MADE EASY

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Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 12

Section A

Q.1 (a) Solution:

Applying KVL to Mesh 1,

$$V_1 - I_1 - 3I_2 - 2(I_1 + I_2) = 0$$

$$V_1 = 3I_1 + 5I_2 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 - 2(I_2 - I_3) - 2(I_1 + I_2) = 0$$

$$V_2 - 2I_2 + 2I_3 - 2I_1 - 2I_2 = 0$$

$$V_2 = 2I_1 + 4I_2 - 2I_3 \quad \text{...(ii)}$$

Writing equation for Mesh 3,

$$I_3 = 2V_3 \quad \text{...(iii)}$$

From figure above $\quad V_3 = 2(I_1 + I_2)$

$$I_3 = 2V_3 = 4I_1 + 4I_2 \quad \text{...(iv)}$$

Substituting the equation (iv) in the equation (ii),

$$V_2 = -6I_1 - 4I_2 \quad \text{...(v)}$$

Comparing Eqs. (i) and (v) with Z-parameter equations, we get

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix} \Omega$$

***Y*-parameters**

We know that, $[Y] = [Z]^{-1}$

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$
$$= (3)(-4) - (5)(-6) = 18$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{\Delta Z}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{-4}{18} = \frac{-2}{9}\,\mho$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-(-6)}{18} = \frac{1}{3}\,\mho$$

$$Y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-5}{18}\,\mho$$

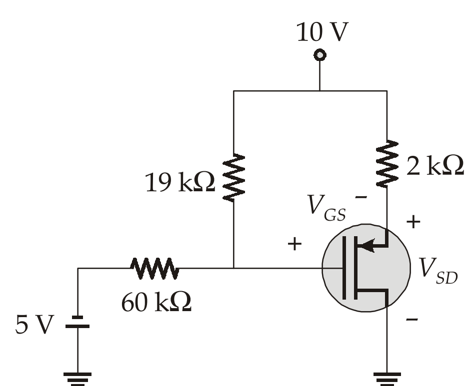
$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{1}{6}\,\mho$$

Hence, *Y*-parameters are

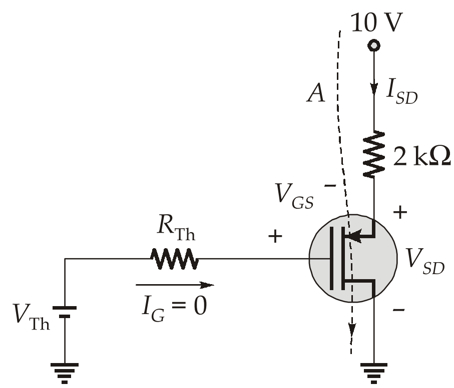
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{-2}{9} & \frac{-5}{18} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}\mho$$

Q.1 (b) Solution:

We have, $V_{TP} = -2$ V; $\dfrac{\mu_p C_{ox} W}{L} = 1\ \text{mA/V}^2$.



We can redraw the circuit in Thevenin equivalent form as shown below:



where, $$V_{Th} = \frac{(10\times 60)+(19\times -5)}{60+19} = \frac{600-95}{79} = 6.4 \text{ Volt}$$

We know that, $$V_{GS} = V_G - V_S$$

$$V_{GS} = V_{Th} - (10 - 2I_{SD}) = V_{Th} - 10 + 2I_{SD}$$

Put $V_{Th} = 6.4$ Volt, $$V_{GS} = 6.4 - 10 + 2I_{SD}$$

$$I_{SD} = \frac{V_{GS}+3.6}{2} \quad \text{...(i)}$$

Assuming MOSFET in saturation,

$$I_{SD} = \frac{\mu_p C_{ox} W}{2L}\left(V_{SG} - |V_{TP}|\right)^2$$

$$I_{SD} = \frac{1}{2}(V_{SG}-2)^2 \quad \text{...(ii)}$$

Put equation (i) in equation (ii),

$$\frac{V_{GS}+3.6}{2} = \frac{1}{2}(V_{SG}-2)^2$$

Here, $$V_{SG} = -V_{GS}$$

We get, $$-V_{SG} + 3.6 = (V_{SG} - 2)^2$$

$$(V_{SG} - 2)^2 + V_{SG} = 3.6$$

$$V_{SG}^2 + 4 - 4V_{SG} + V_{SG} = 3.6$$

$$V_{SG}^2 - 3V_{SG} + 0.4 = 0$$

$$V_{SG} = 2.86 \text{ V}, 0.14 \text{ V}$$

For MOS transistor to be ON, $V_{SG} > |V_{TP}|$

Hence, V_{SG} = 0.14 V is discarded.

Therefore, $V_{SG} = 2.86$ V

Thus,

(i) $V_{GS} = -2.86$ V

(ii) $I_D = \dfrac{V_{GS}+3.6}{2} = \dfrac{-2.86+3.6}{2} = 0.37$ mA

(iii) On applying KVL in loop A, we get,

$$-10 + 2I_{SD} + V_{SD} = 0$$

$$V_{SD} = 10 - 2I_{SD}$$

$$V_{SD} = 10 - (2 \times 0.37) = 9.26 \text{ Volt}$$

Since, $V_{SD} > V_{SG} - |V_{TP}|$ $\Rightarrow$ Our assumption is correct and MOSFET is in saturation region.

Q.1 (c) Solution:

The given circumstances 1, 2, 3, 4 and 5 are expressed in terms of the defined variables, S, M, T and H as $\bar{M}\bar{S}, T\bar{M}S, THS, \bar{T}\bar{M}S$ and $T\bar{H}$ respectively. Thus, the boolean function for turning 'ON' the sprinkler system can be written as:

$$F = \bar{S}\bar{M} + S\bar{M}T + STH + S\bar{M}\bar{T} + T\bar{H}$$

or we can write in canonical form as,

$$F(S, M, T, H) = 00XX + 101X + 1X11 + 100X + XX10$$

$$= 0000 + 0001 + 0010 + 0011 + 1010 + 1011 + 1011$$

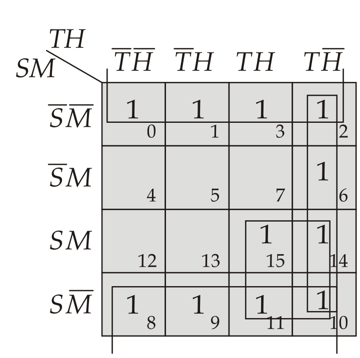
$$+ 1111 + 1000 + 1001 + 0010 + 0110 + 1010 + 1110$$

The expression in minterm and maxterm are, thus, obtained as

$$F(S, M, T, H) = \Sigma m(0, 1, 2, 3, 6, 8, 9, 10, 11, 14, 15)$$

$$= \pi M(4, 5, 7, 12, 13)$$

Minimization:

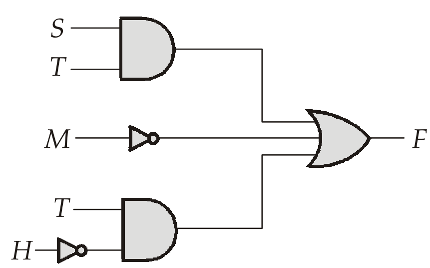


In SOP Form, $F = \overline{M} + ST + T\overline{H}$

$SM \backslash TH$	$\overline{T}\overline{H}$	$\overline{T}H$	TH	$T\overline{H}$
$\overline{S}\overline{M}$	(0)	(1)	(3)	(2)
$\overline{S}M$	0 (4)	0 (5)	0 (7)	1 (6)
SM	0 (12)	0 (13)	(15)	(14)
$S\overline{M}$	(8)	(9)	(11)	(10)

In POS Form, $F = (\overline{M} + T)(S + \overline{M} + \overline{H})$

Logic diagram:



Q.1 (d) Solution:

Given,
$$t_{ox} = 50 \text{ Å} = 50 \times 10^{-8} \text{ cm}$$
$$N_d = 1 \times 10^{18} \text{ cm}^{-3}$$
$$\text{fixed charge, } Q_i = 2 \times 10^{10} \text{ q C/cm}^2$$
$$\phi_{ms} = -0.1 \text{ V}, \in_{ox} = 3.9 \in_0, \in_s = 11.9 \in_0$$

(i) Threshold voltage,
$$V_T = 2\phi_F - \frac{Q_d}{C_{ox}} + \phi_{ms} - \frac{Q_i}{C_{ox}}$$

where,
$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}}{50 \times 10^{-8}} = 6.9 \times 10^{-7} \text{ F/cm}^2$$

$$\phi_F = -kT \ln \frac{N_a}{n_i} = -0.0259 \ln\left(\frac{10^{18}}{1.5 \times 10^{10}}\right)$$

$$\therefore \quad \phi_F = -0.467 \text{ V}$$

Depletion charge, $Q_d = +qN_aW$

where,
$$W = 2\left[\frac{\in_s(-\phi_F)}{qN_q}\right]^{\frac{1}{2}} = 2\left[\frac{11.8 \times 8.85 \times 10^{-14} \times (0.467)}{1.6 \times 10^{-19} \times 1 \times 10^{18}}\right]^{\frac{1}{2}}$$

$\therefore \quad W = 3.49 \times 10^{-6}$ cm

$\therefore \quad Q_d = 1.6 \times 10^{-19} \times 3.49 \times 10^{-6} \times 10^{18} = 5.536 \times 10^{-7}$ C/cm²

$Q_i = 2 \times 10^{10} \times 1.6 \times 10^{-19} = 3.2 \times 10^{-9}$ C/cm²

$$\therefore \quad V_T = -0.934 - \frac{5.536 \times 10^{-7}}{6.9 \times 10^{-7}} - 0.1 - \frac{3.2 \times 10^{-9}}{6.9 \times 10^{-7}}$$

$\therefore \quad V_T = -1.85$ V

(ii) The given MOS is enhancement Mode p-channel device since V_T is negative.

(iii) To achieve $V_T = 0$ V, $\Delta V_T = 1.85$ V is required

so that $V_T + \Delta V_T = 0$

$$\text{Boron dose} = \frac{Q_{Boron}}{q}$$

where, $Q_{\text{Boron}} = \Delta V_T \cdot C_{ox}$

$= 1.85 \times 6.9 \times 10^{-7} = 1.2765 \times 10^{-6}$ C/cm²

$$\therefore \quad \text{Boron dose} = \frac{1.2765 \times 10^{-6}}{1.6 \times 10^{-19}} = 7.98 \times 10^{12}/\text{cm}^2$$

Q.1 (e) Solution:

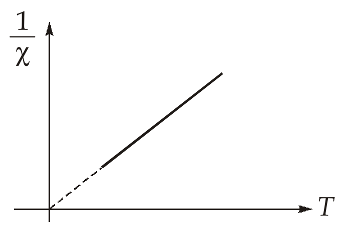
Diamagnetic material: As diamagnetism is due to the orbital motion of the electrons and it depends upon the electronic configuration of the element. As electronic configuration is independent of temperature, diamagnetic susceptibility is independent of temperature.

Paramagnetic material: Paramagnetism is due to the presence of unpaired electrons in the material, so paramagnetic atoms and ions include particles having one electron over and above a complete shell (alkali group), atoms of transition elements, ions of the rare earth elements with incomplete shell, etc. The alignment of atomic dipoles with external field direction is limited by temperature. As temperature increases, thermal agitation of atoms increases and magnetization decreases, thereby decreasing susceptibility. Thus, paramagnetic susceptibility is inversely proportional to temperature.

$$\chi \propto \frac{1}{T} \text{ (Curie's law)}$$

The paramagnetic substance posses a characteristic temperature, called curie temperature T_C, above which they become diamagnetic.

The plot of $\left(\frac{1}{\chi}\right)$ vs T is shown in the figure which is a straight line which illustrates the curie's law.



Ferromagnetic materials: Ferromagnetic materials are characterized by spontaneous magnetization i.e, they posses a large amount of magnetization even in the absence of external magnetic field.

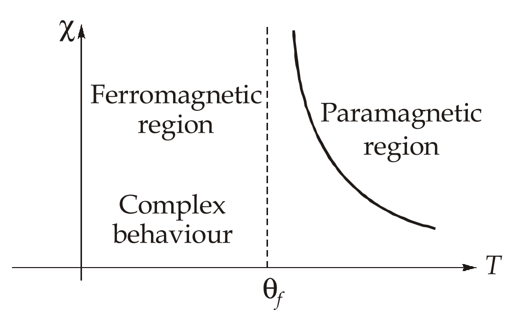
A ferromagnetic material has a characteristic temperature above which its properties are quite different from those below that temperature. It is called the ferromagnetic curie temperature θ_f.

For $T > \theta_f$, a ferromagnet behaves like a paramagnet. The susceptibility obeys the curie law as

$$\chi = \frac{C}{T - \theta_f}; \; C \text{ is called the curie constant.}$$

This expression is not valid for $T < \theta_f$.

The variation of susceptibility with temperature is as shown in the figure below:



Q.2 (a) Solution:

State diagram reduction:

- Let us assume that, the input variable of the circuit is X and the output variable of the circuit is Z.

- The state table for the given Mealy type state diagram can be given as follows:

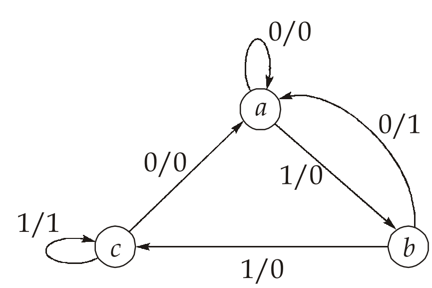
Present State	Next State		Output	
	X = 0	X = 1	X = 0	X = 1
a	*a*	*b*	0	0
b	*a*	*c*	1	0
c	*d*	*c*	0	1
d	*a*	*b*	0	0

For states "*a*" and "*d*", next state and output are same for an applied input. Hence, state "*a*" and "*d*" are said to be equivalent and the state "*d*" can be replaced with state "*a*".

- After replacing state "*d*" with state "*a*", the resultant state table can be given as follows:

Present State	Next State		Output	
	X = 0	X = 1	X = 0	X = 1
a	*a*	*b*	0	0
b	*a*	*c*	1	0
c	*a*	*c*	0	1

- In the above state table, no two states have same next-state and output for an applied input. Hence, it is not possible to reduce the state table further.
- So, the reduced state diagram can be given as follows:



State assignment:

- There are three states in the above reduced state diagram. If n is the number of flip-flops required,

  So, $2^n \geq 3 \quad \Rightarrow \quad n_{min} = 2$
- The state assignment can be done easily, by assuming the outputs of the two D flip flops as Q_1 and Q_0, as follows:

State	Q_1	Q_0
a	0	0
b	0	1
c	1	0

Excitation Table:

Present State Q_1	Present State Q_0	Input X	Next State Q_1^+	Next State Q_0^+	Output Z	Excitations D_1	Excitations D_0
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	1
0	1	0	0	0	1	0	0
0	1	1	1	0	0	1	0
1	0	0	0	0	0	0	0
1	0	1	1	0	1	1	0
1	1	0	X	X	X	X	X
1	1	1	X	X	X	X	X

Minimization

$Q_1 \backslash Q_0X$	$\overline{Q}_0\overline{X}$	$\overline{Q}_0X$	Q_0X	$Q_0\overline{X}$
$\overline{Q}_1$	(0)	(1)	1 (3)	(2)
Q_1	(4)	1 (5)	X (7)	X (6)

$$D_1 = Q_1X + Q_0X$$

K-map for D_1

$Q_1 \backslash Q_0X$	$\overline{Q}_0\overline{X}$	$\overline{Q}_0X$	Q_0X	$Q_0\overline{X}$
$\overline{Q}_1$	(0)	1 (1)	(3)	(2)
Q_1	(4)	(5)	X (7)	X (6)

$$D_0 = \overline{Q}_1\overline{Q}_0X$$

K-map for D_0

K-map for Z

$Q_1 \backslash Q_0X$	$\overline{Q}_0\overline{X}$	$\overline{Q}_0X$	Q_0X	$Q_0\overline{X}$
$\overline{Q}_1$	(0)	(1)	(3)	1 (2)
Q_1	(4)	1 (5)	X (7)	X (6)

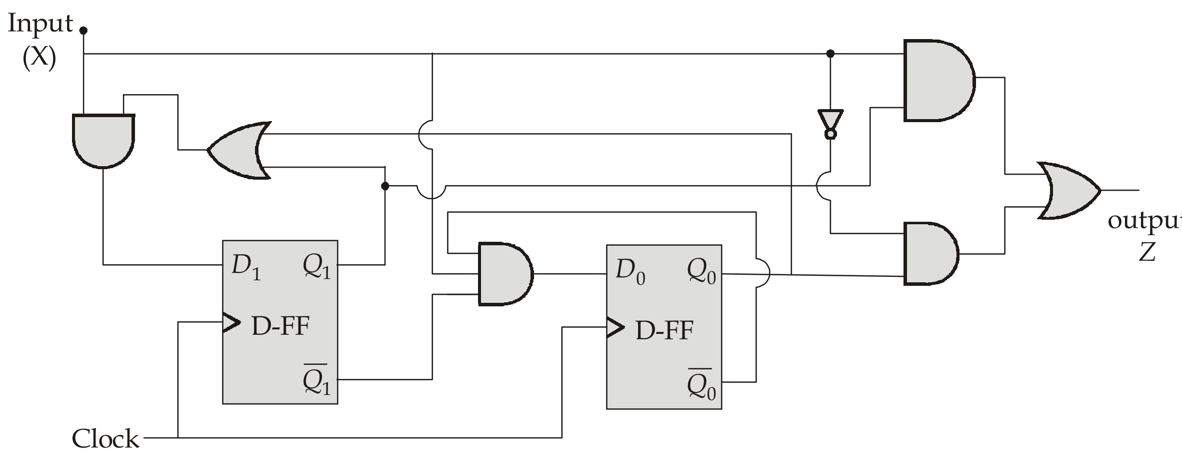
$$Z = Q_1X + Q_0\overline{X}$$

Thus, $D_1 = Q_1X + Q_0X = X(Q_1 + Q_0)$

$$D_0 = \overline{Q}_1\overline{Q}_0X$$

$$Z = Q_1X + Q_0\overline{X}$$

Logic Circuit:



Q.2 (b) Solution:

(i) Case-I: When the two resistances are connected in series, the equivalent resistance is:

$$R_{eq} = R_1 + R_2 = 100 + 50 = 150\ \Omega$$

$$\frac{\partial R}{\partial R_1} = 1 \text{ and } \frac{\partial R}{\partial R_2} = 1$$

Hence, uncertainty in the equivalent resistance is

$$W_R = \pm\sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 W_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 W_{R_2}^2}$$

$$= \pm\sqrt{(1)^2\,(0.1)^2 + (1)^2\,(0.03)^2} = \pm\,0.1044\ \Omega$$

The equivalent resistance for series combination can be expressed as

$$R_{eq} = 150 \pm 0.1044\ \Omega$$

Case-II: When two resistances are connected in parallel, the equivalent resistance is

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{100 \times 50}{100 + 50} = 33.33\ \Omega$$

$$\frac{\partial R}{\partial R_1} = \frac{R_2(R_1 + R_2) - R_1 R_2(1)}{(R_1 + R_2)^2} = \frac{R_2^2}{(R_1 + R_2)^2} = 0.111\ \Omega$$

$$\frac{\partial R}{\partial R_2} = \frac{R_1(R_1 + R_2) - R_1R_2(1)}{(R_1 + R_2)^2}$$

$$= \frac{R_1^2}{(R_1+R_2)^2} = \frac{100^2}{(100+50)^2} = 0.444\ \Omega$$

Hence, uncertainty in the equivalent resistance,

$$W_R = \pm\sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 W_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 W_{R_2}^2}$$

$$= \sqrt{(0.111)^2\,(0.1)^2 + (0.444)^2\,(0.03)^2} = \pm\,0.01734\ \Omega$$

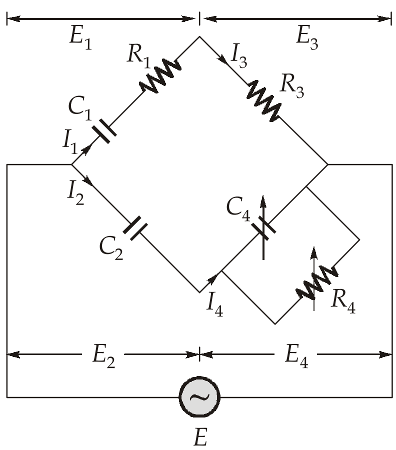
Equivalent resistance for parallel combination can be written as

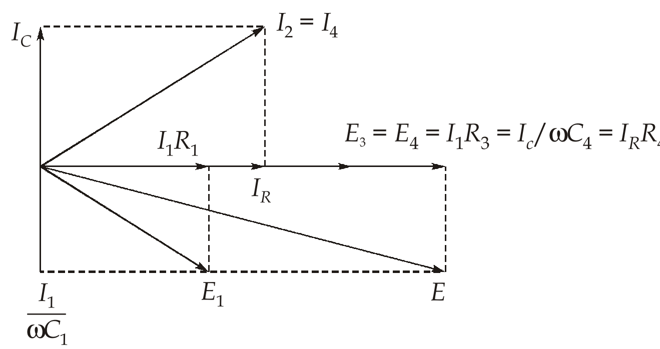
$$= (33.33 \pm 0.01734)\ \Omega$$

(ii) The Schering's bridge measure the value of unknown capacitance in terms of standard capacitance.

The circuit diagram of Schering bridge is shown below:

Given: $C_4 = 0.35\ \mu F$; $R_4 = 318\ \Omega$; $C_2 = 106$ pF ; $R_3 = 130\ \Omega$; $E = 25$ kV, 60 Hz





At balance,

$$\left(R_1 + \frac{1}{j\omega C_1}\right)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_3 \times \frac{1}{j\omega C_2}$$

$$\frac{(1 + j\omega C_1 R_1) R_4}{j\omega C_1\,(1 + j\omega C_4 R_4)} = \frac{R_3}{j\omega C_2}$$

$$j\omega C_2(R_4 + j\omega C_1 R_1 R_4) = j\omega C_1 R_3\,(1 + j\omega C_4 R_4)$$

$$(-\omega^2 C_1 C_2 R_1 R_4 + j\omega R_4 C_2) = j\omega C_1 R_3 - \omega^2 C_1 C_4 R_4 R_3$$

Equating the real and imaginary terms, we get,

$$R_1 = \frac{R_3 C_4}{C_2} \text{ and } C_1 = \frac{R_4 C_2}{R_3}$$

According to given values,

$$R_1 = \frac{130 \times 0.35 \times 10^{-6}}{106 \times 10^{-12}} = 429.245 \text{ k}\Omega$$

$$C_1 = \frac{318 \times 106 \times 10^{-12}}{130} = 2.592 \times 10^{-10} = 259.29 \text{ pF}$$

$$C_1 = 259.29 \text{ pF}$$

Power factor of series RC circuit,

$$\cos\theta_1 = \frac{R_1}{|Z_1|}$$

where $$|Z_1| = \sqrt{R_1^2 + \left(\frac{1}{\omega C_1}\right)^2}$$

$$\left(\frac{1}{\omega C_1}\right)^2 >> R_1^2$$

$\therefore$ $$|Z_1| = \frac{1}{\omega C_1}$$

$$\text{Power factor} = \frac{R_1}{Z_1} = R_1 \omega C_1$$

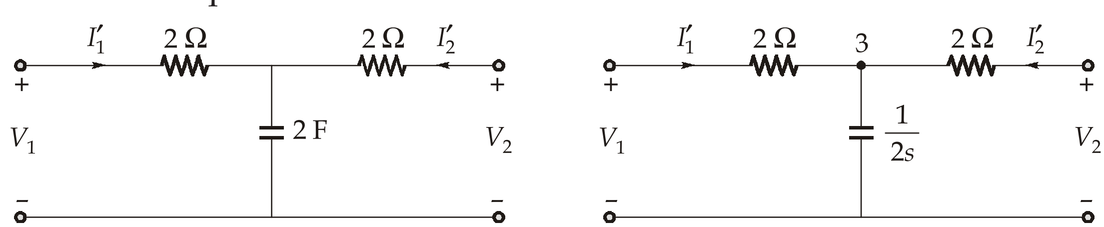
$$\text{P.F.} = \omega R_1 C_1 = 2\pi \times 60 \times 429.245 \times 10^3 \times 259.29 \times 10^{-12}$$

$$\text{P.F.} = 0.0419 \cong 0.042$$

Q.2 (c) Solution:

The above network can be considered as a parallel connection of two networks, N_1 and N_2.

For the network N_1



(a) (b)

Applying KCL at Node 3,

$$I_1' + I_2' = 2s(V_3) \quad \text{...(i)}$$

From above figure,

$$I_1' = \frac{V_1 - V_3}{2} = \frac{1}{2}V_1 - \frac{1}{2}V_3 \quad \text{...(ii)}$$

$$I_2' = \frac{V_2 - V_3}{2} = \frac{1}{2}V_2 - \frac{1}{2}V_3 \quad ...(iii)$$

Substituting the Eq. (ii) and Eq. (iii), in the Eq. (i)

$$\frac{V_1}{2} - \frac{V_3}{2} + \frac{V_2}{2} - \frac{V_3}{2} = (2s)V_3$$

$$(2s+1)V_3 = \frac{V_1}{2} + \frac{V_2}{2}$$

$$V_3 = \frac{1}{2(2s+1)}V_1 + \frac{1}{2(2s+1)}V_2 \quad ...(iv)$$

Substituting the Eq. (iv) in the Eq. (ii),

$$I_1' = \frac{V_1}{2} - \frac{1}{2}\left[\frac{1}{2(2s+1)}V_1 + \frac{1}{2(2s+1)}V_2\right]$$

$$= \left(\frac{4s+1}{8s+4}\right)V_1 - \left(\frac{1}{8s+4}\right)V_2 \quad ...(v)$$

Substituting the Eq. (iv) in the Eq. (iii),

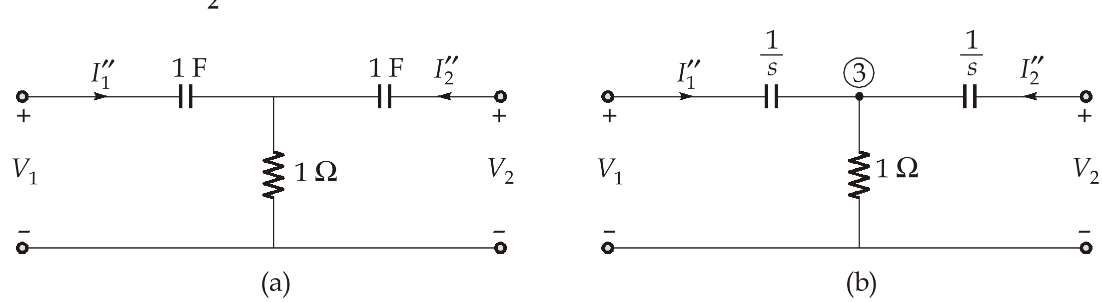
$$I_2' = \frac{V_2}{2} - \frac{1}{2}\left[\frac{1}{2(2s+1)}V_1 + \frac{1}{2(2s+1)}V_2\right]$$

$$= -\left(\frac{1}{8s+4}\right)V_1 + \left(\frac{4s+1}{8s+4}\right)V_2 \quad ...(vi)$$

Comparing Eqs (v) and (vi) with Y-parameter equations, we get

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} \frac{4s+1}{8s+4} & \frac{-1}{8s+4} \\ \frac{-1}{8s+4} & \frac{4s+1}{8s+4} \end{bmatrix}$$

For the network N_2



Applying KCL at Node 3,

$$I_1'' + I_2'' = V_3 \quad ...(i)$$

From above figure,

$$I_1'' = \frac{V_1 - V_3}{\frac{1}{s}} = sV_1 - sV_3 \quad \text{...(ii)}$$

$$I_2'' = \frac{V_2 - V_3}{\frac{1}{s}} = sV_2 - sV_3 \quad \text{...(iii)}$$

Substituting the eqs. (ii) and (iii) in the equation (i),

$$sV_1 - sV_3 + sV_2 - sV_3 = V_3$$

$$(2s+1)V_3 = sV_1 + sV_2$$

$$V_3 = \left(\frac{s}{2s+1}\right)V_1 + \left(\frac{s}{2s+1}\right)V_2 \quad \text{...(iv)}$$

Substituting the Eq. (iv) in the Eq. (ii),

$$I_1'' = sV_1 - s\left[\left(\frac{s}{2s+1}\right)V_1 + \frac{s}{(2s+1)}V_2\right]$$

$$= \left[\frac{s(s+1)}{2s+1}\right]V_1 - \left(\frac{s^2}{2s+1}\right)V_2 \quad \text{...(v)}$$

Substituting the Eq. (iv) in the Eq. (iii),

$$I_2'' = sV_2 - s\left[\left(\frac{s}{2s+1}\right)V_1 + \left(\frac{s}{2s+1}\right)V_2\right]$$

$$= -\left(\frac{s^2}{2s+1}\right)V_1 + \left[\frac{s(s+1)}{2s+1}\right]V_2 \quad \text{...(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations, we get

$$\begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} = \begin{bmatrix} \frac{s(s+1)}{2s+1} & -\left(\frac{s^2}{2s+1}\right) \\ -\left(\frac{s^2}{2s+1}\right) & \frac{s(s+1)}{2s+1} \end{bmatrix}$$

Therefore, the overall Y-parameters of the network are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{4s^2+8s+1}{4(2s+1)} & \dfrac{-(4s^2+1)}{4(2s+1)} \\ \dfrac{-(4s^2+1)}{4(2s+1)} & \dfrac{4s^2+8s+1}{4(2s+1)} \end{bmatrix}$$

Q.3 (a) Solution:

(i) 1. Nodal equation at node 'X'.

$$\frac{V_s-0}{R_1/\beta} = \frac{0-V_1}{R_1}+\frac{0-V_0}{R_1/\alpha}$$

$$\beta V_s = -V_1 - V_0\alpha$$

2. V_{2+} = Voltage at non-inverting terminal of op-amp A_2

$$V_{2+} = \left[V_1\left(R_2+\frac{1}{sC}\right)\right]\Big/\left[R_2+\frac{1}{sC}+\left(R_2 \,\|\, \frac{1}{sC}\right)\right] \quad ...(1)$$

Now applying KCL at inverting terminal of op-amp A_2.

$$\frac{V_1-V_{2+}}{R_3} = \frac{V_{2+}-V_0}{2R_3}$$

$(\because V_{2+}=V_{2-})$ due to virtual short circuit.

$$2V_1 = 3V_{2+} - V_0 \quad ...(2)$$

Substitute the value of V_{2+} from eqn. (1) into eqn. (2).

$$2V_1 = \frac{3V_1\left[R_2+\dfrac{1}{sC}\right]}{R_2+\dfrac{1}{sC}+\dfrac{R_2}{sC\left(R_2+\dfrac{1}{sC}\right)}}-V_0$$

$$2V_1 = \frac{3V_1(1+R_2sC)}{1+R_2sC+\dfrac{R_2}{\left(R_2+\dfrac{1}{sC}\right)}}-V_0$$

$$2V_1 = \frac{3V_1(1+R_2sC)^2}{(1+R_2sC)^2+R_2sC}-V_0$$

$$V_0 = V_1\left[\frac{3(1+R_2sC)^2}{(1+R_2sC)^2+R_2sC}-2\right]$$

$$= V_1\left[\frac{3(1+R_2sC)^2-2(1+R_2sC)^2-2R_2sC}{\left[(1+R_2sC)^2+R_2sC\right]}\right]$$

$$= V_1\left[\frac{(1+R_2sC)^2 - 2R_2sC}{1+(R_2sC)^2+3R_2sC}\right]$$

$$V_0 = V_1\left[\frac{1+(R_2sC)^2}{1+(R_2sC)^2+3R_2sC}\right]$$

3. $$\beta V_s = -V_1 - V_0\alpha \quad \text{[from part (i)]}$$

Substitute value of V_1 from part (ii)

$$V_1 = V_0\left[\frac{1+(R_2sC)^2+3R_2sC}{1+(R_2sC)^2}\right]$$

$\therefore$ $$\beta V_s = -V_0\left[\frac{1+(R_2sC)^2+3R_2sC}{1+(R_2sC)^2}\right] - V_0\alpha$$

$$\beta V_s = -V_0\left[\frac{1+(R_2sC)^2+3R_2sC+\alpha+(R_2sC)^2\alpha}{1+(R_2sC)^2}\right]$$

$$\beta V_s = -V_0\left[\frac{\left[1+(R_2sC)^2\right][1+\alpha]+3R_2sC}{1+(R_2sC)^2}\right]$$

$$\frac{V_0}{V_s} = \frac{-\beta\left[1+(R_2sC)^2\right]}{\left[(1+\alpha)[1+(R_2sC)^2]+3R_2sC\right]}$$

Given, $$\omega_0 = \frac{1}{R_2C} \text{ and } p = \frac{s}{\omega_0}$$

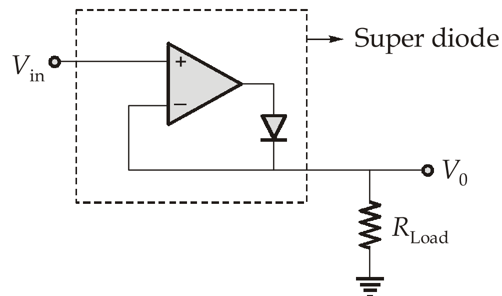
$$\frac{V_0}{V_s} = \frac{-\beta\left[1+\left(\frac{s}{\omega_0}\right)^2\right]}{\left[(1+\alpha)\left[1+\left(\frac{s}{\omega_0}\right)^2\right]+3\left(\frac{s}{\omega_0}\right)\right]}$$

again put $p = \dfrac{s}{\omega_0}$

$$\frac{V_0}{V_s} = \frac{-\beta\left[1+p^2\right]}{(1+\alpha)(1+p^2)+3p}$$

(ii) A regular diode has a voltage drop of near 0.7 V across it. To eliminate this diode drop voltage, we use an op-amp to create a "super-diode" which eliminate the 0.7 V (dead-band) and fairly simple to use. It is also called a precision rectifier.

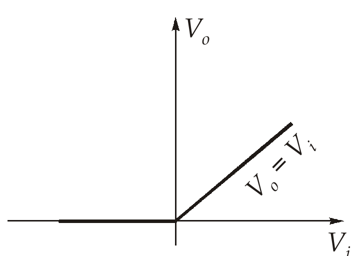
The circuit diagram of super diode is shown below:



When V_{in} is positive, due to the very high gain of the opamp, the output tries to go towards $+V_{sat}$ which forward biases the diode. Finally, the conducting diode closes the feedback loop such that $V_{out} = V_{in}$ (virtual short). In a regular diode, this relation would have been $V_{out} = V_{in} - 0.7$ V. When V_{in} is negative (less than inverting pin voltage), due to high gain, the output tries to go to $-V_{sat}$, which makes the diode reverse biased. So, the opamp output eventually saturates to $-V_{sat}$. V_{out} remains as high-impedance terminal like a regular diode circuit.

Super diode is used generally where the signal to be rectified is small (e.g. of the order of 100 mV or so) and thus, clearly insufficient to turn on a practical diode. A super diode behaves as ideal diode i.e. a very small voltage drop is sufficient to turn on the diode. Hence, super diode can be used in the rectification of a sine wave whose amplitude is very small.

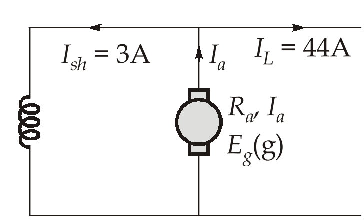
The transfer characteristics of super diode is shown below:



$$V_o = \begin{cases} V_i & \text{when } V_i > 0 \\ 0 & \text{when } V_i < 0 \end{cases}$$

Q.3 (b) Solution:

(i) As generator



$$I_a = I_{sh} + I_L$$

$$I_a = 3 + 44 = 47A$$

Armature voltage drop (E_a) = 5% of 460 V

$$= \frac{5}{100} \times 460 = 23 \text{ volt}$$

$$\text{Armature resistance, } R_a = \frac{E_a}{I_a} = \frac{23}{47} = 0.49\ \Omega$$

$$\text{Now, Generated emf, } E_g(g) = 460 \text{ V} + 23 \text{ V}$$

$$E_g(g) = 483 \text{ volts}$$

(ii)

$$\text{efficiency, } \eta = \frac{\text{Output}}{\text{Input}}$$

$$0.75 = \frac{460 \times 44}{\text{Input}}$$

$$\text{Input} \approx 27 \text{ kWatt}$$

$$\text{Rotational Losses, } P_{\text{rot}} = \text{Input} - E_g I_0$$

$$= 27000 - (483 \times 47)$$

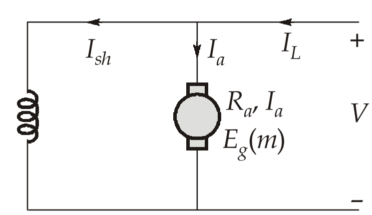
$$P_{\text{rot}} \approx 4.3 \text{ kW}$$

When working as Motor,

$$I_a = I_L - I_{sh}$$

$$I_a = 44 - 3$$

$$I_a = 41 \text{ A}$$



Here,

$$E_g(m) = V - I_a R_a$$

$$= 460 - 41 \times 0.49$$

$$= 439.91 \text{ volt}$$

Since,

$$E_g = \frac{NP\phi Z}{60A} \text{ it implies } E_g \propto N$$

Thus,

$$\frac{E_g(m)}{E_g(g)} = \frac{N_m}{N_g}$$

$$N_m = \frac{439.91 \times 1200}{483} \text{rpm}$$

$$N_m = 1092.94 \text{ rpm}$$

It is given that $P_{\text{rot}} \propto N^2$

$$\therefore \quad \frac{P_{rot}(m)}{P_{rot}(g)} = \left(\frac{N_m^2}{N_g^2}\right)$$

Rotational Losses in motor,

$$P_{rot(m)} = (4.3\times10^3)\left(\frac{1092.94}{1200}\right)^2$$

$$P_{rot(m)} = 3566.96 = 3.566 \text{ kW}$$

$$\text{Output power, } P_{out(m)} = (439.91 \times 41) - 3.566 \times 10^3$$

$$P_{out(m)} = 14.469 \text{ kW}$$

$$\eta_{motor} = \frac{P_{out(m)}}{P_{in(m)}}\times100$$

$$= \frac{14.469\times1000}{44\times460}\times100$$

$$\eta_{motor} = 71.49\%$$

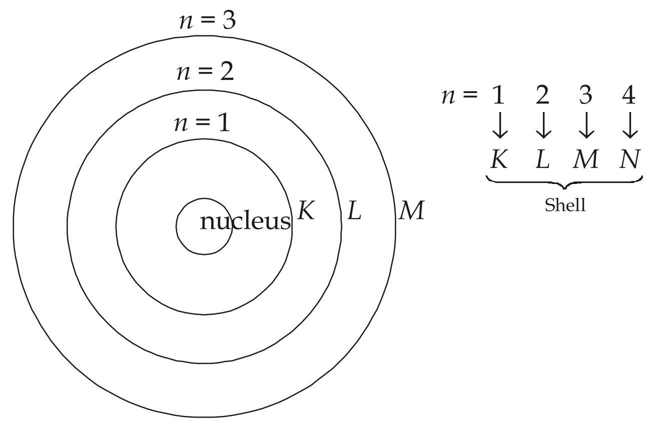
Q.3 (c) Solution:

Quantum numbers are the sets of numbers that describe an electron's orbit and movement within an atom. When the quantum numbers of all the electrons in a given atom are added together, they must satisfy the Schrodinger equation.

Quantum numbers are set of numbers used to describe the position and energy of an electron in an atom. There are four types of quantum numbers: Principal, azimuthal, magnetic and spin, which can be used to fully describe all of the properties of a given electron in an atom.

(a) Principal quantum number: (*n*)

The principal quantum number is one of four quantum numbers assigned to each electron in an atom to describe that electron's state. 'n' designates the principal electron shell. The value of the principal quantum number 'n' can be any integer with a positive value that is equal to or greater than one. The value $n = 1$ denotes the innermost electron shell of an atom, which corresponds to the lowest energy state of an electron. As 'n' increases, the electron is also at a higher energy and is, therefore less tightly bound to the nucleus.



(b) Azimuthal Quantum number (*l*):

- The azimuthal quantum number is also known as angular momentum quantum number and it is denoted by symbol '*l*'.
- The azimuthal quantum number '*l*' is a quantum number of an atomic orbital that determines its orbital angular momentum and describes the aspects of the angular shape of the orbital. The azimuthal quantum number is the second of a set of quantum numbers that describe the unique quantum state of an electron.
- For a given value of the principal quantum number '*n*' (electron shell), the possible values of '*l*' are the integers from 0 to '$n - 1$'.

  i.e., for $n = 1, l = 0 \Rightarrow$ *S*-shell

  $n = 2; l = 0$ to $1 \Rightarrow l = 0$ (*s* sub-shell), $l = 1$ (*p* sub-shell) and so on.

(c) Magnetic quantum number (m_l):

- Magnetic quantum number is used to determine the number of orbitals and the orientation of orbitals in a given subshell. It is denoted by 'm_l'. For each value of '*l*', quantum number m_l varies between +l to –l (including zero)

  i.e., $l = 0 \rightarrow m_l = 0;$ s: [s]

  $l = 1 \rightarrow m_l = -1$, to $1 \Rightarrow -1, 0, 1;$ p: [p_y | p_z | p_x]

  $l = 2 \rightarrow m_l = -2$ to $2 \Rightarrow -2, -1, 0, 1, 2;$ d: [d_{x2-y2} | d_{yz} | d_{z2} | d_{xz} | d_{xy}]

(d) Magnetic spin quantum number (m_s):

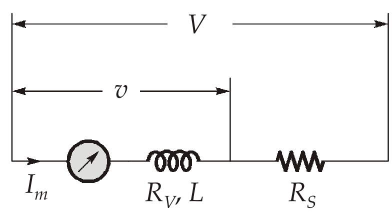
- The spin quantum number is a quantum number that describes the intrinsic angular momentum of an electron or other particle. The component of the spin along a specified axis is given by the spin magnetic quantum number (m_s).

Electron can spin either in clockwise $\left(m_s = \frac{1}{2}\right)$ or anti-clockwise $\left(m_s = \frac{-1}{2}\right)$.

Spin quantum number helps to describe the magnetic properties of material.

Q.4 (a) Solution:

(i)



To extend the range of moving iron voltmeter, a series resistance of high value is added in the voltmeter circuit.

Let the resistance of voltmeter be R_V and inductance be L.

Let, the external series resistance to increase the range be R_s. Thus,

Voltmeter current,
$$I = \frac{V}{\sqrt{(R_S + R_V)^2 + \omega^2 L^2}}$$
$$I = \frac{320}{\sqrt{(2200 + R_S)^2 + (2\pi \times 50 \times 0.6)^2}}$$

Voltage drop across instrument,
$$v = I\sqrt{R_V^2 + (\omega L)^2}$$
$$v = \frac{V}{\sqrt{(R_S + R_V)^2 + \omega^2 L^2}}\sqrt{R_V^2 + (\omega L)^2}$$

The maximum voltage drop across the instrument can be 150 V. Hence,
$$\frac{V}{v} = \frac{\sqrt{(R_S + R_V)^2 + (\omega L)^2}}{\sqrt{R_V^2 + (\omega L)^2}}$$
$$\frac{320}{150} = \frac{\sqrt{(R_S + 2200)^2 + (2\pi \times 50 \times 0.6)^2}}{\sqrt{(2200)^2 + (2\pi \times 50 \times 0.6)^2}}$$
$$(R_s + 2200)^2 = 22.189 \times 10^6 - (2\pi \times 50 \times 0.6)^2$$
$$(R_s + 2200)^2 = 22.153 \times 10^6$$
$$R_s = \sqrt{22.153 \times 10^6} - 2200 = 2506.755\ \Omega$$
$$R_s = 2.506\ \text{k}\Omega$$

The multiplier resistance is shunted by a capacitor in order to compensate for the frequency errors introduced due to inductance of the operating coil given by

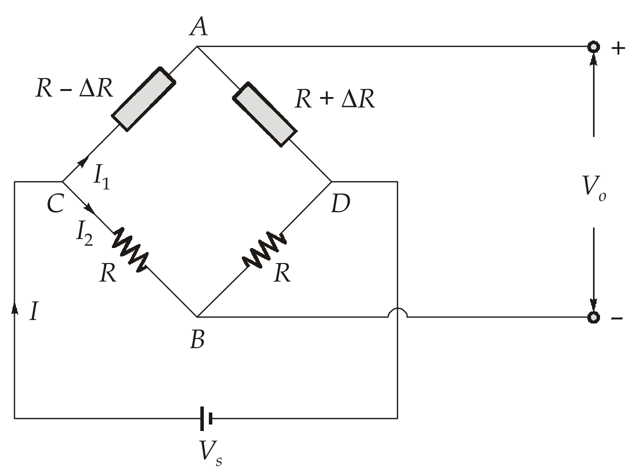
$$C = 0.41\frac{L}{R_S^2} = 0.41 \times \frac{0.6}{(2.506 \times 10^3)^2} = 39.171 \times 10^{-9}\ \text{F}$$

$$C = 39.171\ \text{nF}$$

(ii) Strain gauge 1 is in compression mode, hence resistance of strain gauge 1 after application of strain = $R - \Delta R$.

Similarly, resistance of strain gauge 2 after application of strain = $R + \Delta R$.

The circuit can be redrawn as below:



$$V_o = V_A - V_B \quad \text{...(i)}$$

$$V_o = I_1(R + \Delta R) - I_2(R) \quad \text{...(ii)}$$

Total resistance in arm CAD = Total resistance in arm CBD = $2R$

$$I_1 = I_2 = \frac{I}{2}$$

Putting these values in equation (ii), we get,

$$V_o = \frac{I}{2}(R + \Delta R) - \frac{I}{2}(R)$$

$$V_o = \frac{I}{2}\Delta R \quad \text{...(iii)}$$

1. For strain gauge, change in resistance ΔR due to application of strain ($\in$) is given by

$$\frac{\frac{\Delta R}{R}}{\text{Strain}\ (\in)} = G_f$$

$$\Delta R = G_f \times R \times \text{Strain}\ (\in)$$

$$= 3 \times 130 \times 120 \times 10^{-6} = 0.0468\ \Omega$$

From equation (iii),

$$V_o = \frac{I}{2}\Delta R = \frac{100\times10^{-3}}{2}\times 0.0468 = 2.34 \text{ mV}$$

2. **Sensitivity:** It is defined as output voltage for unit microstrain,

$$V_o = \frac{I}{2}\Delta R = \frac{I}{2}\times R\times G_f \times \in$$

Here, $\in = 1 \times 10^{-6}$

$$\text{Sensitivity} = \frac{100\times10^{-3}}{2}\times 130\times 3\times 10^{-6} = 0.0195 \text{ mV/}\mu\text{strain}$$

$$\text{Sensitivity} = 0.0195 \text{ mV/}\mu\text{strain}$$

3. **Resolution:** It is defined as the minimum input quantity that can be measured accurately.

Output of galvanometer per scale division = 1 mV.

Since $\frac{1}{10^{\text{th}}}$ of division can be read with accuracy,

The minimum output voltage that can read by the galvanometer

$$= 1 \text{ mV}\times\frac{1}{10} = 0.1 \text{ mV}$$

We have, $V_o = \frac{I}{2}(\Delta R) = \frac{I}{2}(R\times G_f \times \in)$

$$\in = \frac{V_o\times 2}{I\times R\times G_f} = \frac{0.1\times10^{-3}\times 2}{100\times10^{-3}\times3\times130} = 5.128\times10^{-6} \text{ Strain}$$

$$\in = 5.128 \text{ Microstrain} = \text{Resolution}$$

Q.4 (b) Solution:

(i) Given, $n_{n0} = 10^{16} \text{ cm}^{-3}$

$G_L = 10^{13}$ electron-hole pairs/cm^3/μs

1. $$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_n}{n_i}\right)$$

$$n_n = n_{n0} + \tau_e G_L$$

$$= 10^{16} + 2\times10^{-9}\times\frac{10^{13}}{10^{-6}} \simeq 10^{16} \text{ cm}^{-3}$$

$$n_n = 10^{16} \text{ cm}^{-3}$$

$\therefore$ $$E_{Fn} - E_{Fi} = 0.0259\ln\left(\frac{10^{16}}{2.25\times10^{6}}\right)$$

$$E_{Fn} - E_{Fi} = 0.575 \text{ eV}$$

2. $$E_i - E_{Fp} = kT \ln\left(\frac{P_n}{n_i}\right)$$

where $$P_n = P_{n0} + \tau_h G_L$$

We have, $$P_{n0} = \frac{n_i^2}{n_{n0}} = \frac{(2.25\times10^6)^2}{10^{16}} = 5.06\times10^{-4}\,\text{cm}^{-3}$$

Hence, $$P_n = 5.06\times10^{-4} + 2\times10^{-9}\left(\frac{10^{13}}{10^{-6}}\right)$$

$$P_n = 2\times10^{10}\ \text{cm}^{-3}$$

∴ $$E_i - E_{Fp} = 0.0259\ln\left(\frac{2\times10^{10}}{2.25\times10^6}\right)$$

$$E_i - E_{Fp} = 0.235 \text{ eV}$$

(ii) 1. The photon energy

$$E = hf = \frac{hc}{\lambda}$$

Therefore, $$\lambda = \frac{hc}{E}$$

$$\lambda = \frac{6.626\times10^{-34}\times3\times10^8}{1.5\times10^{-19}}$$

$$\lambda = 1.325\times10^{-6} = 1.32\ \mu\text{m}$$

The photodiode is operating at a wavelength of 1.32 μm.

2. Responsivity, $$R = \frac{\eta e}{hf}$$

$$R = \frac{0.65\times1.6\times10^{-19}}{1.5\times10^{-19}} = 0.693\ \text{A/W}$$

Also, $$R = \frac{I_P}{P_0} = \frac{\text{Photodiode output current}}{\text{Incident optical power}}$$

∴ $$P_0 = \frac{2.5\times10^{-6}}{0.693}$$

$$P_0 = 3.60\ \mu\text{W}$$

The incident optical power required is 3.60 μW.

Q.4 (c) Solution:

Applying KVL to the circuit, we get

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{0} i(t)dt + \frac{1}{C}\int_{0}^{t} i(t)dt = v(t)$$

$$500i(t) + \frac{-q_0(0^+)}{0.5\times10^{-6}} + \frac{1}{0.5\times10^{-6}}\int_0^t i(t)dt = 100\sin(1000t + 30°)$$

$$500i(t) - \frac{25\times10^{-6}}{0.5\times10^{-6}} + 2\times10^6\int_0^t i(t)dt = 100\left[0.5\cos1000t + \frac{\sqrt{3}}{2}\sin1000t\right]$$

$$500i(t) - 50 + 2\times10^6\int_0^t i(t)dt = 50\left[\cos1000t + \sqrt{3}\sin1000t\right]$$

Taking Laplace transform on both sides, we get

$$500I(s) - \frac{50}{s} + \frac{2\times10^6 I(s)}{s} = 50\left[\frac{s}{s^2+(1000)^2} + \frac{\sqrt{3}\times1000}{s^2+(1000)^2}\right]$$

$$10I(s) - \frac{1}{s} + \frac{4\times10^4 I(s)}{s} = \frac{s}{s^2+(1000)^2} + \frac{\sqrt{3}\times1000}{s^2+(1000)^2}$$

Dividing both sides by 2, we obtain

$$\left[5 + \frac{2\times10^4}{s}\right]I(s) = \frac{0.5}{s} + \frac{s}{2(s^2+1000^2)} + \frac{\sqrt{3}\times1000}{2(s^2+1000^2)}$$

or

$$I(s) = \frac{0.5}{5s+2\times10^4} + \frac{0.5\left(s+\sqrt{3}\times1000\right)}{\left(s^2+1000^2\right)}\times\frac{s}{(5s+2\times10^4)}$$

$$= \frac{0.1}{s+4000} + \frac{0.1\left(s+\sqrt{3}\times1000\right)s}{(s+4000)(s^2+1000^2)}$$

Using partial fraction expansion, we get

$$\frac{0.1s\left(s+\sqrt{3}\times1000\right)}{(s+4000)(s^2+10^6)} = \frac{0.1s\left(s+\sqrt{3}\times1000\right)}{(s+4000)(s+j1000)(s-j1000)}$$

$$= \frac{A_1}{(s+4000)} + \frac{A_2}{(s+j1000)} + \frac{A_3}{(s-j1000)}$$

$$= \frac{0.05336}{s+4000} + \frac{0.0233 - j0.00667}{s+j1000} + \frac{0.0233 + j0.00667}{s-j1000}$$

Therefore, $I(s) = \dfrac{0.1}{s+4000} + \dfrac{0.05336}{s+4000} + \dfrac{0.0233 - j0.00667}{s+j1000} + \dfrac{0.0233 + j0.00667}{s-j1000}$

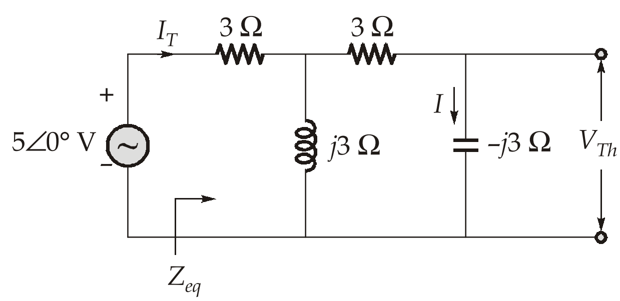
Taking inverse Laplace transform, we get

$$i(t) = 0.15336e^{-4000t} + (0.0233 - j0.00667)e^{-j1000t} + (0.0233 + j0.00667)e^{j1000t}$$
$$= 0.15336e^{-4000t} + 0.0233\,[e^{j1000t} + e^{-j1000t}] + j0.00667[e^{j1000t} - e^{-j1000t}]$$
$$= 0.15336e^{-4000t} + 0.0466 \cos 1000t - 0.01334 \sin 1000t \text{ A}$$

Section B

Q.5 (a) Solution:

Step I: *Calculation of V_{th}*



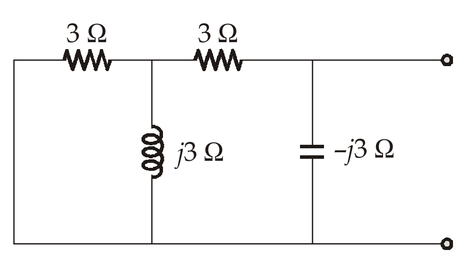
$$Z_{eq} = 3 + \frac{j3(3-j3)}{3+j3-j3} = 3 + \frac{9+j9}{3} = 3 + 3 + j3$$
$$= 6 + j3 = 6.71\angle 26.57° \ \Omega$$
$$I_T = \frac{5\angle 0°}{6.71\angle 26.57°} = 0.75\angle -26.57° \text{ A}$$

Using current division rule, $I = 0.75\angle -26.57° \times \dfrac{j3}{3+j3-j3} = 0.75\angle 63.43° \text{ A}$

$$V_{Th} = (-j3)(0.75\angle 63.43°)$$
$$= 2.25\angle -26.57° \text{ V}$$

Step II *Calculation of Z_{Th}*

The impedance seen from the open terminal after short-circuiting the voltage source is shown below:



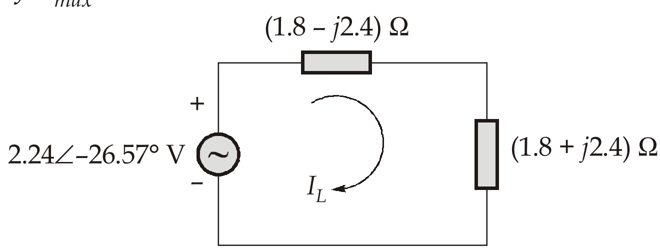
$$Z_{Th} = [(3 \,||\, j3) + 3] \,||\, (-j3)$$
$$= 3 \angle -53.12° \ \Omega = 1.8 - j2.4 \ \Omega$$

Step III *Calculation of* Z_L

For maximum power transfer, the load impedance should be a complex conjugate of the source impedance.

$\therefore$ $$Z_L = 1.8 + j2.4 \ \Omega$$

Step IV *Calculation of* P_{max}

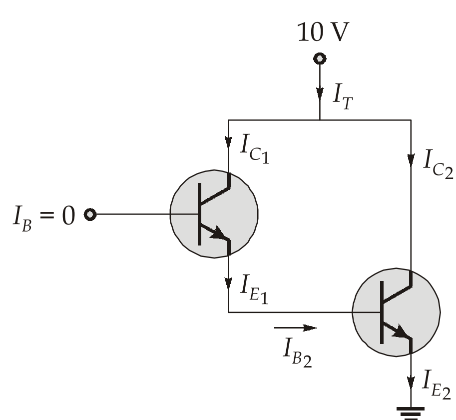


$$I_L = \frac{2.24\angle -26.57°}{1.8 - j2.4 + 1.8 + j2.4} = 0.621\angle -26.57° \text{ A}$$

$$P_{\max} = I_L^2 R_L$$
$$= (0.621)^2 \times 1.8 = 0.694 \text{ W}$$

Q.5 (b) Solution:

We have the circuit as



Now, from the above circuit diagram, we get,

$$I_{C_1} + I_{C_2} = I_T$$
$$I_{B_2} = I_{E_1}$$

Since, $$I_B = 0 \Rightarrow I_{C_1} = I_{E_1} = I_{B_2}$$

and $$I_{C_2} + I_{B_2} = I_{E_2}$$

$$I_{E_2} = I_{C_2} + I_{C_1}$$

$\therefore$ $$I_T = I_{E_2} \quad ...(i)$$

We know that, $$I_C = \alpha I_E + I_{CO}$$

$$I_{CO} = I_C - \alpha I_E$$

For the first transistor, we put $I_C = I_{C_1}$; $I_E = I_{E_1} = I_{C_1}$

$$I_{CO} = I_{C_1} - \alpha I_{C_1}$$

$$I_{C_1} = \left(\frac{I_{CO}}{1-\alpha}\right) \quad ...(ii)$$

Similarly for second transistor, we put $I_C = I_{C_2}$; $I_E = I_{E_2} = (I_{C_1} + I_{C_2})$

$$I_{C_2} = \alpha I_{E_2} + I_{CO}$$

$$I_{C_2} = \alpha(I_{C_1} + I_{C_2}) + I_{CO}$$

$$I_{C_2}(1-\alpha) = \alpha I_{C_1} + I_{CO}$$

$$I_{C_2} = \frac{\alpha I_{C_1}}{(1-\alpha)} + \frac{I_{CO}}{(1-\alpha)} \quad ...(iii)$$

Since, $$I_T = I_{C_1} + I_{C_2} \quad ...(iv)$$

Put equations (ii) and (iii) in (iv), we get,

$$I_T = \frac{I_{CO}}{(1-\alpha)} + \frac{\alpha I_{C_1}}{(1-\alpha)} + \frac{I_{CO}}{(1-\alpha)}$$

$$I_T = \frac{I_{CO}}{1-\alpha} + \frac{\alpha I_{CO}}{(1-\alpha)^2} + \frac{I_{CO}}{1-\alpha}$$

$$I_T = \frac{2I_{CO}}{1-\alpha} + \frac{\alpha I_{CO}}{(1-\alpha)^2} = \frac{I_{CO}}{1-\alpha}\left[2 + \frac{\alpha}{(1-\alpha)}\right] = \left[\frac{(2-\alpha)}{(1-\alpha)^2}\right] I_{CO}$$

We get the required relation as

$$I_T = \left[\frac{(2-\alpha)}{(1-\alpha)^2}\right] I_{CO}$$

Now, we have, $$\alpha = 0.98$$

$$I_{CO} = 5 \text{ mA}$$

$$I_T = \left[\frac{(2-0.98)}{(1-0.98)^2}\right]\times 5\times 10^{-3}\text{ A} = 12.75\text{ A}$$

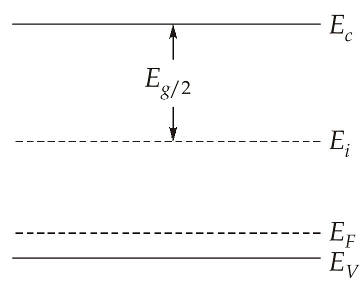
Q.5 (c) Solution:

Work function of metal, $\phi_m = 4.3$ eV

electron effinity, $\chi_e = 4$ eV

acceptor doping cocentration, $N_a = 10^{17}$ cm^{-3}

Energy band diagram of p-type Si,



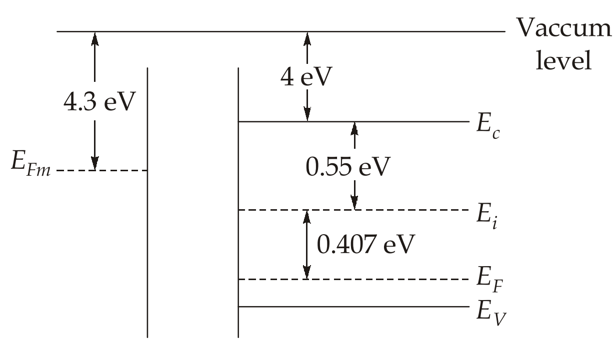
$$\frac{E_g}{2} = \frac{1.1}{2} = 0.55\text{ eV}$$

We have, $$E_i - E_F = kT\ln\frac{N_A}{n_i}$$

$$= 0.0259\ln\frac{10^{17}}{1.5\times 10^{10}}$$

∴ $$E_i - E_F = 0.407\text{ eV}$$

(i) The equilibrium band diagram of metal-semiconductor junction is as below:



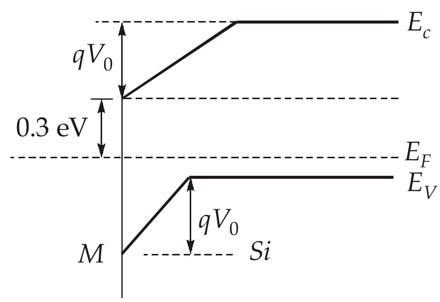
∴ Work function of semiconductor,

$$\phi_s = \chi_e + \frac{E_g}{2} + (E_i - E_F)$$

$$= 4 + 0.55 + 0.407$$

∴ $$\phi_s = 4.957\text{ eV}$$

The band diagram after bending is as shown below:



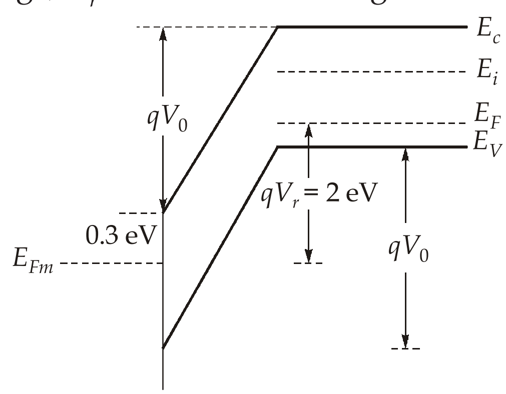
$\therefore$ Barrier height, $qV_0 = (E_c - E_F) - 0.3\text{ eV}$

$$= \left[\frac{E_g}{2} + (E_i - E_F)\right] - 0.3\text{ eV}$$

$$= 0.55\text{ eV} + 0.407\text{ eV} - 0.3\text{ eV}$$

$\therefore$ $qV_0 = 0.657\text{ eV}$

(ii) For reverse bias voltage, $V_r = 2$ V the band diagram is as given below:



$\therefore$ From the above energy band diagram,

$$0.3\text{ eV} + qV_0 = 2\text{ eV} + \frac{E_g}{2} + (E_i - E_F)$$

$$0.3 + qV_0 = 2 + 0.55 + 0.407$$

$$qV_0 = 2.957 - 0.3$$

$\therefore$ $qV_0 = 2.657\text{ eV}$

Q.5 (d) Solution:

We know that,

$$\text{Synchronous speed, } N_s = \frac{120f}{P}\text{ rpm}$$

We have two systems operating at 25 Hz and 60 Hz. For motor generator set to run

$$N_{sm} = N_{sg}$$

$$\frac{120 f_1}{P_1} = \frac{120 f_2}{P_2}$$

$$\frac{f_1}{P_1} = \frac{f_2}{P_2}$$

$$\frac{f_1}{f_2} = \frac{P_1}{P_2}$$

Since, $f_1 = 25$ Hz and $f_2 = 60$ Hz then

$$\frac{P_1}{P_2} = \frac{25}{60}$$

$$P_2 = \frac{60}{25} P_1$$

$$P_2 = 2.4 P_1$$

As, poles exists in pair. Hence, number of poles must be even integer.

So, the lowest three values of P_1 can be 10, 20 and 30.

Given corresponding synchronous speeds are

For $P_1 = 10$, $$N_{S_1} = \frac{120 \times f_1}{P_1} = \frac{120 \times 25}{10} = 300 \text{ rpm}$$

For $P_2 = 20$, $$N_{S_2} = \frac{120 \times f_1}{P_1} = \frac{120 \times 25}{20} = 150 \text{ rpm}$$

For $P_2 = 30$, $$N_{S_3} = \frac{120 \times 25}{30} = 100 \text{ rpm}$$

Q.5 (e) Solution:

(i) Inductive reactance of pressure coil,

$$X_L = 2\pi \times 120 \times 15 \times 10^{-3} = 11.309\ \Omega$$

Resistance of pressure coil circuit $R = 2300\ \Omega$

So phase angle of pressure coil,

$$\beta = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{11.309}{2300}\right) = 0.28^\circ$$

For lagging inductive load with power factor $\cos\phi$ and pressure coil inductance with phase angle β,

$$P_m = V_s \times I_L \times \cos\beta \times \cos(\phi - \beta)$$
$$28 = 260 \times 5.2 \times \cos(0.28°) \times \cos(\phi - 0.28°)$$
$$\phi = 89.090° \cong 89°$$

Without pressure coil inductance, the current in the pressure coil is in phase with the voltage across the coil, hence, True Power, $P = V_s \times I_L \times \cos\phi$.

Therefore,
$$\% \text{ error} = \frac{\cos\beta\cos(\phi-\beta)-\cos\phi}{\cos\phi}\times 100$$
$$= \frac{\cos(0.28°)\cos(89° - 0.28°) - \cos(89°)}{\cos(89°)}\times 100$$
$$= 27.994 \cong 28\%$$
$$\%\text{error} = 28\%$$

(ii) The deflection on the screen is given by $D = \dfrac{LI_d E_d}{2d E_a}$

where E_a = anode voltage = 3200 V

L = Distance between screen and the centre of the deflecting plates

= 38 cm = 0.38 m

I_d = length of deflecting plates = 2.6 cm

= 2.6×10^{-2} m

d = Distance between deflecting plates = 6.3 mm

= 6.3×10^{-3} m

and E_d = Potential between deflecting plates

To obtain a deflection of 2.8 cm on the screen i.e. for $D = 2.8 \times 10^{-2}$ m,

$$E_d = \frac{2d\, E_a D}{L\, I_d} = \frac{2\times 6.3\times 10^{-3}\times 3200\times 2.8\times 10^{-2}}{0.38\times 2.6\times 10^{-2}}$$
$$= 114.267 \text{ V}$$

∴ Input voltage required for a deflection of 2.8 cm

$$E_{\text{in}} = \frac{E_d}{\text{gain}} = \frac{114.267}{140} = 0.816 \text{ V}$$

Q.6 (a) Solution:

(i) From construction point of view, there are two types of rotor construction.

1. Salient pole type
2. Cylindrical rotor type (Non-salient)

Features of salient pole type:

- They are larger in diameter but smaller in axial length.
- The pole shoes are wide and usually cover (2/3) pole pitch.
- Poles of the rotor are laminated in order to reduce eddy current loss.
- This type of rotor construction are suitable for medium and low speed operation. (120 – 500 rpm).
- Due to low speed operation, they are used in hydroelectric plant.

Features of cylindrical rotor machines:

- They are smaller in diameter and larger in axial length.
- Usually number of poles on rotor is usually two.
- Windage loss is less.
- Mechanically more balanced and suitable for high speeds (1000 to 3000 rpm).
- Due to high speed, cylindrical rotor generator are also called as turbogenerators.

(ii) 1. Given, open circuit voltage, $V_{OC} = 450$ V

short circuit current, I_{SC}=200 A

Effective armature resistance,

$$R = 0.2\ \Omega$$

Impedance angle, $$\theta = \cos^{-1}\frac{R}{Z}$$

$$Z = \frac{V_{OC}}{I_{SC}} = \frac{450}{200} = 2.25\ \Omega$$

So, $$\theta = \cos^{-1}\left(\frac{0.2}{2.25}\right) = 84.9^\circ$$

$$Z = |Z| \angle \theta = 2.25 \angle 84.9^\circ\ \Omega$$

∴ $$R + jX_s = 0.2 + j2.241\ \Omega$$

Synchronous reactance,

$$X_s = 2.241\ \Omega$$

2. Full load current $= \dfrac{55\times10^3}{550} = 100$ A

At pf 0.8 lagging, $I_a = 100\angle{-\cos^{-1}(0.8)} = 100\angle{-36.87°}$

$$\vec{E_F} = \vec{V} + \vec{I_a}\vec{Z_s}$$

$$= 550 + (100\angle{-36.87°})(2.25\angle{84.9°})$$

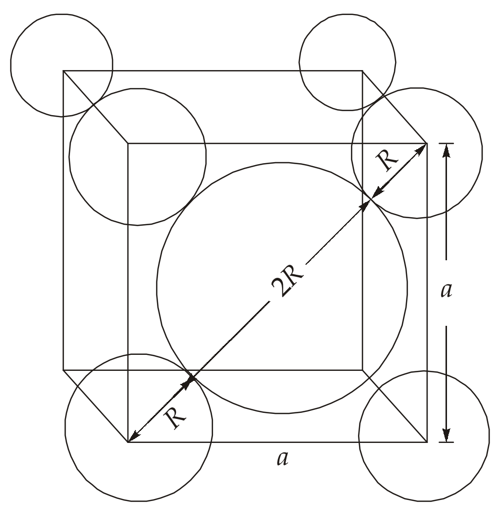
$$= 720.17\angle{13.43°}\text{ V}$$

$$\text{Voltage regulation} = \frac{E_f - V}{V} = \frac{720.17-550}{550} = 0.3090$$

$$= 30.94\%$$

Q.6 (b) Solution:

(i) As Cu crystal is arranged in FCC unit cell structure, we can draw the FCC unit cell structure with the Cu atom at each corner shared with eight other adjoining unit cells and one Cu atom at each face center shared with the neighbouring unit cell.

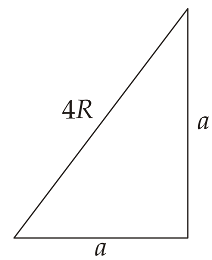


1. Number of atoms in the unit cell

$$= \left(8\times\frac{1}{8}\right) + \left(6\times\frac{1}{2}\right)$$

$$= 1 + 3 = 4 \text{ atoms}$$

2. From the unit cell shown in figure with lattice parameter 'a' and radius of Copper atom 'R', we have



From the Pythagoras theorem, we can say that

$$\sqrt{a^2 + a^2} = 4R$$

$$\sqrt{2}\,a = 4R$$

$$R = 2\sqrt{2}\,a$$

3. In general, if there are x atoms in the unit cell, the atomic concentration is

$$n_{at} = \frac{\text{Number of atoms in unit cell}}{\text{Volume of unit cell}} = \frac{x}{a^3}$$

Thus, for Cu crystal, $n_{at} = \dfrac{4}{a^3}$

We have, $\dfrac{4R}{\sqrt{2}} = a$

$$a^3 = \left(\frac{4R}{\sqrt{2}}\right)^3 = \left(\frac{4 \times 0.128}{\sqrt{2}}\right)^3 \qquad \text{[Given, } R = 0.128 \text{ nm]}$$

$$a^3 = 0.0474 \times 10^{-21}\ \text{cm}^3$$

Thus, $n_{at} = \dfrac{4}{0.0474 \times 10^{-21}}$

$$n_{at} = 8.43 \times 10^{22}\ \text{cm}^{-3}$$

Now,density of crystal, $\rho = \dfrac{\text{Mass of all atoms in unit cell}}{\text{Volume of unit cell}}$

$$\rho = \frac{n_{at} \cdot M_{at}}{N_A a^3}$$

$$\rho = \frac{8.43 \times 10^{22} \times 63.55}{6.022 \times 10^{23} \times 0.0474 \times 10^{-21}}$$

$$\rho = 1.876 \times 10^{23}\ \text{g cm}^{-3}$$

(ii) 1. Super paramagnetism: When the grain size of ferromagnetic and ferrimagnetic materials falls below a certain critical size, these materials behave as if they are paramagnetic. The magnetic dipole energy of each particle becomes comparable to the thermal energy. As a result of thermal energy, this small magnetic moment changes its direction randomly. Thus, the material behaves as if it has no net magnetic moment. This is known as super paramagnetism. Thus, if we produce iron oxide (Fe_3O_4) particles in a 3 to 5 nm size, they behaves as superparamagnetic materials. Such iron-oxide superparamagnetic particles are used to form dispersions in aqueous or organic carrier phases or to form "liquid magnets" or ferrofluids. The particle in the fluid move in response to a gradient in the magnetic field. Since the particles form a stable solution, the entire dispersion moves and hence, the material behaves as a liquid magnet. Such materials are used as seals in computer hard drives and in loudspeakers as heat transfer (cooling) media. The permanent magnet used in the loudspeaker holds the liquid magnets in place. Super paramagnetic particles of iron oxide (Fe_3O_4) also can be coated with different chemicals and used to separate DNA molecules, proteins and cells from other molecules.

2. Wiedemann Franz Lorentz law:

The Wiedemann Franz law gives relation between thermal conductivity and Electrical conductivity at a particular temperature. It states that ratio of thermal conductivity (K) to electrical conductivity (σ) is directly proportional to temperature.

$$\frac{K}{\sigma} \propto T$$

$$\Rightarrow \quad \frac{K}{\sigma} = LT$$

where, K = Thermal conductivity (W/m-K)

σ = Electrical conductivity

L = Lorentz constant = 2.45×10^{-8} WΩ/K^2

T = Temperature

Q.6 (c) Solution:

(i) A 16 bit word represents 2^{16} or 65536 levels. Since, these levels are equally spaced across the 2 V range, thus each step is given by

$$\text{step size} = \frac{2\text{ V}}{65536} = 30.52\ \mu\text{V}$$

Therefore, the system can resolve voltage changes as low as 30.52 μV.

The dynamic range of a system represents the ratio of the largest value obtainable to the smallest value. Therefore,

$$\text{Dynamic range} = \frac{2\text{ V}}{30.52\ \mu\text{V}} = 65536$$

$$(\text{DR})_{\text{dB}} = 20 \log 65536 \cong 96 \text{ dB}$$

(ii) For the binary input $b_1\, b_2\, b_3\, b_4 = 0111$,

$$I_1 = \frac{b_1 V_R}{2R} = \frac{10}{2\times 10\times 10^3} = 0.5 \text{ mA}$$

$$I_2 = \frac{b_2 V_2 / 2}{2R} = \frac{V_R}{4R} = \frac{I_1}{2}$$

$$\Rightarrow \quad I_2 = 0.25 \text{ mA}$$

$$I_3 = \frac{b_3 V_R / 4}{2R} = \frac{V_R}{8R} = \frac{I_1}{4}$$

$$\Rightarrow \quad I_3 = \frac{0.5}{4} = 0.125 \text{ mA}$$

Since $b_4 = 0$, hence $I_4 = 0$

Therefore, the current I_0 delivered to the op-amp is given by

$$I_0 = I_1 + I_2 + I_3 + I_4 = 0.5 + 0.25 + 0.125 = 0.875 \text{ mA}$$

The output voltage $V_0 = -I_0 R_f = -0.875 \times 10^{-3} \times 10 \times 10^3$

$$V_0 = -8.75 \text{ V}$$

(iii) **1.** The decimal equivalent value for binary input 10001010 is given by,

$$D = b_7 2^7 + b_6 2^6 + b_5 2^5 + \ldots + b_0 2^0$$

$$D = 1.2^7 + 0 + 0 + 0 + 1.2^3 + 0.2^2 + 1.2^1 + 0$$

$$D = 128 + 8 + 2 = 138$$

Therefore, $V_0 = 138 \times 10 \dfrac{\text{mV}}{\text{bit}} = 1.38 \text{ V}$

2. For binary output = 00010000

$$D = 0 + 0 + 0 + (1)2^4 + 0 + 0 + 0 + 0$$

$$D = 16$$

Therefore, $V_0 = 16 \times 10 \dfrac{\text{mV}}{\text{bit}} = 0.16 \text{ V}$

(iv) $$f(A, B, C) = \pi M(1, 2, 5)$$

The equivalent sum of product function can be written as,

$$f(A, B, C) = \Sigma m(0, 3, 4, 6, 7)$$

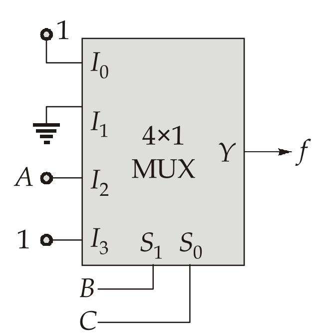
The truth table for the given Boolean function is given below:

A	B	C	$f(A, B, C)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

The given function can be implemented with a 4 to 1 multiplexer with 2 select lines. Variables B and C are chosen for selection lines. The implementation table as drawn with the help of truth table is given below:

BC	00	01	10	11
$\overline{A}$	(0)	1	2	(3)
A	(4)	5	(6)	(7)
	$I_0 = 1$	$I_1 = 0$	$I_2 = A$	$I_3 = 1$

Figure below shows hardware implementation of the given Boolean function using 4 × 1 MUX:



Q.7 (a) Solution:

(i) We have,

Open loop gain, $A_o = 4000 \pm 10$

The percentage change in closed loop gain, $\dfrac{\partial A_f}{A_f} \leq 0.05$ ..(i)

1. We know that,

$$A_f = \frac{A_o}{1 + A_o\beta}$$

Now, differentiate A_f with respect to A_o,

$$\frac{\partial A_f}{\partial A_o} = \frac{1}{(1 + A_o\beta)^2}$$

On rearranging, we get,

$$\%\frac{\partial A_f}{A_f} = \frac{\%\left(\dfrac{\partial A_o}{A_o}\right)}{(1 + A_o\beta)}$$

where, $\dfrac{\partial A_o}{A_o} = \dfrac{10}{4000} \times 100 = 0.25$

Using equation (i), we get,

$$\frac{\left(\dfrac{\partial A_o}{A_o}\right)}{(1 + A_o\beta)} \leq 0.05$$

$$\frac{0.25}{1 + 4000\beta} \leq 0.05$$

$$\frac{0.25}{0.05} \leq 1 + 4000\beta$$

$$0.001 \leq \beta$$

Thus, feedback factor, $\beta \geq 0.001$

2. We know that gain with feedback is given as $\dfrac{A_o}{1 + A_o\beta}$

$$A_f = \frac{A_o}{1 + A_o\beta}$$

$$A_f = \frac{4000}{1 + (4000 \times 0.001)} = 800$$

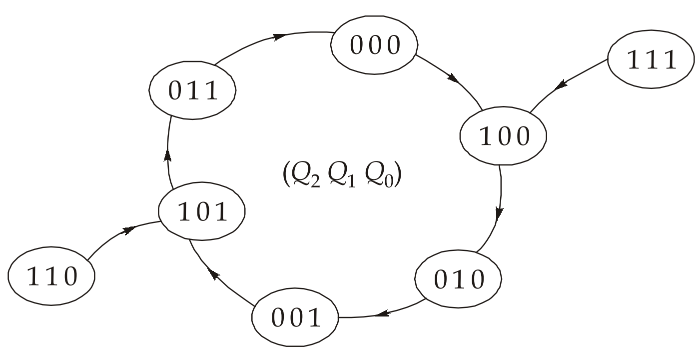
(ii) Excitation of flip flops are

$$T_2 = \overline{Q_1}$$
$$T_1 = Q_2 + Q_1$$
$$T_0 = Q_1$$

The sequence table of the given circuit can be given by

Present State			Excitation			Next State		
Q_2	Q_1	Q_0	$T_2 = \overline{Q_1}$	$T_1 = Q_2 + Q_1$	$T_0 = Q_1$	Q_2^+	Q_1^+	Q_0^+
0	0	0	1	0	0	1	0	0
0	0	1	1	0	0	1	0	1
0	1	0	0	1	1	0	0	1
0	1	1	0	1	1	0	0	0
1	0	0	1	1	0	0	1	0
1	0	1	1	1	0	0	1	1
1	1	0	0	1	1	1	0	1
1	1	1	0	1	1	1	0	0

The complete sequence diagram of the given counter circuit is,



- The modulus of the counter = 6
- Number of unused states = 2
- Since the counter comes back to the used state from the unused states, hence no lock out problem exists in the counter.

Q.7 (b) Solution:

If a voltage source is located between two non reference nodes, it is considered as a super node.

In the given circuit, V_1 and V_2 is forming a super node. Applying KCL for both nodes at a time,

$$\frac{V_1}{4}+\frac{V_2}{1}+\frac{V_1-V_3}{1} = 1 + 2V_0$$

where, $V_0 = V_1 - V_3$

$$\frac{V_1}{4}+\frac{V_2}{1}+\frac{V_1-V_3}{1} = 1 + 2V_1 - 2V_3$$

$$V_1 + 4V_2 + 4V_1 - 4V_3 = 4 + 8V_1 - 8V_3$$

$$-3V_1 + 4V_2 + 4V_3 = 4 \qquad ...(1)$$

Here, $V_2 - V_1 = 4I_0$, where $I_0 = \frac{V_3}{4}$

$$V_2 - V_1 = 4\left(\frac{V_3}{4}\right)$$

$$-V_1 + V_2 - V_3 = 0 \qquad ...(2)$$

Apply KCL at node (3)

$$2V_0 + \frac{V_3}{4} + \frac{V_3-10}{2} + \frac{V_3-V_1}{1} = 0$$

$$8V_0 + V_3 + 2V_3 - 20 + 4V_3 - 4V_1 = 0$$

where, $V_0 = V_1 - V_3$

$$8V_1 - 8V_3 + V_3 + 2V_3 + 4V_3 - 4V_1 = 20$$

$$4V_1 - V_3 = 20 \qquad ...(3)$$

Writing equations (1), (2) and (3) in matrix form,

$$\begin{bmatrix} -3 & 4 & 4 \\ -1 & 1 & -1 \\ 4 & 0 & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -3 & 4 & 4 \\ -1 & 1 & -1 \\ 4 & 0 & -1 \end{vmatrix} \Rightarrow \begin{aligned} \Delta &= -3(-1) - 4(1+4) + 4(-4) \\ \Delta &= 3 - 20 - 16 \\ \Delta &= -33 \end{aligned}$$

$$\Delta_1 = \begin{vmatrix} 4 & 4 & 4 \\ 0 & 1 & -1 \\ 20 & 0 & -1 \end{vmatrix} \Rightarrow \begin{aligned} \Delta_1 &= 4(-1) - 4(20) + 4(-20) \\ \Delta_1 &= -4 - 80 - 80 \\ \Delta_1 &= -164 \end{aligned}$$

$$\Delta_2 = \begin{vmatrix} -3 & 4 & 4 \\ -1 & 0 & -1 \\ 4 & 20 & -1 \end{vmatrix} \Rightarrow \begin{aligned} \Delta_2 &= -3(20) - 4(1+4) + 4(-20) \\ \Delta_2 &= -60 - 20 - 80 \\ \Delta_2 &= -160 \end{aligned}$$

$$\Delta_3 = \begin{vmatrix} -3 & 4 & 4 \\ -1 & 1 & 0 \\ 4 & 0 & 20 \end{vmatrix} \Rightarrow \begin{aligned} \Delta_3 &= -3(20) - 4(-20) + 4(-4) \\ \Delta_3 &= -60 + 80 - 16 \\ \Delta_3 &= 4 \end{aligned}$$

Using Cramer's rule,

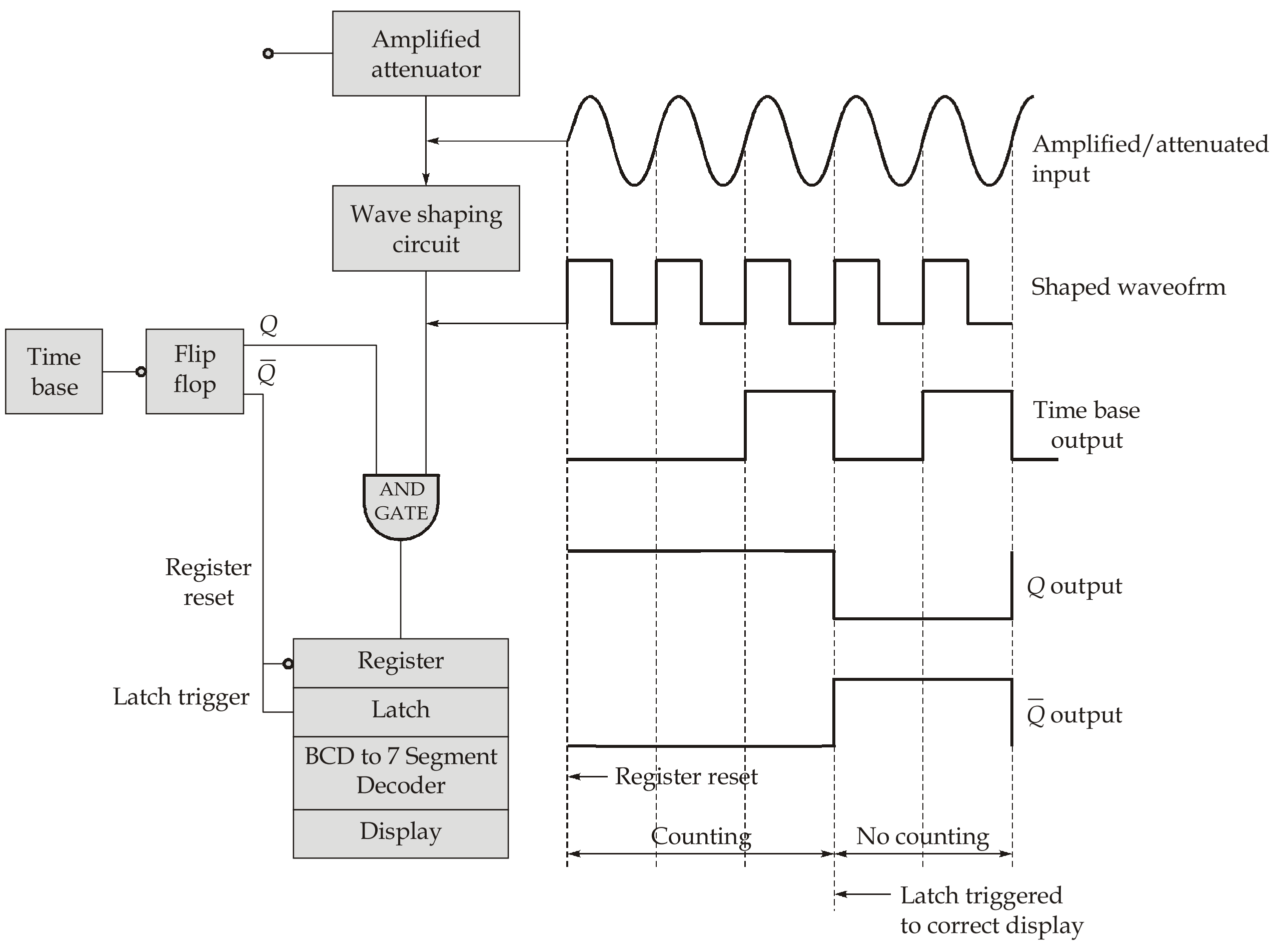
$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-164}{-33} = 4.97 \text{ V}$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-160}{-33} = 4.85 \text{ V}$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{4}{-33} = -0.12 \text{ V}$$

Q.7 (c) Solution:

(i) A basic digital frequency meter is shown below:



Digital frequency meter (DFM) consists of an accurate timing source (or time base), digital counting circuits to count the waveform cycles, circuitry for shaping the input waveform to square wave, and a circuit for gating the shaped waveform to the counter. The input is first amplified or attenuated as necessary and then fed to the wave-shaping circuit, which converts it into a square or pulse waveform with the same frequency as the input.

The presence of this wave-shaping circuit means that the input can be sinusoidal, square, triangular or can have any other repetitive-type waveform. The shaped waveform is fed to one input terminals of a 2-input AND gate, and the other AND gate input is controlled by the Q output from a flip-flop. Consequently, the pulses to be counted pass through the AND gate only when the flip-flop Q-terminal is high.

The flip-flop is controlled by the timing circuit, changing state each instant the timer output waveform goes in a negative direction (a negative going edge). When the timing circuit output frequency is 1 Hz, the flip-flop Q-output terminal is alternatively high for a period of 1 s and low for 1 s. In this case, the counting circuits are toggled for a period of 1 s, and the total count indicates the frequency directly in hertz. The counting circuits are reset to the zero count condition by the negative edge of the $\bar{Q}$ output from the flip-flop, so that the count always starts from zero.

Latch or display enable circuits are employed to make the digital display readable. The latch circuits are briefly triggered at the end of the counting time by the positive-going of the flip-flop $\bar{Q}$ output. The display is corrected at this instant and then remains constant until the next latch trigger input. Different time base frequencies could be used to give several range of frequency measurements.

(ii) The governing equation of the temperature-resistance characteristics of the thermistor is given by

$$R = R_o e\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

The given data is,

$$R_o = 1050\ \Omega$$
$$T_o = 273 + 27 = 300\text{ K}$$
$$\beta = 3140\text{ K}$$
$$R = 2330\ \Omega$$

Taking the logarithm on both sides of equation and rearranging, we get,

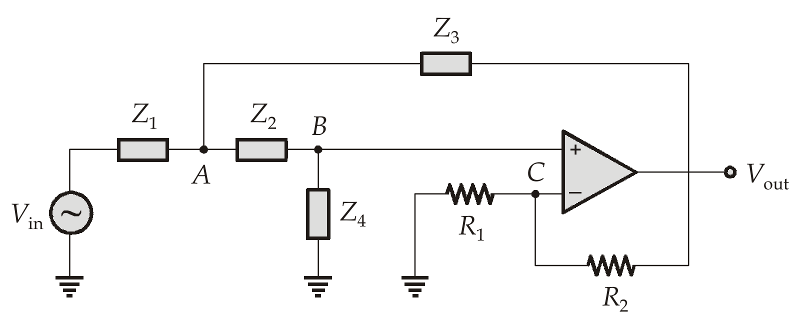
$$\frac{1}{T} = \frac{\ln R - \ln R_o}{\beta} + \frac{1}{T_o}$$

$$= \frac{7.754 - 6.957}{3140} + \frac{1}{300} = 3.587 \times 10^{-3}$$

$$T = 278.77 \text{ K} = 5.77°\text{C}$$

Q.8 (a) Solution:

(i) We have the circuit



Assuming the op-amp to be ideal, using the concept of virtual ground,

$$V_- = V_+ = V_B$$

On applying KCL at node C, we get,

$$\frac{V_C}{R_1} + \frac{V_C - V_{out}}{R_2} = 0$$

$\because$

$$V_C = V_- = V_B$$

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_B$$

$\Rightarrow$

$$V_B = \frac{V_{out}}{\left(1 + \frac{R_2}{R_1}\right)} \quad \text{...(i)}$$

Applying KCL at node A,

$$\frac{V_A - V_{in}}{Z_1} + \frac{V_A - V_{out}}{Z_3} + \frac{V_A - V_B}{Z_2} = 0$$

$$V_A\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) - \frac{V_B}{Z_2} - \frac{V_{out}}{Z_3} - \frac{V_{in}}{Z_1} = 0 \quad \text{...(ii)}$$

Similarly applying KCL at node B,

$$\frac{V_B - V_A}{Z_2} + \frac{V_B}{Z_4} = 0$$

$$V_B\left[\frac{1}{Z_2}+\frac{1}{Z_4}\right] = \frac{V_A}{Z_2}$$

$$\Rightarrow \qquad V_A = V_B\left[1+\frac{Z_2}{Z_4}\right] \qquad ...(iii)$$

Using equations (i) and (iii) in equation (ii), we get,

$$\frac{V_{out}}{\left(1+\frac{R_2}{R_1}\right)}\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{V_{out}}{Z_2\left(1+\frac{R_2}{R_1}\right)}-\frac{V_{out}}{Z_3} = \frac{V_{in}}{Z_1}$$

$$= V_{out}\left[\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{1}{Z_2}-\frac{\left(1+\frac{R_2}{R_1}\right)}{Z_3}\right] = \frac{V_{in}}{Z_1}\left[1+\frac{R_2}{R_1}\right]$$

$$\frac{V_{out}}{V_{in}} = \frac{\left(1+\frac{R_2}{R_1}\right)}{\frac{Z_1}{Z_4}-\left(\frac{R_2}{R_1}\right)\frac{Z_1}{Z_3}+\frac{Z_1 Z_2}{Z_3 Z_4}+\frac{Z_2}{Z_4}+1}$$

(ii) For $Z_1 = Z_2 = Z_3 = Z_4 = 10$ kΩ and $R_1 = 5$ kΩ; $R_2 = 10$ kΩ, we get,

$$\frac{V_{out}}{V_{in}} = \frac{\left(1+\frac{10}{5}\right)}{\frac{10}{10}-\left(\frac{10}{5}\right)\left(\frac{10}{10}\right)+\left(\frac{10\times 10}{10\times 10}\right)+\frac{10}{10}+1}$$

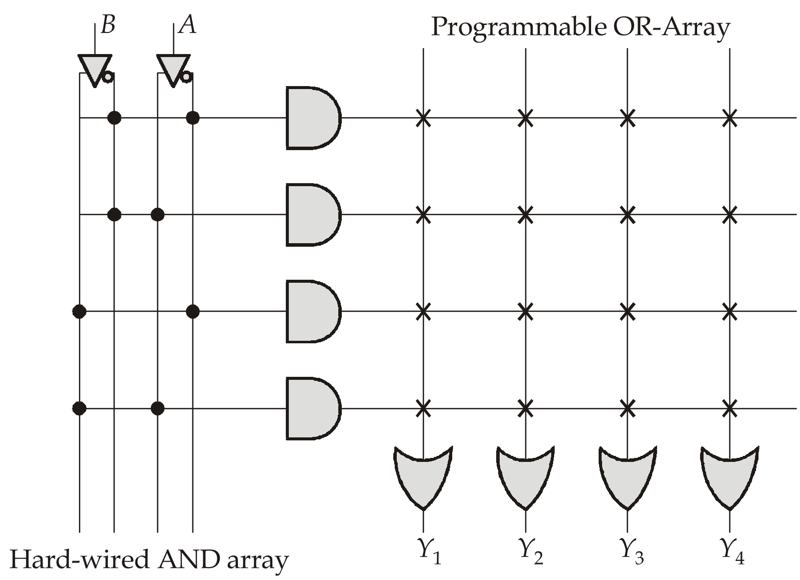
$$\frac{V_{out}}{V_{in}} = \frac{3}{1-2+1+1+1}$$

$$\frac{V_{out}}{V_{in}} = 1.5$$

Q.8 (b) Solution:

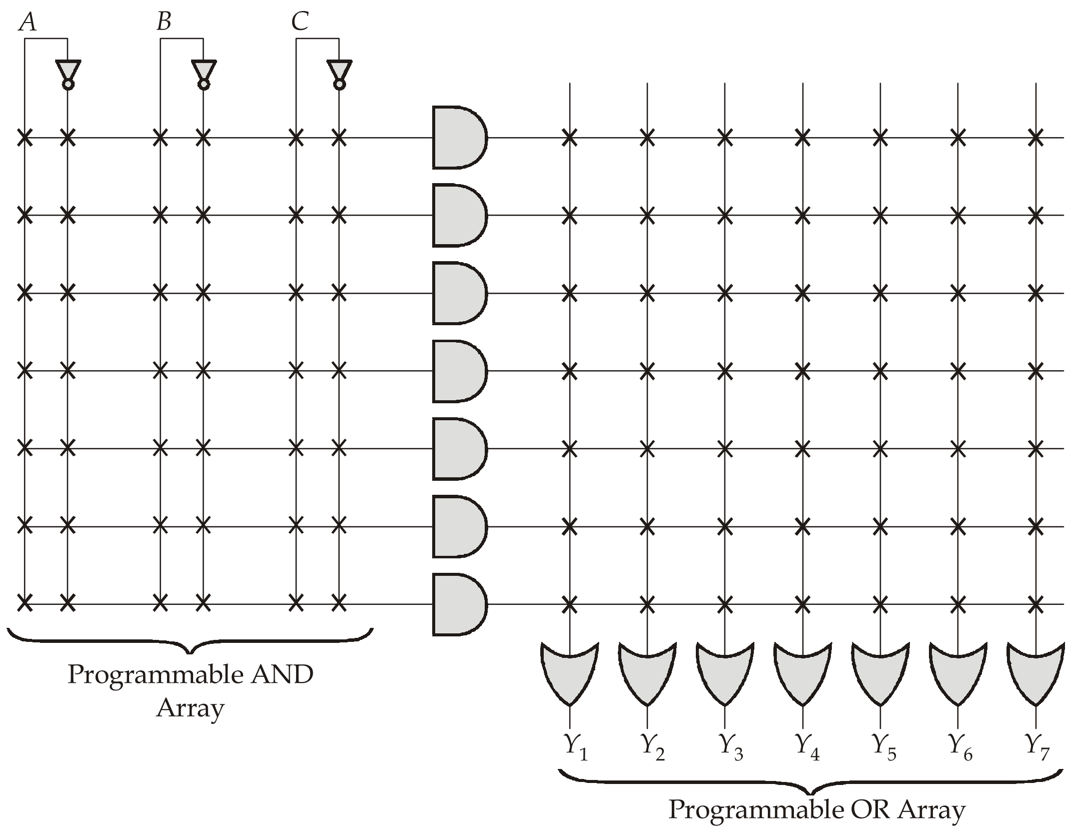
(i) 1. Programmable ROMs:

The architecture of a PROM (Programmable Read Only Memory) allows the user to hardware implement an arbitrary combinational function of a given number of inputs. An n-input and m-output PROM can be used to implement *m* different combinational functions with each function being chosen function of n variables. A generalized PROM device with n inputs and m outputs has 2^n hardwired AND gates at input and m programmable OR gates at the output. Thus each OR gate can be used to generate any conceivable Boolean function of n-variables and this generalized ROM can be used to produce m arbitrary n-variable Boolean functions. The AND array produces all possible minterms for a given number of input variables, and the programmable OR array allows only the desired minterms to appear at their inputs. Figure below shows the architecture of a PROM having 2 input lines, hardwired array of 4 AND gates and programmable array of 4 OR gates.



PROM with 2 input and 4 output lines

A X indicates an intact or unprogrammed fusible link and a dot(·) indicates hardwired interconnection. PROM are slower than dedicated logic circuit and consume more power. Also they can not be used to implement sequential logic owing to absence of flip flops.



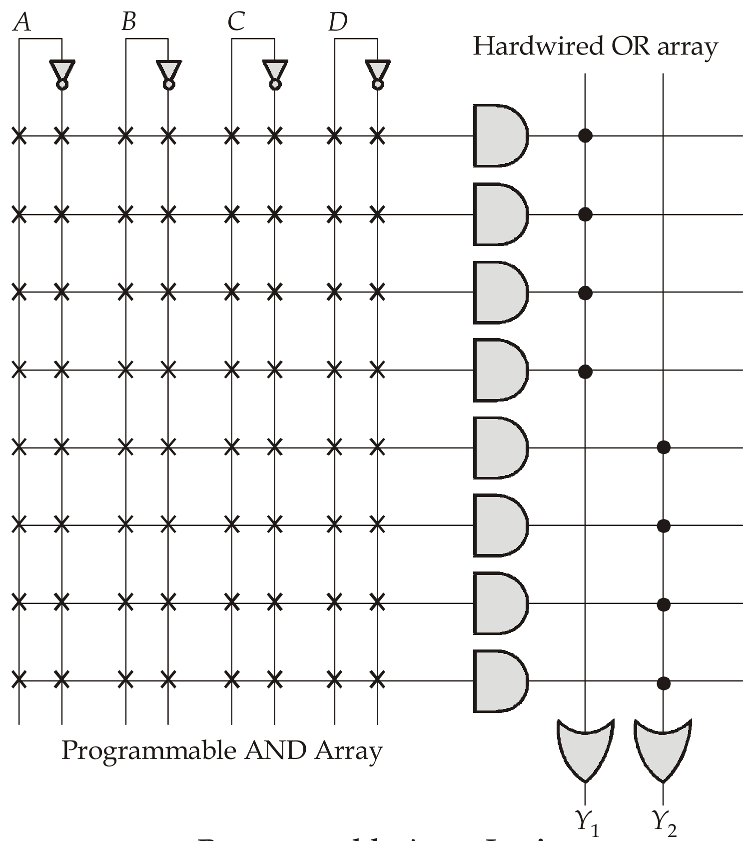
Programmable Logic Array (PLA)

2. **Programmable Logic Array (PLA):**

PLA has a programmable AND array at the input and a programmable OR array at the output.. The number of AND gates in the programmable AND array for m input variables is usually less than 2^m and the number of inputs of each of the OR gates can generate an arbitrary Boolean function with a maximum of minterms equals to the number of AND gates. A PLA device makes more efficient use of logic capacity than PROM. However, two sets of programmable fuses makes it relatively difficult to manufacture, program and test. The figure above shows the internal architecture of a PLA device with three input lines, a programmable array of seven AND gates at the input, and a programmable array of seven OR gates at the output.

3. **Programmable Array Logic (PAL):**

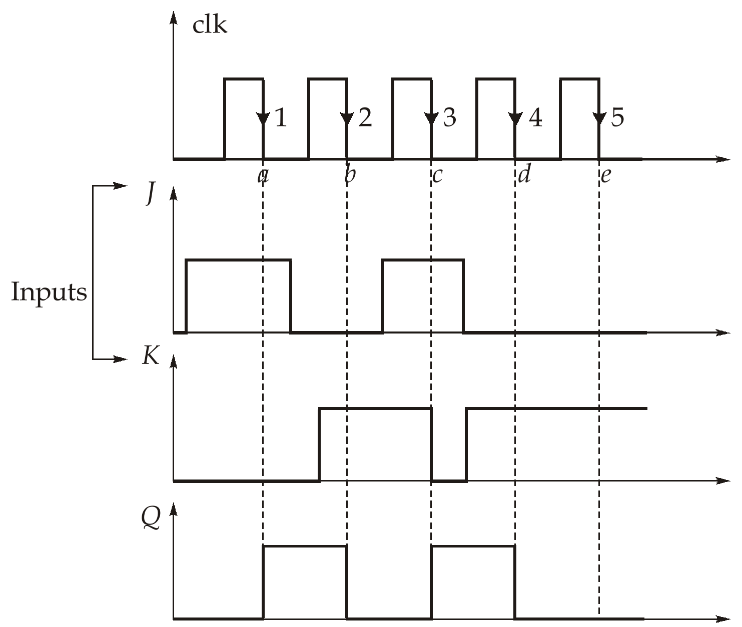
It has a programmable AND array at the input and a fixed OR array at the output. The programmable AND array of a PAL device is similar to that of a PLA device. The OR array is fixed and AND outputs are equally divided between available OR gates. For example, a practical PAL device may have eight input variables, 64 programmable AND gates and four fixed OR gates, with each OR gate having 16 inputs. That is, each OR gate is fed from 16 of the 64 AND outputs. Figure below shows an internal architecture of a PAL device that has four input lines, an array of eight AND gates at the input and two OR gates at the output.



Programmable Array Logic

(ii) The output waveform is drawn with following explanations:

1. Initially $J = 0$, $K = 0$, clk = 0. Assume that the initial state of the flip flop is a 1 i.e., $Q = 0$ initially.
2. At the negative going edge of the first clock pulse (i.e., at a), $J = 1$, $K = 0$ so Q is set to 1 and therefore $\bar{Q} = 0$.
3. At the trailing edge of second clock (i.e., at b) $J = 0$, $K = 1$ so flip-flop resets. It means $Q = 0$ and $\bar{Q} = 1$.
4. At c, $J = 1$, $K = 1$, so flip flop toggles i.e. $Q = 1$, $\bar{Q} = 0$, (or) $Q = 0, \bar{Q} = 1$
5. At d, $J = 0$, $K = 1$, so flip flops resets i.e., $Q = 0$, $\bar{Q} = 1$.



Q.8 (c) Solution:

Considering the

Open circuit test data

$$V_{OC} = 220\text{ V};\ I_{OC} = 0.4\text{ A and } P_{OC} = 26\text{ W}$$

We know that

$$P_{OC} = V_{OC} I_{OC} \cos\theta_0$$

where $\cos\theta_0$ is the power factor of the transformer at no-load.

$$\cos\theta_0 = \frac{P_{OC}}{V_{OV} I_{OC}} = \frac{26}{220 \times 0.4}$$

$$\cos\theta_0 \approx 0.3$$

Thus, Core loss resistance, $R_C = \frac{V_0}{I_0 \cos\theta_0} = \frac{220}{0.4 \times 0.3}$

$$R_C = 1.833\text{ k}\Omega$$

and Magnetizing inductance, $X_m = \frac{V_0}{I_0 \sin\theta_0}$ $\cos\theta_0 = 0.3;\quad \sin\theta_0 = 0.95$

$$X_m = \frac{220}{0.4 \times 0.95}$$

$$X_m = 0.578\text{ k}\Omega$$

Considering the short-circuit test data,

$$V_{SC} = 18 \text{ V}$$
$$I_{SC} = 8.2 \text{ A}$$
$$P_{SC} = 41.3 \text{ W}$$

As,
$$P_{SC} = V_{SC} \times I_{SC} \cos\theta$$
$$41.3 = 18 \times 8.2 \cos\theta$$
$$\cos\theta = 0.28$$

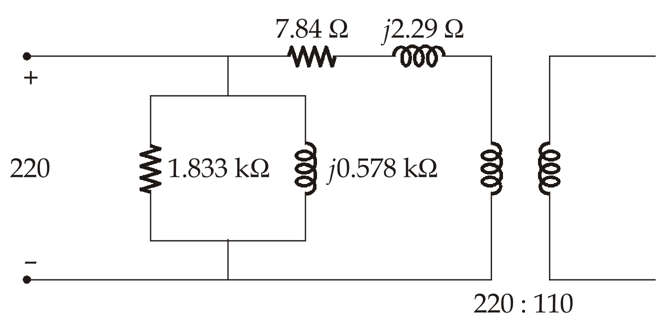
Equivalent resistance referred to primary side,

Thus,
$$R = \frac{V_{SC}}{I_{SC}\cos\theta} = \frac{18}{8.2 \times 0.28} = 7.84\ \Omega$$

Equivalent reactance referred to primary side,

$$X = \frac{V_{SC}}{I_{SC}\sin\theta} = \frac{18}{8.2 \times 0.96} = 2.29\ \Omega$$

Hence, the equivalent circuit can be drawn like this



(i) Equivalent circuit referred to low voltage side,

$$a = \frac{220}{110} = 2$$

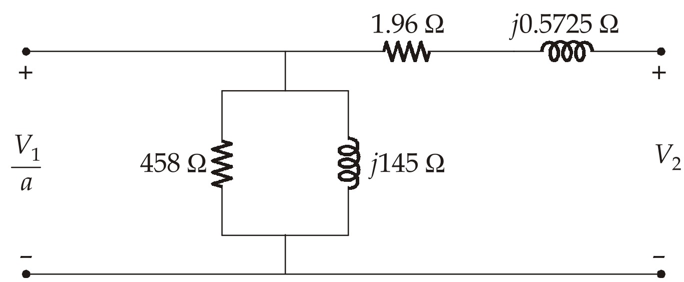
$$Z'_{eq} = \frac{Z_{eq}}{a^2} = \frac{7.84 + j2.29}{(2)^2}$$

$$Z'_{eq} = (1.96 + 0.5725i)\Omega$$

Similarly,
$$R'_c = \frac{1.833k}{2^2} = 0.458 \text{ k}\Omega$$

$$X'_m = \frac{0.578k}{2^2} = 0.145 \text{ k}\Omega$$

Hence, equivalent circuit referred to low voltage side is as below,



(ii) 1. For 0.8 pf lagging,

$$V_1' = V_2 + I_2 Z_{eq}$$

$$V_1' = 110 + (8.2\angle{-\cos^{-1} 0.8})(1.96 + j0.5725)$$

$$V_1' = 125.812 \angle{-2.68°} \text{ Volt}$$

$$\text{V.R} = \frac{125.812-110}{110}\times 100 = 14.37\%$$

2. For Unity pf,

$$V_1' = V_2 + I_2 Z_{eq}$$

$$V_1' = 110 + (8.2\angle{-\cos^{-1} 1})(1.96 + j0.5725)$$

$$V_1' = 126.159\angle 2.13°$$

$$\text{V.R.} = \frac{126.159-110}{110}\times 100$$

$$= 0.1469 \times 110 = 14.69\%$$

3. At rated conditions,

$$P_{out} = 900 \times 0.8$$

$$= 720 \text{ W}$$

Copper loss, $P_{cu} = I_s^2 R_{eq}$

$$= (8.2)^2 (1.96) = 131.79 \text{ W}$$

Using the value obtained in part (ii),

$$\text{Iron Loss, } P_i = \frac{V_i^2}{R_C} = \frac{(125.812)^2}{458} = 34.56 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}}\times 100 = \frac{720}{720+34.56+131.12}\times 100$$

$$\eta = 81.29\%$$

❍❍❍❍