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Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

Electrical Engineering Test No : 10

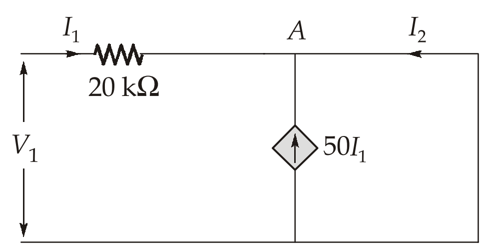
Section-A

Q.1 (a) Solution:

The h-parameters of a two-port network are given by

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

To calculate h_{11} and h_{21}, we put $V_2 = 0$. Thus, the circuit can be drawn as:



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \text{ and } h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

From the circuit, $V_1 = (20\text{ k}\Omega)\cdot I_1$

$\Rightarrow$ $h_{11} = \frac{V_1}{I_1} = 20\text{ k}\Omega$

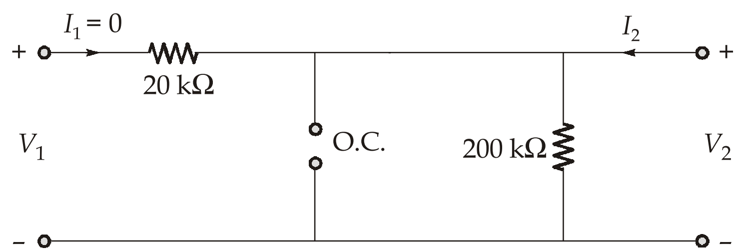
To find I_2, applying KCL at node A,

$$I_1 + 50I_1 + I_2 = 0$$

$\Rightarrow$ $I_2 = -51I_1$

$$\Rightarrow \qquad h_{21} = \frac{I_2}{I_1} = -51$$

To calculate h_{12} and h_{22}, we put $I_1 = 0$



$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \text{ and } h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

For $I_1 = 0$, the current source acts as open-circuit

We have, $V_2 = (200\text{k}) \cdot I_2$

$$\Rightarrow \qquad h_{22} = \frac{I_2}{V_2} = \frac{1}{200k} = 5\times10^{-6}\,℧$$

Since $I_1 = 0$, there is no voltage drop across 20 kΩ resistance.

Hence, $h_{12} = \dfrac{V_1}{V_2} = 1$

Hence, the h-parameters are given by

$$[h] = \begin{bmatrix} 20\times10^3 & 1 \\ -51 & 5\times10^{-6} \end{bmatrix}$$

Q.1 (b) Solution:

Given signal, $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}u(2t-n)$

For $x(t)$ to be periodic,

$$x(t + T) = x(t)$$

Therefore, $x(t+T) = \sum_{n=-\infty}^{\infty} e^{-[2(t+T)-n]}u[2(t+T)-n]$

$$= \sum_{n=-\infty}^{\infty} e^{-(2t+2T-n)} u(2t+2T-n)$$

Let $2T - n = -m$ (m is an integer)

as $n = -\infty \Rightarrow$ $m = -\infty$

$n = \infty \Rightarrow$ $m = \infty$

Therefore, $$x(t+T) = \sum_{m=-\infty}^{\infty} e^{-(2t-m)} u(2t-m)$$

$$= x(t)$$

Thus, $x(t)$ is periodic if

$$2T - n = -m$$

$$T = \frac{n-m}{2}$$

Fundamental time period, $$T = \frac{1}{2} \text{ sec}$$

(as least possible meaningful value of $n - m$ is 1).

Now for power,

$$u(2t-n) = \begin{cases} 1, & (2t-n) > 0 \rightarrow t > \frac{n}{2} \\ 0, & \text{otherwise} \end{cases}$$

To determine $x(t)$ over the interval $0 < t < \frac{1}{2}$, $n = 0, -1, -2, ...$ Therefore, we have

$$x(t) = \sum_{n=-\infty}^{0} e^{-(2t-n)}, \; 0 < t < \frac{1}{2}$$

$$= e^{-2t} \sum_{n=-\infty}^{0} e^{n} = e^{-2t} \sum_{n=0}^{\infty} (e^{-1})^{n}$$

$$x(t) = \frac{e^{-2t}}{1-e^{-1}}, \; 0 < t < \frac{1}{2}$$

Average power contained in $x(t)$ is given by

$$P(x) = \frac{1}{T}\int_0^T |x(t)|^2 \, dt = 2\int_0^T \left[\frac{e^{-2t}}{1-e^{-1}}\right]^2 dt$$

$$= \frac{2}{(1-e^{-1})^2}\int_0^{1/2} e^{-4t}dt = \frac{2}{(1-e^{-1})^2}\times\left.\frac{e^{-4t}}{-4}\right|_0^{1/2}$$

$$= \frac{2}{(1-e^{-1})^2}\times\left[\frac{1}{4}-\frac{e^{-2}}{4}\right]$$

$$= \frac{1}{2}\times\frac{(1-e^{-2})}{(1-e^{-1})^2} = \frac{1}{2}\frac{(1-e^{-1})(1+e^{-1})}{(1-e^{-1})^2}$$

$$= \frac{1}{2}\cdot\frac{(e+1)}{(e-1)} \text{ Units}$$

Q.1 (c) Solution:

As we know, diode current equation is,

$$I = I_0(e^{V_D/\eta V_T}-1)$$

Case-1:

When, $V_{D_1} = 0.4$ V

$I_1 = 10$ mA

$\Rightarrow$ $10\times10^{-3} = I_0(e^{0.4/\eta\times0.026}-1)$

$\because$ $1 << e^{0.4/\eta\times0.026}$

$\Rightarrow$ $10\times10^{-3} = I_0\,(e^{15.385/\eta})$ ...(i)

Case-2:

When, $V_{D_2} = 0.42$ V

$I_2 = 2I_1$

$\Rightarrow$ $2\times10\times10^{-3} = I_0(e^{0.42/\eta\times0.026}-1)$

$\because$ $1 << e^{0.42/\eta\times0.026}$

$\Rightarrow$ $2\times10\times10^{-3} = I_0(e^{16.153/\eta})$ ...(ii)

Dividing equation (i) by (ii),

$$\frac{1}{2} = \frac{e^{15.385/\eta}}{e^{16.153/\eta}}$$

$$\frac{16.153}{\eta} = \frac{15.385}{\eta}+\ln 2$$

Solving, $\eta = 1.1079$

Using this value in equation (2),

$$2\times10\times10^{-3} = I_0\,(e^{16.153/1.1079})$$

$\Rightarrow$ $I_0 = 9.312$ nA

Q.1 (d) Solution:

$$(1 + y^2)dx = (\tan^{-1} y - x)\, dy$$

$$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$$

This is a linear differential equation

$$\text{I.F.} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$$x \cdot e^{\tan^{-1} y} = \int e^{\tan^{-1} y} \frac{\tan^{-1} y}{1+y^2} dy + c$$

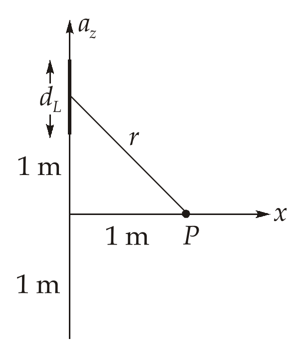
Put, $\tan^{-1} y = t$ on R.H.S.

So that, $\frac{1}{1+y^2} dy = dt$

$$x \cdot e^{\tan^{-1} y} = \int e^t \cdot t\, dt + c = t \cdot e^t - e^t + c = e^{\tan^{-1} y}(\tan^{-1} y - 1) + c$$

$$x = \left(\tan^{-1} y - 1\right) + c\, e^{-\tan^{-1} y}$$

Q.1 (e) (i) Solution:



The potential due to linear charge distribution,

$$V = \frac{1}{4\pi \in_0} \int \frac{\rho_L}{r} dL \text{ V} = \frac{\rho_L}{4\pi \in_0} \int_{-1}^{1} \frac{1}{r} dz$$

where, $r = \sqrt{z^2 + x^2}$ $= \dfrac{10^{-10}}{4\pi \times \dfrac{10^{-9}}{36\pi}} \displaystyle\int_{-1}^{1} \frac{1}{r} dz$

$$= 0.9\left[\ln\left(z+\sqrt{z^2+x^2}\right)\right]_{-1}^{1} = 0.9\times 2\ln\left[1+\sqrt{2}\right]$$

$$V = 1.59 \text{ V}$$

Q.1 (e) (ii) Solution:

Given, $V_i = 10$ V, $Z_0 = 50\ \Omega$

$$Z_L = 200\ \Omega$$

1. The reflection coefficient,

$$\mu = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200-50}{200+50} = \frac{3}{5}$$

2. The reflection coefficient is equal to the ratio of the reflected voltage to the input voltage at load,

i.e.,
$$\mu = \frac{V_r}{V_i}$$

∴
$$V_r = \mu \times V_i = \frac{3}{5}\times 10 = 6 \text{ V}$$

3. Let P_i be the power incident, percentage of incident power transmitted,

$$P_T = P_i[1 - \mu^2]$$

$$= \left[1-\left(\frac{3}{5}\right)^2\right] = P_i\left[1-\frac{9}{25}\right]$$

$$= P_i\left[\frac{16}{25}\right]$$

∴
$$P_T = 0.64\, P_i$$

∴ 64% of the incident power is transmitted.

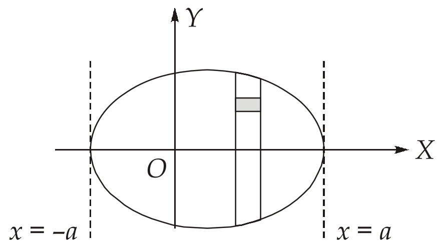
Q.2 (a) Solution:

For the ellipse
$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

$$\frac{y}{b} = \pm\sqrt{1-\frac{x^2}{a^2}}$$

$$y = \pm\frac{b}{a}\sqrt{a^2-x^2}$$

∴ The region of integration can be expressed as



$$-a \le x \le a \text{ and } -\frac{b}{a}\sqrt{a^2 - x^2} \le y \le \frac{b}{a}\sqrt{a^2 - x^2}$$

$$\therefore \quad \iint (x+y)^2 \, dx\, dy = \iint (x^2 + y^2 + 2xy)\, dx\, dy$$

$$= \int_{-a}^{a} \int_{(-b/a)\sqrt{a^2-x^2}}^{b/a\sqrt{a^2-x^2}} (x^2 + y^2 + 2xy)\, dx\, dy$$

$$= \int_{-a}^{a} \int_{(-b/a)\sqrt{a^2-x^2}}^{b/a\sqrt{a^2-x^2}} (x^2 + y^2)\, dx\, dy + \int_{-a}^{a} \int_{(-b/a)\sqrt{a^2-x^2}}^{b/a\sqrt{a^2-x^2}} 2xy\, dy\, dx$$

$$= \int_{-a}^{a} \int_{0}^{b/a\sqrt{a^2-x^2}} 2(x^2 + y^2)\, dy\, dx + 0$$

[Since $(x^2 + y^2)$ is an even function of y and $2xy$ is an odd function of y]

$$= \int_{-a}^{a} \left[2\left(x^2 y + \frac{y^3}{3} \right) \right]_0^{\left(\frac{b}{a}\right)\sqrt{a^2-x^2}} dx$$

$$= 2\int_{-a}^{a} \left[x^2 \times \frac{b}{a}\sqrt{a^2 - x^2} + \frac{1}{3}\frac{b^3}{a^3}(a^2 - x^2)^{3/2} \right] dx$$

$$= 4\int_{0}^{a} \left[\frac{b}{a} x^2 \sqrt{a^2 - x^2} + \frac{b^3}{3a^3}(a^2 - x^2)^{3/2} \right] dx$$

On putting $x = a \sin \theta$ and $dx = a \cos \theta \, d\theta$

$$= 4\int_{0}^{\pi/2} \left(\frac{b}{a} \cdot a^2 \sin^2 \theta \cdot a \cos \theta + \frac{b^3}{3a^3} a^3 \cos^3 \theta \right) \times a \cos \theta \, d\theta$$

$$= 4\int_{0}^{\pi/2} \left(a^3 b \sin^2 \theta \cos^2 \theta + \frac{ab^3}{3} \cos^4 \theta \right) d\theta$$

$$= 4\left[a^3 b \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} + \frac{ab^3}{3} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}\right]$$

$$= \frac{\pi}{4}(a^3 b + ab^3) = \frac{\pi}{4} ab(a^2 + b^2)$$

Q.2 (b) Solution:

Given circuit,

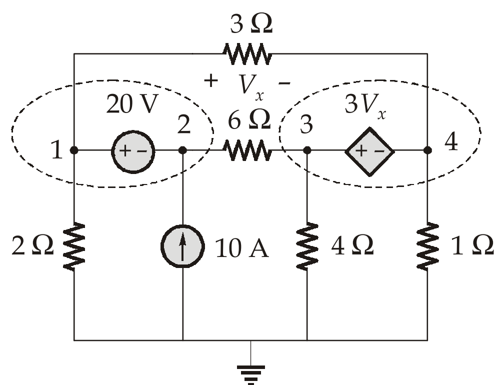


Figure 1

Nodes 1 and 2 as well as nodes 3 and 4 form supernode

We apply KCL to the two supernodes shown in below figure.

At supernode 1 - 2,

$$i_3 + 10 = i_1 + i_2$$

Expressing this in terms of the node voltages,

$$\frac{V_3 - V_2}{6} + 10 = \frac{V_1 - V_4}{3} + \frac{V_1}{2}$$

or $$5V_1 + V_2 - V_3 - 2V_4 = 60 \quad \text{...(i)}$$

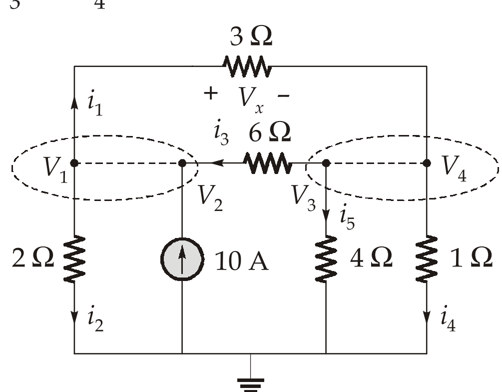


Figure 2

At supernode 3 - 4, $$i_1 = i_3 + i_4 + i_5$$

$$\frac{V_1 - V_4}{3} = \frac{V_3 - V_2}{6} + \frac{V_4}{1} + \frac{V_3}{4}$$

or $$4V_1 + 2V_2 - 5V_3 - 16V_4 = 0 \quad \text{...(ii)}$$

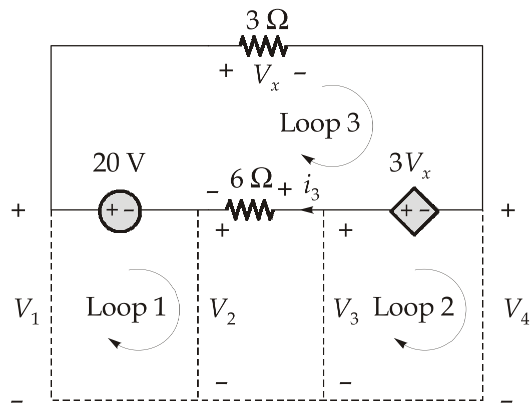


Figure 3

For loop 1, $$V_1 - V_2 = 20 \text{ V} \quad \text{...(iii)}$$

For loop 2, $$-V_3 + 3V_x + V_4 = 0$$

But $V_x = V_1 - V_4$,

So that $$3V_1 - V_3 - 2V_4 = 0 \quad \text{...(iv)}$$

After solving equation (i), (ii), (iii) and (vii), we get,

$$V_1 = 26.67 \text{ V}, \qquad V_2 = 6.67 \text{ V}$$
$$V_3 = 173.33 \text{ V} \qquad V_4 = -46.67 \text{ V}$$

Q.2 (c) Solution:

(i) **Critical temperature :** It is the temperature below which the material changes from conductor to super conductors. The transition from conductor to super conductor is sudden. i.e., when the temperature of material is reduced below the critical temperature, its resistance suddenly reduces to zero.

Critical Magnetic Field : The value of magnetic field beyond which the super conductors return to normal conductor state is called critical magnetic field. The value of critical magnetic field is inversely proportional to the temperature.

Properties of super conductors are:

1. Resistivities of super conductors are greater than other elements at room temperature.
2. In superconducting state, materials do not show thermoelectric effect.
3. The heat loss (I^2R loss) in super conducting material is zero. Below critical temperature, if strong magnetic field is applied to a super conducting material its super conducting property is destroyed.

(ii) Given :

Maximum current, $I_m = 0.005$ mA at 4 K

$$\text{Critical temperature} = 8.60 \text{ K}$$

$$\text{Critical magnetic field, } H_c = 7.2 \times 10^{-4} \text{ A/m}$$

We know,
$$H_c = H_o\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$H_{c(4\text{ K})} = 7.2\times10^{-4}\left[1-\left(\frac{4}{8.6}\right)^2\right] = 5.642 \times 10^{-4} \text{ A/m}$$

We know,
$$I = 2\pi r H_c$$

Given maximum current

$$0.005 \times 10^{-3} = 2\pi r \times 5.642 \times 10^{-4}$$

$$\text{Radius of wire, } r = \frac{0.005\times10^{-3}}{2\pi\times5.642\times10^{-4}} = 1.41 \times 10^{-3} \text{ m}$$

So,
$$\text{Diameter of wire} = 2.82 \times 10^{-3} \text{ m} = 2.82 \text{ mm}$$

Q.3 (a) Solution:

(i)
$$I_d = \frac{5-V_o}{2 \text{ k}\Omega}$$

1. Assume transistor is in saturation

$$I_d = \frac{5-V_o}{2 \text{ k}\Omega} = \frac{(0.3 \text{ mA/V}^2)}{2}(V_{GS}-V_T)^2$$

$$5 - V_o = 0.3(V_{GS} - V_T)^2$$

Since $I_G = 0$,

$$\therefore \quad 5 - V_o = 0.3(V_o - 1)^2$$

$$5 - V_o = 0.3(V_o^2 + 1 - 2V_o)$$

$$0.3V_o^2 - 0.6V_o + 0.3 + V_o - 5 = 0$$

$$0.3V_o^2 + 0.4V_o - 4.7 = 0$$

$$V_o = 3.35 \text{ V}, -4.683 \text{ V}$$

Now taking, $V_o = -4.683$ V

Since V_o is negative, then MOSFET is in cut off region, so negative value of V_o is discarded.

Now taking, $V_o = 3.35$ V

$$\because \quad V_D = V_G = V_o = 3.35 \text{ V}$$

Condition for saturation,

$$V_D \geq V_G - V_T$$

$$3.35 \geq (3.35 - 1)$$

$$3.35 \geq 2.35$$

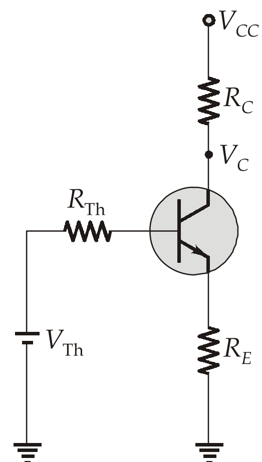
So V_o = 3.35 V is valid and MOSFET is in saturation.

2.
$$I_D = \frac{5-2}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(2-1)^2$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right) = \frac{3}{2} \times 2 = 3 \text{ mA/V}^2$$

(ii) Given : V_{CC} = 20 V, V_{CE} = 8 V, I_C = 5 mA, V_E = 6 V, S = 10, β = 200

We have to find R_1, R_2, R_C and R_E. With equivalent Thevenins' circuit, we have



$$V_{TH} = V_{CC} \times \frac{R_1}{R_1 + R_2}$$

$$R_{Th} = R_1 \parallel R_2$$

$$V_E = I_E . R_E$$

$$R_E = \frac{V_E}{I_E} \simeq \frac{V_E}{I_C} = 1.2 \text{ k}\Omega \quad \text{(Since } \beta \text{ is quite large)}$$

$$V_C = V_E + V_{CE} = 6 + 8 = 14 \text{ V}$$

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{20 - 14}{5 \times 10^{-3}} = 1.2 \text{ k}\Omega$$

For voltage divider circuit,

$$S = \frac{1+\beta}{1 + \beta \times \frac{R_E}{R_E + R_{Th}}}$$

$$1+(200)\times\left(\frac{1.2K}{1.2K+R_{Th}}\right) = \frac{1+200}{10}$$

$$R_{Th} = 11.365K$$

$$\frac{R_1R_2}{R_1+R_2} = 11.365K \quad \text{...(i)}$$

The current, $I_B = \frac{I_C}{\beta} = \frac{5}{200} = 0.025$ mA

So V_{th} can be given as,

$$V_{th} = I_BR_{th} + V_{BE} + V_E$$
$$= 0.025 \times 11.365 + 0.7 + 6$$
$$\approx 6.98 \text{ Volt}$$

and $V_{th} = \frac{R_1}{R_1+R_2}V_{CC}$

So $6.98 = \frac{R_1}{R_1+R_2}(20)$ ...(ii)

From equation (i) and (ii),

$$\frac{R_2}{20} = \frac{11.365K}{6.98}$$

$$R_2 = 32.545 \text{ k}\Omega$$

and $R_1 = 17.46$ kΩ

Q.3 (b) Solution:

(i) A current of 1 μA causes a deflection of 85.4 mm

∴ Current sensitivity, S_i = 85.4 mm/μA

(ii) Voltage across the galvanometer for current of 1 μA

$$= 1 \times 10^{-6} \times 120 = 120 \times 10^{-6} \text{ V}$$

∴ Voltage sensitivity, $S_V = \frac{85.4}{120\times10^{-6}} = 0.712 \times 10^6$ mm/V

$$= 0.712 \text{ mm/}\mu\text{V}$$

(iii) Megaohm sensitivity, $S_0 = \frac{\text{Deflection}}{i\times10^6} = \frac{85.4}{1\times10^{-6}\times10^6} = 85.4$ MΩ/mm

(iv) The deflection of galvanometer is governed by the equation given below:

$$J\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}+K\theta = 0$$

Frequency of damped oscillations

$$\omega_d = \frac{\sqrt{4KJ - D^2}}{2J}$$

$$= \frac{\sqrt{4 \times 12 \times 10^{-9} \times 50 \times 10^{-9} - (5 \times 10^{-9})^2}}{2 \times 50 \times 10^{-9}}$$

$$= 0.487 \text{ rad/s}$$

(or) $$f_d = \frac{\omega_d}{2\pi} = \frac{0.487}{2\pi} = 0.0775 \text{ Hz}$$

(v) Frequency of natural oscillations,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{J}} = \frac{1}{2\pi}\sqrt{\frac{12 \times 10^{-9}}{50 \times 10^{-9}}} = 0.078 \text{ Hz}$$

Period of natural oscillations, $T = \frac{1}{f_n} = \frac{1}{0.078} = 12.82$ sec

(vi) Damping constant required for critical damping,

$$D_c = 2\sqrt{KJ} = 2\sqrt{12 \times 10^{-9} \times 50 \times 10^{-9}}$$

$$= 49 \times 10^{-9} \text{ Nm/rad-s}^{-1}$$

Relative damping, $\xi = \frac{D}{D_c} = \frac{5 \times 10^{-9}}{49 \times 10^{-9}} = 0.102$

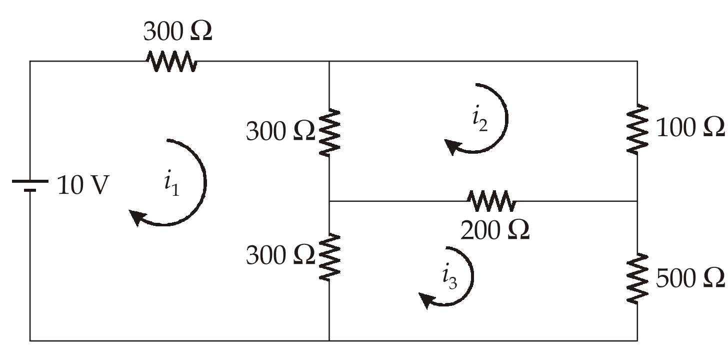
(vii) First maximum deflection,

$$\theta_1 = \theta_F\left[1 + e^{-\pi\xi/\sqrt{1-\xi^2}}\right] = \theta_F\left[1 + e^{(-\pi \times 0.102/\sqrt{1-0.102^2})}\right]$$

$$= 1.724\,\theta_F = 1.724 \times 85.4 = 147.2 \text{ mm}$$

Q.3 (c) Solution:

Using the superposition theorem, analyzing the circuit with dc input only:



Applying KVL in three loops, we get

$$10 = 300(i_1 - i_2) + 300(i_1 - i_3) + 300 i_1$$

$$\Rightarrow \quad 90i_1 - 30i_2 - 30i_3 = 1 \quad \text{...(i)}$$

$$300(i_2 - i_1) + 100i_2 + 200(i_2 - i_3) = 0$$

$$\Rightarrow \quad -3i_1 + 6i_2 - 2i_3 = 0 \quad \text{...(ii)}$$

$$300(i_3 - i_1) + 500i_3 + 200(i_3 - i_2) = 0$$

$$\Rightarrow \quad -3i_1 - 2i_2 + 10i_3 = 0 \quad \text{...(iii)}$$

Writing in Matrix form,

$$\begin{bmatrix} 90 & -30 & -30 \\ -3 & 6 & -2 \\ -3 & -2 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

We have,

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 90 & 1 & -30 \\ -3 & 0 & -2 \\ -3 & 0 & 10 \end{vmatrix}}{\begin{vmatrix} 90 & -30 & -30 \\ -3 & 6 & -2 \\ -3 & -2 & 10 \end{vmatrix}} = \frac{36}{3240}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 90 & -30 & 1 \\ -3 & 6 & 0 \\ -3 & -2 & 0 \end{vmatrix}}{3240} = \frac{24}{3240}$$

Hence, $i_{dc}(t) = i_2 - i_3 = \dfrac{12}{3240} = 3.7 \text{ mA}$

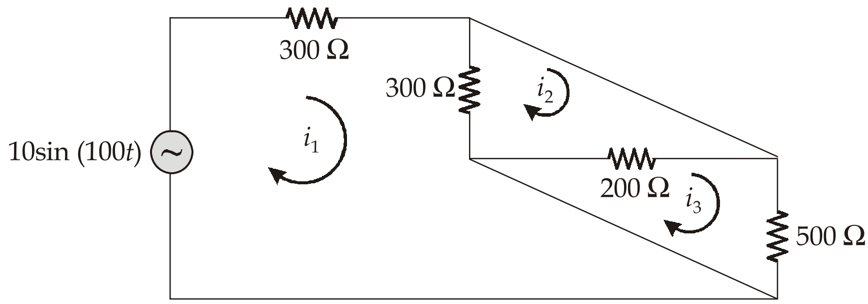
For ac input, $V(t) = 10 \sin 100t, \ \omega = 100 \text{ rad/sec}$

Impedance of LC Branch, $Z_1 = j\left(\omega L - \dfrac{1}{\omega C}\right)$

$$Z_1 = j\left(100 - \frac{1}{100 \times 10^{-4}}\right) = 0$$

Hence, Z_1 acts as short-circuit at $\omega = 100$ rad/sec.

The circuit can be reduced as below:



Applying KVL in three loops, we get

$$10\sin(100t) = 600i_1 - 300i_2$$

$$\Rightarrow \quad \sin(100t) = 60i_1 - 30i_2 \quad \text{...(iv)}$$

$$300(i_2 - i_1) + 200(i_2 - i_3) = 0$$

$$\Rightarrow \quad -3i_1 + 5i_2 - 2i_3 = 0 \quad \text{...(v)}$$

$$200(i_3 - i_2) + 500i_3 = 0$$

$$\Rightarrow \quad -2i_2 + 7i_3 = 0 \quad \text{...(vi)}$$

Writing equations (iv), (v) and (vi) in matrix form,

$$\begin{bmatrix} 60 & -30 & 0 \\ -3 & 5 & -2 \\ 0 & -2 & 7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \sin(100t) \\ 0 \\ 0 \end{bmatrix}$$

where,

$$\Delta = \begin{vmatrix} 60 & -30 & 0 \\ -3 & 5 & -2 \\ 0 & -2 & 7 \end{vmatrix} = 60(31) + 30(-21) = 1230$$

$$\Delta_2 = \begin{vmatrix} 60 & \sin(100t) & 0 \\ -3 & 0 & -2 \\ 0 & 0 & 7 \end{vmatrix} = 21\sin(100t)$$

$$\Delta_3 = \begin{vmatrix} 60 & -30 & \sin(100t) \\ -3 & 5 & 0 \\ 0 & -2 & 0 \end{vmatrix} = 6\sin(100t)$$

Therefore,

$$i_{ac}(t) = \frac{\Delta_2 - \Delta_3}{\Delta} = \frac{15\sin(100t)}{1230} = 12.2\sin(100t)\text{mA}$$

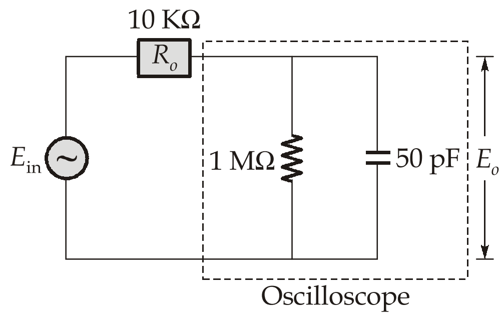
Hence, total current $i(t)$, by superposition theorem

$$i(t) = i_{dc}(t) + i_{ac}(t)$$

$$i(t) = (3.7 + 12.2\sin 100t)\text{ mA}$$

Q.4 (a) Solution:

Equivalent circuit for the system is shown as:



(a) When $f = 100$ kHz,

Value of capacitive reactance at 100 kHz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 100 \times 10^3 \times 50 \times 10^{-12}}$$

$$= 31.8309 \times 10^3\ \Omega$$

Net impedance of the oscilloscope

$$Z_L = \frac{R(-jX_C)}{R - jX_C} = \frac{10^6 \times (-31.8309 j \times 10^3)}{10^6 - 31.8309 \times 10^3 j}$$

$$= 31.814 \times 10^3 \angle -88.176$$

Voltage across the oscilloscope = E_o

$$E_o = E_{in} \times \frac{Z_L}{R_o + Z_L}$$

$$= 1.0\angle 0° \times \frac{31.814 \times 10^3 \angle -88.176}{10 \times 10^3 + 31.814 \times 10^3 \angle -88.176}$$

$$= 0.9454\angle -17.27°\ \text{V (peak)}$$

$$\text{The error in voltage} = \frac{0.9454 - 1}{1} \times 100 = -5.46\%$$

The voltage under loaded conditions lags the voltage under open circuit conditions by an angle of 17.217°.

(b) When $f = 1$ MHz

Value of capacitive reactance = X_C

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 10^6 \times 50 \times 10^{-12}} = 3.183 \times 10^3\ \Omega$$

Net impedance across the oscilloscope,

$$Z_L = \frac{R(-jX_C)}{R - jX_C} = \frac{10^6 \times (-3.183 \times 10^3 j)}{10^6 - (3.183 \times 10^3 j)}$$

$$= 3.1829 \times 10^3 \angle -89.1817\ \Omega$$

Voltage across the oscilloscope

$$= E_{\text{in}} \times \frac{Z_L}{R_o + Z_L}$$

$$= 1.0\angle 0° \times \frac{3.1829 \times 10^3 \angle -89.18°}{10 \times 10^3 + 3.1829 \times 10^3 \angle -89.18}$$

$$= 0.303\angle -71.54°\ \text{V (peak)}$$

$$\text{The error in voltage} = \frac{0.303 - 1}{1} \times 100 = -69.7\%$$

Here, voltage under loaded condition lags by 71.5°.

From (i) and (ii) part, we can see that error in voltage increases with increase in frequency. This indicates the effect of distortion of signal on account of increased shunting effect due to increase in frequency.

Q.4 (b) (i) Solution:

The resolution of a $3\frac{1}{2}$ digit voltmeter on 10 V range is $10^{-3} \times 10 = 10^{-2} = 0.01$. So, the error due to ±1 digit on 10 V range is ±0.01 V.

1. When the instrument is reading 5 V on the 10 V range,

$$\text{error} = (\pm 0.5\% \text{ of } 5\text{ V}) \pm 0.01\text{ V}$$

$$= \pm\left(\frac{0.5}{100} \times 5\right) \pm 0.01\text{ V} = \pm 0.035\text{ V}$$

2. When the instrument is reading 0.1 V on 10 V range,

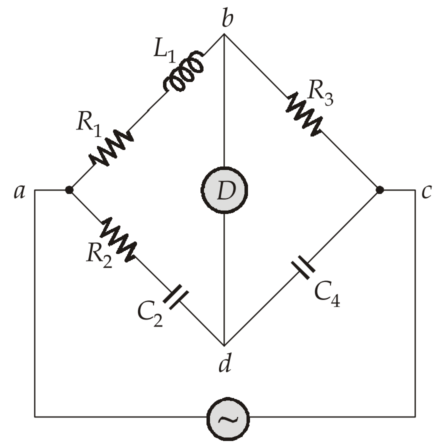
$$\text{error} = (\pm 0.5\% \text{ of } 0.1\text{ V}) \pm 0.01\text{ V}$$

$$= \pm\left(\frac{0.5}{100} \times 0.1\right) \pm 0.01\text{ V} = \pm 0.0105\text{ V}$$

$$\%\text{error} = \frac{0.0105}{0.1} \times 100 = 10.5\%$$

Q.4 (b) (ii) Solution:

Diagram from the given data



$$Z_1 = R_1 + j\omega L_1, Z_3 = R_3$$

$$Z_2 = R_2 - \frac{j}{\omega C_2}, Z_4 = \frac{-j}{\omega C_4}$$

At balance condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\left(\frac{-j}{\omega C_4}\right) = R_3\left(R_2 - \frac{j}{\omega C_2}\right)$$

$$\frac{-jR_1}{\omega C_4} + \frac{L_1}{C_4} = R_2 R_3 - \frac{jR_3}{\omega C_2}$$

On equating Real and Imaginary parts,

$$\frac{L_1}{C_4} = R_2 R_3$$

$$L_1 = R_2 R_3 C_4$$

$$\frac{R_1}{\omega C_4} = \frac{R_3}{\omega C_2}$$

$$R_1 = R_3\left(\frac{C_4}{C_2}\right)$$

Given:

$$R_2 = 834\ \Omega, R_3 = 100\ \Omega$$

$$C_2 = 0.124\ \mu F, C_4 = 0.1\ \mu F, f = 2\ kHz$$

$$L_1 = R_2 R_3 C_4 = 834 \times 100 \times 0.1 \times 10^{-6}$$

$$L_1 = 8.34\ mH$$

$$R_1 = R_3\left(\frac{C_4}{C_2}\right) = 100\left(\frac{0.1}{0.124}\right) = 80.65\ \Omega$$

$$R_1 = 80.65\ \Omega$$

Impedance of specimen,

$$Z_1 = R_1 + j\omega L_1 = 80.65 + j2\pi \times 2 \times 10^3 \times 8.34 \times 10^{-3}$$

$$= 80.65 + j104.8\ \Omega$$

$$Z_1 = \sqrt{(80.65)^2 + (104.8)^2}$$

$$Z_1 = 132.24\ \Omega$$

Q.4 (c) Solution:

The characteristic equation of the matrix A is

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} = 0$$

$$(6 - \lambda)[9 + \lambda^2 - 6\lambda - 1] + 2[-6 + 2\lambda + 2] + 2[2 - 6 + 2\lambda] = 0$$

$$(6 - \lambda)(\lambda^2 - 6\lambda + 8) - 8 + 4\lambda - 8 + 4\lambda = 0$$

$$6\lambda^2 - 36\lambda + 48 - \lambda^3 + 6\lambda^2 - 8\lambda - 16 + 8\lambda = 0$$

$$-\lambda^3 + 12\lambda^2 - 36\lambda + 32 = 0$$

$$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$(\lambda - 2)^2 (\lambda - 8) = 0$$

$$\lambda = 2, 2, 8$$

Eigen vector for $\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_1 + R_2$ and $R_3 \to R_2 + R_3$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 - x_2 + x_3 = 0$$

This equation is satisfied by

$$x_1 = 0, \quad x_2 = 1, \quad x_3 = 1$$

$$X_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

and again

$$x_1 = 1, \quad x_2 = 3, \quad x_3 = 1$$

$$X_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Eigen vector for $\lambda = 8$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - 2x_2 + 2x_3 = 0$$

$$-2x_1 - 5x_2 - x_3 = 0$$

$$\frac{x_1}{2+10} = \frac{x_2}{-4-2} = \frac{x_3}{10-4}$$

$$\frac{x_1}{12} = \frac{x_2}{-6} = \frac{x_3}{6}$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$X_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore$

$$P = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix}, \qquad P^{-1} = \frac{-1}{6}\begin{bmatrix} 4 & 1 & -7 \\ -2 & -2 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Now,

$$P^{-1}AP = \frac{-1}{6}\begin{bmatrix} 4 & 1 & -7 \\ -2 & -2 & 2 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Section-B

Q.5 (a) Solution:

Energy consumed in one minute with rated current and 0.8 p.f. lagging

$$= 240\times10\times0.8\times\frac{1}{60}\times10^{-3} = 0.032 \text{ kWh}$$

∴ Revolutions made in one minute

$$= 0.032 \times 600 = 19.2$$

When lag adjustment is altered:

$$\text{Steady speed, } N = KVI \sin(\Delta-\phi)$$

If the lag adjustment is correctly done,

$$\Delta = 90°$$

Under this condition, steady speed

$$N = KVI \sin(90° - \phi) = KVI \cos\phi$$

∴ Error introduced because of incorrect lag adjustment

$$= \frac{KVI[\sin(\Delta-\phi)-\cos\phi]}{KVI\cos\phi}\times100$$

We have, $\Delta = 86°$

(i) At unity power factor, $\phi = 0$

$$\therefore \quad \text{Error} = \frac{\sin(86°-0°)-1}{1}\times100 = -0.24\%$$

(ii) At 0.5 power factor lagging,

$$\phi = 60°$$

$$\therefore \quad \text{Error} = \frac{\sin(86°-60°)-\cos60°}{\cos60°}\times100 = -12.3\%$$

Comment: Thus it is evident that at low power factors, the errors due to voltage flux not being in quadrature with applied voltage is very serious even though the phase displacement differs from 90° by a small value of 4°.

Q.5 (b) Solution:

Let,
$$A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$$

On transposing A, we have

$$A' = \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix}$$

It A is orthogonal, then $AA' = I$

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}\begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equating the corresponding elements, we have

$$4\beta^2 + \gamma^2 = 1$$

$$2\beta^2 - \gamma^2 = 0$$

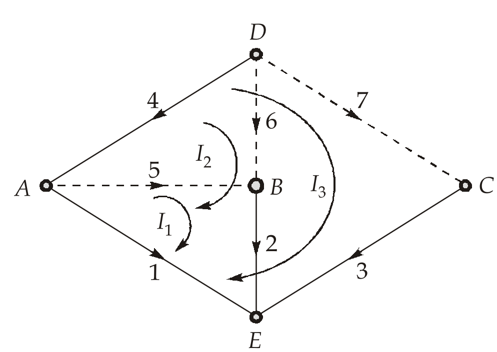
$$\beta = \pm\frac{1}{\sqrt{6}}, \quad \gamma = \pm\frac{1}{\sqrt{3}}$$

But $\alpha^2 + \beta^2 + \gamma^2 = 1$

as $\beta = \pm\frac{1}{\sqrt{6}}, \quad \gamma = \pm\frac{1}{\sqrt{3}}, \quad \alpha = \pm\frac{1}{\sqrt{2}}$

Q.5 (c) Solution:

The tree is arbitrarily selected, which is shown in figure below with branches 4-1-2-3. Thus the twigs are these branches while the links are dotted lines.



Tie-set-1 (loop current I_1): Formed by twigs 1 and 2 with link 5.

Tie-set-2 (loop current I_2): Formed by twigs 1, 2 and 4 with link 6.

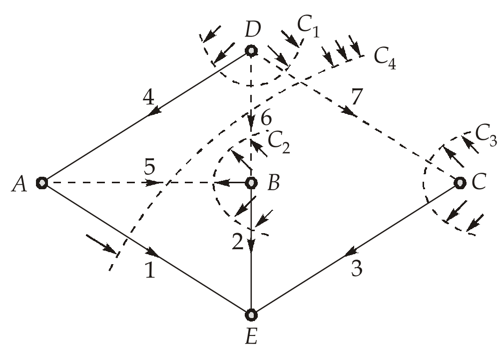
Tie-set-3 (loop current I_3): Formed by twigs 3, 1, 4 and link 7.

[Each fundamental loop contains only one link]

The Tie-set matrix is shown below:

Loop currents	Branches						
	1	2	3	4	5	6	7
I_1	–1	1	0	0	1	0	0
I_2	–1	1	0	–1	0	1	0
I_3	–1	0	1	–1	0	0	1

To obtain the cut-set matrix, the graph is redrawn with twigs in bold and links in dotted lines as shown in figure below.



Cut-sets are formed by taking one twig at a time.

C_1, Cut-set-1: Twig 4, links 6 and 7

C_2, Cut-set-2: Twig 2, links 5 and 6

C_3, Cut-set-3: Twig 3, links 7

C_4, Cut-set-4: Twig 1, links 5, 6, 7.

[Note that total number of fundamental cut-sets = Number of nodes –1 = 5 – 1 = 4]

The necessary cut-set matrix is shown below:

Cut-Sets	Branches						
	1	2	3	4	5	6	7
C_1	0	0	0	1	0	1	1
C_2	0	1	0	0	–1	–1	0
C_3	0	0	1	0	0	0	–1
C_4	1	0	0	0	1	1	1

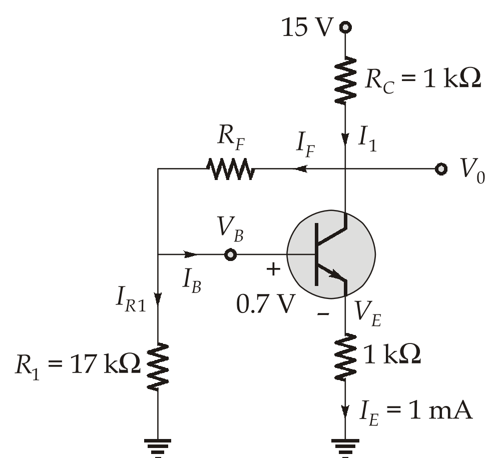
Q.5 (d) Solution:

(i) Consider the transistor is inactive region,

$$I_C = \alpha I_E,$$

$$= \frac{\beta}{(\beta+1)} I_E$$

$$I_C = \frac{99}{(99+1)} \times 1\text{ mA} = 0.99\text{ mA}$$



$$V_{BE} = 0.7\text{ V}$$

$$V_E = I_E \times 1\text{ k}\Omega = 1\text{ V}$$

$$V_B - V_E = 0.7$$

$$V_B = 0.7 + 1 = 1.7\text{ V}$$

$$I_{R1} = \frac{V_B}{17\text{ k}\Omega} = \frac{1.7}{17\text{ k}\Omega} = 100\ \mu\text{A}$$

$$I_B = \frac{I_E}{\beta+1} = \frac{1\text{ mA}}{(99+1)} = 10\ \mu\text{A}$$

By writing KCL at base,

$$I_{RF} = I_B + I_{R1}$$

$$= 10 + 100 = 110\ \mu\text{A}$$

$$I_1 = I_C + I_F$$

$$= 0.99\text{ mA} + 110\ \mu\text{A} = 1.1\text{ mA}$$

(ii)

$$V_0 = 15 - I_1 R_C = 15 - (1.1\text{ mA})(1\text{ k}\Omega)$$

$$= 13.9\text{ V}$$

(iii) Current in R_F,

$$I_F = \frac{V_0 - V_B}{R_E}$$

$$0.11 \text{ mA} = \frac{13.9 - 1.7}{R_F}$$

$$R_F = 110.9 \text{ k}\Omega$$

Q.5 (e) Solution:

Comparison of Crystalline Allotropes of Carbon : Graphite and Diamond

	Graphite	**Diamond**
Structural Property	Covalent and Vander Waal bonding within layers is present. Hexagonal unit cell.	Covalently bonded network. Diamond crystal structure.
Electrical Property	It is good electrical conductor.	It is very good electrical insulator.
Thermal Property	Thermal conductivity comparable to metal.	Excellent thermal conductor about five times that of silver or copper.
Mechanical Property	It has good lubricating property. It is stable at atmospheric pressure.	It is hardest material and a High pressure allotrope.
Applications	Graphite is used in Metallurgical crucibles, welding electrodes, heating elements, electrical contacts, refractory applications	Diamond is used in Cutting tool applications, diamond film coated drills, blades, bearing, jewellery etc.

Carbon Nanotube : It is a very thin filament like carbon molecule whose diameter is in nanometer range but length can be quite long. e.g., 10 to 100 micron depending on how it is grown or prepared.

Carbon nanotube is made by rolling a graphite sheet into a tube and then capping the ends with hemispherical buckminister fullerence molecules. The high aspect ratio (length/diameter) of the CNT makes it an efficient electron emitter. CNTs have high thermal conductivity and high electrical conductivity. CNTs has very high tensile strength. CNTs are very elastic and flexible in nature.

Following are the applications areas of carbon nanotubes :

- CNTs field emission
- CNTs thermal conductivity
- CNTs energy storage
- Molecular electronics using CNTs

- CNTs fibre and fabrics
- CNTs biomedical application
- CNTs air and water filteration
- CNT catalyst support

Q.6 (a) Solution:

Here, we have

$$\lambda x^2 - \mu yz = (\lambda + 2)x \quad \text{...(i)}$$

$$4x^2y + z^3 = 4 \quad \text{...(ii)}$$

Normal to the surface (i),

$$= \nabla[\lambda x^2 - \mu yz - (\lambda + 2)x]$$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\left[\lambda x^2 - \mu yz - (\lambda + 2)x\right]$$

$$= \hat{i}(2\lambda x - \lambda - 2) + \hat{j}(-\mu z) + \hat{k}(-\mu y)$$

Normal at (1, –1, 2)

$$= \hat{i}(2\lambda - \lambda - 2) - \hat{j}(-2\mu) + \hat{k}\mu$$

$$= \hat{i}(\lambda - 2) + \hat{j}z(2\mu) + \hat{k}\mu \quad \text{...(iii)}$$

Normal at the surface (2),

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(4x^2y + z^3 - 4\right)$$

$$= \hat{i}(8xy) + \hat{j}(4x^2) + \hat{k}(3z^2)$$

Normal at the point (1, –1, 2)

$$= -8\hat{i} + 4\hat{j} + 12\hat{k} \quad \text{...(iv)}$$

Since (iii) and (iv) are orthogonal so

$$\left[\hat{i}(\lambda - 2) + \hat{j}(2\mu) + \hat{k}\mu\right]\cdot\left[-8\hat{i} + 4\hat{j} + 12\hat{k}\right] = 0$$

$$-8(\lambda - 2) + 4(2\mu) + 12\mu = 0$$

$$-8\lambda + 16 + 8\mu + 12\mu = 0$$

$$-8\lambda + 20 + 16 = 0$$

$$2\lambda - 5\mu = 4 \quad \text{...(v)}$$

Point (1, –1, 2) will satisfy (i),

$$\lambda(1)^2 - \mu(-1)(2) = (\lambda + 2)(1)$$

$$\lambda + 2\mu = \lambda + 2$$

$$\mu = 1$$

Put $\mu = 1$ in equation (v), we get

$$2\lambda - 5 = 4$$

$$\lambda = \frac{9}{2}$$

Hence $\lambda = \frac{9}{2}$ and $\mu = 1$.

Q.6 (b) Solution:

Pipeline hazards are the situation that prevent the next instruction in the instruction stream from executing during its designated clock cycles. Any condition that causes a stall in the pipeline operations can be called a hazard.

There are primarily three types of hazards:

- **Data hazards:** A data hazard is any condition in which either the source or the destination operands of an instruction are not available at the time expected in the pipeline. As a result of which some operations has to be delayed and the pipeline stalls.
- **Structural Hazards:** This situation arises mainly when two instruction require a given hardware resource at the same time and hence for one of the instructions the pipeline needs to be stalled.
- **Control hazard:** The instruction fetch unit of the CPU is responsible for providing a stream of instructions to the execution unit. The instructions fetched by the fetch unit are in consecutive memory locations and they are executed. However the problem aries when one of the instructions is a branching instruction to some other memory location. Thus all the instructions fetched in the pipeline from consecutive memory locations are invalid now and need to be removed (Also called flushing of the pipeline). This induced a stall till new instructions are again fetched from the memory address specified in branch instruction. Thus the time lost as result of this is called a branch penalty. Often dedicated hardware is incorporated in the fetch unit to identify branch instructions and compute branch addresses as soon as possible and reducing the resulting delay as a result.

Q.6 (c) Solution:

Given, Nominal ratio, $K_n = \frac{6900}{115} = 60$

Primary winding turns, $N_D = 22500$

Secondary winding turns, $N_S = 375$

Turn ratio, $n = \frac{22500}{375} = 60$

No load current, $I = 0.005$ A

No load power factor, $\cos 73° = 0.292$

$\sin 73° = 0.96$

Primary power factor, $\cos 53° = 0.60$

$\sin 53° = 0.798$

Now taking primary voltage,

$$V_P = 6900 + j0$$

$$I_P = 0.0125\,(0.6 - j0.798)$$

$$= 0.0075 - j0.01$$

$$I_0 = 0.005(0.29 - j(0.96))$$

$$= 0.0014 - j0.0048$$

Phase $\frac{I_s}{n}$ is the phasor difference of I_P and I_0

$$\frac{I_s}{n} = 0.0075 - j0.01 - (0.0014 - j0.0048)$$

$$= 0.0061 - j0.0052$$

$$I_s\text{ (reversed)} = n \times \frac{I_s}{n} = 60 \times (0.0061 - j0.0052)$$

$$= 0.366 - j0.312$$

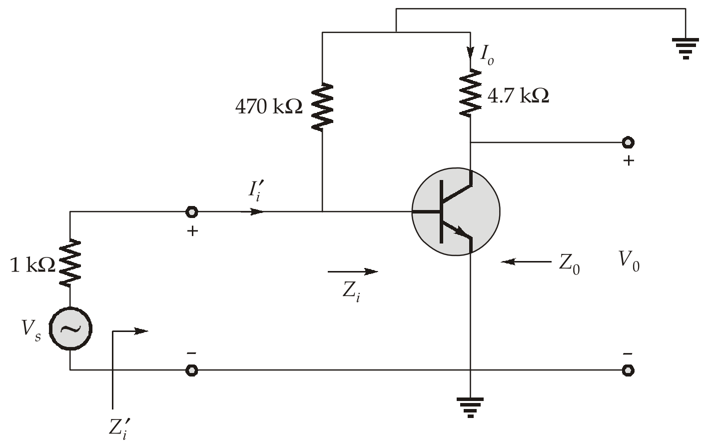
$$I_s = -(0.366 - j0.312)$$

$$= -0.366 + j0.312$$

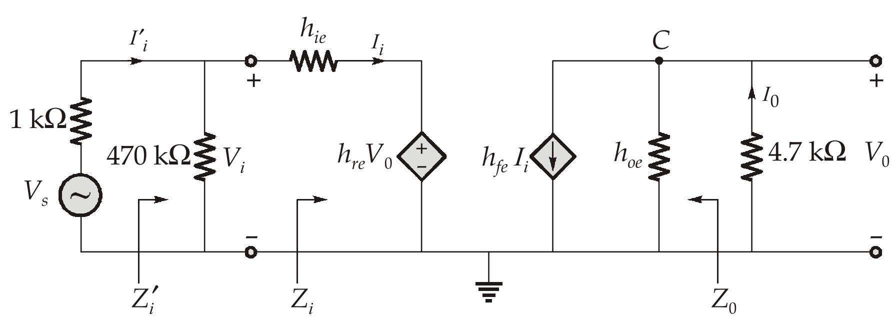
$$I_s = 0.48 \text{ A}$$

Q.7 (a) Solution:

Drawing the AC equivalent circuit,



Substituting h-parameter equivalent circuit:



Current gain (A_i):

$$I_o = \frac{+h_{fe}\, I_i \dfrac{1}{h_{oe}}}{\dfrac{1}{h_{oe}} + 4.7\text{ k}\Omega}$$

$$\Rightarrow \quad \frac{I_o}{I_i} = A_i = \frac{+h_{fe}}{1 + h_{oe} \times 4.7\text{ k}\Omega} = \frac{+110}{1 + 20 \times 10^{-6} \times 4.7 \times 10^3}$$

$$\Rightarrow \quad A_i = \frac{110}{1 + 0.094} = 100.548$$

Input impedance (Z_i):

Applying KVL in input loop,

$$V_i = h_{ie}\, I_i + h_{re}\, V_o = h_{ie}\, I_i - h_{re} \times (I_o \times 4.7\text{ k}\Omega)$$

$$= h_{ie}\, I_i - h_{re}\, (+4.7\text{ k}\Omega) \cdot A_i \cdot I_i$$

$$\Rightarrow \quad \frac{V_i}{I_i} = Z_i = h_{ie} - h_{re} \times 4.7\text{ k}\Omega \times A_i$$

$$= 1600 - 2 \times 10^{-4} \times 4.7 \times 10^3 \times 100.548$$

$$Z_i = 1.5 \text{ k}\Omega$$

Voltage gain (A_v):

$$A_v = \frac{V_o}{V_i} = \frac{I_o \times 4.7\text{ k}}{V_i} = \frac{A_i I_i \times 4.7\text{ k}}{V_i}$$

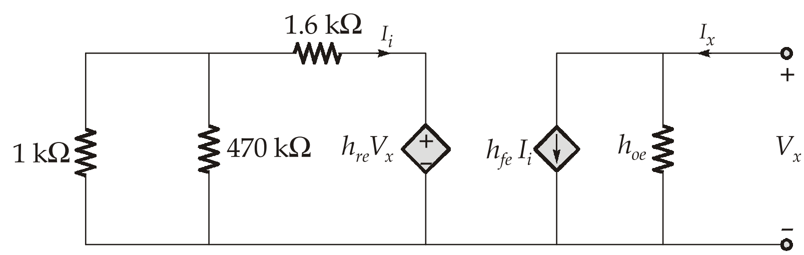
$$= \frac{A_i \times 4.7}{Z_i} = \frac{100.548 \times 4.7}{1.5} \simeq 315$$

Output impedance (Z_o):

Step-1: Disable external sources present in the input.

Step-2: Disconnect load resistance R_L from output circuit.

Step-3: Assume that voltage V_x is applied at output port and current I_x is flowing into output node then R_o is calculated as V_x/I_x.



KCL at output node,

$$I_x = h_{oe} V_x + h_{fe} I_i$$

$$\Rightarrow \quad \frac{I_x}{V_x} = h_{oe} + h_{fe} \cdot \frac{I_i}{V_x}$$

$$\frac{1}{Z_o} = \frac{I_x}{V_x} = h_{oe} + h_{fe} \cdot \frac{I_i}{V_x} \quad \text{...(i)}$$

KVL in input loop,

$I_i(1 \text{ k}\Omega \parallel 470 \text{ k}\Omega) + 1.6 \text{ k}\Omega\, I_i + h_{re}V_x = 0$

$$\Rightarrow \quad 0.9978\, I_i + 1.6\, I_i = h_{re} V_x$$

$$\Rightarrow \quad \frac{I_i}{V_x} = \frac{h_{re}}{2.59 \text{ k}\Omega} = \frac{2 \times 10^{-4}}{2.59 \times 10^3}$$

Now,

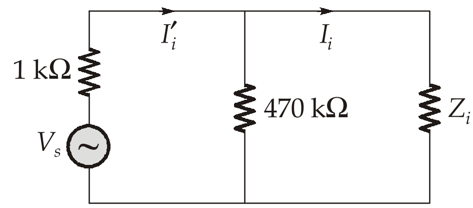
$$\frac{1}{Z_o} = 20 \times 10^{-6} + 110 \times \frac{2 \times 10^{-4}}{2.56 \times 10^3}$$

$$\frac{1}{Z_o} = 2.85 \times 10^{-5}$$

$$\therefore \quad Z_o \simeq 35 \text{ k}\Omega$$

Z_i':

$$Z_i' = 470 \text{ k}\Omega \parallel Z_i = \frac{470 \times 1.5}{470 + 1.5} = 1.495 \text{ k}\Omega$$



A_i':

$$A_i' = \frac{I_o}{I_i'} = \frac{I_o}{I_i} \cdot \frac{I_i}{I_i'} = A_i \cdot \frac{I_i}{I_i'}$$

$$I_i = I_i' \cdot \frac{470\text{ k}\Omega}{Z_i + 470\text{ k}\Omega}$$

$$\Rightarrow \quad \frac{I_i}{I_i'} = \frac{470\text{ k}\Omega}{1.5\text{ k}\Omega + 470\text{ k}\Omega} = 0.9968$$

$$\therefore \quad A_i' = 100.548 \times 0.9968 = 100.226$$

Q.7 (b) Solution:

Given,

$$\text{Electric field wave, } E = \left[\hat{a}_x + E_y\hat{a}_y + (2 + j5)\hat{a}_z\right]e^{-j2.3(-0.6x+0.8y)}$$

(i) The general form of uniform place E_m wave propagates in an arbitrary direction is given as,

$$E = E_m e^{-j\beta r}$$

where, E_m has components in the X, Y and Z direction

$$\beta r = \beta_x x + \beta_y y + \beta_z z$$

$$= \beta\left(\cos\theta_x x + \cos\theta_y y + \cos\theta_z z\right)$$

Where as, given, $\beta = 2.3$

$\therefore$ $\beta r = 2.3\,(-0.6x + 0.8y + 0)$

the direction of propagation,

$$n_\beta = -0.6\hat{a}_x + 0.8\hat{a}_y$$

Since, the electric field, E must be perpendicular to the direction of propagation n_β, therefore it must satisfy $n_\beta \cdot E = 0$ relation

$$\therefore \left(-0.6\hat{a}_x + 0.8\hat{a}_y\right)\cdot\left[\hat{a}_x + E_y\hat{a}_y + (2 + j5)\hat{a}_z\right] = 0$$

$$-0.6 + 0.8\,E_y = 0$$

$$\therefore \quad E_y = 0.75$$

(ii) From the Maxwell's equation,

$$\therefore \quad \bar{H} = \frac{1}{\eta} n_\beta \times \bar{E} \qquad \left(\because \frac{E}{H} = \eta\right)$$

where, n_β is the propagation direction of E

$$\bar{H} = \frac{1}{\eta} n_B \times \bar{E} = \frac{1}{377}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -0.6 & 0.8 & 0 \\ 1 & 0.75 & 2+j5 \end{vmatrix} \qquad \left(\therefore \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi = 377\right)$$

$$\therefore \quad \bar{H} = \frac{1}{\eta}\left[0.8(2+j5)\hat{a}_x + \hat{a}_y(0.6(2+j5) + \hat{a}_z(-0.6\times 0.75 - 0.8)\right]$$

$$= \left[(4.24 + j10.6)\times 10^{-3}\hat{a}_x + (3.18 - j7.95)\times 10^{-3}\hat{a}_y - 3.31\times 10^{-3}\hat{a}_z\right]$$

∴ The magnetic field,

$$= \left[(4.24 + j10.6)\times 10^{-3}\hat{a}_x + (3.18 - j7.95)\times 10^{-3}\hat{a}_y - 3.31\times 10^{-3}\hat{a}_z\right] e^{-j2.3(-0.6x+0.8y)}$$

(iii) The wavelength,

$$\beta = \frac{2\pi}{\lambda}$$

$$\therefore \quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.3} = 2.73\,\text{m}$$

The frequency, $f = \dfrac{C}{\lambda} = \dfrac{3\times 10^8}{2.73} = 0.11\,\text{GHz}$

Q.7 (c) (i) Solution:

Given,

$$f = 14\,\text{GHz}$$

$$\mu_r = 1$$

$$\vec{E} = E_A e^{j(\omega t - 280\pi)}\hat{a}_z \text{ V/m}$$

$$\vec{H} = 3e^{j(\omega t - 280\pi)}\hat{a}_x \text{ A/m}$$

$$V = 3 \times 10^8 \text{ m/sec}$$

Phase constant, $\beta = 280\pi = \dfrac{2\pi}{\lambda}$

$$\therefore \quad \lambda = \frac{1}{140}\,\text{m}$$

$$v = f\lambda$$

$$v = 14\times10^9 \times \frac{1}{140} = 10^8 \text{ m/sec}$$

but $$v = \frac{C}{\sqrt{\mu_r \in_r}}$$

$$10^8 = \frac{3\times10^8}{\sqrt{\in_r}}$$

$\therefore$ $$\sqrt{\in_r} = 3$$

$\Rightarrow$ $$\in_r = 9$$

We know that, $$\frac{E_A}{H_A} = \eta = \sqrt{\frac{\mu_0\mu_r}{\in_0\in_r}}$$

$$\frac{E_A}{3} = \sqrt{\frac{\mu_0}{\in_0}}\cdot\frac{1}{\sqrt{\in_r}} = 120\pi\times\frac{1}{3}$$

$\therefore$ $$E_A = 3\times120\pi\times\frac{1}{3}$$

$\therefore$ E-field, $$E = 120\pi\, e^{j(\omega t-280\pi)}\hat{a}_z \text{ V/m}$$

Q.7 (c) (ii) Solution:

For the toroidal solenoid,

The average flux density is given by,

$$B = \frac{\mu_0 NI}{l_m}$$

Where, l_m = mean length of path of flux

$$l_m = 2\pi\, R_m$$

Where R_m is mean diameter of toroid,

$$B = \frac{\mu_0 NI}{2\pi R_m}$$

$$\lambda = N\phi = NBA$$

$$= \left(\frac{\mu_0 NI}{2\pi R_m}\right)N\pi r^2 = \frac{\mu_0 N^2 I\pi r^2}{2\pi R_m} = \frac{\mu_0 N^2 r^2 I}{2R_m}$$

$$L = \frac{\lambda}{I} = \frac{\mu_0 N^2 r^2}{2R_m}$$

Using above relation,

$$\mu_0 = 4\pi \times 10^{-7}$$
$$N = 1000$$
$$\text{radius, } r = \left(\frac{2}{10^2}\right)m$$

$$\therefore \quad L = \frac{4\pi \times 10^{-7} \times 800 \times (1000)^2}{2 \times \frac{20}{100}} \times \left(\frac{2}{10^2}\right)^2$$

$$= \frac{4\pi \times 10^{-7} \times 800 \times 1000 \times 1000}{2 \times \frac{20}{100}} \times \frac{2}{100} \times \frac{2}{100}$$

$$= \frac{4\pi \times 10^{-7} \times 8 \times 10^7 \times 1}{100} = 4\pi \times 10^{-7} \times 8 \times 10^5$$

$$= 4\pi \times 8 \times 10^{-2} = 1 \text{ H}$$

For air core, $$L = \frac{4\pi \times 10^{-7} \times (1000)^2}{2 \cdot \frac{20}{100}} \times \left(\frac{2}{100}\right)^2 \approx 1.25 \text{ mH}$$

Q.8 (a) Solution:

(i) Let, Z_1 = Impedance of the 2-mF capacitor

$$= \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

Z_2 = Impedance of the 3 Ω resistor in series with the 10 mF capacitor

$$= 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

Z_3 = Impedance of the 0.2 H inductor in series with the 8 Ω resistor

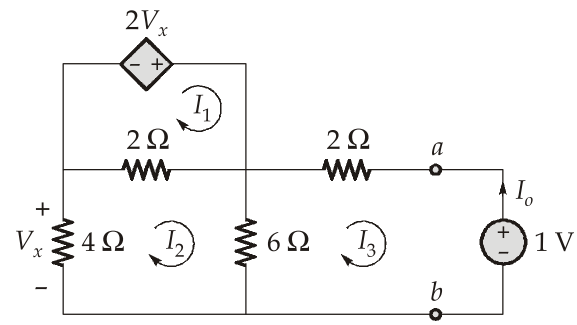
$$= 8 + j50 \times 0.2$$
$$= (8 + j10)\ \Omega$$

The input impedance is

$$Z_{in} = Z_1 + Z_2 \| Z_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8} = (3.22 - j11.07)\ \Omega$$

(ii) To find R_{Th}, we set the independent source equal to zero but leave the dependent source.

Applying 1 V voltage source at terminal *a-b*.



Applying mesh analysis to loop 1

$$-2V_x + 2(I_1 - I_2) = 0$$

$$V_x = I_1 - I_2$$

But, $-4I_2 = V_x = I_1 - I_2$

Hence, $I_1 = -3I_2$ ...(i)

For loops 2 and 3 applying KVL

$$4I_2 + 2(I_2 - I_1) + 6(I_2 - I_3) = 0 \quad \text{...(ii)}$$

$$6(I_3 - I_2) + 2I_3 + 1 = 0 \quad \text{...(iii)}$$

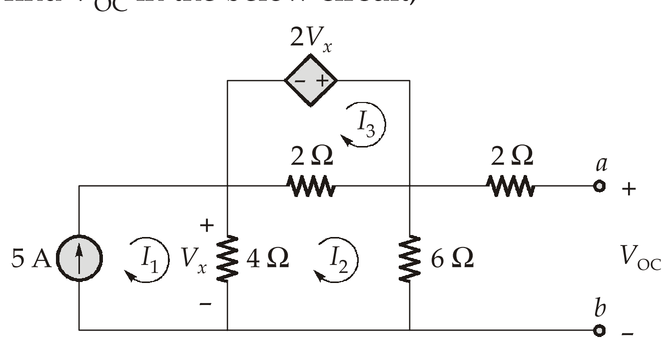
Solving these equations

$$I_3 = -\frac{1}{6}\text{ A}$$

But, $I_o = -I_3 = \frac{1}{6}\text{ A}$

Hence, $R_{Th} = \frac{1\text{ V}}{I_o} = 6\ \Omega$

To get V_{Th}, we find V_{OC} in the below circuit,



$$I_1 = 5\text{ A} \quad \text{...(iv)}$$

$$-2V_x + 2(I_3 - I_2) = 0$$

$$V_x = I_3 - I_2 \quad \text{...(v)}$$

$$4(I_2 - I_1) + 2(I_2 - I_3) + 6I_2 = 0$$

$$12I_2 - 4I_1 - 2I_3 = 0 \quad \text{...(vi)}$$

But, $4(I_1 - I_2) = V_x$

$$4I_1 - 4I_2 = I_3 - I_2$$

$$4I_1 - 3I_2 - I_3 = 0 \qquad \text{...(vii)}$$

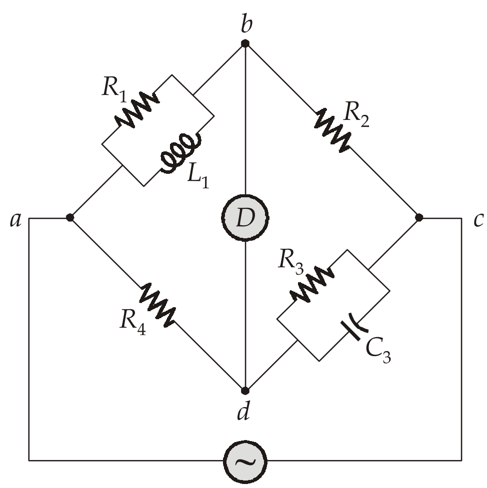
Solving equations (iv), (vi) and (vii)

We get, $I_2 = \dfrac{10}{3}$

Hence, $V_{Th} = V_{OC} = 6I_2 = 20\text{ V}$

Q.8 (b) Solution:

(i) The circuit as described in question can be drawn as :



Impedances of four arms

$$Z_1 = R_1 \| j\omega L_1 = \frac{j\omega R_1 L_1}{R_1 + j\omega L_1}$$

$$Z_2 = R_2$$

$$Z_3 = R_3 \| \frac{1}{j\omega C_3} = \frac{R_3}{1 + j\omega R_3 C_3}$$

$$Z_4 = R_4$$

At balance condition,

$$Z_1 Z_3 = Z_2 Z_4$$

i.e., $$\frac{j\omega R_1 L_1}{R_1 + j\omega L_1} \times \frac{R_3}{1 + j\omega R_3 C_3} = R_2 R_4$$

$$\frac{j\omega R_1 R_3 L_1}{(R_1 + j\omega L_1)(1 + j\omega R_3 C_3)} = R_2 R_4$$

$$j\omega R_1R_3L_1 = R_2R_4[R_1 + j\omega L_1 + j\omega R_1R_3C_3 - \omega^2L_1R_3C_3]$$

$$j\omega R_1R_3L_1 = (R_1R_2R_4 - \omega^2L_1R_2R_3R_4C_3) + j\omega[R_2R_4L_1 + R_1R_2R_3R_4C_3]$$

On equating real and imaginary part equal to zero,

Real part = 0

$$R_1R_2R_4 - \omega^2L_1R_2R_3R_4C_3 = 0$$

$$R_3 = \frac{R_1}{\omega^2L_1C_3} \quad \text{...(i)}$$

Imaginary part = 0

$$R_1R_3L_1 = L_1R_2R_4 + R_1R_2R_3R_4C_3 \quad \text{...(ii)}$$

$$C_3 = \frac{(R_1R_3L_1 - R_2R_4L_1)}{R_1R_2R_3R_4}$$

$$C_3 = L_1\left[\frac{1}{R_2R_4} - \frac{1}{R_1R_3}\right] \quad \text{...(iii)}$$

Put C_3 in eqn. (i)

$$R_3 = \frac{R_1}{\omega^2L_1 \cdot L_1\left[\frac{1}{R_2R_4} - \frac{1}{R_1R_3}\right]}$$

$$\left[\frac{1}{R_2R_4} - \frac{1}{R_1R_3}\right]R_3 = \frac{R_1}{(\omega L_1)^2}$$

$$\frac{R_3}{R_2R_4} - \frac{1}{R_1} = \frac{R_1}{(\omega L_1)^2}$$

$$R_3 = \frac{R_2R_4}{R_1}\left[\left(\frac{R_1}{\omega L_1}\right)^2 + 1\right]$$

and

$$C_3 = \frac{R_1}{\omega^2L_1\left[\frac{R_2R_4}{R_1}\left\{\left(\frac{R_1}{\omega L_1}\right)^2 + 1\right\}\right]}$$

By putting R_3 in eqn. (iii),

$$C_3 = \frac{R_1^2}{\omega^2L_1R_2R_4\left[1 + \left(\frac{R_1}{\omega L_1}\right)^2\right]}$$

(ii) Given :

$$f = 50 \text{ Hz}, R_1 = 500\ \Omega, L_1 = 0.1 \text{ H},$$

$$R_2 = 100\ \Omega, R_4 = 1000\ \Omega$$

$$R_3 = \frac{100 \times 1000}{500}\left[1 + \left(\frac{500}{2\pi \times 50 \times 0.1}\right)^2\right]$$

$$R_3 = 50.86 \text{ k}\Omega$$

$$C_3 = \frac{R_1}{\omega^2 L_1 R_3} = \frac{500}{(2\pi \times 50)^2 \times 0.1 \times 50.86 \times 10^3}$$

$$= 0.997 \times 10^{-5} \text{ F}$$

$$C_3 = 0.997\ \mu\text{F}$$

Q.8 (c) Solution:

$$n = N_D = 4.5 \times 10^{15}/\text{cm}^3$$

$$P = \frac{n_i^2}{N_D} = \frac{2.25 \times 10^{20}}{4.5 \times 10^{15}} = 5 \times 10^4/\text{cm}^3$$

$$\tau_P = \tau_n = 10^{-6} \text{ sec}$$

$$D_P = V_T\, \mu_P = 0.026 \times 500 = 13 \text{ cm}^2/\text{sec}$$

$$L_P = \sqrt{D_P \tau_P} = \sqrt{13 \times 10^{-6}}$$

$$= 3.6 \times 10^{-13} \text{ cm}$$

$$D_n = V_T\, \mu_n = 0.026 \times 1300 = 33.8 \text{ cm}^2/\text{sec}$$

$$L_N = \sqrt{D_n \tau_n} = 5.81 \times 10^{-3} \text{ cm}$$

$$\Delta n\big|_{x=0} = \Delta P\big|_{x=0} = G_L \tau_P = 10^{21} \times 10^{-6} = 10^{15}/\text{cm}^3$$

Excess hole concentration at any distance x ;

$$\Delta P(x) = 10^{15} e^{\frac{-x}{L_P}} \qquad \text{for } x > 0$$

$$\Delta n(x) = 10^{15} e^{\frac{-x}{L_n}} \qquad \text{for } x > 0$$

Total hole concentration:

$$P(x) = P + \Delta P(x) = P + 10^{15} e^{-\frac{x}{L_P}}$$

Hole diffusion current, $$I_p(x) = -AeD_P \frac{dP}{dx}$$

$$= -10^{-2} \times 1.6 \times 10^{-19} \times 13 \times 10^{15} \left(\frac{1}{L_P}\right) e^{\frac{-x}{L_P}}$$

$$= \frac{10^{-2} \times 1.6 \times 10^{-19} \times 13 \times 10^{15}}{3.6 \times 10^{-3}} e^{\frac{-x}{(3.6 \times 10^{-13})}}$$

$$= 5.77 \times 10^{-3} \times e^{-\frac{34.6 \times 10^{-4}}{3.6 \times 10^{-3}}} = 2.2 \text{ mA}$$

Electron diffusion current,

$$I_n(x) = Ae\Delta_n \frac{dn}{dx}$$

$$= 10^{-2} \times 1.6 \times 10^{-19} \times 33.8 \times 10^5 \times \left(\frac{-1}{L_n}\right) e^{-\frac{x}{L_n}}$$

$$= \frac{10^{-2} \times 1.6 \times 10^{-19} \times 33.8 \times 10^{15} \times e^{\frac{-34.6 \times 10^{-4}}{5.51 \times 10^{-3}}}}{5.81 \times 10^{-3}}$$

$$= -5.13 \text{ mA}$$

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