MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

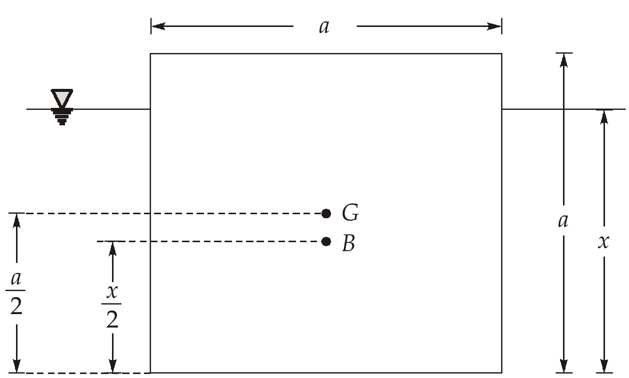
Mechanical Engineering Test No : 12

Full Syllabus Test (Paper-1)

Section : A

1. (a)

Refer figure,



For equilibrium, weight of the cube = Buoyant force on the cube

$$\gamma a^3 s = \gamma a^2 x$$

where γ is specific weight of water

$$\Rightarrow \quad x = as \qquad \text{(i)}$$

CG of cube is at $\frac{a}{2}$ and CB at $\frac{x}{2}$ from bottom.

$$BG = \frac{a}{2} - \frac{x}{2} = \left(\frac{a-x}{2}\right)$$

$$BM = \frac{I}{\forall} = \frac{a^4/12}{a^2 \times x} = \frac{a^2}{12x}$$

$$GM = BM - BG = \frac{a^2}{12x} - \left(\frac{a-x}{2}\right)$$

For stability, GM > 0

$$\Rightarrow \quad \frac{a^2}{12(x)} - \frac{(a-x)}{2} > 0$$

$$\Rightarrow \quad \frac{a^2}{12(x)} > \frac{(a-x)}{2}$$

$$\Rightarrow \quad 1 > 6\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$$

$$\Rightarrow \quad 6s - 6s^2 < 1 \qquad \left[\because \frac{x}{a} = s \text{ from equation (i)}\right]$$

$$\Rightarrow \quad 6s^2 - 6s + 1 > 0$$

By solving equality of above equation,

$$6s^2 - 6s + 1 = 0$$

We get, $s = 0.789 \text{ or } 0.211$

$\therefore \quad s < 0.211 \text{ or } s > 0.789$

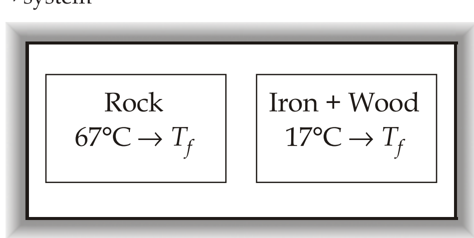
This is the condition to be satisfied for stability.

1. (b)

Given : $m_{\text{rock}} = 6400$ kg; $m_{\text{iron}} = 1050$ kg; $m_{\text{wood}} = 12060$ kg; $c_{p,\text{ rock}} = 1.014$ kJ/kgK; $c_{p,\text{ iron}} = 0.45$ kJ/kgK; $c_{p,\text{ wood}} = 1.38$ kJ/kgK; $T_0 = 17°C = 290$ K; $T_{\text{wood}} = T_{\text{iron}} = 17°C = 290$ K; $T_{\text{Rock}} = 67°C = 340$ K

From energy conservation,

$$(dU)_{\text{system}} = 0$$



$\Rightarrow \quad (dU)_{rock} + (dU)_{wood} + (dU)_{iron} = 0$

$\Rightarrow \quad (mc_p)_{rock}\,(T_f - T_{rock}) + (mc_p)_{wood}\,(T_f - T_{wood}) + (mc_p)_{iron}\,(T_f - T_{iron}) = 0$

$\Rightarrow \quad 6400 \times 1.014 \times (T_f - 340) + 12060 \times 1.38 \times (T_f - 290) + 1050 \times 0.45 \times (T_f - 290) = 0$

$$T_f = 303.75 \text{ K}$$ **Ans.**

$$(\Delta s)_{univ} = (\Delta s)_{system} + (\Delta s)_{surr}$$

$$(\Delta s)_{univ} = 0 \qquad [\because \text{ System is insulated from surrounding}]$$

$$(\Delta s)_{univ} = (\Delta s)_{system} = (\Delta s)_{rock} + (\Delta s)_{wood} + (\Delta s)_{iron}$$

$$= 6400 \times 1.014 \times \ln\left(\frac{303.75}{340}\right) + 12060 \times 1.38 \times \ln\left(\frac{303.75}{290}\right) + 1050 \times 0.45 \times \ln\left(\frac{303.75}{290}\right) = 0$$

$$\Rightarrow \quad (\Delta s)_{univ} = (\Delta s)_{system} = 61.21 \text{ kJ/K}$$

$$\therefore \quad \text{Irreversibility, } I = T_0 \times (\Delta s)_{uni}$$

$$= 290 \times 61.21 = 17750.90 \text{ kJ}$$ **Ans.**

1. (c)

Given : $Q_{total} = 0.16$ kW; $T_1 - T_\infty = 48$ K; $k = 0.15$ kW/mK; $h = 0.04$ kW/m^{2}K; $L = 0.15$ m; $B = 0.10$ m; $z = 0.15$ m; $b = 0.001$ m; $n = 8$

Let 'l' be the height of the fins required in order to achieve the given heat dissipation.

Unfinned surface area, $A_{UF} = (L \times B) - (z \times b \times n)$

$$= (0.15 \times 0.1) - (0.15 \times 0.001 \times 8)$$

$$= 0.0138 \text{ m}^2$$

Heat transfer from unfinned surface,

$$Q_{UF} = h \cdot A_{UF} \cdot (T_1 - T_\infty)$$

$$= 0.04 \times 0.0138 \times 48 = 0.0265 \text{ kW}$$

Heat transfer finned surface, $Q_F = Q_{Total} - Q_{UF} = 0.16 - 0.0265 = 0.1335$ kW

$$\text{Heat transfer from one fin} = \frac{Q_F}{n} = \frac{0.1335}{8} = 0.0167 \text{ kW}$$

$$m = \sqrt{\frac{hP}{kA}} \cdot \sqrt{\frac{h \times (2z)}{k \times b \times z}} = \sqrt{\frac{2h}{kb}} = \sqrt{\frac{2 \times 0.04}{0.15 \times 0.001}} \qquad (\because z >>> b)$$

$$= 23.094 \text{ m}^{-1}$$

$$A = z \times b = 0.15 \times 0.001 = 1.5 \times 10^{-4} \text{ m}^2$$

$$Q_f = \sqrt{hPkA} \cdot \tanh(mL) \cdot (T_1 - T_\infty)$$

$$= mkA(T_1 - T_\infty) \cdot \tanh(\text{mL})$$

$$\Rightarrow \quad 0.0167 = 23.094 \times 0.15 \times 1.5 \times 10^{-4} \times 48 \times \tanh(23.094 \times L)$$

$$L = 0.03507 \text{ m} = 35.07 \text{ mm}$$ **Ans.**

1. (d)

Morse test underestimates the friction power because the peak pressure and temperature inside the inoperative cylinder is different from those values when that cylinder was operating (firing). Further, all engines, approximately, have a common exhaust manifold for all cylinders and by making one cylinder inoperative, the pressure pulsation in the exhaust system are likely to vary and it can change the performance of the engine having different back pressure.

Power output where all cylinder is firing = $2050 \times \frac{240}{180} = 2733.33$ kW

Power output, BP when k^{th} cylinder is cut-off = $\frac{N}{180}\sum_{k=1}^{4} W_k$

Cylinder number cut-off	IP of k^{th} cylinder $IP_k = 2733.33 - \frac{N}{180}\sum_{k=1}^{4} W_k$
1 or 5 or 8	$2733.33 - \left(\frac{240}{180} \times 1830\right) = 293.33$ kW
2 or 6 or 9	$2733.33 - \left(\frac{240}{180} \times 1850\right) = 266.66$ kW
3 or 7 or 11	$2733.33 - \left(\frac{240}{180} \times 1860\right) = 253.33$ kW
4 or 10 or 12	$2733.33 - \left(\frac{240}{180} \times 1820\right) = 306.66$ kW

$\therefore$ Total indicated power, IP $= 3 \times (293.33 + 266.66 + 253.33 + 306.66)$

$= 3359.94$ kW

$$\therefore \quad \eta_{mech} = \frac{BP}{IP} = \frac{2733.33}{3359.94} = 0.8135 \text{ or } 81.35\%$$ **Ans.**

$$\text{bmep} = \frac{BP \times 60000}{LAnk} = \frac{2733.33 \times 60000}{0.5 \times \frac{\pi}{4} \times 0.5^2 \times 240 \times 12}$$

$$= 5.8 \times 10^5 \text{ Pa} = 5.8 \text{ bar}$$ **Ans.**

1. (e)

Cooling load estimate:

Room Load:

A. Room sensible heat (RSH)

(i) Solar and transmission heat transfer through wall, roof, glass.

(ii) Infiltration

(iii) Internal heat gains, (people, lighting, appliance etc.)

(iv) Additional heat gains.

(v) Supply duct heat gain supply duct leakage loss, fan horsepower.

(vi) By passed outside air load

B. Room latent heat (RLH)

(i) Infiltration

(ii) Internal heat gain

(iii) Vapour transmission

(iv) Additional heat gain

(v) Supply duct leakage loss

Heating load estimate:

1. Transmission heat loss calculated from wall, roof etc. due to difference between outside air and inside air temperature.
2. Solar radiation
3. Internal heat gain

Duct heat gain:

Normally supply air is at temperature of 10-15°C. The duct may pass through an unconditioned space having ambient temperature of 40°C. This results in significant heat gain by the time air reaches conditioned space even though when duct is insulated.

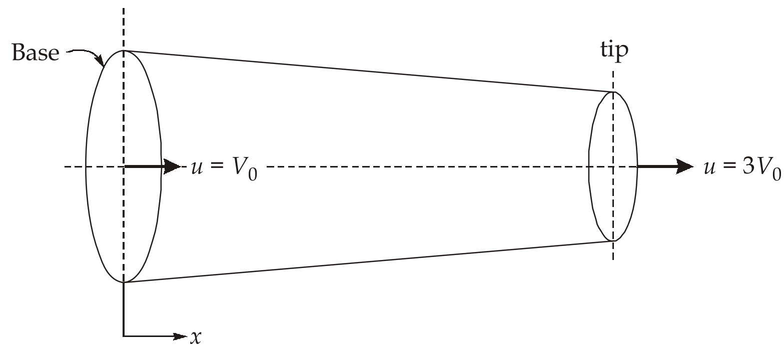
$$\text{Heat gain } (Q) = UA(t_a - t_s)$$

where, U = Heat transfer coefficient; A = Surface area

t and t_s = ambient and supply temperature

As rough estimate, a maximum of 5% of room sensible heat may be added to total heat load.

2. (a) (i)



At distance x from the nozzle base,

$$u(x) = V_0 + \left(\frac{3V_0 - V_0}{l}\right)x = V_0 + \frac{2V_0}{l}x = V_0\left(1 + \frac{2x}{l}\right)$$

For one dimensional flow through the nozzle, acceleration

$$\vec{a} = \frac{\partial u}{dt} + u\frac{du}{dx}$$

The discharge is constant and the flow is steady. Therefore, the local acceleration,

$$\frac{du}{\partial t} = 0$$

Convective acceleration is,

$$a_{\text{convective}} = u\frac{\partial u}{\partial x}$$

$$= V_0\left(1 + \frac{2x}{L}\right) \times \frac{\partial}{\partial x}\left[V_0\left(1 + \frac{2x}{L}\right)\right]$$

$$= V_0\left(1 + \frac{2x}{L}\right) \times V_0 \times \left(\frac{2}{L}\right)$$

$$= \frac{2V_0^2}{L}\left(1 + \frac{2x}{L}\right)$$

At the nozzle base, i.e. at $x = 0$

$$a = 2 \times \frac{4^2}{0.4} \times \left(1 + \frac{2 \times 0}{0.4}\right) = 80 \text{ m/s}^2$$ **Ans.**

At the nozzle tip, i.e. at $x = 0.4$ m

$$a = 2 \times \frac{4^2}{0.4} \times \left(1 + \frac{2 \times 0.4}{0.4}\right) = 240 \text{ m/s}^2$$

2. (a) (ii)

Given : $N = 112$ rpm; $H = 0.9$; $D = 0.6$ m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 112}{60} = 11.73 \text{ rad/s}$$

Let 'y' be the height of the paraboloid, i.e. the depth of the vertex from the rim of the tank, then,

$$y = \frac{\omega^2 x^2}{2g} = \frac{11.73^2 \times (0.3)^2}{2 \times 9.81} = 0.63 \text{ m}$$

Amount of water thrown out = Volume of paraboloid of revolution = $\frac{1}{2}$ × (Volume of circumscribing cylinder)

$$= \frac{1}{2} \times \left(\frac{\pi}{4} \times 0.6^2 \times 0.63\right) = 0.089 \text{ m}^3$$

Original volume of water in the tank = $\frac{\pi}{4} D^2 H = \frac{\pi}{4} \times 0.6^2 \times 0.9 = 0.254 \text{ m}^3$

∴ Volume of water left in the tank = 0.254 – 0.089 = 0.165 m^3 **Ans.**

Let 'θ' be the angle with the horizontal which the tangent to the water surface makes at a point where it meets the rim of the tank. Then,

$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\omega^2 x^2}{2g}\right) = \frac{\omega^2 x}{g} = \frac{11.73^2 \times 0.3}{9.81}$$

$$\tan\theta = 4.21°$$

$$\Rightarrow \quad \theta = \tan^{-1}(4.21) = 76.64°$$ **Ans.**

2. (b)

Given : $V_{N_2} = 0.25$ m^3; $m_{\text{He}} = 0.15$ kg; $(T_{N_2})_1 = (T_{He})_1 = 20°\text{C} = 293\text{K}$,

$(P_{N_2})_1 = (P_{He})_1 = 95\text{kPa}$; $T_R = 500°\text{C} = 773$ K; $(P_{He})_2 = 120\text{kPa}$

Helium undergoing isentropic compression during the process,

$$(T_{He})_2 = (T_{He})_1 \times \left(\frac{P_2}{P_1}\right)_{He}^{\frac{\gamma-1}{\gamma}} = 293 \times \left(\frac{120}{95}\right)^{\frac{0.667}{1.667}}$$

$$= 321.71 \text{ K}$$ **Ans.**

Initial of final volume of He are

$$(V_{He})_1 = \left(\frac{mRT_1}{P_1}\right)_{He} = \frac{0.15 \times 2.0769 \times 293}{95} = 0.9608 \text{ m}^3$$

$$(V_{He})_2 = \left(\frac{mRT_2}{P_2}\right)_{He} = \frac{0.15 \times 2.0769 \times 321.71}{120} = 0.8352 \text{ m}^3$$

Total volume of cylinder, $V_T = (V_{He})_1 + (V_{N_2})_1$

$$= 0.9608 + 0.25 = 1.2108 \text{ m}^3$$

Final volume of N_2, $(V_{N_2})_2 = V_T - (V_{He})_2 = 1.2108 - 0.8352$

$$= 0.3756 \text{ m}^3$$ **Ans.**

Mass of nitrogen, $M_{N_2} = \left(\frac{P_1V_1}{RT_1}\right)_{N_2} = \frac{95 \times 0.25}{0.2968 \times 293} = 0.2731 \text{ kg}$

Final temperature of nitrogen,

$$(T_{N_2})_2 = \left(\frac{P_2V_2}{Rm}\right)_{N_2} = \frac{120 \times 0.3756}{0.2968 \times 0.2731} = 556.06 \text{ K}$$

For He undergoing state 1 to 2:

$$(Q_{1\text{-}2})_{\text{He}} = 0 \qquad [\because \text{ Adiabatic process}]$$

$$\Rightarrow \quad (W_{1\text{-}2})_{\text{He}} = -\left[(\Delta U)_{1-2}\right]_{He} = -\left[mc_v(T_2 - T_1)\right]_{He}$$

$$= -[0.15 \times 3.1156 \times (321.71 - 293)]$$

$$= -13.417 \text{ kJ}$$

$$(W_{1\text{-}2})_{\text{N}_2} = -(W_{1-2})_{He} = 13.417 \text{ kJ}$$

$$(Q_{1\text{-}2})_{\text{N}_2} = (\Delta U_{1-2})_{N_2} + (W_{1-2})_{N_2}$$

$$= \left[mc_v(T_2 - T_1) + (W_{1-2})_{N_2}\right]$$

$$= [0.2731 \times 0.743 \times (556.06 - 293)] + 13.417$$

$$= 66.795 \text{ kJ}$$ **Ans.**

Entropy generation during the process is given by,

$$s_{gen} = \Delta s_{uni} = \Delta s_{sys} + \Delta s_{surr} = (\Delta s)_{N_2} + (\Delta s)_{He} + (\Delta s)_{surr}$$

As helium undergoes isentropic process, so $(\Delta s)_{He} = 0$

$$\therefore \quad s_{gen} = \Delta s_{N_2} + \Delta s_{surr}$$

$$\Delta s_{surr} = -\frac{(Q_{1-2})_{N_2}}{T_R} = \frac{-66.795}{773} = -0.0864 \text{ kJ/K}$$

$$\Delta s_{N_2} = m_{N_2}\left[c_p \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{P_1}{P_2}\right)\right]_{N_2}$$

$$= 0.2731 \times \left[1.039 \ln\left(\frac{556.06}{293}\right) + 0.2968 \times \ln\left(\frac{95}{120}\right)\right]$$

$$= 0.1629 \text{ kJ/K}$$

$$\therefore \quad s_{gen} = 0.1629 - 0.0864 = 0.0765 \text{ kJ/K}$$ **Ans.**

2. (c)

Let us consider the conical cavity, as shown in figure below, which is of diameter D, height H, lateral length L, semi-vertex angle α and surface area A_1. The temperature T_1 of the surface is uniform. Part of the radiation from the surface falls on itself, of which a portion is absorbed and the remainder is reflected.

Rate of emission from the surface = $A_1 \varepsilon_1 \sigma T_1^4$

Of this, the amount falling on A_1 and absorbed by it = $A_1 \varepsilon_1 \sigma T_1^4 F_{11} \varepsilon_1$, where F_{11} is the shape factor of the conical surface with respect to itself.

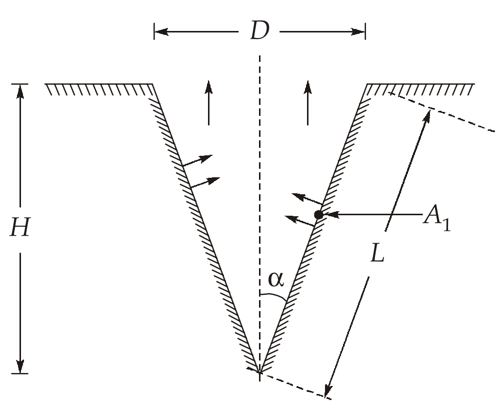


Fig (a) : Conical cavity

The amount reflected = $(1-\varepsilon_1) A_1 \varepsilon_1 \sigma T_1^4 F_{11}$. Of this reflected energy, quantity falling on

A_1 and absorbed = $(1-\varepsilon_1)A_1\varepsilon_1\sigma T_1^4 F_{12}^2\varepsilon_1$.

$$\text{Reflected} = (1-\varepsilon_1)A_1\varepsilon_1\sigma T_1^4 F_{11}^2(1-\varepsilon_1)$$

$$\text{Absorbed} = (1-\varepsilon_1)A_1\varepsilon_1\sigma T_1^4 F_{11}^3(1-\varepsilon_1)\varepsilon_1$$

$$\text{Reflected} = (1-\varepsilon_1)A_1\varepsilon_1\sigma T_1^4 F_{11}^3(1-\varepsilon_1)^2 \text{ and so on}$$

Net rate of emission from the surface

$$Q = \text{Total emission rate} - \text{Total absorption rate}$$

$$= A_1\varepsilon_1\sigma T_1^4\begin{bmatrix} A_1\varepsilon_1\sigma T_1^4 F_{11}\varepsilon_1 + (1-\varepsilon_1)A_1\varepsilon_1\sigma T_1^4 F_{12}^2\varepsilon_1 + \\ (1-\varepsilon_1)A_1\varepsilon_1\sigma T_1^4 F_{11}^3(1-\varepsilon_1)\varepsilon_1 + \ldots \end{bmatrix}$$

$$= A_1\varepsilon_1\sigma T_1^4\left[1-\varepsilon_1 F_{11} - \varepsilon_1(1-\varepsilon_1)F_{11}^2 - \varepsilon_1(1-\varepsilon_1)^2 F_{11}^3 - \ldots\right]$$

$$= A_1\varepsilon_1\sigma T_1^4\left[1-\varepsilon F_{11}(1+(1-\varepsilon_1)F_{11} - (1-\varepsilon_1)^2 F_{11}^2 + \ldots\right]$$

$$= A_1\varepsilon_1\sigma T_1^4\left[1-\frac{\varepsilon_1 F_{11}}{1-(1-\varepsilon_1)F_{11}}\right]$$

$$= A_1\varepsilon_1\sigma T_1^4 \frac{1-F_{11}}{1-F_{11}(1-\varepsilon_1)} \qquad \ldots(i)$$

Let us consider an imaginary flat surface A_2 closing the cavity as shown in figure below. Since A_1 and A_2 together form an enclosure.

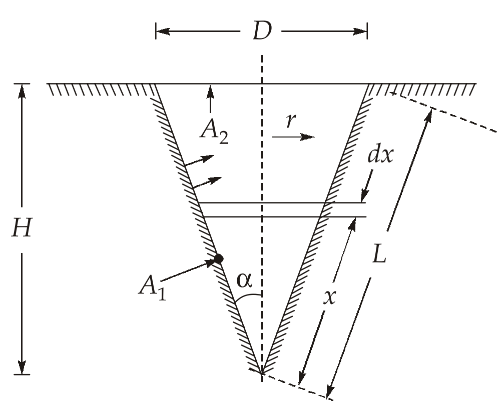


Fig (b) : Conical cavity with a flat plate on top

$$F_{11} + F_{12} = 1$$

$$F_{22} + F_{21} = 1$$

Since, $F_{22} = 0, \quad F_{21} = 1$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{12} = \frac{A_2}{A_1}$$

$\therefore$ $F_{11} = 1 - F_{12} = 1 - \frac{A_2}{A_1}$

$$A_2 = \frac{\pi}{4} D^2$$

$$\frac{r}{x} = \frac{D}{2L}, \ \tan\alpha = \frac{D}{2H}, \ \sin\alpha = \frac{D}{2L}$$

$$D = 2H\tan\alpha, \ L = \frac{D}{2\sin\alpha}, \ L = \frac{H}{\cos\alpha}$$

$$A_1 = \int_0^L 2\pi r dx = \int_0^L 2\pi \frac{D}{2L} x dx$$

$$= \frac{\pi DL}{2} = \text{Surface area of the cone}$$

$$F_{11} = 1 - \frac{\left(\frac{\pi}{4}\right) D^2}{\left(\frac{\pi DL}{2}\right)} = 1 - \frac{1}{2}\frac{D}{L}$$

or $F_{11} = 1 - \sin\alpha$ ...(ii)

Substituting in equation (i),

$$Q = \sigma A_1 \varepsilon_1 T_1^4 \frac{\sin\alpha}{1 - (1-\varepsilon_1)(1-\sin\alpha)} \quad \text{...(iii)}$$

Putting for $A_1 = \frac{\pi DL}{2} = \frac{\pi}{2} 2H\tan\alpha \frac{H}{\cos\alpha}$

$$Q = \varepsilon_1 \sigma T_1^4 \pi H^2 \tan^2\alpha \frac{1}{1 - (1-\varepsilon_1)(1-\sin\alpha)} \quad \text{...(iv)}$$

3. (a)

$$\text{Stoichiometric air-fuel ratio} = \frac{0.86\times\frac{32}{12}+0.13\times\frac{8}{1}}{0.23} = 14.49$$

$$\frac{A}{F} = \left(1+\frac{110}{100}\right)\times 14.49 = 30.58$$

Molecular weight of air = 0.23 × 32 + 0.769 × 28 = 28.9 kg/kmol

Ley y mol of air to be supplied per kg of fuel and the equation of combustion per kg of fuel is $\frac{0.86}{12}C+\frac{0.13}{2}H_2+0.21\times yO_2+0.79\times yN_2 \rightarrow aCO_2+bH_2O+cO_2+dN_2$

From carbon balance : $\frac{0.86}{12} = a$

$\Rightarrow$ $a = 0.0717$

From hydrogen balance : $\frac{0.13}{2} = b$

$\Rightarrow$ $b = 0.065$

From oxygen balance: $0.21y = a+\frac{b}{2}+c$

$\Rightarrow$ $0.21y = 0.0717+\frac{0.065}{2}+c$

$\Rightarrow$ $0.21y = 0.1042 + c$ ...(i)

Number of kilo moles of air for per kg of fuel

$$\frac{30.58}{28.9} = 1.05 = y$$

From equation (i), we get

$\Rightarrow$ $0.21 \times 1.05 = 0.1042 + c$

$\Rightarrow$ $c = 0.116$

From nitrogen balance:

$0.79 \times y = d$

$\Rightarrow$ $d = 0.79 \times 1.05 = 0.8295$

Volumetric analysis of dry exhaust gas :

Constituent	Moles	%Volume
CO_2	0.0717	7.05
O_2	0.1160	11.40
N_2	0.8295	81.55
Total	1.0172	100.00

$$\text{Indicated power, IP} = \frac{BP}{\eta_{mech}} = \frac{124}{0.85} = 145.88 \text{ kW}$$

$$\text{Heat input per sec} = \frac{\text{Heat equivalent to IP}}{\eta_{ith}}$$

$$= \frac{145.88 \times 60}{0.38} = 23033.68 \text{ kJ/min}$$

$$\therefore \quad \dot{m}_f = \frac{23033.68}{43000} = 0.536 \text{ kg/min}$$

$$\Rightarrow \quad \dot{m}_a = \dot{m}_f \times \left(\frac{A}{f}\right)_{actual} = 0.536 \times 30.58 = 16.39 \text{ kg/min}$$

$$\Rightarrow \quad \dot{V}_a = \dot{m}_a v_a = 16.39 \times 0.77 = 12.62 \text{ m}^3\text{/min}$$

$$\Rightarrow \quad \dot{V}_s = \frac{12.62}{0.80} = 15.7 \text{ m}^3\text{/min}$$

$$\Rightarrow \quad \dot{V}_s = \frac{\pi}{4} \times D^2 L \times nK$$

$$\Rightarrow \quad 15.7 = \frac{\pi}{4} \times D^3 \times 1.2 \times \left(\frac{1500}{2 \times 60}\right) \times 6$$

$$\Rightarrow \quad D = 0.6056 \text{ m} = 60.56 \text{ cm}$$

$$L = 1.2 \times 60.56 = 72.67 \text{ cm}$$

Ans.

3. (b)

Mean mass-concentration is given by,

$$x_w = \frac{\text{Mass of } NH_3}{\text{Mass of } NH_3 + \text{Mass of water}}$$

$$= \frac{17x}{17x + 18(1-x)} = \frac{17x}{18-x}$$

(where x = molal fraction of NH_3 in aqua)

$$= \frac{17 \times 0.30}{18 - 0.30} = 0.288$$

$$Q_{absorbed} = 804(1 - x_w) - 930 \times x_w^2$$
$$= 804 \times (1 - 0.288) - 930 \times (0.288)^2$$
$$= 495.31 \text{ kJ/kg of anhydrous } NH_3 \text{ generated}$$

Let m be the mass of weak aqua-ammonia solution per kg of anhydrous NH_3.

For heat balance of the heat exchanger.

$$mc_p(120 - 40) = (m + 1)c_p(90 - 20)$$
$$m = 7 \text{ kg}$$

For heat balance of absorber

Heat of NH_3 at entry to absorber + Heat of weak aqua-ammonia + $Q_{absorbed}$ = Heat of strong aqua-ammonia + Heat to coolant

$$\Rightarrow \quad 1682 + 7[200 + 4.8 \times \{40 - (-50)\}] + 495.31 = Q_{coolant} + 8[200 + 4.8 \times \{20 - (-50)\}]$$

$$\Rightarrow \quad Q_{coolant} = 2313.31 \text{ kJ/kg of anhydraus } NH_3$$ **Ans.**

For heat balance of the generator, we have

$$8[200 + 4.8 \times (90 + 50)] + Q_s = 1840 + 7[200 + 4.8(120 + 50)] + 495.31$$
$$Q_s = 2471.3 \text{ kJ/kg of anhydrous } NH_3$$ **Ans.**

$$\text{Refrigerating effect} = 1682 - 600 = 1082 \text{ kJ/kg}$$

$$\therefore \quad COP = \frac{1082}{2471.31} = 0.439$$ **Ans.**

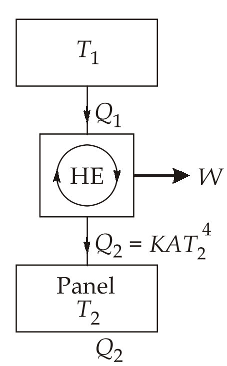
3. (c) (i)

For the heat engine, the heat rejected Q_2 to the panel (at T_2) is equal to the energy emitted from the panel to the surroundings by radiation. If A is the area of panel, $Q_2 \propto AT_2^4$, or $Q_2 = KAT_2^4$, where K is a constant.

Now,
$$\eta = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}$$

or
$$\frac{W}{T_1 - T_2} = \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{KAT_2^4}{T_2} = KAT_2^3$$

$$A = \frac{W}{KT_2^3(T_1 - T_2)} = \frac{W}{K\left(T_1T_2^3 - T_2^4\right)}$$



For a given W and T_1, A will be minimum when

$$\frac{dA}{dT_2} = -\frac{W}{K}\left(3T_1T_2^2 - 4T_2^3\right)\left(T_1T_2^3 - T_2^4\right)^{-2} = 0$$

Since, $\left(T_1T_2^3 - T_2^4\right)^{-2} \neq 0, \; 3T_1T_2^2 = 4T_2^3$

$$\frac{T_2}{T_1} = 0.75 \qquad \text{Proved}$$

$$A_{min} = \frac{W}{K(0.75)^3 T_1^3 (T_1 - 0.75T_1)}$$

$$= \frac{W}{K\frac{27}{256}T_1^4} = \frac{256W}{27KT_1^4}$$

For, W = 1200 W; K = 5.67 × 10^{-8} W/m^{2}K^4 and T_1 = 100 K

$$A_{min} = \frac{256 \times 1200}{27 \times 5.67 \times 10^{-8} \times (1100)^4} = 0.137 \text{ m}^2$$

3. (c) (ii)

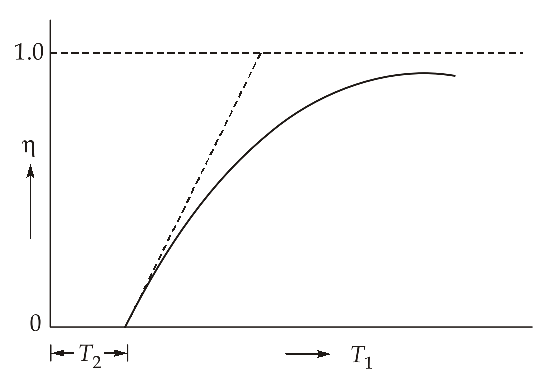
The efficiency of a Carnot engine is given by,

$$\eta = 1 - \frac{T_2}{T_1}$$

If T_2 is constant

$$\left(\frac{\partial \eta}{\partial T_1}\right)_{T_1} = \frac{T_2}{T_1^2}$$

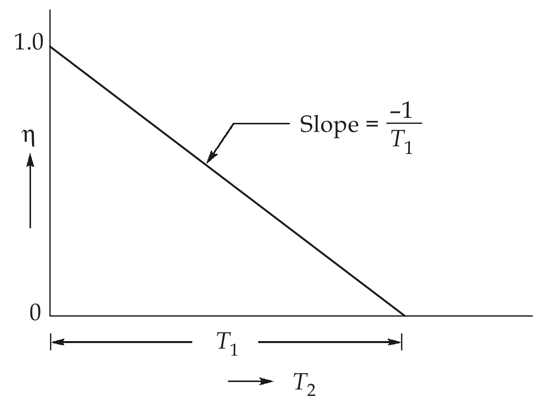
As T_1 increases, η increases, and the slope $\left(\frac{\partial \eta}{\partial T_1}\right)_{T_2}$ decreases as shown in figure (a).



If T_1 is constant

$$\left(\frac{\partial \eta}{\partial T_2}\right)_{T_1} = -\frac{1}{T_1}$$

As T_2 decreases, η increases, but the slope $\left(\frac{\partial \eta}{\partial T_2}\right)_{T_2}$ remains constant as shown in figure (b).



Also $$\left(\frac{\partial \eta}{\partial T_1}\right)_{T_1} = \frac{T_2}{T_1^2} \text{ and } \left(\frac{\partial \eta}{\partial T_2}\right)_{T_1} = -\frac{T_1}{T_1^2}$$

Since, $$T_1 > T_2, \quad \left(\frac{\partial \eta}{\partial T_2}\right)_{T_1} > \left(\frac{\partial \eta}{\partial T_1}\right)_{T_2}$$

So, the more effective way to increase the efficiency is to decrease T_2. Alternatively, let T_2 be decreased by ΔT with T_1 remaining the same.

$$\eta_1 = 1 - \frac{T_2 - \Delta T}{T_1}$$

If T_1 is increased by the same ΔT, T_2 remaining the same

$$\eta_2 = 1 - \frac{T_2}{T_1 + \Delta T}$$

Then $$\eta_1 - \eta_2 = \frac{T_2}{T_1 + \Delta T} - \frac{T_2 - \Delta T}{T_1}$$

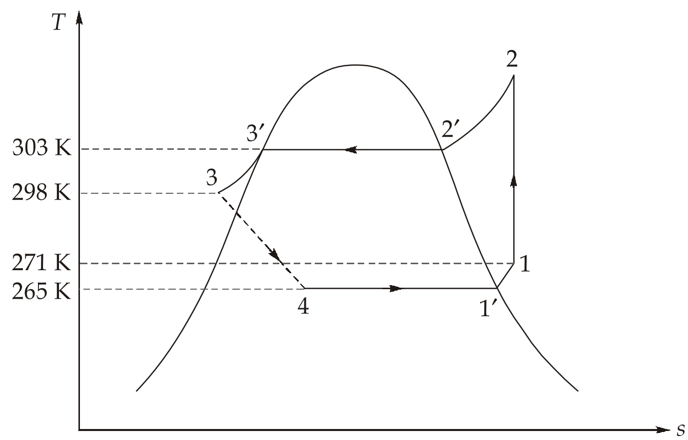
$$= \frac{(T_1 - T_2)\Delta T + (\Delta T)^2}{T_1(T_1 + \Delta T)}$$

Since, $$T_1 > T_2, \quad (\eta_1 - \eta_2) > 0$$

The more effective way to increase the cycle efficiency is to decrease T_2.

4. **(a)**

Refer figure,



$$h_{3'} = 64.59 \text{ kJ/kg}; \quad h_{1'} = 184.07 \text{ kJ/kg}$$
$$v_{1'} = 0.0790 \text{ kJ/kg}; \quad h_{2'} = 199.02 \text{ kJ/kg}$$
$$s_{1'} = 0.7007 \text{ kJ/kg}; \quad s_{2'} = 0.6853 \text{ kJ/kgK}$$
$$c_{pl} = 1.235 \text{ kJ/kg}; \quad c_{pv} = 0.733 \text{ kJ/kgK}$$

For isentropic compression process 1 - 2 :

$$\Rightarrow \quad s_1 = s_2$$

$$\Rightarrow \quad s_{1'} + c_{pv} \times \ln\left(\frac{T_1}{T_{1'}}\right) = s_{2'} + c_{pv} \times \ln\left(\frac{T_2}{T_{2'}}\right)$$

$$\Rightarrow \quad 0.7007 + 0.733 \times \ln\left(\frac{271}{265}\right) = 0.6853 + 0.733 \times \ln\left(\frac{T_2}{303}\right)$$

$$\Rightarrow \quad T_2 = 316.44 \text{ K}$$

$$\Rightarrow \quad h_2 = h_{2'} + c_{pv}(T_2 - T_{2'}) = 199.02 + 0.733 \times (316.44 - 303)$$
$$= 208.87 \text{ kJ/kg}$$

$$\Rightarrow \quad h_1 = h_{1'} + c_{pv}(T_1 - T_{1'}) = 184.07 + 0.733 \times (271 - 265)$$
$$= 188.47 \text{ kJ/kg}$$

$$\Rightarrow \quad h_3 = h_{3'} + c_{pl}(T_{3'} - T_3) = 64.59 - 1.235 \times (303 - 298)$$
$$= 58.42 \text{ kJ/kg} = h_4$$

$$\text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{188.47 - 58.42}{208.87 - 188.47} = 6.375$$ **Ans.**

Refrigeration effect, $RE = h_1 - h_4 = 188.47 - 58.42 = 130.05$ kJ/kg

Refrigeration capacity, $RC = 12 \times 3.5 = \dot{m} \times 130.05$

$\Rightarrow \quad \dot{m} = 0.323$ kg/s $= 19.38$ kg/min

Work done during compression,

$$w_c = \dot{m}(h_2 - h_1)$$

$$= 0.323(208.87 - 188.47) = 6.59 \text{ kW}$$

Theoretical power per tonne of refrigeration $= \dfrac{6.59}{12} = 0.55$ kW/TR **Ans.**

$$v_1 = v_1' \times \frac{T_1}{T_{1'}} = 0.079 \times \frac{271}{265} = 0.081 \text{ m}^3/\text{kg}$$

$$v_2 = v_2' \times \frac{T_2}{T_{2'}} = 0.0235 \times \frac{316.44}{303} = 0.0245 \text{ m}^3/\text{kg}$$

$$\Rightarrow \quad \eta_v = 1 + c - c\left(\frac{v_1}{v_2}\right) = 1 + 0.02 - 0.02 \times \left(\frac{0.081}{0.0245}\right) = 0.9539$$

$$\Rightarrow \quad \eta_v = \frac{\dot{m} \times v_1}{\frac{\pi}{4} \times D^2 \times (1.5 \times D) \times N \times 2}$$

$$\Rightarrow \quad 0.9539 = \frac{19.38 \times 0.081 \times 4}{\pi \times D^3 \times 1000 \times 2 \times 1.5}$$

$\Rightarrow \quad D = 0.0887$ m or 8.87 cm

$L = 1.5 \times 8.87 = 13.30$ cm **Ans.**

4. (b) (i)

Reynolds number (Re) can be defined as the ratio of inertia forces to viscous forces in a fluid flow.

By definition,

$$\text{Re} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{VL}{\nu}$$

Reynolds number signifies which force-inertia or viscous is more dominant in a flow regime. This decides whether flow is turbulent (high inertia forces and high Re) or laminar (high viscous forces and low Re).

Prandtl Number can be defined as the ratio of molecular diffusivity of momentum and thermal diffusivity.

$$Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

Hence, Prandtl number signifies the relative thickness of thermal and velocity boundary layers. High and low values of Prandtl number signify a thicker velocity and thermal boundary layers, respectively. Prandtl number is strictly a property of fluid alone.

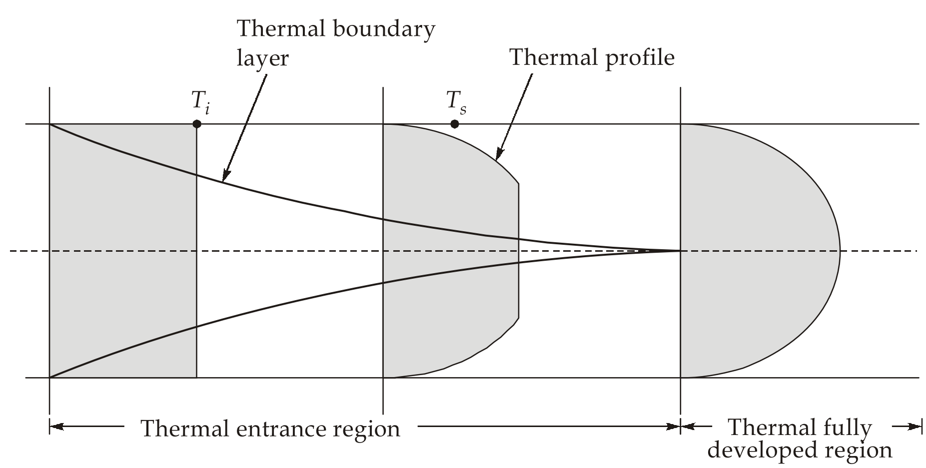
Nusselt Number can be defined as the ratio of heat transfer by convection to the heat transfer by conduction.

$$Nu = \frac{\dot{q}_{convection}}{\dot{q}_{conduction}} = \frac{h\Delta T}{k\Delta T / L} = \frac{hL}{k}$$

Nusselt number signifies enhancement of heat transfer due to convection. High Nusselt number implies convection is more dominant than conduction at fluid-solid surface.

In case of internal forced convection, fully developed flow means that hydrodynamic (velocity) and thermal (temperature) profiles do not change with axial distance along the tube.

For example, as shown in figure below, post thermal entry length, temperature profile does not change in x-direction and depends on radial distance from axis alone.



Parameters upon which entrance length depends:

(a) Velocity and Reynolds number-nature of flow laminar or turbulent

(b) Smoothness or roughness of pipe/tube

(c) Dimensions and geometry of tube.

4. (b) (ii)

$$\text{Bulk mean temperature, } T_f = \frac{10+30}{2} = 20°\text{C} = 293\text{ K}$$

∴ The properties of air at 20°C can be used for calculation.

$$\text{Grashof number, Gr} = \frac{g\beta(T_1 - T_2)\times\delta^3}{\nu^2}$$

$$= \frac{9.81\times\left(\frac{1}{293}\right)\times 0.01^3\times 20}{(15.06\times10^{-6})^2} = 2952.4$$

$$\text{Gr.Pr} = 2952.4 \times 0.703 = 2075.57$$

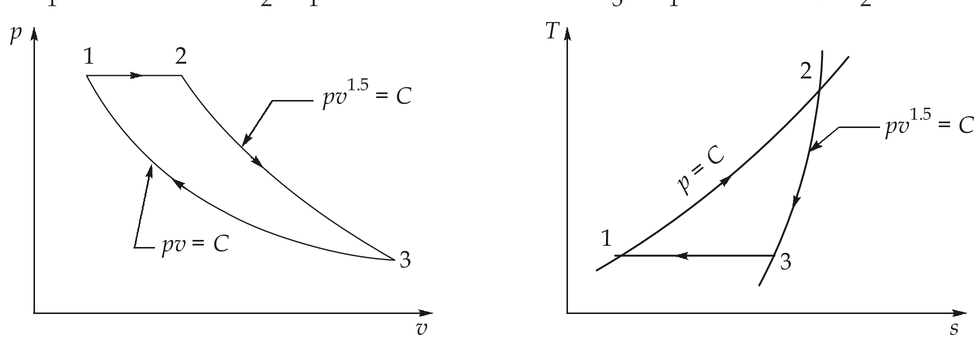
$$\therefore \quad \text{Nu} = 0.42(Gr\,\text{Pr})^{0.25}\,\text{Pr}^{0.012}\left(\frac{L}{\delta}\right)^{-0.30} = \frac{h\delta}{k}$$

$$\Rightarrow \quad h = 0.42(2075.57)^{0.25}(0.703)^{0.012}\times\left(\frac{1.2}{0.01}\right)^{-0.3}\times\frac{0.02593}{0.01}$$

$$\Rightarrow \quad h = 1.74\text{ W/m}^2\text{K}$$ **Ans.**

4. (c) (i)

Given : P_1 = 700 kPa = P_2; T_1 = 267°C = 540 K = T_3; V_1 = 0.03 m^3; V_2 = 0.09 m^3



(a) (b)

From ideal gas equation of state,

$$P_1V_1 = mRT_1$$

$$\Rightarrow \quad m = \frac{P_1V_1}{RT_1} = \frac{700\times0.03}{0.287\times540}$$

$$\Rightarrow \quad m = 0.1355\text{ kg}$$

For constant pressure process 1 - 2 :

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

$$\Rightarrow \quad T_2 = T_1 \times \frac{V_2}{V_1} = 540 \times \frac{0.09}{0.03} = 1620 \text{ K}$$

For polytropic process 2 - 3 :

$$\frac{P_3}{P_2} = \left(\frac{T_3}{T_2}\right)^{\frac{n}{n-1}}$$

$$\Rightarrow \quad P_3 = 700 \times \left(\frac{540}{1620}\right)^{\frac{1.5}{1.5-1}} = 25.926 \text{ kPa}$$

$$Q_{1\text{-}2} = mc_p (T_2 - T_1) = 0.1355 \times 1.005 \times (1620 - 540)$$
$$= 147.072 \text{ kJ}$$

$$Q_{2\text{-}3} = \Delta U_{2\text{-}3} + W_{2\text{-}3}$$
$$= mc_v (T_3 - T_2) + \frac{mR(T_2 - T_3)}{n-1} = mc_v \left(\frac{n-\gamma}{n-1}\right)(T_3 - T_2)$$
$$= 0.1355 \times 0.718 \times \left(\frac{1.5-1.4}{1.5-1}\right) \times (540 - 1620)$$
$$= -21.014 \text{ kJ}$$

$$Q_{3\text{-}1} = \Delta U_{3\text{-}1} + W_{3\text{-}1} \qquad [\because \; T = c, \text{ so } \Delta U_{3\text{-}1} = 0]$$

$$W_{3\text{-}1} = mRT_1 \ln\left(\frac{V_1}{V_3}\right) = mRT_1 \times \ln\left(\frac{P_3}{P_1}\right)$$
$$= 0.1355 \times 0.287 \times 540 \times \ln\left(\frac{24.989}{700}\right) = -69.985 \text{ kJ}$$

Heat received in the cycle, Q_S = 147.072 kJ **Ans.**

Heat rejected in the cycle, Q_R = 21.014 + 69.985 = 91 kJ **Ans.**

$$\therefore \text{ Efficiency of the cycle, } \eta_{cycle} = 1 - \frac{Q_R}{Q_S} = 1 - \frac{91}{147.072}$$
$$= 0.3813 \text{ or } 38.13\%$$ **Ans.**

4. (c) (ii)

Work transfer is identified only at the boundaries of a system. It is a boundary phenomenon, and a form of energy in transit crossing the boundary. Let us consider a gas separated from the vacuum by a partition as shown in figure. Let the partition be removed. The gas rushes to fill the entire volume. The expansion of a gas against vacuum is called free expansion. If we neglect the work associated with the removal of partition, and consider the gas and vacuum together as our system (figure (a)), there is no work transfer involved here, since no work crosses the system boundary, and hence

$\int_1^2 dW = 0,$ although $\int_1^2 pdV \neq 0$

If only the gas is taken as the system (figure (b)), when the partition is removed there is a change in the volume of the gas, and one is tempted to calculate the work from the expression $\int_1^2 pdV$. However, this is not a quasistatic process, although the initial and final end states are in equilibrium. Therefore, the work cannot be calculated from this relation. The two end states can be located on the p-V diagram and these are joined by a dotted line (figure (c)) to indicate that the process had occurred. However, if the vacuum space is divided into a large number of small volumes by partitions and the partitions are removed one by one slowly (figure (d)), then every state passed through by the system is an equilibrium state and the work done can then be estimated from the relation $\int_1^2 pdV$ (figure (e)), Yet, in free expansion of a gas, there is no resistance to the fluid at the system boundary as the volume of the gas increases to fill up the vacuum space. Work is done by a system to overcome some resistance. Since vacuum does not offer any resistance, there is no work transfer involved in free expansion.

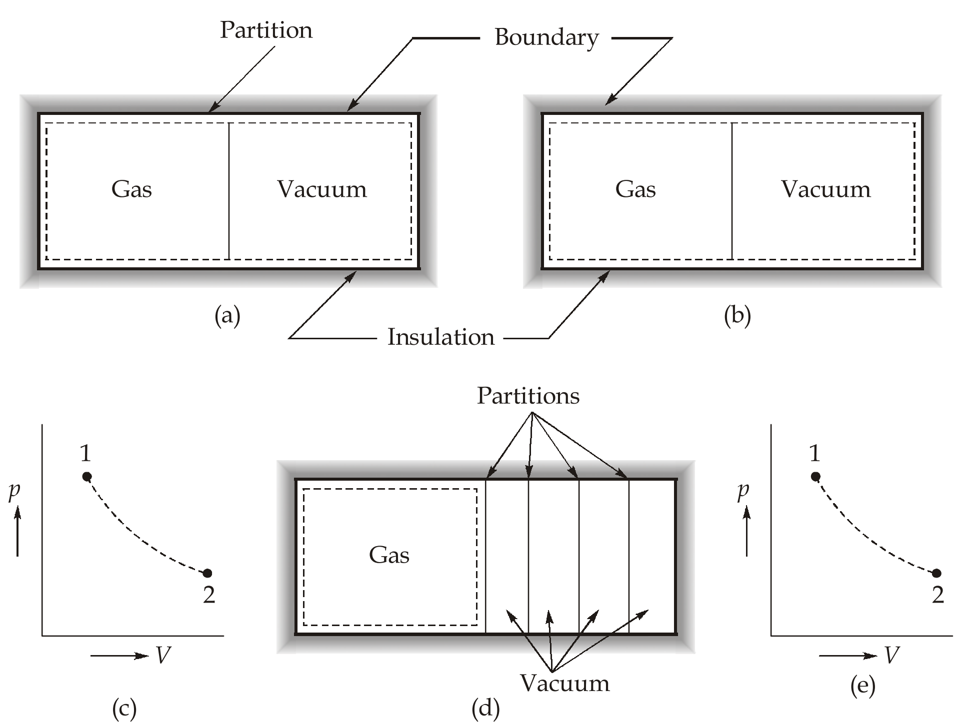


Fig. : Free expansion

Section : B

5. (a)

The cooling of an engine is necessary for the following reasons:

1. The peak gas temperature during the combustion process of an IC engine is of the order of 2500 K. The temperature of the inside surface of the cylinder walls is usually kept below 200°C to prevent deterioration of the oil film. High temperature of the lubricating oil may result in physical and chemical changes in the oil and cause wear and sticking of the piston rings, scoring of the cylinder walls, or seizure of the piston. It is therefore necessary to provide cooling for the walls of combustion space.
2. A large portion of the heat generated in the combustion chamber is transferred from combustion gases to the cylinder head and walls, and piston and valves. Heats absorbed by these components increase their temperatures. The temperature distributions are uneven causing uneven expansion of various engine parts, hence causing thermal stresses in the components of the engine. High thermal stresses cause fatigue and cracking of the components. Therefore, the temperature must be kept less than about 400°C for cast iron and about 300°C for aluminium alloys.
3. The temperature of the cylinder head must also be kept below 220°C. If the cylinder head temperature is high, this may lead to overheated spark-plug electrodes causing pre-ignition in Sl engines.
4. Spark plug and valves must be kept cool to avoid knock and preignition problems which result from overheated spark-plug electrodes or exhaust valves, Preignition results in a loss of efficiency and increases the cylinder head temperature to such an extent that engine failure or complete loss of power may result.

Disadvantages of overcooling:

1. Starting of the engine will be difficult at low temperatures. The engine must be kept sufficiently hot to ensure smooth and efficient operation.
2. Vaporization of the fuel will be reduced at low temperatures, preventing formation of a homogeneous mixture with air. It may cause poor combustion and also increase fuel consumption.
3. Excessive cooling provided to the combustion chamber walls will lower the average combustion gas temperature and pressure and reduce the work per cycle transferred to the piston. Thus, the specific power and efficiency are reduced by excessive cooling.

4. Friction will be increased because of higher viscosity of lubricating oil at lower temperatures.
5. The sulphurous and sulphuric acids are formed from the oxidation of sulphur present in the fuel during the combustion process. These acids may condense at low temperatures and corrode the cylinder surfaces. To prevent condensation of acids, the coolant temperature should be greater than 70°C.

5. (b)

Given : H = 25 m; P = 36750 kW; η_0 = 0.86; ϕ = 2.0; ψ = 0.7; $\frac{d}{D} = 0.35$

∴ Overall efficiency, $\eta_0 = \frac{P}{\rho g Q H}$

or $$Q = \frac{36750 \times 10^3}{10^3 \times 9.81 \times 0.86 \times 25} = 174.24 \text{ m}^3/\text{s}$$

Now, $$V_f = \psi\sqrt{2gH} = 0.7 \times \sqrt{2 \times 9.81 \times 25} = 15.5 \text{ m/s}$$

Also, $$Q = \frac{\pi}{4}\left(D^2 - d^2\right) \times V_f$$

or $$174.24 = \frac{\pi}{4} \times D^2(1 - 0.35^2) \times 15.5$$

∴ $$D^2 = \frac{174.24 \times 4}{\pi(1 - 0.35^2) \times 15.5}$$

∴ $$D = 4.038 \text{ m}$$

and $$d = 0.35 \times D = 4.038 \times 0.35 = 1.413 \text{ m}$$

$$D_m = \frac{D + d}{2} = \frac{4.038 + 1.413}{2} = 2.7255 \text{ m}$$ **Ans.**

Now, $$U = \phi \times \sqrt{2gH} = 2 \times \sqrt{2 \times 9.81 \times 25}$$

or $$\frac{\pi D_m N}{60} = 44.3$$

$$N = \frac{60 \times 44.3}{\pi \times 2.7255} = 310.43 \text{ rpm}$$

5. (c)

Mass of hot gases flowing through the chimney

$$\dot{m}_g = \rho_g A V_g$$

Since the density of hot gases is inversely proportional to its temperature,

$$\rho_g = \frac{C_1}{T_g}, \text{ where } C_1 \text{ is constant and}$$

the velocity of gases, $V_g = C\sqrt{2gH_g}$, we have

$$\dot{m}_g = \frac{C_1}{T_g} \times A \times C\sqrt{2gH_g}, \text{ where } H_g = \text{hot gas column in } m$$

$$\therefore \quad \dot{m}_g = AC_2\sqrt{2gH\left[\frac{m}{m+1}\cdot\frac{T_g}{T_a} - 1\right]} \times \frac{1}{T_g}$$

For a given height of the chimney, H, we can write

$$\dot{m}_g = C_3\left[\frac{m}{m+1}\cdot\frac{1}{T_aT_g} - \frac{1}{T_g^2}\right]^{1/2}$$

Thus the maximum discharge is a function of T_g,

$$\therefore \quad \frac{d\dot{m}_g}{dT_g} = 0 = -\frac{m}{m+1}\frac{1}{T_aT_g^2} + \frac{2}{T_g^3}$$

$$\therefore \quad T_g = 2T_a\frac{m}{m+1} \text{ and } \frac{T_g}{T_a} = 2\left(\frac{m+1}{m}\right)$$

Now, $$\Delta p = \rho_g g H_g = 353gH\left(\frac{1}{T_a} - \frac{m+1}{m}\frac{1}{T_g}\right)$$

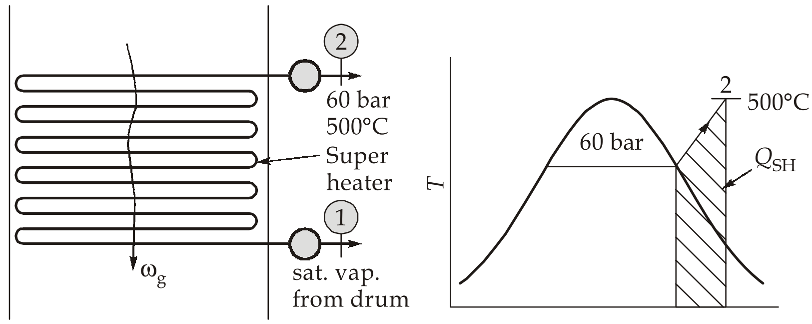
Since, $$\rho_g = \frac{353}{T_g}\frac{m+1}{m}, \text{ on substitution}$$

$$H_g = H\left(\frac{m}{m+1}\frac{T_g}{T_a} - 1\right)$$

$$= H\left(\frac{m}{m+1}\cdot 2\frac{m+1}{m} - 1\right) = H$$

Thus the height of the hot gas column is equal to the chimney height for maximum discharge.

5. (d)



From steam tables, at state points shown in figure, we have

$$h_1 = h_g = 2784.6 \text{ kJ/kg}$$

$$h_2 = 3423.1 \text{ kJ/kg}; v_2 = 0.0566710 \text{ m}^3/\text{kg}$$

Heat absorption rate in superheater coils,

$$Q_{sup} = \dot{m}_s(h_2 - h_1) = 50(3423.1 - 2784.6) = 31925 \text{ kW}$$

$$\text{Surface area required, } A = \frac{31925}{180} = 177.36 \text{ m}^2$$

Now,
$$\dot{m}_s = n \cdot \frac{\pi}{4} d_i^2 \times \frac{V}{v_2}$$

$\therefore$
$$n = \frac{50 \times 4 \times 0.056671}{\pi \times 0.06^2 \times 12}$$

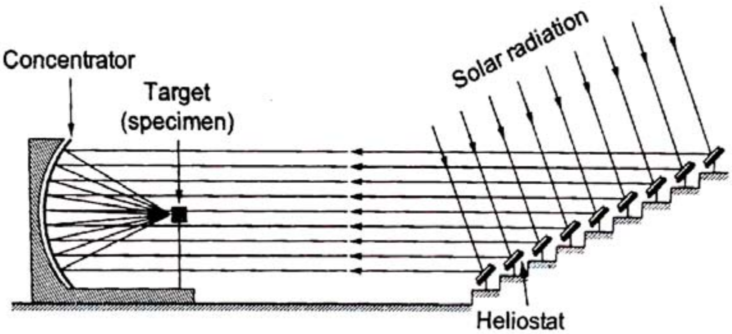
$$= 83.51 \simeq 84 \text{ tubes}$$ **Ans.**

Now,
$$A_0 = n \cdot \pi d_0 \cdot l$$

$\therefore$
$$l = \frac{177.36}{84 \times \pi \times 0.07} = 9.6 \text{ m}$$ **Ans.**

5. (e)

Solar furnaces are ideal tools to study the chemical, optical, electrical and thermodynamic properties of the materials at high temperatures. It is basically an optical system in which solar radiations are concentrated over a small area. It has two main components : (i) a concentrator and (ii) a single piece of a large sized heliostat or a system of large number of small heliostats. Basic principle is shown in figure. Large number of heliostats directs solar radiation onto a paraboloidal reflector surface. The heliostats are adjusted such that they direct the radiation parallel to the optical axis of the paraboloid. For this purpose, accurate sun tracking is required. The concentrators focus the incoming rays at the target placed at its focus.



Solar furnace

There is another possible configuration of solar furnace, where the optical axis is vertical. A large heliostat directs the radiation upwards and the concentrator reflects it downwards sat its focus. In this arrangement, the unmelted portion of specimen forms a crucible to hold the melted portion and is suitable for fusion studies.

Some of the advantages of a solar furnace are:

(i) heating without contamination,

(ii) easy control of temperature,

(iii) working is simple,

(iv) high heat flux is obtainable,

(v) continuous observation possible, and

(vi) absence of electromagnetic field.

In spite of many advantages of solar furnaces, these have not become popular in industries due to following reasons:

(i) Its use is limited to sunny days and that too for 4 - 5 hours only.

(ii) Its cost is high.

(iii) Very high temperatures are obtained only over a very small area.

6. (a)

Manometric efficiency, $\eta_{mano} = \dfrac{gH_m}{V_{w_2}u_2}$

$$H_m = h_s + h_d + h_{fs} + h_{fd}$$

$$= 32 + 1.05 + 6.2 = 39.25 \text{ m}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.305 \times 1450}{60} = 23.156 \text{ m/s}$$

$\therefore \quad V_{w2} = \dfrac{gH_m}{\eta_{mano} \times U_2} = \dfrac{9.81 \times 39.25}{0.8 \times 23.156} = 20.78 \text{ m/s}$

But, $\quad V_{w2} = U_2 - V_{f2} \cot\beta_2$

$$V_{f2} = (U_2 - V_{w2}) \times \tan\beta_2$$

$$V_{f2} = (23.156 - 20.78) \times \tan 35°$$

$$= 1.66 \text{ m/s}$$

$\therefore \quad$ Discharge, $Q = \pi D_2 b_2 V_{f2}$

$$= \pi \times 0.305 \times 0.025 \times 1.66$$

$$= 0.03976 \text{ m}^2/\text{s} = 39.76\ l/\text{s}$$ **Ans.**

Power required to drive the pump,

$$p = \frac{\rho g Q H}{\eta_0} = \frac{10^3 \times 9.81 \times 0.03976 \times 39.25}{0.68}$$

$$= 22.51 \text{ kW}$$ **Ans.**

Velocity in suction and delivery pipe,

$$V_s = V_d = \frac{4Q}{\pi d_s^2} = \frac{4Q}{\pi d_d^2} = \frac{4 \times 0.03976}{\pi \times (0.125)^2} = 3.24 \text{ m/s}$$

Pressure in the suction pipe just before inlet

$$\frac{p_s}{\gamma} = -h_s - h_{fs} - \frac{V_s^2}{2g}$$

$$= -3 - 1.05 - \frac{3.24^2}{2 \times 9.81} = -4.585 \text{ m of water}$$

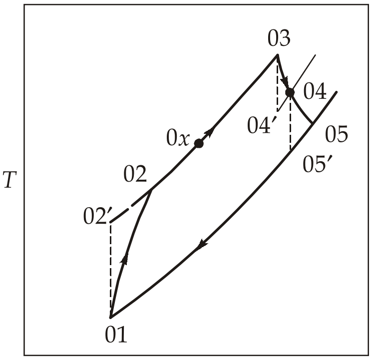
Pressure in the delivery pipe must be sufficient to raise the water through the delivery head

$$\frac{p_d}{\gamma} = h_d + h_{fd} - \frac{V_d^2}{2g}$$

$$= (32 - 3) + 6.2 - \frac{3.24^2}{2 \times 9.81}$$

$$= 34.665 \text{ m of water}$$ **Ans.**

6. (b)



$$T'_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 288(6)^{\frac{0.4}{1.4}} = 480.53$$

$$\eta_{isen,c} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$\therefore \quad T_{02} = 288 + \frac{480.53 - 288}{0.85} = 514.5 \text{ K}$$

As LPT mechanically independent implies HPT power = Compressor power

$$\therefore \quad c_p(T_{03} - T_{04}) = c_p(T_{02} - T_{01})$$

$$\therefore \quad T_{04} = T_{03} - T_{02} + T_{01}$$

$$= 998 - 514.5 + 288 = 771.5 \text{ K}$$ **Ans.**

Now, $$\eta_{HPT} = \frac{T_{03} - T_{04}}{T_{03} - T'_{04}}$$

$$\therefore \quad 0.87 = \frac{998 - 771.5}{998 - T'_{04}}$$

$$\therefore \quad T'_{04} = 737.65 \text{ K}$$

$$\frac{P_{03}}{P_{04}} = \left(\frac{T_{03}}{T'_{04}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{998}{737.65}\right)^{\frac{1.4}{0.4}} = 2.88$$

$$\therefore \quad P_{04} = \frac{6}{2.88} = 2.083 \text{ bar}$$ **Ans.**

Now, $$\frac{T_{04}}{T'_{05}} = \left(\frac{P_{04}}{P_{05}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad T'_{05} = \frac{771.5}{(2.083)^{0.4/1.4}} = 625.57 \text{ K}$$

$$\eta_{LPT} = \frac{T_{04} - T_{05}}{T_{04} - T'_{05}}$$

$$T_{05} = 771.5 - 0.82(771.5 - 625.57)$$

$$= 651.83 \text{ K}$$

Heat exchanger effectiveness,

$$0.7 = \frac{T_{0x} - T_{02}}{T_{05} - T_{02}} = \frac{T_{0x} - 514.5}{651.83 - 514.5}$$

$$\therefore \quad T_{0x} = 610.63 \text{ K}$$

$\therefore$ For LPT, $\dot{P} = \dot{m}c_p(T_{04} - T_{05})$

$$= 10 \times 1.005(771.5 - 651.83)$$

$$= 1202.68 \text{ kW}$$ **Ans.**

Heat supplied, $Q = \dot{m}c_p(T_{03} - T_{0x}) = 10 \times 1.005(998 - 610.63)$

$$= 3893.07 \text{ kW}$$ **Ans.**

Efficiency, $\eta_{th} = \dfrac{\dot{W}_{LPT}}{Q} \times 100$

$$= \frac{1202.68}{3893.07} \times 100 = 30.89\%$$ **Ans.**

6. (c)

Gas required for lighting = $10 \times 0.126 \times 5 = 6.3 \text{ m}^3/\text{day}$

Electrical energy required by eight computers = $\dfrac{8 \times 300 \times 6 \times 3600}{0.25 \times 0.8} = 259.2$ MJ

Required volume of biogas for the engine = $\dfrac{259.2}{23} = 11.27 \text{ m}^3/\text{day}$

Therefore, total daily requirement of biogas = 6.3 + 11.27 = 17.57 m^3

Let the cows required to feed the plant = n

Cow dung produced = 10 × n kg/day

Collectable cow dung per day = 7n kg/day

Weight of solid mass in the cow dung = 7n × 0.18 = 1.26n kg/day

Gas produced per day = 1.26n × 0.22 = 0.2772n

Therefore, total daily requirement of biogas = Gas produced per day

$$17.57 = 0.2772n$$

$\therefore$ $n = 63.38$; 64 cows **Ans.**

Thus 64 cows are required to feed the plant.

Daily feeding of cow dung into the plant = 7 × 64 = 448 kg

Daily slurry produced = 448 + 448 = 896 kg

Volume of slurry added per day = $\frac{896}{1090} = 0.822\ m^3$

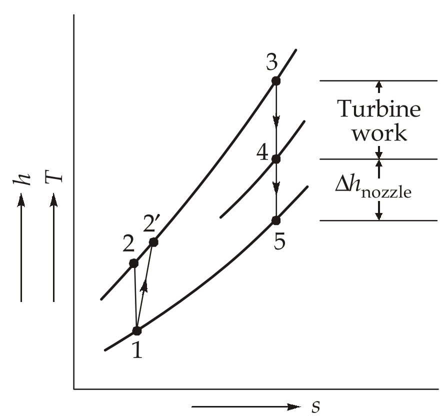
For a 50 days retention time, the total volume of the slurry in the digester

$$= 50 \times 0.822 = 41.1\ m^3$$

As only 90% of the digester volume is occupied by the slurry, the net volume of the digester = $\frac{41.1}{0.9} = 45.67\ m^3$ **Ans.**

7. (a)

Refer to figure:



$$T_1 = 283\ K;\ P_2 = 4.2\ bar;\ P_1 = 1\ bar;\ T_3 = 798\ K$$

We know, $$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 283 \times (4.2)^{\frac{0.4}{1.4}} = 426.44\ K$$

or $$T_2' = 1.2(T_2 - T_1) + T_1$$

$$= 1.2(426.44 - 283) + 283 = 455.13\ K$$

Work required, $W_C = c_p(T_2' - T_1)$

$= 1.005(455.13 - 283)$

$= 173$ kJ/kg of air **Ans.(a)**

Now, $Q = c_p(T_3 - T_2') \times \dot{m}_a$

or $\dot{m}_f \times CV = c_p(T_3 - T_2') \times \dot{m}_a$

or $$\frac{\dot{m}_a}{m_f} = \frac{44000}{1.005(798 - 455.13)}$$

$$\frac{\dot{m}_a}{m_f} = 127.69$$ **Ans.(b)**

Now, $$T_5 = \frac{T_3}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{798}{(4.2)^{\frac{0.4}{1.4}}} = 529.58 \text{ K}$$

Since the work developed by the turbine is consumed by the compressor, hence

$\Delta h_{nozzle} = (h_3 - h_5) - (h_2' - h_1)$

$= 1.005(798 - 529.58) - 173$

$= 96.76$ kJ/kg of air

Velocity at exit of nozzle, $V_s = 44.72\sqrt{96.76} = 439.89$ m/s

$\therefore$ $$\text{Thrust} = \left(1 + \frac{\dot{m}_f}{m_a}\right)\dot{V}_j - \dot{V}_a = \left(1 + \frac{1}{127.69}\right) \times 439.89 - 0$$

$= 443.33$ N/kg of air

7. (b) (i)

The block diagram showing the main components of a fuel-cell power plant is given in Fig. (a). Electrical energy is generated from primary fossil fuels through a fuel cell. Fuel is managed and supplied by a fuel processing unit. In this unit, fuel is received, stored, reformed, purified and supplied to fuel-cell modules. Fuel-cell modules convert fuel energy electrochemically into dc power using ambient air as oxidant. Basic configurations of cell, module and plant are shown in Fig. (b). A number of cells are stacked to form a module. Several modules are interconnected to form a power-generating unit. Fuel gas and air are supplied to modules from common supply pipes. The exhaust is collected in a common pipe and discharged to the atmosphere either directly or after recovery of heat in a cogeneration unit. The power-generating unit generates electrical power as dc. Industrial/commercial loads are rated for standard ac supply such as 3 ph., 400 V, 50/60 Hz or 1 hp, 230/110 V, 50/60 Hz. The electrical power-conditioning unit, converts dc output of fuel cell to ac using inverter and also controls and regulates it.

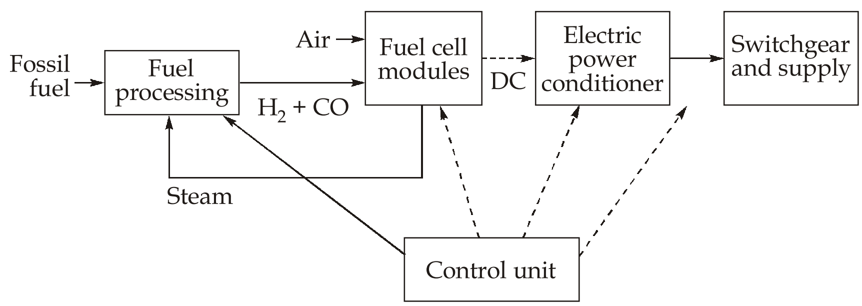


Fig. (a) : Fuel-cell based electrical power-generation scheme

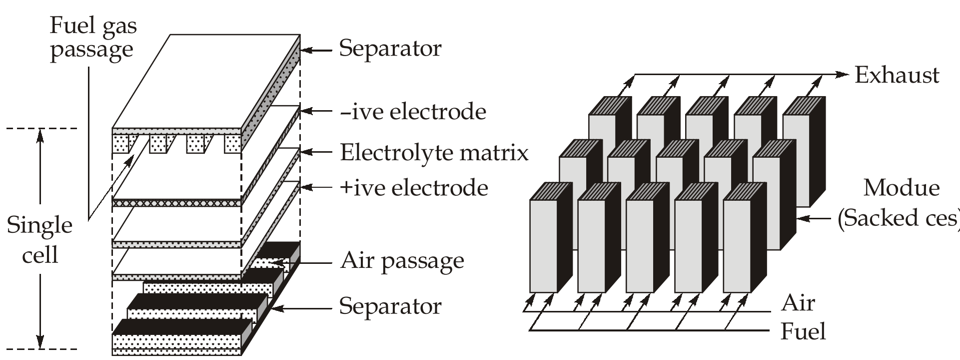


Fig. (b) : Power-generaion unit

7. (b) (ii)

$$\Delta G = -39.59 \text{ kcal/mol} = -166.278 \text{ kJ/mol}$$

$$\Delta H = -56.8 \text{ kcal/mol} = -238.56 \text{ kJ/mol}$$

The electrical work output per mole of fuel,

$$\Delta W_{max} = -\Delta G = 166.278 \text{ kJ}$$

i.e. 166.278 kW electrical power is produced from flow rate of 32 g/s of methanol and 48 g/s of oxygen.

Required flow rate of methanol for electrical output of 200 kW = $\frac{32 \times 200}{166.278} = 38.49$ g/s

Required flow rate of oxygen for electrical output of 200 kW = $\frac{48 \times 200}{166.278} = 57.73$ g/s

Heat transfer is given by, $\Delta Q = \Delta H - \Delta G = -56.8 + 39.59 = -17.21$ kcal/mol

The negative sign indicates that heat is removed from the cell and transferred to the surroundings.

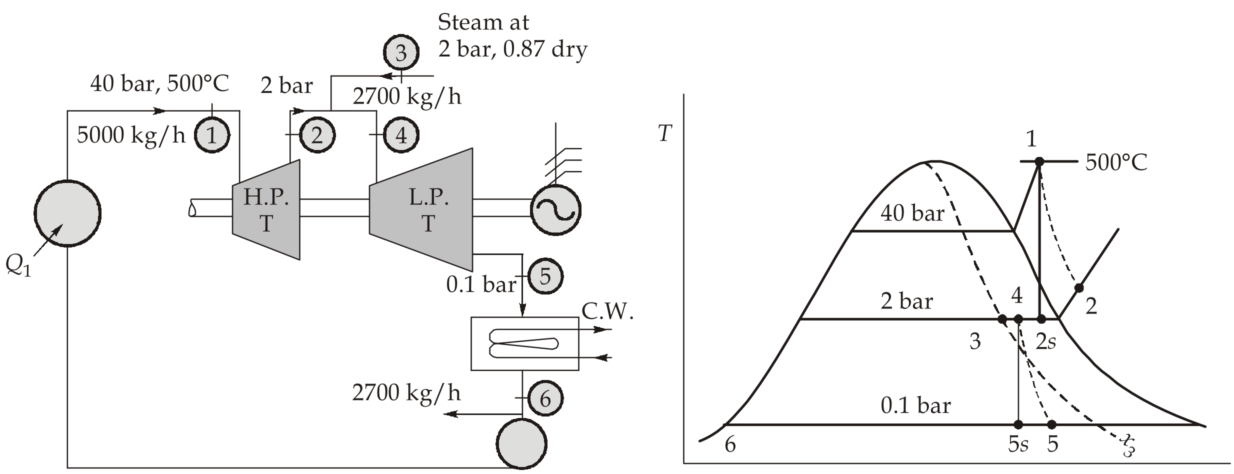
Consumption of fuel at the rate of 38.49 g/s produces heat at the rate

$$= \frac{17.21 \times 38.49}{32} = 20.7 \text{ kcal/s}$$

Thus the required heat removal rate from the cell at electrical output of 200 kW is 20.7 kcal/s **Ans.**

7. (c)

Refer to figure



(a) (b)

From steam table, we have

At 40 bar, 500°C, $h_1 = 3446.0$ kJ/kg; $s_1 = 7.0922$ kJ/kgK

At 2 bar $h_{2f} = 504.7$ kJ/kg; $h_{fg_2} = 2201.5$ kJ/kg

$s_{2f} = 1.5302$ kJ/kg; $s_{fg_2} = 5.5967$ kJ/kgK

At 0.1 bar, $h_6 = 191.81$ kJ/kg; $h_{fg_5} = 2392.1$ kJ/kg

$s_{f6} = 0.6492$ kJ/kgK; $s_{fg_5} = 7.4996$ kJ/kgK

Now, $s_1 = s_{2s} = s_{2f} + x_{2s} s_{fg2}$

$7.0922 = 1.5302 + x_{2s} \times 5.5967$

$\therefore \quad x_{2s} = 0.9938$

$\therefore \quad h_{2s} = h_{2f} + x_{2s} h_{fg2} = 504.7 + 0.9938 \times 2201.5$

$= 2692.55$ kJ/kg

$\therefore \quad h_1 - h_2 = 0.85(3446 - 2692.55) = 640.43$ kJ/kg

$h_2 = 3446 - 640.43 = 2805.57$ kJ/kg

and $\quad h_3 = h_{2f} + x_3 + h_{fg_2}$

$= 504.7 + 0.92 \times 2201.5$

$= 2530.08$ kJ/kg

Now, energy balance gives,

$$2700 \times h_3 + 5000 \times h_2 = (2700 + 5000) \times h_4$$

$$\therefore \quad h_4 = \frac{2700 \times 2530.08 + 5000 \times 2805.57}{2700 + 5000} = 2708.97 \text{ kJ/kg}$$

Also, $\quad h_4 = h_{2f} + x_4 \times h_{fg_2}$

$2708.97 = 504.7 + x_4 \times 2201.5$

$\therefore \quad x_4 = 1.001$

$\therefore \quad s_4 \simeq s_{g2} = s_{5s}$

$7.1269 = 0.6492 \times x_{5_s} \times 7.4996$

$x_{5_s} = 0.864$

$\therefore \quad h_{5_s} = 191.81 + 0.864 \times 2392.1$

$= 2258.58$ kJ/kg

$h_4 - h_5 = 0.82(2708.97 - 2258.58) = 369.32$ kJ/kg

$\Rightarrow \quad h_5 = 2708.97 - 369.32 = 2339.65$ kJ/kg

Now, power output, $\quad \dot{W} = 5000\,(h_1 - h_2) + 7700\,(h_4 - h_5)$

$= 5000(640.43) + 7700(369.32)$

$= 6045914$ kJ/h or 1679.42 kW **Ans.**

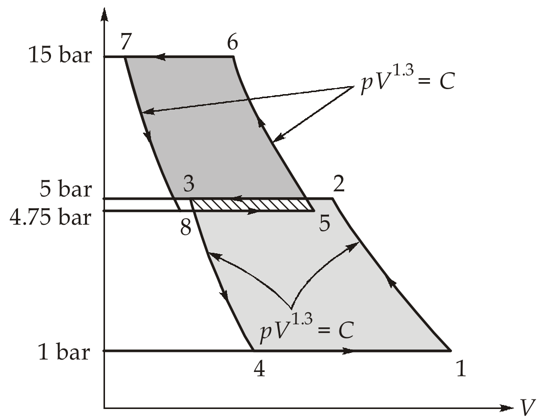
$$\text{Heat supplied, } Q_1 = 5000(h_1 - h_6) \times \frac{1}{3600}$$

$$= 5000(3446 - 191.81) \times \frac{1}{3600} = 4519.708 \text{ kW}$$

$$\therefore \quad \text{Efficiency, } \eta_{cycle} = \frac{1679.42}{4519.708} \times 100 = 37.16\%$$ **Ans.**

8. (a)

Given : $k = 2$; $N = 300$ rpm; $P_1 = 100$ kPa; $T_1 = 300$ K; $P_2 = 6$ bar; $P_5 = 4.75$ bar; $T_3 = 300$ K; $P_6 = 16$ bar; $n = 1.3$; $d_1 = 350$ mm; $L_1 = 400$ mm; $c_1 = c_2 = 0.04$; $c_p = 1.005$ kJ/kgK; $R = 0.287$ kJ/kgK; $L_2 = L_1$



The stroke volume of LP cylinder,

$$\dot{V}_{s,LP} = \dot{V}_1 - \dot{V}_3 = \frac{\pi}{4} d_1^2 L_1 \times N \times k$$

$$= \frac{\pi}{4} \times 0.35^2 \times 0.4 \times \frac{300}{60} \times 2$$

$$= 0.3848 \text{ m}^3/\text{s}$$

The volumetric efficiency of LP cylinder,

$$\eta_{\text{vol,LP}} = 1 + c_1 - c_1 \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$= 1 + 0.04 - 0.04 \times (6)^{1/1.3} = 0.8812$$

or $$\dot{V}_1 - \dot{V}_4 = \dot{V}_{s,LP} \times \eta_{vol,LP}$$

$$= 0.3848 \times 0.8813 = 0.339 \text{ m}^3/\text{s}$$

The mass flow rate of air into LP cylinder

$$\dot{m}_a = \frac{P_1\left(\dot{V}_1 - \dot{V}_4\right)}{RT_1} = \frac{100 \times 0.339}{0.287 \times 300} = 0.394 \text{ kg/s}$$

Temperature after compression in each stage

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 300(6)^{\frac{0.3}{1.3}} = 453.62 \text{ K}$$

∴ Heat transfer in the intercooler

$$\dot{Q} = \dot{m}_a c_p (T_2 - T_1)$$

$$= 0.394 \times 1.005(453.62 - 300)$$

$$= 60.83 \text{ kW}$$ **Ans.**

The effective swept volume rate of HP cylinder

$$\dot{V}_5 - \dot{V}_8 = \frac{\dot{m}_a R T_1}{P_5} = \frac{0.394 \times 0.287 \times 300}{475} = 0.0714 \text{ m}^3/\text{s}$$

$$\eta_{\text{vol,HP}} = 1 + c_2 - c_2\left(\frac{P_3}{P_5}\right)^{1/n}$$

$$= 1 + 0.04 - 0.04\left(\frac{16}{4.75}\right)^{1/1.3} = 0.938$$

$\therefore$
$$\dot{V}_{s,HP} = \frac{\dot{V}_s - \dot{V}_8}{\eta_{vol,HP}} = \frac{0.0714}{0.9382} = 0.0761 \text{ m}^3/\text{s}$$

or
$$\frac{\pi}{4} d_2^2 \cdot \frac{L_1 N \cdot k}{60} = 0.0761$$

$\therefore$
$$d_2^2 = \frac{4 \times 60 \times 0.0761}{\pi \times 0.4 \times 300 \times 2}$$

$$d_2 = 0.155 \text{ m or } 155 \text{ mm}$$ **Ans.**

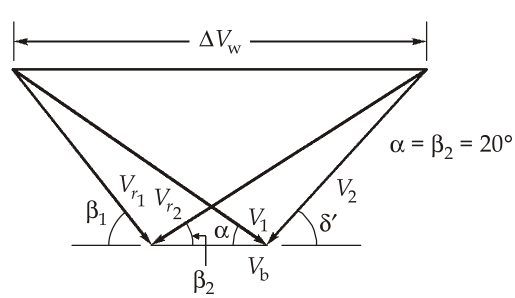
Power required to drive the HP cylinder,

$$\text{I.P.} = \frac{n}{n-1} \times P_5\left(\dot{V}_5 - \dot{V}_8\right)\left[\left(\frac{P_3}{P_8}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3 - 1} \times 4.75 \times 10^2 \times 0.0714\left[\left(\frac{16}{4.75}\right)^{\frac{0.3}{1.3}} - 1\right]$$

$$= 47.54 \text{ kW}$$ **Ans.**

8. (b)



$V_{r_2} = V_1$; $V_{r_1} = V_2$; $\alpha = \beta_2 = 20°$; $U = 120$ m/s; $\frac{U}{V_1} = 0.56$

$\therefore$
$$V_1 = \frac{120}{0.56} = 214.28 \text{ m/s}$$

$$h_b = 0.03 \text{ m}, v = 0.55 \text{ m}^3/\text{kg}; N = 3000 \text{ rpm}$$

$$U = \frac{\pi D_m N}{60}$$

$$\Rightarrow \quad D_m = \frac{60 \times 120}{\pi \times 3000} = 0.764 \text{ m}$$

Now, $$\dot{m}_s \cdot v = \pi D_m h_b \times V_1 \sin\alpha$$

$$\therefore \quad \dot{m}_s = \frac{\pi \times 0.764 \times 0.03 \times 214.28 \times \sin 20°}{0.55}$$

$$\therefore \quad \dot{m}_s = 9.59 \text{ kg/s} = 34524 \text{ kg/hr}$$ **Ans.**

Refer to figure,

$$\tan\beta_1 = \frac{V_1 \sin\alpha}{V_1 \cos\alpha - U} = \frac{214.28 \sin 20°}{214.28 \cos 20° - 120}$$

$$\therefore \quad \beta_1 = 42.01°$$

$$V_{r_1} = \frac{V_1 \sin\alpha}{\sin\beta_1} = \frac{214.28 \sin 20°}{\sin 42.01°} = 109.5 \text{ m/s}$$

Enthalpy drop in moving blades,

$$\Delta h_{mb} = \frac{V_{r_2}^2 - V_{r_1}^2}{2} = \frac{214.28^2 - 109.5^2}{2} \qquad (\because V_1 = V_{r_2})$$

$$= 16.962 \text{ kJ/kg}$$

$$\therefore \quad \Delta h_{stage} = 2\Delta h_{mb} = 2 \times 16.962 = 33.92 \text{ kJ/kg}$$

For 5 pairs of blades,

$$(\Delta h)_T = 5 \times 33.92 = 169.6 \text{ kJ/kg}$$

$$\therefore \quad \text{Diagram power} = \dot{m}_s \times \Delta h_T$$

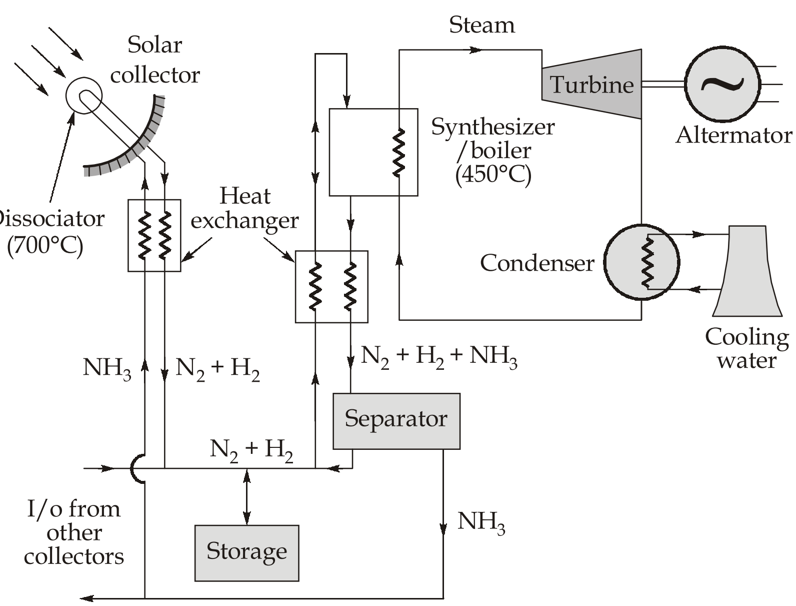
$$= 9.59 \times 169.6$$

$$= 1626.46 \text{ kW}$$ **Ans.**

8. (c) (i)

In a distributed collector system, the solar thermal energy is collected from a large number of sun-tracking solar collectors, cylindrical parabolic trough type or paraboloidal dish type. Each collector transfers heat to a heat-transport fluid. This heat-transfer fluid available at high temperature from the collectors is pooled at some central power station. The heat transfer fluid could be water/steam, to in a steam turbine, or it could be some thermochemical storage be used directs ammonia. The advantage of a later scheme is that thermal energy In stored as chemical energy at ambient temperature and no heat is lost in its transmission over a long distance generation or in storing it overnight for continuous power.

The schematic diagram of a typical distributed collector power generation plant is shown in figure below. The heat collected in collectors is used to dissociate ammonia into nitrogen and hydrogen at high pressure (approx. 300 atm). The heat of reaction, 46 kJ/mole (of NH_3) is provided by the solar energy. This nitrogen-hydrogen mixture is transported to the central plant where N_2 and and H_2 are recombined in a synthesizer, using a catalyst. The heat released during the reaction is utilized in a heat engine to generate electric power through an alternator. In the synthesizer only a part of nitrogen and hydrogen recombine to produce ammonia. The products of the synthesizer are cooled to liquefy ammonia, separate it out and send it to the collector system. The mixture of nitrogen and hydrogen that remains in gaseous state is fed back to the synthesizer.



Distributed collector solar thermal electric-power plant

8. **(c) (ii)**

The main environmental concerns are discussed below:

1. Indirect Energy Use and Emissions Energy is required to produce material used to construct the wind turbine and its installation. This energy is paid back in a period of few months to about a year, what is known as energy payback period. Some pollution (emission of CO, etc.) is caused due to use of energy during construction. But in total, the so-called indirect CO, emission over the total operating life of the 2 wind generator is very low (about 1% of the system using coal).

2. Bird Life Large wind turbines pose a threat to bird life as a result of collision with tower or blades. Their resting and breeding patterns are also affected.
3. The disturbance caused by the noise produced by a wind turbine is one of the important factors that prevent its siting close to inhabited areas. The acoustic noise is composed of (a) mechanical noise due to movement of mechanical parts in the nacelle (mainly gear and also other equipments), which can be reduced by good design and acoustic insulation, and (b) aerodynamic noise (swishing sound from the rotating blades), which is a function of wind speed and which cannot be avoided.
4. In a study, it was found that public appreciation of a landscape decreases as more and more wind turbines are installed. A special case of visual impact is the effect of shadow of the turbine, particularly of the rotor blades. The rotational frequency plays an important role in determining the disturbance level.
5. Telecommunication Interference Wind turbines present an obstacle for incident electromagnetic waves (i.e., TV or microwave signals). These waves can be reflected, scattered and dithered. Thus, they interfere with telecommunication links and badly affect the quality of radio and TV reception. The effect can be mitigated by the use of cable system or by installing powerful antennas.
6. Safety Accidents with wind turbines are rare but they do happen, as in other industrial activities. For example, a detached blade or its fragment may be thrown a considerable distance and can harm people and property.

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