MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

# ESE-2024 Mains Test Series

# Electrical Engineering Test No : 12

## Section-A

**Q.1 (a) Solution:**

```
#include<stdio.h>
#define MAX 5
int main(){
    int i,j;
int space=4;
    /*run loop (parent loop) till number of rows*/
for(i=0; i< MAX; i++) {
    /*loop for initially space, before star printing*/
for(j=0; j< space; j++) {
    printf(" ");
}
for(j=0; j<=i; j++){
    printf(" * ");
}
printf("\n");
space --;
}
/*repeat it again*/
```

```
space=0;
    /*run loop (parent loop) till number of rows*/
for(i=MAX; i>0; i--){
    /*loop for initially space, before star printing*/
for(j=0; j< space; j++){
    printf(" ");
}
for(j=0;j< i;j++){
    printf(" * ");
}
printf("\n");
space++;
}
return 0;
}
```

**Q.1 (b) Solution:**

**(i)** Given, electric flux density,

$$\bar{D} = D_m \sin(\omega t + \beta)\hat{a}_x$$

From Maxwell's equation,

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

and $$\bar{D} = \epsilon_0 \bar{E}$$

$\Rightarrow$ $$\bar{E} = \frac{\bar{D}}{\epsilon_0} \text{ for free space}$$

$$-\frac{\partial \bar{B}}{\partial t} = \bar{\nabla} \times \bar{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{D_m}{\epsilon_0}\sin(\omega t + \beta z) & 0 & 0 \end{vmatrix}$$

$$= \frac{D_m}{\epsilon_0}\frac{\partial}{\partial z}(\sin(\omega t + \beta z))\hat{a}_y = \frac{D_m \beta}{\epsilon_0}\cos(\omega t + \beta z)\hat{a}_y$$

$$\bar{B} = \int(\bar{\nabla}\times\bar{E})\cdot dt = -\frac{D_m\beta}{\epsilon_0}\int\cos(\omega t+\beta z)\hat{a}_y\,dt$$

$$= -\frac{D_m\beta}{\omega\epsilon_0}\sin(\omega t+\beta z)\hat{a}_y$$

Also for free space,

$$\frac{\omega}{\beta} = v = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\Rightarrow \quad \frac{1}{\epsilon_0} = \mu_0\left(\frac{\omega}{\beta}\right)^2$$

$$\therefore \quad \bar{B} = -\frac{D_m\beta}{\omega\epsilon_0}\sin(\omega t+\beta z)\hat{a}_y$$

$$= -\frac{D_m}{\omega}\beta\mu_0\left(\frac{\omega}{\beta}\right)^2\sin(\omega t+\beta z)\hat{a}_y$$

$$\therefore \quad \bar{B} = -\frac{D_m\omega\mu_0}{\beta}\sin(\omega t+\beta z)\hat{a}_y$$

**(ii)** Given,

Magnetic flux density, $\bar{B} = B_m e^{j(\omega t+\beta z)}\hat{a}_y$

From Maxwell's equation,

$$\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t} = -\frac{\partial}{\partial t}\left[B_m e^{j(\omega t+\beta z)}\hat{a}_y\right]$$

or

$$\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ E_x & E_y & E_z\end{vmatrix} = -B_m j\omega e^{j(\omega t+\beta z)}\hat{a}_y$$

By comparing both sides,

$$\left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)\hat{a}_y = -B_m j\omega e^{j(\omega t+\beta z)}\hat{a}_y$$

$$\frac{\partial E_x}{\partial z} = -B_m j\omega e^{j(\omega t+\beta z)} \qquad (\because E_z = 0)$$

$$E_x = \int -B_m j\omega e^{j(\omega t+\beta z)} dz$$

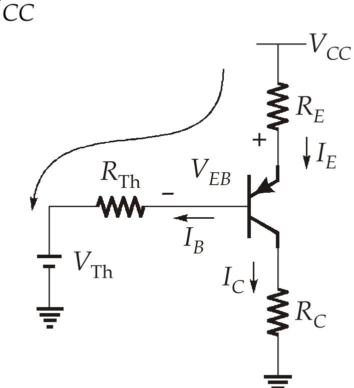
$$= -B_m j\omega \frac{1}{j\beta} e^{j(\omega t+\beta z)}$$

$$\bar{E} = -\frac{B_m \omega}{\beta} e^{j(\omega t+\beta z)} = -\frac{\omega B_m}{\beta} e^{j(\omega t+\beta z)} \hat{a}_x$$

**Q.1 (c) Solution:**

Thevenin equivalent

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2}, \quad V_{Th} = \frac{R_1}{R_1 + R_2} V_{CC}$$




Thevenin equivalent circuit is drawn below,

Applying KVL in loop shown,

$$-V_{CC} + I_E R_E + V_{EB} + R_{Th} I_B + V_{Th} = 0$$

As $\beta = \infty$, $I_B = 0$

$$I_E = \frac{1}{R_E}(V_{CC} - V_{Th} - V_{EB})$$

$$I_E = I_C = \frac{V_{CC} - V_{Th} - V_{EB}}{R_E}$$

$$I_C = \frac{V_{CC} - \frac{R_1}{R_1 + R_2} V_{CC} - V_{EB}}{R_E}$$

$$I_C = \frac{1}{R_E}\left(\frac{R_2}{R_1 + R_2} V_{CC} - V_{EB}\right)$$

The maximum allowable value of $R_C$ is obtained by equating the base and collector voltages. For $R_C > R_{C\,max}$, the transistor operates in saturation region as $V_C > V_B$.

i.e.,
$$V_B = V_C$$

$$V_B = V_{Th} = \frac{R_1}{R_1 + R_2} V_{CC}$$

$$V_C = I_C R_{C\,max}$$

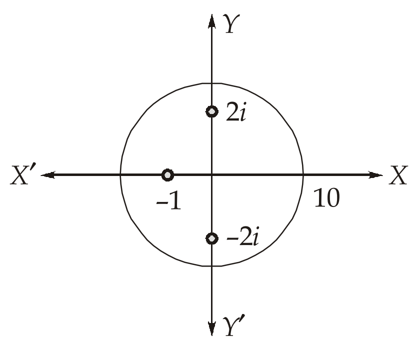
$$\frac{R_1}{R_1 + R_2} V_{CC} = \frac{1}{R_E}\left(\frac{R_2}{R_1 + R_2} V_{CC} - V_{EB}\right) \cdot R_{C\,max}$$

$$\frac{R_E R_1}{R_1+R_2}V_{CC} = \left(\frac{R_2}{R_1+R_2}V_{CC}-V_{EB}\right)R_{C\max}$$

$$R_{C\max} = \frac{R_E R_1}{R_1+R_2}V_{CC}\left(\frac{1}{\frac{R_2}{R_1+R_2}V_{CC}-V_{EB}}\right)$$

**Q.1 (d) Solution:**

Here, we have $\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}dz$




The poles are determined by putting the denominator equal to zero,

i.e., $(z+1)^2(z^2+4) = 0$

$$z = -1, -1 \text{ and } z = \pm 2i$$

The $|z| = 10$ with centre at origin and radius = 10 encloses a pole at $z = -1$ of second order and simple poles $z = \pm 2i$

The given integral $= I_1 + I_2 + I_3$

$$I_1 = \int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}dz = \int_{C_1} \frac{\frac{z^2-2z}{z^2+4}}{(z+1)^2}dz$$

$$= 2\pi i\left[\frac{d}{dz}\frac{z^2-2z}{z^2+4}\right]_{z=-1}$$

$$= 2\pi i\left[\frac{(z^2+4)(2z-2)-(z^2-2z)2z}{(z^2+4)^2}\right]_{z=-1}$$

$$= 2\pi i\left[\frac{(1+4)(-2-2)-(1+2)2(-1)}{(1+4)^2}\right]$$

$$= 2\pi i\left(\frac{-14}{25}\right) = \frac{-28\pi i}{25}$$

$$I_2 = \int_C \frac{\dfrac{z^2-2z}{(z+1)^2(z^2+2i)}}{(z-2i)}dz$$

$$= 2\pi i\left[\frac{z^2-2z}{(z+1)^2(z+2i)}\right]_{z=2i}$$

$$= 2\pi i\left[\frac{-4-4i}{(2i+1)^2(2i+2i)}\right]$$

$$= \frac{2\pi i(-4-4i)}{(-4+1+4i)4i} = \frac{2\pi i(-4(1+i)}{-(3+4i)4i}$$

$$= 2\pi i\frac{(1+i)}{4+3i}$$

$$I_3 = \int_C \frac{\dfrac{z^2-2z}{(z+1)^2(z+2i)}}{(z-2i)}dz$$

$$= 2\pi i\left[\frac{z^2-2z}{(z+1)^2(z-2i)}\right]_{z=-2i}$$

$$= 2\pi i\left[\frac{-4+4i}{(-2i+1)^2(-2i-2i)}\right]$$

$$= 2\pi i\frac{(i-1)}{(3i-4)}$$

$$\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}dz = I_1 + I_2 + I_3$$

$$= \frac{-28\pi i}{25} + 2\pi i\left(\frac{1+i}{4+3i}\right) + 2\pi i\left(\frac{1-1}{3i-4}\right)$$

$$= 2\pi i\left[\frac{-14}{25} + \frac{1+i}{4+3i} + \frac{(i-1)}{3i-4}\right]$$

$$= 2\pi i\left[\frac{-14}{25} + \frac{(1+i)(3i-4)+(i-1)(4+3i)}{-9-16}\right]$$

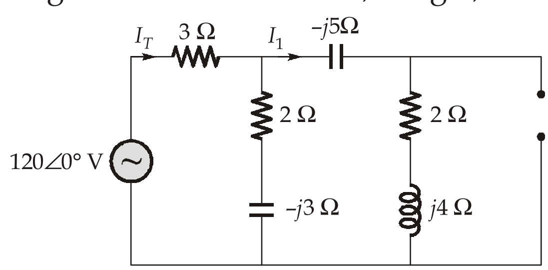
$$= \frac{2\pi i}{-25}[14 + (3i - 4 - 3 - 4i) + (4i - 3 - 4 - 3i)]$$

$$= 0$$

**Q.1 (e) Solution:**

**Superposition Theorem:** According to this theorem, voltage across or current through any element due to multiple sources present in a linear network is equal to the algebraic sum of voltage across or current through that element due to individual source with all other sources replaced by their internal impedances.

Now, taking only the voltage source into account, we get,




Hence, Total current, $I_T = \dfrac{120\angle 0°}{3+[(2-3j)\,||\,(2+4j-5j)]}$

$$= \frac{120\angle 0°}{3+[(2-3j)\,||\,(2-j)]}$$

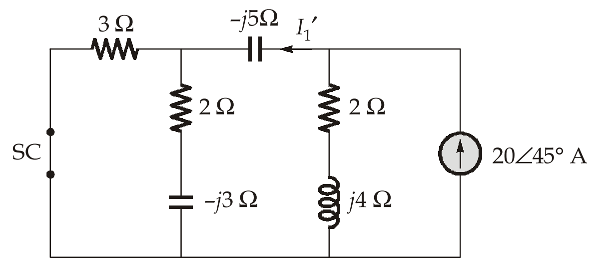
$$I_T = 27.84 + 5.9j = 28.46\angle 11.97° \text{ A}$$

Using current division rule, $I_1 = I_T\left[\dfrac{2-3j}{2+4j-5j+2-3j}\right]$

$$= 28.46\angle 11.97°\left[\frac{2-3j}{4(1-j)}\right]$$

$$I_1 = 18.138 + 0.208j \text{ A}$$

Now, taking only the current source into account, we get,




Using current division rule, $I_1' = 20\angle 45°\left[\dfrac{(2+j4)}{(2+j4)-j5+[3\,\|\,(2-3j)]}\right]$

$$I_1' = -15.28 + j15.62 \text{ A}$$

Now, Net current through the capacitor is

$$I_c = I_1 - I_1'$$

$$I_c = 18.138 + 0.208j - (-15.28 + j15.62)$$

$$= 33.418 - j15.412 = 36.8\angle -24.76° \text{ A}$$

**Q.2 (a) Solution:**

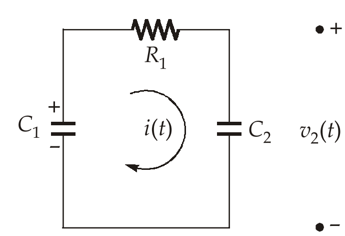
At $t = 0^-$, the network has attained steady-state condition. Hence, the capacitor, $C_1$ acts as an open circuit, full charged to $V_o$ volt.

$$v(0^-) = V_o$$

Since the voltage across the capacitor cannot change instantaneously,

$$v(0^+) = V_o$$

At $t = 0^+$, switch is at position '$b$'.

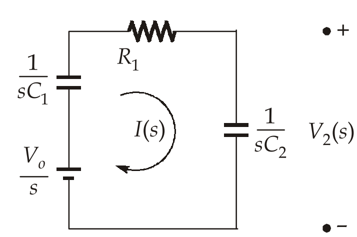



$C_2$ is uncharged at $t = 0$. Hence, it acts as short-circuit at $t = 0^+$

Hence, $$i(0^+) = \frac{V_o}{R_1}$$

Now converting the above circuit in Laplace domain,

Writing KVL equation,




$$-\frac{V_o}{s} + I(s)\times\frac{1}{sC_1} + I(s)R_1 + I(s)\times\frac{1}{sC_2} = 0$$

$$I(s)\left[\frac{1}{sC_1} + \frac{1}{sC_2} + R_1\right] = \frac{V_o}{s}$$

$$I(s)\left[\frac{C_1+C_2}{C_1C_2}+sR_1\right]\times\frac{1}{s} = \frac{V_o}{s}$$

$$I(s) = \frac{V_o}{\left[sR_1+\dfrac{C_1+C_2}{C_1C_2}\right]} = \frac{V_o}{R_1\left[s+\dfrac{C_1+C_2}{C_1C_2R_1}\right]}$$

Taking inverse Laplace transform, we get,

$$i(t) = \frac{V_o}{R_1}e^{-\frac{1}{R_1}\left[\frac{C_1+C_2}{C_1C_2}\right]t} \qquad t>0$$

Now,
$$v_2(s) = I(s)\times\frac{1}{sC_2}$$

$$v_2(s) = \frac{V_o}{R_1C_2s\left[s+\dfrac{C_1+C_2}{C_1C_2R_1}\right]}$$

Using partial fraction, we get

$$V_2(s) = \frac{A}{s}+\frac{B}{\left[s+\dfrac{C_1+C_2}{C_1C_2R_1}\right]}$$

$$A = \left.\frac{V_o}{R_1C_2\left[s+\dfrac{C_1+C_2}{C_1C_2R_1}\right]}\right|_{s=0} = \frac{V_oC_1C_2R_1}{R_1C_2\times(C_1+C_2)}$$

$$= \frac{V_oC_1}{(C_1+C_2)}$$

$$B = \left.\frac{V_o}{R_1C_2s}\right|_{s=-\frac{(C_1+C_2)}{C_1C_2R_1}} = \frac{-V_oC_1C_2R_1}{R_1C_2(C_1+C_2)}$$

$$= \frac{-V_oC_1}{C_1+C_2}$$

$\therefore$
$$V_2(s) = \frac{V_oC_1}{(C_1+C_2)s}-\frac{V_oC_1}{(C_1+C_2)\left[s+\dfrac{C_1+C_2}{C_1C_2R_1}\right]}$$

Taking inverse Laplace transform, we get,

$$v_2(t) = \frac{V_o C_1}{(C_1 + C_2)} - \frac{V_o C_1}{(C_1 + C_2)} e^{-\frac{1}{R_1}\left[\frac{C_1+C_2}{C_1 C_2}\right]t} \quad ; t > 0$$

$$v_2(t) = \frac{V_o C_1}{C_1 + C_2}\left[1 - e^{-\frac{1}{R_1}\left[\frac{C_1+C_2}{C_1 C_2}\right]t}\right] \quad ; t > 0$$

Let, $C = \dfrac{C_1 C_2}{C_1 + C_2}$ i.e. series combination of $C_1$ and $C_2$.

$$\frac{C_1}{C_1 + C_2} = \frac{C}{C_2}$$

$$\therefore \quad v_2(t) = \frac{V_o C}{C_2}\left[1 - e^{-\frac{t}{R_1 C}}\right], \quad t > 0$$

**Q.2 (b) Solution:**

Given : $f(x) = x + x^2; \quad -\pi < x < \pi$

The fourier series expansion,

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2)dx = \frac{1}{\pi}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\{\pi^2 - (-\pi)^2}{2} + \frac{\{\pi^3 - (-\pi)^3\}}{3}\right]$$

$$= \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2)\cos nx\, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\,dx + \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos nx\,dx$$

$$= 0 + \frac{2}{\pi}\int_{0}^{\pi} x^2\cos nx\,dx \quad (\because\ x \cos nx \text{ is an odd function})$$

$$= \frac{2}{\pi}\left[x^2\frac{\sin nx}{n} - \frac{2x(-\cos nx)}{n^2} + \frac{2(-\sin nx)}{n^3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\pi^2\frac{\sin n\pi}{n} - \frac{2\pi(-\cos n\pi)}{n^2} + 2\left(\frac{-\sin n\pi}{n^3}\right) - 0\right]$$

$$a_n = \frac{4(-1)^n}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx\,dx + \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\sin nx\,dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx + 0 \qquad \{x^2 \sin nx \text{ is an odd function}\}$$

$$= \frac{2}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\pi\frac{\cos n\pi}{n} + \frac{\sin n\pi}{n^2} - 0\right]$$

$$= \frac{-2(-1)^n}{n} = \frac{2}{n}(-1)^{n+1}$$

So, fourier series expansion,

$$f(x) = \frac{\pi^2}{3} + 4\left[-\cos x + \frac{1}{2^2}\cos 2x - \frac{1}{3^2}\cos 3x + \ldots\right] +$$

$$2\left[\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \ldots\right] \qquad \ldots(i)$$

On putting $x = \pi$

$$f(\pi) = \frac{\pi^2}{3} + 4\left[1 + \frac{1}{2^2}\cos 2\pi - \frac{1}{3^2}\cos 3\pi + .....\right] + 2 \times 0$$

$$\pi + \pi^2 = \frac{\pi^2}{3} + 4\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + .....\right] \quad ...(ii)$$

Putting $x = -\pi$

$$-\pi + (-\pi)^2 = \frac{\pi^2}{3} + 4\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + .....\right] \quad ...(iii)$$

Adding eqn. (ii) and (iii),

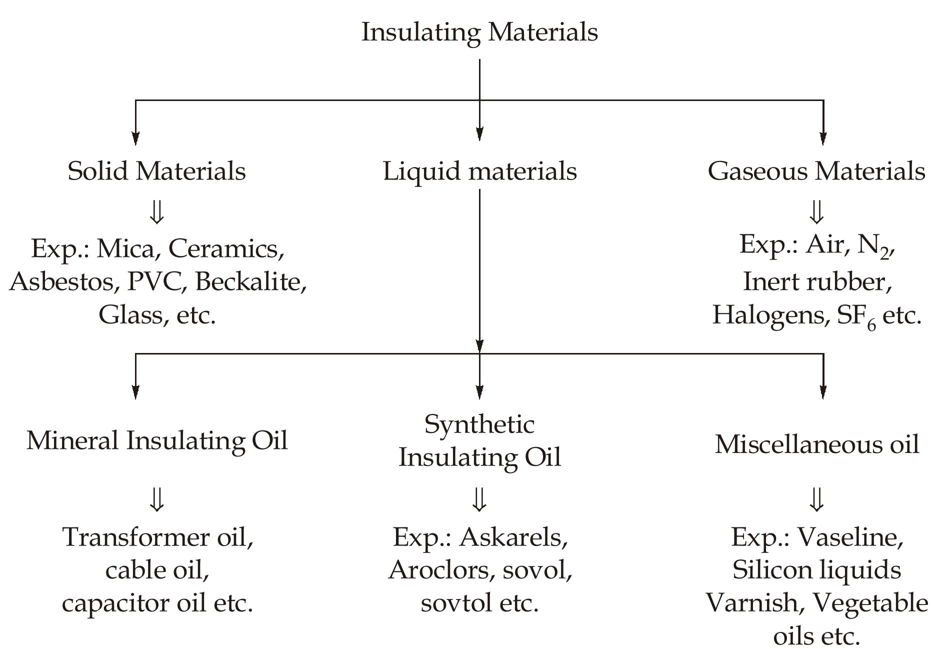
$$2\pi^2 = \frac{2\pi^2}{3} + 8\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + .....\right]$$

$$\frac{4\pi^2}{3} = 8\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + .....\right]$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + ..... = \frac{\pi^2}{6}$$

**Q.2 (c) (i) Solution:**

Classification of insulating materials is given below:



**Insulation Resistance:**

The function of insulation resistance is to prevent the flow of current through it and to conduct the heat generated. Insulation resistance (IR) must be very high to prevent failure of electrical equipment. It is of 2 types:

1. Volume Resistance 2. Surface resistance.

The resistance offered to the current which flows over the surface of the insulating material is called surface resistance. The resistance offered to the current which flows through the volume of the insulating material is called volume resistance.

Factors affecting insulating resistance:

1. **Temperature :** IR decreases with increase in temperature.
2. **Moisture:** Exposure to moisture decreases insulation resistance.
3. **Voltage:** IR decreases with increase in applied voltage.
4. **Age:** IR decreases with increasing age.

**Q.2 (c) (ii) Solution:**

Given: $a_0 = 0.45258$ nm, $b_0 = 0.45186$ nm, $c_0 = 0.76570$ nm

$$\text{Atomic Radius } R = 0.1218 \text{ nm}$$

$$\text{Density } d = 5.904 \text{ gm/cm}^3$$

$$M_{at} = 69.72 \text{ gm/mol}$$

**1.** The number of atoms in each unit cell.

The volume of unit cell is $V_c = a_0\, b_0\, c_0$

$$= (0.45258) \times (0.45186) \times (0.76570)$$

$$V_c = 1.566 \times 10^{-22} \text{ cm}^3$$

We know,

$$\frac{n}{V_c} = \frac{N_A \times d}{M_{at}}$$

$$n = \frac{V_c \times N_A \times d}{M_{at}} = \frac{1.566 \times 10^{-22} \times 6.023 \times 10^{23} \times 5.904}{69.72}$$

$$n = 7.99$$

$$n \cong 8 \text{ atoms/cell}$$

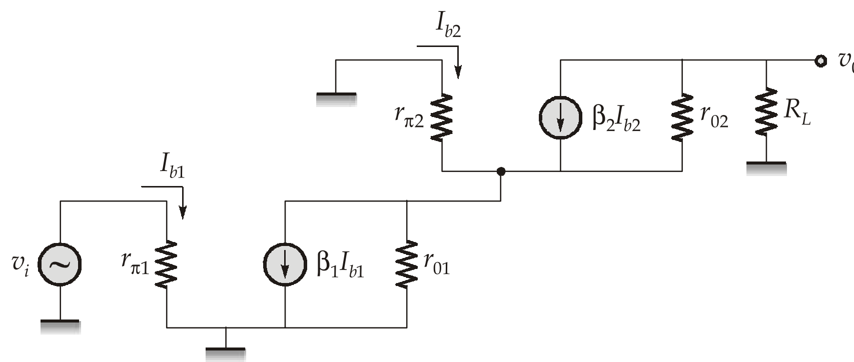
**2.** Atomic packing fraction

$$= \frac{n \times \left(\frac{4\pi}{3} \times R^3\right)}{V_c} = \frac{8 \times 4\pi \times (0.1218 \times 10^{-7})^3}{3 \times 1.566 \times 10^{-22}}$$

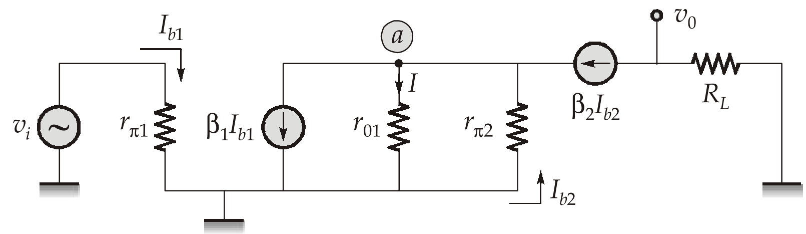
$$\text{APF} = 0.387$$

**Q.3 (a) Solution:**

Replace $Q_1$ and $Q_2$ by their small signal models




Since $r_{02}$ is ignored therefore,




From the circuit, $$v_0 = -\beta_2 I_{b2} R_L \quad \text{...(i)}$$

at node $a$, current flowing through $r_{01}$ is

$$I = (1 + \beta_2)I_{b2} - \beta_1 I_{b1}$$

Also, $$r_{\pi 2} \cdot I_{b2} = -I r_{01}$$

$$r_{\pi 2} \cdot I_{b2} = \beta_1 I_{b1} r_{01} - (1 + \beta_2)I_{b2} \cdot r_{01}$$

$$[(1 + \beta_2)r_{01} + r_{\pi 2}]I_{b2} = \beta_1 r_{01} I_{b1}$$

$\Rightarrow$ $$I_{b2} = \frac{\beta_1 r_{01} I_{b1}}{[(1+\beta_2)r_{01} + r_{\pi 2}]} \quad \text{...(ii)}$$

Also $$v_i = I_{b1} r_{\pi 1} \quad \text{...(iii)}$$

On putting equation (ii), in equation (i),

$$v_0 = \frac{-\beta_2 \cdot \beta_1 \cdot r_{01} \cdot R_L \cdot I_{b1}}{[(1+\beta_2)r_{01} + r_{\pi 2}]}$$

$$v_0 = \frac{-\beta_2 \cdot \beta_1 \cdot r_{01} \cdot R_L \cdot v_i}{r_{\pi 1}[(1+\beta_2)r_{01} + r_{\pi 2}]}$$ using equation (iii)

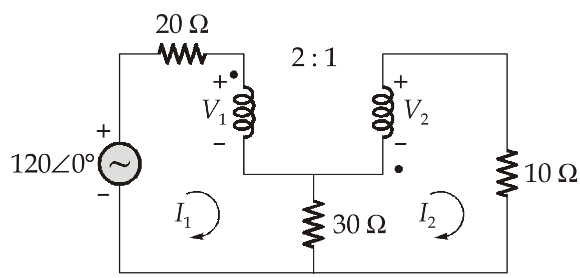
Therefore voltage gain, $$A_v = \frac{V_0}{V_i} = \frac{-\beta_1 \beta_2 \cdot r_{01} \cdot R_L}{r_{\pi 1}[(1+\beta_2)r_{01} + r_{\pi 2}]}$$

Now, given that,

$$r_{01} = 50\ \text{k}\Omega,$$
$$r_{\pi1} = r_{\pi2} = 5\ \text{k}\Omega,$$
$$\beta_1 = \beta_2 = 100$$
$$R_L = 5\ \text{k}\Omega$$

Voltage gain, $$\frac{v_0}{v_i} = \frac{-100\times100\times50\times10^3\times5\times10^3}{5\times10^3(101\times50\times10^3+5\times10^3)} = -98.91\ \text{V/V}$$

**Q.3 (b) (i) Solution:**




Applying mesh analysis for mesh - 1,

$$-120 + (20 + 30)I_1 - 30I_2 + V_1 = 0$$
$$50I_1 - 30I_2 + V_1 = 120 \quad \text{...(i)}$$

For mesh - 2,

$$-V_2 + (10 + 30)I_2 - 30I_1 = 0$$
$$-30I_1 + 40I_2 - V_2 = 0 \quad \text{...(ii)}$$

From transformer action, $$V_2 = \frac{-1}{2}V_1 \quad \text{...(iii)}$$
$$I_2 = -2I_1 \quad \text{...(iv)}$$

Put equation (iii) and (iv) in equation (i) and (ii),

$$50I_1 + 60I_1 + V_1 = 120$$
$$110\ I_1 + V_1 = 120 \quad \text{...(v)}$$
$$-30I_1 - 80I_1 + \frac{1}{2}V_1 = 0$$
$$V_1 - 220I_1 = 0$$
$$V_1 = 220I_1 \quad \text{...(vi)}$$

Put equation (vi) in equation (v),

$$330I_1 = 120$$

$$I_1 = \frac{4}{11}\text{A(rms)}$$

$$V_1 = 220 \times \frac{4}{11} = 80 \text{ V (rms)}$$

$$I_2 = \frac{-8}{11} = -0.7272 \text{ A (rms)}$$

The power absorbed by the 10 Ω resistor

$$P = (-0.7272)^2 \times 10 = 5.289 \text{ W}$$

**Q.3 (b) (ii) Solution:**

Given: $R = 500\ \Omega$, $L = 0.06$ H, $C_s = 0.01$ pF, $C_p = 10$ pF. Therefore, series resonant frequency,

$$f_s = \frac{1}{2\pi\sqrt{LC_s}} = \frac{1}{2\pi\sqrt{0.06 \times 0.01 \times 10^{-12}}} = 6.497 \text{ MHz}$$

Parallel resonant frequency,
$$f_p = \frac{1}{2\pi\sqrt{L\left(\frac{C_s C_p}{C_s + C_p}\right)}} = \frac{1}{2\pi\sqrt{0.06\left(\frac{0.01 \times 10}{0.01 + 10}\right) \times 10^{-12}}}$$

$$f_p = 6.5007 \text{ MHz}$$

**Q.3 (c) Solution:**

$$\text{Reactance of pressure coil} = 2\pi \times 50 \times 10 \times 10^{-3} = 3.14\ \Omega$$

$$\text{Resistance of pressure coil} = 362\ \Omega$$

$$\text{Phase angle of pressure coil} = \beta = \tan^{-1}\left(\frac{3.14}{362}\right) = 0.5° \approx 30°$$

Let, phase angle of current transformer = θ

Phase angle of potential transformer = δ

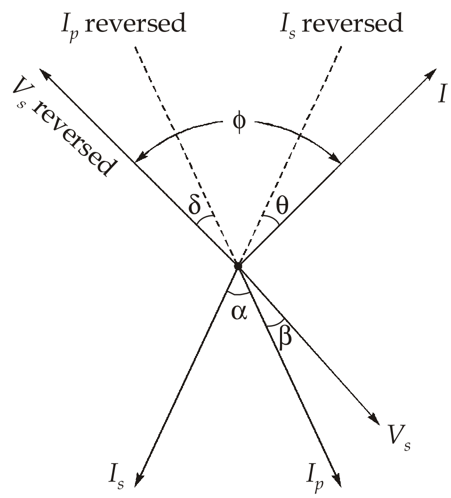
$$\theta = +90',$$

$$\delta = -45'$$

$$\text{Phase angle of load} = 50° = \phi$$

Phase angle between pressure coil current, $I_p$ and a current, $I_s$ of wattmeter current coil,

$$\alpha = \phi - \theta - \beta - \delta = 50° - 90' - 30' - 45'$$
$$= 47.25°$$




$$\text{Correction factor, } K = \frac{\cos\phi}{\cos\beta\cos\alpha} = \frac{\cos 50°}{\cos 30' \times \cos(47.25°)} = 0.947$$

$$\% \text{ ratio error} = \frac{K_n - R}{R} \times 100$$

$$\text{Actual ratio, } R = \frac{K_n \times 100}{100 + \% \text{ ratio error}}$$

$$\therefore \quad \text{Actual ratio of C.T.} = \frac{20 \times 100}{100 - 0.2} = 20.04$$

$$\text{Actual ratio of P.T.} = \frac{160 \times 100}{100 + 0.8} = 99.2$$

∴ Power of load = $K$ × actual ratio of C.T. × actual ratio of P.T. × Wattmeter reading

= 0.947 × 20.04 × 99.2 × 350

= 658.899 kW ≈ 658.9 kW

**Q.4 (a) (i) Solution:**

RMS value of voltage supplied, $V = 100$ V

Maximum value of voltage, $V_m = \sqrt{2} \times 100$ V

Instantaneous value of voltage, $V = \sqrt{2} \times 100 \sin\theta$

Resistance in the forward direction, $R_f = 50 + 50 = 100\ \Omega$

Resistance in the reverse direction, $R_x = 300\ \Omega$

Instantneous value of current in the forward direction,

$$i_f = \frac{V_m}{R_f} = \frac{\sqrt{2}\times 100}{100}\sin\theta = \sqrt{2}\sin\theta$$

Instantaneous value of current in the reverse direction,

$$i_r = \frac{V_m}{R_x} = \frac{\sqrt{2}\times 100}{300}\sin\theta = \frac{\sqrt{2}}{3}\sin\theta$$

$$\text{RMS value of current} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}\left(\sqrt{2}\sin\theta\right)^2 d\theta + \frac{1}{2\pi}\int_{\pi}^{2\pi}\left(\frac{\sqrt{2}}{3}\sin\theta\right)^2 d\theta}$$

$$= 0.745 \text{ A}$$

Average value of current, $$I_{av} = \frac{1}{2\pi}\left[\int_0^{\pi}\sqrt{2}\sin\theta\, d\theta + \int_{\pi}^{2\pi}\frac{\sqrt{2}}{3}\sin\theta\, d\theta\right] = 0.3 \text{ A}$$

Power taken from the mains = Power supplied in the forward half cycle + power supplied in the backward half cycle

$$= \frac{1}{2}\left(\frac{V^2}{R_f}+\frac{V^2}{R_r}\right) = \frac{1}{2}\left[\frac{(100)^2}{100}+\frac{(100)^2}{300}\right]$$

$$= 66.7 \text{ W}$$

Power consumed in 50 Ω resistor = $I^2R$

$$= (0.745)^2 \times 50 = 27.7 \text{ W}$$

Power dissipated in rectifier = Total power supplied – Power consumed in resistor

$$= 66.7 - 27.7 = 39 \text{ W}$$

**Q.4 (a) (ii) Solution:**

Characteristics of ceramics:

1. High Brittleness
2. High hardness
3. High melting point
4. Chemical Inertness
5. Electrical insulation
6. Low water absorption
7. Low thermal expansion
8. High temperature withstand capacity

Mainly ceramic materials are divided into two groups depending whether permittivity ($\in_r$) is less than or greater than 12.

Materials with ($\in_r < 12$) are more used in insulators, bushing, housing and as like Example Porcelain, alumina etc. Porcelain is used as insulators in transmission lines and distribution lines, fuse links, plugs and socket.

Material with permittivity ($\in_r$) > 12 is mainly used as dielectric in capacitor because size of dielectric should be as small as possible. Example $BaTiO_3$.

**Q.4 (b) Solution:**

**Magnetostatic deflection in CRT:**

When a charged particle passes through the magnetic field or electric field, in both of the cases there will be a mechanical force acting on the charged particle.

In magnetic field, when the $e^-$ passes, there will be a Lorentz force acting on it perpendicular to the direction of motion as well as the direction of the magnetic field.

When this electron will pass through electric field, due to Coulombic force, there will be a deflection.

We can use both electrostatic deflection and magnetic deflection in Cathode Ray Display (CRD) but we generally use the electrostatic deflection in CRD which is used for viewing electrical waveforms.

But when CRD is used as picture tube (CRT) of television, then we use magnetic deflection of the electron beams.

$$\text{Lorentz force, } F = \text{BIL Newtons}$$

and
$$q = ne \text{ coulombs}$$

$$I = \frac{dq}{dt} = \frac{ne}{t}$$

$$\therefore \quad F = \frac{BneL}{t}$$

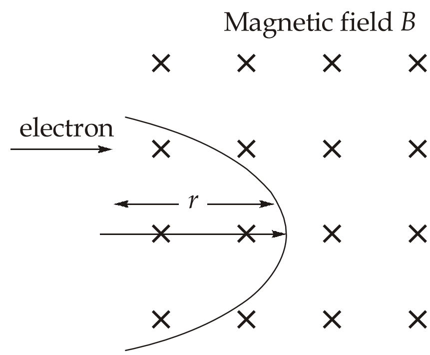
Now drift velocity of $e^-$ $v_d = \dfrac{L}{t}$

$$\therefore \quad F = B\,ne\,v_d$$

$\therefore$ Force acting on single $e^-$ $\Rightarrow$ $\dfrac{F}{n} = Be\,v_d$

The electron will have circular motion in magnetic field.

$$\therefore \quad \frac{v_d^2}{r} = \text{angular acceleration.}$$

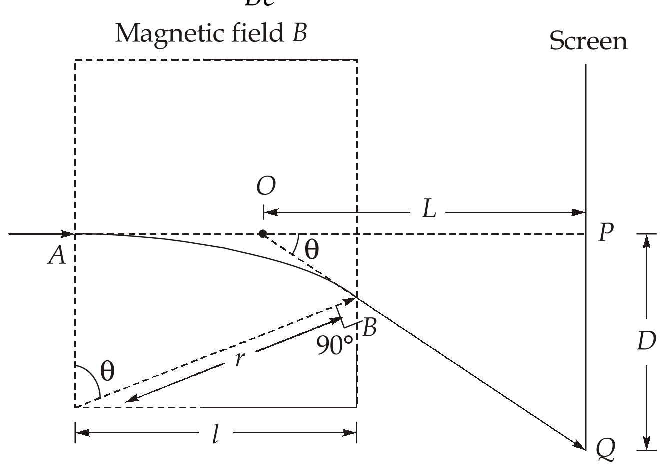



Now force acting on electron is $\frac{mv_d^2}{r}$ where $m$ is mass of one single $e^-$.

Equating both the forces,

$$\frac{mv_d^2}{r} = Bev_d$$

$$r = \frac{mv_d}{Be} \quad ...(i)$$




Now from $\Delta OPQ$

$$\tan\theta = \frac{D}{L} \approx \theta$$

where, $l \rightarrow$ width of magnetic field region

$D \rightarrow$ Deflection of $e^-$ from centre of the screen

$L \rightarrow$ The distance of screen from the intersecting point.

From the arc $AB$, we can write its angle as also $\theta$.

$\because$ $\theta$ is very small.

$\therefore$ $$\tan\theta \cong \theta = \frac{D}{L} \cong \frac{l}{r}$$

$\therefore \qquad D = \dfrac{Ll}{r}$

Now, from equation (i)

$$D = \frac{Ll \times Be}{mv_d} \qquad \text{...(ii)}$$

Again, Energy of $e^-$ when passing through magnetic field.

$$E = \frac{mv_d^2}{2}$$

and $V_a$ is the accelerating voltage, the voltage difference between anode and cathode of an electron gun system.

$\therefore \qquad E = eV_a$

Now, we can write, $\qquad \dfrac{mv_d^2}{2} = eV_a$

$$v_d = \sqrt{\frac{2eV_a}{m}}$$

Put in equation (ii)

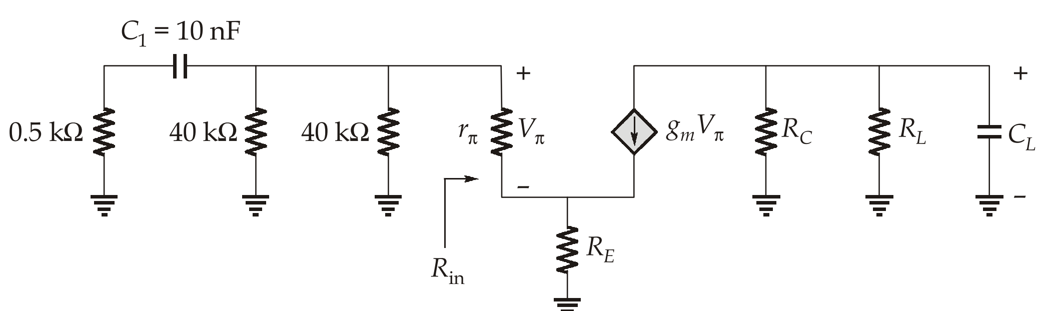
$\therefore \qquad D = \dfrac{Ll \times Be \times \sqrt{m}}{m \times \sqrt{2eV_a}} = \dfrac{LlB}{\sqrt{V_a}} \times \sqrt{\dfrac{e}{2m}}$

$$D = LlB\sqrt{\frac{e}{2mV_a}}$$

**Q.4 (c) Solution:**

The cutoff frequency due to capacitor $C_1$ can be given as

$$f_L = \frac{1}{2\pi C_1 R_{eq}}$$




$$R_{in} = r_\pi + (1+\beta)R_E$$

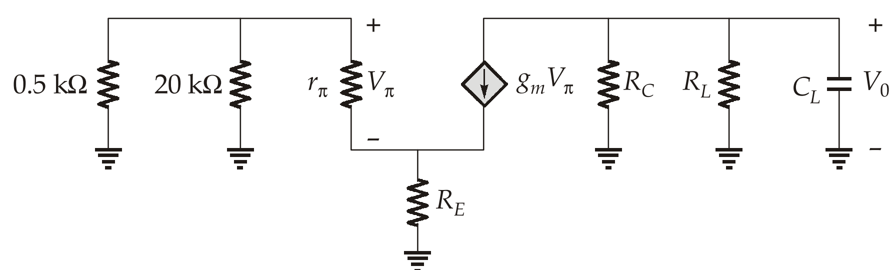
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} \times 10^3$$

$$r_\pi = 2.5\text{ k}\Omega$$
$$R_{in} = 2.5 + 101 \times 0.5$$
$$R_{in} = 53\text{ k}\Omega$$

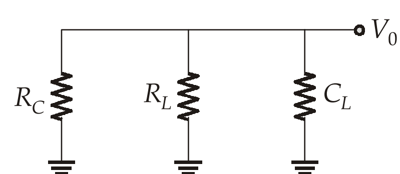
Therefore,
$$R_{eq} = \left[R_{in} \,||\, 40 \,||\, 40 + 0.5\text{k}\Omega\right]$$
$$= [53 \,||\, 20 + 0.5]$$
$$R_{eq} = 15.02\text{ k}\Omega$$

Now,
$$f_L = \frac{1}{2\pi C_1 R_{eq}} = \frac{1}{2\pi \times 10 \times 10^{-9} \times 15.02 \times 10^3}$$
$$f_L = 1.059\text{ kHz}$$

The upper cutoff frequency $f_H$ can be calculated as




Thus, the equivalent circiut can be drawn as




∴
$$f_H = \frac{1}{2\pi C_L R_L'}$$
$$R_L' = R_C \,||\, R_L = 2 \,||\, 2 = 1\text{k}\Omega$$

∴
$$f_H = \frac{1}{2\pi \times 15 \times 10^{-9} \times 10^3}$$
$$f_H = 10.61\text{ kHz}$$

∴ 3-dB bandwidth can be given as
$$BW = f_H - f_L$$
$$= (10.61 - 1.059)\text{ kHz}$$
$$BW = 9.551\text{ kHz}$$

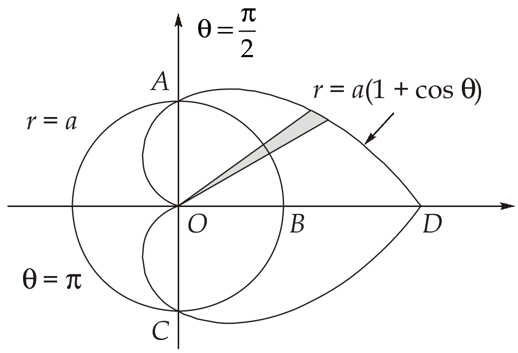
**Section-B**

**Q.5 (a) Solution:**

$$r = a(1 + \cos\theta) \quad \text{...(i)}$$

$$r = a \quad \text{...(ii)}$$

Solving (i) and (ii), by eliminating, we get




$$a(1 + \cos\theta) = a$$

$$1 + \cos\theta = 1$$

$$\cos\theta = 0$$

$$\theta = \frac{-\pi}{2} \text{ or } \frac{\pi}{2}$$

Limits of $r$ are $a$ and $a(1 + \cos\theta)$

Limits of $\theta$ are $\frac{-\pi}{2}$ to $\frac{\pi}{2}$

$$\text{Required area} = \text{Area } A\,BCDA$$

$$= \int_{-\pi/2}^{\pi/2} \int_{r \text{ for circle}}^{\text{for cardiode}} r\, d\theta\, dr$$

$$= \frac{a^2}{2} \int_{-\pi/2}^{\pi/2} \left[(1+\cos\theta)^2 - 1\right] d\theta$$

$$= \frac{a^2}{2} \int_{-\pi/2}^{\pi/2} (\cos^2\theta + 2\cos\theta)\, d\theta$$

$$= a^2 \left[\int_0^{\pi/2} \cos^2\theta\, d\theta + 2\int_0^{\pi/2} \cos\theta\, d\theta\right]$$

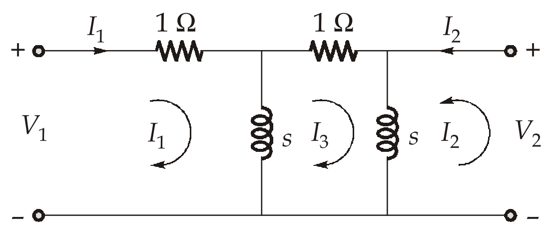
$$= a^2\left[\frac{\pi}{4} + 2(\sin\theta)_0^{\pi/2}\right]$$

$$= a^2\left[\frac{\pi}{4} + 2\right] = \frac{a^2}{4}(\pi + 8)$$

**Q.5 (b) Solution:**

Open circuit parameter are also known as $z$-parameters.

Transform the given two port network into Laplace domain,




By applying KVL in Mesh 1;

$$V_1 - I_1 \times 1 - s(I_1 - I_3) = 0$$

$$V_1 = (s + 1)I_1 - sI_3 \qquad \text{...(i)}$$

By applying KVL in Mesh 3;

$$-s(I_3 - I_1) - I_3 \times 1 - s(I_3 + I_2) = 0$$

$$-sI_3 + sI_1 - I_3 - sI_3 - sI_2 = 0$$

$$I_3(2s + 1) = -sI_2 + sI_1$$

$$\therefore \qquad I_3 = \frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2 \qquad \text{...(ii)}$$

By applying KVL in Mesh 2;

$$V_2 - s(I_2 + I_3) = 0$$

$$V_2 = sI_2 + sI_3 \qquad \text{...(iii)}$$

Substituting equation (ii) in equation (i),

$$V_1 = (s+1)I_1 - s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$V_1 = \left(\frac{s^2+3s+1}{2s+1}\right)I_1 + \left(\frac{s^2}{2s+1}\right)I_2 \qquad \text{...(iv)}$$

Substituting equation (ii) in equation (iii),

$$V_2 = sI_2 + s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$V_2 = \left(\frac{s^2}{2s+1}\right)I_1 + \left(\frac{s^2+s}{2s+1}\right)I_2 \quad ...(v)$$

Comparing equation (iv) and (v),

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{s^2+3s+1}{2s+1} & \frac{s^2}{2s+1} \\ \frac{s^2}{2s+1} & \frac{s^2+s}{2s+1} \end{bmatrix}$$

**Q.5 (c) Solution:**

For resonant circuit frequency,

$$f_1 = \frac{1}{2\pi\sqrt{L_1C_1}}$$

∴ Inductance of circuit-1,

$$L_1 = \frac{1}{4\pi^2 f_1^2 C_1} = \frac{1}{4\pi^2 \times 60 \times 60 \times 1 \times 10^{-6}} = 7.05 \text{ H}$$

Impedance of circuit-1 at 50 Hz,

$$Z_1 = R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right)$$

$$= 100 + j\left(2\pi \times 50 \times 7.05 - \frac{1}{2\pi \times 50 \times 1 \times 10^{-6}}\right)$$

$$= 100 - j968.28\ \Omega$$

Impedance of circuit-2 at 50 Hz,

$$Z_2 = R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right)$$

$$= 100 + j\left(2\pi \times 50 \times L_2 - \frac{1}{2\pi \times 50 \times 1.5 \times 10^{-6}}\right)$$

$$= 100 + j(314L_2 - 2122.67)$$

For equal current in two circuits,

$$Z_1 = Z_2$$

or $$314L_2 - 2122.67 = 968.28$$

Inductance of circuit-2,

$$L_2 = 3.676 \text{ H}$$

Resonant frequency of circuit-2,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{3.676\times1.5\times10^{-6}}} = 67.77 \text{ Hz}$$

**Q.5 (d) Solution:**

Given :

Forward voltage, $V_D = 0.5$ V

The current equation of diode,

$$I_D = I_o\left[e^{\frac{V_D}{\eta V_T}} - 1\right]$$

$I_o$ = Reverse saturation current

$V_T$ = Thermal voltage

$\eta$ = Ideality factor

At $T = 400°$ K, for diode $D_1$

Thermal voltage, $V_T = \dfrac{T}{11600}$

$$V_T = \frac{400}{11600} = 0.0345 \text{ V}$$

$$I_{D1} = I_o\left[e^{\frac{V_D}{\eta V_T}} - 1\right] = I_o\left[e^{\frac{0.5}{2\times0.0345}} - 1\right]$$

$$I_{D1} = 1402.01 I_o$$

At $T = 300°$K; for diode $D_2$

Thermal voltage, $V_T = \dfrac{T}{11600}$

$$V_T = \frac{300}{11600} = 0.0258 \text{ V}$$

$$I_{D2} = I_o\left[e^{\frac{V_D}{\eta V_T}} - 1\right]$$

$$= I_o\left[e^{\frac{0.5}{2\times0.0258}} - 1\right]$$

$$I_{D2} = 16152.99 I_o$$

Required ratio,
$$\frac{I_{D1}}{I_{D2}} = \frac{1402.01 I_o}{16152.99 I_o}$$

$$\frac{I_{D1}}{I_{D2}} = 0.0868$$

**Q.5 (e) Solution:**

The distance between two consecutives slits = $\frac{1}{\text{No. of slits per mm}}$

$$= \frac{1}{500}\times10^{-3}\,\text{m}$$

$$d = 2\times10^{-6}\text{ m}$$

The diffraction angle for red light can be calculated as,

$$\sin\theta = \frac{n\lambda}{d}$$

for $n = 1$ (first order diffraction) and $\lambda = 7\times10^{-7}$ m

$$\sin\theta = \frac{1\times7\times10^{-7}}{2\times10^{-6}} = 0.35$$

$$\theta = 20.48°$$

The diffraction angle for green light can be calculated as,

$$\sin\theta = \frac{n\lambda}{d}$$

for ($n = 1$) first order diffraction and $\lambda = 5.38\times10^{-7}$ m

$\therefore$
$$\sin\theta = \frac{1\times5.38\times10^{-7}}{2\times10^{-6}} = 0.269$$

$$\theta = 15.6°$$

The angle of deviation (angle of diffraction) for different wavelengths are not the same. Hence, it is possible to examine the contents of an incident wave by looking at the different angles of deviation produced by the different components of a beam.

**Q.6 (a) (i) Solution:**

$$f(x) = c\,e^{-|x|}, -\infty < x < \infty$$

We have, $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} c\,e^{-|x|}\,dx = 1$$

$\because e^{-|x|}$ is an even function,

$$2c\int_{0}^{\infty} e^{-x}dx = 1$$

$$2c\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$$

$$2c\,(1 - 0) = 1$$

$$\Rightarrow \quad c = \frac{1}{2}$$

Mean of the distribution, $$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx = \frac{1}{2}\int_{-\infty}^{\infty} x\,e^{-|x|}dx,$$

Since in $x\,e^{-|x|}$ function is odd

$$\therefore \quad \mu = 0$$

Variance of the distribution, $$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \int_{-\infty}^{\infty} (x-0)^2\cdot\frac{1}{2}e^{-|x|}\,dx$$

$$= 2\cdot\frac{1}{2}\int_{0}^{\infty} x^2 e^{-x}dx, \qquad \because x^2 e^{-|x|} \text{ is an even function,}$$

$$= \left[-x^2e^{-x} - 2x\,e^{-x} - 2e^{-x}\right]_0^{\infty}$$

$$= (0 - (-2)) = 2$$

$$\therefore \quad \sigma^2 = 2$$

Required probability $= P(0 \le x \le 4)$

$$= \frac{1}{2}\int_{0}^{4} e^{-|x|}\,dx = \frac{1}{2}\int_{0}^{4} e^{-x}\,dx = -\frac{1}{2}(e^{-4} - 1) = 0.4908$$

**Q.6 (a) (ii) Solution:**

Given,
$$u - v = (x - y)(x^2 + 4xy + y^2)$$
$$f(z) = u + iv \quad \text{...(i)}$$
$$i\,f(z) = iu - v \quad \text{...(ii)}$$

Add equation (i) and (ii),
$$(1 + i)\,f(z) = (u - v) + i(u + v)$$

Let,
$$F(z) = U + iV = (1 + i)\,f(z) \quad \text{...(iii)}$$
$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}\left((x-y)(x^2+4xy+y^2)\right)$$
$$= x^2 + 4xy + y^2 + (x - y)(2x + 4y)$$
$$= x^2 + 4xy + y^2 + 2x^2 - 2xy + 4xy - 4y^2$$
$$= 3x^2 + 6xy - 3y^2$$
$$\frac{\partial U}{\partial y} = -(x^2 + 4xy + y^2) + (x - y)(4x + 2y)$$
$$= -x^2 - 4xy - y^2 + 4x^2 - 4xy + 2xy - 2y^2$$
$$= 3x^2 - 6xy - 3y^2$$

Now,
$$F'(x) = \frac{\partial U}{\partial x} + i\frac{\partial V}{\partial x} = \frac{\partial U}{\partial x} - i\frac{\partial U}{\partial y}$$
$$= 3x^2 + 6xy - 3y^2 - i(3x^2 - 6xy - 3y^2)$$

$\therefore$
$$F'(z, 0) = 3z^2 - i\,3z^2 = 3(1 - i)z^2$$

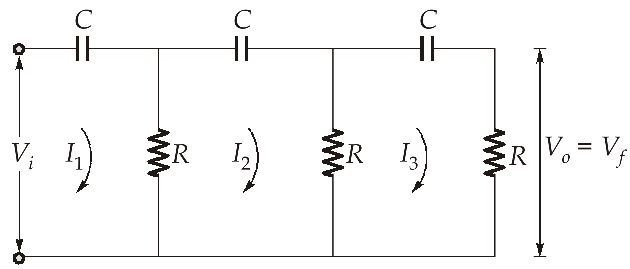
$\Rightarrow$
$$\int F'(z,0) = F(z) = \int 3(1-i)z^2 dz$$

$\Rightarrow$
$$F(z) = (1 - i)z^3 + c \quad \text{...(iv)}$$

From equation (iii) and (iv),
$$(1 + i)\,f(z) = (1 - i)z^3 + c$$
$$f(z) = \left(\frac{1-i}{1+i}\right)z^3 + \frac{c}{1+i}$$

$\Rightarrow$
$$f(z) = -iz^3 + c'$$

**Q.6 (b) Solution:**




Applying KVL in the three loops, we get

$$I_1\left(R+\frac{1}{j\omega C}\right)-I_2R = V_i$$

$$-I_1R+I_2\left(2R+\frac{1}{j\omega C}\right)-I_3R = 0$$

$$0-I_2R+I_3\left(2R+\frac{1}{j\omega C}\right) = 0$$

$$\begin{bmatrix} R+\frac{1}{sC} & -R & 0 \\ -R & 2R+\frac{1}{sC} & -R \\ 0 & -R & 2R+\frac{1}{sC} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_i \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad \Delta = \frac{(1+sRC)(1+2sRC)^2}{s^3C^3}-\frac{R^2(1+2sRC)}{sC}-\frac{R^2(1+sRC)}{sC}$$

$$= \frac{(1+sRC)(1+4sRC+4s^2C^2R^2)-R^2s^2C^2(1+2sRC+1+sRC)}{s^3C^3}$$

$$= \frac{1+5sRC+8s^2C^2R^2+4s^3C^3R^3-3s^3R^3C^3-2R^2s^2C^2}{s^3C^3}$$

$$= \frac{1+5sRC+6s^2C^2R^2+s^3C^3R^3}{s^3C^3}$$

$$\Delta_3 = \begin{vmatrix} \frac{1+sRC}{sC} & -R & V_i \\ -R & \frac{1+2sRC}{sC} & 0 \\ 0 & -R & 0 \end{vmatrix} = V_iR^2$$

$$\therefore \qquad I_3 = \frac{\Delta_3}{\Delta} = \frac{V_i R^2 s^3 C^3}{1+5sRC+6s^2C^2R^2+s^3C^3R^3}$$

$$\therefore \qquad V_0 = V_f = I_3 R = \frac{V_i R^3 s^3 C^3}{1+5sRC+6s^2C^2R^2+s^3C^3R^3}$$

$$\beta = \frac{V_0}{V_i} = \frac{R^3 s^3 C^3}{1+5sCR+6s^2C^2R^2+s^3C^3R^3}$$

Replacing $s$ by $(j\omega)$ we get,

$$\beta = \frac{-j\omega^3R^3C^3}{1+5j\omega CR-6\omega^2C^2R^2-j\omega^3C^3R^3}$$

Dividing the numerator and denominator by $-j\omega^3R^3C^3$ and replacing $\frac{1}{\omega RC}$ by $\alpha$ we get,

$$\beta = \frac{1}{1-6j\alpha-5\alpha^2+j\alpha^3}$$

$$\beta = \frac{1}{(1-5\alpha^2)-j\alpha(6-\alpha^2)}$$

For 'β' to be real

$$\alpha(6-\alpha^2) = 0$$

$$\alpha = \sqrt{6}$$

$$\omega = \frac{1}{RC\sqrt{6}}$$

∴ At this frequency,

$$\beta = \frac{1}{1-5(\sqrt{6})^2} = \frac{-1}{29}$$

To have sustained oscillation

$$|A\beta| \geq 1$$

$$|A| \geq 29$$

**Q.6 (c) (i) Solution:**

Nominal ratio,

$$K_n = \frac{100}{5} = 20$$

In the absence of any other data, the turns ratio is taken equal to the nominal ratio,

$$n = 20$$

Neglecting the burden of the secondary winding, the total burden of the secondary circuit is equal to the burder of the meter,

∴ Burden of secondary circuit = 25 VA

Voltage across primary winding,

$$E_P = \frac{V_A}{I_P} = \frac{25}{100} = 0.25 \text{ V}$$

Load component of current, $I_e = \dfrac{\text{Iron loss}}{E_p} = \dfrac{0.2}{0.25} = 0.8 \text{ A}$

Magnetising current, $I_m = 1.5 \text{ A}$

Secondary circuit phase angle, $\delta = \tan^{-1}\dfrac{X}{R}\tan^{-1}\dfrac{1}{5} = 11°18'$

$\therefore$ $\cos\delta = 0.98$

and $\sin\delta = 0.196$

$$\text{Actual ratio, } R = n + \frac{I_e\cos\delta + I_m\sin\delta}{I_s}$$

$$\text{Ratio error} = \frac{K_n - R}{R}\times 100$$

$$= \frac{20 - 20.125}{20.215}\times 100 = -1.075\%$$

Phase angle error, $\theta = \dfrac{180}{\pi}\left[\dfrac{I_m\cos\delta - I_e\sin\delta}{nI_s}\right]$

$$= \frac{180}{\pi}\left[\frac{15\times 0.98 - 0.98\times 0.196}{20\times 5}\right] = 0.75°$$

**Q.6 (c) (ii) Solution:**

The special arrangements incorporated in electrodynamometer wattmeter to make it a low power type of wattmeter are as follows:

- **Pressure coil current:** The pressure coil circuit is designed to have a low value of resistance; so that the current flowing through it is increased to give an increased operating torque. The pressure coil current in a low power factor wattmeter may be as much as 10 times the value employed for high power factor wattmeters.
- **Compensation for pressure coil current:** The power being measured in a low power factor circuit is small and current is high on account of low power factor.

  If ordinary wattmeter is used, it result into large power loss in the current coil and therefore, it will give large error.

Hence, it is absolutely necessary to compensate for the pressure coil current in a low power factor wattmeter.

- **Compensation for inductance of pressure coil:** The error caused by pressure coil inductance is $VI \sin\phi \tan\beta$. Now, with the low power factor, the value of $\phi$ is large and therefore, the error is correspondingly large. Hence, in a low power factor wattmeter, this is compensated by connecting a capacitor across a part of series resistance of the pressure coil circuit.
- **Small control torque:** Low power factor wattmeter are designed to have a small control torque so that they give full scale deflection for power factor as low as 0.1.

**Q.7 (a) Solution:**

Given : $\mu_n = 7.5 \text{ cm}^2/\text{V-s}; \rho = 9.43 \times 10^{-6}\ \Omega\text{-m}$

**(i)** We know, $\sigma = ne\mu_n$

$$n = \frac{1}{\rho\mu_n e}$$

$$= \frac{1}{9.43\times10^{-6}\times7.5\times1.6\times10^{-19}}$$

$$= 8.84 \times 10^{22} \text{ cm}^{-3}$$

Atomic concentration, $n_{at} = \dfrac{dN_A}{M_{at}}$

$$n_{at} = \frac{7.3\times6.023\times10^{23}}{115} = 3.82\times10^{22} \text{ cm}^{-3}$$

Effective number of electrons denoted by per In atom

$$n_{\text{eff}} = \frac{n}{n_{at}} = \frac{8.84\times10^{22}}{3.82\times10^{22}}$$

$$\eta_{\text{eff}} = 2.3141$$

**(ii)** Given : $l_e = 8.2 \text{ nm}$

Drift mobility, $\mu_m = \dfrac{e\tau}{m_e}$

$$\tau = \frac{\mu_n m_e}{e} \quad (m_e = \text{mass of electron})$$

$$\tau = \frac{7.5\times10^{-4}\times9.1\times10^{-31}}{1.6\times10^{-19}}$$

$$\tau = 4.266 \times 10^{-15} \text{ s}$$

We know, $l_e = v_d \tau$

$$v_d = \frac{8.2 \times 10^{-9}}{4.266 \times 10^{-15}} \text{ m/s}$$

$$v_d = 1.922 \times 10^{6} \text{ m/s}$$

**(iii)** According to Wiedemann-Franz law,

Thermal conductivity, $K = \sigma T C_{\text{WFL}}$

where $C_{WFL}$ = Lorentz number (or) Wiedemann-Frantz-Lorentz coefficient

$$C_{WFL} = \frac{\pi^2 K_B^2}{3e^2} = \frac{\pi^2 (1.38 \times 10^{-23})^2}{3 \times (1.6 \times 10^{-19})^2}$$

$$= 2.45 \times 10^{-8} \text{ W}\Omega/\text{K}^2$$

$$K = \sigma T C_{WFL} = \frac{T C_{WFL}}{\rho}$$

$$= \frac{300 \times 2.45 \times 10^{-8}}{9.43 \times 10^{-6}}$$

$$K = 0.78 \text{ W/K}$$

**Q.7 (b) Solution:**

Given : $I_{D(\text{ON})} = 3 \text{ mA at } V_{GS(\text{ON})} = 10 \text{ V}$

$$V_{GS(\text{TH})} = 5 \text{ V}$$

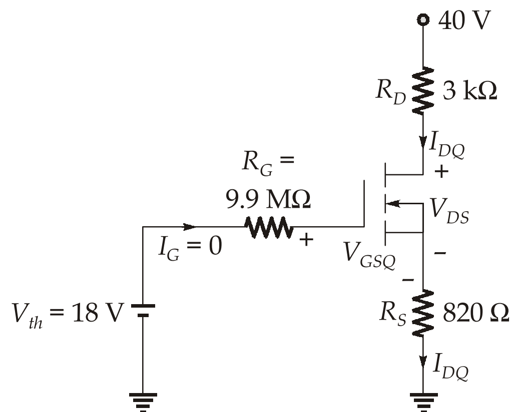
We know, $k = \dfrac{I_{D(\text{ON})}}{[V_{GS(\text{ON})} - V_{GS(\text{TH})}]^2}$

$$k = \frac{3}{(10-5)^2} = 0.12 \text{ mA/V}^2$$

Thevenin equivalent circuit, $V_{\text{Th}} = \dfrac{40 \times 18}{18 + 22} = 18 \text{ V}$

$$R_{\text{Th}} = R_G = 18 \,||\, 22 = \frac{18 \times 22}{18 + 22} = 9.9 \text{ M}\Omega$$

Thevenin equivalent circuit,




Apply KVL in G-S loop,

$$-V_{Th} + I_G R_G + V_{GS} + I_{DQ} R_S = 0$$

$$I_D = \frac{18 - V_{GS}}{820} \text{ A} \quad \text{...(i)}$$

Also, the current in MOSFET

$$I_D = K[V_{GS} - V_{GS(TH)}]^2$$

$$I_D = 0.12 \times 10^{-3}[V_{GS} - 5]^2 \quad \text{...(ii)}$$

From eqn. (i) and (ii),

$$\frac{18 - V_{GS}}{820} = 0.12 \times 10^{-3}[V_{GS}^2 - 10V_{GS} + 25]$$

$$9.84V_{GS}^2 - 98.4V_{GS} + 246 = 1800 - 100V_{GS}$$

$$9.84V_{GS}^2 + 1.6V_{GS} - 1554 = 0$$

$$V_{GS} = 12.486, -12.648$$

$$V_{GS} = 12.486 \text{ V}$$

$$I_{DQ} = \frac{18 - 12.486}{820} = 6.724 \times 10^{-3} \text{ A}$$

$$I_{DQ} = 6.724 \text{ mA}$$

Apply KVL in D-S loop,

$$-40 + R_D I_{DQ} + V_{DS} + R_S I_{DQ} = 0$$

$$V_{DS} = 40 - (3 + 0.82) \times 10^3 \times 6.724 \times 10^{-3}$$

$$V_{DS} = 14.314 \text{ V}$$

**Q.7 (c) Solution:**

Let, $f(x) = 3x - \cos x - 1$

$f(0) = -2$ = negative,

$f(1) = 3 - 0.5403 - 1 = 1.4597$ = positive

so a root of $f(x) = 0$ lies between 0 and 1. It is nearer to 1.

Let us take $x_0 = 0.6$

Also, $f'(x) = 3 + \sin x$

∴ Newton's iteration formula gives,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{3x_n - \cos x_n - 1}{3 + \sin x_n}$$

$$= \frac{x_n \sin x_n + \cos x_n + 1}{3 + \sin x_n} \quad \text{...(i)}$$

Putting $n = 0$, the first approximation $x_1$ is given by

$$x_1 = \frac{x_0 \sin x_0 + \cos x_0 + 1}{3 + \sin x_0} = \frac{(0.6)\sin\left(0.6 \times \frac{180}{\pi}\right) + \cos\left(0.6 \times \frac{180}{\pi}\right) + 1}{3 + \sin\left(0.6 \times \frac{180}{\pi}\right)}$$

$$= \frac{0.6 \times 0.5646 + 0.82533 + 1}{3 + 0.5646} = 0.6071$$

Putting $n = 1$ in (*i*), the second approximation is

$$x_2 = \frac{x_1 \sin x_1 + \cos x_1 + 1}{3 + \sin x_1} = \frac{0.6071 \sin\left(0.6071 \times \frac{180}{\pi}\right) + \cos\left(0.6071 \times \frac{180}{\pi}\right) + 1}{3 + \sin\left(0.6071 \times \frac{180}{\pi}\right)}$$

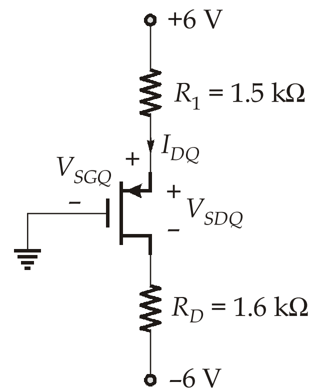
$$= \frac{0.6071 \times 0.57049 + 0.8213 + 1}{3 + 0.57049} = 0.6071$$

Clearly, $x_1 = x_2$

Hence the desired root is 0.6071 correct to four decimal places.

**Q.8 (a) Solution:**

**(i)** For DC analysis of the given circuit, all the coupling capacitor can be open circuited and the resultant equivalent circuit will be as shown below:




By assuming that the transistor is in saturation mode and taking the numerical value of $I_{DQ}$ in mA, we get,

$$I_{DQ} = K_p\left(V_{SGQ} - |V_{tp}|\right)^2$$

where, $$V_{SGQ} = 6 - 1.5\, I_{DQ}$$

So, $$I_{DQ} = 2.5(6 - 1.5I_{DQ} - 1.8)^2$$

$$= 2.5(4.2 - 1.5I_{DQ})^2$$

$$I_D = 2.5(4.2^2 - 2 \times 4.2 \times 1.5I_{DQ} + 2.25\, I_{DQ}^2)$$

$$I_D = 2.5\left(17.64 - 12.6I_{DQ} + 2.25I_{DQ}^2\right)$$

$$5.625I_{DQ}^2 - 32.5I_{DQ} + 44.1 = 0$$

By solving the above equation, we get

$$I_{DQ} = 3.6 \text{ mA}, 2.177 \text{ mA}$$

For $$I_{DQ} = 3.6 \text{ mA}, V_{SGQ} = 6 - 1.5 \times 3.6$$

$$= 0.6 \text{ V} < |V_{tp}|$$

$$I_{DQ} = 2.177 \text{ mA}, V_{GSQ}$$

$$= 6 - 1.5 \times 2.177 = 2.7345 \text{ V} > |V_{tp}|$$

So, for the assumed case, the valid value of $I_{DQ}$ is 2.177 mA.

$$V_{SDQ} = 12 - (1.5 + 1.6)I_{DQ}$$

$$= 12 - 3.1 \times 2.177 = 5.2513 \text{ V}$$

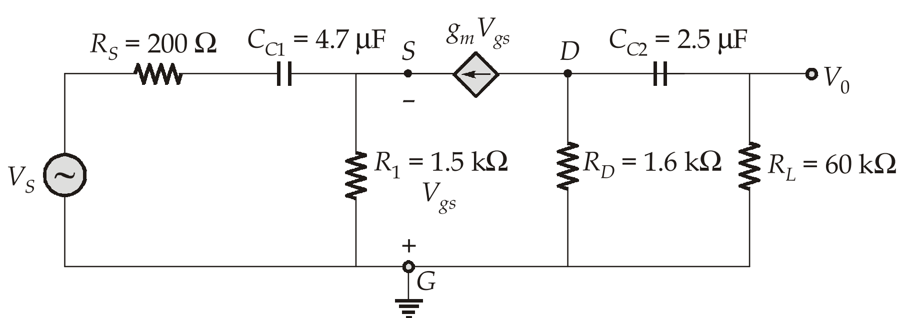
$$V_{SGQ} - |V_{TP}| = 2.734 - 1.8 = 0.934 \text{ V}$$

Since, $V_{SDQ} > V_{SGQ} - |V_{tp}|$, so, the initial assumption is correct about the mode of operation of transistor.

The small-signal parameters of the transistor are,

$$g_m = 2K_p(V_{SGQ} - |V_{tp}|)$$
$$= 2 \times 2.5 \times 0.934$$
$$= 4.67 \text{ mA/V}$$
$$r_0 = \frac{V_A}{I_{DQ}} = \frac{1}{\lambda I_{DQ}} = \infty$$

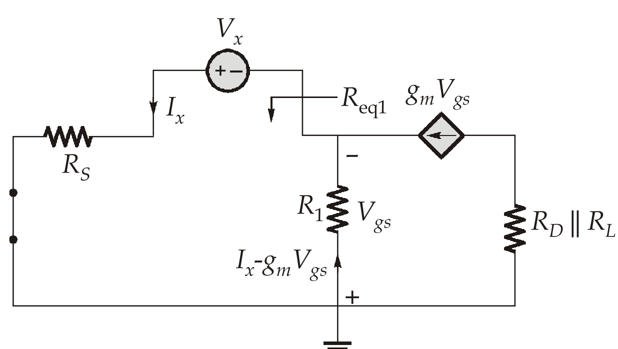
**(ii)** The small signal equivalent of the given amplifier will be




Calculation of time constant ($\tau_1$) associated with $C_{C1}$:

$$\tau_1 = R_{eq1} C_{C1}$$

While calculating $R_{eq1}$, $C_{C2}$ must be short circuited and voltage source $V_s$ should be deactivated. To calculate $R_{eq1}$, a voltage source $V_x$ is connected across the capacitor $C_{C1}$ as shown below:




$$V_x = R_s I_x + V_{gs} \quad \text{...(i)}$$

and

$$V_{gs} = R_1(I_x - g_m V_{gs})$$

$\therefore$

$$V_{gs} = \frac{R_1 I_x}{1 + g_m R_1}$$

Put in equation (i) we get,

$$\frac{V_x}{I_x} = \left(R_s + \frac{R_1}{1+g_m R_1}\right)$$

$$R_{eq1} = \frac{V_x}{I_x} = R_s + \frac{R_1}{1+g_m R_1} = 200 + \frac{1500}{1+4.67\times1.5}$$

$$= 387.382\ \Omega$$

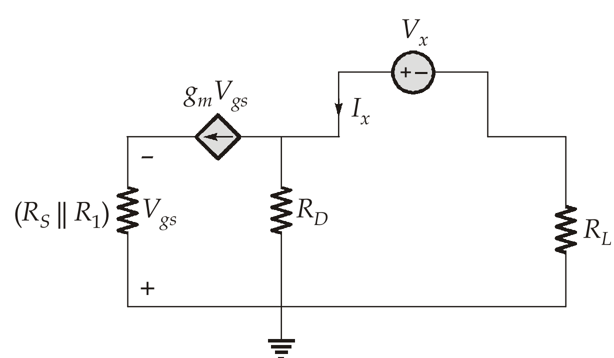
$$\tau_1 = R_{eq1}\, C_{C1} = 387.382 \times 4.7 \times 10^{-6}$$

$$= 1.82\text{ ms}$$

Calculation of time constant ($\tau_2$) associated with $C_{C2}$:

$$\tau_2 = R_{eq2} C_{C2}$$

While calculating $R_{eq2}$, $C_{C1}$ must be short circuited and the voltage source must be deactivated. To calculate $R_{eq2}$, a voltage source $V_x$ is connected across the capacitor $C_{C2}$ as shown below:




$$V_{gs} = -g_m\, V_{gs}(R_s \parallel R_1)$$

So,
$$V_{gs} = 0$$

$$R_{eq2} = \frac{V_x}{I_x} = R_D + R_L = 60 + 1.6 = 61.6\text{ k}\Omega$$

$$\tau_2 = R_{eq2}C_{C2} = 61.6 \times 10^3 \times 2.5 \times 10^{-6}$$

$$= 0.154 = 154\text{ ms}$$

**(iii)** The corner frequency associated with $C_{C1}$ is

$$f_{C1} = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi\times1.82\times10^{-3}} = 87.447\text{ Hz}$$

The corner frequency associated with $C_{C2}$ is

$$f_{C2} = \frac{1}{2\pi\tau_2} = \frac{1}{2\pi\times154\times10^{-3}} = 1.033\text{ Hz}$$

So, the corner frequency due to $C_{C1}$ dominates that due to $C_{C2}$. Hence, the lower cut-off frequency of the amplifier can be given by

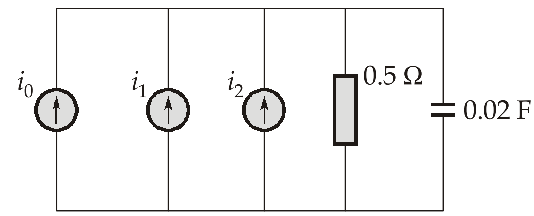
$$f_L = f_{C1} = 87.447 \text{ Hz}$$

**Q.8 (b) Solution:**

**(i)** Each source is sinusoidal and hence

$$V = ZI$$

where $V$ is the phasor voltage, $I$ is the phasor current, and $Z$ is the driving point impedance of the RC circuit, which is given by




$$Z = \frac{1}{Y} = \frac{1}{2 + j\omega(0.02)}$$

Using superposition theorem,

With the dc source, $V_0 = \frac{1}{2} \times 5 = 2.5 \text{ V}$

With the first harmonic source,

$$\omega = 100, I_1 = 3 \angle 45° \text{ A}$$

and $V_1 = \frac{1}{2+2j} 3\angle 45° = \frac{3\angle 45°}{2.828\angle 45°} = 1.06\angle 0°$

Thus, $v_1(t) = 1.06 \cos 100t \text{ V}$

With the second harmonic,

$$\omega = 200, I_2 = 2 \angle -10° \text{ A}$$

and $V_2 = \frac{1}{2+j4} 2\angle -10° = \frac{2\angle -10°}{4.472\angle 63°} = 0.447\angle -73°$

Thus, $v_2(t) = 0.447 \cos(200t - 73°) \text{ V}$

By superposition, the output voltage is:

$$v(t) = 2.5 + 1.06 \cos 100t + 0.447 \cos(200t - 73°) \text{ V}$$

**(ii)** $P_{av} = v(t)i(t)$

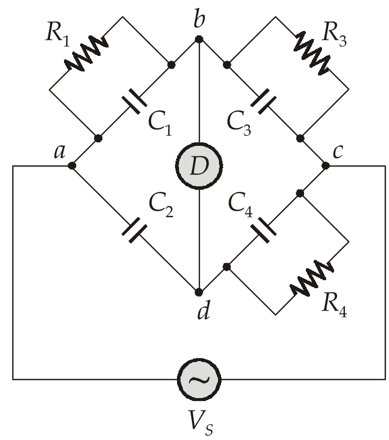
$$= 5 \times 2.5 + \left(\frac{3}{\sqrt{2}}\right) \times \left(\frac{1.06}{\sqrt{2}}\right) \cos 45° + \left(\frac{2}{\sqrt{2}}\right) \times \left(\frac{0.477}{\sqrt{2}}\right) \cos 63°$$

$$= 12.5 + 1.59 \cos 45° + 0.477 \cos 63°$$

$$= 13.84 \text{ W}$$

**Q.8 (c) (i) Solution:**

The voltage circuit is shown below :




For balance, $Y_1Y_4 = Y_2Y_3$

or, $$\left(\frac{1}{R_1}+j\omega C_1\right)\left(\frac{1}{R_4}+j\omega C_4\right) = (j\omega C_2)\left(\frac{1}{R_3}+j\omega C_3\right)$$

or, $$\left(\frac{1}{R_1R_4}-\omega^2C_1C_4\right)+j\omega\left(\frac{C_4}{R_1}+\frac{C_1}{R_4}\right) = j\omega\frac{C_2}{R_3}-\omega^2C_2C_3$$

Equating the real and imaginary parts, we have

$$\frac{1}{R_1R_4}-\omega^2C_1C_4 = -\omega^2C_2C_3 \quad ...(i)$$

and $$\frac{C_4}{R_1}+\frac{C_1}{R_4} = \frac{C_2}{R_3} \quad ...(ii)$$

From (i) and (ii), we have

$$C_1 = \frac{\frac{C_2R_4}{R_3}+\omega^2C_2C_3C_4R_4^2}{1+\omega^2C_4^2R_4^2}$$

Now $$\omega^2C_2C_3C_4R_4^2 << \frac{C_2R_4}{R_3}$$

and $$\omega^2C_4^2R_4^2 << 1$$

Hence we can write, $$C_1 = \frac{C_2R_4}{R_3}$$

When the capacitor $C_1$ is without specimen dielectric let its capacitance be $C_0$ .

$$\therefore \qquad C_0 = \frac{C_2R_4}{R_3} = 150 \times \frac{5000}{5000} = 150 \text{ pF}$$

When the specimen is inserted as dielectric, let the capacitance be $C_S$ .

$$\therefore \qquad C_S = \frac{C_2R_4}{R_3} = 900 \times \frac{5000}{5000} = 900 \text{ pF}$$

Now, $$C_0 = \frac{\varepsilon_0 A}{d} \text{ and } C_S = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Hence relative permittivity of specimen

$$\varepsilon_r = \frac{C_S}{C_0} = \frac{900}{150} = 6$$

**Q.8 (c) (ii) Solution:**

Primary winding turns, $N_p = 1$

Secondary winding turns, $N_s = 199$

$\therefore$ Turns ratio, $n = 199$

$$\text{Nominal ratio, } K_n = \frac{1000}{5} = 200$$

Magnetizing current, $I_m = 7$ A

Loss component, $I_e = 4$ A

**Case I:** for 0.8 p.f. lagging

For lagging p.f. the secondary phase angle $\delta$ is positive

$$\cos\delta = \text{secondary p.f.} = 0.8$$

$$\sin\delta = \sqrt{(1)^2 - (0.8)^2} = 0.6$$

From equation, actual ratio, $$R = n + \frac{I_e\cos\delta + I_m\sin\delta}{I_s}$$

$$= 199 + \frac{4\times 0.8 + 7\times 0.6}{5} = 200.48$$

$$\therefore \qquad \text{Ratio error} = \frac{K_n - R}{R}\times 100 = \frac{200 - 200.48}{200.48}\times 100 = -0.24\%$$

and $$\text{phase angle, } \theta = \frac{180}{\pi}\left[\frac{I_m\cos\delta - I_e\sin\delta}{nI_s}\right]$$

$$= \frac{180}{\pi}\left[\frac{7\times 0.8 - 4\times 0.6}{199\times 5}\right] = +0.185°$$

**Case II:** for 0.8 p.f. leading:

For leading p.f. δ is negative:

$$\cos\delta = 0.8$$

and $$\sin\delta = -0.6$$

$$\text{Actual ratio, } R = n + \frac{I_e\cos\delta + I_m\sin\delta}{I_s}$$

$$= 199 + \frac{4\times0.8 - 7\times0.6}{5} = 198.8$$

$$\therefore \quad \text{Ratio error} = \frac{K_n - R}{R}\times100 = \frac{200-198.8}{198.8}\times100$$

$$= +0.603\%$$

and $$\text{phase angle, } \theta = \frac{180}{\pi}\left[\frac{I_m\cos\delta - I_e\sin\delta}{nI_s}\right]$$

$$= \frac{180}{\pi}\left[\frac{7\times0.8+4\times0.6}{199\times5}\right] = 0.46°$$

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