text{sign}} \times 1.\text{mantissa} \times 2^{(\text{exponent-bias})}$ or $\pm M * B^{\pm e}$, where M is Mantissa, B is base and e is exponent.

  In floating-point representation, the mantissa is the part of a number that contain its significant digits. Together with the exponent, it forms a floating-point number.

  Now, for **Single Precision Formate:**

  It consists of 32 bits with 23 bits for Mantissa, 8 bits for Bias exponent and 1 sign bit.

32 bit		
Sign	Bias Exponent	Mantissa
1 bit	8 bit	23 bit

Exponent range:

–Bias to Bias +1 where $\text{Bias} = 2^{n-1} - 1$

–127 to 128 $= 2^{8-1} - 1 = 127$

Mantissa range:

1 to $(2 - 2^{-M})$ $\therefore M = 23$ bit

1 to $(2 - 2^{-23})$

Data range:

$\pm M * B^{\pm e}$

$\pm 1 * 2^{-127}$ to $\pm(2 - 2^{-23}) * 2^{128}$

Considering the **Double Precision Format,**

It is of 64 bit as shown in figure below:

64 bit		
Sign	Bias Exponent	Mantissa
1 bit	11 bit	52 bit

- Exponent range

  –Bias to Bias +1, where $\text{Bias} = (2^{n-1} - 1)$

  –1023 to 1024 $= 2^{11-1} - 1 = 1023$

- Mantissa range

  $\{1 \text{ to } (2 - 2^{-M})\}$

  $\{1 \text{ to } (2 - 2^{-52})\}$

- Range of Data:

  $\pm M * B^{\pm e}$

  $\pm 1 * 2^{-1023}$ to $\pm(2 - 2^{-52}) \times 2^{1024}$

(ii)
- For –∞; Sign bit is 1;

  Bias Exponent - All 1's

  Mantissa - All 0's

S	BE	Mantissa
1	All 1's	All 0's

- For ∞; Sign bit is 0;

  Bias Exponent - All 1's

  Mantissa - All 0's

S	BE	Mantissa
1	All 1's	All 0's

- For Bias Exponent All 1's and
  Mantissa not all zero's

As, for Bias Exponent all 1's, it represent a special number. When Mantissa is all 0's along with Bias Exponent all 1's it represent +∞/–∞ whereas when Mantissa is not all zero then it does not represent any number.

Q.6 (b) Solution:

```
      LXI H, 2060 H  ;  Load HL pair with 2060 H
      LXI D, 0000 H  ;  Load DE pair with 0000 H
      MOV C, M       ;  Load C register with the length of set
NEXT: INX H          ;  Increment the content of HL pair
      MOV A, M       ;  Move the byte to accumulator
      RRC            ;  Rotate accumulator right, D0 in CY
      JC SKIP        ;  Jump to skip if the number is odd
      RLC            ;  else Rotate accumulator left to get back the original number
      RLC            ;  Rotate accumulator left; D7 in CY
      JC SKIP        ;  Jump to skip if the number is negative
      RRC            ;  Rotate accumulator right to get back the signal number
      ADD E          ;  Add the content of register E to accumulator
      MOV E, A       ;  Store the lower byte of sum in E register
      JNC SKIP       ;  Check for overflow
      INR D          ;  Increment register D if overflow occurred
SKIP: DCR C          ;  Decrement the content of register C
      JNZ NEXT       ;  Jump to NEXT to access next byte
      MOV A, E       ;  Load lower byte of sum to accumulator
      STA 3001 H     ;  Store the lower byte of sum to accumulator
      MOV A, D       ;  Load higher byte of sum to accumulator
      STA 3002H      ;  Store the higher byte of sum of 3002 H
      HLT            ;  Halt the execution
```

Q.6 (c) Solution:

(i) During the *CZ* process of crystal growth, several impurities are added into the crystal. The crucible used to hold the molten silicon during the *CZ* process is usually fused silica (SiO_2).

At 1500°C, silica release a considerable amount of oxygen into molten silicon. Over 95% of the dissolved oxygen will escape from the surface of the melt as SiO. Some of oxygen will be incorporated into the growing crystal. Oxygen in silicon is an unintentional impurity. As an impurity, oxygen has three effects in silicon; donor formation, yield strength improvement and defect generation by oxygen precipitation. The ability of defects to capture harmful impurities is called gettering.

Carbon is another impurity in polysilicon and is transported to the melt from the graphite parts in the furnace. Another method to produce high purity form of silicon crystal is float zone method. Since, in the float zone process molten silicon is not contained in crucible, the crystal is not subjected to oxygen contamination as in the *CZ* process.

(ii) Oxidation plays a crucial role in creating insulating layers, forming gate oxides, providing isolation between components, passivating semiconductor surfaces, and controlling semiconductor properties. These processes are essential for the manufacturing of reliable and high performance integrated circuits used in various electronic devices.

1. **Insulator Formation:** One of the primary purposes of oxidation in IC fabrication is to create insulating layers, typically of silicon dioxide (SiO_2). Silicon dioxide is an excellent insulator and is used to electrically isolate different regions of the semiconductor device, such as transistor and interconnects. By oxidizing silicon wafers, a layer of SiO_2 is formed on the surface, serving as an insulating barrier between different components.
2. **Gate Oxide Formation:** Oxidation is crucial for forming the gate oxide layer in metal-oxide-semiconductor (MOS) transistors, which are the building blocks of modern ICs. The gate oxide serves as the dielectric layer between the gate electrode and the semiconductor channel. It enables the control of the transistor's conductivity of modulating the electric field across the channel. The thickness and quality of the gate oxide layer significantly influence the performance and reliability of the transistor.
3. **Isolation between Components:** Oxidation is also used to create isolation structures between different components on the IC, such as isolation trenches or wells. These structures prevent electrical interference between neighbouring devices, reducing crosstalk and improving device performance.
4. **Surface Passivation:** Oxidation can be employed for surface passivation, which involves the formation of a thin oxide layer on the semiconductor surface to reduce density of surface states and surface defects. Passivation helps improve

the reliability and stability of semiconductor devices by reducing leakage currents and enhancing the interface between the semiconductors and the surrounding materials.

5. **Control of Semiconductor Properties:** Oxidation can be used to manipulate the electrical properties of semiconductors. For example, by selectively oxidizing certain regions of a semiconductor wafer, dopants can be driven into the substrate, creating regions of controlled conductivity or adjusting the threshold voltage of transistors.

Q.7 (a) Solution:

(i) Given: $h = 400$ km ; $\mu = 0.9$; $f_{muf} = 10$ MHz

We know that, $$\mu = \sqrt{1 - \frac{81N}{f^2}}$$

For $f = 10$ MHz, $$0.9 = \sqrt{1 - \frac{81N_{max}}{(10 \times 10^6)^2}}$$

$$N_{max} = \frac{\left[1 - (0.9)^2\right] \times (10 \times 10^6)^2}{81} = 2.345 \times 10^{11} \ /\text{m}^3$$

Therefore, $$f_c = 9\sqrt{N_{max}} = 9\sqrt{2.345 \times 10^{11}} = 4.358 \text{ MHz}$$

When earth's curvature is taken into account, considering $R = 6371$ km and h as the height of ionospheric layer from the earth, we have

$$\frac{f_{muf}}{f_c} = \sqrt{\frac{D^2}{4\left(h + \frac{D^2}{8R}\right)^2} + 1}$$

$$\frac{D^2}{4\left(h + \frac{D^2}{8R}\right)} = \left(\frac{f_{muf}}{f_c}\right)^2 - 1$$

Skip Distance, $$D = 2\left\{h + \frac{D^2}{8R}\right\}\sqrt{\left(\frac{f_{MUF}}{f_c}\right)^2 - 1}$$

$$D = 2\left\{400 + \frac{D^2}{8 \times 6371}\right\}\sqrt{\left(\frac{10 \times 10^6}{4.358 \times 10^6}\right)^2 - 1}$$

$$D = 2\left\{400 + \frac{D^2}{8 \times 6371}\right\} \times 2.065$$

$$D = 1652.16 + 0.8103 \times 10^{-4} D^2$$

$D^2 - 12341.108D + 20.389 \times 10^6 = 0$

After solving the equation,

$$D = 10376.114, 1964.993 \text{ km}$$

Since skip distance is the shortest distance from the transmitter and represents the ground range, hence

$$D \cong 1965 \text{ km}$$

(ii) **Given data:** Propagation losses = 200 dB ; Margin and other losses = 3 dB

G/T = 11 dB ; EIRP = 45 dBW ; $[C/N]$ = ?

$$[C/N] = [\text{EIRP}] + \left[\frac{G}{T}\right] - [\text{Losses}] - [K] - [B_N] \quad \text{...(i)}$$

$$= [\text{EIRP}] + \left[\frac{G}{T}\right] - [\text{Losses}] - [K] - [B_N]$$

$$= 45 + 11 - 200 - 3 - 10\log(1.38 \times 10^{-23}) - 10\log(36 \times 10^6)$$

$$= 6.038 \text{ dB}$$

$$\therefore \quad \left[\frac{C}{N}\right](\text{dB}) = 6.038 \text{ dB}$$

Q.7 (b) Solution:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+5}{(s+2)(s+3)^2} = \frac{A}{s+2} + \frac{B}{(s+3)} + \frac{C}{(s+3)^2}$$

where,

$$A = \left.\frac{s+5}{(s+3)^2}\right|_{s=-2} = \frac{3}{1} = 3$$

$$B = \frac{d}{ds}\left[\frac{s+5}{(s+2)}\right] = \left.\frac{s+2-s-5}{(s+2)^2}\right|_{s=-3} = \frac{-3}{1} = -3$$

$$C = \left.\frac{s+5}{s+2}\right|_{s=-3} = \frac{2}{-1} = -2$$

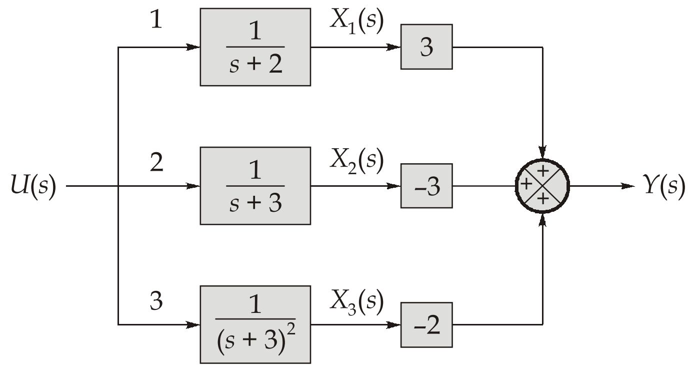
$$\therefore \quad G(s) = \frac{Y(s)}{U(s)} = \frac{3}{s+2} - \frac{3}{s+3} - \frac{2}{(s+3)^2}$$

Let

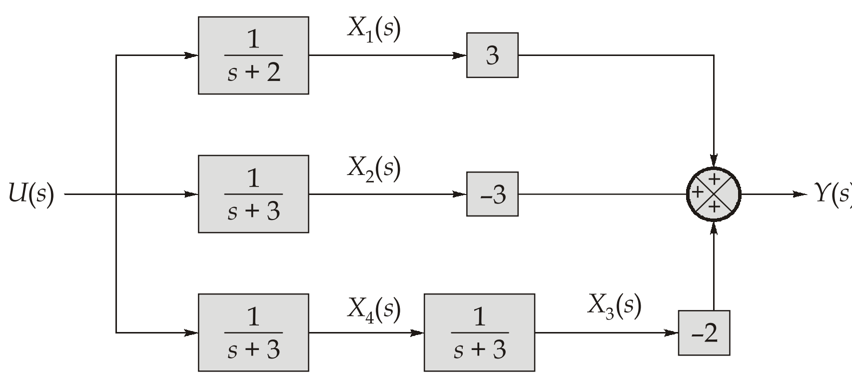
$$X_1(s) = \frac{U(s)}{s+2};\ X_2(s) = \frac{U(s)}{s+3} \text{ and } X_3(s) = \frac{U(s)}{(s+3)^2}$$

So, $$Y(s) = 3X_1(s) - 3X_2(s) - 2X_3(s)$$

The above system can be depicted by four number of integrals as shown below:



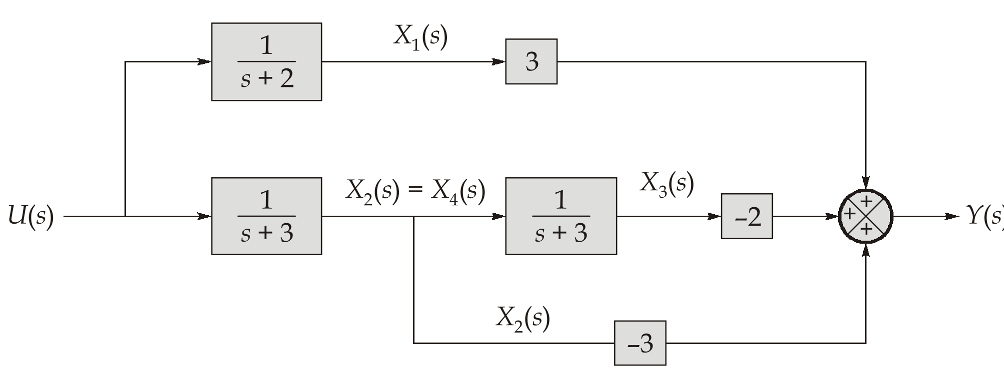
The blocks in branch 1 and 2 can be depicted by one integrator each. However, the block in branch 3 has a squared term and hence, requires two integrators as shown below:



where, $$X_3(s) = \frac{X_4(s)}{s+3} \text{ and } X_4(s) = \frac{U(s)}{s+3}$$

Also, $$X_2(s) = \frac{U(s)}{s+3} \text{ and } X_1(s) = \frac{U(s)}{s+2}$$

Thus, it is seen that $X_2(s)$ and $X_4(s)$ are similar and hence, we can get rid of one integrator and draw the state diagram with three integrators as shown below:



State Model/State Equations:

Now, $$X_1(s) = \frac{U(s)}{s+2}$$

i.e., $$sX_1(s) + 2X_1(s) = U(s)$$

$$\dot{x}_1 + 2x_1 = u \quad \text{...(i)}$$

or $$\dot{x}_1 = -2x_1 + u$$

Similarly,

$$X_2(s) = \frac{U(s)}{s+3}$$

$$sX_2(s) + 3X_2(s) = U(s)$$

$$\dot{x}_2 = -3x_2 + u \quad \text{...(ii)}$$

$$X_3(s) = \frac{X_2(s)}{s+3}$$

$$sX_3(s) + 3X_3(s) = X_2(s)$$

$$\dot{x}_3 = x_2 - 3x_3 \quad \text{...(iii)}$$

Also, $$Y = 3x_1 - 2x_3 - 3x_2$$

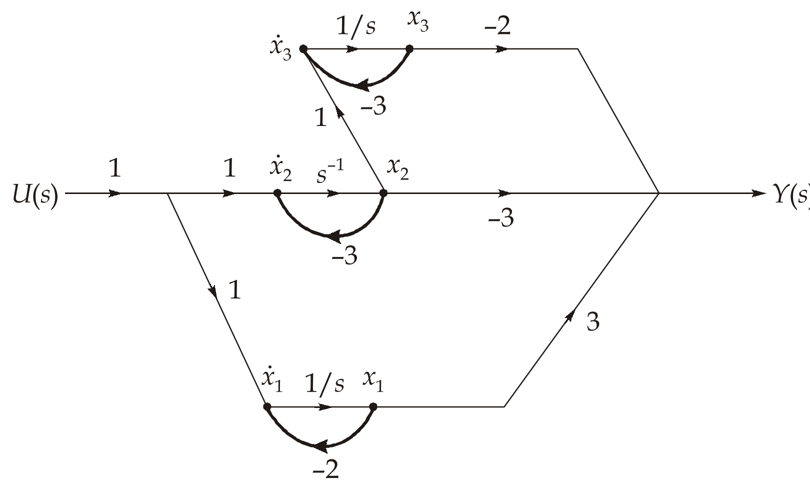
$$= 3x_1 - 3x_2 - 2x_3 \quad \text{...(iv)}$$

Using equations (i), (ii), (iii) and (iv), the state model in Jordan canonical form is obtained as below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 3 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The state signal flow graph can be drawn as



Q.7 (c) Solution:

(i) Magnitude of reflection coefficient is,

$$\rho = |\Gamma_L| = \frac{S-1}{S+1} = \frac{4-1}{4+1} = \frac{3}{5} \qquad [\because S = 4]$$

First voltage minimum occurs when,

$$2\beta d_{min} = \pi + \theta; \quad \theta = \text{angle of reflection coefficient}$$

Given that, $$d_{min} = \frac{\lambda}{6}, \text{ so,}$$

$$2\times\frac{2\pi}{\lambda}\times\frac{\lambda}{6} = \pi + \theta$$

$$\frac{2\pi}{3} - \pi = \theta$$

$$\frac{-\pi}{3} = \theta$$

$$\theta = \frac{-\pi}{3} \text{ or } (-60°)$$

Hence, $$\Gamma_L = \rho e^{j\theta} = \frac{3}{5}e^{-j\frac{\pi}{3}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

So, the load impedance can be given by,

$$Z_L = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) = 80\left[\frac{1+\frac{3}{5}e^{-j\frac{\pi}{3}}}{1-\frac{3}{5}e^{-j\frac{\pi}{3}}}\right]\Omega$$

$$= 80\left[\frac{1+\frac{3}{5}\cos 60° - j\frac{3}{5}\sin 60°}{1-\frac{3}{5}\cos 60° + j\frac{3}{5}\sin 60°}\right]$$

$$= 80\left[\frac{1.3 - j0.5196}{0.7 + j0.5196}\right]$$

$$= 80(0.842 - 1.367j) = (67.36 - 109.36j)\ \Omega$$

The series equivalent of the load impedance is

$$Z_L = R_L + jX_L$$

where $R_L = 67.36\ \Omega$

$$|X_L| = 109.36\ \Omega \text{ (capacitive)}$$

The parallel equivalent of the load impedance is,

$$Y_L = G_L + jB_L$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{(67.36 - 109.36j)}$$

$$= 4.08 \times 10^{-3} + j6.63 \times 10^{-3}$$

$$Y_L = (4.08 + j6.63)\ \text{m℧}$$

$$R_L = \frac{1}{G_L} = 245.1\ \Omega$$

$$|X_L| = \frac{1}{|B_L|} = 150.83\ \Omega \quad \text{(capacitive)}$$

So, the load impedance can be described as,

- A resistance of 67.36 Ω in series with a capacitive reactance of 109.36 Ω.
- A resistance of 245.1 Ω in parallel with a capacitive reactance of 150.83 Ω.

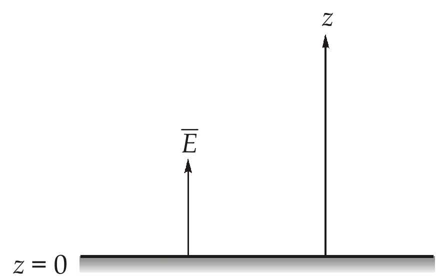
(ii) The electric field inside a conductor is 0 i.e., $E = 0$ for $z < 0$. Also, E for $z > 0$ is normal to the surface of the conductor. The surface charge density is therefore,

$$\rho_s = \epsilon_0 \epsilon_r E_n = \epsilon_0 (5)\ 10\cos(3 \times 10^8 t - 20x)$$

At $x = 2$ m and $t = 0.5 \times 10^{-9}$ sec

$$\rho_s = 50\,\epsilon_0 \cos(3 \times 10^8 \times 0.5 \times 10^{-9} - 20 \times 2)$$

$$= 3.394 \times 10^{-10}\ \text{C/m}^2$$



From the Faraday's law,

$$\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t} = -\mu_0\mu_r\frac{\partial\vec{H}}{\partial t}$$

$$\vec{H} = -\frac{1}{\mu_0\mu_r}\int\vec{\nabla}\times\vec{E}\,dt$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z\end{vmatrix}$$

$$= \frac{\partial E_z}{\partial y}\hat{a}_x - \frac{\partial E_z}{\partial x}\hat{a}_y$$

Since E_z is not a function of y, hence $\frac{\partial E_z}{\partial y} = 0.$ We therefore get,

$$\vec{\nabla}\times\vec{E} = -\frac{\partial E_z}{\partial x}\hat{a}_y = -200\sin(3\times10^8 t - 20x)\hat{a}_y$$

Thus,

$$\vec{H} = \frac{200}{\mu_0\mu_r}\int\sin(3\times10^8 t - 20x)\hat{a}_y dt$$

$$= \frac{-200}{\mu_0\mu_r}\frac{\cos(3\times10^8 t - 20x)}{3\times10^8}\hat{a}_y$$

The surface current,

$$\vec{J}_s = \hat{n}\times\overline{H} = \hat{a}_z\times\frac{-200\cos(3\times10^8 t - 20x)}{3\times10^8\mu_0\mu_r}\hat{a}_y$$

$$= \frac{200\cos(3\times10^8 t - 20x)}{3\times10^8\mu_0\mu_r}\hat{a}_x$$

At $x = 2$ m and $t = 0.5\times10^{-9}$ sec,

$$\vec{J_s} = \frac{200\cos(3\times10^8\times0.5\times10^{-9}-20\times2)}{4\pi\times10^{-7}\times20\times3\times10^8}$$

$$\vec{J_s} = -0.0145\hat{a}_x \text{ A/m}$$

Q.8 (a) Solution:

(i) Simple Input/Output Mode:

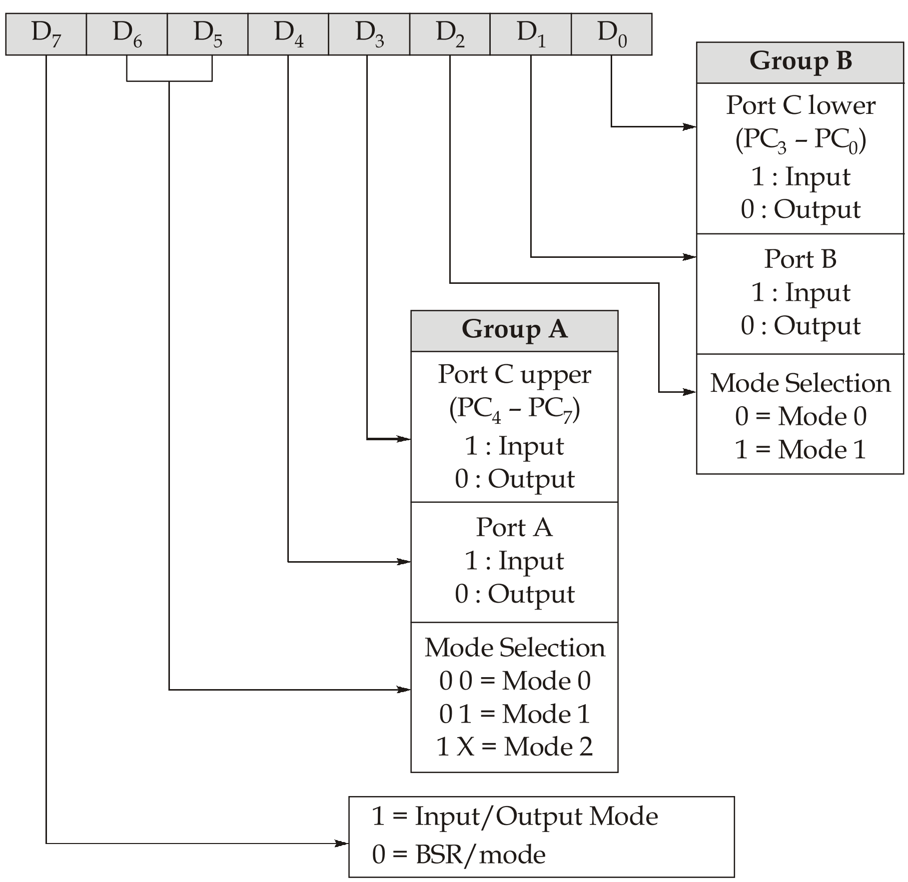
In this mode, ports A and B are used as two simple 8 bit input/output ports and port C as two 4-bit input/output ports. Each port can be programmed to function as simply an input/output port.

This mode is concerned only with the eight bits of port C, which can be set or reset by writing an appropriate control word in the control register.

The input/output features in this are:

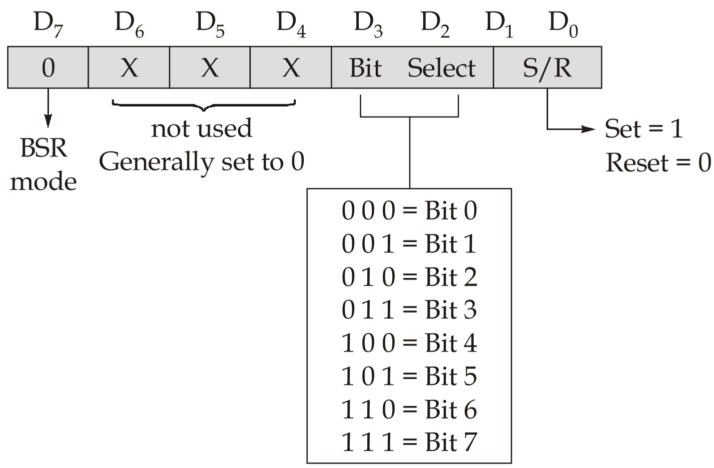
1. Outputs are latched.
2. Inputs are not latched.
3. Ports don't have hand shake or interrupt handling capability.

Control word for input/output mode is given below:



Bit Set/Reset Mode:

A control word with bit $D_7 = 0$ is recognized as BSR control word. In this mode, any of the 8-bits of port C can be set or reset depending on D_0 of the control word. The bit to be set or reset is selected by bit select flags D_3, D_2 and D_1 BSR control word is given below:



(ii) 1. For direct mapping:

Since, Block size = 16 Words = 2^4 words

So, word field consists of 4 bits to identify words in a block.

$$\text{Number of blocks in cache} = \frac{\text{Catche memory size}}{\text{Words in one block}}$$

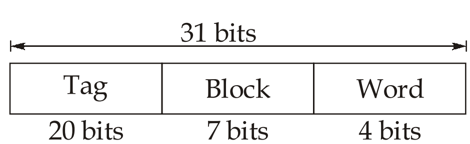
$$= \frac{2K}{16} = 2^7 \text{ blocks}$$

So, 7 bits are required in block field

$$\text{Tag bits} = \text{Physical address size} - 7 - 4$$

For a 2 GB = 2^{31} Bytes physical memory, number of bits in physical address = $\log_2 2^{31}$ = 31 bits. Hence, Tag bits = 31 – 7 – 4 = 20 bits.

Thus, the main memory address for direct mapping is divided as follows:



For a 2-way set associative mapping:

Block size = 16 words

Hence, number of Bits in word field = 4 bits

$$\text{Number of sets} = \frac{\text{Number of blocks}}{\text{Set associativity}}$$

$$= \frac{128}{2} = 64 \text{ sets}$$

So, 6 bits are needed,

$$\text{Tag bits} = 31 - 4 - 6 = 21 \text{ bits}$$

Main memory address has the format as below:

21 bits	6 bits	4 bits
Tag	Set	Word

2. $$\text{Speedup factor} = \frac{\text{Execution time of non-pipelined processor } (t_{p1})}{\text{Execution time of pipelined processor } (t_{p2})}$$

where, $$t_{p1} = \text{CPI} \times \text{cycle time}_1 \times \text{Number of instructions}$$

$$= 4 \times \frac{1}{2.5} \times 100 = 160 \text{ nsec}$$

and $$t_{p2} = (K + n - 1) \text{ cycle time}_2$$

$$= 52 \text{ nsec} \qquad (\because K = 5, n = 100)$$

So, $$\text{speedup factor} = \frac{160}{52} = 3.07$$

For a pipelined processor, the CPI can be approximated to 1. Hence, MIPS rate of processor is given by

$$\frac{1}{\text{Cycle Time} \times 10^6} = \frac{2 \times 10^9}{10^6}$$

$$= 2 \times 10^3 \text{ million instructions per second}$$

Q.8 (b) Solution:

(i) Applying Kirchhoff's Voltage law, we get

$$L\frac{di(t)}{dt} + Ri(t) = v(t)$$

Therefore, $$\frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{1}{L}v(t)$$

The difference equation approximation of this equation is

$$\frac{i(nT) - i(nT - T)}{T} = \frac{1}{L}v(nT) - \frac{R}{L}i(nT)$$

Simplifying, we get

$$i(nT) = \frac{\frac{T}{L}}{1+\frac{R}{L}T}v(nT) + \frac{1}{1+\frac{R}{L}T}i(nT-T)$$

Assume $T = 1$ sec. Then this equation becomes

$$i(n) = \frac{\frac{1}{L}}{1+\frac{R}{L}}v(n) + \frac{1}{1+\frac{R}{L}}i(n-1)$$

Substituting $R = 1\ \Omega$ and $L = 1$ H and $v(n) = e^{j\omega n}$, we get

$$2i(n) - i(n-1) = e^{j\omega n}$$

Taking DTFT,

$$2I(e^{j\omega}) - e^{-j\omega}I(e^{j\omega}) = V(e^{j\omega})$$

$$I(e^{j\omega}) = \frac{1}{2-e^{-j\omega}}V(e^{j\omega})$$

Since $I(e^{j\omega}) = H(e^{j\omega})V(e^{j\omega})$, thus the system function is

$$H(e^{j\omega}) = \frac{1}{2-e^{-j\omega}} = \frac{1}{(2-\cos\omega)+j\sin\omega}$$

(ii) Given
$$H(e^{j\omega}) = \frac{e^{j\omega}-a}{e^{j\omega}-b} = \frac{(\cos\omega-a)+j\sin\omega}{(\cos\omega-b)+j\sin\omega}$$

$$\left|H(e^{j\omega})\right|^2 = \frac{(\cos\omega-a)^2+\sin^2\omega}{(\cos\omega-b)^2+\sin^2\omega} = \frac{1-a^2-2a\cos\omega}{1+b^2-2b\cos\omega}$$

Substituting $b = \frac{1}{a}$ [since $ab = 1$], we have

$$\left|H(e^{j\omega})\right|^2 = \frac{1+a^2-2a\cos\omega}{1+\frac{1}{a^2}-\frac{2}{a}\cos\omega} = a^2\cdot\frac{1+a^2-2a\cos\omega}{1+a^2-2a\cos\omega} = a^2$$

To find phase response $\phi(\omega)$,

$$H(e^{j\omega}) = \frac{e^{j\omega}-a}{e^{j\omega}-\frac{1}{a}} = a\cdot\frac{e^{j\omega}-a}{ae^{j\omega}-1} = -ae^{j\omega}\frac{1-ae^{-j\omega}}{1-ae^{j\omega}}$$

$$= -ae^{j\omega}\frac{1-a\cos\omega+ja\sin\omega}{1-a\cos\omega-ja\sin\omega}$$

$$\phi(\omega) = \omega - \pi + \tan^{-1}\left[\frac{a\sin\omega}{1 - a\cos\omega}\right] - \tan^{-1}\left[\frac{-a\sin\omega}{1 - a\cos\omega}\right]$$

$$= \omega - \pi + 2\tan^{-1}\left[\frac{a\sin\omega}{1 - a\cos\omega}\right]$$

To find time delay $\tau(\omega)$,

i.e.,
$$\tau(\omega) = -\frac{d\phi(\omega)}{d\omega} = -1 - 2\frac{(1 - a\cos\omega)a\cos\omega - (a\sin\omega)a\sin\omega}{(1 - a\cos\omega)^2 + (a\sin\omega)^2}$$

$$= -1 - \frac{a\cos\omega - a^2}{1 - 2a\cos\omega + a^2} = \frac{-1 + a\cos\omega}{1 - 2a\cos\omega + a^2}$$

Q.8 (c) Solution:

The phase modulated signal can be given by,

$$s(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right]$$

For the given carrier signal $A_c = 1$ V and modulator has $K_p = 1$ rad/V, so

$$s(t) = \cos\left[2\pi f_c t + m(t)\right]$$

$$= \text{Re}\left[e^{j(2\pi f_c t + m(t))}\right] = \text{Re}\left[e^{j2\pi f_c t} \cdot e^{jm(t)}\right]$$

$e^{jm(t)}$ is a periodic signal with period $T_m = \dfrac{1}{f_m}$ and let us take its Fourier series coefficient represented as C_K.

So,
$$e^{jm(t)} = \sum_{K=-\infty}^{\infty} C_K\, e^{j2\pi K f_m t}$$

$$s(t) = \text{Re}\left[e^{j2\pi f_c t} \sum_{K=-\infty}^{\infty} C_k\, e^{j2\pi K f_m t}\right]$$

$$= \sum_{K=-\infty}^{\infty} C_K \cos[2\pi(f_c + K f_m)t]$$

By taking the Fourier transform of $s(t)$, we get,

$$S(f) = \sum_{K=-\infty}^{\infty} \frac{C_K}{2}\left[\delta(f + f_c + K f_m) + \delta(f - f_c - K f_m)\right]$$

The Fourier series coefficient C_K of $e^{jm(t)}$ can be calculated as,

$$C_K = \frac{1}{T_m}\int_{-T_m/2}^{T_m/2} e^{jm(t)} e^{-j\frac{2\pi}{T_m}Kt}\,dt$$

$$C_K = \frac{1}{T_m}\left[\int_{-T_m/2}^{0} e^{-j} e^{-jK\frac{2\pi}{T_m}t}\,dt + \int_{0}^{T_m/2} e^{j}\cdot e^{-jK\frac{2\pi}{T_m}t}\,dt\right]$$

$$= \frac{1}{T_m}\times\frac{T_m}{j2\pi K}\left[e^{-j}\left(-e^{-jK\frac{2\pi}{T_m}t}\right)_{-T_m/2}^{0} + e^{j}\left(-e^{-jK\frac{2\pi}{T_m}t}\right)_{0}^{T_m/2}\right]$$

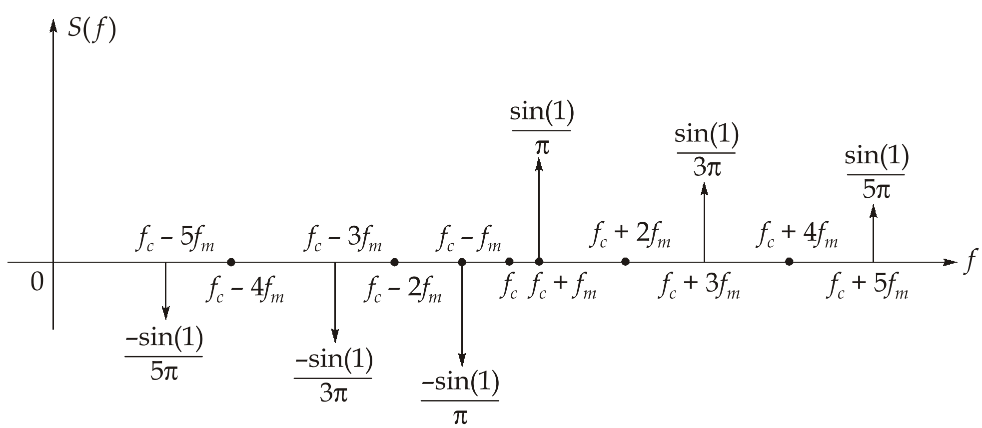
$$= \frac{-j}{2\pi K}\left[e^{-j}(-1+e^{jK\pi}) + e^{j}(1-e^{-jK\pi})\right]$$

$$C_K = \begin{cases} 0 & ; \quad K \text{ is even} \\ \dfrac{2\sin(1)}{K\pi} & ; \quad K \text{ is odd} \end{cases}$$

So, by substituting the value of C_K in equation (i), we get

$$S(f) = \sum_{K=-\infty}^{\infty} \frac{\sin(1)}{(2K+1)\pi}\left[\delta(f+f_c+(2K+1)f_m) + \delta(f-f_c-(2K+1)f_m)\right]$$

The spectrum of $s(t)$ for positive frequencies can be plotted as,



The spectrum of $s(t)$ for negative frequencies will be mirror image of the spectrum for positive frequencies. From the above spectrum, it is clear that the carrier frequency component of PM signal, for the given $m(t)$, contains no power. So, the total power of the modulated signal is contained by the sidebands for the given $m(t)$.

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