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Detailed Solutions

ESE-2024
Mains Test Series

Electrical Engineering
Test No : 7

Section A : Computer Fundamentals + Electrical & Electronic Measurements

Q.1 (a) Solution:

Pipelined computers employ various hardware techniques to minimize the performance degradation caused by instruction branching.

(i) Prefetch target instruction

(ii) Use a branch target buffer

(iii) Use a loop buffer

(iv) Use branch prediction

(i) **Prefetch target instruction:** One way of handling a conditional branch is to prefetch the target instruction in addition to the instruction following the branch. Both are saved until the branch is executed. If the branch condition is successful, the pipeline continues from the branch target instruction. An extension of this procedure is to continue fetching instructions from both places until the branch decision is made. At that time control chooses the instruction stream of the correct program flow.

(ii) **Branch target buffer:** Another possibility is the use of a branch target buffer or BTB. The BTB is an associative memory included in the fetch segment of the pipeline. Each entry in the BTB consists of the address of a previously executed branch instruction and the target instruction for that branch. It also stores the next few instructions after the branch target instruction. When the pipeline decodes a branch instruction, it searches the associative memory BTB for the address of the instruction.

If it is in the BTB, the instruction is available directly and prefetch continues from the new path. If the instruction is not in the BTB, the pipeline shift to a new instruction stream and stores the target instruction in the BTB. The advantage of this scheme is that branch instructions which have occurred previously are readily available in the pipeline without interruption.

(iii) Loop buffer: A variation of the BTB is the loop buffer. This is a small very high speed register file maintained by the instruction fetch segment of the pipeline. When a program loop is detected in the program, it is stored in the loop buffer in its entirety, including all branches. The program loop can be executed directly without having to access memory until the loop mode is removed by the final branching out.

(iv) Branch prediction: Another procedure that some computers use is branch prediction. A pipeline with branch prediction uses some additional logic to guess the outcome of a conditional branch instruction before it is executed. The pipeline then begins prefetching the instruction stream from the predicted path. A correct prediction eliminates the wasted time caused by branch penalties.

Q.1 (b) Solution:

Let balanced load is star connected and V and I be the phase voltage and phase current respectively cos ϕ is the power factor of the load.

$$\text{Phase voltage, } V = 231 \text{ V}$$

and $$\text{phase current, } I = 30 \text{ A}$$

If the first case, the wattmeter measures the power in one phase,

$$\therefore \quad VI\cos\phi = 5.54 \times 10^3$$

(or) $$\cos\phi = \frac{5.54\times 10^3}{231\times 30} = 0.8$$

and $$\sin\phi = 0.6$$

When the current coil is connected in the R phase and the pressure coil circuit is connected across the Y and B phases, reading of wattmeter is:

$$= V_{YB}I_R\cos(90-\phi)$$

$$= \sqrt{3}VI\sin\phi$$

But VI sin ϕ is the reactive power of each phase and therefore the wattmeter indicates

$\sqrt{3}\times$ Reactive power of each phase

$$\text{Magnitude of reading} = \sqrt{3}\times 231\times 30\times 0.6\times 10^{-3}$$

$$= 7.2 \text{ kW}$$

Q.1 (c) Solution:

Strain Gauge: If a metal conductor is stretched or compressed, its resistance changes on account of the fact that both length and diameter of conductor change. Also, there is a change in the value of resistivity of the conductor when it is strained and this property is called piezoresistive effect. Therefore, resistance strain gauge are known as piezoresistive gauges. The strain gauge are used for measurement of strain and associated stress in experimental stress analysis. Also, many other detectors and transducers notably the load cells, torque meters, diaphragm type pressure gauges, temperature sensors, accelerometers and flow meters, employ strain gauges as secondary transducers.

The Gauge factor is given as

$$G_f = 1 + 2\nu + \frac{\Delta\rho/\rho}{\in}$$

Since, the change in value of resistivity of a material is neglected.

$$G_f = 1 + 2\nu$$

$$\text{Poission ratio, } \nu = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta D/D}{\Delta L/L}$$

where, $\Delta D = 0.02$ mm, $D = 1.5$ mm

$\Delta L = 1$ mm, $L = 24$ mm

$$\therefore \qquad \nu = \frac{\frac{0.02}{1.5}}{\frac{1}{24}} = 0.32$$

$$\text{Gauge factor } G_f = 1 + 2(0.32) = 1.64$$

Now, the change in the value of resistance of the gauge when strained is given by

$$\frac{\Delta R}{R} = G_f \times \frac{\Delta L}{L}$$

$$\Delta R = 1.64 \times \left(\frac{1}{24}\right) \times 120$$

$$\Delta R = 8.2\ \Omega$$

Q.1 (d) Solution:

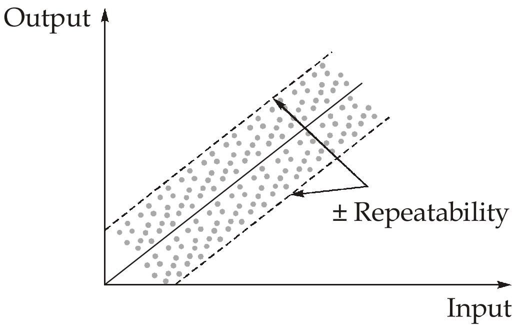
(i) Reproducibility:

It is the degree of closeness with which a given value may be repeatedly measured or in other words measured value does not change with time.

(ii) Repeatability:

Repeatability is defined as the variation of scale reading and is random in nature.

In other words, Repeatability is defined as the closeness of a number of measured values of the same quantity under the same conditions (such as the same observer, the same method, the same apparatus, and the same environment). This is affected by internal noise and drift. The repeatability is expressed in percentage of the true value.



(Input-output relationship with ± Repeatability)

(iii) Accuracy:

Accuracy is the closeness to the true value of the quantity being measured.

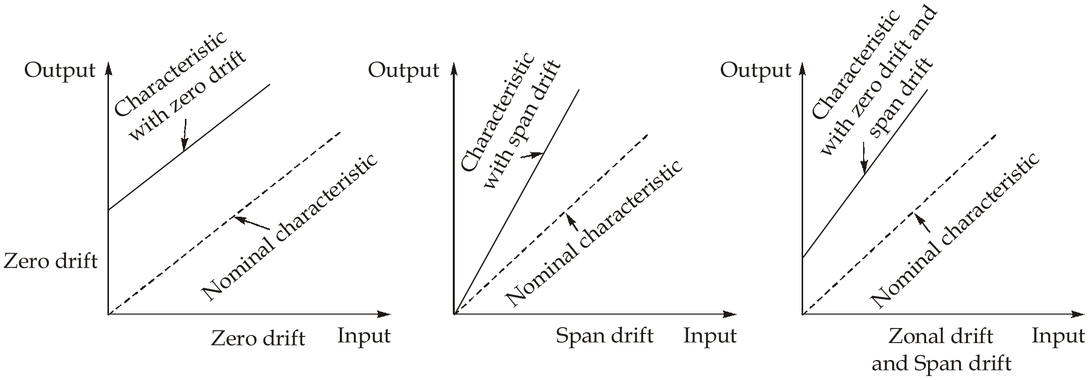
- Accuracy requires precision but precision does not require accuracy.
- Accuracy can be improved by re-calibration but precision cannot.

(iv) Drift:

Drift means variation in reading with respect to time. Drift may be classified into three categories:

- Zero drift
- Span drift or sensitivity drift
- Zonal drift

The input-output characteristics with drift are shown in figure below:



(v) Precision:

Precision is used in measurements to describe the consistency or the reproducibility of results.

Q.1 (e) Solution:

(i) Given : $(27.35)_8 = 2 \times 8^1 + 7 \times 8^0 + 3 \times 8^{-1} + 5 \times 8^{-2}$

$$= 16 + 7 + \frac{3}{8} + \frac{5}{64}$$

$$= 16 + 7 + 0.375 + 0.078125$$

$$= (23.45)_{10}$$

(ii) Given : $(110111.101)_2$

Decimal equivalent of $(110111.101)_2$

$$= 2^5 \times 1 + 2^4 \times 1 + 2^3 \times 0 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 1$$
$$+ 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$

$$= 55.625$$

(iii) $(3D5)_{16} = 0011\ 1101\ 0101$

$$= 2^9 \times 1 + 2^8 \times 1 + 2^7 \times 1 + 2^6 \times 1 + 2^4 \times 1 + 2^2 \times 1 + 2^0 \times 1$$

$$= 512 + 256 + 128 + 64 + 16 + 4 + 1 = (981)_{10}$$

Alternative:

$$(3D5)_{16} = 3 \times 16^2 + D \times 16^1 + 5 \times 16^0$$

$$= 3 \times 16^2 + 13 \times 16 \times 5 \times 1$$

$$= (981)_{10}$$

Q.2 (a) Solution:

Given, Memory unit = 256 K words

$$= 2^8 \times 2^{10} = 2^{18} \text{ words}$$

(i) An instruction code is stored in one word of memory.

∴ Instruction size = 32 bits

The instruction has 4 field.

32 bits

Indirect bit	Opcode	Register code	Address

There are 64 registers.

∴ bits required to specify one register = $\log_2 64 = 6$ bits

For address, bits required = $\log_2 2^{18} = 18$ bits

For indirect bit = 1 bit (given)

∴ Bits required for opcode = $32 - 1 - 6 - 18$

= 7 bits for opcode

(ii) For register code part, bits required = 6 bits.

For address part, bits required = 18 bits

(iii)

32 bits

Indirect bit	Opcode	Register code	Address
1 bit	7 bits	6 bits	18 bits

(iv) The memory unit has a capacity of 2^{18} x 32 bits. Hence, in the data input, No. of bits = 32 bits.

In address input, No. of bits = 18 bits

Q.2 (b) Solution:

Resistance of fixed (field) coils,

$$R_1 = 3\ \Omega$$

Reactance of fixed coils at 50 Hz,

$$X_1 = 2\pi \times 50 \times 0.12 = 37.7\ \Omega$$

Reactance of moving coil at 50 Hz,

$$X_2 = 2\pi \times 50 \times 0.003 = 0.9425\ \Omega$$

Let the current being measured be I

and current flowing through fixed coil be I_1 ,

$$I_1 = \frac{IR_2}{R_1 + R_2} = \frac{I \times 30}{30 + 3} = 0.909\ I$$

Let current flowing through moving coil be I_2 ,

$$I_2 = \frac{I\,R_1}{R_1 + R_2} = \frac{3I}{33} = \frac{I}{11}$$

Deflection, $\theta \propto I_1 I_2$

$$= KI_1 I_2 = 0.0826\ KI^2$$

with A.C impedance of fixed coils,

$$Z_1 = \sqrt{(3)^2 + (37.7)^2} = 37.8\ \Omega$$

Phase angle, $\alpha_1 = \tan^{-1}\frac{37.7}{3} = 85.4°$

Impedance of moving coil $Z_2 = \sqrt{(30)^2 + (0.9425)^2} = 30\ \Omega$

Phase angle, $\alpha_2 = \tan^{-1}\frac{0.9425}{30} = 1.8°$

Current through fixed coil, $I_1 = \frac{Z_2\angle\alpha_2}{Z_1\angle\alpha_1 + Z_2\angle\alpha_2} \times I = 0.5904I\angle{-47.7°}$

Current through moving coil, $I_2 = \frac{Z_1\angle\alpha_1}{Z_1\angle\alpha_1 + Z_1\angle\alpha_2} \times I = 0.7431I\angle 36°$

Phase difference between I_1 and I_2 is

$$\phi = -47.7° - 36°$$
$$= -83.7°$$

$\therefore$ $\cos\phi = 0.1097$

Deflection with a.c.

$$\theta = KI_1I_2\cos\phi$$
$$= K \times 0.5904 \times I \times 0.743 \times I \times 0.1097$$
$$= 0.048\ KI^2$$

$$\text{Percentage error} = \frac{\text{Reading on a.c.} - \text{Reading on d.c.}}{\text{Reading on d.c.}}$$

$$= \frac{0.048KI^2 - 0.0826KI^2}{0.0826KI^2} \times 100 = -41.96\%$$

Q.2 (c) Solution:

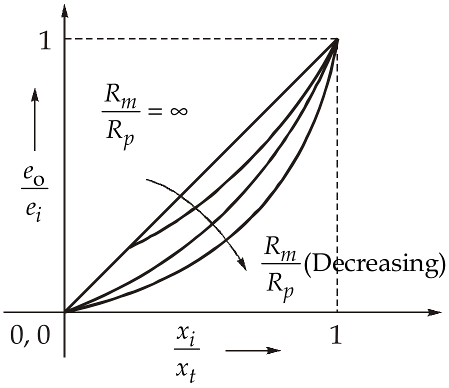
(i) Merits of Potentiometer:

1. They are inexpensive.
2. They are simple to operate and very useful for applications where the requirements are not particularly severe.
3. They are very useful for measurement of large amplitudes of displacement (in cm scale).
4. Their electrical efficiency is very high and they provide sufficient output to permit control operation without further amplification.
5. It should be understood that while the frequency response of wire wound potentiometers is limited, the other types of potentiometers are free from this problem.
6. In wire wound potentiometers the resolution is limited, while in cermet and metal film potentiometers, the resolution in infinite.

Demerits:

1. The main disadvantage of using a linear potentiometer is that they require a large force to move their sliding contacts (wipers).

2. The other problems with sliding contacts are that they can be contaminated, can wear out, become misaligned and generate noise. So the life of the transducer is limited. However, recent developments have produced a roller contact wiper which (it is claimed that it) increases the life of the transducer upto 40 times.

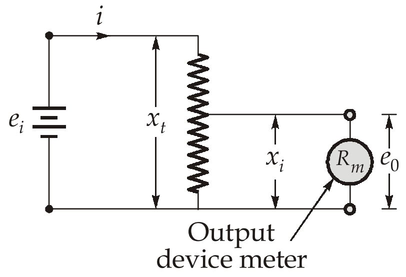


$$\text{Sensitivity, } S = \frac{\text{output}}{\text{input}} = \frac{e_0}{x_i} = \frac{e_i}{x_t}$$

$$e_0 = \frac{x_i}{x_t} e_i$$

$$\frac{e_0}{e_i} = \frac{x_i}{x_t}$$

Let, $$K = \frac{x_i}{x_t}$$



The total resistance seen by the source is:

$$R = R_p(1-K) + \frac{KR_pR_m}{KR_p + R_m}$$

$$= \frac{KR_p^2(1-K) + R_pR_m}{KR_p + R_m}$$

$$\therefore \quad \text{Current, } i = \frac{e_i}{R} = \frac{e_i(KR_p + R_m)}{KR_p^2(1-K) + R_pR_m}$$

The output voltage under load condition is:

$$e_0 = i\frac{KR_pR_m}{KR_p + R_m} = \frac{e_i(KR_p + R_m)}{KR_p^2(1-K) + R_pR_m} \times \frac{KR_pR_m}{(KR_p + R_m)}$$

$$= \frac{e_i K}{K(1-K)(R_p / R_m)+1}$$

The ratio of output voltage to input voltage under load condition is:

$$\frac{e_0}{e_i} = \frac{K}{K(1-K)(R_p / R_m)+1}$$

(ii) We know,

Deflecting torque due to a weight is

$$T_d = \text{mg} \sin\theta \times d$$

Where, d = distance of the weight from spindle

$$d = \frac{T_d}{mg\sin\theta} = \frac{1.13\times10^{-3}}{5\times10^{-3}\times9.81\times\sin60°}$$

$$= 0.0266 \text{ m} = 26.6 \text{ mm}$$

Q.3 (a) Solution:

(i) Multiplying power of shunt,

$$m = \frac{I}{I_m} = \frac{50}{5} = 10$$

In order that the meter may read correctly at all frequencies the time constants of meter and shunt circuits should be equal. Under this condition multiplying power m is,

$$m = 1+\frac{R}{R_{sh}}$$

∴ Resistance of shunt,

$$R_{sh} = \frac{R}{m-1} = \frac{0.09}{10-1} = 0.01\ \Omega$$

Also, $$\frac{L}{R} = \frac{L_{sh}}{R_{sh}}$$

∴ Inductance of shunt,

$$L_{sh} = \frac{L}{R}R_{sh} = \frac{90}{0.09}\times0.01 = 10\ \mu\text{H}$$

If the shunt is made non-inductive:

With d.c. the current through the meter for a total current of 50 A is,

$$I_m = \frac{R_{sh}}{R+R_{sh}} \times I = \frac{0.01}{0.09+0.01} \times 50 = 5.0 \text{ A}$$

With 50 Hz, the current through the meter for a total current of 50 A is

$$I_m = \frac{R_{sh}}{\sqrt{(R+R_{sh})^2+\omega^2 L^2}} \times I$$

$$= \frac{0.01 \times 50}{\sqrt{(0.09+0.01)^2+(2\pi \times 50 \times 90 \times 10^{-6})^2}} = 4.81 \text{ A}$$

Since the meter reading is proportional to the current,

$$\text{Error} = \frac{4.81-5}{5} \times 100\% = -3.8\%$$

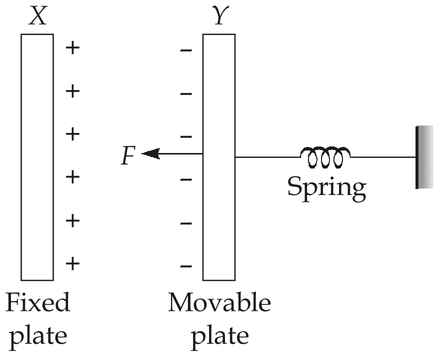
The meter reads 3.8% low.

(ii) Let,

V = Potential difference applied to plate X and Y

C = Capacitance between two plates

F = Force between plates



$$\text{Energy stored} = \frac{1}{2} CV^2$$

Let there be small change by dV in the applied voltage, the plate Y move towards X by small distance 'dx'.

For increase in applied voltage, current flow will be

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt}$$

$$\text{Input energy} = Vi\,dt = V^2\,dC + CV \cdot dV$$

Change in stored energy

$$= \frac{1}{2}(C+dC)(V+dV)^2 - \frac{1}{2}CV^2 = \frac{1}{2}V^2 dC + CV \cdot dV$$

(After neglecting higher order terms)

Using principle of energy conservation,

Input electrical energy = Increased in stored energy + Mechanical work done

$$V^2\,dC + CV\cdot dV = \frac{1}{2}V^2 dC + CV\cdot dV + F\cdot dx$$

On solving further,

$$F = \frac{1}{2}V^2\frac{dC}{dx}$$

Q.3 (b) Solution:

(i) Round Robin with time quantum of 2 unit.

Ready Queue

P_1	P_2	P_3	P_1	P_4	P_5	P_2	P_6	P_3	P_6	P_3

Gantt Chart:

P_1	P_2	P_3	P_1	P_4	P_5	P_2	P_6	P_3	P_6	P_3

0 2 4 6 8 9 11 12 14 16 18 19

Process id	Service Time	Arrival Time	C.T	TAT	W.T
P_1	4	0	8	8	4
P_2	3	1	12	11	8
P_3	5	2	19	17	12
P_4	1	3	9	6	5
P_5	2	4	11	7	5
P_6	4	6	18	12	8

$$\text{TAT} = \text{CT} - \text{AT}$$

$$\text{WT} = \text{TAT} - \text{BT} \qquad \text{(BT = Service/Burst Time)}$$

$$\text{Avg TAT} = \frac{\text{Sum of TAT of all process}}{\text{Total no. of process}}$$

$$= \frac{8+11+17+6+7+12}{6} = 10.167$$

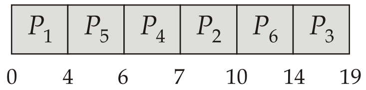
$$\text{Avg. WT} = \frac{\text{Sum of WT of all process}}{\text{Total no. of process}}$$

$$= \frac{4+8+12+5+5+8}{6} = 7$$

(ii) Non-preemptive priority scheduling:

In non-preemptive scheduling, if some resource is allocated to a process then that resource will not be taken back until the completion of the process. In the Non Preemptive Priority scheduling, the Processes are scheduled according to the priority number assigned to them. Once the process gets scheduled, it will run till the completion. The lower the priority number, the higher is the priority of the process.

Gantt Chart:



Process id	B.T	A.T	C.T	TAT	W.T
P_1	4	0	4	4	0
P_2	3	1	10	9	6
P_3	5	2	19	17	12
P_4	1	3	7	4	3
P_5	2	4	6	2	0
P_6	4	6	14	8	4

$$\text{Avg. TAT} = \frac{4+9+17+4+2+8}{6} = 7.33$$

$$\text{Avg. WT} = \frac{0+6+12+3+0+4}{6} = 4.1667$$

So, on comparing the both result, the average turn-around time and waiting time of non-preemptive CPU scheduling is less than the round robin cpu scheduling.

Q.3 (c) Solution:

Given $N_1 = 1$, $N_2 = 199$, $n = 199$ = CT turns ratio

Nominal CT ratio; $k_n = \frac{1000}{5} = 200, I_W = 4A \;\; I_\mu = 7\,\text{A}$

CT secondary rated current, $I_S = 5$ A

For 0.8 pf lagging: (δ = +ve)

$$\cos\delta = 0.8$$

$$\therefore \quad \sin\delta = 0.6$$

Actual CT ratio, $R = n + \left(\frac{I_W \cos\delta + I_\mu \sin\delta}{I_S}\right) = \left[199 + \left(\frac{4\times0.8+7\times0.6}{5}\right)\right]$

or, $R = (199 + 1.48) = 200.48$

$$\therefore \qquad \text{Ratio error} = \left(\frac{k_n - R}{R}\right)\times 100$$

$$= \left(\frac{200 - 200.48}{200.48}\right)\times 100 = -0.24\% \approx -0.239\%$$

$$\text{And phase angle error} = \left[\left(\frac{I_\mu \cos\delta - I_W \sin\delta}{n I_S}\right)\times\frac{180}{\pi}\right]^\circ$$

$$= \left[\left(\frac{7\times 0.8 - 4\times 0.6}{199\times 5}\right)\times\frac{180}{\pi}\right]^\circ = 0.18426^\circ$$

For 0.8 pf leading: (δ = –ve)

$$\therefore \qquad \cos\delta = 0.8$$

$$\sin\delta = -0.6$$

$$\therefore \quad \text{Actual CT ratio,} \qquad R = n + \left(\frac{I_W \cos\delta + I_\mu \sin\delta}{I_S}\right)$$

$$= \left[199 + \left(\frac{4\times 0.8 - 7\times 0.6}{5}\right)\right] = 198.8$$

$$\therefore \qquad \%\ \text{Ratio error} = \left(\frac{k_n - R}{R}\right)\times 100 = \left(\frac{200 - 198.8}{198.8}\right)\times 100 = 0.603\%$$

$$\text{and} \qquad \text{Phase angle error} = \left[\left(\frac{I_\mu \cos\delta - I_W \sin\delta}{n I_S}\right)\times\frac{180}{\pi}\right]^\circ$$

$$= \left[\left(\frac{7\times 0.8 + 4\times 0.6}{199\times 5}\right)\times\frac{180}{\pi}\right]^\circ = 0.46^\circ$$

Q.4 (a) (i) Solution:

Virtual memory: Virtual memory is the separation of user logical memory from physical memory. This technique provides larger memory to the user by creating virtual memory space. It facilitates the user to create a process which is larger than the physical memory space. We can have more processes executing from the memory at a time.

It increases degree of multiprogramming. With the virtual memory technique, we can execute a process which is only partially loaded in the memory.

This concept states that, declare some portion of a secondary memory as main memory and store the program into secondary memory. Later, transfer the program from secondary memory to the main memory in a form of pages based on the CPU demand called Demand Paging.

Virtual memory concept is implemented using

(i) Paging (ii) Segmentation

Advantages of Virtual memory:

1. A process can execute without having all its pages in physical memory.
2. A user process can be larger than physical memory.
3. Higher degree of multiprogramming.
4. Less I/O for loading and unloading for individual user processes.
5. Higher CPU utilization and throughput.
6. Allows address spaces to be shared by several processes.
7. Memory is used more efficiently because the only sections of a job stored in memory are those needed immediately while those not needed remain in secondary storage.

Disadvantages of Virtual memory:

1. Increased processor hardware costs.
2. Increased overheads for handling pages interrupts.
3. Increased software complexity to prevent thrashing.

Q.4 (a) (ii) Solution:

Given sequence:

A, B, C, D, B, E, D, A, C, E, C, E

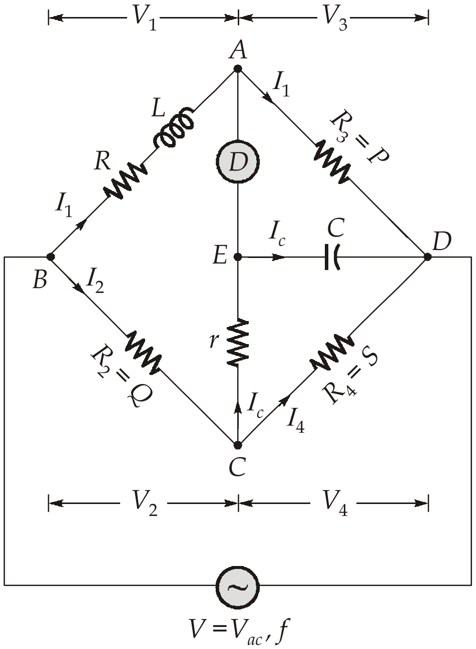
Replacement Algorithm used → LRU

LRU

Count Value →	3	2	1	0
Initial words →	A	B	C	D
B is a hit	A	C	D	B
E is a miss	C	D	B	E
D is a hit	C	B	E	D
A is a miss	B	E	D	A
C is a miss	E	D	A	C
E is a hit	D	A	C	E
C is a hit	D	A	E	C
E is a hit	D	A	C	E

Q.4 (b) Solution:

Given circuit is Anderson's bridge and can be redrawn as,



L_1 : Self inductance to be measured
R_1 : Resistance of self inductor
r, R_4, R_2, R_3 : Known non inductive resistances
C : Fixed standard capacitor

$$I_1 = I_3$$

and $$I_2 = I_C + I_4$$

We know, by using KVL, $$I_1R_3 = I_C \times \frac{1}{j\omega C}$$

$$I_C = I_1 j\omega CR_3$$

Writing the balance equations for bridge,

$$I_1(R_1 + j\omega L_1) = I_2R_2 + I_cr \quad \text{...(i)}$$

and $$I_C\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_c)R_4 \quad \text{...(ii)}$$

Substituting value of I_C in above equation (i) and (ii),

$$I_1(R_1 + j\omega L_1) = I_2R_2 + I_1\ j\omega CR_3r$$

or $$I_1(R_1 + j\omega L_1 - j\omega CR_3r) = I_2R_2 \quad \text{...(iii)}$$

and $$j\omega CR_3I_1\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_1j\omega CR_3)R_4$$

$$I_1(j\omega CR_3r + j\omega CR_3R_4 + R_3) = I_2R_4 \qquad \text{...(iv)}$$

Using equation (iii) and (iv),

$$I_1(R_1 + j\omega L_1 - j\omega CR_3r) = I_1\left(j\omega CR_3R_2 + j\omega\frac{CR_2R_3r}{R_4} + \frac{R_2R_3}{R_4}\right)$$

Equating real and imaginary parts,

$$R_1 = \frac{R_2R_3}{R_4} \qquad \text{...(v)}$$

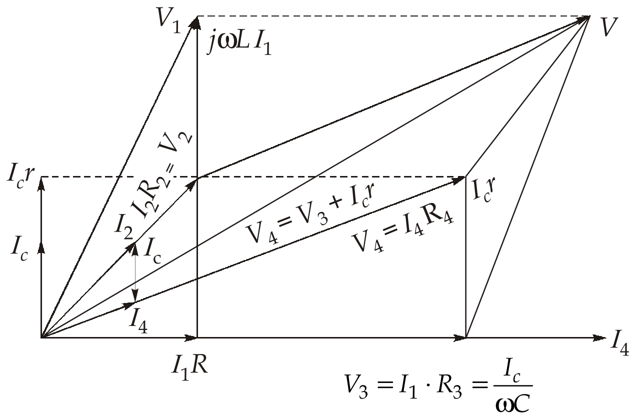
$$L_1 = C\frac{R_3}{R_4}[r(R_4+R_2)+R_2R_4] \qquad \text{...(vi)}$$

(i)

$$R = \frac{PQ}{S} = \frac{100\times1000}{1000} = 100\ \Omega$$

$$L = \frac{CP}{S}[r(S+Q)+QS] = 0.11\ \text{H}$$

(ii) The phasor diagram can be drawn as follows:



Q.4 (c) (i) Solution:

Measured value = $VI\sin(\Delta - f)$

where, Δ = phase angle between voltage and flux

$\cos\phi$ = power factor

true value = $VI\cos\phi$

error = measured value – true value

Case-I: $\Delta = 85°$, pf = $\cos\phi = 1$, $\phi = 0°$

$V = 220$ V, $I = 5$ A

Error = $VI\sin(\Delta - \phi) - VI\cos\phi$

$$= 220 \times 5 \sin(85 - 0) - 220 \times 5 \times 1 = -4.2 \text{ W}$$

Case-II:

$$\Delta = 85°, \text{pf} = \cos\phi = 0.5 \Rightarrow \phi = 60°$$

$$V = 220 \text{ V}, I = 5 \text{ A}$$

$$\text{Error} = VI \sin(\Delta - \phi) - VI \cos\phi$$

$$= 220 \times 5 \times \sin(85° - 60°) - 220 \times 5 \times 0.5$$

$$= -85.1 \text{ W}$$

Q.4 (c) (ii) Solution:

Data given :

$$I_m = 1 \times 10^{-3} \text{ A}$$

$$R_m = 50 \ \Omega$$

$$I_1 = 1 \text{ A}; \ I_2 = 5 \text{ A}; \ I_3 = 10 \text{ A}; \ I_4 = 20 \text{ A}$$

$$m_1 = \frac{I_1}{I_m} = 1000$$

$$m_2 = \frac{I_2}{I_m} = 5000$$

$$m_3 = \frac{I_3}{I_m} = 10000$$

$$m_4 = \frac{I_4}{I_m} = 20000$$

$$R_{sh1} = \frac{R_m}{m_1 - 1} = \frac{50}{1000 - 1} = 0.05 \ \Omega$$

$$R_{sh2} = \frac{R_m}{m_2 - 1} = \frac{50}{5000 - 1} = 0.01 \ \Omega$$

$$R_{sh3} = \frac{R_m}{m_3 - 1} = \frac{50}{10000 - 1} = 0.005 \ \Omega$$

$$R_{sh4} = \frac{R_m}{m_4 - 1} = \frac{50}{20000 - 1} = 0.0025 \ \Omega$$

∴ The resistance of various sections of universal shunt are :

$$R_1 = R_{sh1} - R_{sh2} = 0.05 - 0.01 = 0.04 \ \Omega$$

$$R_2 = R_{sh2} - R_{sh3} = 0.01 - 0.005 = 0.005 \ \Omega$$

$$R_3 = R_{sh3} - R_4 = 0.005 - 0.0025 = 0.0025 \ \Omega$$

$$R_4 = R_{sh4} = 0.0025 \ \Omega$$

Section B : Power Electronics & Drives-1 + Engineering Mathematics-1 + Basic Electronics Engg.-2 + Analog Electronics-2 + Electrical Materials-2

Q.5 (a) Solution:

$$A = \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix} \quad \text{...(i)}$$

$$\bar{A} = \begin{bmatrix} 1-i & 2 & 5+5i \\ -2i & 2-i & 4-2i \\ -1-i & -4 & 7 \end{bmatrix}$$

$$(\bar{A})' = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix}$$

$$A^{\theta} = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix} \quad \text{...(ii)}$$

On adding (i) and (ii), we get

$$A + A^{\theta} = \begin{bmatrix} 2 & 2-2i & 4-6i \\ 2+2i & 4 & 2i \\ 4+6i & -2i & 14 \end{bmatrix}$$

Let, $$R = \frac{1}{2}(A + A^{\theta})$$

$$= \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix} \quad \text{...(iii)}$$

On subtracting (ii) from (i), we get

$$A - A^{\theta} = \begin{bmatrix} 2i & 2+2i & 6-4i \\ -2+2i & 2i & 8+2i \\ -6-4i & -8+2i & 0 \end{bmatrix}$$

Let, $$S = \frac{1}{2}(A - A^{\theta}) = \begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix} \quad \text{...(iv)}$$

From (iii) and (iv), we have

$$A = \underbrace{\begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix}}_{\text{Hermitian matrix}} + \underbrace{\begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}}_{\text{Skew-Hermitian matrix}}$$

Q.5 (b) Solution:

$$f(x) = x \log_{10} x - 1.2$$

$$f(1) = -1.2 = -\text{ve}$$

$$f(2) = 2 \log_{10} 2 - 1.2$$

$$= 0.59794 = -\text{ve}$$

$$f(3) = 3 \log_{10} 3 - 1.2$$

$$= 1.4314 - 1.2 = 0.23136 = +\text{ve}$$

So a root of $f(x) = 0$ lies between 2 and 3,

Let us take, $x_0 = 2$

Also $f'(x) = \log_{10} x + x \cdot \dfrac{1}{x \ln 10}$

$$= \log_{10} x + 0.43429$$

∴ Newton's iteration formula gives,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = \frac{0.43429 \times x_n + 1.2}{\log_{10} x_n + 0.43429} \quad \text{...(i)}$$

Putting $n = 0$, the first approximation is

$$x_1 = \frac{0.43429 \times x_0 + 1.2}{\log_{10} x_0 + 0.43429}$$

$$= \frac{0.43429 \times 2 + 1.2}{\log_{10} 2 + 0.43429}$$

$$= \frac{0.86858 + 1.2}{0.30103 + 0.43429} = 2.813169$$

Similarly, putting $n = 1, 2, 3, 4$ in equation (i), we get

$$x_2 = 2.741109$$

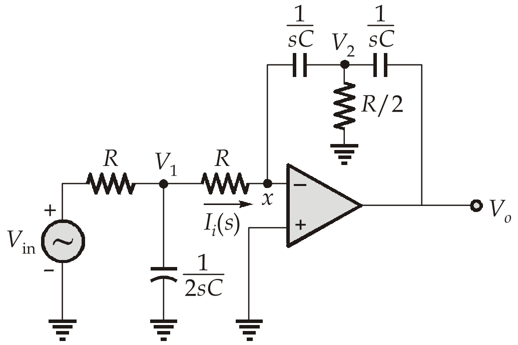
$$x_3 = 2.740646$$

$$x_3 = 2.740646$$

∴ $x = 2.74065$ (rounded off to five decimal places)

Q.5 (c) Solution:

The above figure can be represented as :



$$V_x = 0 \text{ due to virtual ground}$$

Applying KCL at node V_1, we get

$$(2sC)V_1 + \frac{V_1 - V_{in}}{R} + \frac{V_1}{R} = 0 \quad \text{...(i)}$$

$$\frac{V_1}{R} = I_i(s) \quad \text{...(ii)}$$

On putting equation (2) in equation (1)

$$(2sRC + 2)I_i(s) = \frac{V_{in}}{R} \quad \text{...(iii)}$$

On applying KCL at node V_2, we get

$$\frac{2V_2}{R} + sCV_2 + (V_2 - V_0)(sC) = 0 \quad \text{...(iv)}$$

Let $I_f(s)$ is the current across the capacitor.

Therefore, $$I_f(s) = V_2(sC)$$

$$\left(\frac{2}{sRC} + 2\right) I_f(s) = (sC)V_0(s)$$

$$I_f(s) = \frac{(sC)V_0(s)}{2\left[1 + \frac{1}{sRC}\right]}$$

$$= \frac{s^2RC^2V_0(s)}{2[1 + sRC]} \quad \text{...(v)}$$

From the above circuit, it is clear that

$$I_i(s) = -I_f(s)$$ [∵ Since Op-amp is ideal]

So, $$\frac{V_{\text{in}}(s)}{2R(1+sRC)} = \frac{-s^2RC^2V_0(s)}{2(1+sRC)}$$

So, $$\frac{V_0(s)}{V_{\text{in}}(s)} = -\frac{1}{(sRC)^2}$$

Q.5 (d) Solution:

$$x^2 + y^2 + z^2 = 4a^2 \quad \text{...(i)}$$

$$x^2 + y^2 - 2ay = 0 \quad \text{...(ii)}$$

Considering the section in the positive quadrant of the xy-plane and taking z to be positive (that is volume above the xy-plane) and changing to polar co-ordinates,

(i) Becomes,

$$r^2 + z^2 = 4a^2$$

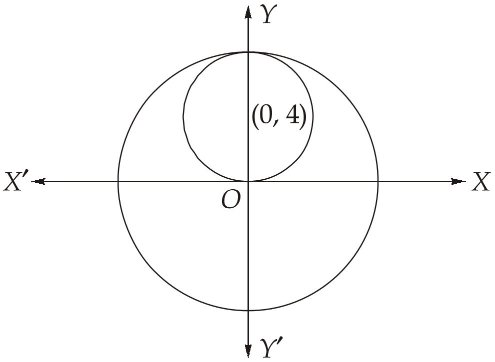
$$z^2 = 4a^2 - r^2$$

$$\therefore \quad z = \sqrt{4a^2 - r^2}$$

(ii) becomes, $r^2 - 2\,ar\sin\theta = 0$

$$r = 2\,a\sin\theta$$

$$\text{Volume} = \iiint dx\,dy\,dz = 4\int_0^{\pi/2} d\theta \int_0^{2a\sin\theta} r\,dr \int_0^{\sqrt{4a^2-r^2}} dz$$



$$= 4\int_0^{\pi/2} d\theta \int_0^{2a\sin\theta} r\,dr\,[z]_0^{\sqrt{4a^2-r^2}} = 4\int_0^{\pi/2} d\theta \int_0^{2a\sin\theta} r\sqrt{4a^2-r^2}\,dr$$

Let, $4a^2 - r^2 = t$

$$-2r\,dr = dt$$

For $r = 0$, $t = 4a^2$

For $r = 2\,a\sin\theta$, $t = 4a^2\cos^2\theta$

$$= 4\int_0^{\pi/2} d\theta \int_{4a^2}^{4a^2\cos^2\theta} \frac{1}{(-2)}\sqrt{t}\,dt$$

$$= 4\int_0^{\pi/2} d\theta\left(\frac{-1}{2}\times\frac{2}{3}\times(t)^{3/2}\right)_{4a^2}^{4a^2\cos^2\theta}$$

$$= \frac{-4}{3}\int_0^{\pi/2}(4a^2\cos^2\theta)^{3/2}-(4a^2)^{3/2}\,d\theta$$

$$= \frac{-4}{3}\int_0^{\pi/2}8a^3(\cos^3\theta-1)\,d\theta = \frac{32a^3}{3}\int_0^{\pi/2}(1-\cos^3\theta)\,d\theta$$

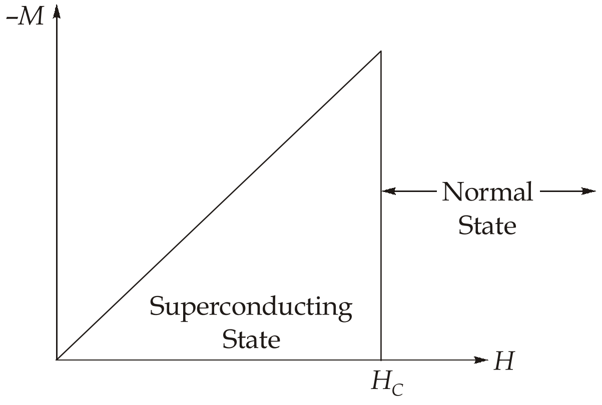
$$= \frac{32a^3}{3}\int_0^{\pi/2}\left(1-\frac{1}{4}\cos 3\theta-\frac{3}{4}\cos\theta\right)d\theta$$

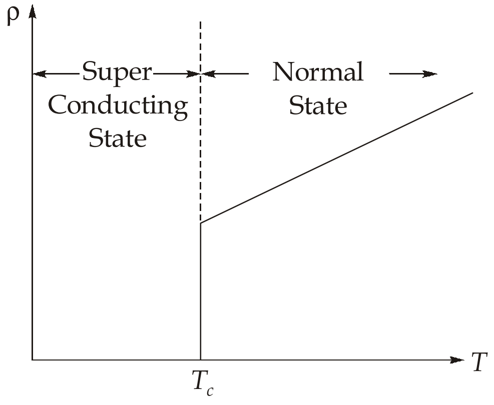
$$= \frac{32a^3}{3}\left[\theta-\frac{1}{12}\sin 3\theta-\frac{3}{4}\sin\theta\right]_0^{\pi/2}$$

$$= \frac{32a^3}{3}\left(\frac{\pi}{2}+\frac{1}{12}-\frac{3}{4}\right) = \frac{32a^3}{3}\left(\frac{\pi}{2}-\frac{2}{3}\right)$$

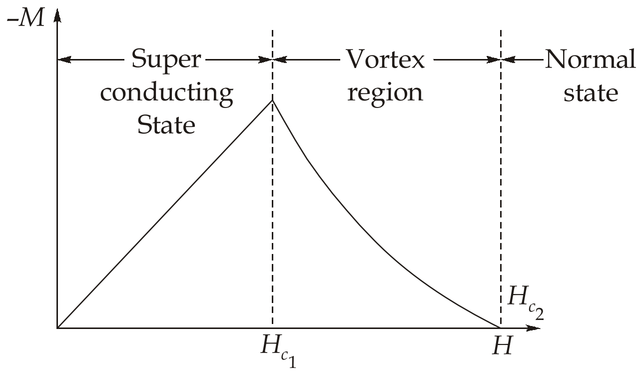
Q.5 (e) Solution:

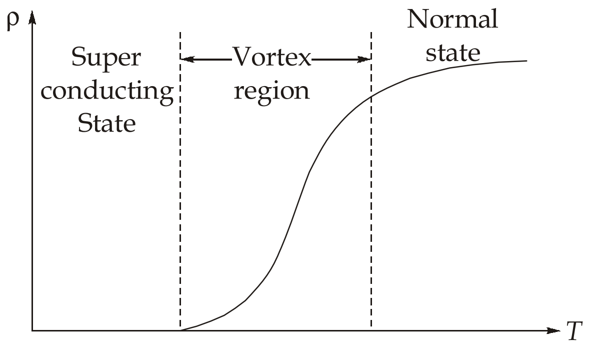
(i) **Type-I Superconductors :** Those superconductors which show perfect diamagnetism upto the critical field (H_c) and acquire normal state beyond critical field are known as type-1 or ideal superconductors. The critical field of these superconductors are low. These materials obey Silsbee's rule and Meissner effect. The transition from superconducting state to normal state is abrupt. e.g., Pb, In, Pd, Hg, etc.



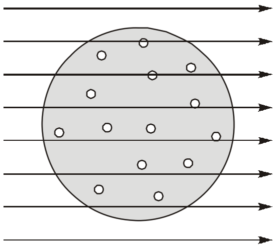


(ii) Type-II Superconductors : Those superconductors in which the ideal behaviour is seen upto lower critical field H_{c1} beyond which the magnetization gradually changes and attains zero at an upper critical field H_{c2}. They have high critical field and critical temperature. These material incompletely follow Silsbee's rule and Meissner effect. e.g., NbTi, Nb_3Al.

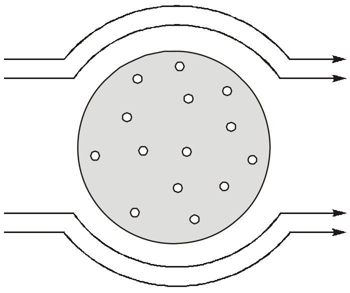




(iii) Meissner Effect : If a superconductor is cooled in a magnetic field to below critical temperature T_c (transition temperature), then at the transition the magnetic lines are pushed out. The expulsion of magnetic flux from interior of a superconductor material as material undergoes the transition to the superconducting phase is known as Meissner effect. Meissner effect is reversible in nature.



$T > T_c$



$T < T_c$

$$\vec{B} = \mu_o(\vec{H}+\vec{M}) = \mu_o(1+\chi)H$$

$$\vec{\chi} = \frac{\vec{M}}{\vec{H}}$$

where,

H = applied magnetic field

M = magnetization

χ = magnetic susceptibility

B = $0, \chi = -1, \mu_r = 0$

Q.6 (a) Solution:

The average output voltage V_o :

$$V_o = \frac{2V_m}{\pi}\cos\alpha = \frac{2\times 230\sqrt{2}}{\pi}\cos 50^\circ$$

$$V_o = 133.10 \text{ Volts}$$

Now the average load current can be given as,

$$I_o = \frac{V_o - E}{R} \qquad \text{(load current is continuous)}$$

$$I_o = \frac{133.10 - 60}{6} = 12.18 \text{ A}$$

Now, the rms value fundamental component of supply current is

$$I_{s1} = \frac{2\sqrt{2}I_o}{\pi} = 0.9 \times 12.18$$

$$= 10.962 \text{ A}$$

Active power input to the converter is

$$P_{\text{in}} = V_{sr}.I_{s1}\cos\alpha$$

$$= 230 \times 10.962 \times \cos 50^\circ$$

$$P_{\text{in}} = 1620.63 \text{ Watts}$$

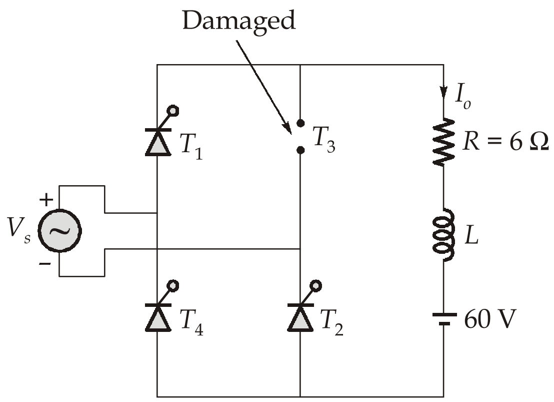
Reactive power input to converter,

$$Q_{\text{in}} = P_{\text{in}}\tan\alpha$$

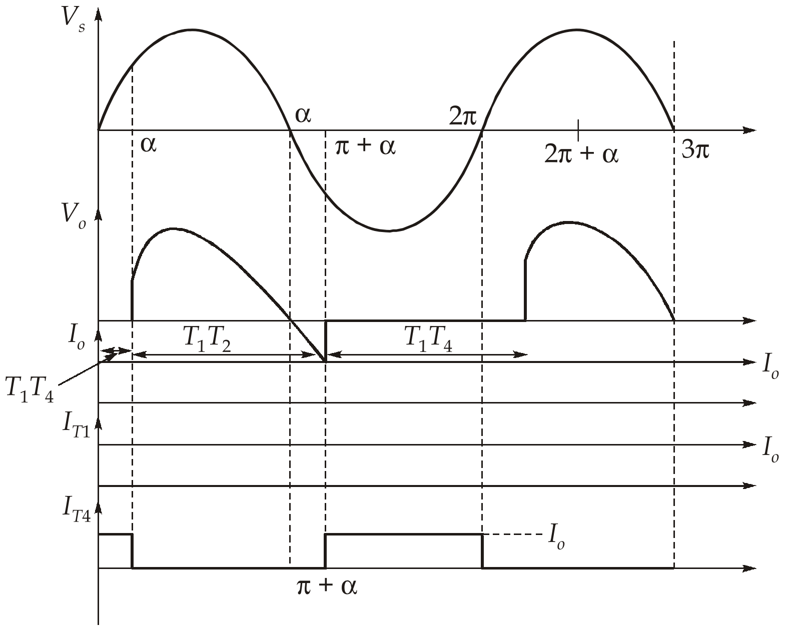
$$= 1620.63 \times \tan 50^\circ$$

$$Q_{\text{in}} = 1931.397 \text{ VAR}$$

Now, one of thyristor is damaged :



Waveform :



Now, average load current is given as

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin \omega t \cdot d\omega t$$

$$V_o = \frac{V_m}{\pi} \cos \alpha$$

$$V_o = \frac{\sqrt{2} \times 230}{\pi} \cos 50°$$

$$V_o = 66.552 \text{ V}$$

Load current,

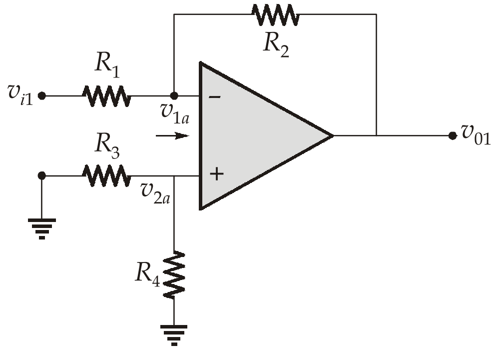
$$I_o = \frac{V_o - E}{R} = \frac{66.552 - 60}{6}$$

$$I_o = 1.092 \text{ A}$$

Q.6 (b) Solution:

Using superposition and the virtual short concepts.

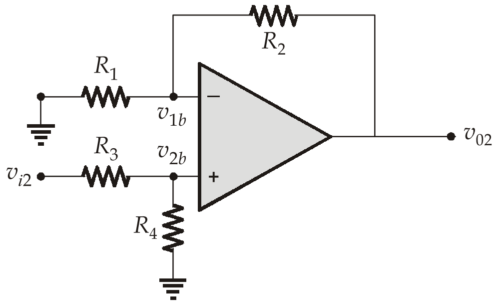
(i) When v_{i1} is acting alone:



When input v_{i2} = 0, no current will flow in R_3 and R_4. There v_{2a} = 0. For remaining circuit

$$v_{01} = -\frac{R_2}{R_1} \cdot v_{i1} \qquad \text{...(i)}$$

When v_{i2} is acting alone:



As current through Op-amp input terminals is zero, R_3 and R_4 form a voltage divider.

$$v_{2b} = \frac{R_4}{R_3 + R_4} v_{i2}$$

Also, $\quad v_{2b} = v_{1b} \quad$ (Virtual short)

So, $$v_{02} = \left(1 + \frac{R_2}{R_1}\right) v_{1b} = \left(1 + \frac{R_2}{R_1}\right) v_{2b}$$

$$= \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) v_{i2}$$

Which can be arranged as

$$v_{02} = \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4/R_3}{1+R_4/R_3}\right)v_{i2} \quad \text{... (ii)}$$

Net output, $v_0 = v_{01} + v_{02}$

From equations (i) and (ii),

$$v_0 = \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4/R_3}{1+R_4/R_3}\right)v_{i2} - \frac{R_2}{R_1}v_{i1} \quad \text{... (iii)}$$

Given: $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

Putting this in equation (iii), we get,

$$v_0 = \frac{R_2}{R_1}(v_{i2} - v_{i1})$$

(ii) The circuit is called difference amplifier. It is used in instrumentation amplifier also. Although simple Op-amp is also a difference amplifier, this particular circuit gives us flexibility where output is only ratios of resistors.

(iii)

$$v_{cm} = \frac{v_{i1}+v_{i2}}{2} \quad \text{... (iv)}$$

$$A_{cm} = \frac{v_0}{v_{cm}}$$

$$\frac{R_4}{R_3} = 11, \frac{R_2}{R_1} = 10$$

From equation (iii),

$$v_0 = (1+10)\left(\frac{11}{1+11}\right)v_{i2} - 10v_{i1}$$

$$= 10.0833\, v_{i2} - 10\, v_{i1} \quad \text{... (v)}$$

Also $v_d = v_{i2} - v_{i1}$ ...(vi)

From equations (iv) and (vi),

$$v_{i1} = v_{cm} - \frac{v_d}{2}$$

$$v_{i2} = v_{cm} + \frac{v_d}{2}$$

Put this in equation (v),

$$v_0 = 10.0833\left(v_{cm} + \frac{v_d}{2}\right) - (10)\left(v_{cm} - \frac{v_d}{2}\right)$$

$$= 10.04\, v_d + 0.0833\, v_{cm}$$

$$= A_d v_d + A_{cm} v_{cm}$$

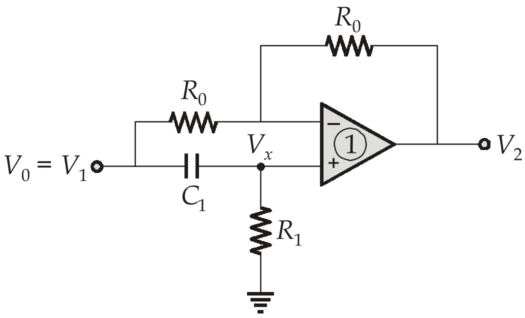
Comparing, $A_d = 10.04165$

$$A_{cm} = 0.0833$$

$$\text{CMRR(dB)} = 20\log_{10}\frac{A_d}{A_{cm}} = 20\log_{10}\frac{10.04165}{0.0833} = 41.62 \text{ dB}$$

Q.6 (c) Solution:

Consider the circuit element (1)



$$V_x = \frac{R_1}{R_1 + \frac{1}{sC_1}} \cdot V_1 \qquad \text{...(1)}$$

Also, $\dfrac{V_1 - V_x}{R_o} = \dfrac{V_x - V_2}{R_o}$

$$V_1 = 2V_x - V_2$$

$$V_1 = 2\times\frac{R_1}{R_1 + \frac{1}{sC_1}} \cdot V_1 - V_2$$

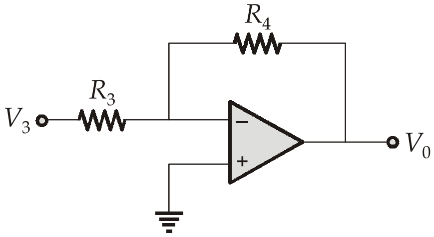
Therefore, $V_2 = \left[\dfrac{2R_1}{R_1 + \frac{1}{sC_1}} - 1\right] V_1$

$$V_2 = \left[\frac{R_1 - \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}}\right] V_1 \qquad \text{...(2)}$$

Similarly for circuit element (2),

$$V_3 = \left[\frac{R_2 - \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}}\right] V_2 \qquad \text{...(3)}$$

For circuit element (3),



$$V_o = -\frac{R_4}{R_3} V_3 \qquad \text{...(4)}$$

Therefore, total loop gain of oscillator

$$A\beta = \frac{V_2}{V_1}\cdot\frac{V_3}{V_2}\cdot\frac{V_0}{V_3}$$

$$= \left[\frac{R_1 - \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}}\right]\cdot\left[\frac{R_2 - \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}}\right]\left[-\frac{R_4}{R_3}\right]$$

$$= \left[\frac{R_1 - \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}\right]\cdot\left[\frac{R_2 - \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}\right]\left[-\frac{R_4}{R_3}\right]$$

$$|A\beta| = \frac{R_4}{R_3}$$

$$\angle A\beta = 180° - 2\tan^{-1}\left(\frac{1}{\omega R_1 C_1}\right) - 2\tan^{-1}\left(\frac{1}{\omega R_2 C_2}\right)$$

According to Barkhausen criteria,

$$|A\beta| = 1 \;\Rightarrow\; \frac{R_4}{R_3} = 1 \;\Rightarrow\; R_4 = R_3$$

$$\angle A\beta = 0° \text{ or } 360°$$

Therefore,
$$2\tan^{-1}\left(\frac{1}{\omega R_1C_1}\right)+2\tan^{-1}\left(\frac{1}{\omega R_2C_2}\right) = 180°$$

$$\tan^{-1}\left(\frac{1}{\omega R_1C_1}\right)+\tan^{-1}\left(\frac{1}{\omega R_2C_2}\right) = 90°$$

$$\tan^{-1}\left(\frac{\frac{1}{\omega R_1C_1}+\frac{1}{\omega R_2C_2}}{1-\frac{1}{\omega^2 R_1R_2C_1C_2}}\right) = 90°$$

$$1-\frac{1}{\omega^2 R_1R_2C_1C_2} = 0$$

Therefore, frequency of oscillation

$$\omega = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

if $R_1 = R_2 = R$ and $C_1 = C_2 = C$,

then
$$\omega = \frac{1}{RC}$$

Q.7 (a) Solution:

During storage time - $0 \le t \le t_s$

$$i_c(t) = I_{cs}$$

and
$$V_{CE}(t) = V_{CES}$$

Instantaneous power loss during t_s is

$$P_s(t) = i_c(t)\cdot V_{CE}(t) = I_{CS}\cdot V_{CES} = 60 \times 2.5$$

$$P_s(t) = 150 \text{ W}$$

Average power loss during t_s is

$$P_s = \frac{1}{T}\int_0^{t_s} I_{cs}\cdot V_{CE}\cdot dt = f\cdot I_{CE}\cdot V_{CES}\cdot t_s$$

$$= 10 \times 10^3 \times 60 \times 2.5 \times 4 \times 10^{-6}$$

$$P_s = 6 \text{ W}$$

During fall time, $0 \le t \le t_f$

$$i_c(t) = I_{cs}\left[1-\frac{t}{t_f}\right]$$

and $$V_{CE}(t) = \frac{V_{CC} - V_{CES}}{t_f} \cdot t$$

Average power loss during fall time

$$P_t = \frac{1}{T}\int_0^{t_f} I_{cs}\left(1 - \frac{t}{t_f}\right)\left[\frac{V_{CC} - V_{CES}}{t_f}\right] t \cdot dt$$

$$= f \cdot [V_{CC} - V_{CES}] \cdot I_{CS} \int_0^{t_f}\left(\frac{t}{t_f} - \frac{t^2}{t_f^2}\right) \cdot dt$$

$$= f[V_{CC} - V_{CES}] \cdot I_{CS} \int_0^{t_f}\left(\frac{t}{t_f} - \frac{t^2}{t_f^2}\right) \cdot dt$$

$$= f[V_{CC} - V_{CES}] \cdot I_{CS} \cdot \left[\frac{t_f^2}{2t_f} - \frac{1}{t_f^2} \cdot \frac{t_f^3}{3}\right]$$

$$= \frac{f[V_{CC} - V_{CES}]I_{CS} \cdot t_f}{6}$$

$$= \frac{10 \times 10^3[200 - 2.5] \times 60 \times 3 \times 10^{-6}}{6} = 59.25 \text{ W}$$

Instantaneous power loss during fall time is

$$P_f(t) = I_{CS}\left[1 - \frac{t}{t_f}\right]\left[\frac{V_{CC} - V_{CES}}{t_f} \cdot t\right]$$

Peak instantaneous power,

$$\frac{d\,P_f(t)}{dt} = 0$$

Gives time t_m at which instantaneous power loss during t_f, would be maximum

Here, $$P_f(t) = (V_{CC} - V_{CES})I_{CS}\left(\frac{t}{t_f} - \frac{t^2}{t_f^2}\right)$$

$$\frac{dP_f(t)}{dt_f} = (V_{CC} - V_{CES})I_{CS}\left[\frac{1}{t_f} - \frac{2t_m}{t_f^2}\right] = 0$$

$$\frac{1}{t_f} = \frac{2t_m}{t_f^2}$$

$$t_m = \frac{t_f}{2}$$

Peak instantaneous power loss during t_f is

$$P_f = I_{CS}\left[1-\frac{t_m}{t_f}\right]\left[\frac{V_{CC}-V_{CES}}{t_f}\cdot t_m\right]$$

$$P_{fm} = I_{CS}\left[1-\frac{1}{2}\right]\left[\frac{V_{CC}-V_{CES}}{2}\right] = \frac{I_{CS}(V_{CC}-V_{CE})}{4}$$

$$P_{fm} = \frac{60(200-2.5)}{4} = 2962.5 \text{ W}$$ (Peak instantaneous power during fall time)

Total average power loss during turn off procedure is

$$P_{\text{off}} = P_s + P_f = 6 + 59.25 = 65.25 \text{ W}$$

During off period, $i_c(t) = I_{\text{CEO}}$ and $V_{CE}(t) = V_{CC}$

instantaneous power loss during t_0 is

$$P_0(t) = V_{CE}\cdot i_c = V_{CC} \times I_{CEO} = 1.5 \times 10^{-3} \times 200 = 0.3 \text{ W}$$

Average power during t_0 is

$$P_0 = \frac{1}{T}\int_0^{t_0} P_0(t)\cdot dt = f\cdot I_{\text{CEO}}\cdot V_{CC}\cdot t_0$$

$$= 10 \times 10^3 \times 1.5 \times 10^{-3} \times 200 \times 30 \times 10^{-6} = 0.090 \text{ W}$$

Q.7 (b) Solution:

Here, we have,

$$(D+4)x + 3y = t \quad \text{...(i)}$$

$$2x + (D+5)y = e^t \quad \text{...(ii)}$$

To eliminate y operating (i) by $(D + 5)$ and multiplying (ii) by 3 then subtracting, we get

$$(D+5)(D+4)x + 3(D+5)y - 3(2x) - 3(D+5)y = (D+5)t - 3e^t$$

$$[(D+4)(D+5) - 6]x = (D+5)t - 3e^t$$

Auxiliary equation is,

$$m^2 + 9m + 14 = 0$$

$$m = -2, -7$$

$$\text{C.F.} = C_1\, e^{-2t} + C_2 e^{-7t}$$

$$\begin{aligned}
\text{P.I.} &= \frac{1}{D^2+9D+14}(1+5t-3e^t)\\
&= \frac{1}{D^2+9D+14}e^{0t} + \frac{1}{D^2+9D+14}t - 3\frac{1}{D^2+9D+14}e^t\\
&= \frac{1}{0^2+9(0)+14}e^{0t} + 5\frac{1}{14\left(1+\frac{9D}{14}+\frac{D^2}{14}\right)}t - 3\frac{1}{1^2+9(1)+14}e^t\\
&= \frac{1}{14}+\frac{5}{14}\left[1+\left(\frac{9D}{14}+\frac{D^2}{14}\right)\right]^{-1} t - \frac{1}{8}e^t\\
&= \frac{1}{14}+\frac{5}{14}\left[1-\left(\frac{9D}{14}+\frac{D^2}{14}\right)+\ldots\right]t - \frac{1}{8}e^t\\
&= \frac{1}{14}+\frac{5}{14}\left(t-\frac{9}{14}\right)-\frac{1}{8}e^t\\
&= \frac{5}{14}t-\frac{31}{196}-\frac{1}{8}e^t
\end{aligned}$$

$$x = C_1e^{-2t}+C_2e^{-7t}+\frac{5}{14}t-\frac{31}{196}-\frac{1}{8}e^t$$

$$\begin{aligned}
3y &= t-\frac{dx}{dt}-4x\\
&= t-\frac{d}{dt}\left[C_1e^{-2t}+C_2e^{-7t}+\frac{5}{14}t-\frac{31}{196}-\frac{1}{8}e^t\right]\\
&\quad -4\left[C_1e^{-2t}+C_2e^{-7t}+\frac{5}{14}t-\frac{31}{196}-\frac{e^t}{8}\right]
\end{aligned}$$

$$\begin{aligned}
3y &= t+2C_1e^{-2t}+7C_2e^{-7t}-\frac{5}{14}+\frac{1}{8}e^t-4C_1e^{-2t}-4C_2e^{-7t}-\frac{10}{7}t\\
&\quad +\frac{31}{49}+\frac{1}{2}e^t
\end{aligned}$$

$$\therefore \qquad y = \frac{1}{3}\left[-2C_1e^{-2t}+3C_2e^{-7t}-\frac{3}{7}t+\frac{27}{98}+\frac{5}{8}e^t\right]$$

Hence,
$$x = C_1e^{-2t} + C_2e^{-7t} + \frac{5}{14}t - \frac{1}{8}e^t - \frac{31}{196}$$
$$y = -\frac{2}{3}C_1e^{-2t} + C_2e^{-7t} - \frac{1}{7}t + \frac{5}{24}e^t + \frac{9}{98}$$

Q.7 (c) (i) Solution:

Given equation in symbolic form is
$$(D^2 + 4D + 4)y = 3\sin x + 4\cos x$$

To find C.F. :

Its A.E. is $(D + 2)^2 = 0$

where, $D = -2, -2$
$$\text{C.F.} = (C_1 + C_2x)e^{-2x}$$

To find P.I. :
$$\text{P.I.} = \frac{1}{D^2 + 4D + 4}(3\sin x + 4\cos x)$$

We know, $\frac{1}{f(D^2)}\sin(ax+b) = \frac{1}{f(-a^2)}\sin(ax+b)$; $f(-a^2) \neq 0$

and $\frac{1}{f(D^2)}\cos(ax+b) = \frac{1}{f(-a^2)}\cos(ax+b)$; $f(-a^2) \neq 0$

$$= \frac{1}{-1 + 4D + 4}(3\sin x + 4\cos x)$$
$$= \frac{4D - 3}{16D^2 - 9}(3\sin x + 4\cos x)$$
$$= \frac{(4D - 3)}{-16 - 9}(3\sin x + 4\cos x)$$
$$= -\frac{1}{25}[3(4\cos x - 3\sin x) + 4(-4\sin x - 3\cos x)]$$
$$= \sin x$$

Complete solution is
$$y = (C_1 + xC_2)e^{-2x} + \sin x$$

when $x = 0, y = 1.\ \therefore$ $1 = C_1$

Also, $y' = C_2e^{-2x} + (C_1 + C_2x)(-2)(e^{-2x}) + \cos x$

When $x = 0$, $y' = 0$

$\therefore$ $$0 = C_2 - 2C_1 + 1$$

i.e., $$C_2 = 1$$

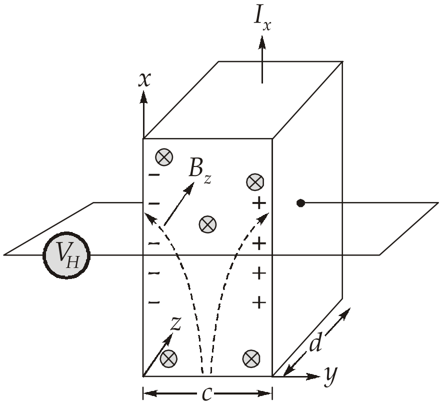
Hence, the required solution is

$$y = (1 + x)e^{-2x} + \sin x$$

Q.7 (c) (ii) Solution:

Hall effect is observed when a potential difference (Hall voltage) is generated across an electric material, that is transverse to an electric current in the material and to an applied magnetic field perpendicular to current.

In given specimen when magnetic field is imposed in positive z-direction, the resulting force brought to bear on the charge carrier will cause them to be deflected in the y-direction (for holes in right direction in specimen and for electrons to the left in specimen).



Where $$V_H = \text{Hall voltage}$$
$$B_z = \text{Magnetic field}$$
$$I_x = \text{Current } (x\text{-direction})$$

$$V_H = \frac{R_H I_x B_z}{d}$$

R_H is Hall coefficient, n is number of charge carrier.

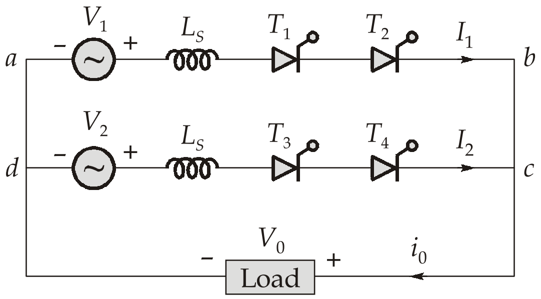
$$R_H = \frac{1}{n|e|}$$

Electron mobility, $$\mu_e = \frac{\sigma}{n|e|}$$

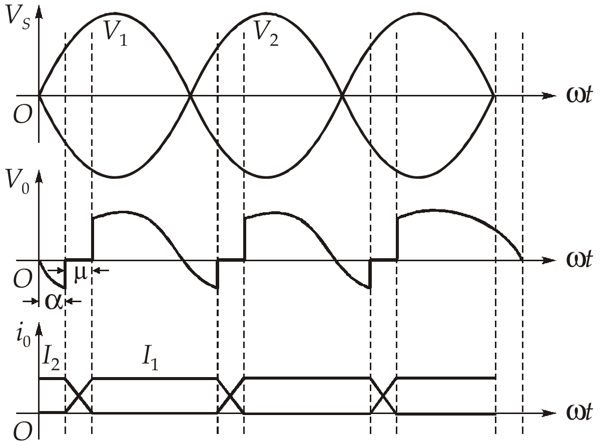
or $$\mu_e = |R_H|\sigma$$

Q.8 (a) Solution:

(i) Equivalent circuit of 1 - ϕ full converter with source inductance is given as:



and its typical current and voltage wave form is given as:



[Note : Here μ = overlap angle]

Now, applying KVL in *abcda* in figure (a) gives

$$V_1 - L_S\frac{di_1}{dt} = V_2 - L_S\frac{di_2}{dt}$$

or
$$V_1 - V_2 = L_S\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right)$$

It is seen that if $V_1 = V_m \sin \omega t$, then $V_2 = -V_m \sin \omega t$

∴
$$L_S\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) = 2V_m \sin \omega t \qquad \text{...(i)}$$

As the load current is assumed constant,

So,
$$i_1 + i_2 = I_0$$

or
$$\frac{di_1}{dt} + \frac{di_2}{dt} = 0 \qquad \text{...(ii)}$$

Now equation (i) become,

$$\frac{di_1}{dt} - \frac{di_2}{dt} = \frac{2V_m}{L_S}\sin\omega t \qquad \text{...(iii)}$$

adding equation (ii) in (iii), we get

$$\frac{di_1}{dt} = \frac{V_m}{L_S}\sin\omega t \qquad \text{...(iv)}$$

Load current i_1 through thyristor pair T_1 , T_2 builds up from zero to I_0 . during the overlap angle μ ,

i.e. at $\omega t = \alpha$

$i_1 = 0$

and at $\omega t = (\alpha + \mu)$, $i_1 = I_0$

So,
$$\int_0^{I_1} di_1 = \frac{V_m}{L_S}\int_{\frac{\alpha}{\omega}}^{\frac{(\alpha+\mu)}{\omega}} \sin\omega t \cdot dt$$

or
$$I_0 = \frac{V_m}{\omega L_S}[\cos\alpha - \cos(\alpha+\mu)] \qquad \text{...(v)}$$

or
$$\cos(\alpha+\mu) = -\frac{\omega L_S I_0}{V_m} + \cos\alpha$$

or
$$\cos(\alpha+\mu) = \cos\alpha - \frac{\omega V_S I_0}{V_m} \qquad \text{...(vi)}$$

(ii) It is seen that output voltage V_0 is zero form α to $(\alpha + \mu)$. Thus average output voltage V_0 is given by

$$V_0 = \frac{V_m}{\pi}\int_{\alpha+\mu}^{(\alpha+\pi)} \sin\omega t \cdot d(\omega t)$$

$$V_0 = \frac{V_m}{\pi}[\cos\alpha + \cos(\alpha+\mu)] \qquad \text{...(vii)}$$

Since,
$$\cos(\alpha+\mu) = \cos\alpha - \frac{\omega L_s I_0}{V_m}$$

So, equation (vii) become,

$$V_0 = \frac{2V_m}{\pi}\cos\alpha - \frac{\omega L_S}{\pi} I_0 \qquad \text{...(viii)}$$

Given, $I_0 = 5$ A,

$$\alpha = \frac{\pi}{4}\text{rad},$$

$$V_0 = 140 \text{ V}$$

$$V_m = 330 \text{ V} = 330.00 \text{ V}, \omega = 100\,\pi$$

So,
$$V_0 = \frac{2V_m}{\pi}\cos\alpha - \frac{\omega L_S I_0}{\pi}$$

or,
$$140 = \frac{2\times 330.00}{\pi}\cos\frac{\pi}{4} - \frac{100\pi L_S \times 5}{\pi}$$

$$= 148.68 - \frac{500\pi L_S}{\pi}$$

or,
$$\frac{500\pi L_S}{\pi} = 148.68 - 140$$

or,
$$L_S = \frac{8.68\times\pi}{500\pi}$$

or,
$$L_S = \frac{8.68}{500}\text{H}$$

or,
$$L_S = 0.01710 \text{ H}$$

or,
$$L_S = 17.10 \text{ mH}$$

$\because$
$$\cos(\alpha + \mu) = \cos\alpha - \frac{\omega L_S I_0}{V_m}$$

or,
$$\cos(\alpha + \mu) = \cos 45° - \frac{100\pi\times 17.10\times 10^{-3}\times 5}{330}$$

or,
$$\cos(\alpha + \mu) = 0.625$$

or,
$$(\alpha + \mu) = \cos^{-1}(0.625)$$

$$(\alpha + \mu) = 51.26°$$

or,
$$45 + \mu = 51.26°$$

or,
$$\mu = 51.26° - 45°$$

or,
$$\mu = 6.26°$$

So, angle of overlap = 6.26°

$$\text{Load resistance} = \frac{140\text{ V}}{5\text{ A}} = 28\ \Omega$$

$\therefore$
$$R_L = 28\ \Omega$$

Q.8 (b) Solution:

Characterisitc equation of the matrix A is

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\,[(1-\lambda)\,(2-\lambda] - 1(0) + 1(0-1+\lambda) = 0$$

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

According to Cayley-Hamilton theorem,

$$A^3 - 5A^2 + 7A - 3I = 0 \quad \text{...(i)}$$

We have to verify the equation (i),

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 13 & 13 \\ 0 & 1 & 6 \\ 13 & 13 & 14 \end{bmatrix}$$

$$A^3 - 5A^2 + 7A - 3I = \begin{bmatrix} 14 & 13 & 13 \\ 0 & 1 & 0 \\ 13 & 13 & 14 \end{bmatrix} - 5\begin{bmatrix} 5 & 4 & 5 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + 7\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 14-25+14-3 & 13-20-7+0 & 13-20+7+0 \\ 0+0+0+0 & 1-5+7-3 & 0-0+0-0 \\ 13-20+7+0 & 13-20+7-0 & 14-25+14-3 \end{bmatrix} = 0$$

Hence Cayley Hamilton theorem is verified

Now, $= A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$$= A^5(A^3 - 5A^2 + 7A - 3I) + A(A^3 - 5A^2 + 7A - 3I) + A^2 + A + I$$

$$= \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

Q.8 (c) (i) Solution:

KCL at node-A, we have

$$\frac{0-V_A}{1\text{ K}} = \frac{V_A-V_0}{2\text{ K}}$$

$$-2\,V_A = V_A - V_0$$

$$V_0 = 3\,V_A$$

Now, apply KCL at node-B, we have

$$\frac{V_B-V_0}{R+\frac{1}{Cs}} + \frac{V_B-0}{R \parallel \frac{1}{Cs}} = 0 \qquad \{V_A = V_B\}$$

$$\frac{(V_A-V_0)Cs}{(RCs+1)} + \frac{V_A(RCs+1)}{R\times 1} = 0$$

$$V_A\left[\frac{Cs}{(1+RCs)} + \frac{(1+RCs)}{R}\right] = \frac{V_0Cs}{(1+RCs)}$$

$$V_A[RCs + 1 + R^2C^2s^2 + 2RCs] = 3V_A\,RCs$$

$$R^2C^2s^2 + 3RCs + 1 = 3RCs \qquad (\text{Put } s = j\omega)$$

$$-R^2C^2s^2 + 1 = 0$$

$$\omega^2 = \frac{1}{R^2C^2}$$

$$\omega = \frac{1}{RC}$$

$$2\pi f = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC}$$

Where R_1 = 1 kΩ and C = 4.7 μF

$$f = \frac{1}{2\pi\times 1\times 10^3\times 4.7\times 10^{-6}}$$

$$= \frac{1000}{2\pi\times 4.7} = 33.88\text{ Hz}$$

Alternatively, we can solve the problem directly since it a Wein Bridge, the frequency of oscillation is given by

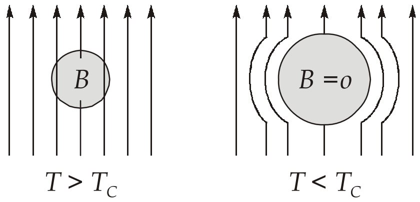
$$f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$

Here, $R_1 = R_2 = 1\ \text{k}\Omega$ and $C_1 = C_2 = 4.7\ \mu\text{F}$

$$f = \frac{1}{2\pi\sqrt{R^2C^2}} = \frac{1}{2\pi RC} = 33.88\ \text{Hz}$$

Q.8 (c) (ii) Solution:

Superconductors exhibit zero resistance but also spontaneously expel all magnetic flux when cooled through the superconducting transition, and behave as perfect diamagnetic material. This phenomena is called "Meissner effect" represented by figure below,



Meissner effect

It has been observed that when a long superconductor is cooled in a longitudinal magnetic field as shown in figure, below the transition temperature, the lines of induction are pushed out of material hence magnetic field becomes zero inside the specimen.

We know from magnetic properties of materials that

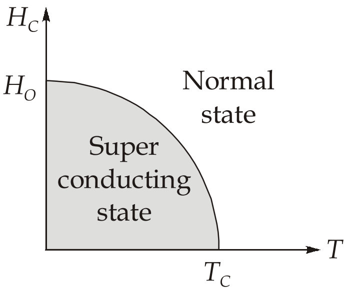
$$B = \mu_0(H + M),$$
$$\text{for } B = 0,\ H = -M,$$

Consequently, since $\chi_m = \frac{M}{H}$, we may state that magnetic susceptibility in a superconductor is negative. This is referred to as perfect diamagnetism is justified by Meissner effect.

With only small deviations, the critical field H_c varies with temperature according to the parabolic law,

$$H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

H_0 is the critical field at absolute zero and T_c is the transition temperature. For any particular superconductor the shape of variation of H_c with temperature is shown in figure below.



Factors affecting transition temperature of superconductor are as follows:

- **Magnetic field:**
  Transition temperature can be reduced with increment in critical magnetic field value.
- **Isotropic mass:**
  Transition temperature varies in materials found in isotopic form having different isotopic mass.

$$T_C \propto \frac{1}{\sqrt{m}}$$

- **Mechanical stress or pressure:**
  Transition temperature can be varied with application of mechanical stress or pressure.

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