MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

Mechanical Engineering Test No : 11

Full Syllabus Test (Paper-2)

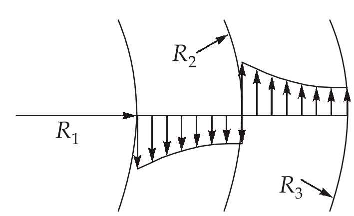
Section : A

1. (a)

Due to shrinkage fitting there will be shrink pressure – external pressure for inner cylinder and internal pressure for outer cylinder. Hoop stress will be compressive for inner cylinder and tensile for outer cylinder.

Let p be the shrinkage pressure.

According to given data



$$R_1 = 5 \text{ cm}$$
$$R_3 = 10 \text{ cm}$$
$$R_2 = ?$$

Hoop stress for inner cylinder

$$\sigma_{hi} = \frac{-pR_2^2}{(R_2^2 - R_1^2)} + \frac{-pR_1^2R_2^2}{(R_2^2 - R_1^2)r^2}$$

$$(\sigma_{hi})_{max} = \frac{-pR_2^2}{(R_2^2 - R_1^2)} + \frac{-pR_1^2R_2^2}{(R_2^2 - R_1^2)R_1^2}$$

$$(\sigma_{hi})_{max} = \frac{-2pR_2^2}{\left(R_2^2 - R_1^2\right)} \quad ...(1)$$

For outer cylinder

$$\sigma_{ho} = \frac{pR_2^2}{\left(R_3^2 - R_2^2\right)} + \frac{pR_2^2 R_3^2}{\left(R_3^2 - R_2^2\right) r^2}$$

$$(\sigma_{ho})_{max} = \frac{p\left(R_2^2 + R_3^2\right)}{\left(R_3^2 - R_2^2\right)} \quad ...(2)$$

As given $\left|(\sigma_{hi})_{max}\right| = \left|(\sigma_{ho})_{max}\right|$

$$\Rightarrow \frac{2pR_2^2}{\left(R_2^2 - R_1^2\right)} = \frac{p\left(R_2^2 + R_3^2\right)}{R_3^2 - R_2^2}$$

$$\Rightarrow \frac{2R_2^2}{R_2^2 - 25} = \frac{\left(R_2^2 + 100\right)}{100 - R_2^2}$$

$$\Rightarrow 200R_2^2 - 2R_2^4 = R_2^4 + 100R_2^2 - 25R_2^2 - 2500$$

$$\Rightarrow 3R_2^4 - 125R_2^2 - 2500 = 0$$

$$\Rightarrow R_2^2 = 56.43$$

$$\Rightarrow R_2 = 7.51 \text{ cm}$$

$$\Rightarrow D_2 = 15.02 \text{ cm}$$

So junction diameter will be 15.02 cm.

1. (b)

Given : $S_f = \sigma_b = 300$ N/mm²; $S_{ut} = 600$ N/mm²; $R = 90\%$

$$k_a = 0.44; k_b = 0.75; k_c = 0.897$$

For steel, $S'_e = 0.5S_{ut} = 0.5 \times 600 = 300 \text{ N/mm}^2$

$$S_e = k_a k_b k_c S'_e$$

$$= 0.44 \times 0.75 \times 0.897 \times 300$$

$$= 88.803 \text{ N/mm}^2$$

$$0.9S_{ut} = 0.9 \times 600 = 540 \text{ N/mm}^2$$

$$\log_{10}(0.9 S_{ut}) = \log_{10}(540) = 2.7324$$

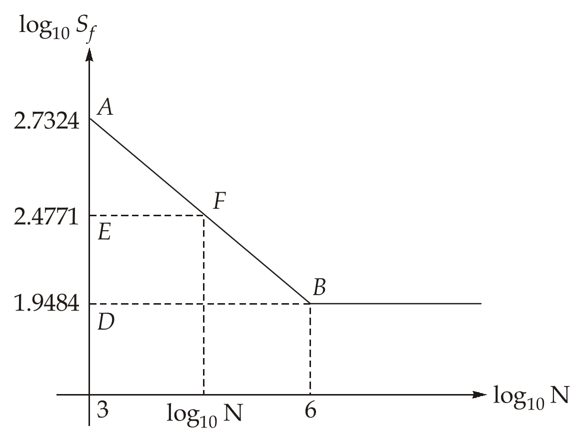
$$\log_{10}(S_e) = \log_{10}(88.803) = 1.9484$$

$$\log_{10}(S_f) = \log_{10}(300) = 2.4771$$

Also, $\log_{10}(10^3) = 3$

and, $\log_{10}(10^6) = 6$

The S-N curve for the bar is shown in the figure below.



As AB is a straight line, so we can write

$$\frac{2.7324 - 1.9484}{3-6} = \frac{2.4771 - 1.9484}{\log_{10} N - 6}$$

$$\Rightarrow \quad \log_{10}N = 6 + \frac{(2.4771 - 1.9484)}{(2.7324 - 1.9484)} \times (3-6)$$

$$\Rightarrow \quad \log_{10}N = 3.97691$$

$$\Rightarrow \quad N = 10^{3.97691}$$

$$\Rightarrow \quad N = 9482.2 \text{ cycles}$$ **Ans.**

Hence, the life of the bar for 90% reliability is 9482.2 cycles.

1. (c)

Mass moment of inertia of the beam,

$$I = \left[\frac{1}{12} \times (160) \times (240)^3 - \frac{\pi}{64} \times (80)^4\right] = 182309380.7 \text{ mm}^4$$

$$\text{Weight of the beam, } w = \left[(160 \times 240) - \frac{\pi}{4} \times 80^2\right] \times 10^{-6} \times 5 \times (12 \times 10^3)$$

$$= 2002.4 \text{ N}$$

The maximum bending moment in the beam is at the mid-span.

$$M_{max} = (M_{mid\ span})_{due\ to\ W} + (M_{mid\ span})_{due\ to\ w}$$

$$= \frac{WL}{4} - \frac{wL^2}{8} = \frac{W \times 5}{4} - \frac{2002.4 \times 5^2}{8}$$

$$= (1.25W - 6257.5) \text{ Nm}$$

Using bending equation, $\frac{M}{I_{NA}} = \frac{\sigma}{y}$

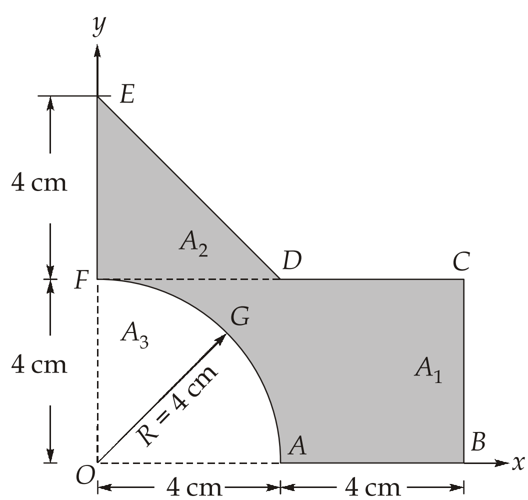
$$\Rightarrow \quad \frac{M_{max}}{I_{NA}} = \frac{\sigma_{per}}{y_{max}}$$

$$\Rightarrow \quad \frac{(1.25W - 6257.5) \times 10^3}{(182309380.7)} = \frac{15}{120}$$

$$\Rightarrow \quad W = 23.2 \text{ kN}$$

Ans.

1. (d)



Dividing the shaded area portion into 3 portions as rectangular area (A_1 : OBCF), a triangular area (A_2 : DEF) and a quadrant of a circular area (A_3 : OAGF).

Areas:

$$A_1 = OB \times CB = 8 \times 4 = 32 \text{ cm}^2$$

$$A_2 = \frac{1}{2} \times EF \times FD = \frac{1}{2} \times 4 \times 4 = 8 \text{ cm}^2$$

$$A_3 = \frac{\pi}{4} \times 4^2 = 4\pi \text{ cm}^2$$

Coordinate of centroids $G_1(4, 2)$, $G_2\left(\frac{4}{3}, \frac{16}{3}\right) \equiv (1.333, 5.333)$

$$G_3 = \frac{4R}{3\pi} = \frac{4 \times 4}{3\pi} = 1.6976 \text{ cm}$$

For the shaded area:

$$\bar{x} = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 - A_3}$$

$$\bar{x} = \frac{(32\times 4)+(8\times 1.333)-(4\pi\times 1.6976)}{(32+8-4\pi)}$$

$$\bar{x} = 4.2769 \text{ cm}$$ **Ans.**

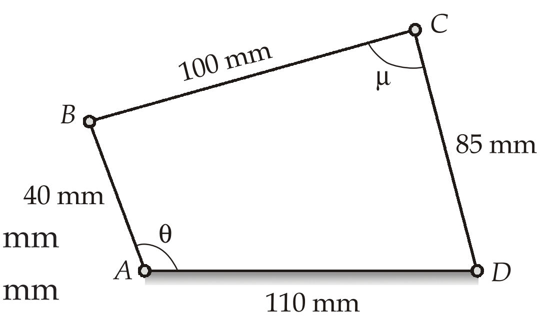
$$\bar{y} = \frac{Ay_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3}$$

$$\bar{y} = \frac{(32\times 2)+(8\times 5.333)-(4\pi\times 1.6976)}{(32+8-4\pi)}$$

$$\bar{y} = 3.1104 \text{ cm}$$

1. (e)

$$AB = s = 40 \text{ mm}$$
$$AD = l = 110 \text{ mm}$$
$$BC = p = 100 \text{ mm}$$
$$CD = q = 85 \text{ mm}$$
$$s + l = 40 + 110 = 150 \text{ mm}$$
$$p + q = 100 + 85 = 185 \text{ mm}$$



Grashoff's law: $s + l < p + q$ is satisfied

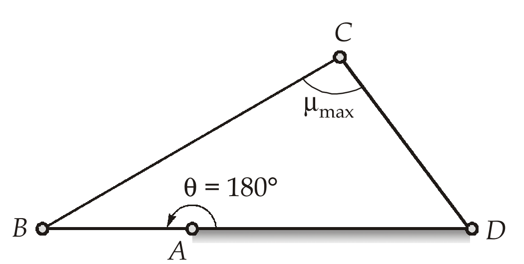
Since the Grashoff's law is satisfied and link adjacent to the shortest link is fixed. So, the mechanism is crank-rocker.

Let, θ = crank angle and μ = transmission angle

For maximum transmission angle,

$$\theta = 180°$$

From ΔBCD,



$$BD^2 = BC^2 + CD^2 - 2 \times BC \times CD \times \cos(\mu_{max})$$

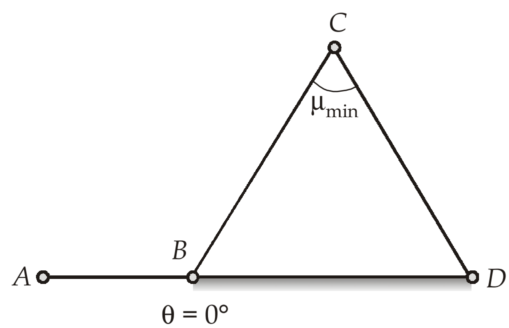
$$\Rightarrow \quad (40 + 110)^2 = (100)^2 + (85)^2 - 2 \times (100) \times 85 \times \cos(\mu_{max})$$

$$\Rightarrow \quad \mu_{max} = 108.077° \qquad \textbf{Ans.}$$

For minimum transmission angle,

$$\theta = 0°$$

From ΔBCD,

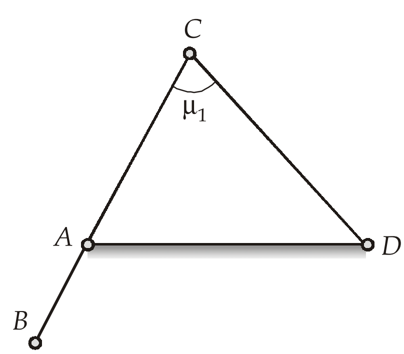


$$BD^2 = BC^2 + CD^2 - 2 \times BC \times CD \times \cos(\mu_{min})$$

$$\Rightarrow \quad (110 - 40)^2 = (100)^2 + (85)^2 - 2 \times (100) \times 85 \times \cos(\mu_{min})$$

$$\Rightarrow \quad \mu_{min} = 43.531° \qquad \textbf{Ans.}$$

The two toggle positions:



From ΔACD;

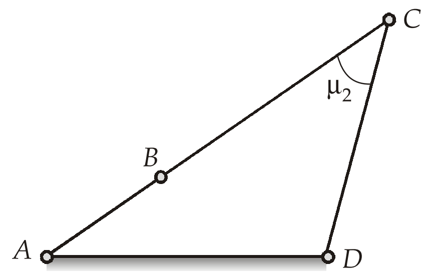
$$AD^2 = AC^2 + CD^2 - 2 \times AC \times CD \times \cos\mu_1$$

$$(110)^2 = (100 - 40)^2 + (85)^2 - 2 \times (100 - 40) \times 85 \times \cos\mu_1$$

$$\Rightarrow \quad \mu_{min} = 97.181° \qquad \textbf{Ans.}$$

Similarly, for other toggle position,

From ΔACD;



$$AD^2 = AC^2 + CD^2 - 2 \times AC \times CD \times \cos\mu_2$$

$$110^2 = (100 + 40)^2 + (85)^2 - 2(100 + 40)(85)(\cos\mu_2)$$

$$\Rightarrow \quad \mu_2 = 51.779°$$ **Ans.**

Hence, Transmission angle corresponding to two toggle positions are 97.181° and 51.779°.

2. (a)

$T_G = 30; T_F = 60; N_F = N_P = 0$

From the figure, $R_F = R_G + 2R_H$

$$\Rightarrow \quad T_F = T_G + 2T_H$$

$$\Rightarrow \quad 60 = 30 + 2T_H$$

$$\Rightarrow \quad T_H = 15$$

Action	C or E	D or G	H	F or P
C is fixed	0	x	$-x\cdot\frac{T_G}{T_H}$	$-x\cdot\frac{T_G}{T_H}\cdot\frac{T_H}{T_F}$
C is free	y	$y + x$	$y - x\cdot\frac{T_G}{T_H}$	$y - x\cdot\frac{T_G}{T_H}\cdot\frac{T_H}{T_F}$

For pointer P remains stationary,

$$N_P = N_F = 0$$

$$\Rightarrow \quad y - x\cdot\frac{T_G}{T_H}\cdot\frac{T_H}{T_F} = 0$$

$$\Rightarrow \quad y - x\cdot\frac{30}{15}\cdot\frac{15}{60} = 0$$

$$\Rightarrow \quad y - \frac{x}{2} = 0$$

$$\Rightarrow \quad y = \frac{x}{2}$$

$$\therefore \quad \frac{T_C}{T_D} = \frac{N_D}{N_C} = \frac{y+x}{y}$$

$$= \frac{\left(\frac{x}{2}\right)+x}{\left(\frac{x}{2}\right)} = \frac{\left(\frac{1}{2}\right)+1}{\left(\frac{1}{2}\right)} = 3$$ **Ans.**

Now, when B and C rotates at different speeds,

$$N_C = y = 120 \text{ rpm}$$

$$N_A = y \cdot \frac{T_C}{T_A}$$

$$N_B = \left(y \cdot \frac{T_C}{T_A}\right) \times 1.08$$

$$N_D = N_B \cdot \frac{T_B}{T_D} = \left(y \cdot \frac{T_C}{T_A}\right) \times 1.08 \times \frac{T_B}{T_D}$$

$$\Rightarrow \quad N_D = 120 \times 1.08 \times \frac{T_C}{T_D} \qquad [\because \quad T_A = T_B]$$

$$= 120 \times 1.08 \times 3 = 388.8 \text{ rpm}$$

or $$y + x = 388.8$$

$$\Rightarrow \quad 120 + x = 388.8$$

$$\Rightarrow \quad x = 268.8 \text{ rpm}$$

$$\therefore \quad N_P = y - x \cdot \frac{T_G}{T_H} \cdot \frac{T_H}{T_F}$$

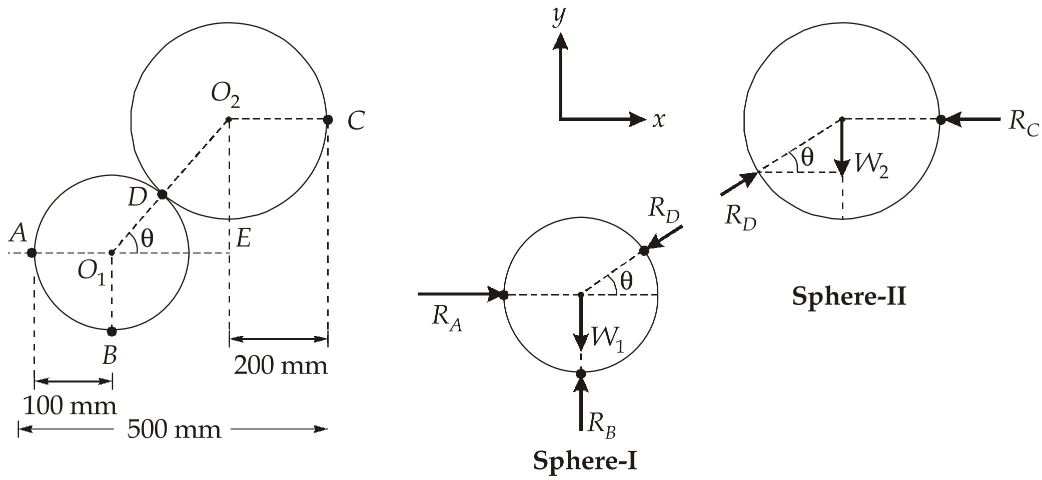
$$= 120 - (268.8) \times \frac{30}{15} \times \frac{15}{60} = -14.4 \text{ rpm}$$ **Ans.**

i.e., P rotates at 14.4 rpm in direction opposite to that of C

© Copyright: MADE EASY

2. (b) (i)

Given : r_1 = 100 mm; r_2 = 200 mm; W_1 = 100 N; W_2 = 200 N



From ΔO_1O_2E; $\cos\theta = \dfrac{O_1E}{r_1 + r_2}$

$$\Rightarrow \qquad \cos\theta = \frac{(500 - 100 - 200)}{100 + 200}$$

$$\Rightarrow \qquad \cos\theta = \frac{2}{3}$$

$$\Rightarrow \qquad \theta = 48.1896° \simeq 48.19°$$

From free-body diagram of sphere II;

$\Sigma F_y = 0;$ $\quad R_D \sin\theta - W_2 = 0$

$$\Rightarrow \qquad R_D = \frac{W_2}{\sin\theta} = \frac{200}{\sin(48.19°)} = 268.327 \text{ N}$$ **Ans.**

$\Sigma F_x = 0;$ $\quad R_D \cos\theta - R_C = 0$

$\Rightarrow \qquad R_C = R_D \cos\theta = 268.327 \times \cos(48.19°) = 178.88$ N **Ans.**

From free-body diagram of sphere-I;

$\Sigma F_y = 0;$ $\quad R_B - W_1 - R_D \sin\theta = 0$

$\Rightarrow \qquad R_B = W_1 + R_D \sin\theta = 100 + 268.327 \times \sin(48.19°)$

$= 300$ N **Ans.**

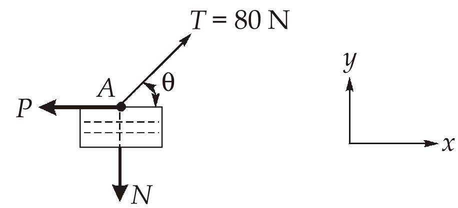
$\Sigma F_x = 0;$ $\quad R_A - R_D \cos\theta = 0$

$\Rightarrow \qquad R_A = R_D \cos\theta = 268.327 \times \cos(48.19°) = 178.88$ N **Ans.**

Hence, the reactions at the point of contact *A*, *B*, *C* and *D* are 178.883 N, 300 N, 178.88 N and 268.327 N, respectively.

(ii)

Free body diagram of slider:



$\Sigma F_x = 0;$ $\quad T\cos\theta - P = 0$

$$\Rightarrow \quad \cos\theta = \frac{P}{T} = \frac{60}{80}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{60}{80}\right) = 41.4096^\circ$$

From geometry, $\tan\theta = \dfrac{1200}{x}$

$$\Rightarrow \quad x = \frac{1200}{\tan\theta} = \frac{1200}{\tan(41.4096^\circ)} = 1360.67 \text{ cm}$$ **Ans.**

2. (c) (i)

The Lewis equation is based on the following assumptions:

(i) The effect of the radial component (F_r), which induces compressive stress, is neglected.

(ii) It is assumed that the tangential component (F_t) is uniformly distributed over the face width of the gear. This is possible when the gears are rigid and accurately machined.

(iii) The effect of stress concentration is neglected.

(iv) It is assumed that at any time only one pair of teeth is in contact and takes the total load.

2. (c) (ii)

Given : P = 8 kN; τ_{per} = 80 N/mm^2; D = 60 mm

Primary shear stress:

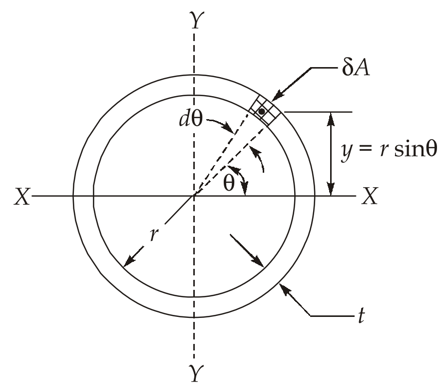
The primary shear stress in the weld is given by

$$\tau_1 = \frac{P}{A} = \frac{P}{\pi D t}$$

$$= \frac{(8 \times 10^3)}{\pi \times 60 \times t} = \left(\frac{42.4413}{t}\right) \text{N/mm}^2$$

Bending stress:

Consider an elemental section of area δA as shown in the figure. It is located at an angle θ with x-axis and subtends an angle $d\theta$.



$$\delta A = r d\theta t$$

and
$$\delta(I_{xx}) = (\delta A)(y^2) = (r d\theta t)(r\sin\theta)^2$$
$$= tr^3 \sin^2\theta \, d\theta$$

The moment of inertia of an annular fillet weld is obtained by integrating the above expression. Thus,

$$I_{xx} = 2\int_0^{\pi} tr^3 \sin^2\theta \, d\theta = 2tr^3 \int_0^{\pi} \sin^2\theta d\theta$$

$$I_{xx} = 2tr^3 \int_0^{\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$I_{xx} = 2tr^3 \left[\int_0^{\pi} \frac{d\theta}{2} - \int_0^{\pi} \frac{\cos 2\theta d\theta}{2}\right] = 2tr^3 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$I_{xx} = 2tr^3 \left(\frac{\pi}{2}\right); \; I_{xx} = \pi tr^3$$

For the given welded joint,

$$I_{xx} = \pi tr^3 = \pi(t)(30)^3 \text{ mm}^4$$

$$M_b = P_e = (8 \times 10^3)(240) \text{ N-mm}$$

$$y = \frac{D}{2} = \frac{60}{2} = 30 \text{ mm}$$

Bending moment,
$$\sigma_b = \frac{M_b y}{I_{xx}} = \frac{(8\times 10^3)(240)\times(30)}{\pi(t)(30)^3}$$

$$= \frac{679.061}{t} \text{ N/mm}^2$$

Maximum shear stress:

The maximum shear stress in the weld is given by,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2}$$

$$= \sqrt{\left(\frac{679.061}{2t}\right)^2 + \left(\frac{42.4413}{t}\right)^2} = \frac{342.1728}{t}\ \text{N/mm}^2$$

Size of weld:

Since the permissible shear stress in the weld is $\tau_{per} = 80\ \text{N/mm}^2$

$$\Rightarrow \quad \frac{342.1728}{t} = 80$$

$$\Rightarrow \quad t = 4.27716$$

Now, $h = \frac{t}{0.707}\ \frac{4.27716}{0.707} = 6.0497\text{mm} \simeq 6.05\ \text{mm}$ **Ans.**

3. (a)

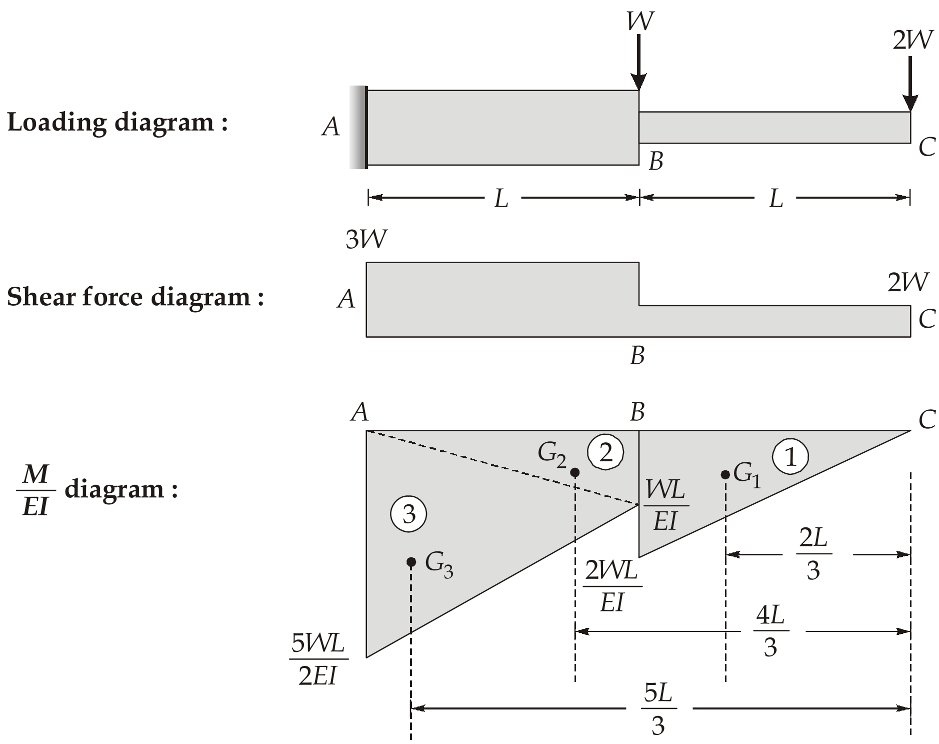
Calculating support reactions:

$\Sigma F_V = 0;$

$$\Rightarrow \quad R_A = W + 2W = 3W \quad \text{(Upwards)}$$

$\Sigma M_A = 0;$

$$\Rightarrow \quad M_A = W(L) + 2W(2L) = 5WL$$



The $\frac{M}{EI}$ diagram is divided into three parts (triangle A_1, A_2 and A_3). Their centre of gravities G_1, G_2 and G_3 are located at $\frac{2L}{3}$, $\frac{4L}{3}$ and $\frac{5L}{3}$, respectively from the point C.

The $\frac{M}{EI}$ areas are:

$$A_1 = \frac{1}{2} \times \frac{2WL}{EI} \times L = \frac{WL^2}{EI}$$

$$A_2 = \frac{1}{2} \times \frac{WL}{EI} \times L = \frac{WL^2}{2EI}$$

and,
$$A_3 = \frac{1}{2} \times \frac{5WL}{2EI} \times L = \frac{5WL^2}{4EI}$$

The slope and deflection at the fixed end of the beam are zero.

$$\theta_A = 0 \text{ and } y_A = 0$$

Using first moment-area theorem, the slope at C is given as

$$\theta_C + 0 = A_1 + A_2 + A_3$$

$$\Rightarrow \qquad \theta_C = \frac{WL^2}{EI} + \frac{WL^2}{2EI} + \frac{5WL^2}{4EI} = \frac{11WL^2}{4EI} \qquad \textbf{Ans.}$$

Using second moment-area theorem, the deflection at C is given as,

$$y_C - y_A = \text{Moment of the } \left(\frac{M}{EI}\right) \text{ diagram between } A \text{ and } C \text{ about } C$$

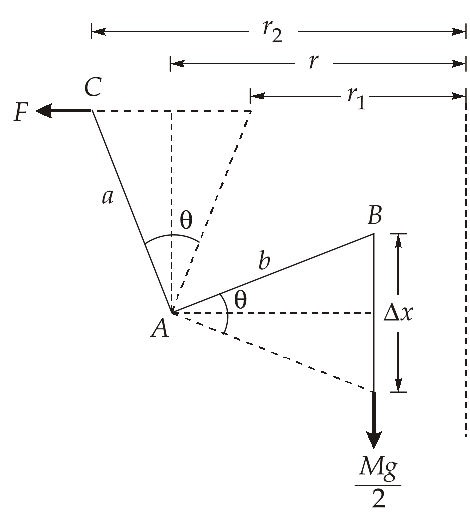
$$y_C - 0 = \left(A_1 \times \frac{2L}{3}\right) + \left(A_2 \times \frac{4L}{3}\right) + \left(A_3 \times \frac{5L}{3}\right)$$

$$\Rightarrow \qquad y_C = \left(\frac{WL^2}{EI} \times \frac{2L}{3}\right) + \left(\frac{WL^2}{2EI} \times \frac{4L}{3}\right) + \left(\frac{5WL^2}{4EI} \times \frac{5L}{3}\right)$$

$$= \frac{41WL^3}{12EI} \qquad \textbf{Ans.}$$

3. (b)

Given : a = 120 mm, b = 60 mm, Δx = 50 mm, r_1 = 70 mm, N_1 = 280 rpm,
s = 25 N/mm = 25 × 10^3 N/m, m = 3.5 kg, M = 5 kg



$$\omega_1 = 2\pi\frac{N_1}{60} = 2\pi \times \frac{280}{60} = 29.3215 \text{ rad/s}$$

$$\frac{r_2 - r_1}{a} = \theta = \frac{\Delta x}{b}$$

$$\Rightarrow \qquad r_2 = r_1 + \frac{ah}{b} = 70 + \frac{120 \times 50}{60} = 170 \text{ mm}$$

$$s = 2 \times \frac{a^2}{b^2}\left(\frac{F_2 - F_1}{r_2 - r_1}\right)$$

Now, $F_1 = mr_1\omega_1^2 = 3.5 \times 0.070 \times (29.3215)^2 = 210.639 \text{ N}$

or $(25 \times 10^3) = 2 \times \dfrac{(120)^2}{(60)^2}\left(\dfrac{F_2 - 210.639}{0.170 - 0.070}\right)$

$\Rightarrow$ $F_2 = 523.139 \text{ N}$

Now, $F_2 = mr_2\omega_2^2$

$\Rightarrow$ $523.139 = 3.5 \times 0.170 \times \left(\dfrac{2\pi N_2}{60}\right)^2$

$\Rightarrow$ $N_2 = 283.153 \text{ rpm}$ **Ans.**

The speed when the sleeve is lifted by 50 mm is 283.153 rpm.

Now, $mr_1\omega_1^2 a = \dfrac{1}{2}(Mg + F_{s1})b$

$(3.5) \times (0.070) \times (29.3215)^2 \times (0.120) = \dfrac{1}{2} \times \left(5 \times 9.81 + F_{s_1}\right) \times 0.060$

$\Rightarrow$ $F_{s_1} = 793.505 \text{ N}$

Initial compression $= \dfrac{F_{s_1}}{s} = \dfrac{793.505}{25 \times 10^3} = 0.03174 \text{ m} = 31.74 \text{ mm}$ **Ans.**

Effort of the governor is also the average force applied on the spring.

Hence, Effort $= \dfrac{1}{2} \times s \times h$

$= \dfrac{1}{2} \times \left(25 \times 10^3\right) \times 0.050 = 625 \text{ N}$ **Ans.**

Power of the governor is given as,

Power = Effort × Displacement

= 625 × 0.05

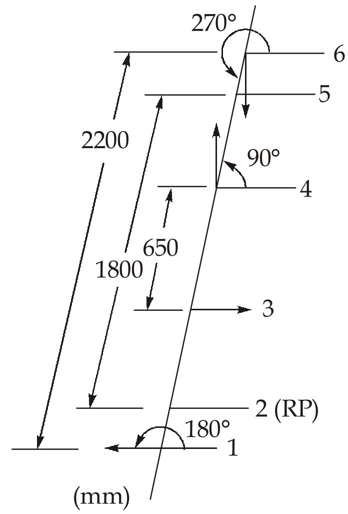
= 31.25 Nm **Ans.**

3. **(c)**

Leading wheels

Balance mass on each leading wheel $= m_p + \dfrac{1}{2}cm$

$= 300 + \dfrac{1}{2} \times \left(\dfrac{2}{3} \times 345\right) = 415 \text{ kg}$



Taking the plane 2 as the reference plane and $\angle\theta_3 = 0°$

$$m_1 = m_6 = 140 \text{ kg} \qquad m_3 = m_4 = 415 \text{ kg}$$
$$r_1 = r_6 = 0.250 \text{ m} \qquad r_3 = r_5 = 0.780 \text{ m}$$
$$r_3 = r_4 = 0.320 \text{ m} \qquad l_1 = -0.2 \text{ m}$$
$$l_3 = 0.575 \text{ m} \qquad l_4 = 1.225 \text{ m}$$
$$l_5 = 1.8 \text{ m} \qquad l_6 = 2 \text{ m}$$
$$m_1 r_1 l_1 = 140 \times (0.250) \times (-0.2) = -7 \text{ kg.m}^2$$
$$m_3 r_3 l_3 = 415 \times (0.320) \times (0.575) = 76.36 \text{ kg.m}^2$$
$$m_4 r_4 l_4 = 415 \times (0.320) \times (1.225) = 162.68 \text{ kg.m}^2$$
$$m_5 r_5 l_5 = m_5 \times (0.780) \times (1.8) = 1.404 m_5 \text{ kg.m}^2$$
$$m_6 r_6 l_6 = 140 \times (0.250) \times 2 = 70 \text{ kg.m}^2$$

Now, $\Sigma mrl\cos\theta = 0, \quad \Sigma mrl\sin\theta = 0$

We get,
$$1.404 m_5 = \begin{bmatrix} (-7\cos 180° + 76.36\cos 0° + 162.68\cos 90° + 70\cos 270°)^2 \\ +(-7\sin 180° + 76.36\sin 0° + 162.68\sin 90° + 70\sin 270°)^2 \end{bmatrix}^{0.5}$$

$\Rightarrow$
$$1.404 m_5 = [(83.36)^2 + (92.68)^2]^{0.5}$$

$\Rightarrow$
$$1.404 m_5 = 124.6534$$

$\Rightarrow$
$$m_5 = 88.78 \text{ kg}$$

$$\tan\theta_5 = \frac{-92.68}{-83.36} = 1.112$$

or
$$\theta_5 = 228.04°$$

(ii) Trailing wheels

The arrangement remains the same except that only half of the required reciprocating masses have to be balanced at the cranks.

i.e., $$m_3 = m_4 = \frac{1}{2}\left(\frac{2}{3}\times 345\right) = 115 \text{ kg}$$

then, $$m_3 r_3 l_3 = 115 \times (0.320) \times (0.575) = 21.16 \text{ kg.m}^2$$

and $$m_4 r_4 l_4 = 115 \times (0.320) \times (1.225) = 45.08 \text{ kg.m}^2$$

$$1.404 m_5 = \begin{bmatrix} (-7\cos 180° + 21.16\cos 0° + 45.08\cos 90° + 70\cos 270°)^2 \\ +(-7\sin 180° + 21.16\sin 0° + 45.08\sin 90° + 70\sin 270°)^2 \end{bmatrix}^{0.5}$$

$\Rightarrow$ $$1.404 m_5 = [(28.16)^2 + (-24.92)^2]^{0.5}$$

$\Rightarrow$ $$1.404 m_5 = 37.60$$

$\Rightarrow$ $$m_5 = 26.78 \text{ kg}$$

$$\tan\theta_5 = \frac{-(-24.92)}{-28.16} = \frac{+24.92}{-28.16}$$

$$= -0.885 \text{ or } \theta_5 = 138.49°$$

By symmetry, $$m_2 = m_5 = 26.78 \text{ kg}$$

and $$\tan\theta_2 = \frac{-28.16}{+24.92} = -1.130 \text{ or } \theta_2 = 311.51°$$

4. (a)

Given : P = 20 kN; e = 600 + 200 = 800 mm; τ = 65 N/mm^2; n = 7

Primary shear forces on rivets,

$$P_1' = P_2' = P_3' = P_4' = P_5' = P_6' = P_7' = \frac{P}{n}$$

$$= \frac{20\times 10^3}{7} = 2857.14 \text{ N}$$

By symmetry, the centre of gravity G is located at the centre of rivet-4. The radial distance of rivets centres from the centre of gravity G are as follows:

$$r_1 = r_2 = r_6 = r_7 = \sqrt{100^2 + 200^2} = 223.61 \text{ mm}$$

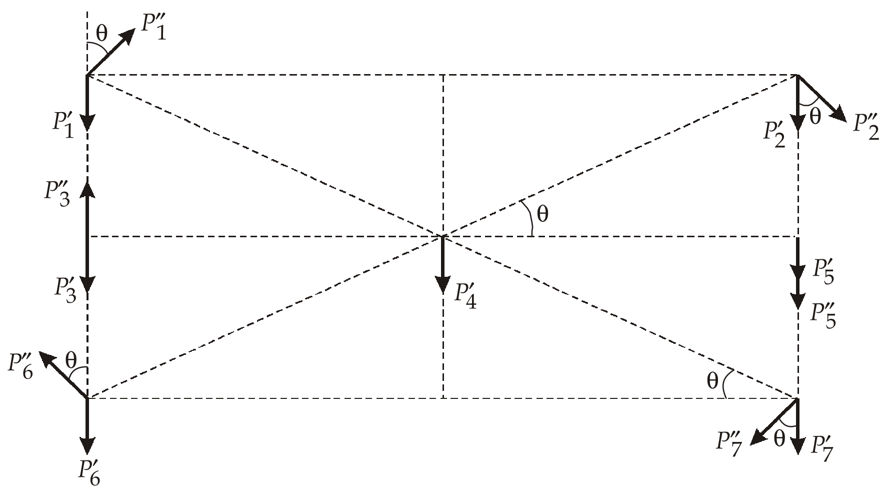
$$r_4 = 0$$

$$r_3 = r_5 = 200 \text{ mm}$$

Secondary sher forces on rivets:

$$P_4'' = 0$$

On rivet - 2,
$$P \cdot e = \frac{P_2''}{r_2}\left[r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + r_7^2\right]$$
$$\Rightarrow \quad (20 \times 10^3) \times 800 = \frac{P_2''}{223.61}\left[4 \times (223.61)^2 + 0 + 2(200)^2\right]$$
$$\Rightarrow \quad P_2'' = 12777.45 \text{ N}$$
Now,
$$P_2'' = P_1'' = P_6'' = P_7'' = 12777.45 \text{ N}$$
On rivet-5,
$$P \cdot e = \frac{P_5''}{r_5}\left[r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + r_7^2\right]$$
$$\Rightarrow \quad \left(20 \times 10^3\right) \times 800 = \frac{P_5''}{200}\left[4(223.61)^2 + 0 + 2(200)^2\right]$$
$$\Rightarrow \quad P_5'' = 11428.34 \text{ N}$$
Now,
$$P_5'' = P_3'' = 11428.34 \text{ N}$$



$$\tan\theta = \frac{100}{200}$$
$$\Rightarrow \quad \theta = 26.57°$$

The resultant shear force on the worst rivet, i.e. rivet-2 or rivet-7 as these are subjected to maximum shear force.

$$P_2 = \sqrt{(2857.14)^2 + (12777.45)^2 + 2(2857.14)(12777.45)\cos(26.57°)} = 15386.01 \text{ N} = P_7$$

Now, on equating the maximum shear force to the shear strength of the rivet.

$$P_2 = \frac{\pi}{4} d^2 \tau$$

$$\Rightarrow \quad 15386.01 = \frac{\pi}{4} \times d^2 \times 65$$

$$\Rightarrow \quad d = 17.36 \text{ mm}$$ **Ans.**

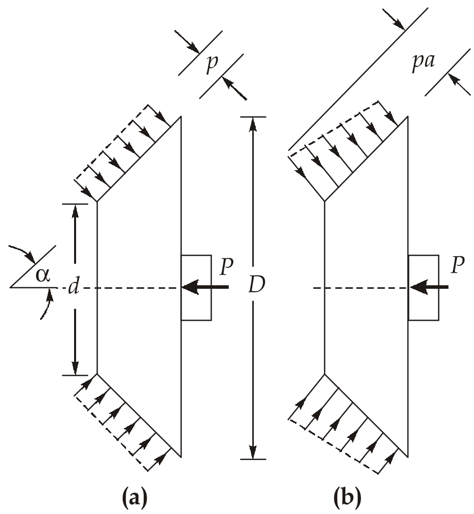
4. (b) (i)

1. **Ductile materials under static load:** Under static load, ductile materials are not affected by stress concentration, to the extent that photo- elastic analysis might indicate. When the stress in the vicinity of the discontinuity reaches the yield point, there is plastic deformation, resulting in a redistribution of stresses. This plastic deformation or yielding is local and restricted to very small area in the component. There is no perceptible damage to the part as a whole. Therefore, it is common practice to ignore the theoretical stress concentration factor for components that are made of ductile materials and subjected to static load.
2. **Ductile materials under fluctuating load:** However, when the load is fluctuating, the stress at the discontinuities may exceed the endurance limit and in that case, the component may fail by fatigue. Therefore, endurance limit of the components made of ductile material is greatly reduced due to stress concentration. This accounts for the use of stress concentration factors for ductile components. However, some materials are more sensitive than others to stress raising notches under fluctuating load. To account for this effect, a parameter called notch sensitivity factor is found for each material. The notch sensitivity factor is used to modify theoretical stress concentration factor.
3. **Brittle materials:** The effect of stress concentration is more severe in case of brittle materials, due to their inability to plastic deformation. Brittle materials do not yield locally and there is no readjustment of stresses at the discontinuities. Once the local stress at the discontinuity reaches the fracture strength, a crack is formed. This reduces the material available to resist external load and also increases the stress concentration at the crack. The part then quickly fails. Therefore, stress concentration factors are used for components made of brittle materials subjected to both static load as well as fluctuating load.

(ii)

Two theories are used to obtain the torque capacity of the cone clutch.

1. **Uniform pressure theory :** According to uniform pressure theory, pressure 'p' at radius 'r' is constant. This constant pressure distribution is illustrated in figure (a) and (b).



Pressure distribution

$$P = 2\pi \int_{d/2}^{D/2} pr\,dr = 2\pi p \int_{d/2}^{D/2} r\,dr$$

$$= 2\pi p \left[\frac{r^2}{2}\right]_{d/2}^{D/2} = 2\pi p \left[\frac{D^2 - d^2}{8}\right]$$

$$\therefore \quad P = \frac{\pi p}{4}\left(D^2 - d^2\right) \quad \text{...(i)}$$

From equation,

$$M_t = \frac{2\pi\mu}{\sin\alpha} \int_{d/2}^{D/2} pr^2\,dr = \frac{2\pi\mu p}{\sin\alpha} \int_{d/2}^{D/2} r^2\,dr$$

$$= \frac{2\pi\mu p}{\sin\alpha}\left[\frac{r^3}{3}\right]_{d/2}^{D/2} = \frac{2\pi\mu p}{\sin\alpha}\left[\frac{\left(D^3 - d^3\right)}{24}\right];$$

$$\therefore \quad M_t = \frac{\pi\mu p}{12\sin\alpha}\left(D^3 - d^3\right) \quad \text{...(ii)}$$

Dividing equation (ii) and (i),

$$\frac{M_t}{P} = \frac{\pi\mu p}{12\sin\alpha}\left(D^3 - d^3\right) \times \frac{4}{\pi p\left(D^2 - d^2\right)}$$

$$\frac{M_t}{P} = \frac{\mu}{3\sin\alpha}\left(D^3 - d^3\right) \times \frac{1}{\left(D^2 - d^2\right)}$$

$$M_t = \frac{\mu P}{3\sin\alpha} \frac{\left(D^3 - d^3\right)}{\left(D^2 - d^2\right)} \quad \text{...(iii)}$$

2. **Uniform wear theory :** According to uniform wear theory, the product (pr) is constant. The pressure distribution as per this theory is illustrated in figure (b). Since 'pr' is constant,

$$pr = p_a\left(\frac{d}{2}\right)$$

In above expression, p_a is intensity of pressure at the inner diameter. It is also the permissible intensity of pressure.

$$P = 2\pi\int_{d/2}^{D/2} pr\,dr = 2\pi\left(p_a\frac{d}{2}\right)\int_{d/2}^{D/2} dr$$

$$= 2\pi\left(p_a\frac{d}{2}\right)[r]_{d/2}^{D/2} = 2\pi\left(p_a\frac{d}{2}\right)\left[\frac{(D-d)}{2}\right]$$

$$\therefore \quad P = \frac{\pi p_a d}{2}(D-d) \quad \text{...(iv)}$$

From equation,

$$M_t = \frac{2\pi\mu}{\sin\alpha}\int_{d/2}^{D/2} pr^2 dr = \frac{2\pi\mu}{\sin\alpha}\left(p_a\frac{d}{2}\right)\int_{d/2}^{D/2} r\,dr$$

$$= \frac{2\pi\mu}{\sin\alpha}\left(p_a\frac{d}{2}\right)\left[\frac{r^2}{2}\right]_{d/2}^{D/2}$$

or

$$M_t = \frac{2\pi\mu}{\sin\alpha}\left(p_a\frac{d}{2}\right)\left[\frac{(D^2-d^2)}{8}\right]$$

$$\therefore \quad M_t = \frac{\pi\mu p_a d}{8\sin\alpha}(D^2-d^2) \quad \text{...(v)}$$

Dividing equation (v) by (iv),

$$\frac{M_t}{P} = \frac{\pi\mu p_a d}{8\sin\alpha}(D^2-d^2) \times \frac{2}{\pi p_a d(D-d)}$$

$$\frac{M_t}{P} = \frac{\mu}{4\sin\alpha}\frac{(D^2-d^2)}{(D-a)}$$

$$M_t = \frac{\mu P}{4\sin\alpha}(D+d)$$

4. **(c)**

$$N = 600 \text{ rpm}; \ \omega = 2\pi\frac{N}{60} = 2\pi\times\frac{600}{60} = 20\pi \text{ rad/s};$$

$$C_S = \pm 2\% = 4\% = 0.04; \ \sigma_{per} = 7 \text{ MPa}; \ \rho = 7200 \text{ kg/m}^3$$

Let the diameter of the flywheel rim be D and the peripheral velocity of the flywheel rim by v.

Limiting peripheral velocity,

$$v = \sqrt{\frac{\sigma}{\rho}} = \sqrt{\frac{7 \times 10^6}{7200}} = 31.18 \text{ m/s}$$

Also,

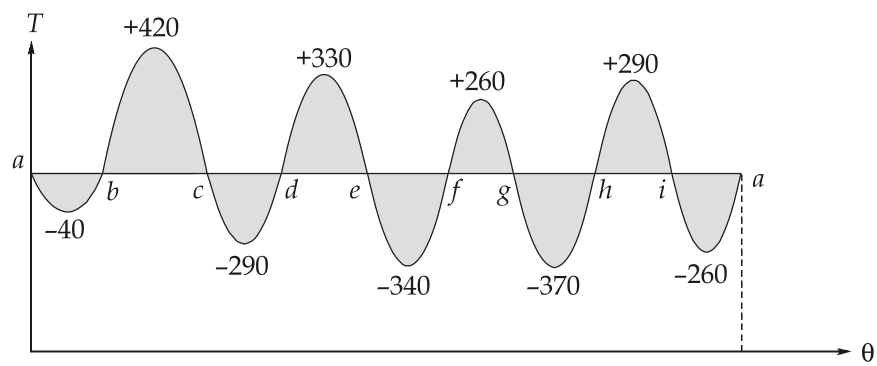
$$v = \frac{\pi D N}{60}$$

$\Rightarrow$

$$D = \frac{v}{\pi N} \times 60 = \frac{31.18}{\pi \times 600} \times 60$$

$$= 0.9925 \text{ m} = 992.5 \text{ mm}$$

Turning moment diagram,



Let energy at point a be E,

$$E_a = E$$

$$E_b = E_a - 40 = E - 40 \text{ (Minimum)}$$

$$E_c = E_b + 420 = (E - 40) + 420 = E + 380$$

$$E_d = E_c - 290 = (E + 380) - 290 = E + 90$$

$$E_e = E_d + 330 = (E + 90) + 330 = E + 420 \text{ (Maximum)}$$

$$E_f = E_e - 340 = (E + 420) - 340 = E + 80$$

$$E_g = E_f + 260 = (E + 80) + 260 = E + 340$$

$$E_h = E_g - 370 = (E + 340) - 370 = E - 30$$

$$E_i = E_h + 290 = (E - 30) + 290 = E + 260$$

The maximum fluctuation of energy,

$$(\Delta E)_{max} = E_{max} - E_{min}$$

$$= (E + 420) - (E - 40)$$

$$= 460 \text{ mm}^2$$

Scale: x-axis: $1 \text{ mm} = 5° = 5° \times \frac{2\pi}{360°} = \frac{\pi}{36}$ radians

y-axis: $1 \text{ mm} = 500$ N-m

$$\Rightarrow \quad 1 \text{ mm}^2 = \left(\frac{\pi}{36}\right) \times 500 = 43.6332 \text{ Nm}$$

Now, $(\Delta E)_{max} = 460 \text{ mm}^2 = (460 \times 43.6332) \text{ Nm} = 20071.272$ N-m

Also, $(\Delta E)_{max} = I\omega^2 C_s = I\left(\frac{2\pi N}{60}\right)^2 C_s$

$$20071.272 = I \times \left(\frac{2\pi \times 600}{60}\right)^2 \times 0.04$$

$$\Rightarrow \quad I = 127.103 \text{ kg-m}^2$$

Also, $I = mR^2 = m\left(\frac{D}{2}\right)^2$

$$\Rightarrow \quad 127.103 = m\left(\frac{0.9925}{2}\right)^2$$

$$\Rightarrow \quad m = 516.12 \text{ kg}$$

Let thickness of rim be t and width be $b = 5t$

Now, $m = (\text{Width} \times \text{Thickness} \times 2\pi R) \times \text{Density}$

$$\Rightarrow \quad m = (5t) \times t \times (2\pi R) \times \rho$$

$$\Rightarrow \quad 516.12 = 5t^2 \times \left(2\pi \times \frac{0.9925}{2}\right) \times 7200$$

$$\Rightarrow \quad t = 0.0678 \text{ m} = 67.8 \text{ mm}$$

Width, $b = 5t = 5 \times 67.8 = 339$ mm

Cross-sectional area, $A = bt = 339 \times 67.8 = 22984.2 \text{ mm}^2$ **Ans.**

Section : B

5. (a)

A systematic approach is required to establish an efficient and effective maintenance strategy. Further, this system should also consider the following points:

- The operational and structural complexity of the plant.
- The relationship between production and plant engineering and its dynamic nature.
- The relationship between maintenance strategy, maintenance objective, workload, and resource availability.
- Requirements, bottleneck, and influencing factors.

The top down-bottom up approach accommodates all the above points for consideration. This system is briefed out here under.

Step 1: Understand the characteristics of plant operation.

(i) Construct a process flowchart.

(ii) Understand the type of production and production policy.

(iii) Rank the units according to their criticality.

(iv) Identify the maintenance scheduling characteristics.

(v) Identify the prevailing maintenance strategy and maintenance objective.

Step 2: Establish a maintenance plan for each unit.

(i) Analyse the plant units into their 'maintenance causing items'.

(ii) Determine the best maintenance method for each item and divide into 'on-line' and 'off- line' methods.

(iii) Establish an off-line plan for each unit and frequency.

(iv) Identify the corrective maintenance guidelines.

Step 3: Prepare a maintenance schedule for the plant.

(i) Prepare the list of all machines or units.

(ii) Prepare the on-line schedules and frequency.

(iii) Prepare the off-line schedule and frequency with the help of Step 2 (iii).

(iv) Estimate the planned workload for on-line and off-line schedules based on the information in Step 3 (ii) and (iii). Also forecast the expected and unexpected unplanned workload by experience.

(v) Review the schedules once again in accordance with the actual resources available.

Step 4: Establish the initial policy for spare parts and for reconditioning.

(i) Identify the parts that are to be maintained as spares with the help of Step 2 (ii) and Step 3 (i), (ii), and (iii).

5. (b)

Carburizing

Carburizing is the addition of carbon to the surface of low-carbon steels at temperatures generally between 850 and 950°C (1560 and 1740°F), at which austenite, with its high solubility for carbon, is the stable crystal structure. Hardening is accomplished when the high-carbon surface layer is quenched to form martensite so that a high-carbon martensitic case with good wear and fatigue resistance is superimposed on a tough, low-carbon steel core. Case depth of carburized steel is a function of carburizing time and the available carbon potential at the surface.

Advantages:

- High quality carburized case can be produced.
- Time consume is also less.
- Production cost is low.
- Heat consume is also less.
- Process is clean.

Disadvantages:

- High labour cost for packing and unpacking.
- It is difficult to quench directly from carburizing temperature.
- Much time is needed in heating and cooling.

Cyaniding

In cyaniding a case of high hardness and wear resistance is produced on C-alloy steels. Work is immersed in molten salt bath containing 20-30% NaCN which is heated to 820-860°C. this is usually followed by water quenching. The time required is 30-90 min depending upon the depth of case which is 0.15-0.5 mm.

The atomic C diffuses into steel, the work can be directly quenched as soon as it taken out of bath, then low temperature tempering is done at 200°C.

Cyaniding is used to produce light cases on small shafts, worms, nuts, springs, pins, etc.

Advantage:

Resistance to corrosion and wear is high.

Disadvantages:

- High cost.
- Cyaniding bath is toxic hence the worker needs protection.

Flame Hardening

The process consists of heating the surface of medium carbon steel by high temperature gas flame at 2400°C-3300°C and immediately cooling in air or water. Heat may be supplied by oxyacetylene torch. The flame rapidly imparts large amounts of heat to the surface. The heat supply so quickly to the surface and for a short time, the core remains unaffected. As soon as the desire temperature is achieved water immediately sprayed which cools the surface. By proper control and cooling temperature. The core is not affected by treatment. The thickness of hardened layer is 2-4 mm and its surface structure is martensite.

© Copyright: MADE EASY

Advantages:

- There is practically no distortion of the work piece because only small sections of the work piece are heated.
- As heating rate is high the work surface remains clean.
- The process rate can be automated
- The process is more efficient and economical for large work.

Disadvantages:

- Very thin section may get distorted extensively.
- Over heating may cause crack.

Uses

- It is used to increase wear resistance and surface hardness Eg: piston pins, large gears, hand tools, shaft, mill rolls etc.

5. (c)

Table : Selection of cutting fluid based on work material

Material	Characteristic	Cutting Fluid to be Used
Grey cast iron	Grey cast iron could be machined dry since the graphite flakes act as solid lubricants in cutting.	Soluble oils and thinner neat oils are satisfactory for flushing swarf and metal dust.
Copper alloys	Better machinability. Some could be machined dry.	Water-based fluids could be conveniently used. For tougher alloys, a neat oil blended with fatty or inactive EP additive is used.
Aluminium alloys	They are generally ductile and can be machined dry. However, in combination with steel they have a high friction coefficient. Generally, BUE forms on the tool and prevents the chip flowing smoothly away from the work.	It is necessary to have the tool surface highly polished. Generally, a soluble oil to which an oiliness agent has been added is used. For more difficult application, straight neat oil or fatty oil or kerosene could be used.
Mild steel and low-to medium-carbon steels	Easiest to machine. Low carbon steels have lower tensile strength and hence may create problems because of their easy tearing.	Milky-type soluble oil or mild EP neat cutting oil could be used.

©Copyright: MADE EASY

Table : Selection of cutting fluid based on tool material

Tool material	Characteristics	Cutting-fluid requirements
High-carbon steel	Not widely used. They should be well cooled.	Water-based coolants are generally used.
High-speed steels	Have better hot hardness characteristics	For general machining water-based cutting fluids can be used. For heavy-duty work, EP neat oils are preferable.
Nonferrous materials	During a cut they should never be overcooled or subjected to intermittent cooling because they are brittle and are likely to suffer thermal shock.	Neat cutting oils are the most suitable choice for most applications.
Carbides, ceramics and diamond	These are used for very high speeds. Hence the requirement is to reduce the large amount of heat produced to reduce the thermal distortion of the workpiece.	Water-based coolant is recommended. In low-speed applications, EP based oils could reduce the problem of adhesion of chip with tool.

5. (d) (i)

The microprocessor, as controllers, claim the following advantages over analog controllers:

1. The form of controlling action (e.g., proportional or three mode) can be changed by purely a change in the computer software.
2. No alteration in hardware or electrical wiring is required.
3. Whereas with analog control, separate controllers are required for each process being controlled, however, with a microprocessor many separate processes can be controlled by sampling processes with a multiplexer.

- As compared to analog control, digital control gives better accuracy because the amplifiers and other components used with analog systems change their characteristics with time and temperature and so show drift, while digital control does not suffer from drift in the same way since it operates on signals in only the on-off mode.

(ii)

Closed loop transfer function is given by,

$$\text{CLTF} = \frac{C}{R} = \frac{2k_p}{1+2k_p}$$

$$\text{Error} = R - C$$

$$= \left[R - R\cdot\frac{2k_p}{1+2k_p}\right] = R\left[1 - \frac{2k_p}{1+2k_p}\right]$$

Now the input is 5V. Therefore the error is given by

$$\text{Error} = 5\left[1 - \frac{2\times 50}{1+2\times 50}\right] = 0.0495V$$ **Ans.**

5. (e)

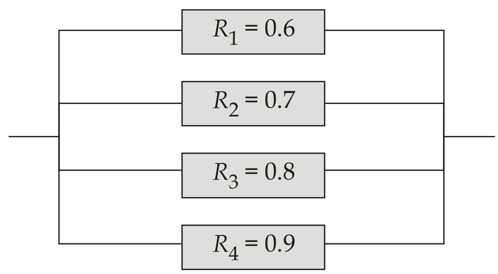


Fig. : Parallel system of four components

The reliability of the system functioning at any time t is given by

$$R_p = 1 - [(1 - R_1)(1 - R_2)(1 - R_3)(1 - R_4)]$$
$$= 1 - [(1 - 0.6)(1 - 0.7)(1 - 0.8)(1 - 0.9)]$$
$$= 1 - [0.4 \times 0.3 \times 0.2 \times 0.1]$$
$$= 0.9976$$

(a) If the reliability of the third component is increased to 0.9, then the system reliability is given by

$$R_a = 1 - 0.4 \times 0.3 \times 0.1 \times 0.1 = 1 - 0.0012$$
$$= 0.9988$$

i.e. overall reliability increased by 0.0012.

(b) If the reliability of the third component is decreased to 0.6, then the system reliability is given by

$$R_b = 1 - 0.4 \times 0.3 \times 0.3 \times 0.1$$
$$= 0.9964$$

i.e. overall reliability decreased by 0.0012.

6. (a) (i)

The difference between TIG and MIG welding processes is given in tabular form below:

S. No.	Aspects	TIG welding	MIG welding
1.	Name of the process	Tungsten inert-gas welding.	Metal inert-gas welding.
2.	Type of electrode used	Non-consumable tungsten electrode.	Consumable metallic electrode.
3.	Electrode feed	Electrode feed not required.	Electrode need to be fed at a constant speed from a wire reel.
4.	Electrode holder	It is called welding torch and has got a cap filled on the back to cover the tungsten electrode. It has also got connections for shielding gas, cooling water and control cable. It may be air-cooled also.	It is called welding gun or torch. It has facility to continuously feed wire electrodes; shielding inert-gas, cooling water and control table.
5.	Welding current	Both A.C. and D.C. can be used.	D.C. with reverse polarity is used.
6.	Feed metal	Filler metal may or may not be used.	Filler metal in the form of spool wire is used.
7.	Bases metal thickness	Metal thickness which can be welded is limited to about 5 mm.	Thickness limited to about 40 mm.
8.	Welding speed	Slow.	Fast.

(ii)

Given : $h_1 = 4.15$ mm; $h_2 = 3.65$ mm; $D = 520$ mm; $R = 260$ mm; $\mu = 0.05$, $\sigma_0 = 220$ N/mm^2

The roll pressure at entry and exit is given as

$$p = \sigma_0' = \frac{2}{\sqrt{3}}\sigma_0 = \frac{2}{\sqrt{3}} \times 220 = 254.03 \text{ N/mm}^2$$ **Ans.**

Now,
$$h_1 - h_2 = 2R(1 - \cos\alpha)$$
$$4.15 - 3.65 = 2 \times 260 \times (1 - \cos\alpha)$$
$$\Rightarrow \quad \alpha = 2.513° \text{ or } 0.04386 \text{ rad}$$

Now,
$$H_0 = 2\sqrt{\frac{R}{h_2}}\tan^{-1}\left(\sqrt{\frac{R}{h_2}}\alpha\right)$$
$$= 2 \times \sqrt{\frac{260}{3.65}} \times \left[\tan^{-1}\left(\sqrt{\frac{260}{3.65}} \times 0.04386°\right)\right] \times \frac{\pi}{180°}$$
$$= 5.9845 \text{ mm}$$

At neutral point,
$$H_n = \frac{1}{2}\left[H_0 - \frac{1}{\mu}\ln\left(\frac{h_1}{h_2}\right)\right]$$
$$= \frac{1}{2}\left[5.9845 - \frac{1}{0.05}\ln\left(\frac{4.15}{3.65}\right)\right] = 1.7084 \text{ mm}$$

Now, $$\theta_n = \sqrt{\frac{h_2}{R}} \times \tan\left(\sqrt{\frac{h_2}{R}} \cdot \frac{H_n}{2}\right)$$

$$= \sqrt{\frac{3.65}{260}} \times \tan\left(\sqrt{\frac{3.65}{260}} \times \frac{1.7084}{2} \times \frac{180°}{\pi}\right)$$

$$= 0.012514 \text{ radians or } 0.71699°$$

Now, $$h_n = h_2 + 2R(1 - \cos\theta_n)$$

$$= 3.65 + 2 \times 260 \times [1 - \cos(0.71699)]$$

$$= 3.6907 \text{ mm}$$

∴ $$p_n = \sigma_0' \cdot \frac{h_n}{h_2} \cdot e^{\mu H_n}$$

$$= (254.03) \times \frac{3.6907}{3.65} \times e^{(0.05 \times 1.7084)}$$

$$= 279.77 \text{ N/mm}^2$$ **Ans.**

Hence, the pressure at the entrance and at the exit to the rolls is 254.03 N/mm^2 and pressure at the neutral plane is 279.77 N/mm^2.

6. (b)

Given : D = 8400 units, C_0 = ₹45 per order; C_h = 30% of ₹12 = ₹3.6 per unit per year; C = ₹12 per unit

As the order is placed according to the EOQ model

Economic order quantity, $$Q^* = \sqrt{2D\frac{C_0}{C_h}} = \sqrt{2 \times 8400 \times \frac{45}{3.6}}$$

$$= 458.26 \text{ units/order} \simeq 458 \text{ units/order}$$

Total annual cost associated with economic order quantity,

$$TAC = D \cdot C + \frac{D.C_0}{Q^*} + \frac{Q^* \cdot C_h}{2}$$

$$= 8400 \times 12 + \frac{8400 \times 45}{458} + \frac{458 \times 3.6}{2}$$

$$= 100800 + 825.33 + 824.4$$

$$= ₹102449.73$$

When 1% discount is offered, the unit price will be 99% of ₹12, i.e. ₹11.88.

In this case, $$Q = \frac{8400}{4} = 2100 \text{ units/order; } C_h = 30\% \text{ or ₹}11.88$$

$$T_{AC} = 8400 \times 12 + \frac{8400 \times 45}{458} + \frac{2100 \times (0.30 \times 11.88)}{2}$$
$$= 99792 + 180 + 3742.2$$
$$= ₹103714.2$$

Since, the total cost comes out to be higher when one percent discount is offered, so the company should not accept the offer.

Now, let x be the percentage minimum discount acceptable to the company. Then, it can be determined by setting the total cost with discount under consideration will be equal to the total cost associated with the policy of ordering EOQ.

$$\left[8400 \times \left(\frac{100 - x}{100}\right) \times 12 + \frac{8400 \times 45}{2100} + \frac{2100 \times 0.30 \times \left(\frac{100 - x}{100}\right) \times 12}{2}\right] = ₹102449.73$$

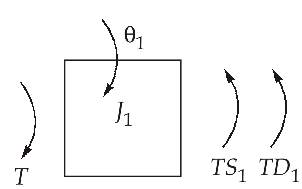
$$\Rightarrow \quad 1008(100 - x) + 180 + (37.8)(100 - x) = 102449.73$$
$$\Rightarrow \quad (1045.8)(100 - x) = 102449.73$$
$$\Rightarrow \quad 100 - x = 97.79$$
$$\Rightarrow \quad x = 2.21$$ **Ans.**

Hence, the minimum discount acceptable to the company is 2.21%.

6. (c)

For J_1

The free body diagram for J_1 is illustrated in figure below.



Free body diagram for J_1

Spring torque due to K_1

$$TS_1 = K_1(\theta_1 - \theta_2)$$

Damping torque due to B_1

$$TD_1 = B_1\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right)$$

Applying Newton's second law of motion,

$$J\alpha = \Sigma T$$

$$J_1 \frac{d^2\theta_1}{dt} = -TD_1 - TS_1 + T$$

$$J_1 \frac{d^2\theta_1}{dt} + TD_1 + TS_1 = T \quad \text{...(i)}$$

Substituting for TS_1 and TD_1 in equation (i), we have

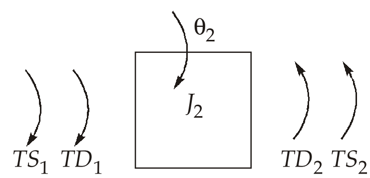
$$J_1 \frac{d^2\theta_1}{dt} + B_1 \frac{d\theta_1}{dt} + K_1\theta_1 = B_1 \frac{d\theta_2}{dt} + K_1\theta_2 + T \quad \text{...(ii)}$$

Taking Laplace transform, we get

$$\left[J_1 s^2 + B_1 s + K_1\right]\theta_1(s) = \left[B_1 s + K_1\right]\theta_2(s) + T(s)$$

For J_2

The free body diagram for the rotor J_2 is given in figure below.



Free body diagram for rotor J_2

Spring torque due to K_2

$$TS_2 = K_2\theta_2$$

Damping torque due to B_2

$$TD_2 = B_2 \frac{d\theta_2}{dt}$$

Applying Newton's second law of motion,

$$J_2 \frac{d^2\theta_2}{dt} = TS_1 + TD_1 - TS_2 - TD_2$$

$$J_2 \frac{d^2\theta_2}{dt} + TD_2 + TS_2 = TS_1 + TD_1$$

Substituting for various damping and spring torques

$$J_2 \frac{d^2\theta_2}{dt} + (B_1 + B_2)\frac{d\theta_2}{dt} + (K_1 + K_2)\theta_2 = B_1 \frac{d\theta_1}{dt} + K_1\theta_1 \quad \text{...(iii)}$$

Taking Laplace transform

$$\left[J_2 s^2 + (B_1 + B_2)s + (K_1 + K_2)\right]\theta_2(s) = [B_1 s + K_1]\theta_1(s) \quad \text{...(iv)}$$

From equation (iv), substituting for $\theta_1(s)$ in equation (ii) and rearranging, we get

$$[J_1 J_2 s^4 + \{(B_1 + B_2)J_1 + J_2 B_1\}s^3 + \{(K_1 + K_2)J_1 + B_1(B_1 + B_2) + K_1 J_2\}s^2$$

$$+\{(K_1 + K_2)B_1 + K_1(B_1 + B_2)\}s + K_1 K_2 - (B_1^2 s^2 + 2K_1 B_1 s)]\theta_2(s) = (K_1 + B_1 s)T(s)$$

Transfer function is given by

$$\frac{\theta_2(s)}{T(s)} = \frac{K_1 + B_1 s}{J_1 J_2 s^4 + \{(B_1 + B_2)J_1 + J_2 B_1\}s^3 + \{(K_1 + K_2)J_1 + B_1(B_1 + B_2) + K_1 J_2\}s^2 + \{(K_1 + K_2)B_1 + K_1(B_1 + B_2)\}s + K_1 K_2 - (B_1^2 s^2 + 2K_1 B_1 s)}$$

7. (a)

The forward kinematic model is obtained first. For the forward kinematic model, the frame assignment for the home position is carried out first. While assigning frames it is observed that the link dimension L_1 can be eliminated from the kinematic model by choosing the origin of frame {0} to coincide with origin of frame {1} at joint 2. The link dimension L_2 can be made zero by modifying the design slightly as shown in figure below such that the axis of prismatic link passes through the origin of frame {1}.

The final frame assignment with the origin of three frames, frame {0}, frame {1} and frame {2} at the same point is shown in figure. This minimizes the number of non-zero parameters as well as satisfies the necessary condition for existence of closed form solutions.

The joint-link parameters are tabulated in Table below.

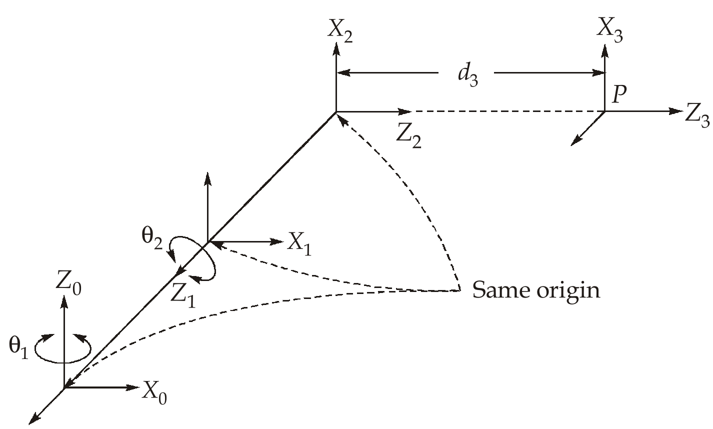


Fig. : Frame assignment for spherical arm with horizontal home position

Joint-link parameters for spherical arm									
1	0	90°	0	θ_1	θ_1	C_1	S_1	0	1
2	0	–90°	0	θ_2	θ_2	C_2	S_2	0	1
3	0	0	d_3	0	d_3	1	1	1	0

The three link transformation matrices ${}^0T_1$, ${}^1T_2$ and ${}^2T_3$ and the overall arm transformation matrix ${}^0T_3$ are obtained as

$$ {}^0T_1(\theta_1) = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{...(i)} $$

$$ {}^1T_2(\theta_2) = \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{...(ii)} $$

$$ {}^2T_3(d_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{...(iii)} $$

and, thus,

$$ {}^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3 = \begin{bmatrix} C_1C_2 & -S_1 & -C_1S_2 & -d_3C_1S_2 \\ S_1C_2 & C_1 & -S_1S_2 & -d_3S_1S_2 \\ S_2 & 0 & C_2 & d_3C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{...(iv)} $$

7. (b)

Let, x = amount invested in portfolio-1; y = amount invested in portfolio-2

Objective function :

Maximize, $z = 0.08x + 0.16y$

Constraints,

$x + y \leq 10000$ ...(i)

$x \leq 66000$ ...(ii)

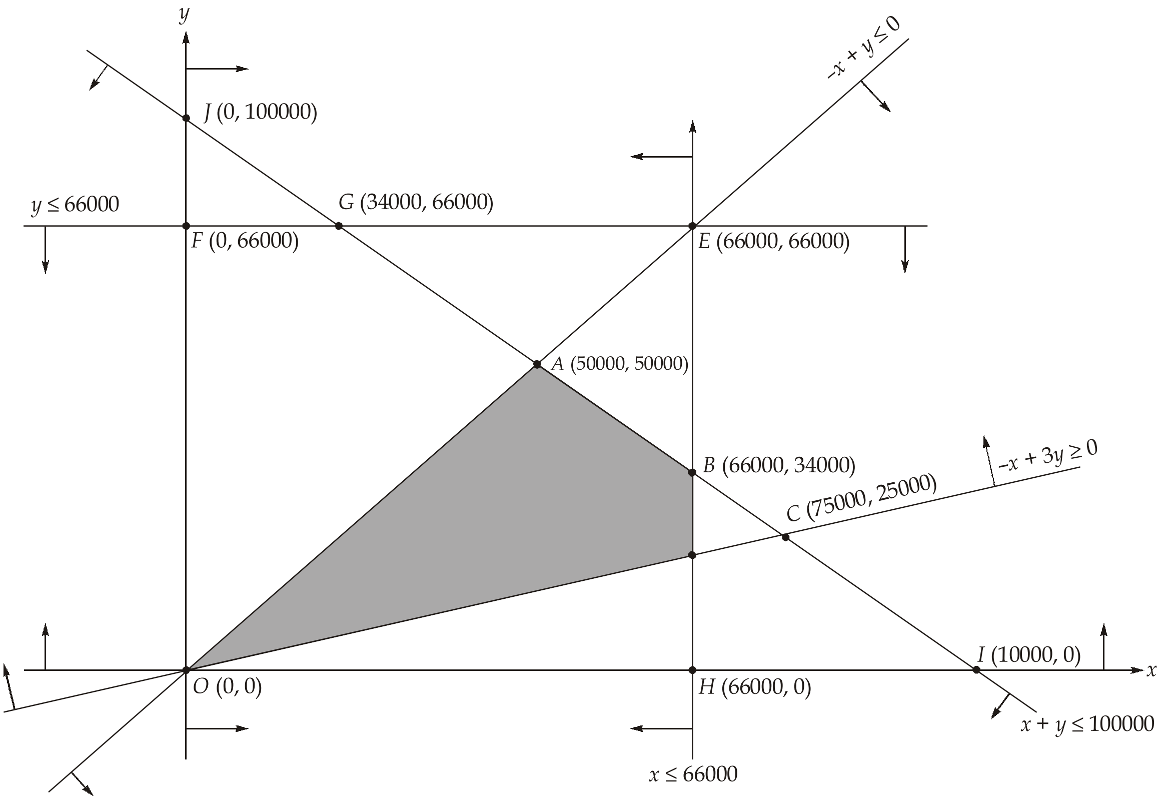
$y \leq 66000$ ...(iii)

$0.08x + 0.16y \leq 0.10(x + y)$ or $-x + 3y \geq 0$ ...(iv)

$5x + 9y \leq 7(x + y)$ or $-x + y \leq 0$ ...(v)

$$x \geq 0 \quad \text{...(vi)}$$
$$y \geq 0 \quad \text{...(vii)}$$

Thus, the feasible region as per the given constraint is shown in the graph below:



The value of z corresponding to the extreme points of feasible region are:

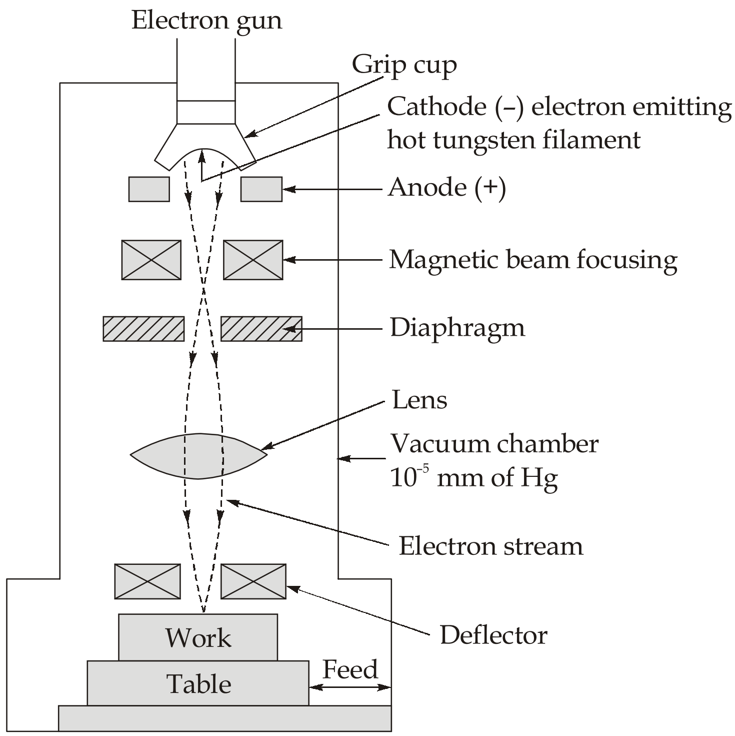
Extreme point	$z = 0.08x + 0.16y$
O (0, 0)	$z_O = 0.08(0) + 0.16(0) = 0$
A (50000, 50000)	$z_A = 0.08(50000) + 0.16(50000) =$ `12000 (maximum)
B (66000, 34000)	$z_B = 0.08(66000) + 0.16(34000) =$ `10720
D (000, 2000)	$z_C = 0.08(66000) + 0.16(22000) =$ `8800

Hence, the optimum amount the person should invest is ₹50000 in each portfolios, to get a maximum return of ₹12000.

7. **(c) (i)**

Electron beam machining is a process of machining materials with the use of a high velocity beam of electrons. This process is best suited for micro cutting of material (in mg/s) because the evaporated area is function of the beam power and the method of focusing which can be easily controlled.

Principle and Working: In this process the material is removed with the help of a high velocity (travelling at half the speed of light. i.e., 160,000 km/s) focused stream of electrons which are focused magnetically upon a very small area. These electrons heat and raise the temperature locally above the boiling point and thus melt and vaporise the work material at the point of bombardment.



Schematic diagram of EBM

The electrons are obtained in free state by heating the cathode metal in vacuum to the temperature at which they attain sufficient speed for escaping to the space around the cathode. These can then be made to move under the effect of electric or magnetic field and can be accelerated greatly. The acceleration is carried out by electric field and focusing and concentration is done by controllable magnetic fields.

Advantages:

1. It is excellent strategy for micro-machining. It can drill holes or cut slots which cannot be otherwise made.
2. It can cut any known material, metal or non-metal that would exist in vacuum.
3. No physical or metallurgical damage.
4. There is no contact between the work and tool.
5. Heat can be concentrated on a particular spot.
6. Close dimensional tolerances can be achieved because problem of tool wear is non-existent.

Disadvantages:

1. Low metal removal rate.
2. High equipment cost.
3. High operator skill required.
4. Only small cuts are possible.
5. High power consumption.
6. Unsuitable for producing perfectly cylindrical deep holes.
7. Workpiece size is limited due to requirement of vacuum in the chamber.

Applications:

1. Micro-machining operations on workpieces of thin sections.
2. Micro-drilling operations (upto 0.002 mm) for thin orifices, dies for wire drawing parts of electron microcopes, fibre spinners, injector holes for diesel engines etc.
3. Very effective for machining of metals of low heat conductivity and high melting point.

(ii)

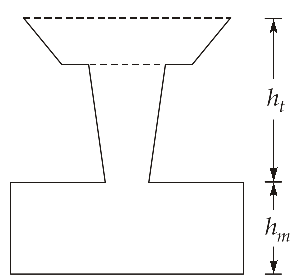
Base area of mould cavity, A_m = 1250 mm × 1000 mm

Height of mould cavity, h_m = 300 mm

Area of gate, A_g = 350 mm²

Available head for filling metal.

Case I : Filling the mould cavity using top gate



Top gate

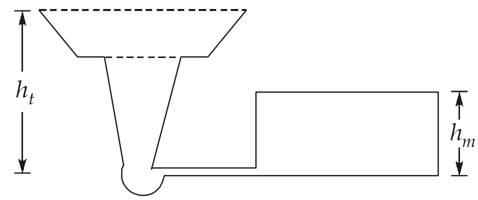
Filling time for top gate is given as,

$$t_f = \frac{V_m}{A_g v_g} = \frac{V_m}{A_g\sqrt{2gh_t}}$$

$$= \frac{1250 \times 1000 \times 300}{350 \times \sqrt{2 \times 9.81 \times 0.400} \times 10^3}$$

$$= 382.46 \text{ s}$$ **Ans.**

Case II : Filling the mould cavity using bottom gate



Bottom gate

Filling time for bottom gate is given as,

$$t_f = \frac{2A_m}{A_g} \cdot \frac{1}{\sqrt{2g}} \left(\sqrt{h_t} - \sqrt{h_t - h_m}\right)$$

$$= \frac{2 \times (1250 \times 1000)}{350} \times \frac{1}{\sqrt{2 \times 9.81}} \times \left(\sqrt{0.400} - \sqrt{0.400 - 0.300}\right)$$

$$= 509.94 \text{ s}$$ **Ans.**

8. (a)

The kinematic model of the manipulator will be

$$^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 = T_E$$

$$\begin{bmatrix} C_1C_4 & -C_1S_4 & -S_1 & -d_3S_1 + 5C_1 \\ S_1C_4 & -S_1S_4 & C_1 & d_3C_1 + 5S_1 \\ -S_4 & -C_4 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.250 & 0.433 & -0.866 & -89.10 \\ 0.433 & -0.750 & -0.500 & -45.67 \\ -0.866 & -0.500 & 0.000 & 50.00 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The solutions for joint-variables are found by using the direct approach. The solution for first joint variable θ_1 is obtained by comparing elements (1, 3) and (2, 3).

$$-S_1 = -\sin\theta_1 = -0.866$$

$$C_1 = \cos\theta_1 = -0.5$$

or $\theta_1 = A\tan2(0.866, -0.5) = -60°$ (that is: $\theta_1 = A\tan2(-a_x, a_y)$)

The solution for second variable d_2 is obtained from element (3, 4) as

$$d_2 = 50 \text{(that is } d_2 = d_z\text{)}$$

Similarly, the third joint variable d_3 is obtained from elements (1, 4) and elements (2, 4) by squaring, adding and simplifying

$$d_3 = \pm\sqrt{d_x^2 + d_y^2 - 25}$$

or $d_3 = 100$

Note that d_3 cannot be negative. The fourth joint variable θ_4 is computed from elements (3, 1) and (3, 2) as

$$\theta_4 = \text{Atan2}(0.866, 0.5) = 60° \text{ (that is; } \theta_4 = \text{Atan2}(-n_z, -o_z))$$

The given end-effector position and orientation, T_E will be achieved by setting the joint variable vector to

$$q = \begin{bmatrix} -60° & 50 & 100 & 60° \end{bmatrix}$$

8. (b)

Diameter, D = 70 mm; Cutting force, F_C = 2600 N; Feed force, F_x = 1200 N; Orthogonal rake angle, α = 25°; Cutting speed, v = 0.6 m/s; Feed rate, f = 0.12 mm/rev

Length of continuous chip in one revolution, l_c = 125 mm

For orthogonal cutting, $\lambda = 90°$

Chip thickness ratio is given as

$$r = \frac{t}{t_c} = \frac{l_c}{l}$$

$$\Rightarrow \quad r = \frac{l_c}{l} = \frac{l_c}{\pi D} = \frac{125}{\pi \times 70} = 0.5684$$

Using shear angle relation, $\tan\phi = \dfrac{r\cos\alpha}{1 - r\sin\alpha}$

$$\Rightarrow \quad \tan\phi = \frac{(0.5684) \times \cos(25°)}{1 - (0.5684) \times \sin(25°)}$$

$$\Rightarrow \quad \tan\phi = 0.6780$$

$$\Rightarrow \quad \text{Shear angle, } \phi = 34.138°$$ **Ans.**

$$\text{Thrust force, } F_t = \frac{F_x}{\sin\lambda} = \frac{1200}{\sin(90°)} = 1200 \text{ N}$$

Using force relation,

Frictional force along the rake face,

$$F = F_c \sin\alpha + F_t \cdot \cos\alpha$$
$$= 2600 \times \sin(25°) + 1200 \times \cos(25°)$$
$$= 2186.37 \text{ N}$$

Normal force perpendicular to the rake face,

$$N = F_c \cos\alpha - F_t \cdot \sin\alpha$$
$$= 2600 \times \cos(25°) - 1200 \times \sin(25°)$$
$$= 1849.26 \text{ N}$$

Force along the shear plane, $F_s = F_c \cos\phi - F_t \cdot \sin\phi$

$$= 2600 \times \cos(34.138°) - 1200 \times \sin(34.138°)$$
$$= 1478.56 \text{ N}$$ **Ans.**

Force normal to the shear plane,

$$F_n = F_c \sin\phi + F_t \cdot \cos\phi$$
$$= 2600 \times \sin(34.138°) + 1200 \times \cos(34.138°)$$
$$= 2452.31 \text{ N}$$ **Ans.**

$$\text{Coefficient of friction, } \mu = \frac{F}{N} = \frac{2186.37}{1849.26} = 1.1823$$ **Ans.**

Using velocity relation,

$$\frac{v}{\cos(\phi - \alpha)} = \frac{v_s}{\cos\alpha}$$

$$\Rightarrow \quad v_s = v \cdot \frac{\cos\alpha}{\cos(\phi - \alpha)} = 0.6 \times \frac{\cos 25°}{\cos(34.138° - 25°)}$$
$$= 0.551 \text{ m/s}$$ **Ans.**

8. (c)

Resolution is given by,

$$r = \frac{y_{\max}}{N} = \frac{180°}{1000} = 0.18°$$ **Ans.**

For $y = 50°$, calculate v_0 for each voltmeter, taking into account its R_V in kΩ :

$$v_0(R_V) = v_i\left[1 + \left(\frac{R}{R_V} + \frac{y_{\max}}{y}\right)\left(1 - \frac{y}{y_{\max}}\right)\right]^{-1}$$

$$= 10\left[1 + \left(\frac{1}{R_V} + \frac{180}{50}\right)\left(1 - \frac{50}{180}\right)\right]^{-1}$$

For this value of $v_0(R_V)$, calculate the linear approximation of the angle.

$$y(R_V) = \left(\frac{y_{\max}}{v_i}\right)v_0(R_V) = \frac{180}{10}v_0(R_V)$$

The angular measurement error $e(R_V)$ due to R_V is

$$e(R_V) = 50 - y(R_V)$$

and this error can be compared to the resolution of $r = 0.18°$

(a) For $R_V = 30$ kΩ

$$v_0(30) = 10\left[1+\left(\frac{1}{30}+\frac{180}{50}\right)\left(1-\frac{50}{180}\right)\right]^{-1} = 2.759 \text{ V}$$

and $$y(30) = \frac{180°}{10}\times 2.759 = 49.662°$$

$\therefore$ $$e(30) = 50 - 49.662 = 0.338° > 0.18°$$

This error is larger than the resolution of the potentiometer r = 0.18° and the analog voltmeter is rejected.

(b) For R = 10 MΩ = 10000 kΩ

$$v_0(10000) = 10\left[1+\left(\frac{1}{10^4}+\frac{180}{50}\right)\left(1-\frac{50}{180}\right)\right]^{-1} = 2.7777 \text{ V}$$

and $$y(10^4) = \left(\frac{180}{10}\right)\times 2.7777$$

$\therefore$ $$e(10^4) = 50 - 49.9986 = 0.0014 < 0.18°$$

The digital voltmeter introduces an error of 0.0014°, which is much smaller than the resolution of the potentiometer r = 0.18°, and the digital voltmeter is chosen.

❍❍❍❍