MADE EASY
Leading Institute for ESE, GATE & PSUs

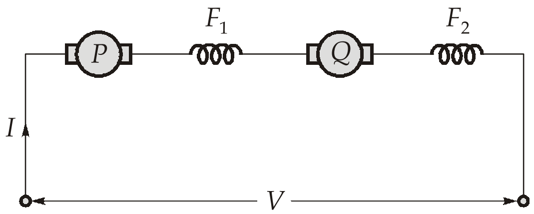
Detailed Solutions

# ESE-2024
# Mains Test Series

# Electrical Engineering
# Test No : 13

## Section-A

**Q.1 (a) Solution:**



Both the motors are carrying current $I$

Induced emf $E_1$ in machine, $$P = \left(\frac{PZ}{2\pi A}\right)\phi \cdot \omega_P$$

i.e. $$E \propto \phi \cdot \omega$$

Where flux/pole $(\phi) = KI$, where $K$ is a constant

Similarly, $$E_2 = \left(\frac{PZ}{2\pi A}\right)\phi \cdot \omega_Q$$

Where,
$P$ = Number of poles
$Z$ = Number of conductors
$A$ = Number of parallel path

Torque developed in either machine,

$$T = \left(\frac{PZ}{2\pi A}\right)K \cdot I^2$$

Neglecting losses,

$$\text{Mechanical power developed} = \text{Developed torque} \times \text{Speed}$$

$$\text{Power output of motor, } P = P_1 = \frac{PZ}{2\pi A}K \cdot I^2(n_p) \cdot \frac{2\pi}{60}$$

$$= K_P \cdot n_P{}^a \qquad \text{(given)}$$

Similarly for motor $Q$,

$$P_2 = \left(\frac{PZ}{2\pi A}\right)K \cdot I^2 \cdot (n_Q) \cdot \frac{2\pi}{60}$$

$$= K_Q \cdot n_Q^b \qquad \text{(given)}$$

$$\frac{\text{Power output of motor } P}{\text{Power output of motor } Q} = \frac{K_P \cdot n_P^a}{K_a \cdot n_Q^b} = \frac{n_P}{n_Q}$$

$$\frac{K_P}{K_Q} = \frac{n_P^{1-a}}{n_Q^{1-b}}$$

Hence proved.

**Q.1 (b) Solution:**

$$\text{Closed loop transfer function} = \frac{G(s)}{1+G(s)H(s)}$$

$$\text{Open loop transfer function, } G(s) = \frac{1}{s(s+3)}$$

$$H(s) = 1$$

$$\text{CLTF} = \frac{\frac{1}{s(s+3)}}{1+\frac{1}{s(s+3)}} = \frac{1}{s^2+3s+1}$$

Comparing with standard second order form,

$$s^2 + 2\xi\omega_n s + \omega_n{}^2 = 0$$

$$\Rightarrow \qquad \omega_n{}^2 = 1$$

$$\omega_n = 1 \text{ rad/sec}$$

$$2\xi\omega_n = 3$$

$$\xi = 1.5$$

**(i)** Resonant frequency $\omega_r = 0$ for $\xi > 0.707$

**(ii)** Resonant peak $M_r = 1$, for $\xi > 0.707$

**(iii)** Bandwidth $\left|\frac{G(s)}{1+G(s)H(s)}\right|_{(j\omega)} = \frac{1}{\sqrt{2}}$

$$= \left|\frac{\omega_n^2}{-\omega^2 + j2\xi\omega_n\omega + \omega_n^2}\right| = \frac{1}{\sqrt{2}}$$

$$= \frac{\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + (2\xi\omega_n\omega)^2}} = \frac{1}{\sqrt{2}}$$

$$= \frac{\omega_n^4}{\left(\omega_n^2 - \omega^2\right)^2 + (2\xi\omega_n\omega)^2} = \frac{1}{2}$$

Put $\omega_n = 1, \xi = 1.5$

$$\frac{1}{\left(1-\omega^2\right)^2 + (3\omega)^2} = \frac{1}{2}$$

$$2 = (1 - \omega^2)^2 + (3\omega)^2$$

$$2 = 1 + \omega^4 - 2\omega^2 + 9\omega^2$$

$$2 = 1 + \omega^4 + 7\omega^2$$

$$\omega^4 + 7\omega^2 - 1 = 0$$

$$\omega^2 = 0.14, -7.1400$$

As –7.1400 is not feasible,

$$\omega^2 = 0.14$$

$$\omega = \sqrt{0.14} = 0.3741 \text{ rad/ sec}$$

Thus, B.W. = 0.3741 rad/sec.

**Q.1 (c) Solution:**

Given that,

Number of poles $(P) = 4$, $V_L = 400$ V

$P_{\text{out}} = 7.5$ kW, $s_{fL} = 0.04$

$$T_{fL} = \frac{P_{\text{out}}}{\omega_r} = \frac{7.5\times10^3\times4}{4\pi\times50(1-0.04)}$$

$$T_{fL} = 49.73 \text{ N-m}$$

**(i)** Full load torque,
$$T_{fL} = \frac{3}{\omega_{sm}} \cdot \frac{V_{\text{ph}}^2 \times \frac{r_2'}{s}}{\left[\left(r_1 + \frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2\right]}$$

$$49.73 = \frac{3}{50\pi} \cdot \frac{(231)^2 \times 25 r_2'}{\left[(1.08 + 25 r_2')^2 + (2.82)^2\right]}$$

$$(2.82)^2 + (1.08 + 25\ r_2')^2 = \frac{25478 r_2'}{49.73}$$

$$625 r_2'^2 + 9.1188 + 54 r_2' = 512.3\ r_2'$$

$$625 r_2'^2 - 458.33 r_2' + 9.1188 = 0$$

On solving above equation,

$$r_2' = 0.7128\ \Omega,\ \ 0.020466\ \Omega$$

Smaller value is rejected as it would regain to large current for development of the required torque.

Hence, $r_2' = 0.71\ \Omega \rightarrow$ Rotor resistance as referred to stator.

**(ii)** $r_1 = 1.08\ \Omega,\qquad R_2' = 0.71\ \Omega$

$x_1 = 1.41\ \Omega,\qquad x_2' = 1.41\ \Omega$

Maximum torque always occurred at maximum slip,

$$s_{MT} = \frac{r_2'}{\sqrt{r_1^2 + (x_1 + x_2')^2}} = \frac{0.71}{\sqrt{(1.08)^2 + (2.82)^2}} = 0.2351$$

$$T_{\max} = \frac{3}{\omega_s} \cdot \frac{V_{\text{ph}}^2 \cdot \frac{r_2'}{s_{MT}}}{\left(r_1 + \frac{r_2'}{s_{MT}}\right) + (x_1 + x_2')^2}$$

$$T_{\max} = \frac{3}{50\pi} \cdot \frac{(231)^2 \times 3.02}{(1.08 + 3.02)^2 + (2.82)^2} = 124.29 \text{ N-m}$$

Speed at maximum torque,

$$N_r\,(T_{\max}) = N_s(1 - s_{MT}) = 1500\,(1 - 0.2351)$$

$$N_r\,(T_{\max}) = 1147.35 \text{ rpm}$$

**Q.1 (d) Solution:**

Given that,

$$X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+4z^{-2}} = \frac{z^2+2z}{z^2-2z+4}$$

We know that,

$$ZT\{A^n \cos \omega_0 n\} = \frac{z(z-A\cos\omega_0)}{z^2-2Az\cos\omega_0+A^2}$$

$$ZT\{A^n \sin \omega_0 n\} = \frac{Az\sin\omega_0}{z^2-2Az\cos\omega_0+A^2}$$

On comparing the denominator of $X(z)$,

$$z^2 - 2z + 4 = z^2 - Az \cos \omega_0 + A^2$$

$$A \cos \omega_0 = 1 \quad \text{and} \quad A = 2$$

$$2 \cos \omega_0 = 1$$

$$\cos \omega_0 = \frac{1}{2} = \cos 60°$$

$$\omega_0 = 60° = \frac{\pi}{3}$$

$$\sin \omega_0 = \sin 60° = \frac{\sqrt{3}}{2}$$

Compare the numerator of $X(z)$,

$$z^2 + 2z = z^2 - Az \cos \omega_0 + \alpha(Az \sin \omega_0)$$

or $$z^2 + 2z = z^2 - 2z\left(\frac{1}{2}\right) + \alpha\left(2z\frac{\sqrt{3}}{2}\right)$$

or $$\alpha = \sqrt{3}$$

Therefore, $$X(z) = \frac{\left[z^2-2z\cos\frac{\pi}{3}\right]+\sqrt{3}\left[2z\sin\frac{\pi}{3}\right]}{z^2-2z\left[2\cos\frac{\pi}{3}\right]+4}$$

$$X(z) = \frac{z^2-2z\cos\frac{\pi}{3}}{z^2-2z\left[2\cos\frac{\pi}{3}\right]+4} + \frac{\sqrt{3}\left[2z\sin\frac{\pi}{3}\right]}{z^2-2z\left[2\cos\frac{\pi}{3}\right]+4}$$

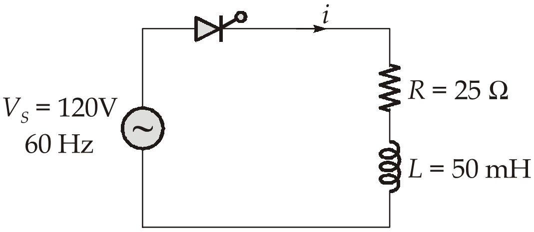
$$x(n) = 2^n \cos\frac{n\pi}{3}u(n) + 2^n\sqrt{3}\sin\frac{n\pi}{3}u(n)$$

$$\therefore \quad x(n) = 2^n\left[\cos\frac{n\pi}{3} + \sqrt{3}\sin\frac{n\pi}{3}\right]u(n)$$

**Q.1 (e) Solution:**

```
          LXI   H, C100H;
          MOV   E, M
          MVI   D, 00H       Load DE with the byte at C100H
          LXI   H, C200H
          MOV   A, M         Load A with the number at location C200H
          LXI   H, 0000H;    Initialise HL with 0000H
          CPI   00H
          JZ    EXIT;        If A value is 00H jump to EXIT
AGAIN:    DAD   D;           Add HL and DE contents
          DCR   A;           Decrement A
          JNZ   AGAIN;       If results of decrement is not zero jump to AGAIN
EXIT:     HLT
```

**Q.2 (a) Solution:**



Applying Kirchoffs law,

$$V_m \sin\omega t = Ri + L\frac{di}{dt}$$

$$i(\omega t) = i_F(\omega t) + i_N(\omega t)$$

where $i_F$ = Forced response and $i_N$ = Natural response.

Now for natural response

$$Ri(t) + L\frac{di(t)}{dt} = 0$$

By laplace transform,

$$RI(s) + LsI(s) = 0$$

$$i_N(t) = Ae^{-\frac{R}{L}t}$$

For forced response

$$Ri + L\frac{di}{dt} = V_m \sin\omega t$$

$$\left(D+\frac{R}{L}\right)i = \frac{V_m}{L}\sin(\omega t)$$

$$i = \frac{V_m/L}{D+\frac{R}{L}}\sin(\omega t)$$

By multiplying and dividing by term $\left(D-\frac{R}{L}\right)$ in numerator and denominator

$$i = \frac{V_m/L}{D+\frac{R}{L}}\sin\omega t\frac{\left(D-\frac{R}{L}\right)}{\left(D-\frac{R}{L}\right)}$$

$$i = \frac{V_m/L}{D^2-\frac{R^2}{L^2}}\sin\omega t\left(D-\frac{R}{L}\right)$$

Put $D^2 = -\omega^2$

$$i = \frac{V_m/L}{-(\omega)^2-\frac{R^2}{L^2}}\sin\omega t\left(D-\frac{R}{L}\right)$$

$$i = \frac{-V_m \cdot L}{(\omega L)^2+R^2}\sin\omega t\left(D-\frac{R}{L}\right) = \frac{-V_m L}{Z^2 L}\sin\omega t(LD-R)$$

$$= \frac{V_m}{Z^2}\sin\omega t(R-LD)$$

$$= \frac{V_m}{Z^2}[R\sin\omega t-\omega L\cos\omega t]$$

$$= \frac{V_m Z}{Z^2}\left[\frac{R}{\sqrt{R^2+(\omega L)^2}}\sin\omega t-\frac{\omega L}{\sqrt{R^2+(\omega L)^2}}\cos\omega t\right]$$

$$= \frac{V_m}{Z}[\sin\omega t\cos\theta - \cos\omega t\sin\theta]$$

where, $$\theta = \tan^{-1}\frac{\omega L}{R}$$

Using identity sinA cosB – cosA sin B = sin(A – B)

$$= \frac{V_m}{Z}\sin[\omega t - \theta]$$

Now, $$i(t) = \frac{V_m}{Z}\sin[\omega t - \theta] + Ae^{-t/\tau}$$

Now, at $\alpha = 30°$ or $\alpha = 0.524$ radians

$$i(\omega t) = 0$$

$$i(t) = \frac{V_m}{Z}\sin[\omega t - \theta] + Ae^{-tR/L}$$

$$i(\omega t) = \frac{V_m}{Z}\sin[\omega t - \theta] + Ae^{-\omega tR/\omega L}$$

Now, at $\omega t = 30°$ or 0.524 radian

$$0 = \frac{V_m}{Z}\sin[\omega t - \theta] + Ae^{-\alpha R/\omega L}$$

$$A = -\frac{V_m}{Z}\sin[\alpha - \theta]e^{\frac{\alpha R}{\omega L}}$$

Now, $$i(\omega t) = \frac{V_m}{Z}\sin[\omega t - \theta] - \frac{V_m}{Z}\sin[\alpha - \theta]\times e^{\frac{\alpha R}{\omega L}} \times e^{-\frac{\omega tR}{\omega L}}$$

For, $$R = 25\ \Omega,\ f = 60\text{ Hz},\quad L = 50\text{ mH}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = 0.6460\text{ radian}$$

$$V_m = 120\sqrt{2}\text{ V}$$

$$Z = \sqrt{R^2 + \omega^2 L^2} = 31.31\ \Omega$$

So, $$i(\omega t) = \frac{120\sqrt{2}}{31.31}\sin(\omega t - 0.6460) - \frac{120\sqrt{2}}{31.31}\sin(\alpha - 0.6460)e^{\frac{-\omega Rt}{\omega L}} \times e^{\frac{\alpha R}{\omega L}}$$

Putting $\alpha = 0.524$ radian,

$$i(\omega t) = 5.42\sin(\omega t - 0.6460) + 1.33\ e^{-\omega t/0.754}\text{ Amp}$$

**Q.2 (b) (i) Solution:**

Let the current flowing through $R_1$ is $I_1$ and that of Zener diode is $I_Z$.

Since $Q_1$ and $Q_2$ are identical and β is very large,

$$I_Z = I_1 = \frac{V_s - V_{BE(\text{on})}}{R_1}$$

$$V_Z = V_{Z0} + I_Z r_z$$

$$V_0 = \left(1+\frac{90}{10}\right)V_Z = 10\,V_Z$$

$$\Delta V_0 = 10\,\Delta V_Z = 10(r_z \Delta I_Z) = 10\,r_z \Delta I_1$$

$$\Delta I_1 = \frac{\Delta V_s}{R_1}$$

So,
$$\Delta V_0 = \frac{10\,r_z}{R_1}\Delta V_s$$

$$\%\text{Line regulation} = \frac{\Delta V_0}{\Delta V_s}\times 100 = \frac{10\times 15}{9300}\times 100 = 1.613\,\%$$

**Q.2 (b) (ii) Solution:**

Transfer function,

$$T(s) = \frac{V_0(s)}{V_1(s)} \qquad \text{...(i)}$$

$$V_0(s) = V_1(s)\left(\frac{-R_1}{R_1}\right) + V_1(s)\left(\frac{1/Cs}{R+\dfrac{1}{Cs}}\right)\left(1+\frac{R_1}{R_1}\right)$$

$$\Rightarrow \quad \frac{V_0(s)}{V_1(s)} = -1+\left(\frac{2}{1+RCs}\right) = -1+\frac{2}{1+RCs}$$

$$\Rightarrow \quad \frac{V_0(s)}{V_1(s)} = \frac{-1-RCs+2}{1+RCs} = \frac{1-RCs}{1+RCs} = T(s)$$

From this $T(s)$ we can say,

The given circuit is an all-pass filter.

**Q.2 (c) Solution:**

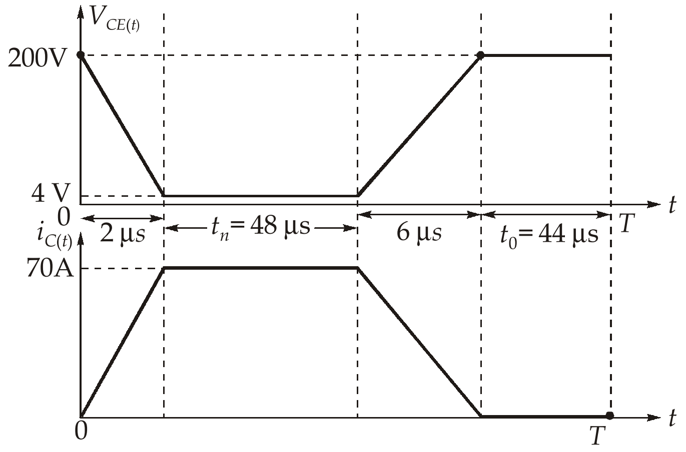
Duty ratio is given as $\alpha = 0.5$

$T_{\text{on}} = 0.05$ ms = 50 µs

conduction period = 50 – 2 = 48 µs

off period = 50 – 6 = 44 µs

Redrawing the waveforms, we get



Average power loss is given by,

$$P_C = \frac{1}{T}\int_0^t V_{CE}(t) i_C(t) dt \quad \text{...(i)}$$

**(i)** During turn on process

$$V_{CE(t)} = \left(\left(\frac{4-200}{t_{on}}\right)t + 200\right)\text{V}$$

and $$i_{C(t)} = \left(\frac{70}{t_{on}}\right)t \text{ A}$$

From equation (i), we get

Average power loss, $$P_C = \frac{1}{T}\int_0^{t_{on}}\left(-\frac{196}{t_{on}}t + 200\right)\cdot\left(\frac{70}{t_{on}}\cdot t\right)dt$$

$$= \frac{1}{T}\int_0^{t_{on}}\left(-\frac{196\times 70}{t_{on}{}^2}\cdot t^2 + \frac{200\times 70}{t_{on}}\cdot t\right)dt$$

$$= \frac{1}{T}\left[-\frac{196\times 70}{t_{on}{}^2}\cdot\frac{t^3}{3} + \frac{200\times 70}{t_{on}}\cdot\frac{t^2}{2}\right]_0^{t_{on}}$$

$$= f\left[-\frac{196\times 70\times t_{on}}{3} + \frac{200\times 70\times t_{on}}{2}\right]$$

$$= 10\times 10^3\times\left[-\frac{196\times 70\times 2\times 10^{-6}}{3} + \frac{200\times 70\times 2\times 10^{-6}}{2}\right]$$

$$P_C = 48.533 \text{ W.}$$

**(ii)** During conduction period

$$V_{CE}(t) = 4\text{ V}$$

$$i_C(t) = 70\text{ A}$$

Average power loss, $$P_C = \frac{1}{T}\int_0^{t_n}(4\times 70)\,dt$$

$$= 10 \times 10^3 \times 4 \times 70 \times 48 \times 10^{-6}$$

$$P_C = 134.4\text{ W}$$

**(iii)** During turn off process

$$V_{ce}(t) = \left(\frac{200-4}{t_{off}}\right).t+4$$

$$i_c(t) = -\frac{70}{t_{off}}t+70$$

Power loss, $$P_c(t) = \frac{1}{T}\int_0^{t_{off}}\left(\left(\frac{196}{t_{off}}\right)\cdot t+4\right)\cdot\left(-\frac{70}{t_{off}}t+70\right)dt$$

$$= \frac{1}{T}\int_0^{t_{off}}\left[-\frac{196\times 70}{t_{off}^2}\cdot t^2+\frac{196\times 70\times t}{t_{off}}+4\times 70+\left(\frac{-4\times 70t}{t_{off}}\right)\right]dt$$

$$= \frac{1}{T}\left[\frac{-196\times 70\times t^3}{t_{off}^2.3}+\frac{196\times 70\times t^2}{2t_{off}}+4\times 70t-\frac{4\times 70}{2t_{off}}t^2\right]_0^{t_0}$$

$$= \frac{1}{T}\times t_{off}\left[-\frac{196\times 70}{3}+\frac{196\times 70}{2}+4\times 70-\frac{4\times 70}{2}\right]$$

$$= 145.6\text{ W.}$$

**(iv)** During off period

$$V_{CE}(t) = 200\text{ V}$$

$$i_c(t) = 0$$

$$P_C = 0$$

**(v)** Peak power loss during turn on process

$$P_C(t) = V_{CE}(t)\cdot i_C(t)$$

$$= \left[\left(-\frac{196}{t_{on}}\right)\cdot t+200\right]\cdot\left[\frac{70}{t_{on}}\cdot t\right]$$

$$P_C(t) = \frac{-196\times70}{t_{on}{}^2}\cdot t^2+\frac{200\times70}{t_{on}}.t$$

Power loss will be maximum at $t = t_m$ where $\frac{dP_C}{dt}=0$

Therefore,

$$\frac{dP_C(t)}{dt} = \frac{-196\times70}{t_{on}^2}.2t+\frac{200\times70}{t_{on}}$$

$$0 = -\frac{196\times70}{t_{on}^2}.2t_m+\frac{200\times70}{t_{on}}$$

$$t_m = \frac{200\times70\times t_{on}}{196\times70\times2}=1.0204\ \mu s$$

Peak power loss will be

$$P_C(t) = -\frac{196\times70}{t_{on}^2}.t_m^2+\frac{200\times70}{t_{on}}.t_m$$

$$P_C(t) = -3571.37 + 7142.8$$

$$P_c(t) = 3571.43 \text{ W}$$

**(vi)** Total power loss in one cycle will be,

$$P_T = 48.533 + 134.4 + 145.6 + 0$$

$$P_T = 328.533 \text{ W}$$

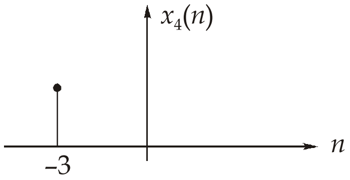
**Q.3 (a) (i) Solution:**

$$y[n] = x_1[n] * x_2[n] * x_3[n]$$

$$y[n] = 0.5^n \,.\, u[n] * [u(n+3) * (\delta[n] - \delta(n-1))]$$

$$= 0.5^n\, u(n) * [u[n+3] - u[n+2]]$$

$$x_4(n) = u[n+3] - u[n+2]$$

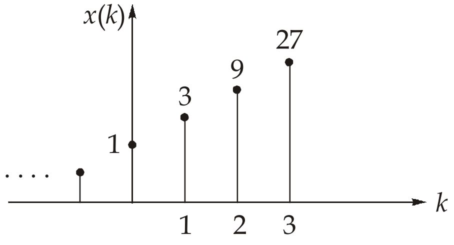


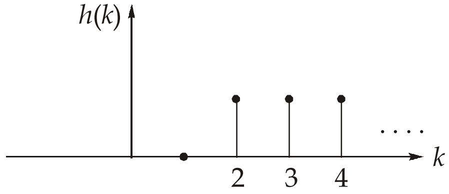
$$y[n] = 0.5^n\, u[n] * \delta[n+3]$$

$$y[n] = 0.5^{n+3}\, u[n+3]$$

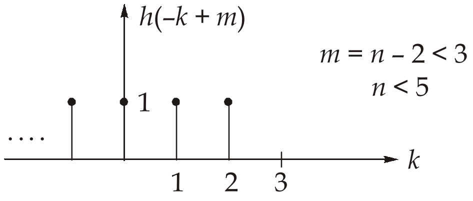
**Q.3 (a) (ii) Solution:**

$$x(k) = 3^k\, u(-k+3)$$
$$h(k) = u(k-2)$$

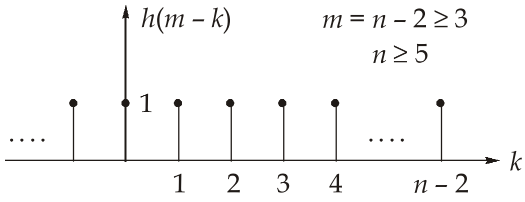




$$y(k) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(n-k)$$
$$h(n-k) = u(n-k-2)$$



$m = n-2 < 3$
$n < 5$



$m = n-2 \geq 3$
$n \geq 5$

$n \geq 5$**:**
$$y(n) = \sum_{k=-\infty}^{3} 3^k = \sum_{k=-3}^{\infty}\left(\frac{1}{3}\right)^k$$
$$= 3^3 + 3^2 + 3^1 + 1 + \frac{1}{3} + ....$$
$$= \frac{3^3}{1-\frac{1}{3}} = \left(\frac{81}{2}\right)$$

**$n < 5$:**

$$y(n) = \sum_{k=-\infty}^{n-2} 3^k$$

$$= [3^{n-2} + 3^{n-3} + 3^{n-4} \ldots]$$

$$= \frac{3^{n-2}}{\left(1-\frac{1}{3}\right)} = \frac{1}{2} \cdot 3^{n-1}$$

$$y(n) = \begin{cases} \frac{81}{2} & n \geq 5 \\ \frac{1}{2} \cdot 3^{n-1} & n < 5 \end{cases}$$

**Q.3 (b) Solution:**

Subscripts 1 and 2 are used to denote 11 kV, 25 Hz and 22 kV, 50 Hz systems respectively.

At 50 Hz the voltage is doubled for the same current, therefore the output $P_2$ at 50 Hz is double the output $P_1$ at 25 Hz i.e. $P_2 = 2P_1$

Given that, $$\frac{P_{oh1}}{P_1} = 1.8\%,$$

$$\frac{P_{h1}}{P_1} = 0.8\%;$$

and $$\frac{P_{e1}}{P_1} = 0.3\%$$

**Ohmic loss:**

Since the current is same at both the frequencies and voltages, the ohmic losses in watts remain unaltered,

i.e., $$P_{oh2} = P_{oh1}$$

Percentage ohmic loss at $f_2$, $V_2$ is

$$\frac{P_{oh2}}{P_2} = \frac{P_{oh1}}{2P_1} = \frac{1.8}{2} = 0.9\%$$

**Core loss:**

The voltage is related to $f$, $B_m$ etc by the expression

$$V = \sqrt{2}\pi f\, B_m A_i N_{ph}$$

$$\therefore \quad \frac{V_1}{V_2} = \left(\frac{f_1}{f_2}\right)\left(\frac{B_{m1}}{B_{m2}}\right)$$

or, $$\frac{11000}{22000} = \frac{25}{50}\left(\frac{B_{m1}}{B_{m2}}\right)$$

⇒ $$B_{m1} = B_{m2}$$

The hysteresis loss, $$P_h = K_h f B_m^x \text{ watts}$$

∴ $$\frac{P_{h2}}{P_{h1}} = \left(\frac{f_2}{f_1}\right)\left(\frac{B_{m2}}{B_{m1}}\right)^x$$

or $$\frac{P_{h2}}{P_{h1}} = \left(\frac{50}{25}\right)(1)^x$$

⇒ $$P_{h2} = 2\ P_{h1}$$

Percentage hysteresis loss at $f_2$, $V_2$ is

$$= \frac{P_{h2}}{P_2} = \frac{2P_{h1}}{2P_1} = 0.8\%$$

The eddy current loss, $$P_e = Ke f^2 B_m{}^2 \text{ watts}$$

∴ $$\frac{P_{e2}}{P_{e1}} = \left(\frac{f_2}{f_1}\right)^2 \cdot \left(\frac{B_{m2}}{B_{m1}}\right)^2$$

or, $$P_{e2} = P_{e1}\left(\frac{50}{25}\right)^2 = 4P_{e1}$$

Percentage eddy current loss at $f_2$, $V_2$ is

$$\frac{P_{e2}}{P_2} = \frac{4P_{e1}}{2P_1}$$

$$= \frac{2P_{e1}}{P_1} = 2(0.3) = 0.6\%$$

Efficiency at $f_1$, $V_1$ is, $$\eta_1 = 1 - \frac{\text{Losses}}{\text{Output + losses}}$$

$$= 1 - \frac{\text{Losses/output}}{1 + \text{Losses/output}} = 1 - \frac{\text{p.u. losses}}{1 + \text{p.u. losses}}$$

$$= 1 - \frac{0.018 + 0.008 + 0.003}{1 + 0.018 + 0.008 + 0.003}$$

$$= 1 - \frac{0.029}{1.029} = 0.97182 \text{ (or) } 97.182\%$$

Efficiency at $f_2$, $V_2$ is,

$$\eta_2 = 1 - \frac{0.009 + 0.008 + 0.006}{1 + 0.009 + 0.008 + 0.006}$$

$$= 1 - \frac{0.023}{1.023} = 0.97752\% \text{ (or) } 97.752\%$$

**Q.3 (c) Solution:**

Receiving end voltage is taken as reference,

$$V_R = \frac{132}{\sqrt{3}} = 76.21 \text{ kV}$$

$$A = 0.98\angle 3°, \quad B = 110\angle 75° \ \Omega/\text{phase}$$

$$\text{p.f.} = \cos\phi = 0.8 \text{ lagging}, \quad \sin\phi = 0.6$$

$$|I_R| = \frac{50 \times 10^6}{\sqrt{3} \times (132 \times 10^3)} = 218.69 \text{ A}$$

$$\therefore \quad I_R = 218.69\angle -\cos^{-1} 0.8 = 218.69\angle -36.86° \text{ A}$$

**(i)**

$$V_s = AV_R + BI_R$$

$$V_s = (0.98\angle 3°)\,(76.21 \times 10^3)\,(110\angle 75°)\,(218.69\angle -36.86°)$$

$$\therefore \quad V_s = 95.367\angle 11.35° \text{ kV}$$

$$|V_s| = 95.367 \text{ kV}$$

or, $$|V_s(\text{line})| = \sqrt{3}(95.367) = 165.18 \text{ kV} = 165.18 \text{ kV}$$

and power angle $(\delta) = 11.35°$

**(ii)** Since, $$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$

or, $$AD - BC = 1$$

and $$A = D$$

$$C = \frac{A^2 - 1}{B} = \frac{(0.98)^2\angle 6° - 1}{110\angle 75°} = 9.996 \times 10^{-4}\angle 39.08°$$

$$I_s = CV_R + DI_R$$

$$= (9.996 \times 10^{-4}\angle 39.08°)\,(76.21) \times 10^3 + (0.98\angle 3°)\,(218.69\angle -36.86°)$$

$$\therefore \quad I_s = 247.617\angle -16.756° \text{ A}$$

$$\therefore \quad P_s = 3V_sI_s \cos(11.35° + 16.756°)$$

$$= 3(95.36 \times 10^3)\,(247.617) \cos 28.106°$$

$$= 62.484 \text{ MW}$$

and $$Q_s = 3V_sI_s \sin(11.35° + 16.756°)$$

$$= 3(95.36 \times 10^3)\,(247.617) \sin 28.106°$$

$$= 33.37 \text{ MVAR}$$

**(iii)** Line loss $= P_s - P_R$

$= 62.484 - 50\ (0.8)$

$= 22.484$ MW

VAR absorbed $= Q_s - Q_R$

$= 33.37 - 50(0.6) = 3.37$ MVAR

**(iv)** For unit p.f. load,

$$Q_R = 0$$

and $V_s = V_R = 132 \times 10^3$ kV (line)

Since,
$$Q_R = \frac{|V_R||V_R|}{|B|}\sin(\beta-\delta) - \left|\frac{A}{B}\right||V_R|^2\sin(\beta-\alpha) = 0$$

$$\frac{(132\times10^3)\,(132\times10^3)}{110}\sin(75^\circ-\delta) - \left|\frac{0.98}{110}\right|(132\times10^3)^2\sin(75^\circ-3^\circ) = 0$$

or, $\sin(75^\circ - \delta) - 0.98\sin72^\circ = 0$

$\therefore$ $\delta = 6.25^\circ$

$$P_R = \frac{|V_s||V_R|}{|B|}\cos(\beta-\delta) - \left|\frac{A}{B}\right||V_R|^2\cos(\beta-\alpha)$$

$$= \frac{132\times10^3}{110}\cos(75^\circ-6.25^\circ) - \left(\frac{0.98}{110}\right)(132\times10^3)^2\cos72^\circ$$

$\therefore$ $P_R = 9.44$ MW at upf

**Q.4 (a) (i) Solution:**

**For Machine-1:**

Moment of inertia, $I_1 = 25000$ kg-m$^2$

Rating of machine in MVA, $S_1 = \dfrac{\text{MW}}{pf_1} = \dfrac{45}{0.8} = 56.25$ MVA

Angular velocity, $\omega_1 = 2\pi f_1 = 2\pi \times 50$

$= 100\ \pi$ electrical rad/sec

Kinetic energy stored in the rotor at synchronous speed

$$KE_1 = \frac{1}{2}I\omega_{sm}^2 = \frac{1}{2}I\left(\frac{\omega_1}{2}\right)^2 = \frac{1}{2}\times25000\times(50\pi)^2$$

$$= 308.4251 \text{ MJ}$$

Inertia constant, $H_1 = \dfrac{\text{Kinetic Energy stored}}{S_1 \text{ in MVA}} = \dfrac{308.4251}{56.25} = 5.483$ MJ/MVA

**For Machine-2:**

Moment of inertia, $I_2 = 9000 \text{ kg-m}^2$

Rating of machine in MVA, $S_2 = \frac{MW_2}{pf_2} = \frac{60}{0.75} = 80 \text{ MVA}$

Kinetic energy stored in a rotor at synchronous speed,

$$\omega = 2\pi f_2 = 2\pi \times 50 = 100\pi \text{ electrical rad/sec}$$

$$\text{KE}_2 = \frac{1}{2} I\omega_{sm}^2 = \frac{1}{2} \times 9000 \times (2\pi \times 50)^2 = 444.132 \times 10^6 \text{ J}$$

Inertia constant, $H_2 = \frac{KE_2}{S_2} = \frac{444.132}{80} = 5.55 \text{ MJ/MVA}$

$$\text{base MVA } S = 100 \text{ MVA}$$

So, $H_{1\,\text{new}} = \frac{S_1 H_1}{S} = \frac{56.25 \times 5.483}{100} = \frac{K \cdot E}{S} = 3.084 \text{ MJ/MVA}$

$$H_{2\,\text{new}} = \frac{S_2 H_2}{S} = \frac{80 \times 5.55}{100} = 4.44 \text{ MJ/MVA}$$

As inertia is different, machine will not swing together

So, $H_{\text{eq}} = \frac{H_1 \times H_2}{H_1 + H_2} = \frac{3.084 \times 4.44}{3.084 + 4.44} = 1.8199 \text{ MJ/MVA}$

**Q.4 (a) (ii) Solution:**

Assuming three conditions, healthy fault and post fault clearing represented by *A*, *B* and *C* respectively

Given, $P_{\max 1} = 1.6$ p.u. (pre fault)

$P_{\max 2} = 0.3$ p.u. (at fault)

$P_{\max 3} = 1.2$ p.u. (after fault clearing)

Initial loading, $P_s = 1.0$ p.u.

$$\text{Initial loading angle, } \delta_0 = \sin^{-1}\frac{P_s}{P_{\max 1}} = \sin^{-1}\frac{1}{1.6}$$

$$= 38.682° = 0.675 \text{ rad (elec)}$$

$$\gamma_1 = \frac{\text{Maximum power that can be transferred during fault}}{\text{Maximum power that can be transferred initially}} = \frac{P_{\max 2}}{P_{\max 1}} = \frac{0.3}{1.6} = 0.1875$$

$$\gamma_2 = \frac{P_{\max 3}}{P_{\max 1}} = \frac{1.2}{1.6} = 0.75$$

$$\delta_m = 180° - \sin^{-1}\frac{\sin\delta_0}{\gamma_2} = 180° - \sin^{-1}\left(\frac{1/1.6}{0.75}\right)$$

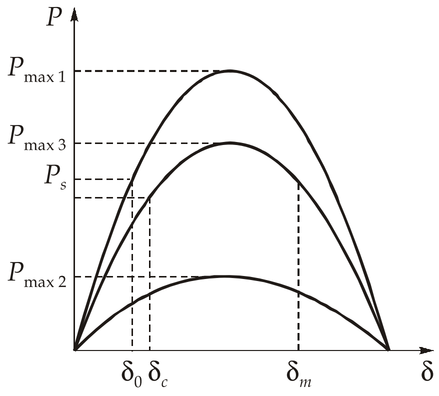
$$= 180° - 56.44° = 123.56° \text{ or } 2.1565 \text{ rad (elec.)}$$

Critical clearing angle, $$\delta_c = \cos^{-1}\left(\frac{\sin\delta_0 \times (\delta_m - \delta_0) - \gamma_1 \cos\delta_0 + \gamma_2 \cos\delta_m}{\gamma_2 - \gamma_1}\right)$$

$$\delta_c = \cos^{-1}\frac{\left[\frac{1}{1.6} \times (2.1565 - 0.675) - 0.1875 \times 0.7806 + 0.75 \times (-0.5528\right]}{0.75 - 0.1875}$$

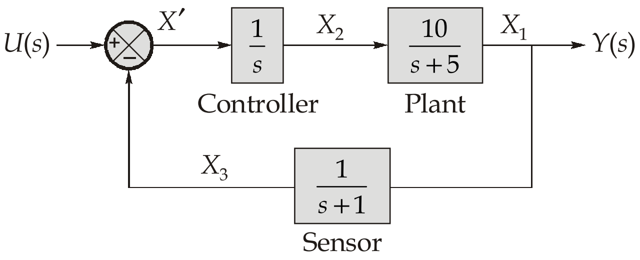
$$= \cos^{-1}\left[\frac{0.9259 - 0.14636 - 0.4146}{0.5625}\right]$$

$$= \cos^{-1}\left[\frac{0.36494}{0.5625}\right] = 49.55° \text{ (electrical)}$$



**Q.4 (b) (i) Solution:**

Let us define the output of the plant s $X_1$ and output of the controller is $X_2$ and output of the sensor as $X_3$.



$$\therefore \qquad \frac{X_1(s)}{X_2(s)} = \frac{10}{s+5}$$

$$\frac{X_2(s)}{X'(s)} = \frac{1}{s}$$

Where, $X'(s) = U(s) - X_3(s)$

$$\frac{X_3(s)}{X_1(s)} = \frac{1}{s+1}$$

$$Y(s) = X_1(s)$$

Therefore we can write,

$$sX_1(s) = -5X_1(s) + 10X_2(s)$$

$$sX_2(s) = -X_3(s) + U(s)$$

$$sX_3(s) = X_1(s) - X_3(s)$$

$$\therefore \qquad Y(s) = X_1(s)$$

By taking the inverse Laplace transform,

$$\dot{x}_1 = -5x_1 + 10x_2$$

$$\dot{x}_2 = -x_3 + U$$

$$\dot{x}_3 = x_1 - x_3$$

$$y = x_1$$

Thus, the state model of the system,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 10 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

**Q.4 (b) (ii) Solution:**

Given: From the characteristic equation

$$\therefore \qquad G(s)H(s) = \frac{K}{(1+s)(1.5+s)(2+s)}$$

Put $s = -1 + j\omega$

$$GH(-1 + j\omega) = \frac{K}{(j\omega)(1.5-1+j\omega)(2-1+j\omega)}$$

$$GH(-1 + j\omega) = \frac{K}{(j\omega)(0.5 + j\omega)(1 + j\omega)}$$

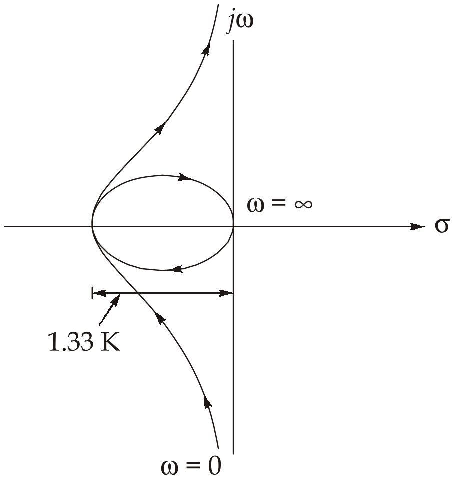
$$M = |GH(-1 + j\omega)| = \frac{K}{\omega\left(\sqrt{1+\omega^2}\right)\left(\sqrt{0.25+\omega^2}\right)}$$

$$M = \frac{2K}{\omega\left(\sqrt{1+4\omega^2}\right)\left(\sqrt{1+\omega^2}\right)}$$

Phase angle $\angle GH(-1 + j\omega) = \phi = -90 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$

| $\omega$ | $M$ | $\phi$ |
|---|---|---|
| 0 | $\infty$ | –90° |
| $\infty$ | 0 | –270° |
| 0.5 | $2.53K$ | –161.56° |
| 0.1 | $19.5K$ | –107° |
| 1 | $0.63K$ | –198° |
| 10 | $9.94 \times 10^{-4} K$ | –261° |
| 0.707 | $1.33K$ | –180° |

Nyquist plot for the system is given as



Now, the frequency at which phase is –180° is given by

$$-180° = -90° - \tan^{-1}\omega - \tan^{-1}(2\omega)$$

$$\frac{\omega + 2\omega}{1 - 2\omega^2} = \frac{1}{0}$$

$$\omega = \frac{1}{\sqrt{2}}\,\text{rad/sec}$$

At $\omega = \frac{1}{\sqrt{2}}$ rad/sec, the value of $M$ is given as,

$$M = \frac{2K}{\frac{1}{\sqrt{2}} \times \sqrt{1+4\times\frac{1}{2}} \times \sqrt{1+\frac{1}{2}}} = \frac{4}{3}\text{K}$$

From the Nyqusit criterion, $Z = P - N$

Here, $Z$ = No. of closed-loop poles inside the right half of $S = -1$ plane.

$P$ = No. of open-loop poles inside the right half of $S = -1$ plane.

$N$ = No. of encirclement around $S = -1$.

As $P = 0,$

$\therefore$ For system to be stable, $N$ should be zero.

For $N$ to be zero, $1.33K < 1$

$$K < \frac{1}{1.33}$$

$$K < 0.75$$

Hence, $K = 0.75$ is the largest value of $K$.

**Q.4 (c) Solution:**

**(i)** $$\text{slip, } s = \frac{-n}{m}\cos\alpha = -a\cos\alpha_m$$

For 25% speed range, $S_m = 0.25$ thus below the synchronous speed

$$0.25 = -a\cos 165°$$

$$a = \frac{-0.25}{\cos 165°} = 0.2588 \approx 0.259$$

$$\frac{n}{m} = a \text{ or } \frac{2}{m} = 0.259 \text{ or } m = 7.722$$

**(ii)** For a speed of 780 rpm

$$\text{slip, } s = \frac{1000-780}{1000} = 0.22$$

$$V_{d1} = \frac{3\sqrt{6}}{\pi}s\frac{V}{n} = \frac{3\sqrt{6}}{\pi} \times \frac{0.22 \times \frac{440}{\sqrt{3}}}{2} = 65.363 \text{ V}$$

$$V_{d2} = \frac{3\sqrt{6}}{\pi}\frac{V}{m}\cos\alpha$$

$$= \frac{3\sqrt{6}}{\pi}\frac{440}{\sqrt{3}\times 7.722}\times\cos 140^\circ = -58.95 \text{ V}$$

$$R'_S = 0.1 \times (0.5)^2 = 0.025,$$

$$R_r = 0.08 \times 0.5 \times 0.5 = 0.02$$

$$I_d = \frac{V_{d1}+V_{d2}}{2(SR'_s + R_r)+R_d} = \frac{65.363-58.95}{2(0.22\times 0.025+0.02)+0.01}$$

$$= 105.11 \text{ A}$$

$$\text{Torque, } T = \frac{|V_{d2}|I_d}{s\,\omega_{ms}} = \frac{58.95\times 105.11}{0.22\times 104.72} \approx 269 \text{ N-m}$$

For 800 rpm $\quad \text{slip, } s = \dfrac{1000-800}{1000} = 0.2$

**(iii)** $\quad \text{Rated slip} = \dfrac{1000-970}{1000} = 0.03$

$$\text{Rated torue} = \frac{\frac{3}{104.72}\times\left(\frac{440}{\sqrt{3}}\right)^2\times\frac{0.08}{0.03}}{\left((0.1)^2+\frac{0.08}{0.03}\right)^2+(0.7)^2} = 605.32 \text{ N-m}$$

$$\text{Half rated torque} = 302.66 \text{ N-m}$$

For 800 rpm, $\quad \text{slip, } s = \dfrac{1000-800}{1000} = 0.2$

$$V_{d1} = \frac{3\sqrt{6}}{\pi}\times\frac{0.2\times\frac{440}{\sqrt{3}}}{2} = 59.42 \text{ V}$$

$$V_{d2} = \frac{3\sqrt{6}}{\pi}\times\frac{\frac{440}{\sqrt{3}}}{7.722}\cos\alpha = 76.95\cos\alpha$$

$$I_d = \frac{59.42+76.95\cos\alpha}{2(0.2\times 0.025+0.02)+0.01} = 990.33 + 1282.5\cos\alpha$$

$$\text{Torque, } T = \frac{|V_{d2}|I_d}{s\omega_{ms}} = \frac{76.95|\cos\alpha|\times(990.33+1282.5\cos\alpha)}{0.2\times 104.72}$$

$$T = (3.673\,|\cos\alpha|)\,(990.33 + 1282.5\cos\alpha)$$

Let, $\quad \cos\alpha = -X,$

$$\text{Torque, } T = (3.673X)\,(990.33 - 1282.5X)$$

This should be equal to half rated torque 302.66 N-m

$$(3.673X)\,(990.33 - 1282.5X) = 302.66$$

$$X^2 - 0.772\,X + 0.06425 = 0$$

$$X = 0.677 \text{ and } 0.0949$$

$$\alpha = 132.6° \text{ and } 95.45°$$

Later value of α corresponds to operation in unstable part of characteristics.

## Section-B

**Q.5 (a) Solution:**

From the given data:

$$\text{Generator armature current} = \text{Motor armature current} - \text{Line current}$$

$$= 360 - 40 = 320 \text{ A}$$

$$\text{Input power} = VI_L = 220 \times 40 = 8800 \text{ W} = 8.8 \text{ kW}$$

$$\text{Armature copper loss} = \left(I_{am}^2 + I_{ag}^2\right)Ra = (360^2 + 320^2) \times 0.02 = 4.64 \text{ kW}$$

$$\text{Stray losses of each machine, } P_{st} = \frac{1}{2}(P_{\text{in}} - P_{\text{cu}}) = \frac{1}{2}(8.8 - 4.64) = 2.08 \text{ kW}$$

**(i) Motor:**

$$I_{fm} = 5.2 \text{ A}$$

$$\text{Input power to motor } P_{\text{in(m)}} = V(I_{am} + I_{fm})$$

$$= 220(360 + 5.2) = 80.344 \text{ kW}$$

Power losses to motor :

$$P_{LM} = P_{st} + I_{am}^2 \cdot R_a + VI_{fm}$$

$$= 2.08 + \frac{(360)^2 \times 0.02}{1000} + \frac{220 \times 5.2}{1000}$$

$$= 2.08 + 2.592 + 1.414 = 5.816 \text{ kW}$$

$$\text{Motor efficiency (in \%)} = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{in}} - P_{\text{loss}}}{P_{\text{in}}} = \frac{80.344 - 5.816}{80.344} \times 100$$

$$\%\eta_m = 92.76\%$$

**(ii) Generator:**

$$I_{fg} = 5 \text{ A}$$

$$\text{Output power} - P_{\text{out, g}} = 220 \times 320 = 70.4 \text{ kW}$$

Power losses in generator,

$$P_{L,g} = P_{st} + I_{ag}^2 \times R_a + V \cdot I_{fg}$$

$$= 2.08 + \frac{(320)^2 \times 0.02}{1000} + \frac{220 \times 6}{1000} = 5.448 \text{ kW}$$

Genertor efficiency, $$\%\eta_a = \frac{70.4}{70.4 + 5.448} \times 100 = 92.81\%$$

**Q.5 (b) Solution:**

The differential equations governing the mechanical translation system,

$$M_1 \frac{d^2X_1}{dt^2} + B_1 \frac{dX_1}{dt} + K_1X_1 + K_2(X_1 - X_2) = f_1(t)$$

$$M_2 \frac{d^2X_2}{dt^2} + K_2(X_2 - X_1) + K_3(X_2 - X_3) + B_3 \frac{d(X_2 - X_3)}{dt} = f_2(t)$$

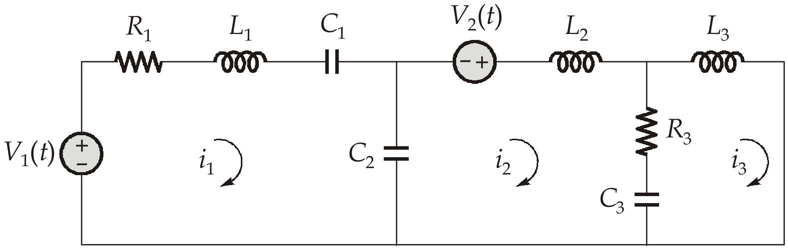
$$M_3 \frac{d^2X_3}{dt^2} + B_2 \frac{d(X_3 - X_2)}{dt} + K_3(X_3 - X_2) = 0$$

1. Force voltage equations to the given mechanical system,

$$V_1(t) = L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1 dt + R_1 i_1 + \frac{1}{C_2}\int (i_1 - i_2)dt$$

$$V_2(t) = L_2 \frac{di_2}{dt} + \frac{1}{C_1}\int (i_2 - i_1)dt + R_3(i_2 - i_3) + \frac{1}{C_3}\int (i_2 - i_3)dt$$

$$L_3 \frac{di_3}{dt} + \frac{1}{C_3}\int (i_3 - i_2)dt + R_3(i_3 - i_2) = 0$$

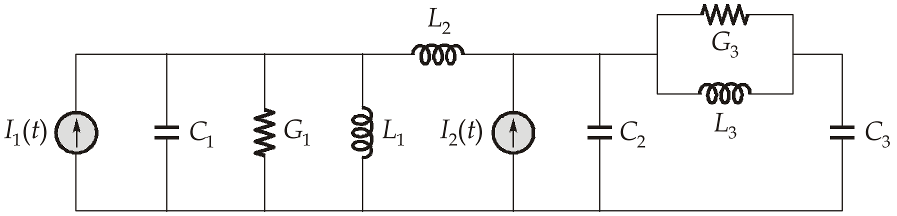


2. Force-current equations to the given mechanical system,

$$I_1(t) = C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int V_1 dt + G_1 V_1 + \frac{1}{L_2}(V_1 - V_2)dt$$

$$I_2(t) = C_2\frac{dV_2}{dt} + \frac{1}{L_2}\int(V_2 - V_1)dt + G_3(V_2 - V_3) + \frac{1}{L_3}\int(V_2 - V_3)dt$$

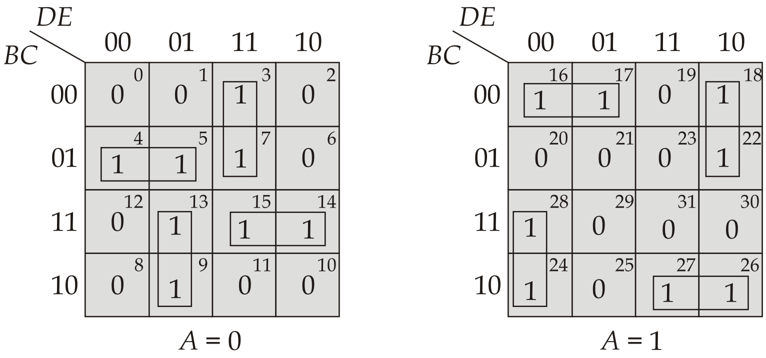
$$C_3\frac{dV_3}{dt} + \frac{1}{L_3}\int(V_3 - V_2)dt + G_3(V_3 - V_2) = 0$$



**Q.5 (c) Solution:**

Given, $F(A, B, C, D, E) = \pi M(0, 1, 2, 6, 8, 10, 11, 12, 19, 20, 21, 23, 25, 29, 30, 31)$

**K-map:**



| BC \ DE | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0$^{0}$ | 0$^{1}$ | 1$^{3}$ | 0$^{2}$ |
| 01 | 1$^{4}$ | 1$^{5}$ | 1$^{7}$ | 0$^{6}$ |
| 11 | 0$^{12}$ | 1$^{13}$ | 1$^{15}$ | 1$^{14}$ |
| 10 | 0$^{8}$ | 1$^{9}$ | 0$^{11}$ | 0$^{10}$ |

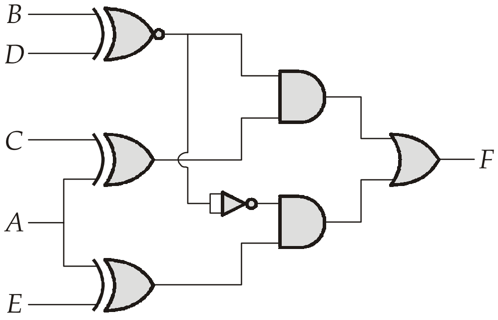
$A = 0$

| BC \ DE | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1$^{16}$ | 1$^{17}$ | 0$^{19}$ | 1$^{18}$ |
| 01 | 0$^{20}$ | 0$^{21}$ | 0$^{23}$ | 1$^{22}$ |
| 11 | 1$^{28}$ | 0$^{29}$ | 0$^{31}$ | 0$^{30}$ |
| 10 | 1$^{24}$ | 0$^{25}$ | 1$^{27}$ | 1$^{26}$ |

$A = 1$

$$\begin{aligned}
F &= \bar{A}(\bar{B}C\bar{D} + \bar{B}DE + B\bar{D}E + BCD) + A(\bar{B}\bar{C}\bar{D} + \bar{B}D\bar{E} + B\bar{D}\bar{E} + B\bar{C}D)\\
&= \bar{A}\left[C(\bar{B}\bar{D} + BD) + E(\bar{B}D + B\bar{D})\right] + A\left[\bar{C}(\bar{B}\bar{D} + BD) + \bar{E}(\bar{B}D + B\bar{D})\right]\\
&= \bar{A}C(B \odot D) + \bar{A}E(B \oplus D) + A\bar{C}(B \odot D) + A\bar{E}(B \oplus D)\\
&= (A \oplus C)(B \odot D) + (A \oplus E)(B \oplus D)
\end{aligned}$$

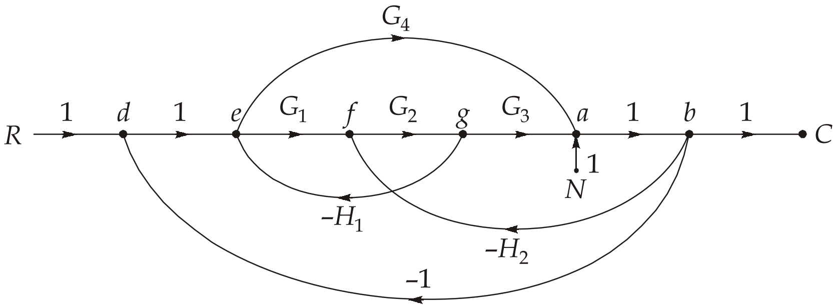
**Circuit:**



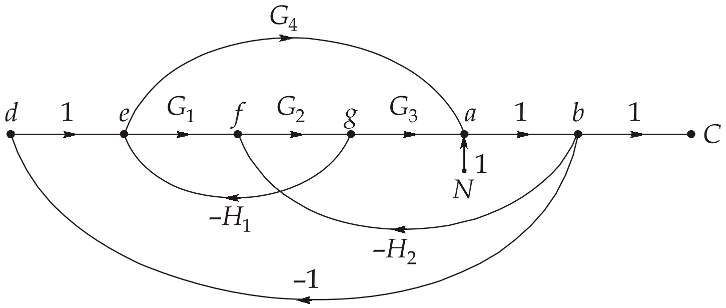
**Q.5 (d) Solution:**

To find $\frac{C}{N}$, assuming $R = 0$

Now converting the block diagram in signal flow graph,



For $R = 0$, the signal flow graph reduces to



**Forward Path:** There is only one forward path.

$$P_1 = Nabc \Rightarrow 1 \times 1 = 1$$

**Loops:**

$$L_1 = abdefga = 1 \times (-1) \times G_1G_2G_3 = -G_1G_2G_3$$

$$L_2 = abfga = 1 \times (-H_2)G_2G_3 = -G_2G_3H_2$$

$$L_3 = gef = -H_1G_1G_2 = -G_1G_2H_1$$

$$L_4 = abdea = 1 \times (-1) \times G_4 = -G_4$$

$$L_5 = fgeabf = G_2(-H_1)\,G_4(-H_2) = G_2G_4H_1H_2$$

$$\Delta_1 = 1 - L_3 = 1 + G_1G_2H_1$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$$

$$\therefore \quad \frac{C}{N} = \frac{P_1\Delta_1}{\Delta} = \frac{1\times(1+G_1G_2H_1)}{1-(L_1+L_2+L_3+L_4+L_5)}$$

$$\frac{C}{N} = \frac{(1+G_1G_2H_1)}{(1+G_1G_2G_3+G_1G_2H_1+G_2G_3H_2+G_4-G_2G_4H_1H_2)}$$

For $\frac{C}{N} = 0,$ the condition is

$$1 + G_1G_2H_1 = 0$$

$$G_1G_2H_1 = -1$$

$\therefore$ **Loop 3** $\Rightarrow$

$$L_3 = -G_1G_2H_1$$

$$L_3 = +1$$

Hence for $\frac{C}{N} = 0,$ the value of loop 3 gain, $L_3$ should be 1 or the value of $G_1G_2H_1$ should be –1.

**Q.5 (e) Solution:**

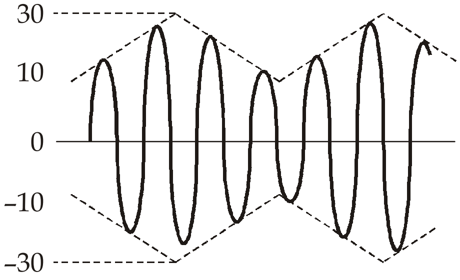
**(i)** Given,

$$\mu = 0.5 = \frac{m_p}{A} = \frac{10}{A}$$

(where, $m_p$ is the peak of triangular signal)

$\therefore$

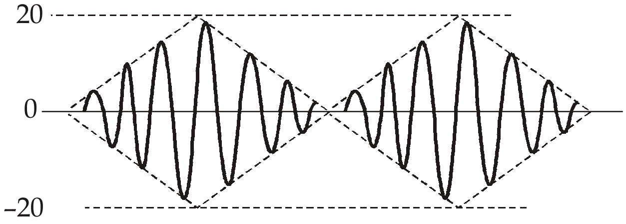
$$A = 20$$



**(ii)** Given,

$$\mu = 1 = \frac{m_p}{A} = \frac{10}{A}$$

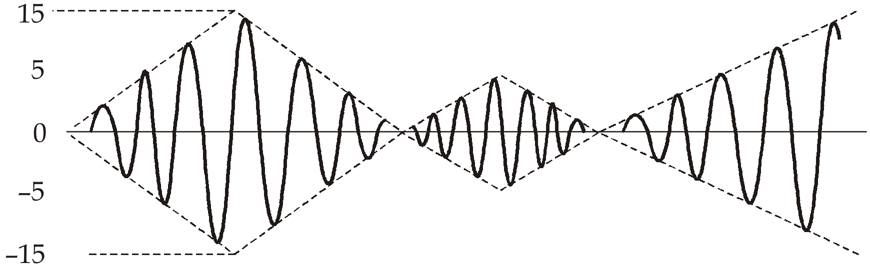
$$A = 10$$



**(iii)** Given,

$$\mu = 2 = \frac{m_p}{A} = \frac{10}{A}$$

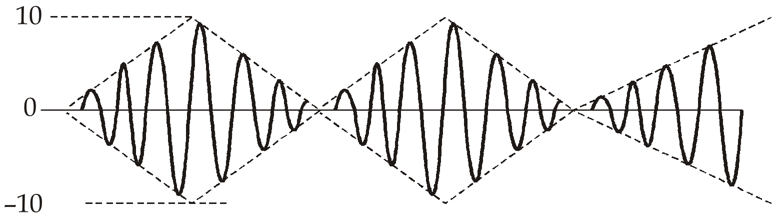
$$A = 5$$



**(iv)**

$$\mu = \infty = \frac{m_p}{A} = \frac{10}{A}$$

$$A = 0$$



### Q.6 (a) (i) Solution:

Output voltage,

$$V_0 = -V_s\left(\frac{D}{1-D}\right) = -24\left(\frac{0.4}{1-0.4}\right) = -16 \text{ V}$$

$$I_L = \frac{V_s D}{R(1-D)^2} = \frac{24\times 0.4}{5(1-0.4)^2} = 5.33 \text{ A}$$

$$\Delta i_L = \frac{V_s DT}{L} = \frac{24\times 0.4}{20\times 10^{-6}\times 10^5} = 4.8 \text{ A}$$

$$I_{L(\max)} = I_L + \frac{\Delta i_L}{2} = 5.33 + \frac{4.8}{2} = 7.73 \text{ A}$$

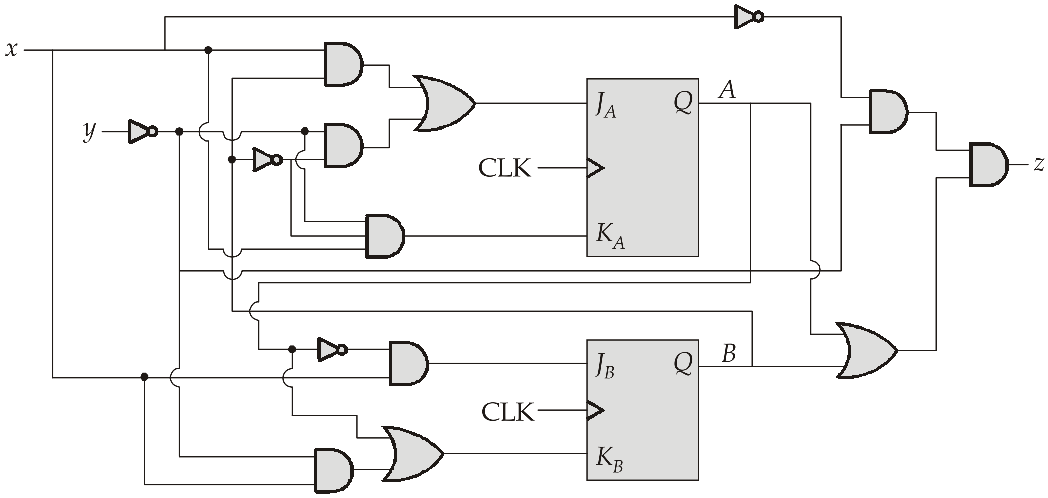
$$I_{L(\min)} = I_L - \frac{\Delta i_L}{2} = 5.33 - \frac{4.8}{2} = 2.93 \text{ A}$$

$$\frac{\Delta V_0}{V_0} = \frac{D}{RCf} = \frac{0.4}{5\times 80\times 10^{-6}\times 100\times 10^3} = 0.01 = 1\%$$

### Q.6 (a) (ii) Solution:

Given, $J_A = Bx + \bar{B}\bar{y}$; $K_A = \bar{B}x\bar{y}$; $J_B = \bar{A}x$; $K_B = A + x\bar{y}$; $Z = A\bar{x}\bar{y} + B\bar{x}\bar{y}$

**1.**



2.

| Present state | | Input | | Next State | | O/p | Flip flop inputs | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | $B$ | $x$ | $y$ | $A(t+1)$ | $B(t+1)$ | $Z$ | $J_A$ | $K_A$ | $J_B$ | $K_B$ |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |

## Q.6 (b) Solution:

**(i)** In order to find the parallel realisation of the given IIR digital filter, the partial fraction expansion of $\frac{H(z)}{z}$ is determined.

Given,
$$H(z) = \frac{3\left[2z^2+5z+4\right]}{(2z+1)(z+2)}$$

$$\therefore \qquad \frac{H(z)}{z} = \frac{\frac{3}{2}\left[2z^2+5z+4\right]}{z\left(z+\frac{1}{2}\right)(z+2)}$$

Now using partial fraction expansion,

$$\frac{H(z)}{z} = \frac{A}{z}+\frac{B}{\left(z+\frac{1}{2}\right)}+\frac{C}{(z+2)}$$

$$A = \left.\frac{\frac{3}{2}(2z^2+5z+4)}{\left(z+\frac{1}{2}\right)(z+2)}\right|_{z=0} = \frac{\frac{3}{2}[4]}{1} = 6$$

$$A = 6$$

$$B = \left.\frac{\frac{3}{2}(2z^2+5z+4)}{(z+2)z}\right|_{z=-\frac{1}{2}} = \frac{\frac{3}{2}\left(2\times\frac{1}{4}-\frac{5}{2}+4\right)}{-0.5\times1.5}$$

$$B = -4$$

$$C = \left.\frac{\frac{3}{2}(2z^2+5z+4)}{\left(z+\frac{1}{2}\right)z}\right|_{z=-2} = \frac{\frac{3}{2}[2\times4-5\times2+4]}{(-2)\times(-1.5)}$$

$$C = 1$$

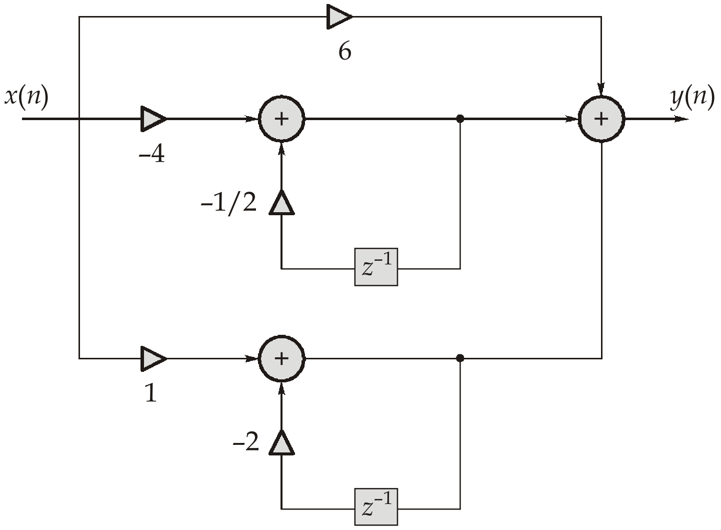
$$\therefore \quad \frac{H(z)}{z} = \frac{6}{z}+\frac{(-4)}{\left(z+\frac{1}{2}\right)}+\frac{1}{(z+2)}$$

$$\frac{H(z)}{z} = \frac{6}{z}-\frac{4}{\left(z+\frac{1}{2}\right)}+\frac{1}{(z+2)}$$

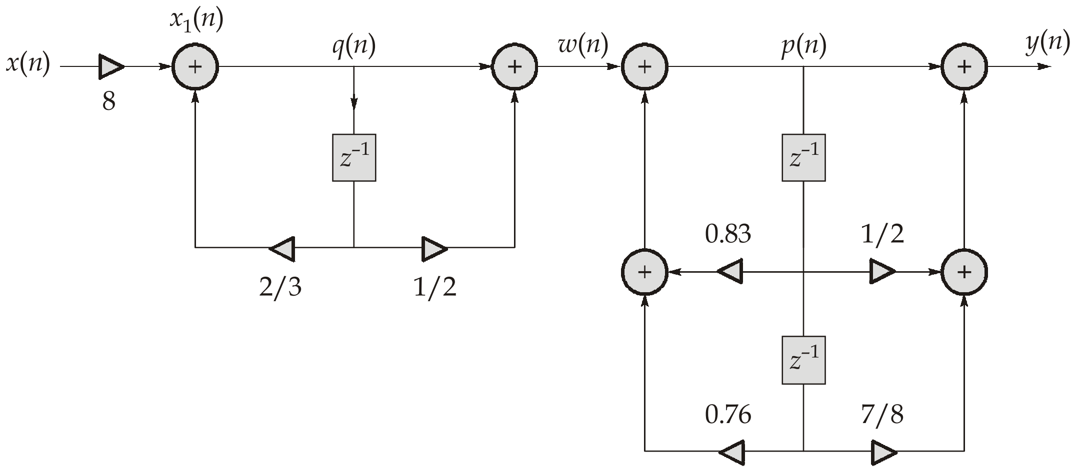
$$H(z) = 6-\frac{4z}{\left(z+\frac{1}{2}\right)}+\frac{z}{(z+2)}$$

$$H(z) = 6-\frac{4}{\left[1+\frac{1}{2}z^{-1}\right]}+\frac{1}{[1+2z^{-1}]}$$

∴ The parallel realization of $H(z)$ is



**(ii)** From the given figure,



Let $\frac{W(z)}{X_1(z)} = H_1(z)$ and $\frac{Y(z)}{W(z)} = H_2(z)$

From the figure,

$$q(n) = x_1(n) + q(n-1) \times \frac{2}{3}$$

$$Q(z) = X_1(z) + \frac{2}{3} z^{-1} Q(z)$$

$$Q(z)\left[1 - \frac{2}{3} z^{-1}\right] = X_1(z)$$

$$\frac{Q(z)}{X_1(z)} = \frac{1}{\left(1 - \frac{2}{3} z^{-1}\right)} \quad \text{...(i)}$$

Also,

$$w(n) = q(n) + q(n-1)\frac{1}{2}$$

$$W(z) = Q(z) + \frac{z^{-1}}{2} Q(z)$$

$$W(z) = Q(z)\left[1 + \frac{1}{2} z^{-1}\right]$$

$$\frac{W(z)}{Q(z)} = \left[1 + \frac{1}{2} z^{-1}\right] \quad \text{...(ii)}$$

From eqn. (i) and (ii),

$$\frac{W(z)}{X_1(z)} = H_1(z) = \frac{\left(1 + \frac{1}{2} z^{-1}\right)}{\left(1 - \frac{2}{3} z^{-1}\right)} \quad \text{...(iii)}$$

Now again from the figure,

$$p(n) = w(n) + p(n-1) \times 0.83 + p(n-2) \times 0.76$$

$$P(z) = W(z) + z^{-1} \times 0.83P(z) + 0.76z^{-2}P(z)$$

$$P(z)[1 - 0.83z^{-1} - 0.76z^{-2}] = W(z)$$

$$\frac{P(z)}{W(z)} = \frac{1}{\left[1-0.83z^{-1}-0.76z^{-2}\right]} \quad \text{...(iv)}$$

Also,

$$y(n) = p(n)+\frac{1}{2}p(n-1)+\frac{7}{8}p(n-2)$$

$$Y(z) = P(z)+\frac{1}{2}z^{-1}P(z)+\frac{7}{8}z^{-2}P(z)$$

$$Y(z) = P(z)\left[1+\frac{1}{2}z^{-1}+\frac{7}{8}z^{-2}\right]$$

$$\frac{Y(z)}{P(z)} = \left[1+\frac{1}{2}z^{-1}+\frac{7}{8}z^{-2}\right] \quad \text{...(v)}$$

From eqn. (iv) and (v),

$$\frac{Y(z)}{W(z)} = H_2(z) = \frac{\left(1+\frac{1}{2}z^{-1}+\frac{7}{8}z^{-2}\right)}{\left[1-0.83z^{-1}-0.76z^{-2}\right]} \quad \text{...(vi)}$$

Now from eqn. (iii) and (vi),

The system function $H(z) = 8.H_1(z) \cdot H_2(z)$

$$H(z) = \frac{8\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{2}{3}z^{-1}\right)} \cdot \frac{\left(1+\frac{1}{2}z^{-1}+\frac{7}{8}z^{-2}\right)}{\left[1-0.83z^{-1}-0.76z^{-2}\right]}$$

## Q.6 (c) Solution:

**(i)**

$$X_s \text{ (adjusted)} = \left.\frac{V_{\text{rated}}/\sqrt{3}}{I_{SC}}\right|_{\text{At } I_f \text{ corresponding to } V_{\text{oc}} = V_{\text{rated}}}$$

Rated armature current,

$$\sqrt{3}\, V_{\text{rated}}\, I_{a\text{ (rated)}} = 10 \text{ MVA}$$

$$I_{a\text{ (rated)}} = \frac{10\times10^3}{\sqrt{3}\times13.8} = 418.4\text{ A}$$

At $I_f = 842$ A, $I_{SC} = \frac{418.4}{226}\times842 = 1558.8$ A

$$X_s \text{ (adjusted)} = \frac{13.8\times10^3/\sqrt{3}}{1558.8} = 5.11\,\Omega$$

Base values, $(\text{MVA})_B = 10,\ (\text{kV})_B = 13.8$

$$X_s \text{ (pu)} = 5.11\times\frac{10}{(13.8)^2} = 0.2683$$

$$R_a = 0.75\,\Omega$$

$$\bar{Z}_s = 0.75 + j5.11 = 5.16\angle 81.65^\circ\,\Omega$$

$$Z_s = 5.16\,\Omega,\ \theta = 81.65^\circ,\ \alpha = 8.35^\circ$$

**(ii) 1.** p.f. = 0.9 lagging;

$$\phi = \cos^{-1} 0.9 = 25.84^\circ$$

$$P_e = 8.75 \text{ MW}$$

$$\tan\phi = \frac{Q_e}{P_e}$$

$$Q_e = P_e \tan\phi = 8.75 \tan 25.84^\circ$$

$= 4.24$ MVA, positive as it is lagging

For calculating field current, we need excitation emf.
Hence on per phase basis,

$$V_t = \frac{13.8}{\sqrt{3}} = 7968 \text{ V}$$

$$\frac{P_e}{3} = V_t I_a \cos\phi$$

$$\frac{8.75\times10^6}{3} = 7968\, I_a \times 0.9$$

or $I_a = 406.7 \text{ A}, \bar{I}_a = 406.7\angle -25.8^\circ$

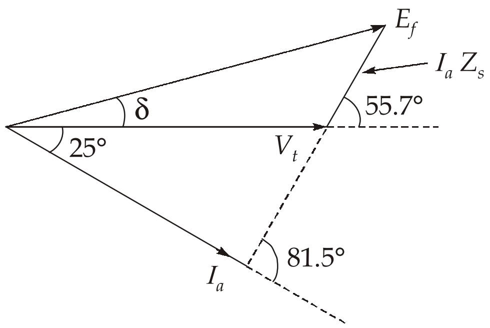
$$\bar{I}_a \bar{Z}_s = 406.7 \times 5.16\angle 81.65^\circ - 25.8^\circ = 2098.67\angle 55.85^\circ \text{ V}$$

Generator equation, $\bar{E}_f = \bar{V}_t + \bar{I}_a \bar{Z}_s$

or $E_f = 7968\angle 0^\circ + 2098.67\angle 55.85^\circ$

$= 9309.56\angle 10.75^\circ$ V

The phasor diagram is drawn in figure below,



$$E_f = 9309.56 \text{ or } 16.124 \text{ kV (line)}$$

From the modified air-gap line

$$I_f = \frac{842}{13.8} \times 16.124 = 983.84 \text{ A}$$

Ohmic loss, $3I_a^2R_a = 3 \times (406.7)^2 \times 0.75 = 0.372 \text{ MW}$

**2.** $$P_m\text{(in)} = \frac{E_f^2}{Z_s^2}R_a + \frac{V_t E_f}{Z_s}\sin(\delta - \alpha) \quad \text{...(i)}$$

Field current adjusted to

$$I_f = 842 \text{ A}$$

$$E_f = V_{OC} = V_t \text{(rated)} = 7968 \text{ V} = 7.968 \text{ kV}$$

$$P_m\text{(in)} = 9.122/3 = 3.04 \text{ MW per phase}$$

Substituting in equation (i)

$$3.04 = \frac{(7.968)^2}{(5.16)^2} \times 0.75 + \frac{(7.968)^2}{(5.16)^2}\sin(\delta - \alpha)$$

$$1.252 = \frac{(7.968)^2}{(5.16)}\sin(\delta - \alpha)$$

$$\sin(\delta - \alpha) = 0.1017$$

$$\delta - \alpha = 5.84°$$

$$\delta = 5.84° + 8.35° = 14.19°$$

$$Q = \frac{V_t^2}{Z_s^2}X_s + \frac{V_t E_f}{Z_s}\cos(\delta + \alpha)$$

or $$Q = -\frac{(7.968)^2}{(5.16)^2} \times 5 + \frac{(7.968)^2}{(5.16)}\cos(14.19° + 8.35°)$$

or $$Q = -0.558 \text{ MVAR/phase}$$

or $$Q = +1.675 \text{ MVAR, leading}$$

## Q.7 (a) (i) Solution:

**Power factor improvement:** Forced commutation techniques for ac-dc converters can improve input power factor. Some methods are:

1. **Extinction angle control:** The switching actions performed by gate-turn-off thyristors (GTOs). The characteristics of GTOs allows turning on and turning off by applying short positive and negative pulses on its gate respectively.

   Performance of semi and full converters with this control method is similar to that with phase angle control except the power factor is leading.

2. **Symmetric angle control:** Allows one quadrant operation. The fundamental component of input current is in phase with the input voltage and the displacement factor is unity. Therefore, the power factor is improved.

   This control can be applied for the same half controlled forced commutated bridge converter with two switches.

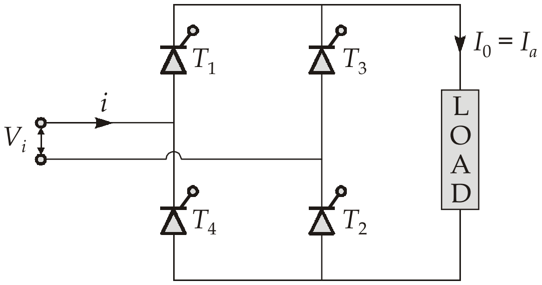
3. **Pulse width modulation control:** The converter switches are turned-on and off several times during a half-cycle. Thus, the harmonics can be eliminated or reduced and the output voltage is controlled by varying width of pulses.

   The lower order harmonics can be eliminated by selecting the number of pulses per half cycle. However, increasing the pulses can increase the magnitude of higher order harmonics, which can be easily filtered out.

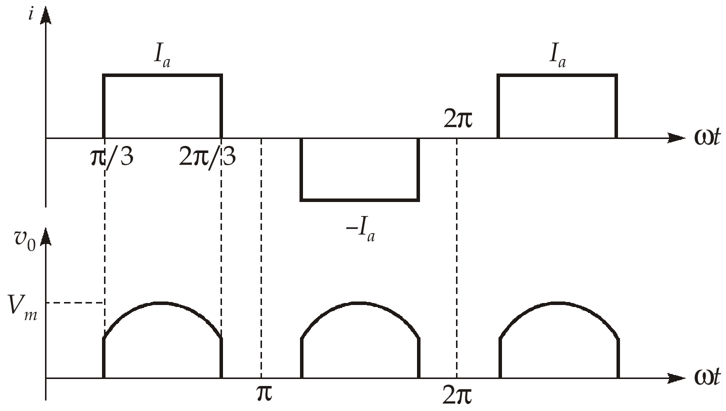
4. **Sinusoidal pulse-width modulation:** In this pulse widths are not uniform (like in PWM), the pulse widths are generated by comparing a triangular voltage with a sinusoidal voltage. The power factor is further improved.

   The width of pulses (and the output voltage) are varied by changing the amplitude of sinusoidal voltage or the modulation index. In sinusoidal PWM control, the displacement factor is unity and power factor is improved.

## Q.7 (a) (ii) Solution:



(Single-phase full converter)



$$i(t) = I_{dc} + \sum_{n=1,2,3}^{2\pi} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$I_{dc} = \frac{1}{2\pi}\left\{\int_{\pi/3}^{2\pi/3} I_a\, d(\omega t) - \int_{4\pi/3}^{5\pi/3} I_a\, d(\omega t)\right\} = 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} i(t)\cos(n\omega t)\, d(\omega t) = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} i(t)\sin(n\omega t)\, d(\omega t)$$

$$= \frac{4I_a}{n\pi}\sin\frac{n\pi}{6}, \quad n = 1, 3, 5.... \text{ and } b_n = 0 \text{ for } n = \text{even}$$

$$i(t) = 4I_a \sum_{n=1,3,5}^{\infty} \frac{\sin\left(\frac{n\pi}{6}\right)}{n\pi}\sin(n\omega t)$$

Rms input current, $I_{s,r} = I_a\sqrt{\frac{\pi/3}{\pi}} = \frac{I_a}{\sqrt{3}}$ $(\because I_a = I_0)$

$$I_{s1} = \left(\frac{4I_a}{\sqrt{2}}\right)\times\frac{1}{\pi}\sin\left(\frac{\pi}{6}\right) = 0.4502\, I_a$$

Hence, $\text{HF} = \left[\left(\frac{I_{sr}}{I_{s1}}\right)^2 - 1\right]^{1/2} = 0.803$

$$\text{Also, harmonic factor,HF} = \left[\left(\frac{I_{s,r}}{I_{s1}}\right)^2 - 1\right]^{1/2} = \left[\frac{\pi\times\frac{\pi}{3}}{4\left(1-\cos\frac{\pi}{3}\right)} - 1\right]^{1/2} = 0.803$$

$$\text{DF} = \cos\phi_1 = \cos 0° = 1$$

$$\text{PF} = \left(\frac{I_1}{I_{s,r}}\right)\text{DF} = \frac{2\sqrt{2}I_a\sin\left(\frac{\pi}{6}\right)}{\left(\pi\cdot\frac{I_a}{\sqrt{3}}\right)}\times 1 = 0.78\ (\text{lag})$$

**Q.7 (b) Solution:**

The given open-loop system has no poles in the right-half of $s$-plane. So, $P = 0$. For the closed-loop system to be stable, the Nyquist plot must not encircle the $(-1 + j0)$ point of the $q(s)$ plane.

The given open-loop transfer function in sinusoidal form is

$$G(j\omega)H(j\omega) = \frac{K}{j\omega\left[(j\omega)^2 + j\omega + 2\right]} = \frac{K}{j\omega\left[\left(2-\omega^2\right)+j\omega\right]}$$

$$= \frac{K\left[\left(2-\omega^2\right)-j\omega\right]}{j\omega\left[\left(2-\omega^2\right)+j\omega\right]\left[\left(2-\omega^2\right)-j\omega\right]}$$

$$= \frac{K\left[\left(2-\omega^2\right)-j\omega\right]}{j\omega\left[\left(2-\omega^2\right)^2+\omega^2\right]} = -\frac{K}{\left(2-\omega^2\right)^2-\omega^2} - j\frac{K\left(2-\omega^2\right)}{\omega\left[\left(2-\omega^2\right)^2+\omega^2\right]}$$

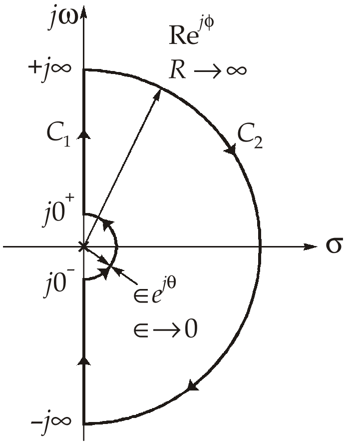


Fig. (a)

Along the segment ($C_1$) of the Nyquist contour on the $j\omega$-axis, $s$ varies from – $j\infty$ to + $j\infty$

At $\omega = -\infty$, $G(j\omega)H(j\omega) = -0 - j0$

At $\omega = 0^-$, $G(j\omega)H(j\omega) = -(K/4) + j\infty$

At $\omega = 0^+$, $G(j\omega)H(j\omega) = -(K/4) - j\infty$

At $\omega = +\infty$, $G(j\omega)H(j\omega) = -0 + j0$

So, we get four points to draw an approximate Nyquist plot. The semicircular indent around the pole at the original Nyquist contour of figure (a) represented by $s = \underset{\in \to 0}{\in e^{j\theta}}$ (where θ varies from – 90° through 0° to +90°) is mapped into

$$\underset{\in\to 0}{\text{Lt}} \frac{K}{\in e^{j\theta}\left[\left(\in e^{j\theta}\right)+\left(\in e^{j\theta}+2\right)\right]} = \underset{\in\to 0}{\text{Lt}} \frac{K}{2\in e^{j\theta}} = \infty e^{-j\theta}$$

$$= \infty \angle 90° \to \angle 0° \to \angle -90°$$

It is an infinite circular arc with clockwise directions.

The infinite semicircular arc of the Nyquist contour (segment $C_2$) of figure (a), represented by $R = \underset{R\to\infty}{\text{Re}^{j\theta}}$ (ϕ varying from +90° through 0° to – 90°) is mapped into

$$\underset{R\to\infty}{\text{Lt}} \frac{K}{\text{Re}^{j\phi}\left[\left(\text{Re}^{j\phi}\right)^2 + \text{Re}^{j\phi} + 2\right]} = \underset{R\to\infty}{\text{Lt}} \frac{K}{R^3 e^{j3\phi}} = 0\, e^{-j3\phi}$$

$$= 0\angle -270° \to \angle 0° \to \angle +270°)$$

The map turns around the origin from ∠–270° → ∠0° → ∠ +270° as sketched figure.

The point of intersection of the Nyquist plot with the real axis is obtained by equating is imaginary part of $G(j\omega)H(j\omega)$ to zero, i.e.

$$-\frac{K\left(2-\omega^2\right)}{\omega\left[\left(2-\omega^2\right)^2+\omega^2\right]} = 0$$

i.e. $\omega^2 = 0$

or $\omega = \sqrt{2}$

The value of $G(j\omega)H(j\omega)$ at that point is obtained by substituting this value of co in the real pan of $G(j\omega)H(j\omega)$, i.e.

$$-\left.\frac{K}{\left(2-\omega^2\right)^2+\omega^2}\right|_{\omega^2=2} = \frac{-K}{2}$$

Based on the above information, an approximate Nyquist plot is drawn as shown in figure (b). It crosses the real axis at ω = $\pm\sqrt{2}$ with an intercept of – $K/2$.

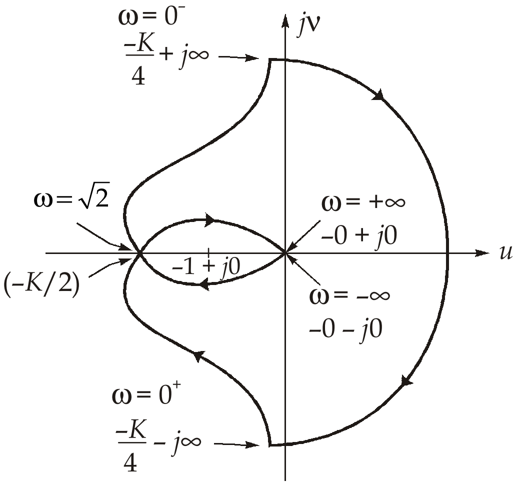


Fig. (b)

From the Nyquist plot

For $K/2 > 1$

i.e. for $K > 2$

the Nyquist plot encircles the (– 1 + $j$0) point twice in the clockwise direction.

$$N = P - Z$$

$$-2 = 0 - Z$$

or $Z = 2$

So the system is unstable. For $K < 2$, the system will be stable.

**Q.7 (c) Solution:**

Given,

Message signal, $m(t) = 2 \sin 2000\pi t - 3 \cos 4000\pi t$

Modulation index, $\mu = 70\% = 0.7$

Carrier frequency, $f_c = 580$ kHz or $A_c \cos(2\pi \times 580 \times 10^3 t)$

The general Am signal is

$$s(t) = A_c[1 + \mu m(t)]\cos 2\pi f_c t$$

∴ $$s(t) = A_c\left[1 + 0.7(2\sin 2000\pi t - 3\cos 4000\pi t)\right] \cos 1160 \times 10^3 \pi t$$

$$= A_c[1 + 1.4\sin 2000\pi t - 2.1\cos 4000\pi t]\cos 1160 \times 10^3 \pi t$$

$$= A_c \cos 1160 \times 10^3 \pi t + 1.4\, A_c \sin 2000\pi t \cdot \cos 1160 \times 10^3 \pi t$$

$$-2.1\, A_c \cos 4000\pi t \cdot \cos 1160 \times 10^3\, \pi t$$

$$s(t) = A_c \cos 1160 \times 10^3 \pi t + \frac{1.4A_c}{2}\left[\sin(1162 \times 10^3 \pi t) + \sin(1158 \times 10^3 \pi t)\right]$$

$$-\frac{2.1A_c}{2}[\cos(1164 \times 10^3 \pi t) + \cos(1156 \times 10^3 \pi t)]$$

$$\therefore \quad s(t) = A_c \cos 11060 \times 10^3 \pi t + 0.7\, A_c[\sin(1162 \times 10^3 \pi t) + \sin(1158 \times 10^3 \pi t)$$

$$- 1.05\, A_c[\cos(1164 \times 10^3 \pi t) + \cos\,(1156 \times 10^3 \pi t)]$$

**(i)** Given,

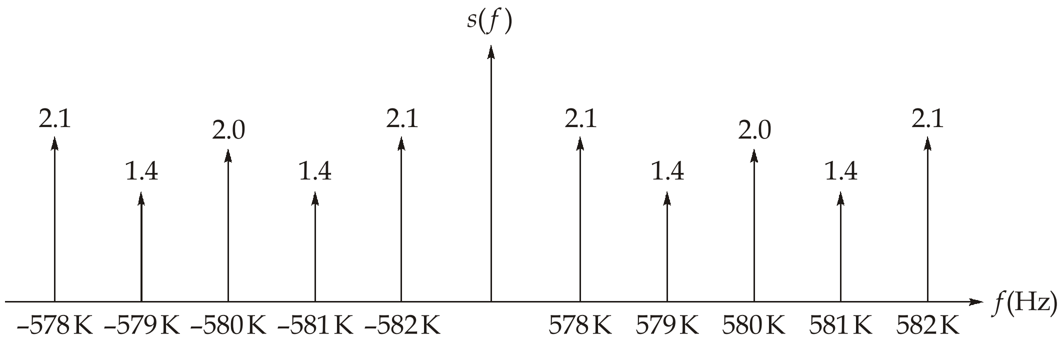
$$P_T = 10 \text{ Watts}$$

The frequency spectrum of the *Am* signal $s(t)$

$$s(t) = 2[\delta(f - 580 \text{ k}) + \delta(f + 580 \text{ k})] + 1.4[\delta(f - 581 \text{ k}) - \delta(f + 581 \text{ k})$$

$$+ \delta(f - 579 \text{ k}) - \delta(f + 579 \text{ k})] - 2.1[\delta(f - 582 \text{ k}) + \delta(f + 582 \text{ k})$$

$$+ \delta(f - 578 \text{ k}) + \delta(f + 578 \text{ k})]$$



New modulation index,

$$\mu' = \sqrt{(0.7)^2 + (1.05)^2} = 1.26$$

$$P_T = P_C\left[1 + \frac{\mu'^2}{2}\right]$$

$$10 = P_C\left[1 + \frac{(1.26)^2}{2}\right]$$

$$\therefore \quad P_C = \frac{10}{1 + \frac{(1.26)^2}{2}} = 5.575 \text{ Watts}$$

but $$P_C = \frac{A_C^2}{2} = 5.575$$

$$\therefore \quad A_C = \sqrt{11.15} = 3.34 \text{ V}$$

∴ The Am signal $s(t)$ is

$$s(t) = 4\cos 1160 \times 10^3\pi t + 2.34[\sin(1162 \times 10^3\pi t) + \sin(1158 \times 10^3\pi t] - 3.507[\cos(1164 \times 10^3\pi t) + \cos(1156 \times 10^3\pi t)]$$

**(ii)** The power efficiency,

$$\eta = \frac{\mu'^2}{2+\mu'^2} = \frac{1.26^2}{2+1.26^2} = 0.4425$$

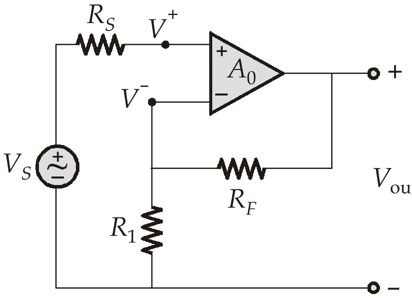
or $$\%\eta = 44.25\%$$

**Q.8 (a) Solution:**

Given, $V_S = 100$ mV, $R_1 = 5$ kΩ,

$R_F = 395$ kΩ

$R_S = 500\ \Omega = 0.5$ kΩ, $A_0 = 2 \times 10^5$



**(i)**
$$V_0 = A_0(V^+ - V^-) \quad \text{...(i)}$$

$$V^+ = V_S \quad \text{...(ii)}$$

Using voltage divider, $$V^- = \frac{R_1}{R_1 + R_F}V_0 \quad \text{...(iii)}$$

$$V_0 = A_0\left[V_S - \frac{R_1}{R_1+R_F}V_0\right]$$

$$\frac{1}{A_0}V_0 + \frac{R_1}{R_1+R_F}V_0 = V_S$$

$$A_f = \frac{V_0}{V_S} = \frac{1}{\left(\frac{1}{A_0} + \frac{R_1}{R_1+R_F}\right)}$$

$$A_f = \frac{A_0(R_1+R_F)}{R_1+R_F+A_0R_1} = \frac{A_0(R_1+R_F)}{R_F+R_1(1+A_0)}$$

$$= \frac{A_0(R_1 + R_F)}{R_F + R_1(1 + A_0)}$$

**(ii)**

$$A_f = \frac{2 \times 10^5(5+395)}{395 + 5(1 + 2 \times 10^5)} = 79.968$$

**(iii)**

$$V_{\text{out}} = A_f \cdot V_S = 79.968 \times 100 \times 10^{-3}$$

$$V_{\text{out}} = 7.9968 \text{ Volt}$$

**(iv)** When $A_0 \to \infty$

$$A_f' = 1 + \frac{R_F}{R_1} = 1 + \frac{395}{5} = 80$$

Error in gain $A_f$,

$$\% \text{ error in gain} = \frac{79.968 - 80}{80} \times 100\%$$

$$\% \, \Delta A_F = -0.04\%$$

Output voltage when $A_f \to \infty$

$$V'_{\text{out}} = A'_f \cdot V_S = 80 \times 100 \times 10^{-3} = 8 \text{ Volt}$$

% error in voltage, $$\Delta V_0 = \frac{7.9968 - 8}{8} \times 100 = -0.04\%$$

**Q.8 (b) Solution:**

Given, $$G(s)H(s) = \frac{K}{s(s^2 + 13s + 121)}$$

$$G(j\omega)\,H(j\omega) = \frac{K}{j\omega\left[(j\omega)^2 + 13(j\omega) + 121\right]} = \frac{K}{j\omega(121 - \omega^2 + j13\omega)}$$

$$\text{Gain margin, G.M.} = 20\log\left\{\frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}\right\}$$

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = \frac{K}{\omega\left[\sqrt{(121-\omega^2) + (13\omega)^2}\right]_{\omega=\omega_{pc}}}$$

For $\omega_{pc}$,

$$\angle G(j\omega)H(j\omega)\big|_{\omega=\omega_{pc}} = -180°$$

$$-90° - \tan^{-1}\left(\frac{13\omega_{pc}}{121-\omega_{pc}^2}\right) = -180°$$

$$-\tan^{-1}\left(\frac{13\omega_{pc}}{121-\omega_{pc}^2}\right) = -90°$$

$$\tan^{-1}\left(\frac{13\omega_{pc}}{121-\omega_{pc}^2}\right) = 90°$$

$$\therefore \quad 121-\omega_{pc}^2 = 0$$

$$\therefore \quad \omega_{pc} = 11 \text{ rad/sec}$$

$$\therefore \quad |G(j\omega)H(j\omega)|_{\omega_{pc}=11\text{ rad/sec}} = \frac{k}{(11)\sqrt{(121-11^2)+(13\times 11)^2}} = \frac{k}{1573}$$

Given, Gain margin, G.M. = +12 dB

$$12 = 20\log\left|\frac{1}{\left(\frac{k}{1573}\right)}\right|$$

i.e.,

$$\frac{12}{20} = \log\left\{\frac{1573}{k}\right\}$$

$$3.9810 = \frac{1573}{k}$$

i.e,

$$k = 395.119$$

For phase margin,

$$\text{P.M.} = 180° + \angle G(j\omega)H(j\omega)|_{\omega=\omega_{gc}}$$

For $\omega_{gc}$,

$$|G(j\omega)H(j\omega)| = 1$$

$$\frac{395.119}{|j\omega(121-\omega^2)+j13\omega|_{\omega=\omega_{gc}}} = 1$$

$$156119.0242 = \omega_{gc}^2\left[(121-\omega_{gc}^2)^2+169\omega_{gc}^2\right]$$

By trial and error method,

$$\omega_{gc} = 3.35 \text{ rad/sec}$$

$$\text{P.M.} = 180° + \left\{\frac{\angle k}{\angle j\omega\angle\left\{\left[121-(3.35)^2\right]+j13\times 3.35\right\}}\right\}$$

$$= 180° + \left\{\frac{0°}{90°+21.638°}\right\} = 180° - 111.638°$$

$$\text{P.M.} = 68.36°$$

**Q.8 (c) Solution:**

Reactance of generators rating 50 MVA = $j\,0.1$ p.u.

Reactance of the transformer = $j\,0.08$ p.u.

Reactance of 60 MVA generator = $j\,0.12$ p.u.

Reactance of the reactor = $j\,0.20$ p.u.

Let us choose 60 MVA and 11 kV as base quantities

∴ Base MVA = 60 MVA, $V_b = 11$ kV

∴ $X_G = j\,0.12$ p.u.

Reactance of the generator having rating 50 MVA or the base of 60 MVA

$$= j0.1\times\frac{\text{MVA}_b}{\text{Rating}}$$

$$X''_{g1} = X''_{g2} = X''_g = j0.1\times\frac{60}{50} = j0.12 \text{ p. u.}$$

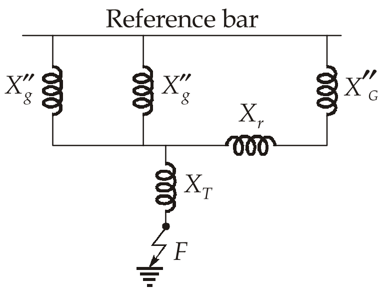
Reactance of the transformer on the base of 50 MVA

$$X_T = j0.08\times\frac{\text{MVA}_b}{\text{Rating}} = j0.08\times\frac{60}{40} = j\,0.12 \text{ p.u.}$$

The reactance of the reactance on the base of 60 MVA = $j0.20\times\dfrac{\text{MVA}_b}{\text{Rating}}$

$$X_r = j0.20\times\frac{60}{80} = j0.15 \text{ p.u.}$$

The reactance diagram can be drawn as



One fault impedance is given by

$$Z_f = (X''_{g1} || X''_{g2}) || (X_r + X''_G) + X_T$$

$$= \frac{X''_g}{2} || (X_r + X''_G) + X_T$$

$$= \frac{j0.12}{2} || (j0.15 + j0.12) + j0.12 = (j0.06 || j0.27) + j0.12$$

$$= \frac{j0.06 \times j0.27}{j(0.06 + 0.27)} + j0.12 = j0.169 \text{ p.u. and } V_f = 1\angle 0° \text{ p.u.}$$

$$\text{Fault current } (I_f) = \frac{V_f}{Z_g} = \frac{1}{j0.169} = -j5.914 \text{ p.u.}$$

Now, base current on H.V. side of the transformer ($I_{bH}$)

$$= \frac{60 \times 10^6}{\sqrt{3} \times 66 \times 10^3} = 524.864 \text{ A}$$

Fault current on H.V. side

= $I_f$ in p.u. × base current

= 5.914 × 524.864 = 3104.046 A

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