MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

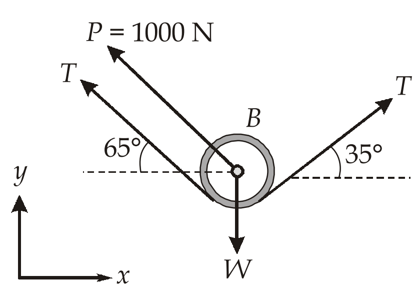
Mechanical Engineering Test No : 13

Full Syllabus Test (Paper-2)

Section : A

1. (a)

Free body diagrams of pulleys B :



Pulley B

From free body diagram of pulley B :

$\Sigma F_x = 0;$

$T\cos(35°) - T\cos(65°) - 1000\cos(65°) = 0$

$$\Rightarrow \quad T = \frac{1000\cos(65°)}{\cos(35°) - \cos(65°)} = 1065.78 \text{ N}$$ **Ans.**

$\Sigma F_y = 0;$

$T\sin(35°) + T\sin(65°) + 1000\sin(65°) - W = 0$

$$\Rightarrow \qquad W = (1065.78) \times [\sin(35°) + \sin(65°)] + 1000 \sin(65°)$$

$$= 2483.54 \text{ N}$$ **Ans.**

1. (b)

Given : $m = 1.5$; $s = 1200$ N-m; $F_0 = 15$ N; $c = 50$ N-s/m

Frequency at resonance,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1200}{1.5}} = 28.284 \text{ rad/s}$$

(i) Phase angle at resonance,

$$\tan\phi = \infty$$

$$\Rightarrow \qquad \phi = \frac{\pi}{2}$$ **Ans.**

(ii) Damping factor,
$$\xi = \frac{c}{2\sqrt{sm}} = \frac{50}{2 \times \sqrt{1200 \times 1.5}} = 0.5893$$

Amplitude at resonance,

$$A_{reso} = \frac{F_0 / s}{2\xi} = \frac{(15/1200)}{2 \times 0.5893}$$

$$= 0.010606 \text{ m} = 10.606 \text{ mm}$$ **Ans.**

(iii) The frequency corresponding to the peak amplitude

$$\omega_{peak} = \omega_n\sqrt{1-2\xi^2} = (28.284) \times \sqrt{1 - 2 \times (0.5893)^2}$$

$$= 15.632 \text{ rad/s}$$ **Ans.**

(iv) The damped frequency, $\omega_d = \omega_n\sqrt{1-\xi^2} = (28.284) \times \sqrt{1-(0.5893)^2}$

$$= 22.851 \text{ rad/s}$$ **Ans.**

1. (c)

Given : $\sigma_x = 200$ MPa; $\sigma_y = -100$ MPa; $\sigma_z = 50$ MPa; $\tau_{xy} = -100$ MPa; $\tau_{yz} = 50$ MPa and $\tau_{xz} = -50$ MPa, $G = 80$ GPa, $E = 200$ GPa

Poisson's ratio, $\mu = \dfrac{E}{2G} - 1 = \dfrac{200}{2(80)} - 1 = 0.25$

Now,
$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_x - \mu\left(\sigma_y + \sigma_z\right)\right]$$

$$= \frac{1}{200 \times 10^3} \times [200 - 0.25(-100+50)] = 1.0625 \times 10^{-3}$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_y - \mu\left(\sigma_x + \sigma_z\right)\right]$$

$$= \frac{1}{200\times10^3}\times[-100 - 0.25(200+50)] = -0.8125\times10^{-3}$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_z - \mu\left(\sigma_x + \sigma_y\right)\right]$$

$$= \frac{1}{200\times10^3}\times[50 - 0.25(200-100)] = 0.125\times10^{-3}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-100}{80\times10^3} = -1.25\times10^{-3}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} = \frac{50}{80\times10^3} = 0.625\times10^{-3}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G} = \frac{-50}{80\times10^3} = -0.625\times10^{-3}$$

The strain tensor is

$$e_{ij} = \begin{bmatrix} \varepsilon_{xx} & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_{yy} & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_{zz} \end{bmatrix}$$

$$= \begin{bmatrix} 1.0625\times10^{-3} & -0.625\times10^{-3} & -0.3125\times10^{-3} \\ -0.625\times10^{-3} & -0.8125\times10^{-3} & 0.3125\times10^{-3} \\ -0.3125\times10^{-3} & 0.3125\times10^{-3} & 0.125\times10^{-3} \end{bmatrix} \textbf{ Ans.}$$

1. (d)

Given: P = 10 kN, τ_{per} = 90 N/mm^2

Primary shear stress:

Let t is the throat of each weld. There are two welded W_1 and W_2 and their throat area is given by,

$$A = 2(60t) = 120t \text{ mm}^2$$

The primary shear stress is given as,

$$\tau_1 = \frac{P}{A} = \frac{10\times10^3}{(120t)} = \left(\frac{250}{3t}\right)\text{N/mm}^2$$

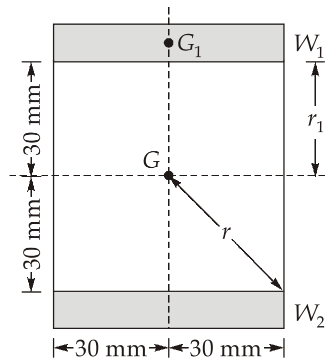
Secondary shear stress:

The two welds are symmetrical and G is the centre of gravity of the two welds,]

$$e = \frac{60}{2} + 120 = 150 \text{ mm}$$

$$M = P \times e = (10 \times 10^3)(150) = 1.5 \times 10^6 \text{ N-mm}$$

The distance r of the farthest point in the weld from the centre of gravity is given by:



$$r = \sqrt{30^2 + 30^2} = 42.43 \text{ mm}$$

The polar moment of inertia of the two welds is given as

$$J = J_1 + J_2 = 60t\left[\frac{60^2}{12} + 30^2\right] + 60t\left[\frac{60^2}{12} + 30^2\right] \quad \because \; J_1 = J_2$$

$$= (144000t) \text{ mm}^4$$

The secondary shear stress is given as:

$$\tau_2 = \frac{Mr}{J} = \frac{(1.5 \times 10^6)(42.43)}{(144000t)} = \left(\frac{441.98}{t}\right) \text{N/mm}^2$$

Resultant shear stress:

$$\tau_R = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos 45°}$$

$$\tau_R = \frac{1}{\tau}\sqrt{441.98^2 + \left(\frac{250}{3}\right)^2 + 2 \times 441.98 \times \frac{250}{3} \times \cos 45°}$$

$$\tau_R = \frac{504.36}{t}$$

Since the permissible shear stress for the weld material is 90 N/mm^2.

$$\Rightarrow \qquad \frac{504.36}{t} = 90$$

$$\Rightarrow \qquad t = 5.604 \simeq 5.6 \text{ mm}$$

1. (e)

- **Hunting :** A governor is said to be hunt if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place.
- **Isochronism :** A governor with a range of speed zero is known as an isochronous governor. This means that for all positions of the sleeve or the balls, the governor has the same equilibrium speed. Any change of speed results in moving the balls and the sleeve to their extreme positions. However, an isochronous governor is not practical due to friction at the sleeve.
- **Stability :** A governor is said to be stable if it brings the speed of the engine to the required value and there is not much hunting. The ball masses occupy a definite position for each speed of the engine within the working range. Obviously, the stability and the sensitivity are two opposite characteristics.
- **Sensitiveness of a governor :** A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of sensitivity.

  As a governor is used to limit the change of speed of the engine between minimum to full-load conditions, the sensitiveness of a governor is also defined as the ratio of the difference between the maximum and the minimum speeds (range of speed) to the mean equilibrium speed. Thus,

$$\text{Sensitiveness} = \frac{\text{Range of speed}}{\text{Mean speed}}$$

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

When N = Mean speed

N_1 = Minimum speed corresponding to full-load conditions

N_2 = Maximum speed corresponding to no-load conditions

- **Effort of a governor :** The effort of a governor is the force it can exert at the sleeve on the mechanism, which controls the supply of fuel to the engine. The mean force exerted during the given change of speed is termed as effort. Generally efforts are defined for 1% change of speed.

- **Power of a governor :** The power of a governor is the work done at the sleeve for a given percentage change of speed, i.e., it is the product of the effort and the displacement of the sleeve.

  For a Porter governor, having all equal arms which intersect on the axis or pivoted at points equidistant from the spindle axis,

$$\text{Power} = \frac{E}{2}\times(2\times\text{height of governor})$$

2. (a)

(i)

Given : $D_s = 200$ mm; $\tau_H = 1.2\tau_S$; $P_H = 1.15P_S$; $N_H = 1.08\,N_S$

We know that, $T \propto \frac{P}{N}$

$$\therefore \quad \frac{T_H}{T_S} = \frac{P_H}{P_S}\times\frac{N_S}{N_H} = \frac{1.15}{1.08} = 1.065 \quad \text{...(i)}$$

Also, $\tau_H = 1.2\,\tau_S \qquad \left(\tau = \frac{TR}{J}\right)$

$$\therefore \quad \frac{T_H R_H}{J_H} = 1.2\frac{T_S R_S}{J_S}$$

$$\therefore \quad \frac{T_H}{T_S} = (1.2)\frac{J_H}{J_S}\times\frac{R_S}{R_H} \qquad \left(\frac{J}{R} = \frac{\pi D^3}{16}\right)$$

$$\therefore \quad \frac{T_H}{T_S} = (1.2)\left[\left(\frac{D_H^4 - d_H^4}{D_H}\right)\times\frac{\pi}{16}\right]\times\frac{16}{\pi D_s^3} \quad \text{...(ii)}$$

From equations (i) and (ii), we get

$$1.065 = (1.2)\frac{D_H^4 - d_H^4}{D_H D_S^3}$$

Now, for the maximum value of internal diameter of the hollow shaft, its external diameter (D_H) will be equal to the diameter D_s of the solid shaft.

$$1.065 = (1.2)\frac{D_H^4 - d_H^4}{D_s^4} = 1.2\left(\frac{200^4 - d_H^4}{200^4}\right)$$

$$\therefore \quad d_H = 115.83 \text{ mm}$$

(ii)

Given : $L = 4$ m; $d = 30$ mm; $E = 200$ GPa; $\delta_{max} = 3.8$ mm; $h = 50$ mm

Maximum stress induced, $\sigma_{max} = E \cdot \varepsilon = E \cdot \frac{\delta_{max}}{L}$

$$= \left(200 \times 10^3\right) \times \frac{3.8}{\left(4 \times 10^3\right)} = 190 \text{ MPa}$$ **Ans.**

On equating work done and strain energy,

$$W(h + \delta_{max}) = \frac{\sigma_{max}^2}{2E} AL$$

$$\Rightarrow \quad W(50 + 3.8) = \frac{(190)^2}{2 \times \left(200 \times 10^3\right)} \times \left(\frac{\pi}{4} \times 30^2\right) \times \left(4 \times 10^3\right)$$

$$\Rightarrow \quad W = 4743.0457 \text{ N} = 4.7430457 \text{ kN}$$

$$\simeq 4.743 \text{ kN}$$ **Ans.**

2. (b)

(i)

The complete S-N curve from 10^0 cycle to 10^8 cycles is shown in figure below.

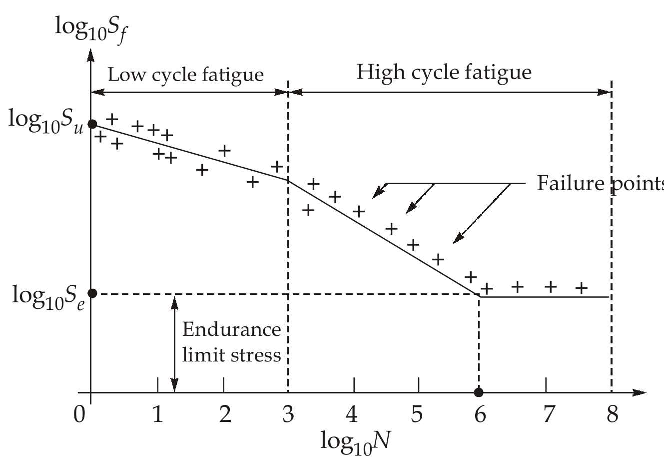


Fig. : Low and high cycle fatigue

There are two regions of this curve namely, low- cycle fatigue and high-cycle fatigue. The difference between these two fatigue failures is as follows:

1. Any fatigue failure when the number of stress cycles are less than 1000, is called low-cycle fatigue. Any fatigue failure when the number of stress cycles are more than 1000, is called high-cycle fatigue.
2. Failure of studs on truck wheels, failure of setscrews for locating gears on shafts or failures of short-lived devices such as missiles are the examples of low cycle fatigue. The failure of machine components such as springs, ball bearings or gears that are subjected to fluctuating stresses, are the examples of high-cycle fatigue.

3. The low-cycle fatigue involves plastic yielding at localized areas of the components. There are some theories of low-cycle fatigue. However, in many applications the designers simply ignore the fatigue effect when the number of stress cycles is less than 1000. A greater factor of safety is used to account for this effect. Such components are designed, on the basis of ultimate tensile strength or yield strength with suitable factor of safety. Components subjected to high-cycle fatigue are designed on the basis of endurance limit stress. S-N curve, Soderberg line, Gerber line or Goodman diagram are used in design such components.

(ii)

Given: R = 16 mm, L = 32 mm, $D = 2R$ = 32 mm, μ = 0.03 Pa-s, N = 1350 rpm, F_r = 1500 N

Diametral clearance between journal and bearing bore is given as

$$C = 32.04 - 32 = 0.04 \text{ mm}$$

$$\text{Rotational speed, } n = \frac{N}{60} = \frac{1350}{60} = 22.5 \text{ rps}$$

$$\text{Pressure, } P = \frac{W}{LD} = \frac{1500}{0.032 \times 0.032} = 1.4648 \times 10^6 \text{ Pa}$$

Bearing characteristic number is given as:

$$\text{BCN} = \frac{\mu n}{P} = \frac{0.03 \times 22.5}{1.4648 \times 10^6} = 4.608 \times 10^{-7}$$

Sommerfeld number is given as;

$$S = \frac{\mu n}{P}\left(\frac{D}{C}\right)^2 = \frac{0.03 \times 22.5}{1.4648 \times 10^6} \times \left(\frac{32}{0.04}\right)^2 = 0.29492$$

Coefficients of friction using petroff's equation is given as:

$$f = 2\pi^2\left(\frac{\mu n}{P}\right) \times \frac{D}{C} = 2\pi^2 \times \left(\frac{0.03 \times 22.5}{1.4648 \times 10^6}\right) \times \left(\frac{32}{0.04}\right)$$

$$= 7.2769 \times 10^{-3}$$

Frictional torque loss is given as

$$(T_f)_{\text{loss}} = fWR = (7.2769 \times 10^{-3})\,(1500)\,(0.016)$$

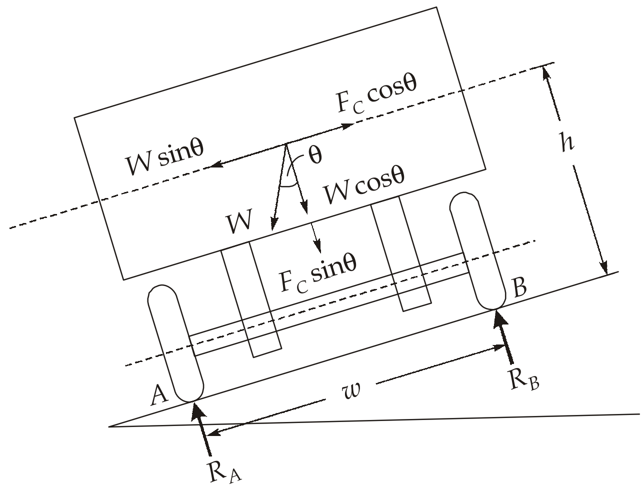
$$= 0.1746456 \text{ N-m} \simeq 0.17465 \text{ N-m}$$

2. (c)

Given : M = 2000 kg; w = 1.6 m; R = 28 m; m = 200 kg; θ = 10°; h = 0.940 m;

$r = \frac{640}{2} = 320$ mm = 0.320 m; k = 0.220

$$\text{Velocity, } v = 45 \text{ km/h} = \frac{45 \times 1000}{60 \times 60} = 12.5 \text{ m/s}$$



First, considering the effect of dead weight ($W = Mg$) of the car and that of the centrifugal force on it, determine the reactions R_A and R_B at the wheels A and B.

Resolving the forces perpendicular to the track

$$R_A + R_B = Mg\cos\theta + \frac{Mv^2}{R}\sin\theta$$

$$= 2000 \times 9.81 \times \cos(10°) + \frac{2000 \times (12.5)^2}{28} \times \sin(10°)$$

$$= 21259.96 \text{ N}$$

Taking moments about B,

$$R_A \times w = Mg\cos\theta + \frac{w}{2} + F_c\sin\theta \times \frac{w}{2} + Mg\sin\theta \times h - F_c\cos\theta \times h$$

$$\Rightarrow \quad R_A = \left(Mg\cos\theta + \frac{Mv^2}{R}\sin\theta\right) \times \frac{1}{2} + \left(Mg\sin\theta - \frac{Mv^2}{R}\cos\theta\right) \times \frac{h}{b}$$

$$\Rightarrow \quad R_A = \left(2000 \times 9.81 \times \cos(10°) + \frac{2000 \times (12.5)^2}{28}\sin(10°)\right) \times \frac{1}{2}$$
$$+ \left(2000 \times 9.81 \times \sin(10°) - \frac{2000 \times (12.5)^2}{28} \times \cos(10°)\right) \times \frac{0.940}{1.6}$$

$$\Rightarrow \quad R_A = 10629.98 - 4455.7$$

$$\Rightarrow \quad R_A = 6174.28 \text{ N}$$

then, $\quad R_A = 21260 - R_B = 21259.96 - 6174.28 = 15085.68 \text{ N}$

Reaction due to gyroscopic couple,

$$C_W = 2I_w\omega_w\cos\theta \times \omega_p$$

$$= 2mk^2 \frac{v^2}{rR} \times \cos\theta$$

$$= 2 \times 200 \times (0.220)^2 \times \frac{(12.5)^2}{0.320 \times 28} \times \cos(10°)$$

$$= 332.48 \text{ N-m}$$

Reaction on each outer wheel,

$$R_{GO} = \frac{C_w}{2w} = \frac{332.48}{2 \times 1.6} = 103.9 \text{ N} \qquad \text{(upwards)}$$

Reaction on each inner wheels,

$$R_{GI} = 103.9 \text{ N (downwards)}$$

Therefore,

Pressure on outer rails $= R_B + R_{GO} = 6174.28 + 103.9$

$= 6278.18$ N (downwards) **Ans.**

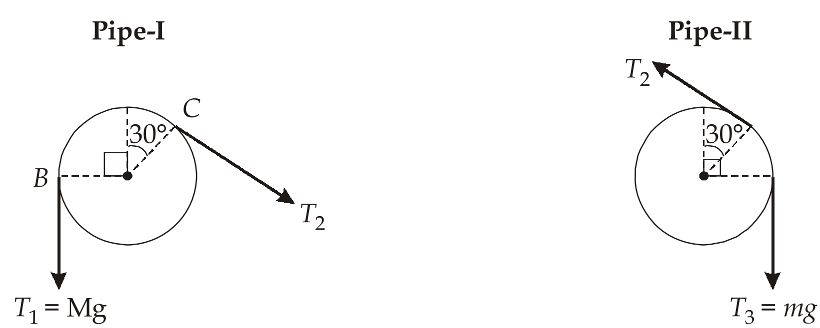
Pressure on inner rails $= R_A + R_{GI} = 15085.68 - 103.9$

$= 14981.78$ N (upwards) **Ans.**

(Pressure is opposite to the reactions)

3. (a)

(i)



Angle of wrap, $\alpha_1 = 90° + 30° = 120° = \frac{2\pi}{3}$; Angle of wrap, $\alpha_2 = 90° - 30° = 60° = \frac{\pi}{3}$

Case 1 : Minimum value of m $[M > m]$

For pipe I: $\frac{T_1}{T_2} = e^{\mu\alpha_1}$

$$\Rightarrow \quad T_2 = \frac{T_1}{e^{\mu\alpha_1}} = \frac{Mg}{e^{\mu\alpha_1}} = \frac{40 \times 9.81}{e^{0.3 \times \frac{2\pi}{3}}} = 209.34 \text{ N}$$

For pipe II : $\frac{T_2}{T_3} = e^{\mu\alpha_2}$

$\Rightarrow$ $T_3 = \frac{T_2}{e^{\mu\alpha_2}} = \frac{209.34}{e^{0.3\times\frac{\pi}{3}}} = 152.90$ N

$\Rightarrow$ $m = \frac{T_3}{g} = \frac{152.90}{9.81} = 15.586 \text{ kg} \simeq 15.59 \text{ kg}$ **Ans.**

Case 2 : Maximum value of m [$M < m$]

For pipe I: $\frac{T_2}{T_1} = e^{\mu\alpha_1}$

$\Rightarrow$ $T_2 = T_1 e^{\mu\alpha_1} = Mge^{\mu\alpha_1} = 40\times 9.81\times e^{0.3\times\frac{2\pi}{3}} = 735.54$ N

For pipe II : $\frac{T_3}{T_2} = e^{\mu\alpha_2}$

$\Rightarrow$ $T_3 = T_2 e^{\mu\alpha_2} = 735.54\times e^{0.3\times\frac{\pi}{3}} = 1007.03$ N

$\Rightarrow$ $mg = 1007.03$

$\Rightarrow$ $m = \frac{1007.03}{9.81} = 102.653 \text{ kg} \simeq 102.65 \text{ kg}$ **Ans.**

Hence, the two values of mass 'm' for which equilibrium is possible is 15.59 kg and 102.65 kg.

(ii)

For 1st stage of the motion,

$v_0 = 0;\ s_0 = 0;\ v = 50$ m/s and $a = 2$ m/s^2

$v = v_0 + at_1$

$\Rightarrow$ $50 = 0 + 2t_1$

$\Rightarrow$ $t_1 = 25$s

and $v^2 = v_0^2 + 2a(s - s_0)$

$\Rightarrow$ $50^2 = 0 + 2(2)(s_1 - 0)$

$\Rightarrow$ $s_1 = 625$ m

For 2nd stage of the motion,

$v_0 = 50$ m/s; $s_0 = 625$ m; $t_2 = 60$s; $a = 0$

$s - s_0 = v_0 t_2 + \frac{1}{2}at_2^2$

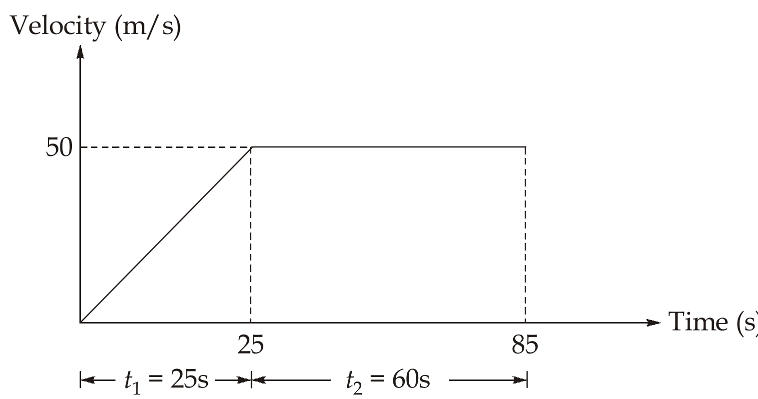
$\Rightarrow$ $s - 625 = 50\times 60 + \frac{1}{2}\times 0\times (60)^2$

$\Rightarrow$ $s = 3625$ m **Ans.**

$$\text{Average speed, } v_{avg} = \frac{s}{t_1 + t_2} = \frac{3625}{25+60} = 42.647 \text{ m/s}$$ **Ans.**

Hence, the total distance traveled is 3625 m and average speed is 42.647 m/s.

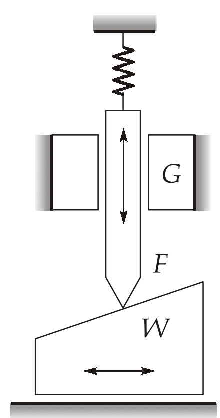
Variation of velocity of the train with time:



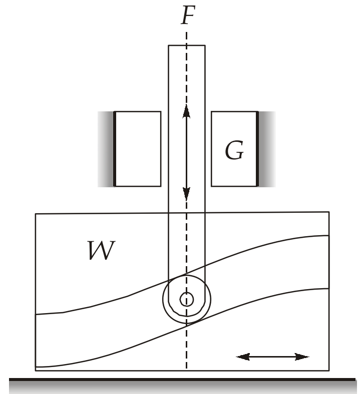
3. (b)

(i)

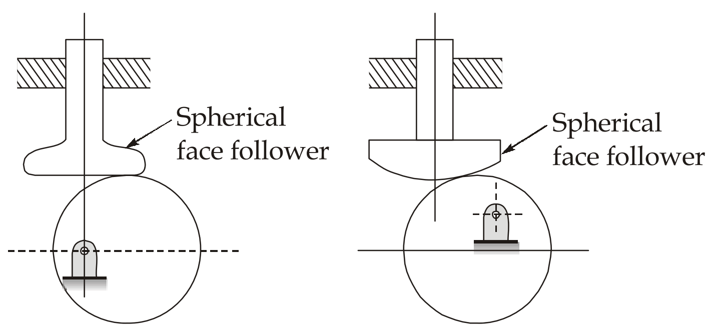
Knife-edge Follower : It is quite simple in construction. Figure shows such a follower. However, its use is limited as it produces a great wear of the surface at the point of contact.



Roller Follower : It is a widely used cam follower and has a cylindrical roller free to rotate about a pin joint. At low speeds, the follower has a pure rolling action, but at high speeds, some sliding also occurs. In case of steep rise, a roller follower jams the cam and, therefore, is not preferred.



Mushroom Follower : A mushroom follower has the advantage that it does not pose the problem of jamming the cam. However, high surface stresses and wear are quite high due to deflection and misalignment if a flat-faced follower is used. These disadvantages are reduced if a spherical-faced follower is used instead of a flat-faced follower.



(ii)

Given : $\phi = 20°$; $t = 30$; $m = 5$ mm; $G = 4$; $v = 1.8$ m/s

$$T = Gt = 4 \times 30 = 120$$

$$r = \frac{mt}{2} = \frac{5 \times 30}{2} = 75 \text{ mm}$$

$$R = rG = 75 \times 4 = 300 \text{ mm}$$

$$R_a = R + m = 300 + 5 = 305 \text{ mm}$$

$$r_a = r + m = 75 + 5 = 80 \text{ mm}$$

$$\text{Path of approach} = \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(305)^2 - (300\cos(20°))^2} - 300\sin(20°)$$

$$= 13.811 \text{ mm}$$

$$\text{Path of recess} = \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(80)^2 - (75\cos(20°))^2} - 75\sin(20°)$$

$$= 12.203 \text{ mm}$$

Path of contact = Path of approach + Path recess

$$= 13.811 + 12.203$$

$$= 26.014 \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi} = \frac{26.014}{\cos(20°)} = 27.684 \text{ mm}$$

$$\text{Angle of action of the pinion} = \frac{\text{Arc of contact}}{r} = \frac{27.684}{75} = 0.3691 \text{ rad} = 21.148° \quad \textbf{Ans.}$$

$$\text{Maximum velocity of sliding} = (\omega_P + \omega_G) \times \text{Path of approach}$$

$$= \left(\frac{v}{r} + \frac{v}{R}\right) \times \text{Path of approach}$$

$$= \left(\frac{1800}{75} + \frac{1800}{300}\right) \times 13.811 = 414.33 \text{ m/s} = 0.41433 \text{ m/s}$$

3. (c)

(i)

Given : d = 2 m; t = 10 mm; P = 2 MPa; E = 200 GPa; μ = 0.3

For spherical pressure vessel, hoop strain in any direction is given as

$$\epsilon_H = \frac{Pd}{4tE}(1-\mu)$$

$$\Rightarrow \quad \epsilon_H = \frac{2 \times 2000}{4 \times 10 \times (200 \times 10^3)} \times (1 - 0.3) = 3.5 \times 10^{-4}$$

$$\text{also,} \quad \epsilon_H = \frac{\Delta d}{d}$$

$$\Rightarrow \quad \Delta d = d \cdot \epsilon_H = 2000 \times (3.5 \times 10^{-4}) = 0.7 \text{ mm} \quad \textbf{Ans.}$$

$$\text{Volumetric strain, } \epsilon_V = 3\epsilon_H$$

$$\Rightarrow \quad \frac{\Delta V}{V} = 3\epsilon_H$$

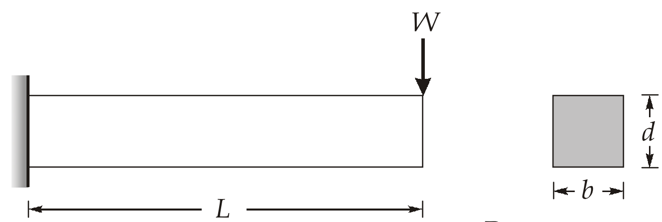
$$\Rightarrow \quad \Delta V = 3\epsilon_H V$$

$$\Rightarrow \quad \Delta V = 3\epsilon_H \frac{\pi d^3}{6} = 3 \times \left(3.5 \times 10^{-4}\right) \times \frac{\pi \times (2000)^3}{6}$$

$$= 4398229.715 \text{ mm}^3 \quad \textbf{Ans.}$$

(ii)

Given : L = 4 m; W = 8 kN; σ_b = 6 MPa; δ_{max} = 10 mm; $E = 2 \times 10^4$ N/mm^2



Beam cross-section

Using bending equation;

$$\sigma_{b,\max} = \frac{M_{\max}}{I}\cdot\frac{d}{2} = \frac{W\cdot L}{\frac{1}{12}bd^3}\cdot\frac{d}{2} = \frac{6WL}{bd^2}$$

$$6 = \frac{6\times\left(8\times10^3\right)\times4000}{bd^2}$$

$\Rightarrow$ $$bd^2 = 3.2 \times 10^7 \quad \text{...(i)}$$

Maximum deflection at the free end,

$$\delta_{\max} = \frac{WL^3}{3EI}$$

$$10 = \frac{\left(8\times10^3\right)(4000)^3}{3\times\left(2\times10^4\right)\times\frac{1}{12}bd^3}$$

$\Rightarrow$ $$bd^3 = 1.024 \times 10^{10} \quad \text{...(ii)}$$

Using equation (i) and (ii),

$$d = \frac{1.024\times10^{10}}{3.2\times10^7} = 320 \text{ mm}$$ **Ans.**

and $$b = \frac{3.2\times10^7}{d^2} = 312.5 \text{ mm}$$ **Ans.**

4. (a) (i)

When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a single slider-crank chain or simply a slider-crank chain.

Taking a different link as the fixed link, the slider-crank mechanism shown in Fig. (a) can be inverted into the mechanism shown in Fig. (b), (c) and (d).

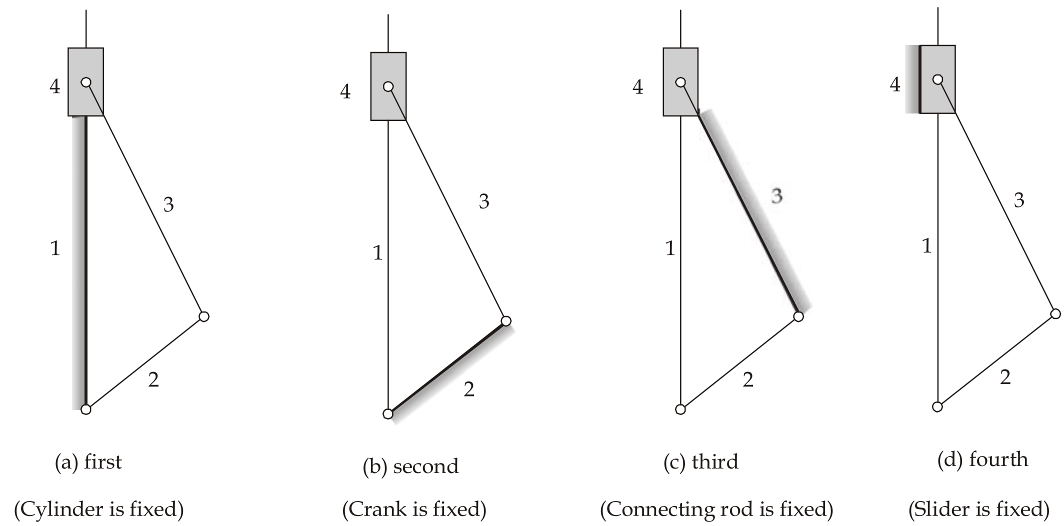


Fig. Inversions of the slider crank mechanism

(i) First Inversion

This inversion is obtained when link 1 (i.e; cylinder) is fixed.

Applications

- Reciprocating engine
- Reciprocating compressor

(ii) Second Inversion

Fixing of the link 2 (i.e; crank) of a slider-crank chain results in the second inversion.

Applications

- Whitworth quick return mechanism
- Rotary engine (GNOME engine)

(iii) Third Inversion

By fixing the link 3 (i.e; connecting rod) of the slider-crank mechanism, the third inversion is obtained.

Applications

- Oscillating cylinder engine
- Crank and slotted-lever mechanism

(iv) Fourth Inversion

If the link 4 (i.e., slider) of the slider-crank mechanism is fixed, the fourth inversion is obtained.

Applications

- Hand pump

(ii)

N = 210 rpm; L = 320 mm; m = 50 kg; m_r = 35 kg; c = 60% = 0.6

$$\text{Angular speed, } \omega = \frac{2\pi N}{60} = \frac{2\pi \times 210}{60} = 7\pi \text{ rad/s}$$

$$\text{Crank radius, } r = \frac{L}{2} = \frac{320}{2} = 160 \text{ mm}$$

(1) Mass to be balanced at the crankpin,

$$m_{cp} = cm + m_r = (0.6 \times 50 + 35) = 65 \text{ kg}$$

Balance mass (m_b) required at a radius r_c = 250 mm,

$$m_b r_b = m_{cp} r$$

$$m_c \times 0.250 = 65 \times 0.160$$

$\Rightarrow$ $$m_c = 41.6 \text{ kg}$$ **Ans.**

(2) Unbalance force when crank has turned θ = 40°,

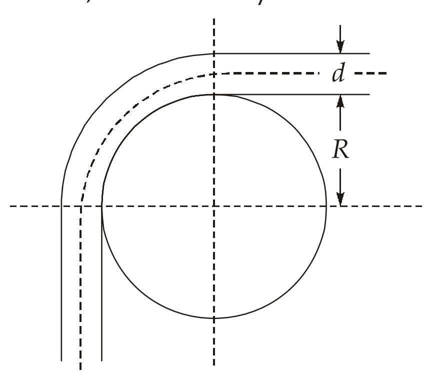
$$F_{un} = \sqrt{((1-c)mr\omega^2 \cos\theta)^2 + (cmr\omega^2 \sin\theta)^2}$$

$$= \sqrt{(1-0.6)\times 50 \times 0.160 \times (7\pi)^2 \times \cos(40°))^2 + (0.6 \times 50 \times 0.160 \times (7\pi)^2 \times \sin(40°))^2}$$

$$= 1905.74 \text{ N}$$ **Ans.**

4. (b) (i)

Given : d = 12 mm; E = 200 GPa; σ = 1600 N/mm²



$$\frac{\sigma_b}{y} = \frac{E}{R} \qquad \because R >> r$$

$\Rightarrow$ $$R = \frac{E}{\sigma_b} \cdot y = \frac{E}{\sigma_b} \cdot \frac{d}{2} \qquad \left[\because y = \frac{d}{2}\right]$$

$\Rightarrow$ $$R = \frac{200 \times 10^3}{1600} \times \frac{12}{2} = 750 \text{ mm}$$ **Ans.**

Again, using bending equation,

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\Rightarrow \quad M = \frac{\sigma_b}{y}.I = \frac{\sigma_b}{\left(\frac{d}{2}\right)}.\frac{\pi}{64}d^4 = \frac{\pi d^3}{32}\sigma_b$$

$$\Rightarrow \quad M = \frac{\pi \times (12)^3 \times 1600}{32} = 271433.6053 \text{ N.mm}$$

$$= 271.4336053 \text{ N.m} \simeq 271.434 \text{ Nm}$$ **Ans.**

(ii)

Given : D_1 = 160 mm; D_2 = 80 mm; ΔT = 100°C; $\alpha = 1.5 \times 10^{-5}$/°C; E = 200 kN/mm^2

If ends of the rod were not fixed, then on heating, the rod would have increased by ΔL, given by,

$$\Delta L = L\alpha\Delta T \quad \text{...(i)}$$

If a compressive force P is applied, the contraction achieved is given by,

$$\Delta L = \frac{4PL}{\pi E D_1 D_2} \quad \text{...(ii)}$$

Equating equation (i) and (ii);

$$\frac{4PL}{\pi E D_1 D_2} = L\alpha\Delta T$$

$$\Rightarrow \quad P = \frac{\pi E D_1 D_2}{4} \times \alpha\Delta T$$

$$= \frac{\pi \times \left(200 \times 10^3\right) \times 160 \times 80 \times \left(1.5 \times 10^{-5}\right) \times 100}{4}$$

$$= 3.016 \times 10^6 \text{ N}$$

The maximum stress is induced at the small end, given by,

$$\sigma_{max} = \frac{P}{A_2} = \frac{P}{\frac{\pi}{4}D_2^2} = \frac{3.016 \times 10^6}{\frac{\pi}{4} \times (80)^2}$$

$$= 600.014 \text{ N/mm}^2 \quad \text{(Compressive)}$$ **Ans.**

4. (c)

(i)

The desirable properties of a good bearing material are as follows:

(i) When metal to metal contact occurs, the bearing material should not damage the surface of the journal. It should not stick or weld to the journal surface.

(ii) It should have high compressive strength to withstand high pressures without distortion.

(iii) In certain applications like connecting rod or crankshaft, bearings are subjected to fluctuating stresses. The bearing material, in these applications, should have sufficient endurance strength to avoid failure due to pitting.

(iv) The bearing material should have the ability to yield and adopt its shape to that of the journal. This property is called conformability. When the load is applied, the journal is deflected resulting in contact at the edges. A conformable material adjusts its shape under these circumstances.

(v) The dirt particles in lubricating oil tend to jam in the clearance space and, if hard, may cut scratches on the surfaces of the journal and bearing. The bearing material should be soft to allow these particles to get embedded in the lining and avoid further trouble. This property of the bearing material is called embeddability.

(vi) In applications like engine bearings, the excessive temperature causes oxidation of lubricating oils and forms corrosive acids. The bearing material should have sufficient corrosion resistance under these circumstances.

The desirable properties of lubricating oil are as follows:

(i) It should be available in a wide range of viscosities.

(ii) There should be little change in viscosity of the oil with change in temperature.

(iii) The oil should be chemically stable with bearing material and atmosphere at all temperatures encountered in the application.

(iv) The oil should have sufficient specific heat to carry away frictional heat, without abnormal rise in temperature.

(v) It should be commercially available at reasonable cost.

(ii)

Given: $n_1 = 7$, $n_2 = 6$, Power = 16 kW, N = 600 rpm, $\frac{D}{d} = \frac{R}{r} = 2.2$, $\mu = 0.1$, $P = 0.25\ \text{N/mm}^2$.

The total number of pairs of contacting surfaces

$$n = n_1 + n_2 - 1 = 7 + 6 - 1 = 12$$

Torque transmitting capacity,

$$\text{Power} = T_f \times \omega = T_f \times \frac{2\pi N}{60}$$

$$\Rightarrow \quad 16 \times 10^3 = T_f \times \frac{2\pi \times 600}{60}$$

$$\Rightarrow \quad T_f = 254.65\ \text{N.m}$$

The torque transmitting capacity under uniform wear theory is given as

$$T_f = n\,\mu\,\pi\,P\,r\,(R^2 - r^2)$$

$$\Rightarrow \quad 254.65 = 12 \times (0.1) \times \pi \times (0.25) \times r\,[(2.2r)^2 - r^2] \times 10^{-3}$$

$$= 12 \times (0.1) \times \pi \times (0.25) \times (3.84r^3) \times 10^{-3}$$

$$\Rightarrow \quad r^3 = 70362.51$$

$$\Rightarrow \quad r = 41.2838 \text{ mm}$$

Hence inner radius is 41.2838 mm

$$\text{Outer radius, } R = 2.2r = (2.2)(41.2838) = 90.8244 \text{ mm}$$

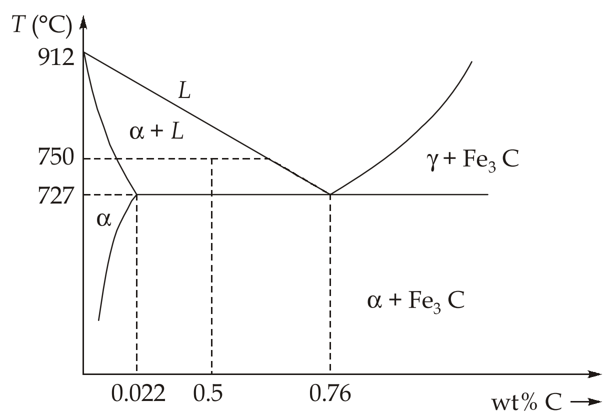
Now,

$$\text{Inner dameter, } d = 2r = 2 \times 41.2838 = 82.5676 \text{ mm}$$

$$\text{Outer dameter, } D = 2R = 2 \times 90.8244 = 181.1352 \text{ mm}$$

Section : B

5. (a)



(i) From the phase diagram,

Mass fraction of total ferrite,

$$M_{\text{total-}\alpha} = \frac{6.67 - 0.5}{6.67 - 0.022} = 0.9281$$

Mass fraction of total cementite,

$$M_{Fe_3C} = \frac{0.5 - 0.022}{6.67 - 0.022} = 0.0719$$

(ii) From the phase diagram,

Mass fraction of proeutectoid ferrite,

$$M_{\text{pro-}\alpha} = \frac{0.76 - 0.5}{0.76 - 0.022} = 0.3523$$

Mass fraction of pearlite,

$$M_P = \frac{0.5 - 0.022}{0.76 - 0.022} = 0.6477$$

(iii) All ferrite is either proeutectoid or eutectoid (in the pearlite), so the sum of these two ferrite fractions will be equal to the fraction of total ferrite.

So, $M_{pro\text{-}\alpha} + M_{eut\text{-}\alpha} = M_{total\text{-}\alpha}$

$\Rightarrow \quad M_{eut\text{-}\alpha} = M_{total\text{-}\alpha} - M_{pro\text{-}\alpha} = 0.9281 - 0.3523 = 0.5758$

5. (b)

Principle and working:

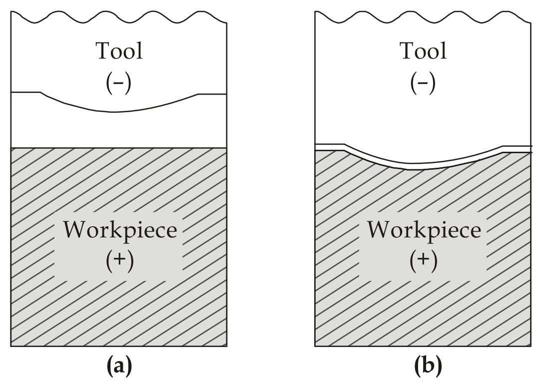


Fig.(1) : The principle of ECM process
(a) Shape of workpiece before machining;
(b) Tool shape is reproduced on workpiece after ECM

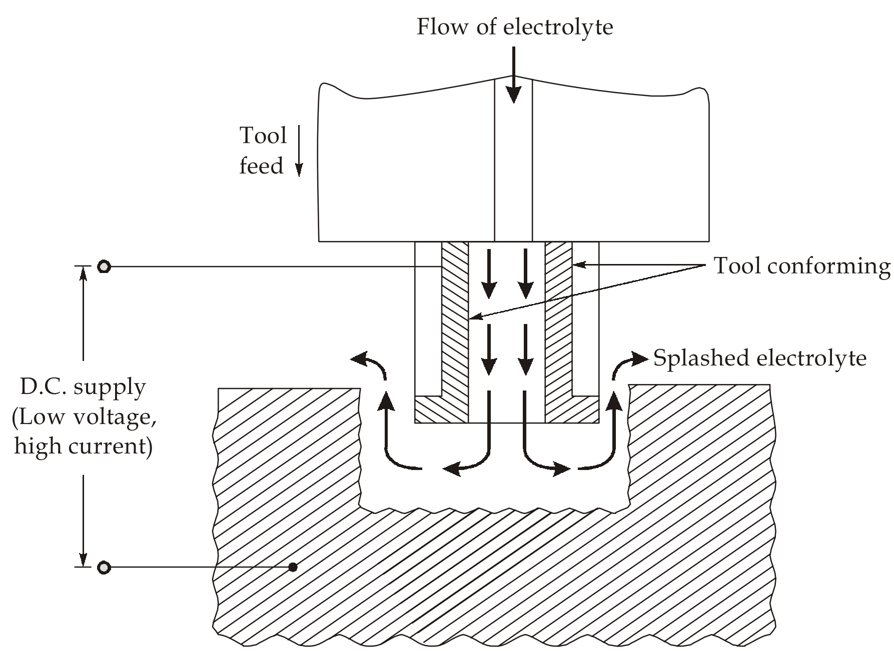


Fig. (2) : Set up for Electro-Chemical Machining (ECM)

It is an inherently versatile process of machining because of its capability of stress free machining of various kinds of metals and alloys. It can produce shapes and cavities which are costly and extremely difficult to machine with the conventional machining processes and a true shape of the tool (or cathode) can be made on the workpiece (or anode) by controlled dissolution of anode of an electrolytic cell.

An electrolyte (usually a neutral salt solution such as sodium chloride, sodium nitrate, sodium chlorate) is passed through a very small gap (0.05 to 0.03 mm) created between the workpiece (or anode) and the tool (or cathode) whereas a direct current flow is made between them. When sufficient electrical energy (about 6 eV) is available, a metallic ion may be pulled out of the workpiece surface. The positive metallic ions will react with negative ions present in the electrolytic solution forming metallic hydroxides and other compounds. Hence the metal will be anodically dissolved with the formation of sludges and precipitates. The metal removal rate is governed by Faraday classical laws of electrolysis.

Advantages:

1. The process is capable of machining metals and alloys irrespective of their strength and hardness.
2. Fragile parts, which are otherwise not easily machinable can be shaped by ECM.
3. Intricate and complex shapes can be machined easily through this process.
4. Metal removal rate is quite high in comparison to traditional machining, specially in respect of high tensile and high temperature resistant materials.
5. Wear on tool is insignificant or almost non-existent.

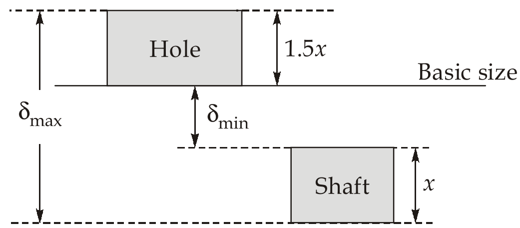
Disadvantages:

1. Non-conductive materials cannot be machined.
2. The process cannot be used to machine sharp interior edges and corners less than 0.2 mm radius because of very high current densities at those points.
3. Very high power consumption.
4. Larger floor space is required.
5. A constant monitoring is required to suitably vary the tool feed rate and supply pressure of electrolyte so as to avoid formation of cavitation.

5. (c)

Given: Basic size = 50 mm, δ_{max} = 0.08 mm, δ_{min} = 0.02 mm, let shaft tolerance is x, then hole tolerance is $1.5x$.

(i) Hole basis system:



$$\delta_{max} = \delta_{min} + x + 1.5x$$

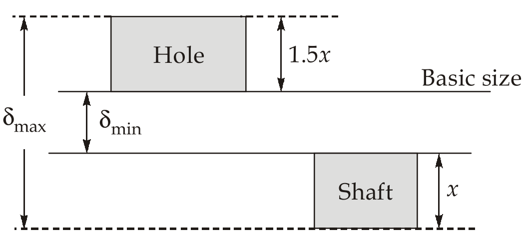
$$\Rightarrow \quad 0.08 = 0.02 + x + 1.5x$$

$$\Rightarrow \quad x = 0.024 \text{ mm}$$

Hole
- maximum size = $50 + 1.5x = 50 + (1.5) \times (0.024)$ = **50.036 mm**
- minimum size = 50 mm

Shaft
- maximum size = $50 - \delta_{min} = 50 - 0.02$ = **49.98 mm**
- minimum size = $50 - \delta_{min} - x = 50 - 0.02 - 0.024$ = **49.956 mm**

(ii) Shaft basis system:



$$\delta_{max} = \delta_{min} + 1.5x + x$$

$$\Rightarrow \quad 0.08 = 0.02 + 1.5x + x$$

$$\Rightarrow \quad x = 0.024 \text{ mm}$$

Hole
- maximum size = $50 + \delta_{min} + 1.5x = 50 + 0.02 + (1.5) \times (0.024)$ = **50.056 mm**
- minimum size = 50 mm + δ_{min} = 50 + 0.02 = 50.02 mm

Shaft
- maximum size = **50 mm**
- minimum size = $50 - x = 50 - 0.024$ = **49.976 mm**

5. (d)

Let integral gain be

$$G_i = \frac{1}{RC} = \frac{1}{80\times10^{-6}\times50\times10^{3}} = 0.25 \text{ s}^{-1} \quad \text{... (i)}$$

Therefore,

$$1\% \text{ of the input for 1 s} = 0.01 + (10 - 0) \times 1 = 0.1 \text{ V.s}$$

Let k_i be percent of the output range. Therefore,

$$\text{Output voltage} = \frac{k_i}{100}(8-0) = 0.08k_iV$$

$$G_i = \frac{\text{Output voltage}}{\text{Input voltage}} = \frac{0.08k_i}{0.1} = 0.8\,k_i\text{ s}^{-1} \quad \text{... (ii)}$$

Equating the value of G_i we get

$$0.25 \text{ s}^{-1} = 0.8\,k_i\text{ s}^{-1}$$

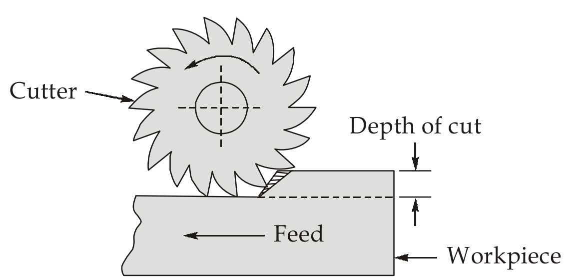
$$\therefore \quad k_i = \frac{0.25}{0.8} = 0.3125\% \;/\; (\%s)$$

5. (e)

(i)

1. **Upmilling (or conventional milling) process.**

In 'upmilling process', the workpiece is fed opposite to the cutter's tangential velocity. Each tooth of the cutter starts the cut with zero depth of cut, which gradually increases and reaches the maximum value as the tooth leaves the cut. The chip thickness at the start is zero increases to maximum at the end of the cut.



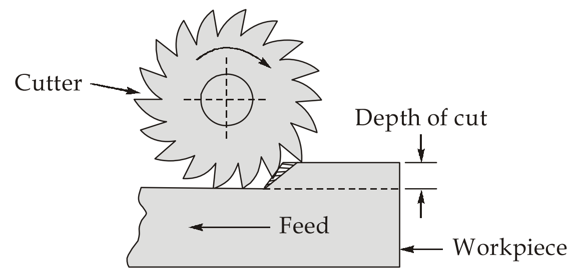
Up milling process

The action of the cutter forces the workpiece and the table against the direction of table feed, thus each 'tooth' enters a clean metal gradually thus the shock load on each tooth is minimised.

When making dep cuts, such as in heavy slotting operations, the cutter tends to pull the workpiece out of the vice or the fixture since the cutting force is directed upward at an angle; This required secured clamping of workpiece.

2. Downmilling (or climb milling) process.

In 'down milling process' the workpiece is fed in the same direction as that of the cutter's tangential velocity. The cutter enters the top of the workpiece and removes the chip that gets progressively thinner as the cutter tooth rotates.



Down milling process

Down or climb milling is used only on materials that are free of scale and other surface imperfections that would damage the cutters.

(ii)

Given: L = 360 mm, D = 140 mm, Z = 20 teeth, d = 5 mm, f_t = 0.1 mm/tooth, v = 40 m/min

$$\text{Cutting velocity, } v = \frac{\pi DN}{1000}$$

$$\Rightarrow \quad 40 = \frac{\pi \times 140 \times N}{1000}$$

$$\Rightarrow \quad N = 90.95 \text{ rpm}$$

$$\text{Milling feed, } f_m = f_t ZN = 0.1 \times 20 \times 90.95 = 181.9 \text{ mm/min}$$

$$\text{Compulsory approach, } x = \sqrt{d(D-d)} = \sqrt{5(140-5)} = 25.98 \text{ mm}$$

The approach and overtravel distance is taken to be the same.

Hence, $x_1 = x_2 = x = 25.98$ mm

$$\text{Total length of cut, } L_m = L + x_1 + x_2 = 360 + 25.98 + 25.98 = 411.96 \text{ mm}$$

$$\text{Machining time, } t_m = \frac{L_m}{f_m} = \frac{411.96}{181.9} = 2.265 \text{ min}$$

6. **(a)**

(i)

Factors affecting tool life:

Tool life depends upon the following factors:

(i) Tool material.

(ii) Hardness of the material.
(iii) Type of material being cut.
(iv) Type of the surface on the metal (scaly or smooth).
(v) Profile of the cutting tool.
(vi) Type of the maching operation being performed.
(vii) Microstructure of the material.
(viii) Finishing required on the workpiece.
(ix) Cutting speed: As the cutting speed is reduced, the tool life increases.
(x) Feed and depth of cut : An increase in feed and depth of cut will shorten tool life but not nearly as much as an increases in cutting speed.
(xi) Cutting temperature : In general, higher temperatures cause shorter tool life.

Tool failure criteria:

The following are some of the possible tool failure criteria that could be used for limiting tool life:

Based on tool wear:

(i) Wear land size.
(ii) Crater depth, width or other parameters.
(iii) A combination of the above two.
(iv) Chipping or fine cracks developing at the cutting edge.
(v) Volume or weight of material worn off the tool.
(vi) Total destruction of the tool.

Based on consequences of worn tool:

(i) Limiting value of change in component size.
(ii) Limiting value of surface finish.
(iii) Fixed increase in cutting force or power required to perform a cut.

(ii)

Given: $VT^{0.15} \times f^{0.78} \times d^{0.39} = C$

On substituting the given values: $V = 29$ m/min, $f = 0.3$ mm/rev, $d = 2.5$ mm and $T = 60$ min we get

$$C = 29 \times (60)^{0.15} \times (0.3)^{0.78} \times (2.5)^{0.39} = 29.95517$$

Now,
$$T^{0.15} = \frac{29.95517}{V \times f^{0.78} \times d^{0.39}}$$

Changes in tool life:

1. **When cutting speed is increased by 30%:**

$$T^{0.15} = \frac{29.95517}{(29\times 1.30)\times(0.3)^{0.78}\times(2.5)^{0.39}}$$

$$T = 10.44 \text{ min}$$

2. **When feed is increased by 30%:**

$$T^{0.15} = \frac{29.95517}{(29)\times(0.3\times 1.30)^{0.78}\times(2.5)^{0.39}}$$

$$T = 15.33 \text{ min}$$

3. **When depth of cut is increased by 30%:**

$$T^{0.15} = \frac{29.95517}{(29)\times(0.3)^{0.78}\times(2.5\times 1.30)^{0.39}}$$

$$T = 30.33 \text{ min}$$

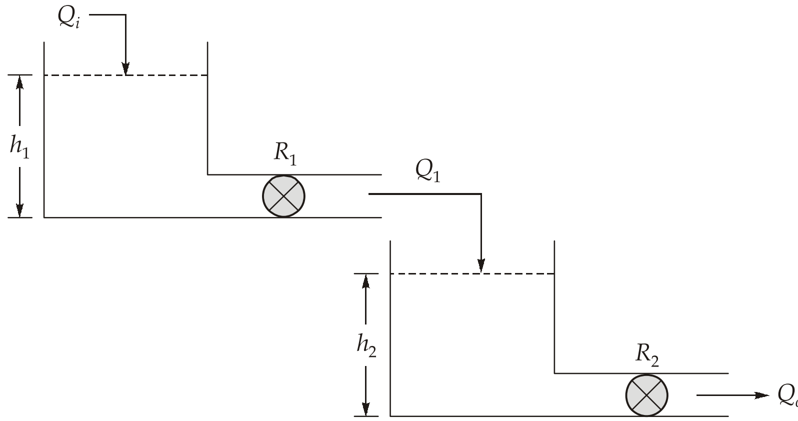
4. **When cutting speed, feed and depth of cut are all increased by 30%, simultaneously.**

$$T^{0.15} = \frac{29.95517}{(29\times 1.30)\times(0.3\times 1.30)^{0.78}\times(2.5\times 1.30)^{0.39}}$$

$$T = 1.35 \text{ min}$$

6. **(b)**

Consider the case of two tank liquid-level systems connected in series as shown in figure. The tanks are set up such that fluid level in the downstream tank does not affect the fluid-level dynamics of the upstream tank. Therefore, the fluid level of the upstream tank can be determined without any consideration of the downstream tank.



Non-interacting tanks

Taking material balance for the first tank, we get

$$Q_i - Q_1 = A_1 \frac{dh_1}{dt}$$

where, $$Q_1 = \frac{h_1}{R_1}$$

Differentiating equation, we have

$$R_1 \frac{dQ_1}{dt} = \frac{dh_1}{dt}$$

Substituting equation, we have

$$Q_i - Q_1 = A_1 R_1 \frac{dQ_1}{dt}$$

$$Q_i = A_1 R_1 \frac{dQ_1}{dt} + Q_1$$

Taking Laplace transform

$$Q_i(s) = A_1R_1sQ_1(s) + Q_1(s) = [AR_1s + 1]Q_1(s)$$

$$\frac{Q_1(s)}{Q_i(s)} = \frac{1}{A_1R_1s+1} = \frac{1}{\tau_1 s+1}$$

where, $\tau_1 = A_1R_1$ or C_1R_1, as $A_1 = C_1$.

Similarly, we can write

$$\frac{Q_o(s)}{Q_1(s)} = \frac{1}{\tau_2 s+1}$$

where $\tau_1 = A_1R_1$ and $\tau_2 = A_2R_2 = R_2C_2$. Multiplying and dividing by $Q_1(s)$ and using equation, we get

$$\frac{Q_o(s)}{Q_i(s)} = \frac{Q_1(s)}{Q_i(s)} \times \frac{Q_o(s)}{Q_1(s)}$$

The overall transfer function is

$$G(s) = \frac{Q_o(s)}{Q_i(s)} = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$$

It is also possible to relate, h_2 and Q_i

$$Q_1 - Q_o = A_2\frac{dh_2}{dt} \text{ and } Q_2 = \frac{h_2}{R_2}$$

Therefore, $$Q_1 - \frac{h_2}{R_2} = A_2\frac{dh_2}{dt}$$

$$Q_1 = A_2\frac{dh_2}{dt} + \frac{h_2}{R_2}$$

$$R_2Q_1 = R_2A_2\frac{dh_2}{dt} + h_2$$

Taking Laplace transform,

$$R_2Q_1(s) = R_2A_2sH_2(s) + H_2(s)$$

Therefore, $$\frac{H_2(s)}{Q_1(s)} = \frac{R_2}{\tau_2 s+1}$$

where $\tau_2 = A_2R_2$. Multiplying equation with, we can write

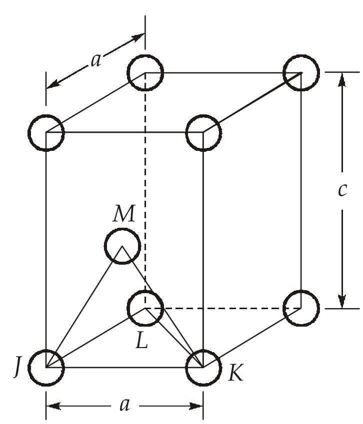
$$\frac{H_2(s)}{Q_i(s)} = \frac{1}{(\tau_1 s+1)}\frac{R_2}{(\tau_2 s+1)}$$

The above equation is the transfer function.

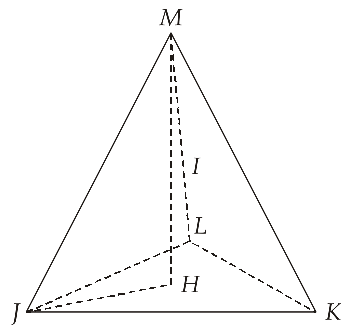
6. (c)

(i)

A sketch of one-third of an HCP unit cell is shown below:



Consider the tetrahedron labelled as JKLM, which is reconstructed as



The atom at point M is midway between the top and bottom faces of the unit cell-that is $\overline{MH} = \frac{c}{2}$. And, since atoms at points J, K, and M all touch one another,

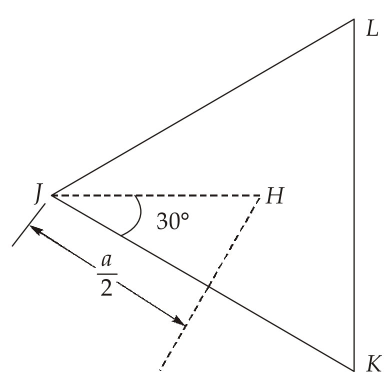
$$\overline{JM} = \overline{JK} = 2R = a$$

where R is the atomic radius. Furthermore, from triangle JHM,

$$(\overline{JM})^2 = (\overline{JH})^2 + (\overline{MH})^2$$

$$a^2 = (\overline{JH})^2 + \left(\frac{c}{2}\right)^2$$

Now, we can determine the $\overline{JH}$ length by consideration of triangle JKL, which is an equilateral triangle,



$$\cos 30° = \frac{a/2}{JH} = \frac{\sqrt{3}}{2}$$

$$\overline{JH} = \frac{a}{\sqrt{3}}$$

Substituting this value for $\overline{JH}$ in the above expression yields,

$$a^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + \left(\frac{c}{2}\right)^2 = \frac{a^2}{3} + \frac{c^2}{4}$$

and, solving for c/a,
$$\frac{c}{a} = \sqrt{\frac{8}{3}} = 1.633$$

(ii)

Given: $R = 0.136$ nm $= 1.36 \times 10^{-8}$ cm, $N_A = 6.023 \times 10^{23}$ atoms/mol, $W = 192.2$ g/mol.

For FCC, number of atoms per unit cell is, $Z = 4$

Density is given as:
$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\left(\frac{ZW}{N_A}\right)}{a^3} = \frac{\left(\frac{ZW}{N_A}\right)}{\left(2\sqrt{2}R\right)^3} = \frac{ZW}{16\sqrt{2}R^3 N_A}$$

$$= \frac{4 \times (192.2)}{16\sqrt{2} \times \left(1.36 \times 10^{-8}\right)^3 \times \left(6.023 \times 10^{23}\right)} = 22.426 \text{ g/cm}^3$$

7. (a)

(i)

(A) Liquid Penetration Inspection: This method is used to detect large cracks or openings, cold shuts, fatigue cracks, and pits. There are two types of liquid penetration inspections :

(a) Dye penetration inspection

(b) fluorescent liquid penetration inspection

(a) Dye penetration inspection: The liquid dye penetrant is sprayed on to the clean surface of test piece. Excess amount of dye is removed, surface is washed with water and thoroughly dried. Then a developer is sprayed on the surface, which brings out the colour in the dye penetrant that has penetrated into cracks or pinholes.

(b) Fluorescent Liquid penetration inspection: Fluorescent liquid is applied to the surface to be inspected. After removing the excess liquid, surface is washed and dried. Then the test surface is viewed under black light whose wavelength lies between the visible and ultra violet regions of the spectrum, which caused the penetrant to glow clearly in the dark. Some solids used in the cleaners and developers contain a high percentage of chlorine to the make the liquid non-inflammable, but requires a great care in view of health hazards associated with chlorine. Penetrant testing has the advantage that it can be used for all types of materials.

(B) Eddy Current Inspection: Eddy current testing is one of the methods of detecting discontinuities and flaws. The method is based on the principle that when the alternate current carrying conductor coil is brought near a metallic/conductive specimen, eddy curents are induced in the specimen due to electromagnetism. These eddy currents produce their own magnetic field that opposes the field of the current carrying coil, thereby increasing the impedance (resistance). Coil impedance can be measured whose variation indicates the crack or flaw.

(C) Ultrasonic Inspection: Ultrasonic inspection is fast, reliable NDT method which employs electronically produced high frequency sound waves that will penetrate metals, liquids, and many other materials at speeds of several thousands feet per second. Piezo electric materials (e.g. quartz) are used to produce ultrasonic waves. (These materials undergo a change in physical dimension when subjected to electric field. This conversion of electrical energy to mechanical energy is known as Piezoelectric effect). If an alternating electric field is applied to a piezoelectric crystal, the crystal will expand during the first half of cycle and contract when the electric field is reversed. By varying the frequency of the alternating electric field, we can vary the frequency of the mechanical vibration (sound wave) produced in the crystal. There are two common ultrasonic test methods, the through-transmission and the pulse-echo methods. Ultrasonic inspection is used to detect and locate defects such as shrinkage in cavities, internal bursts or cracks, porosity, and large non-metallic inclusions. Wall thickness can also be measured in closed vessels or in cases where such measurement cannot otherwise be made.

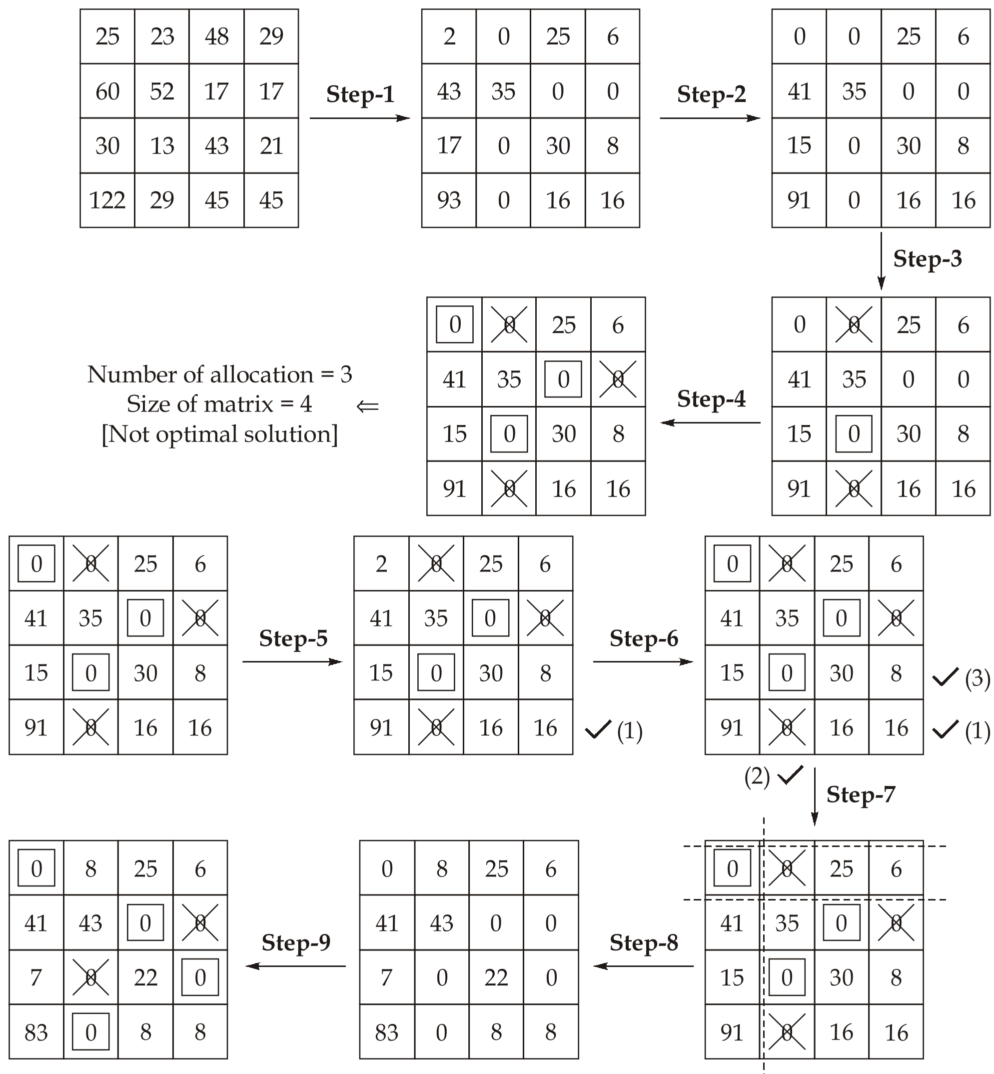
(ii) The comparison between design FMEA and process FMEA is given in a tabular representation here:

Table: Comparison between Design FMEA and Process FMEA

	Design FMEA	**Process FMEA**
Purpose	Meeting specification	Production in compliance with drawing
Responsibility	Design and development	Production
Objective	Avoiding design failure	Avoiding failures during process planning, production implemtation
Timings for commencement	After design selection	With production release
Period of review	Prior to first trial run	Prior to any investment
FMEA first completion	Prior to production release	Prior to bulk production

Design FMEA should be initiated before or at design concept finalisation at component level, sub-system level, and system level whereas process FMEA should be initiated before or at feasibility stage, prior to tooling for production.

7. (b)



Now, Number of allocations = 4

Size of matrix = 4

As number of allocations is equal to the size of matrix, so the solution is optional solution.

The minimum processing cost is

$$\text{Cost} = 25 + 17 + 21 + 29 = ₹92$$

Steps involved in the algorithm:

Step-1: Subtract the minimum element of every row from that corresponding row.

Step-2: If all the rows and all the columns are alerted with atleast one zero, then skip this step. Otherwise , Subtract the minimum element of every column from that corresponding column.

After step 1 and 2 we will get initial basic feasible solution . After this we have to check whether it is optimal solution or not .

Steps for checking optimality of solution :

Step-3: Examine now successively until a row with exactly one unmarked Zero is found. Put that zero in a box , indicating that an assignment will be made there . Cross all the other zeroes (if any) of that Corresponding column , indicating that it can not be used for making other assignment.

Step-4: When you are done for all the rows, go for doing the same for columns . Examine all the columns successively until a column with exactly one unmarked zero is found. Put that Zero in a box and cross all other zeros (if any) of that corresponding now.

Repeat step 3 and 4 untill all the zeros are either assigned or crossed. When all the zeros are assigned or crossed , check whether number of allocations made are equal to the size of matrix or not . If it is equal then the initial basic feasible solution is also an optimal solution . If it is not equal then the initial basic feasible solution is not optimal.

Steps to get optimal solution:

Step-5: Mark all rows for which allocation is not done.

Step-6: Mark all column which have unassigned zero in the marked row. Mark all rows which have assignment in the marked column. Continue doing this until chain of marking is completed.

Step-7: Draw the minimum number of line through unmarked row and through marked column to cover all the zero at least once.

Step-8: Select the smallest element that do not have line through them and subtract it from all the elements that do not have line through them . Add that minimum value to every element that at the intersection of lines and leave the remaining elements unchanged.

Step-9: Repeat step 5 and 6 , after this the final solution obtained must be the optimal solution.

7. (c)

Let the given rotation matrix which specifies the orientation of frame {2} with respect to frame {1} be

$$^{1}R_2 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

The equivalent rotation matrix for a set of ZYX-Euler angle rotation $(\theta_1\theta_2\theta_3)$ is given by equation,

$$^{1}R_2 = \begin{bmatrix} C_2C_3 & S_1S_2C_3 - C_1S_3 & S_1S_3 + C_1S_2C_3 \\ C_2S_3 & S_1S_2S_3 + C_1C_3 & -S_1C_3 + C_1S_2S_3 \\ -S_2 & S_1C_2 & C_1C_2 \end{bmatrix}$$

Equating the corresponding elements of these matrices gives nine equations in three independent variables, θ_1, θ_2, θ_3. Apart from the redundancy in equations, additional complication is that these are transcendental in nature.

Equations elements (1, 1) and (2, 1) in equation with corresponding elements in equation gives,

$$C_2C_3 = r_{11} \text{ and } C_2S_3 = r_{21}$$

Squaring and adding gives

$$C_2 = \cos\theta_2 = \pm\sqrt{r_{11}^2 + r_{21}^2}$$

Combining with the element (3, 1), $(-S_2 = r_{31})$, the angle θ_2 is computed as

$$\tan\theta_2 = \frac{S_2}{C_2}$$

which gives, $$\theta_2 = A\tan 2\left(-r_{31}, \pm\sqrt{r_{11}^2 + r_{21}^2}\right)$$

where A tan2(a, b) is a two-argument arc tangent function.

The solution for θ_1 and θ_3 depends on value of θ_2. Here, two cases arise which are worked out as follows:

Case-1: $\theta_2 \neq 90°$

From the elements (1, 1) and (2, 1) in equation and θ_3 is obtained as

$$\theta_3 = A\tan 2\left(\frac{r_{21}}{C_2}, \frac{r_{11}}{C_2}\right)$$

and from elements (3, 2) and (3, 3), θ_1 is

$$\theta_1 = A\tan 2\left(\frac{r_{32}}{C_2}, \frac{r_{33}}{C_2}\right)$$

Note that there is one set of solution corresponding to each value of θ_2.

Case-2: $\theta_2 \neq \pm 90°$

For $\theta_2 = \pm 90°$, the solution obtained in case 1 degenerates. However, it is possible to find only the sum or difference of θ_3 and θ_1. Comparing elements (1, 2) and (2, 2)

$$r_{12} = S_1S_2C_3 - C_1S_3$$

and $$r_{22} = S_1S_2S_3 + C_1C_3$$

If $\theta_2 = +90°$, these equations reduce to

$$r_{12} = \sin(\theta_1 - \theta_3)$$

$$r_{22} = \cos(\theta_1 - \theta_3)$$

and $$\theta_1 - \theta_3 = A\tan2(r_{12}, r_{22})$$

Choosing $\theta_3 = 0°$ gives the particular solution

$$\theta_2 = 90°,\ \theta_3 = 0° \text{ and } \theta_1 = A\tan2(r_{12}, r_{22})$$

With $\theta_2 = -90°$, the solution is

$$r_{12} = -\sin(\theta_1 + \theta_2)$$

$$r_{22} = \cos(\theta_1 + \theta_2)$$

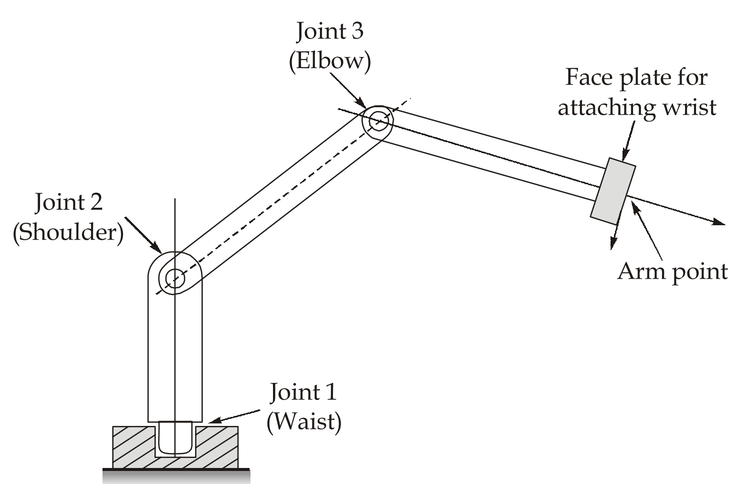
and $$\theta_1 + \theta_2 = A\tan2(-r_{12}, r_{22})$$

Choosing $\theta_2 = 0°$ gives the particular solution

$$\theta_2 = -90°,\ \theta_3 = 0° \text{ and } \theta_1 = A\tan2(-r_{12}, r_{22})$$

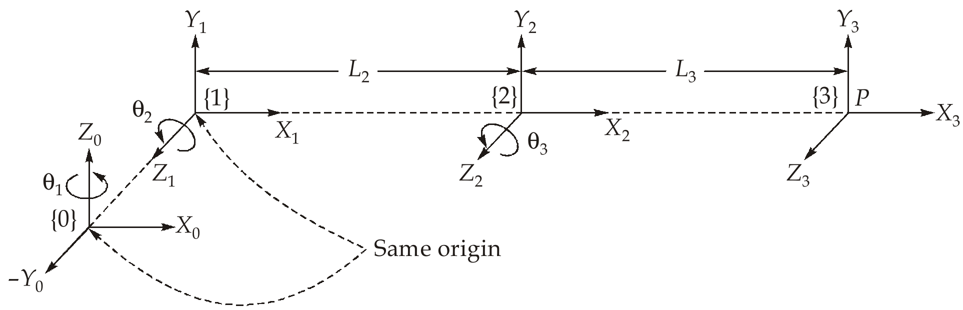
8. (a)

An articulated arm is a 3-DOF manipulator with three revolute joints, that is an RRR arm configuration as shown in figure. The axes of joint 2 and joint 3 are parallel and axis of joint 1 is perpendicular to these two. At the end of the arm, a faceplate is provided to attach the wrist.



A 3-DOF articulated arm with three revolute joints

To dertermine the 'arm point' transformation matrix, the frames are assigned first as shown in figure. The resulting joint-link parameters are tabulated in table. For all the three joints, joint-offsets are assumed to be zero.



Frame assignment for articulated arm

Table: Joint-link parameters for articulated arm

Link *i*	a_i	a_i	d_i	θ_i	q_i	$C\alpha_i$	$S\alpha_i$
1	0	90°	0	θ_1	θ_1	0	1
2	L_2	0	0	θ_2	θ_2	1	0
3	L_3	0	0	θ_3	θ_3	1	0

The link transformation matrices are:

$$^0T_1\,(\theta_1) = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2\,(\theta_2) = \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3\,(\theta_3) = \begin{bmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation matrix for the endpoint of the arm is, therefore,

$$^0T_1 = {}^0T_1{}^1T_2{}^2T_3 = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & C_1\left(L_3C_{23}+L_2C_2\right) \\ S_1C_{23} & -S_1S_{23} & -C_1 & S_1\left(L_3C_{23}+L_2C_2\right) \\ S_{23} & C_{23} & 0 & L_2S_{23}+L_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where C_{23} and S_{23} refer to cos $(\theta_2 + \theta_3)$ and $\sin(\theta_2 + \theta_3)$, respectively.

8. (b)

(i)

1. **Grain Boundaries:** Grain boundary is formed during solidification. A polycrystalline material is made up of many crystals called as crystallites or grains. In the boundary region between two adjacent grains, the atoms are irregularly arranged and this leads to lattice distortion. Grain boundaries are defined as the junction between two randomly growing dendrite.

   When this distortion is slight, of the order of a few degrees, then it is termed as small or low angle grain boundary and for large angle distortions it is termed as high angle grain boundary.

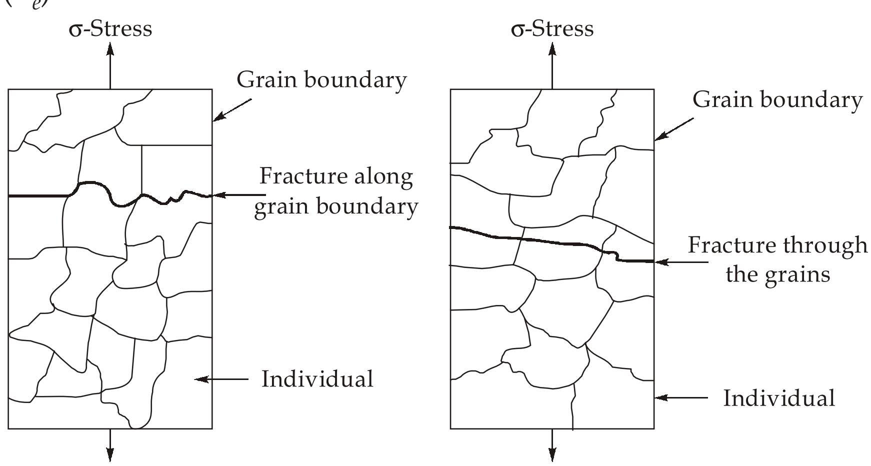
Characteristics of grain boundary:

- Grain boundaries are region of orientation mismatch.
- Grain boundaries are region of high potential energy.
- Grain boundaries are region of low melting point.
- Grain boundaries are region of heavy impurity concentration.

Transgranular fracture: When the fracture in material occurs through grains in small multi plane manner like in brittle material, known as transgranular fracture.

Inter-granular fracture: When the fracture in material occurs through grain boundaries like in ductile material, the fracture is known as inter-granular fracture.

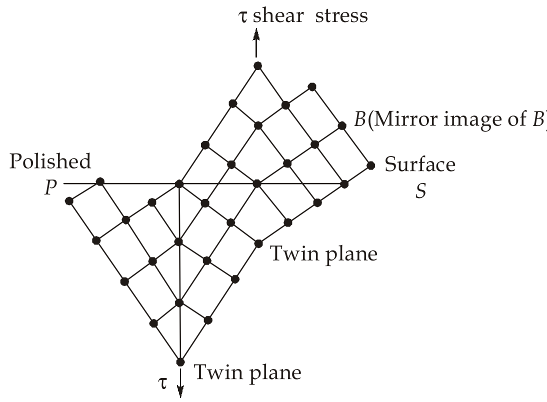
This fracture occurs when temperautre of material is greater than its equi-cohesive temperature (T_e).



Intergranular fracture **Transgranular fracture**

2. **Twin Boundaries:** When a poly-crystalline material is subjected to shear force, the atomic arrangement in the slipped and unslipped portion of the crystal can become mirror image of each other about two boundary known as twin boundaries.

Twins are produced by atomic displacement by mechanical shear force (mechanical twins found in BCC and HCP) and also during annealing heat treatment following deformation (annealing twin found in FCC).



Twin boundaries

Twin boundaries formation results in increase in net potential and hence non-uniform properties will be present in the lattice.

3. **Stacking Faults:** Stacking faults are the faults in stacking sequence. Stacking faults formation results in non-uniform properties and unstability in lattice due to increase in potential energy.

   Stacking faults are found in FCC metals when there is an interruption in the ABCABCABC... stacking sequence of close-packed planes.

(ii) For plane A since the plane passes through the origin of the coordinate system as shown, we will move the origin of the coordinate system one unit cell distance to the right along the y-axis; thus , this is a $(3\bar{2}4)$ plane, as summarized below.

	x	*y*	*z*
Intercepts	2a/3	–b	c/2
Intercepts in terms of *a*, *b* and *c*	2/3	–1	½
Reciprocals of intercepts	3/2	–1	2
Reduction	3	–2	4
Enclosure	$(3\bar{2}4)$		

For plane B we will leave the origin at the unit cell as shown; this is a (221) plane, a summarized below.

	x	y	z
Intercepts	a/2	b/2	c
Intercepts in terms of a, b and c	1/2	1/2	1
Reciprocals of intercepts	2	2	1
Reduction	Not necessary		
Enclosure	(221)		

8. (c)

(i)

1. Buses: Buses are the paths along which digital signals move from one section to another.

A bus is just a number of conductors along which electrical signals can be carried. It might be tracks on a printed circuit board or wires in a ribbon cable.

In a microprocessor system there are the following three forms of bus:

- The data bus carries the data associated with the processing function of the CPU.
- Word lengths used may be 4, 8, 16, 32 or 64.
- Each wire in the bus carries a binary signal, i.e. a0 or a1.
- The more the wires the data bus has the longer the word length that can be used.
- The earliest microprocessors were 4-bit (word length: $2^4 = 16$) devices and such 4-bit microprocessors are still used in such devices as toys, washing machins etc. They were followed by 8-bit microprocessors (e.g., Motorola 6800, the Intel 8085 A and the Zilog Z80). Now 16-bit, 32-bit and 64 bit microprocessors are available.

2. Address bus:

- It carries signals which indicate where data is to be found and so the selection of certain memory locations or input or output ports.
- Each storage location within a memory device has a unique identification, termed its address, so that system is able to select a particular instruction or data item in the memory.
- Each input/output interface also has an address.
- When a particular address is selected by its address being placed on the address bus, only that location is open to the communications from the CPU. The CPU is thus able to communicate with just one location, at a time.

- A computer with an 8-bit data has typically a 16-bit wide address bus, i.e., 16 wires. This size of address enables 2^{16} locations to be addressed. 2^{16} is 65 536 locations and is usually written as 64 K, where K is equal to 1024.

3. Control bus:

- This bus carries the signals relating to control actions.
- It is also used to carry the system clock signals; these are to synchronise all the actions of the microprocessor system.

(ii)

Physical resolution is given by,

$$r_M = \frac{360°}{2M} = \frac{360°}{2(1000)} = 0.18°$$

Ideal design requires that,

$$r_M = r_n,$$

or $$\frac{360°}{2M} = \frac{360°}{2^n - 1}$$

∴ $$2M = 2^n - 1$$

or $$M = 2^n - 1 - 0.5$$

∴ $$1000 = 2^{n-1} - 0.5$$

On solving, we get

$$n = 10.966 \simeq 11$$

i.e. the digital word length has to be $n = 11$ **Ans.**

❍❍❍❍