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Detailed Solutions

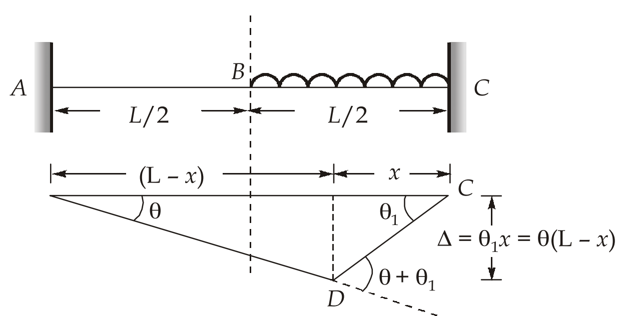
ESE-2024 Mains Test Series

Civil Engineering Test No : 7

Section A : Design of Steel Structure + Hydrology

Q.1 (a) Solution:

For a fixed ended beam and loaded partly for its span as shown in figure, the possible locations of plastic hinges will be at A and C and at a point D at a distance x from the support C.



From mechanism,

$$\Delta = (L - x)\theta = x\theta_1$$

$$\Rightarrow \quad \theta_1 = \frac{L-x}{x}\theta$$

Now, External work done = Intensity of load × Area of collapse mechanism diagram under the load

$$= \frac{w_u}{\frac{L}{2}} \times \left[\left(\frac{x\theta_1 + \frac{L}{2}\theta}{2}\right)\left(\frac{L}{2} - x\right) + \left(\frac{x\theta_1}{2}\right)x\right]$$

$$= \frac{2w_u}{L} \times \left[\frac{1}{2}\left[\left(x \times \frac{L-x}{x} + \frac{L}{2}\right)\left(\frac{L}{2} - x\right)\right]\theta + \left(\frac{x^2}{2} \times \frac{L-x}{x}\right)\theta\right]$$

$$= \frac{w_u}{L} \times \left[\left((L-x) + \frac{L}{2}\right)\left(\frac{L}{2} - x\right) + x(L-x)\right]\theta$$

$$= \frac{w_u}{L} \times \left[\frac{3}{4}L^2 - \frac{3}{2}Lx - \frac{Lx}{2} + x^2 + Lx - x^2\right]\theta$$

$$= \frac{w_u}{L} \times \left[\frac{3}{4}L^2 - Lx\right]\theta$$

$$= w_u\left(\frac{3}{4}L - x\right)\theta$$

Internal work done $= M_p\theta + M_p(\theta + \theta_1) + M_p\theta_1$

$$= 2M_p(\theta + \theta_1)$$

$$= 2M_p\left(\theta + \frac{L-x}{x}\theta\right)$$

$$= 2M_p\left(1 + \frac{L-x}{x}\right)\theta$$

$$= 2M_p\frac{L}{x}\theta$$

By the principal of virtual work

External work done = Internal work done

$$\Rightarrow \quad w_u\left(\frac{3}{4}L - x\right)\theta = 2M_p\left(\frac{L}{x}\right)\theta$$

$$\Rightarrow \quad M_p = \frac{w_u}{2L}\left(\frac{3}{4}Lx - x^2\right)$$

For maximum value of M_p.

$$\frac{dM_p}{dx} = 0 = \frac{w_u}{2L}\left(\frac{3}{4}L - 2x\right)$$

$\Rightarrow$ $$x = \frac{3}{8}L$$

Hence $$M_p = \frac{w_u}{2L}\times\left[\frac{3}{4}L\times\frac{3}{8}L - \left(\frac{3}{8}L\right)^2\right]$$

$\Rightarrow$ $$M_p = \frac{w_u}{2L}\times\left[\frac{9L^2}{32} - \frac{9L^2}{64}\right]$$

$\Rightarrow$ $$w_u = \frac{64\times 2LM_P}{9L^2} = \frac{128}{9}\frac{M_P}{L} = 14.22\frac{M_P}{L}$$

Q.1 (b) Solution:

Hydrologic cycle:

- Water occurs on the earth in all its three states, viz. liquid, solid and gaseous and in various degrees of motion. Evaporation of water from ocean water bodies such as oceans and lakes, formation and movement of clouds, rain and snowfall, stream flow and ground water movement are some examples of the dynamic aspects of water. The various aspects of water related to the earth can be explained in terms of a cycle known as the hydrologic cycle.

Human interference affects various parts of this cycle:

1. **Deforestation and Urbanization**: Removing forest and Paving surface disrupts natural infiltration and increases surface runoff, leading recharge and more to less groundwater flooding.
2. **Irrigation:** Agriculture consumes large amount of water, altering natural distribution and resources reducing available freshwater.
3. **Pollution:** Industrial and agricultural activities introduce pollutants into water bodies, affecting water quality and ecosystems.
4. **Dam construction**: Dams alter the flow of rivers, affecting downstream habitats, sediment transport and ground water recharge.
5. **Climate change**: Human-induced climate change alters precipitation patterns, leading to more frequent droughts, floods and shifts in the timing and intensity of rainfall, impacting the water cycle globally.

Q.1 (c) Solution:

(i) For Fe 410 grade steel: f_u = 410 MPa, f_y = 250 MPa

Partial safety factor for material:

Governed by yielding, $\gamma_{mo} = 1.1$

Governed by ultimate stress, $\gamma_{m1} = 1.25$

Now, gross area in shear, $A_{vg} = (1 \times 100 + 50) \times 8 = 1200 \text{ mm}^2$

$$\text{Net area in shear, } A_{vn} = \left\{(1 \times 100 + 50) - \left(2 - \frac{1}{2}\right) \times 18\right\} \times 8 = 984 \text{ mm}^2$$

Gross area in tension, $A_{tg} = 35 \times 8 = 280 \text{ mm}^2$

$$\text{Net area in tension, } A_{tn} = \left(35 - \frac{1}{2} \times 18\right) \times 8 = 208 \text{ mm}^2$$

The block shear strength will be minimum of T_{db1} and T_{db2} as calculated below.

$$T_{db1} = \frac{A_{vg} f_y}{\sqrt{3}\gamma_{mo}} + \frac{0.9 A_{tn} f_u}{\gamma_{m1}}$$

$$= \left[\frac{1200 \times 250}{\sqrt{3} \times 1.1} + \frac{0.9 \times 208 \times 410}{1.25}\right] \times 10^{-3} \text{ kN}$$

$$= 218.86 \text{ kN}$$

$$T_{db2} = \left[\frac{0.9 f_u A_{vn}}{\sqrt{3} \times \gamma_{m1}} + \frac{f_y A_{tg}}{\gamma_{m0}}\right]$$

$$= \left[\frac{0.9 \times 410 \times 984}{\sqrt{3} \times 1.25} + \frac{250 \times 280}{1.1}\right] \times 10^{-3} \text{ kN} = 231.34 \text{ kN}$$

Hence, the block shear strength of the tension member is 218.86 kN.

(ii) For Fe410 grade steel f_u = 410 MPa, f_y = 250 MPa

$$\gamma_{mo} = 1.1, \gamma_{m1} = 1.25$$

$$A_{vn} = A_{vg} = 2 \times 100 \times 10 = 2000 \text{ mm}^2$$

$$A_{tn} = A_{tg} = 225 \times 10 = 2250 \text{ mm}^2$$

The block shear strength will be minimum of T_{db1} and T_{db2} as calculatedd below.

$$T_{db1} = \frac{A_{vg} f_y}{\sqrt{3}\gamma_{mo}} + \frac{0.9 A_{tn} f_u}{\gamma_{m1}}$$

$$\Rightarrow \qquad T_{db1} = \left[\frac{2000\times 250}{\sqrt{3}\times 1.1} + \frac{0.9\times 2250\times 410}{1.25}\right]\times 10^{-3} \text{ kN}$$

$$= 926.632 \text{ kN}$$

$$T_{db2} = \frac{0.9A_{vn}f_u}{\sqrt{3}\gamma_{m1}} + \frac{A_{tg}f_y}{\gamma_{mo}}$$

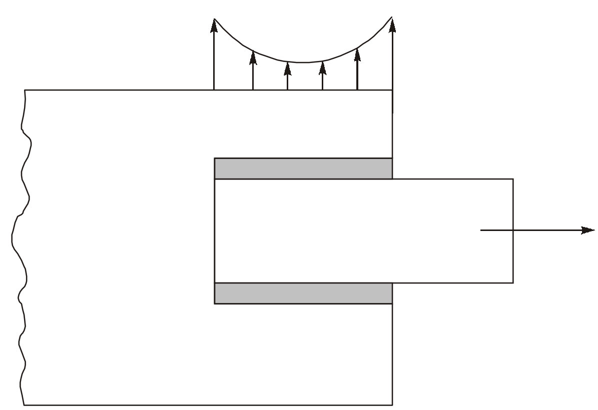
$$= \left[\frac{0.9\times 2000\times 410}{\sqrt{3}\times 1.25} + \frac{2250\times 250}{1.1}\right]\times 10^{-3} \text{kN}$$

$$= 852.23 \text{ kN}$$

Hence, the block shear strength of the tension member is 852.23 kN.

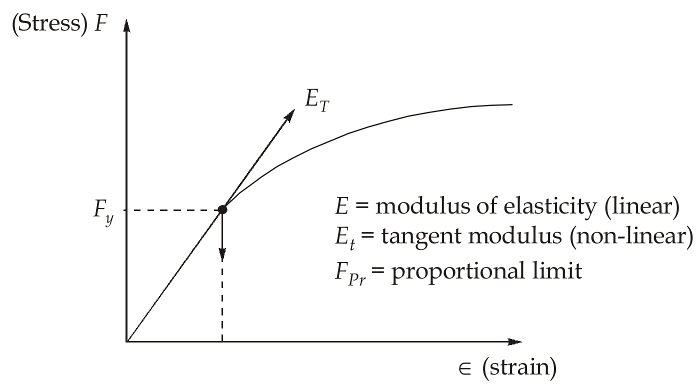
Q.1 (d) Solution:

(i) The stress distribution in longitudinal fillet welds is shown in figure below.



The stress distribution is non-uniform with higher stresses at the two ends. Because of this, the stress concentration occurs in due course of time and the failure of weld initiates from here. Hence, to check the failure, end returns are provided on the either sides. This end return is generally twice the size of the weld.

(ii) In the derivation of buckling load for columns the flexural rigidity (EI) in presumed to be constant but it is not so when the induced level of stress exceeds proportional limit, the variable nature of column, stiffness is caused by the residual stresses. Accordingly the stiffness and the moment curvature relationship will be continually changing as the axial load is increased beyond the proportional limit. If sum of applied compressive stress and residual stress is less than the yield stress, the member deforms inelastically and if this sum exceeds the yield stress, the member in said to deform in elastically. In the inelastic range, the stiffness is more accurately proportional to tangent modulus. The average stress-strain relationship for unbraced length of compression member is shown in figure



Q.1 (e) Solution:

$$\text{Mean precipitation, } \bar{x} = \frac{800+700+900+800+850+1000+600+750}{8}$$

$$= 800 \text{ mm}$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{(800-800)^2+(700-800)^2+(900-800)^2+(800-800)^2 +(850-800)^2+(1000-800)^2+(600-800)^2+(750-800)^2}{8-1}}$$

$$= 122.47 \text{ cm}$$

$$\text{Coefficient of variation, } C_v = \frac{\sigma}{\bar{x}}\times 100$$

$$= \frac{122.47}{800}\times 100 = 15.31\%$$

Now, number of optimum raingauge S, N is given by

$$N = \left(\frac{C_v}{\varepsilon}\right)^2$$

$$= \left(\frac{15.31}{5}\right)^2 = 9.38 \simeq 10 \text{ (say)}$$

∴ 10 raingauges will be required in the catchment.

Q.2 (a) Solution:

For Fe410 grade of steel; f_y = 250 MPa, f_{yw} = 250 MPa

Partial safety factor; γ_{mo} = 1.1

Factored bending moment = 150 kN-m

Factored shear force = 210 kN

Required plastic section modulus, $z_{PZ'\text{req}} = \dfrac{M\gamma_{mo}}{f_y} = \dfrac{150\times10^6\times1.1}{250} = 660\times10^3\ \text{mm}^3$

Try ISLB 325 @422.81 N/m.

$$\in = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

Section classification

Outstand of flange, $b = \dfrac{b_f}{2} = \dfrac{165}{2} = 82.5$ mm

Now, $\dfrac{b}{t_f} = \dfrac{82.5}{9.8} = 8.418 < 9.4 \Rightarrow$ Flange is plastic

Also, $\dfrac{d}{t_w} = \dfrac{273.4}{7.0} = 39.06 < 84 \Rightarrow$ Web is plastic

Hence, the overall section is plastic.

1. Check for shear capacity

Design shear force, V = 210 kN

Design shear strength of the section,

$$V_d = \frac{f_y}{\sqrt{3}\times\gamma_{mo}} ht_w = \frac{250}{\sqrt{3}\times1.1}\times325\times7\times10^{-3}\ \text{kN}$$

$= 298.5$ kN

> 210 kN (OK)

Check for high or low shear

$0.6V_d = 0.6 \times 298.5 = 179.1$ kN > V (= 210 kN)

This case is of high shear since $V > 0.6\ V_d$

2. Check for design bending strength

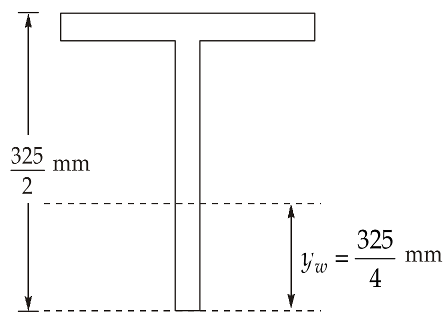
Design bending strength, $M_{dv} = M_d - \beta(M_d - M_{fd})$

$$< 1.2Z_{ez}\frac{f_y}{\gamma_{mo}}$$

where $$\beta = \left(\frac{2V}{V_d} - 1\right)^2 = \left(\frac{2\times 210}{298.5} - 1\right)^2 = 0.1657$$

$$M_d = Z_{pz}\frac{f_y}{\gamma_{mo}} = 687.76\times 10^3 \times \frac{250}{1.1}\times 10^{-6} \text{ kNm}$$

$$= 156.3 \text{ kN-m}$$



Section modulus of flange, $Z_{fd} = Z_{pz} - Z_w y_w$

$$Z_{fd} = 687.76 \times 10^3 - (325 \times 7) \times \frac{325}{4}$$

$$= 502.916 \times 10^3 \text{ mm}^3$$

Plastic design strength of the area of the cross-section excluding the shear area is,

$$M_{fd} = Z_{fd}\times\frac{f_y}{\gamma_{mo}}$$

$$= 502.916\times 10^3 \times \frac{250}{1.1}\times 10^{-6} = 114.3 \text{ kN-m}$$

So, $$M_{dv} = 156.30 - 0.1657 \times (156.30 - 114.29)$$

$$= 149.34 \text{ kN-m} < 150 \text{ kN-m}$$

Hence, the section is unsafe.

Thus try ISLB 350 @ 495 N/m.

Section classification

Outstand of flange element, $b = \frac{b_f}{2} = \frac{165}{2} = 82.5$ mm

Now, $\frac{b}{t_f} = \frac{82.5}{11.4} = 7.23 < 9.4 \Rightarrow$ Flange is plastic

Also, $\frac{d}{t_w} = \frac{295.2}{7.4} = 39.9 < 84 \Rightarrow$ Web is plastic

Hence, the section is plastic.

1. Check for shear capacity

Design shear force, V = 210 kN

Design shear strength of the section.

$$V_d = \frac{f_y}{\sqrt{3}\gamma_{mo}} \times ht_w = \frac{250}{\sqrt{3} \times 1.1} \times 350 \times 7.4 \times 10^{-3}$$

$$\simeq 340 \text{ kN} > 210 \text{ kN} \qquad \text{(OK)}$$

Check for high/low shear

$$0.6V_d = 0.6 \times 340 = 204 \text{ kN}$$

This case is of high shear since V > 0.6 V_d

2. Check for design bending strength

$$M_{dv} = M_d - \beta(M_d - M_{fd})$$

$$\leq 1.2Z_e \frac{f_y}{\gamma_{mo}}$$

where $$\beta = \left(\frac{2V}{V_d} - 1\right)^2 = \left(\frac{2 \times 210}{340} - 1\right)^2 = 0.0554$$

$$M_d = Z_{pz}\frac{f_y}{\gamma_{mo}} = 851.11 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} = 193.43 \text{ kNm}$$

$$Z_{fd} = Z_{pz} - A_w y_w$$

$$= 851.11 \times 10^3 - (350 \times 7.4) \times \frac{350}{4}$$

$$= 624.485 \times 10^3 \text{ mm}^3$$

$$M_{fd} = Z_{fd}\frac{f_y}{\gamma_{mo}} = 624.485 \times 10^3 \times \frac{250}{1.1} \times 10^{-6}$$

$$= 141.93 \times 10^3 \text{ mm}^3$$

$$M_{dv} = 193.43 - 0.0554 \times (193.43 - 141.93)$$

$$= 190.58 \text{ kNm} > 150 \text{ kNm}$$

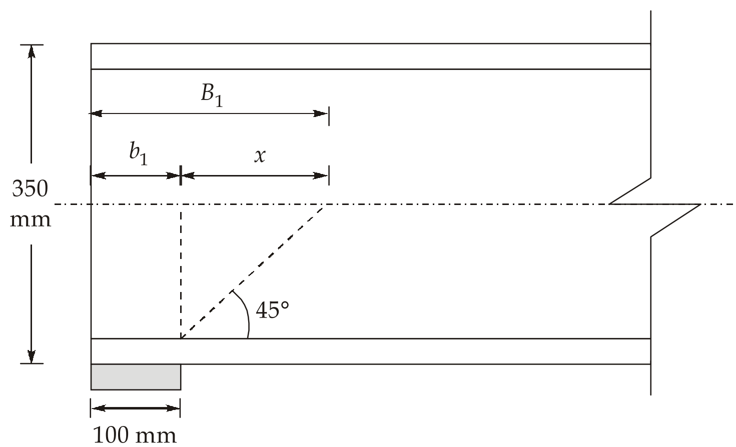
$$< 1.2Z_e \frac{f_y}{\gamma_{mo}} = 1.2 \times 751.9 \times 10^3 \times \frac{250}{1.1} \times 10^{-6}$$

$$= 205.06 \text{ kNm} > 150 \text{ kNm} \quad \text{(OK)}$$

3. Check for web buckling (at support)

Although web buckling check is not required in general $\frac{d}{t_w} < 67 \in$. However since shear is high the check has been applied.

Assume a stiff bearing length, b_1 = 100 mm as shown in figure.



$$A_b = B_1 \times t_w = (b_1 + x)t_w$$

$$\because \quad \text{where } x = \frac{h}{2}\tan 45° = \frac{350}{2} = 175 \text{ mm}$$

$$\therefore \quad A_b = (100 + 175) \times 7.4 = 2035 \text{ mm}^2$$

$$\text{Effective length of web, } kL = 0.7d = 0.7 \times 295.2 = 206.64 \text{ mm}$$

$$I_{\text{eff}} \text{ of web} = \frac{bt_w^3}{12} = \frac{100 \times 7.4^3}{12} = 3376.87 \text{ mm}^4$$

$$A_{\text{eff}} \text{ of web} = bt_w = 100 \times 7.4 = 740 \text{ mm}^2$$

$$\text{So, radius of gyration of web, } r = \sqrt{\frac{3376.87}{740}} = 2.136 \text{ mm}$$

$$\text{Slenderness ratio, } \lambda = \frac{kL}{r} = \frac{206.64}{2.136} = 96.74$$

For $\lambda = 96.74$ and $f_{yw} = 250\ \text{N/mm}^2$, the design compressive stress, f_{cd} can be calculated as

$$\frac{100-96.74}{107-f_{cd}} = \frac{100-90}{107-121}$$

$$\Rightarrow \qquad f_{cd} = 111.56\ \text{N/mm}^2$$

Capacity of the web section, $f_{wb} = A_b \times f_{cd} = 2035 \times 111.56 \times 10^{-3}$ kN

$= 227.00$ kN > 210 kN (OK)

Check for web bearing

$$F_w = (b+n_1)t_w \times \frac{f_{yw}}{\gamma_{mo}}$$

where $n_1 = 2.5(t_f + R_1)$ (assuming dispersion at 1 : 2.5)

$= 2.5 \times (11.4 + 16) = 68.5$ mm

Stiff bearing length has been assumed as $b_1 = 100$ mm

$$\therefore \qquad F_w = (100+68.5)\times 7.4 \times \frac{250}{1.1} \times 10^{-3} = 283.39\ \text{kN}$$

> 210 kN (OK)

Q.2 (b) Solution:

(i)

Time from start of storm t(h)	Time interval Δt (h)	Accumulated rainfall in time, t (in cm)	Depth of rainfall in Δt, (in cm)	$\phi\Delta t$ (cm) = 0.5 × 2	Effective rainfall ordinates (in cm)
0	2	0	0	1.0	0
2	2	0.8	0.8	1.0	0
4	2	3.2	2.4	1.0	1.4
6	2	5.6	2.4	1.0	1.4
8	2	6.8	1.2	1.0	0.2
10	2	7.4	0.6	1.0	0
12	2	9.4	2.0	1.0	1.0
14	2	9.8	0.4	1.0	0.0

In a given time interval Δt, effective rainfall (ER) is given by:

$$ER = \text{maximum}\begin{cases}(i)\ (\text{actual depth of rainfall} - \phi\Delta t) \\ (ii)\ 0\end{cases}$$

(ii)

Unit hydrograph: A unit hydrograph is defined as the hydrograph of different runoff resulting from one unit depth (1 cm) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D hours).

Two basic assumptions that constitute the foundation for the unit hydrograph theory are:

1. **Time invariance**: The first basic assumption is that the direct-runoff response to given effective rainfall in a catchment is time-invariant. This implies that the DRH for a given effective rainfall in a catchment is always the same irrespective of when it occurs.
2. **Linear response**: The direct runoff response to the rainfall excess is assumed to be linear. This is the most important assumption of the unit hydrograph theory. Linear response means that if an input $x_1(t)$ causes an output $y_1(t)$ and an input $x_2(t)$ causes an output $y_2(t)$ then an input $x_1(t) + x_2(t)$ gives an output $y_1(t) + y_2(t)$.

Limitations to the use of unit hydrograph are the following:

1. Precipitation must be from rainfall only. Snowmelt runoff can not be satisfactory responded by unit hydrograph.
2. The catchment should not have unusually large storage in term of tanks, ponds, large flood bank storage etc. which affect the linear relationship between storage and discharge.
3. If the precipitation is decidedly non-uniform, unit hydrograph can not be expected to give good results.

Q.2 (c) Solution:

(i)

$$\text{Shank area of bolt, } A_s = \frac{\pi}{4}\times 16^2 = 201.06 \text{ mm}$$

$$\text{For Fe410 grade of steel, } f_u = 410 \text{ MPa}$$

$$\text{For bolts of grade 4.6, } f_{ub} = 400 \text{ MPa}$$

$$\text{For 16 mm diameter bolt, } A_{nb} = 0.78\times\frac{\pi}{4}\times 16^2 = 156.83 \text{ mm}^2$$

$$\text{Diameter of the bolt hole, } d_0 = 16 + 2 = 18 \text{ mm}$$

Partial safety factor for material of bolt, $\gamma_{mb} = 1.25$

Partial safety factor for resistance governed by ultimate stress, $\gamma_{m1} = 1.25$

Thickness of plate from bearing consideration will be minimum of 10 mm, 16 mm and 16 mm (2 × 8) and will therefore be 10 mm.

The bolts will be in double shear and bearing. Since the two plates to be jointed are of thickness 10 mm and 16 mm, packing plate of thickness (16 – 10) = 6 mm will be required.

As the thickness of packing plate is equal to 6 mm, the shear strength of the joint will not get reduced.

Strength of the bolt in bearing double shear per pitch length

$$\therefore \quad V_{dsb} = (n_n A_{nb} + n_s A_{sb}) \frac{f_{ub}}{\sqrt{3}\gamma_{mb}}$$

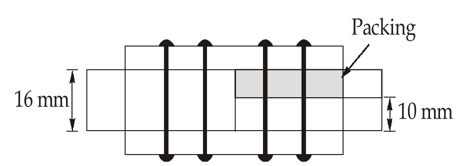
Assuming that one of shear plane is intercepting thread area and other shank area,

$$\therefore \quad V_{dsb} = (1 \times 156.80 + 1 \times 201.06) \times \frac{400}{\sqrt{3} \times 1.25} \text{N} = 66.12 \text{ kN}$$

Assuming the strength of bolt in bearing per pitch length to be more than that in shear, the number of bolts required.

$$= \frac{600}{66.12} = 9.07 \simeq 10$$

Provide the bolts in two rows (chain pattern) as shown in figure.



Since there will be two bolts per pitch length, strength of the joint per pitch length on the basis of shearing strength of bolts = 2 × 65 = 130 kN

Now, equating the strength of bolt in shear per pitch length to the net tensile strength of the plate per pitch length, the pitch of the bolt can be determined.

$$T_{dn} = 0.9 \frac{f_{up}}{\gamma_{m1}} (p - nd_n) t = 2 \times 65 \times 10^3$$

where p is pitch of bolts

$$\therefore \quad T_{dn} = 0.9 \times \frac{410}{1.25} (p - 18) \times 10 = 2 \times 65 \times 10^3$$

$$\Rightarrow \quad p = 62 \text{ mm} > 2.5 \times 16 \quad (= 40 \text{ mm}) \qquad \text{(OK)}$$

Let's provide pitch of 60 mm.

Now, $\quad k_b$ is least of $\dfrac{e}{3d_0} = \dfrac{27}{3 \times 18} = 0.5 \qquad \{e = 1.5 \times d_0 = 1.5 \times 18 = 27\}$

$$\frac{p}{3d_0} - 0.25 = \frac{60}{3 \times 18} - 0.25 = 0.86$$

$$\frac{f_{ub}}{f_u} = \frac{400}{410} = 0.975$$

So, $k_b = 0.5$

Design strength in bearing, $V_{dpb} = 2.5 \times k_b \times d \times t \times \frac{f_u}{\gamma_{mb}}$

$$= 2.5 \times 0.5 \times 16 \times 10 \times \frac{410}{1.25} \times 10^{-3}$$

$$= 65.6 \text{ kN } > 65 \text{ kN}$$

Hence, there is no need to increase the end distance.

(ii)

The probability of occurrence or exceedance (P) of a rainfall in a year is calculated empirically as below:

$$P = \frac{m}{N+1}$$

where, m = Rank of an event obtained by arranging all the available data in decreasing order and ranking them from 1 (highest event) to lowest event.

N = Total number of data available

$(N = 11)$

Now return period, T is given by

$$T = \frac{1}{P} = \frac{N+1}{m}$$

Rainfall	Rank (m)	Probability, P $P = \frac{m}{N+1} = \frac{m}{12}$	Return period, $T = \frac{1}{P} = \frac{12}{m}$
950	1	0.083	12
900	2	0.167	6
850	3	0.250	4
800	4	0.333	3
750	5	0.416	2.4
700	6	0.50	2
650	7	0.583	1.715
600	8	0.667	1.50
500	9	0.750	1.33
400	10	0.833	1.20
350	11	0.916	1.09

By interpolation, maximum annual rainfall which has a recurrence interval of 8 years, and thus p is given by

$$\frac{950-900}{12-6} = \frac{p-900}{8-6}$$

$$\Rightarrow \quad p = 916.67 \text{ mm}$$

Q.3 (a) Solution:

For Fe410 grade of steel, $f_u = 410 \text{ N/mm}^2$

$f_y = 250 \text{ N/mm}^2$

Partial factor of safety for shop weld, $\gamma_{mw} = 1.25$

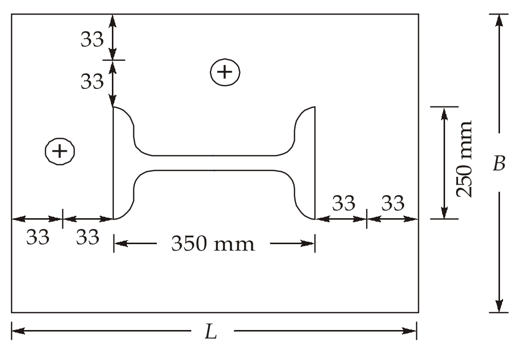
Partial factor of safety for site weld, $\gamma_{mw} = 1.5$

Partial factor of safety for material, $\gamma_{mo} = 1.1$

Assuming truss shoe plate to be connected to column cap by 20 mm diameter bolts.

Minimum edge distance for 20 mm diameter bolts = 22 × 1.5 = 33 mm

Size of plate:



Values in mm

Length of the plate, $L = 350 + 2 \times 33 + 2 \times 33 = 482 \text{ mm} \simeq 500 \text{ mm (say)}$

Width of the plate, $B = 250 + 2 \times 33 + 2 \times 33 = 382 \simeq 400 \text{ mm (say)}$

∴ Provide plate of (500 × 400) mm as column cap.

Thickness of the plate.

Thickness of the base plate required is given by

$$t_C = \sqrt{2.5w\left(a^2 - 0.3b^2\right) \times \frac{\gamma_{mo}}{f_y}}$$

Projected length of the base plate, $a = \frac{L-h}{2} = \frac{500-350}{2} = 75 \text{ mm}$

Projected width of the base plate, $b = \frac{B-b_f}{2} = \frac{400-250}{2} = 75$ mm

Out of 105 kN and 63 kN loads, 105 kN is larger load.

$\therefore$ Uplift pressure, $w = \frac{P}{A_1} = \frac{105\times1000}{500\times400} = 0.525\ \text{N/mm}^2$

$$t_C = \sqrt{2.5\times0.525(75^2-0.3\times75^2)\times\frac{1.1}{250}} = 4.77\ \text{mm}$$

As t_C cannot be less than, $t_f = 11.6$ mm

Thus provide a column cap of 12 mm thickness. Thus, size of column cap adopted is 500 × 400 × 12 mm.

Connection of column base plate with top of column:

The column base plate is required to be welded to the column.

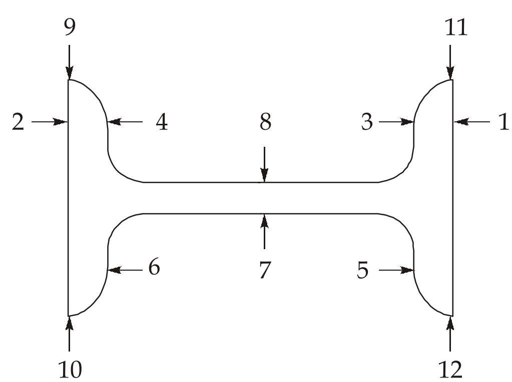
Let the size of the weld, $s = 5$ mm

Length available for welding along the column profile

$= 2 \times 250 + 2(250 - 10.1) + (350 - 2 \times 11.6)$

$= 1306.6$ mm

There are 12 edges available for welding as shown.



Overall effective weld length available

= Total weld length available – (2 × 5 × total number of edges available)

= 1306.6 – (2 × 5 × 12) = 1186.6 mm

Strength of the weld per mm length

$$= l_w \times t_t \times \frac{f_u}{\sqrt{3}\gamma_{mw}} = 1\times0.7\times5\times\frac{410}{\sqrt{3}\times1.5}$$

$= 552.33$ N/mm

$\therefore$ Length of the weld required $= \dfrac{105 \times 1000}{552.33} = 190.1 \text{ mm} < 1186.6 \text{ mm}$ (OK)

Connection of column cap with roof truss shoe plate.

Here the bolts will be designed for the tensile force of 63 kN.

Bolts are of grade 4.6 and hence, $f_{ub} = 400 \text{ N/mm}^2$.

Tensile strength of 20 mm diameter bolt,

$$T_{nb} = 0.9 A_{nb} f_{wb} \leq \frac{f_{yb} \cdot A_{sb}}{\gamma_{mo}} \times \gamma_{mb}$$

For 20 mm diameter bolts,

$$\text{Net area of bolts, } A_{nb} = 245 \text{ mm}^2 \qquad \left(= 0.78 \times \frac{\pi}{4} \times d^2\right)$$

$$\text{Shank area of bolts, } A_{sb} = 314 \text{ mm}^2$$

$$\therefore \quad T_{nb} = 0.9 \times 245 \times \frac{400}{1000} \text{kN} = 88.2 \text{ kN}$$

$$\text{Also,} \quad \frac{f_{yb} A_{sb}}{\gamma_{mo}} \gamma_{mb} = \frac{250 \times 314 \times 1.25}{1.1} N = 89.2 \text{kN}$$

$$\therefore \quad T_{nb} < \frac{f_{yb} \cdot A_{sb}}{\gamma_{mo}} \times \gamma_{mb} \qquad \text{(OK)}$$

Now, design tensile strength of the bolt $= \dfrac{T_{nb}}{\gamma_{mb}} = \dfrac{88.2}{1.25} = 70.56 \text{ kN}$

$\therefore$ Number of bolts required $= \dfrac{63}{70.56} = 0.893$

Let's provide 4 bolts of 20 mm dia. of grade 4.6 (One each at the corner of the cap).

Q.3 (b) Solution:

(i)

The relationship between rainfall and the resulting runoff is quite complex and is influenced by a host of factors relating the catchment and climate one method is to fit a linear regression line between R and p.

The calculations are tabulated below:

Month no.	Rainfall, P (cm)	Runoff, R (cm)	P^2	$P.R$
1	8	0.4	64	3.2
2	20	6	400	120
3	25	8	625	200
4	12	3	144	36
5	10	2.5	100	25
6	8	1.2	64	9.6
7	4	0.4	16	1.6
8	20	4	400	80
9	10	2	100	20
10	5	0.2	25	1.0
	$\Sigma P = 122$	$\Sigma R = 27.7$	$\Sigma P^2 = 1938$	$\Sigma PR = 496.4$

The equation for straight line regression between runoff R and rainfall p is:

Now $$R = aP + b$$

where, $$a = \frac{N\Sigma PR - \Sigma P \Sigma R}{N\Sigma P^2 - (\Sigma P)^2}$$

$\Rightarrow$ $$a = \frac{(10)\times(496.4)-(122)(27.7)}{(10)(1938)-(122)^2} = 0.352$$

and $$b = \frac{\Sigma R - a\Sigma P}{N}$$

$\Rightarrow$ $$b = \frac{27.7-(0.352)(122)}{10} = -1.52$$

Hence, $$R = 0.352P - 1.52$$

(ii)

The evaporation losses from water surface depend upon the following factors:

1. **Vapour pressure or humidity**: The rate of evaporation is proportional to the difference between the saturation vapour pressure at the water temperature (ρ_w) and the actual vapour pressure in the air (ρ_a). Thus,

$$E_L = c(\rho_w - \rho_a)$$

the above equation, also known as Dalton's law of evaporation,

where, E_L = Rate of evaporation (mm/day) and

ρ_w and ρ_a are in mm of mercury.

Evaporation continues till, $\rho_w = \rho_a$, if $\rho_w < \rho_a$ then condensation takes place.

2. **Temperature**: If the temperature is more, then saturation vapour pressure increases and the evaporation increases.
3. **Wind**: Wind aids in removing the evaporated water from the zone of evaporation and consequently create greater scope for evaporation upto a certain extent.
4. **Atmospheric pressure**: Other factors remaining same, rate of evaporation decreases with increase in atmospheric pressure.
5. **Quality of water**: The quality of water in the water body also affects the rate of evaporation. Since the presence of any dissolved salts in water reduces the saturated vapour pressure of water, which consequently reduces the rate of evaporation.
6. **Area of water surface**: The amount of the evaporation is directly proportional to the area of evaporation.
7. **Depth of water**: The depth of water influences the evaporation considerably. More depth reduces the summer evaporation and increases the winter evaporation. In summer the temperature of shallow water bodies rises rapidly and hence more evaporation occurs in shallow water bodies.

Various methods available for reduction of evaporation losses are:

1. By providing mechanical surface covers.
2. By opting for deep reservoirs as against shallow reservoir for storing same volume of water.
3. By adding chemicals like cetyl alcohol (Hexa decanol) or stearyl alcohol (Octa deconal).

Q.3 (c) Solution:

Since, both the ends of column are fixed.

$$\text{Effective length, } kL = 0.65 \times 12 = 7.8 \text{ m}$$

$$\text{Factored load} = 875 \text{ kN}$$

$$\text{Total area of column section} = 4 \times 1540 = 6160 \text{ mm}^2$$

$$\text{Design compressive stress, } f_{cd} = \frac{875 \times 1000}{6160} \simeq 142 \text{ N/mm}^2$$

Using table, interpolating the value of $\frac{kL}{r}$ for $f_{cd} = 142$ N/mm^2

i.e.
$$\frac{152-136}{70-80} = \frac{152-142}{70-\frac{kL}{r}}$$

$$\Rightarrow \quad \frac{kL}{r} = 76.25$$

$$\Rightarrow \quad \frac{0.65 \times 12000}{76.25} = r$$

$$\Rightarrow \quad r = 102.295 \text{ mm}$$

We know that, $$r = \sqrt{\frac{I_z}{A}}$$

where I_z is moment of inertia and A is cross-sectional area of one column.

Now, $$I_z = I_{zz} + \text{Area} \times \left(\frac{S}{2} - C_{zz}\right)^2$$ where S is spacing of angles

$$\Rightarrow \quad I_z = 145 \times 10^4 + 1540 \times (0.5S - 27.6)^2 \quad \text{...(i)}$$

But, $$r = \sqrt{\frac{I_z}{A}} = 102.295 \text{ mm}$$

$$\Rightarrow \quad I_z = Ar^2 = 1540 \times (102.295)^2 = 16114971.22 \text{ mm}^4 \quad \text{...(ii)}$$

Equating eq. (i) and (ii) we get

$$16114971.22 = 145 \times 10^4 + 1540 \times (0.5S - 27.6)^2$$

$$\Rightarrow \quad S = 250.369 \text{ mm} \simeq 250.37 \text{ mm}$$

Let's keep spacing 'S' of angle section as 255 mm.

Design of lacing system:

Angle of inclination = 45°

Transverse shear to be carried by lacing, $V = \frac{2.5}{100} \times 875 \text{ kN} = 21.875 \text{ kN}$

Transverse shear per plane $\frac{V}{n} = \frac{21.875}{2} = 10.94 \text{ kN}$

Minimum width of lacing flat = 3 × 20 = 60 mm

Thickness of lacing flat $\not< \frac{1}{60} \times (255 - 2 \times 50) \operatorname{cosec} 45° = 3.653 \text{ mm}$

Let us take flat of size 60 mm × 8 mm

Minimum radius of gyration $= \frac{t}{\sqrt{12}} = \frac{8}{\sqrt{12}} = 2.309 \text{ mm}$

$$\text{Effective slenderness ratio} = \frac{0.7\times(255-100)\,\text{coses}\,45°}{2.309}$$

$$= 66.45 < 145 \quad \text{(OK)}$$

From table given, design compressive stress f_{cd} for slenderness ratio of 66.45.

$$\frac{70-60}{152-168} = \frac{70-66.45}{152-f_{cd}}$$

$$\Rightarrow \quad f_{cd} = 157.68\ \text{N/mm}^2$$

$$\text{Design compressive strength of the flat} = \frac{157.68\times60\times8}{1000} = 75.69\ \text{kN}$$

$$\therefore \quad 75.68\ \text{kN} > 10.94\ \text{kN} \quad \text{(Hence OK)}$$

Tensile strength of lacing flat:

Diameter of bolt hole for 20 mm diameter bolts, d_o = 22 mm

Tensile strength of lacing flat is minimum of

1. Net section rupture

$$T_{dn} = 0.9(B-d_0)t\times\frac{f_u}{\gamma_{m1}}$$

where d_0 is diameter of bolt hole i.e. 20 + 2 = 22 mm

$$= 0.9(60-22)\times8\times\frac{410}{1.25\times1000}\text{kN} = 89.74\ \text{kN}$$

2. Gross section yielding

$$T_{dg} = \frac{A_g f_y}{\gamma_{mo}} = (60\times8)\times\frac{250}{1.1\times1000}\text{kN} = 109.09\ \text{kN}$$

Hence, tensile strength of the lacing flat is 89.74 kN which is more than 10.94 kN (OK)

Q.4 (a) Solution:

(i) For f_y = 250 MPa, from table 2 of IS 800, maximum outstand (b/t) for the flange to be compact section = 8.4 for plastic sectino.

$$\text{Actual b/t} = \frac{\left(\frac{650}{2}-\frac{15}{2}\right)}{25} = 12.7 > 8.4$$

Maximum b/t for the flange to be semi compact = 13.6

Hence, the section is semi compact and $M_d = Z_e f_y$

Now, $$I_z = \frac{BD^3}{12} - (B - t_w)\frac{d^3}{12}$$

$$= 650 \times \frac{1550^3}{12} - (650 - 15) \times \frac{1500^3}{12}$$

$$= 23116.15 \times 10^6 \text{ mm}^4$$

Elastic section modulus, $$Z_{ez} = \frac{23116.15 \times 10^6}{\left(\frac{1500}{2} + 25\right)} = 29.827 \times 10^6 \text{ mm}^3$$

Now, $$M_d = 29.827 \times 10^6 \times 250$$
$$= 7456.75 \times 10^6 \text{ N-mm} = 7456.75 \text{ kN-m}$$

Plastic section modulus, $$Z_{pz} = \frac{2Bt_f\left(D - t_f\right)}{2} + \frac{t_w d^2}{4}$$

$$= \frac{2 \times 650 \times 25 \times (1550 - 25)}{2} + 15 \times \frac{1500^2}{4} = 33.21875 \times 10^6 \text{ mm}^3$$

$$\simeq 33.219 \times 10^6 \text{ mm}^3$$

Now, $$M_p = 33.219 \times 10^6 \times \frac{250}{10^6} = 8304.75 \text{ kN-m}$$

Hence reduction in capacity from that corresponding to compact behavior

$$= \left(\frac{8304.75 - 7456.75}{8304.75}\right) \times 100 = 10.21\%$$

(ii) For f_y = 410 MPa, maximum $\frac{b}{t}$ for the flange to be semi compact

$$= 13.6 \times \left(\frac{250}{410}\right)^{0.5} = 10.62$$

Therefore the section is slender. Limit for the effective flange wide for semi compact behavior.

$$= 10.62 \times 25 = 265.5 \text{ mm}$$

Hence the top effective width of the flange = 265.5 × 2 + 15 = 546 mm

Location of neutral axis

Taking moments about the top edge, the distance of the neutral axis from the top edge is

$$\bar{y} = \frac{546\times25\times\frac{25}{2}+1500\times15\times\left(\frac{1500}{2}+25\right)+650\times25\times\left(1525+\frac{25}{2}\right)}{546\times25+1500\times15+650\times25}$$

$$= 812.83 \text{ mm}$$

$$I_z = (546\times25)(812.83-12.5)^2+\left(15\times\frac{1500^3}{12}\right)+(15\times1500)$$
$$\times(812.83-775)^2+650\times25\times(737.17-12.5)^2$$

$$= 2.153 \times 10^{10} \text{ mm}^4$$

$$Z_{z(\text{top flange})} = \frac{2.153\times10^{10}}{812.83} = 26487.7 \times 10^3 \text{ mm}^3$$

$$\therefore \quad M_d = 410\times\frac{26487.7\times10^3}{10^6} = 10859.957 \text{ kN-m}$$

$$\simeq 10860 \text{ kNm}$$

Capacity of the whole section is as effective as the semi-compact section.

$$M_d = 410\times26487.7\times\frac{10^3}{10^6} = 10860 \text{ kN-m}$$

Capacity, if the whole section is as effective as the plastic section.

$$M_p = 33.219\times10^6\times\frac{410}{10^6} = 13619.79 \text{ kN-m}$$

Hence reduction in capacity due to slenderness.

$$\frac{13619.79-10860}{13619.79}\times100\% = 20.26\%$$

Thus, 20.26% of the capacity of the cross-section could not be utilized, due to the slenderness of the cross-section.

Q.4 (b) Solution:

(i)

Given $k = 20$ hrs

$x = 0.25$

$$\Delta t = 12 \text{ hrs}$$

Now, $2kx = 2 \times 20 \times 0.25 = 10$ hrs

Now, $k(20\,h) > \Delta t\,(12h) > 2\,kx\,(10\,h)$ (OK)

Now,
$$C_0 = \frac{-kx + 0.5\Delta t}{k - kx + 0.5\Delta t}$$
$$= \frac{-20 \times 0.25 + 0.5 \times 12}{20 - 20 \times 0.25 + 0.5 \times 12} = 0.047$$
$$C_1 = \frac{kx + 0.5\Delta t}{k - kx + 0.5\Delta t}$$
$$= \frac{20 \times 0.25 + 0.5 \times 12}{20 - 20 \times 0.25 + 0.5 \times 12} = 0.524$$

Now, $C_0 + C_1 + C_2 = 1$

$\Rightarrow$ $C_2 = 1 - C_0 - C_1$

$= 1 - 0.047 - 0.524 = 0.429$

Time (hrs)	Inflow (m^3/s)	$C_0I_2 = 0.047I_2$	$C_1I_2 = 0.524I_1$	$C_2Q_1 = 0.429Q_1$	Outflow Q_2 (m^3/s) $Q_2 = C_0I_2 + C_1I_1 + C_2Q_1$
0	50	–	–	–	50
12	70	0.047 × 70	0.524 × 50	0.429 × 50	50.94
24	160	0.047 × 160	0.524 × 70	0.429 × 50.94	66.05
36	250 Peak of inflow	0.047 × 250	0.524 × 160	0.429 × 66.05	123.93
48	230	0.047 × 230	0.524 × 250	0.429 × 123.93	194.98
60	200	0.047 × 200	0.524 × 230	0.429 × 194.98	213.57 Peak of outflow
72	180	0.047 × 180	0.524 × 200	0.429 × 213.57	204.88
84	120	0.047 × 120	0.524 × 180	0.429 × 204.88	187.85
86	100	0.047 × 100	0.524 × 120	0.429 × 187.58	148.17
108	80	0.047 × 80	0.524 × 100	0.429 × 148.17	119.72

Attenuation = (Peak of inflow hydrograph) – (Peak of outflow hydrograph)

$= 250 - 213.57 = 36.43\ m^3/s$

(ii)

The flood forecasting techniques can be broadly divided into three categories:

(a) **Short range forecasts**: In this, the river stages at successive stations on a river are correlated with hydrological parameters, such as rainfall over the local area, antecedent precipitation index, and variation of the stage at the upstream base point during the travel time of a flood. This method can be advance warning of 12 – 40 hrs for flood. The flood forecasting used for a metropolitan city like Delhi is based on this technique.

(b) **Medium -range forecasts**: In this method, rainfall-runoff relationhip are used to predict flood levels with warning of 2-5 days. Coaxial graphical correlations of runoff, with rainfall and other parameters like the time of the year, storm duration and antecedent wetness have been developed to a high stage of refinement by the US Weather Bureau.

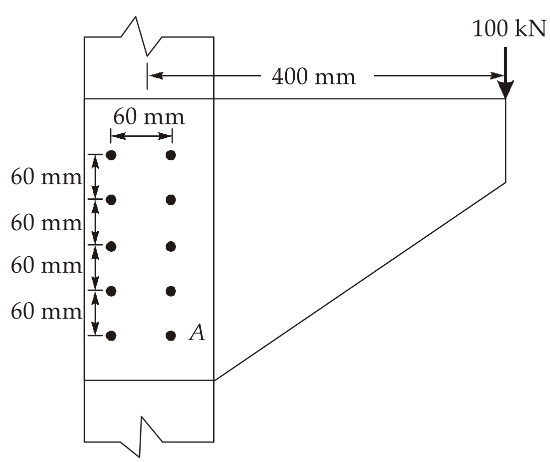
(c) **Long-range forecasts**: Using radars and meteorological satellite data, advance information about critical storm-producing weather system, their rain potential and time of occurrence of the event are predicted well in advance.

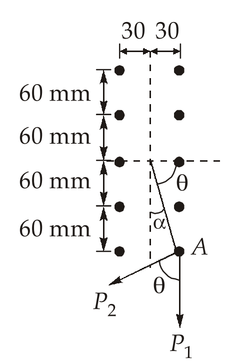
Q.4 (c) Solution:

(i)

The bolt will be subjected to (1) direct shear acting vertically downwards and (2) shear due to torsional moment acting perpendicular to the radius vector.

Assume the bolts arrangement as shown in the figure, having 5 bolts in each of the two rows at a pitch of 60 mm. Equal load will be shared by each bracket i.e. 100 kN.





$$\text{Load on each bracket} = \frac{1}{2}\times 200 = 100 \text{ kN}$$

$$\text{Direct shear force, } P_1 = \frac{100}{10} = 10 \text{ kN}$$

$$\text{Torsional moment, } M = 100 \times 400 = 4 \times 10^4 \text{ kN-mm}$$

Shear force due to torsional moment

$$P_2 = \frac{Mr}{\Sigma r^2}$$

(where r is the radial distance of each bolt from centroid of bolt group).

$$\Sigma r^2 = 4 \times (30^2 + 120^2 + 30^2 + 60^2) + 2 \times 30^2 = 81000 \text{ mm}^2$$

$$\text{For bolt } A, r_A = \sqrt{30^2 + 120^2} = 123.69 \text{ mm}$$

$$\therefore \quad P_2 = \frac{4 \times 10^4 \times 123.69}{81000} = 61.08 \text{ kN}$$

$$\cos\theta = \frac{30}{\sqrt{30^2 + 120^2}} = 0.2425$$

$$\therefore \quad \text{Resultant shear force} = \sqrt{P_1^2 + P_1^2 + 2P_1 P_2 \cos\theta}$$

$$= \sqrt{10^2 + 61.08^2 + 2 \times 10 \times 61.08 \times 0.2425} = 64.24 \text{ kN}$$

Bolts are subjected to single shear.

Now one-way shear strength of bolts,

$$= A_{nb} \times \frac{f_{ub}}{\sqrt{3}\gamma_{mb}}$$

$$= 0.78 \times \pi \times \frac{25^2}{4} \times \frac{400}{\sqrt{3} \times 1.25} \times 10^{-3} \text{ kN}$$

$$= 70.74 \text{ kN} > 64.24 \text{ kN}$$

Hence connection is safe (Assuming bearing strength of the bolts is more than the strength of bolts in single shear).

(ii)

$$\text{Watershed area, } A = 500 \text{ ha} = 5 \text{ km}^2$$

$$t_r = 25 \text{ minutes} = \frac{25}{60} \text{hrs} = 0.4167 \text{ hrs}$$

$$\text{Time of concentration, } t_c = 50 \text{ minutes} = \frac{50}{60} \text{hrs} = 0.833 \text{ hrs}$$

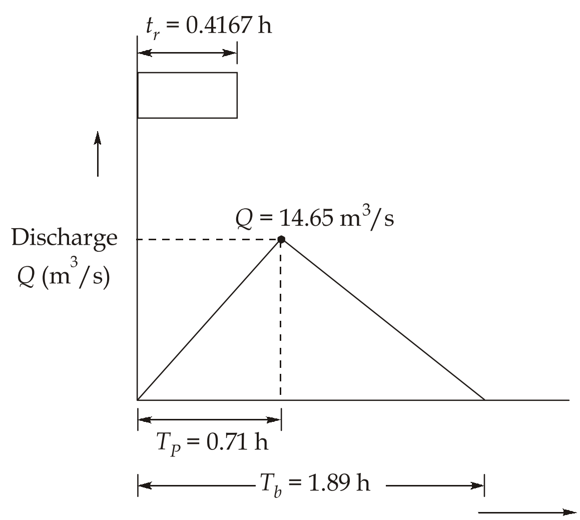
$$\text{Lag time, } t_p = 0.6\, t_c = 0.6 \times 0.833 = 0.50 \text{ hrs}$$

$$T_p = \left(\frac{t_r}{2} + t_p\right) = \left(\frac{0.4167}{2} + 0.50\right) = 0.71 \text{ hrs}$$

$$\text{Discharge, } Q = 2.08\frac{A}{T_p} = 2.08 \times \frac{5}{0.71} = 14.65 \text{ m}^3/\text{s}$$

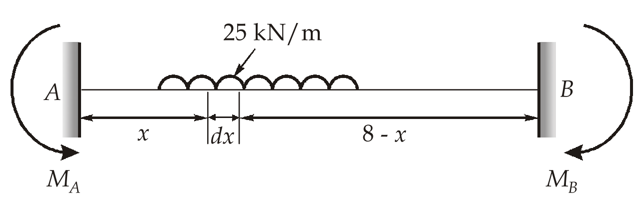
$$\text{Base width, } T_b = 2.67\,T_p = 2.67 \times 0.71 = 1.89 \text{ hrs}$$

The derived triangular unit hydrograph is given as below.



Section B : Structural Analysis-1 + CPM PERT-1 + Flow of fluids, Hydraulic Machines and Hydro Power-2

Q.5 (a) Solution:



Consider an element of length dx at x distance from A,

$$\text{Elemental load, } W_x = 25\,dx$$

$$FEM \text{ at end } A \text{ due to elemental load, } dM_A = \frac{-W_x ab^2}{L^2}$$

$$\Rightarrow \quad dM_A = \frac{-25x \times (8-x)^2\,dx}{8^2}$$

$$\therefore \qquad M_A = \int_1^5 \frac{-25x(8-x)^2\,dx}{64}$$

$$= \frac{-25}{64}\int_1^5 [64x + x^3 - 16x^2]\,dx$$

$$= \frac{-25}{64}\left[32x^2 + \frac{x^4}{4} - \frac{16x^3}{3}\right]_1^5$$

$$= \frac{-25}{64}\left[32(5^2-1^2) + \frac{5^4-1^4}{4} - \frac{16}{3}(5^3-1^3)\right]$$

$$= -\,102.60 \text{ kNm}$$

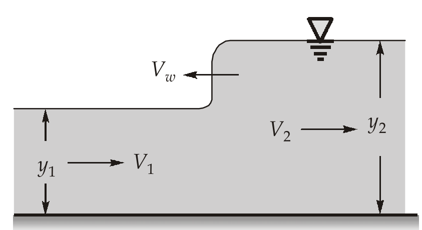
Similarly,
$$\int_0^{M_B} dM_B = \int_1^5 \frac{25dx \times (8-x) \times x^2}{8^2}$$

$$M_B = \frac{25}{64}\int_1^5 [8x^2 - x^3]\,dx$$

$$= \frac{25}{64} \times \left[\frac{8x^3}{3} - \frac{x^4}{4}\right]_1^5 = 68.23 \text{ kN-m}$$

Q.5 (b) Solution:

(i)



Here,
$$y_1 = 1.3 \text{ m}$$
$$V_1 = 0.75 \text{ m/sec.}$$
$$V_w = 4.0 \text{ m/sec.}$$

By continuity equation

$$y_1\,(V_1 + V_w) = y_2\,(V_2 + V_w)$$

$$\Rightarrow \qquad 1.3 \times (0.75 + 4) = y_2\,(V_2 + 4)$$

$$\Rightarrow \qquad V_2 = \frac{6.175}{y_2} - 4$$

By the surge equation,

$$\frac{V_1 + V_w}{\sqrt{g \cdot y_1}} = \left[\frac{1}{2} \times \frac{y_2}{y_1}\left(1 + \frac{y_2}{y_1}\right)\right]^{1/2}$$

$$\Rightarrow \quad \frac{(0.75+4)^2}{9.81\times1.3} = \frac{1}{2}\times\frac{y_2}{y_1}\times\left[1+\frac{y_2}{y_1}\right]$$

$$\Rightarrow \quad \left(\frac{y_2}{y_1}\right)^2+\left(\frac{y_2}{y_1}\right)-3.54 = 0$$

$$\therefore \quad \left(\frac{y_2}{y_1}\right) = \frac{-1\pm\sqrt{1+4\times3.584}}{2} = 1.446$$

$$\therefore \quad y_2 = 1.3 \times 1.446 = 1.88 \text{ m}$$

$$\therefore \quad V_2 = \frac{6.175}{1.88}-4.0 = -\,0.715 \text{ m/sec.}$$

$$= 0.715 \text{ m/sec. in upstream direction.}$$

(ii)

For most efficient trapezoidal section,

$$R = \frac{y_e}{2}$$

But $\quad B + 2\,my_e = 2\sqrt{m^2+1}\cdot y_e$

Here, $\quad m = 2.0$

$$\therefore \quad B + 2 \times 2 \times y_e = 2\sqrt{(2)^2+1}\cdot y_e$$

$$\Rightarrow \quad B_e = 0.4721\, y_e$$

$$\text{Area, } A = (B_e + my_e)y_e$$

$$= (0.4721 + 2) \times y_e^{\,2}$$

$$= 2.4721\, y_e^{\,2}$$

$$R = \frac{y_e}{2}$$

Discharge by Manning's formula

$$Q = \frac{1}{n}AR^{2/3}S_0^{1/2}$$

$$\Rightarrow \quad 15 = \frac{1}{0.014}\times\left(2.4721y_c^2\right)\times\left(\frac{y_e}{2}\right)^{2/3}\left(\frac{1}{5000}\right)^{1/2}$$

$$= 1.573\, y_e^{\,8/3}$$

$$\Rightarrow \quad y_e^{\,8/3} = 9.536$$

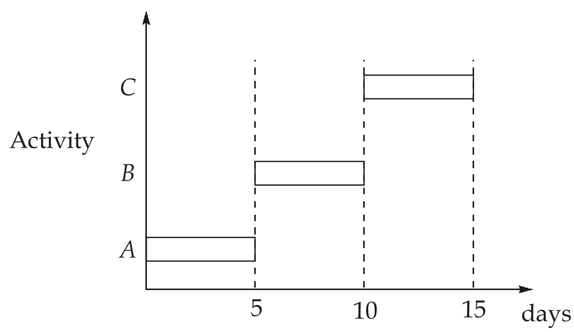
$$\Rightarrow \quad y_e = 2.329 \text{ m}$$

Bottom width, B_e = $2.329 \times 0.4721 = 1.1$ m

Q.5 (c) Solution:

Bar chart which is also popularly known as Gantt chart, was developed by Henry L. Gantt in around 1900 AD.

- A bar chart consists of two co-ordinate axes, one representing activities to be performed and other representing the time elapsed.
- Activities are represented in the form of bars parallel to horizontal axis.
- Each bar represents one activity.
- Length of bar represents time required for the completion of an activity.



Advantages of bar chart are:

1. Simple to draw and easy to understand.
2. No trained or skill person is required.
3. Project progress can be expressed in terms of percentage.
4. It provides a visual representation of the entire project.

Limitations of bar chart are:

1. Lack of degree of details.
2. It does not show the interdependency between the various activities of the project.
3. It can not be used for review of project progress.
4. Controlling, monitoring and updating can not be done for project.

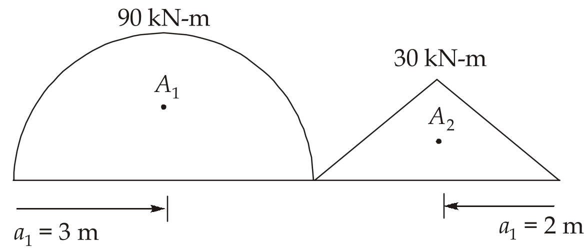
Q.5 (d) Solution:

Free moment diagram for span AB is a parabola.

$$\text{Maximum ordinate} = \frac{wL^2}{8} = \frac{20 \times 6^2}{8} = 90 \text{ kN-m}$$

Maximum ordinate of free moment diagram for span BC,

$$= \frac{PL}{4} = \frac{30 \times 4}{4} = 30 \text{ kN-m}$$



$$A_1 = \frac{2}{3} \times 6 \times 90 = 360 \text{ kN-m}^2$$

$$A_2 = \frac{1}{2} \times 4 \times 30 = 60 \text{ kN-m}^2$$

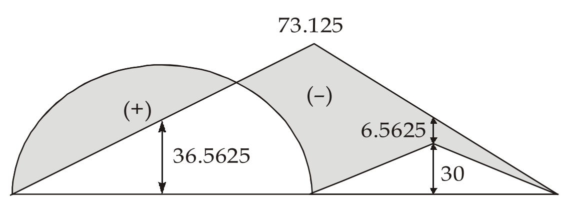
Applying three moment equation,

$$M_A\left[\frac{L_1}{I_1}\right] + 2M_B\left[\frac{L_1}{I_1} + \frac{L_2}{I_2}\right] + M_C\left[\frac{L_2}{I_2}\right] = -\left(\frac{6A_1a_1}{I_1L_1}\right) - \left(\frac{6A_2a_2}{I_2L_2}\right)$$

Here, $M_A = M_C = 0$

$$\Rightarrow \quad 2M_B\left[\frac{6}{I} + \frac{4}{2I}\right] = -\left(\frac{6 \times 360 \times 3}{I \times 6}\right) - \left(\frac{6 \times 60 \times 2}{2I \times 4}\right)$$

$$M_B = -73.125 \text{ kN-m}$$



Values in kNm
BMD

Q.5 (e) Solution:

Given data: C_i = Rs. 1500, C_s = Rs. 300, n = 4 years, r = 8%

Depreciation factor, D is given by

$$D = (C_i - C_s)\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= (1500 - 300)\left[\frac{0.08}{(1+0.08)^4 - 1}\right] = \text{Rs. } 266.3$$

Now, depreciation at the end of m years is given by

$$D_m = D(1+i)^{m-1}$$

Depreciation at the end of 1st year,

$$D_1 = D = \text{Rs. } 266.3$$

Book value at the end of 1st year.

$$B_1 = C_i - D_1 = 1500 - 266.3$$
$$= \text{Rs. } 1233.7$$

Depreciation at the end of 2nd year,

$$D_2 = D(1+i)^{m-1}$$
$$= 266.3(1+0.08)^{2-1} = \text{Rs. } 287.604$$

Book value at the end of 2nd year,

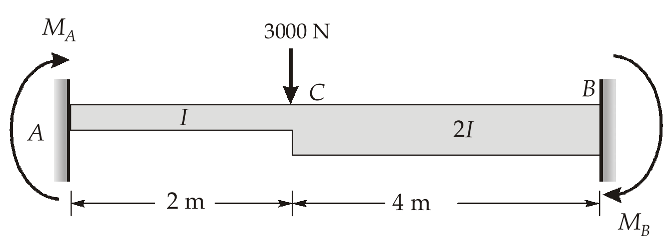
$$B_2 = B_1 - D_2 = 1233.7 - 287.604 = \text{Rs. } 946.096$$

Similarly,

$$D_3 = 266.3(1+0.08)^{3-1} = \text{Rs. } 310.612$$
$$B_3 = 946.096 - 310.612 = \text{Rs. } 635.484$$
$$D_4 = 266.3\,(1+0.08)^{4-1} = \text{Rs. } 335.46$$
$$B_4 = 635.484 - 335.46 = \text{Rs. } 300.024 \simeq \text{Rs. } 300 \text{ (say)}$$

Q.6 (a) Solution:

Using slope deflection method:



Fixed end moments:

$$\bar{M}_{AC} = \bar{M}_{CA} = \bar{M}_{CB} = \bar{M}_{BC} = 0$$

Slope deflection equtions:

Member AC

Here Δ_C is +ve,

$$M_{AC} = \bar{M}_{AC} + \frac{2E(I)}{L}\left(2\theta_A + \theta_C - \frac{3\Delta}{L}\right)$$

$$= 0 + \frac{2EI}{2}\left(\theta_C - \frac{3\Delta}{2}\right) \qquad [\because \text{ End A is fixed}]$$

$$= EI\theta_C - \frac{3EI\Delta}{2} \qquad \text{...(i)}$$

$$M_{CA} = \bar{M}_{CA} + \frac{2EI}{L}\left(2\theta_C + \theta_A - \frac{3\Delta}{L}\right)$$

$$= 0 + \frac{2EI}{2}\left(2\theta_C - \frac{3\Delta}{2}\right)$$

$$= 2EI\theta_C - \frac{3EI\Delta}{2} \qquad \text{...(ii)}$$

Member *CB*:

Here Δ is –ve,

$$M_{CB} = \bar{M}_{CB} + \frac{2E(2I)}{L}\left(2\theta_C + \theta_B + \frac{3\Delta}{L}\right)$$

$$= 0 + \frac{4EI}{4}\left(2\theta_C + \frac{3\Delta}{4}\right) \qquad [\because \text{ End B is fixed}]$$

$$= 2EI\theta_C + \frac{3EI\Delta}{4} \qquad \text{...(iii)}$$

$$M_{BC} = \bar{M}_{BC} + \frac{2E(2I)}{4}\left(2\theta_B + \theta_C + \frac{3\Delta}{L}\right)$$

$$= 0 + EI\left(\theta_C + \frac{3\Delta}{2}\right)$$

$$= EI\theta_C + \frac{3EI\Delta}{4} \qquad \text{...(iv)}$$

Joint equilibrium conditions:

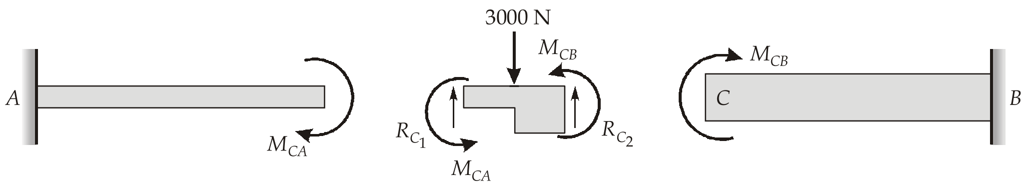
At joint C,

$$M_{CA} + M_{CB} = 0$$

$$\Rightarrow \quad 2EI\theta_C - \frac{3EI\Delta}{2} + 2EI\theta_C + \frac{3EI\Delta}{4} = 0$$

$$\Rightarrow \qquad EI\theta_C = \frac{3EI\Delta}{16} \qquad ...(v)$$

Shear equation:

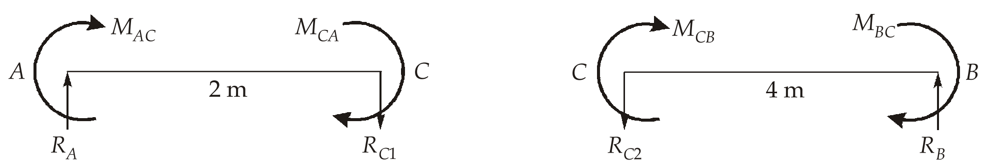


Shear equation at joint C,

$$\Sigma F_y = 0$$

$$\Rightarrow \qquad R_{C1} + R_{C2} = 3000$$

Free body equilibrium of AC and CB are shown below.



$$\Sigma M_A = 0$$

$$\Rightarrow \qquad R_{C1}(2) + M_{AC} + M_{CA} = 0$$

$$\Rightarrow \qquad R_{C1} = -\left(\frac{M_{AC} + M_{CA}}{2}\right)$$

$$\Sigma M_B = 0$$

$$\Rightarrow \qquad R_{C2}(4) - M_{BC} - M_{CB} = 0$$

$$\Rightarrow \qquad R_{C2} = \frac{M_{BC} + M_{CB}}{4}$$

From shear equation,

$$M_{BC} + M_{CB} - 2M_{AC} - 2M_{CA} = 12000$$

$$EI\theta_C + \frac{3EI\Delta}{4} + 2EI\theta_C + \frac{3EI\Delta}{4} - 2EI\theta_C + 3EI\Delta - 4EI\theta_C + 3EI\Delta = 12000$$

$$\Rightarrow \qquad -3EI\theta_C + 7.5EI\Delta = 12000 \qquad ...(vi)$$

From (v) and (vi), we get

$$EI\Delta = 1729.73$$

$$EI\theta_C = 324.32$$

Final end moments

$$M_{AC} = EI\theta_C - \frac{3EI\Delta}{2} = 324.32 - \frac{3}{2}(1729.73)$$

$$= -2270.275 \text{ N-m}$$

$$M_{CA} = 2EI\theta_C - \frac{3EI\Delta}{2} = 2(324.32) - \frac{3}{2}(1729.73)$$

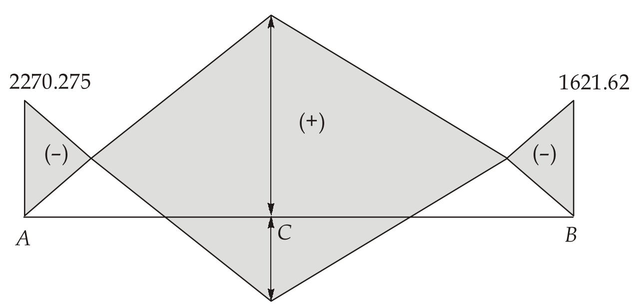
$$= -1945.955 \text{ N-m}$$

$$M_{CB} = 2EI\theta_C + \frac{3EI\Delta}{4} = 2(324.32) + \frac{3}{4}(1729.73)$$

$$= 1945.94 \text{ N-m}$$

$$M_{BC} = EI\theta_C + \frac{3EI\Delta}{4} = 324.32 + \frac{3}{4}(1729.73)$$

$$= 1621.62 \text{ N-m}$$



(Values in kNm)
BMD

Q.6 (b) Solution:

Assuming constant flow velocity and radial discharge at outlet $V_{f1} = V_{f2} = V_2$

Now, flow ratio, $\psi = \dfrac{V_{f1}}{\sqrt{2gH}} = 0.15$

$\Rightarrow$ $V_{f1} = V_{f2} = 0.15 \times \sqrt{2 \times 9.81 \times 70} = 5.56$ m/sec.

Overall efficiency, $\eta_0 = \eta_m \times \eta_H$

$= 0.84 \times 0.95 = 0.798$

Now power developed, $P = \eta_0 \gamma Q H$

Hence, Discharge, $Q = \dfrac{P}{\eta_o \cdot \gamma \cdot H}$

$$= \frac{370 \times 10^3}{0.798 \times 9.81 \times 10^3 \times 70} = 0.675 \text{ m}^3/\text{sec}$$

But discharge, $Q = [\text{Peripheral area}] \times V_{f1}$

$\Rightarrow$ $0.675 = [1 - 0.05] \times \pi \times D_1 B_1 \times 5.56$

$\Rightarrow$ $D_1 B_1 = \dfrac{0.675}{\pi \times 0.95 \times 5.56} = 0.0407$

Since, $B_1 = 0.1 D_1$

Now, $0.1 \times D_1 \times D_1 = 0.0407$

$\Rightarrow$ $D_1^2 = 0.407$

$\Rightarrow$ $D_1 = 0.638 \text{ m}$

$\Rightarrow$ $B_1 = 0.1 \times D_1 = 0.064 \text{ m}$

Also, since, $D_2 = 0.5 \times D_1$

$= 0.5 \times 0.638 \text{ m} = 0.319 \text{ m}$

Pheripheral velocity $u_1 = \dfrac{\pi D_1 N}{60} = \dfrac{\pi \times 0.639 \times 750}{60} = 25.09 \text{ m/sec}$

$u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi \times 0.319 \times 750}{60} = 12.55 \text{ m/sec.}$

Hydraulic efficiency, $\eta_H = \dfrac{u_1 \cdot V_{w1}}{gH} = 0.95$

Hence, swirl velocity at entry, $V_{w1} = \dfrac{\eta_H \cdot g \, H}{u_1}$

$$= \frac{0.95 \times 9.81 \times 70}{25.09} = 26.0 \text{ m/sec.}$$

(i) Guide vane angle at inlet, α_1

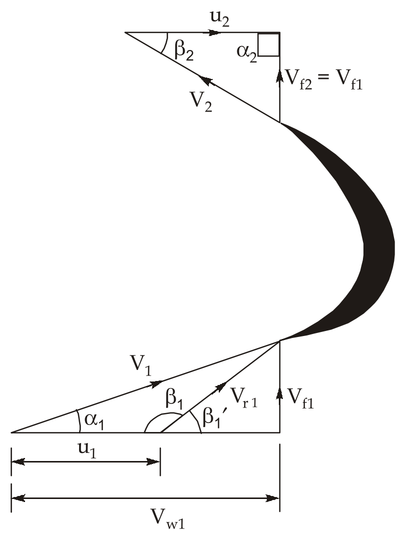
From inlet velocity triangle at inlet:

$$\tan\alpha_1 = \frac{V_{f1}}{V_{w1}} = \frac{5.56}{26} = 0.2138$$

$\therefore$ $\alpha_1 = 12.07°$

(ii) Blade angle at inlet, β_1

Since $V_{w1} > u_1$, angle β_1 is obtuse and the velocity triangle at inlet and outlet will be as shown in figure below.



From inlet triangle,

$$\beta_1 = 180° - \beta_1'$$

$$\tan\beta_1' = \frac{V_{f1}}{V_{w1} - u_1} = \frac{5.56}{26.0 - 25.09} = 6.1099$$

$$\beta_1' = 80.70°$$

$$\beta_1 = 180° - \beta_1' = 99.3°$$

(iii) From outlet velocity triangle,

$$\tan\beta_2 = \frac{V_{f2}}{u_2} = \frac{5.56}{12.53} = 0.4437$$

$\therefore$ $$\beta_2 = 23.93°$$

where β_2 is blade angle at outlet.

Q.6 (c) Solution:

(i)

As there is no load acting at the joint M, considering horizontal force equilibrium at the joint and noting that members (3) and (4) have the same angle of inclination, and thus,

$$N_4 = -N_3$$

By cutting a section y-y as shown in Fig. (a) and noting that N_3 and N_4 are interrelated, there are effectively only three unknown forces, which can be analysed by the method of sections.

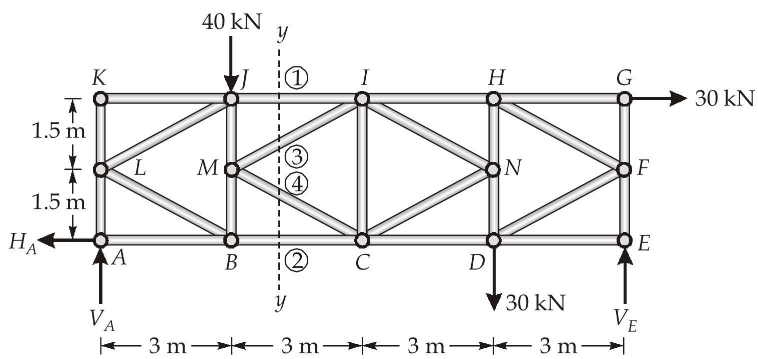


Fig (a) *Overall free-body*

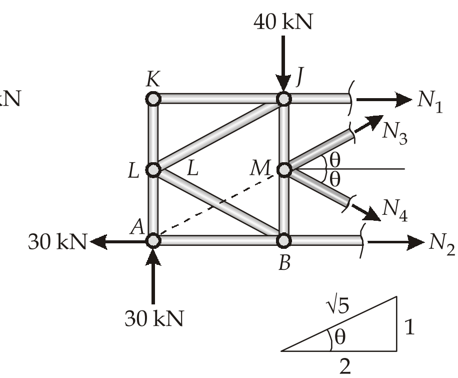


Fig (b) *Section at y-y (free-body)*

Fig. Free-body of truss

Consider the overall free-body Fig. (a), the support reactions can be calculated.

Considering the sectioned free-body in Fig. (b), including the reactions at A and resolving the forces in the vertical direction,

$$\Sigma F_y = 0$$

$$\Rightarrow \quad (30)+(N_3)\left(\frac{1}{\sqrt{5}}\right)-(N_4)\left(\frac{1}{\sqrt{5}}\right)-(40) = 0$$

Substituting $N_3 = -N_4$ and solving,

$$2N_3\left(\frac{1}{\sqrt{5}}\right) = 10$$

$$\Rightarrow \quad N_3 = 11.1803 \text{ kN (tension)}$$

$$\therefore \quad N_4 = -N_3 = -11.1803 \text{ kN (compression)}$$

Taking moments about the point B,

$$\Sigma M_B = 0$$

$$\Rightarrow \quad (30)(3)+(N_1)(3)+(N_3+N_4)\left(\frac{2}{\sqrt{5}}\right)(1.5) = 0$$

$$\Rightarrow \quad N_1 = -30 \text{ kN (compression)}$$

Resolving the forces horizontally,

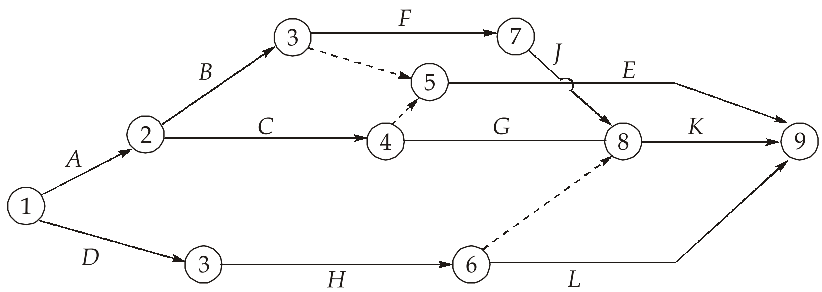
$$\Sigma F_x = 0$$

$$\Rightarrow \quad (N_1)+\left(\frac{N_3}{\sqrt{5}}\right)+\left(\frac{N_4}{\sqrt{5}}\right)+(N_2)-(H_A) = 0$$

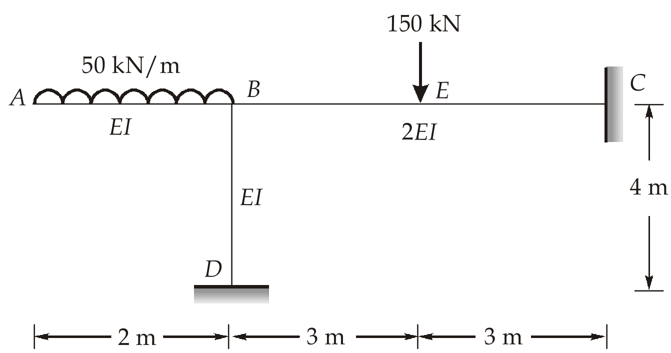
$$\Rightarrow -30 + \frac{1}{\sqrt{5}}(11.1803 - 11.1803) + N_2 - 30 = 0$$

$$\Rightarrow \qquad N_2 = 60 \text{ kN (tension)}$$

(ii)



Q.7 (a) Solution:



Distribution factor (DF):

Joint	Member	Member stiffness	Joint stiffness	D.F. = $\frac{M.S.}{J.S.}$
B	BC	$\frac{4(2EI)}{6}$	$\frac{7EI}{3}$	0.57
	BD	$\frac{4EI}{4}$		0.43

Fixed end moment:

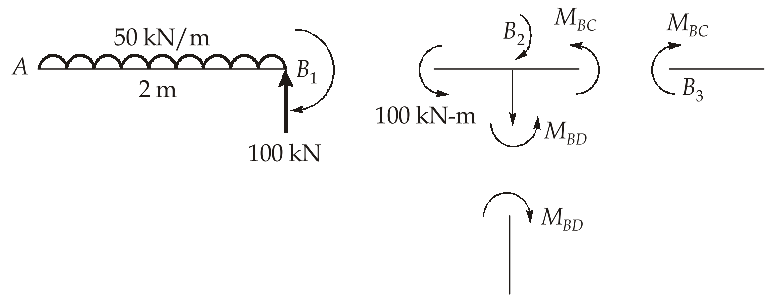
Span BC:

$$\bar{M}_{BD} = \bar{M}_{DB} = 0$$

$$\bar{M}_{BC} = \frac{-150 \times 6}{8} = -112.5 \text{ kN-m}$$

$$\bar{M}_{CB} = \frac{+150 \times 6}{8} = +112.5 \text{ kN-m}$$

Joint equation equilibrium at B:

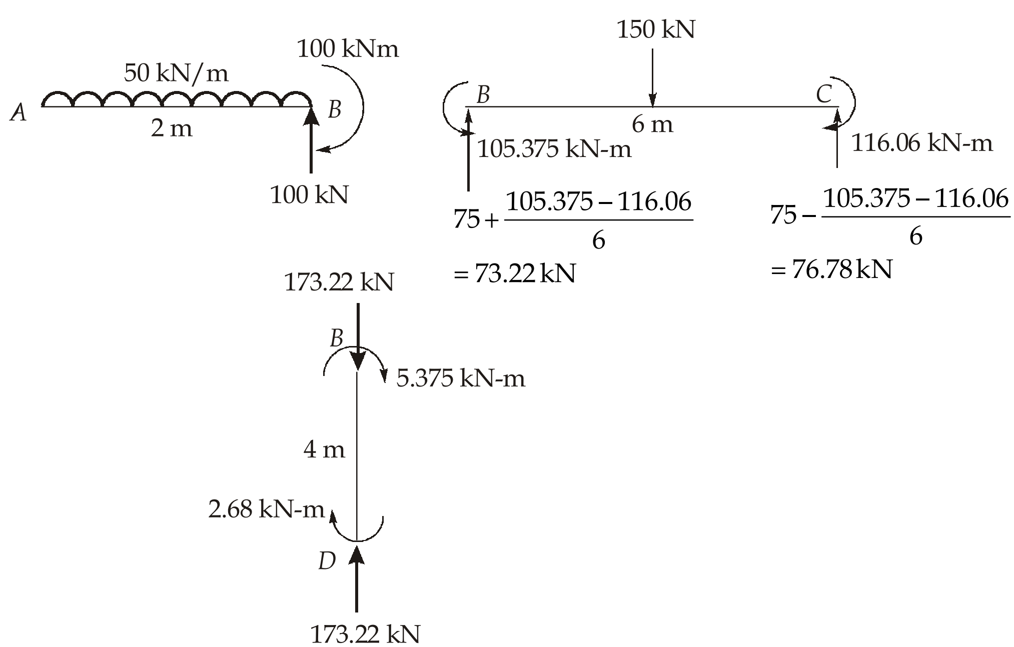


$$M_{BC} + M_{BC} + 100 = 0$$

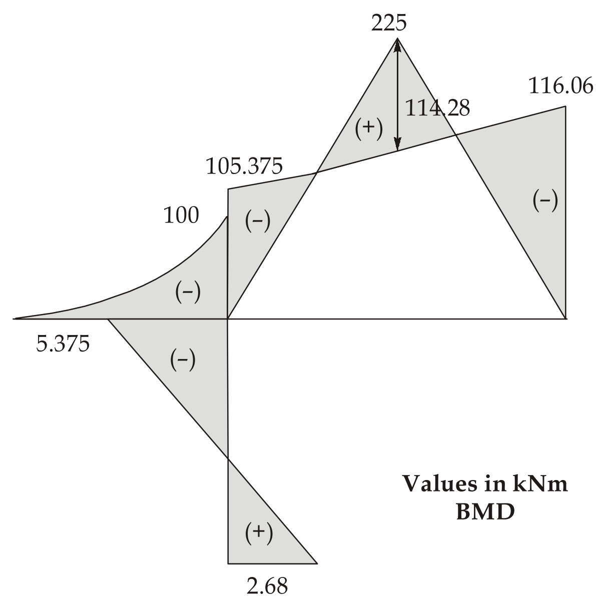
$$\Rightarrow \quad M_{BC} + M_{BD} = -100$$

D	B	B	C
	0.43	0.57	
0	0	–112.5	112.5
	5.375	7.125	
2.68			3.56
2.68	5.375	–105.375	116.06

Free body diagram:

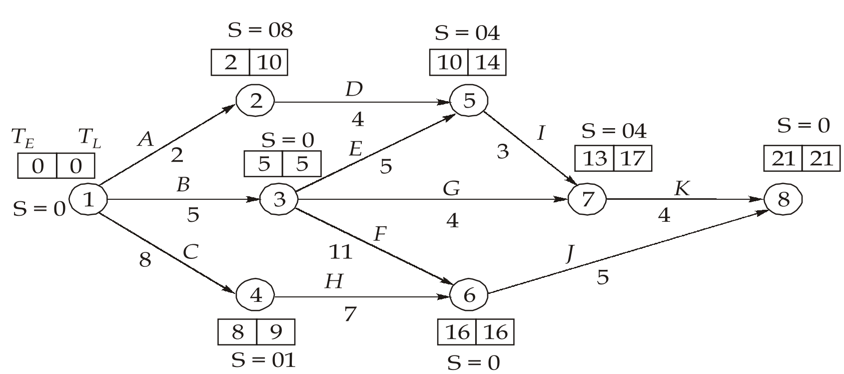


Bending moment diagram:



Q.7 (b) Solution:

The network of the given activities is shown in fiure below.



The earliest event time and latest event time have been calculated as:

$$T_{Ej} = T_{Ei} + t_{ij}$$
$$T_{Li} = T_{Lj} - t_{ij}$$

We also know that,

$$EST = T_{Ei}$$
$$EFT = T_{Ei} + t_{ij}$$
$$\text{LST} = T_{Lj} - t_{ij}$$
$$LFT = T_{Lj}$$

Floats

$$\text{Total float, } F_T = LST - EST$$
$$\text{Free float, } F_F = F_T - S_j$$
$$\text{Independent float, } F_{ID} = F_F - S_i$$

The values of activity times, total float, free float and independent float are tabulated below.

Activitiy (i–j)	Duration (days) t_{ij}	Earliest Time		Latest Time		Total Float (F_T)	Free Float (F_F)	Independent Float (F_{ID})	Remarks
		Start (EST)	Finish (LFT)	Start (LST)	Finish (LFT)				
A	2	0	2	08	10	08	0	0	
B	5	0	5	0	5	0	0	0	Critical
C	8	0	8	01	9	01	0	0	
D	4	2	6	10	14	08	4	– 4	
E	5	05	10	09	14	04	0	0	
F	11	05	16	05	16	0	0	0	Critical
G	4	05	9	13	17	08	4	4	
H	7	8	15	09	16	01	1	0	
I	3	10	13	14	17	04	0	– 4	
J	5	16	21	16	21	0	0	0	Critical
K	4	13	17	17	21	04	4	0	

The activities on the critical path are those activities that have total float (F_T) equal to zero. Thus B – F – J will be the critical path.

Q.7 (c) Solution:

(i)

Given: $D_2 = 0.8$ m, $Q = 1.1$ m^3/sec, $H = 70$ m, $N = 1000$ rpm

$B_2 = 0.08$ m, $\eta_h = 0.82$

Leakage loss = 4%

$$\therefore \quad Q_{th} = 1.1 \times 1.04 = 1.144 \text{ m}^3/\text{sec}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.8 \times 1000}{60} = 41.89 \text{ m/sec}$$

Since,

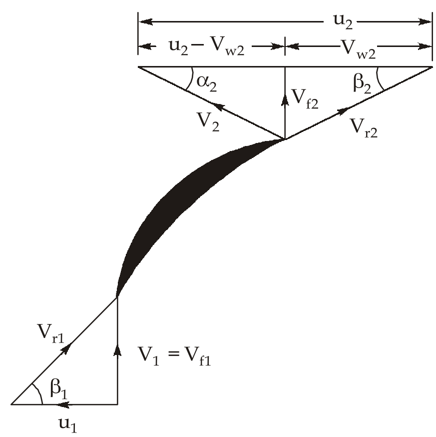
$$Q_{th} = \pi D_2 B_2 V_{f2}$$

$$\Rightarrow \quad V_{f2} = \frac{1.144}{\pi \times 0.8 \times 0.08} = 5.69 \text{ m/sec.}$$

Since $\eta_H = \frac{gH}{u_2 V_{w2}}$

$\Rightarrow$ $V_{w2} = \frac{9.81 \times 70}{41.89 \times 0.82} = 20$ m/sec.

From velocity triangle at the outlet,



$$\tan\beta_2 = \frac{V_{f2}}{u_2 - V_{w2}} = \left(\frac{5.69}{41.89 - 20}\right) = 0.26$$

$\therefore$ $\beta_2 = 14.57°$

$$\text{Power required, } P = \left[\frac{V_{w2} \cdot u_2}{g} \gamma \cdot Q_{th}\right] + 10$$

$$= \left[\frac{20 \times 41.89}{9.81} \times 9.81 \times 1.144\right] + 10 = 968.4 \text{ kW}$$

$$\text{Mechanical efficiency, } \eta_{mech.} = \frac{(968.4 - 10)}{968.4} = 0.99$$

$$\text{Overall efficiency, } \eta_0 = \eta_{mech.} \times \eta_H$$

$$= 0.99 \times 0.82$$

$$= 0.812$$

$$= 81.2\%$$

(ii)

$$\text{Net available head, } H = 400 \times (1 - 0.05) = 380 \text{ m}$$

$$\text{Power per jet} = 500 \text{ kW}$$

$$\text{Specific speed} = \frac{N\sqrt{P}}{H^{5/4}}$$

$$\Rightarrow \quad 14 = \frac{N \times \sqrt{500}}{(380)^{5/4}} = 0.01333 \text{ N}$$

$$\therefore \quad \text{Rotational speed, } N = 1050 \text{ rpm}$$

$$\therefore \quad V_1 = C_v\sqrt{2gH}$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 380} = 84.62 \text{ m/sec.}$$

$$\text{Speed ratio, } \phi = \frac{u}{\sqrt{2gH}} = 0.46$$

$$\Rightarrow \quad u = 0.46 \times \sqrt{2 \times 9.81 \times 380}$$

$$= 39.72 \text{ m/sec.}$$

$$\text{But} \quad u = \frac{\pi DN}{60}$$

$$\Rightarrow \quad 39.72 = \frac{\pi \times D \times 1050}{60}$$

$$\Rightarrow \quad D = 0.722 \text{ m} \qquad \text{(mean diameter of bucket circle)}$$

$$\text{Power developed} = 1000 \text{ kW} = \gamma Q.\ H.\ \eta_O$$

$$\Rightarrow \quad 1000 = 9.81 \times Q \times 380 \times 0.85$$

$$\Rightarrow \quad Q = 0.3156 \text{ m}^3/\text{sec}$$

As there are two jets of diameter d,

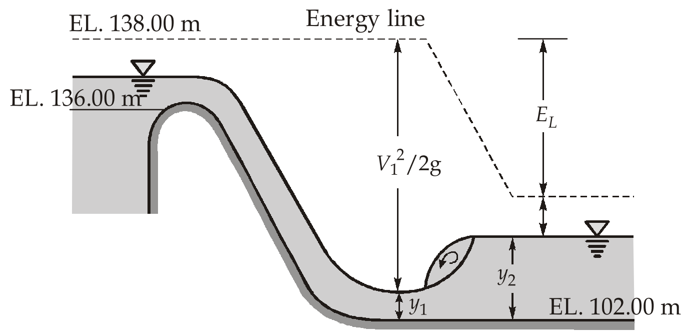
$$2 \times \frac{\pi}{4} \times d^2 \times 84.62 = 0.3156$$

$$\Rightarrow \quad d = 0.04873 \text{ m}$$

$$d = 4.878 \text{ cm}$$

Q.8 (a) Solution:

(i)



The discharge per unit width of the spillway i.e. q is given by

$$q = \frac{2}{3}C_d\sqrt{2g}H^{3/2}$$

where, $H = 138.00 - 136.00 = 2.00$ m

So,
$$q = \frac{2}{3}\times 0.735\times\sqrt{2\times 9.81}\times(2.0)^{3/2}$$
$$= 6.139 \text{ m}^3/\text{s/m}$$
$$E_1 = 138.00 - 102.00 = 36.00 \text{ m}$$

But,
$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2} = y_1 + \frac{(6.139)^2}{19.62\, y_1^2}$$

Hence
$$y_1 + \frac{1.9208}{y_1^2} = 36.00$$

$\Rightarrow$
$$y_1 = 0.2317 \text{ m}$$

$\therefore$
$$V_1 = \frac{q}{y_1} = \frac{6.139}{0.2317} = 26.495 \text{ m/s}$$

For rectangular channel

$$\text{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{26.495}{\sqrt{9.81\times 0.2317}} = 17.574$$

$\therefore$
$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1+\sqrt{1+8\text{Fr}_1^2}\right) = \frac{1}{2}\left(-1+\sqrt{1+8\times(17.574)^2}\right)$$
$$= 24.36$$

$\therefore$
$$y_2 = 0.2317 \times 24.36 = 5.644 \text{ m}$$

$\therefore$ Required tailwater elevation = 102.000 + 5.644 = 107.644 m

(ii)

Assumptions in gradually varies flow:

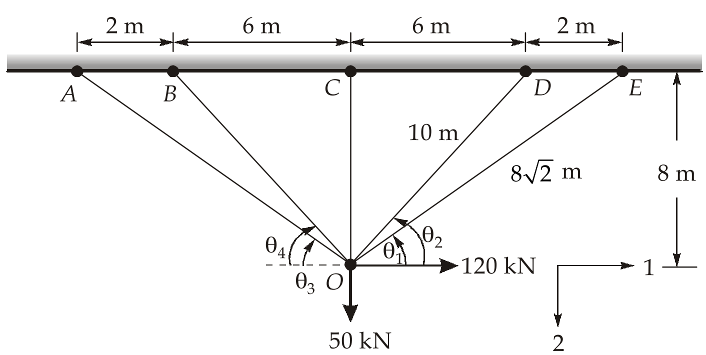
1. The pressure distribution at any section in gradually varied flow is assumed to be hydrostatic. This is because the curvature of stream lines is very small or ngeligible. The exclusion of the region of high curvature from the analysis of GVF is required to meet this assumption.
2. The resistance to flow at any depth is assumed to be given by the corresponding uniform flow equation, such as Manning's or Chezy's equation. The only condition is that the slope term to be used in these equations is the slope of energy line (S_f) instead of bed slope (S_0)

$$\text{Manning's equation, } V = \frac{1}{n}R^{2/3}(S_f)^{1/2}$$

$$\text{and Chezy's equation, } V = C\sqrt{R\,S_f}$$

3. Channel is prismatic with small channel bed slope, i.e. hydraulic gradient line (HGL) will lie at free surface.
4. There is no air entrainment.
5. Velocity distribution is invariant i.e. kinetic energy correction factor, α is 1.
6. Flow is steady i.e. there is no lateral inflow or outflow from the channel.
7. Resistance coefficint (C and n) are constants with the depth of flow.

Q.8 (b) Solution:



$$\tan\theta_1 = \tan\theta_3 = \frac{8}{8} = 1 \qquad \Rightarrow \qquad \theta_1 = \theta_3 = 45°$$

$$\tan\theta_2 = \tan\theta_4 = \frac{8}{6} = \frac{4}{3} \qquad \Rightarrow \qquad \theta_2 = \theta_4 \simeq 53°$$

At 'O' using stiffnes matrix method,

$$\text{Stiffness matrix, } k = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$k_{11} = \frac{\Sigma AE}{L}\cos^2\theta_i$$

$$= 2\times\left[\frac{1}{10}\cos^2 53°\right] + 2\times\left[\frac{1}{8\sqrt{2}}\cos^2 45°\right]$$

$$+ \frac{1}{8}\cos^2 90° = 0.1608$$

$$k_{22} = \frac{\Sigma AE}{L}\sin^2\theta_i = 2\times\left[\frac{1}{10}\sin^2 53°\right] + 2\times\left[\frac{1}{8\sqrt{2}}\sin^2 45°\right]$$

$$+ \frac{1}{8}\sin^2 90° = 0.34095$$

$$k_{12} = k_{21} = \frac{\Sigma AE}{L}\cos\theta_i\sin\theta_i$$

$$= \left[\frac{1}{10}\cos 53°\times\sin 53° - \frac{1}{10}\cos 53°\times\sin 53°\right]$$

$$+\left[\frac{1}{8\sqrt{2}}\cos 45°\times\sin 45° - \frac{1}{8\sqrt{2}}\cos 45°\times\sin 45°\right]$$

$$+\frac{1}{8}\times[\cos 90°\sin 90°] = 0$$

$\therefore$ $$[F] = [k][\Delta]$$

So, $$\begin{bmatrix}120\\50\end{bmatrix} = \begin{bmatrix}0.1608 & 0\\0 & 0.3409\end{bmatrix}\begin{bmatrix}\Delta_1\\\Delta_2\end{bmatrix}$$

$$0.1608\Delta_1 = 120$$

and $$0.34095\Delta_2 = 50$$

Solving above equations:

$$\Delta_1 = \frac{120}{0.1605} = 746.25\ (\rightarrow)$$

$$\Delta_2 = \frac{50}{0.3409} = 146.67\ (\downarrow)$$

Now forces in the members are.

$$P_{OA} = \frac{AE}{L}[\Delta_1\cos\theta_3 + \Delta_2\sin\theta_3]$$

$$= \frac{1}{8\sqrt{2}}[746.27\cos 45° + 146.65\sin 45°]$$

$$= 55.81 \text{ kN (Tensile)}$$

$$P_{OB} = \frac{1}{10}[746.27\cos 53° + 146.65\sin 53°]$$

$$= 56.62 \text{ kN (Tensile)}$$

$$P_{OC} = \frac{1}{8}[146.65\sin 90°] = 18.33 \text{ kN (Tensile)}$$

$$P_{OD} = \frac{1}{10}[-746.27\cos 53° + 146.65\sin 45°]$$

$$= -33.19 \text{ kN (Compressive)}$$

$$P_{OE} = \frac{1}{8\sqrt{2}}[-746.26\cos 45° + 146.67\sin 456]$$

$$= -37.48 \text{ kN (Compressive)}$$

Q.8 (c) Solution:

(i)

For given project, best alternative is determined using net present value method.

For alternative 'A',

Construction cost, C_A = 350 Lacs

Running cost per year, C_A' = 10 Lacs

Benefit per year, B_A = 40 Lacs

Single series present worth factor, F_A = 18.20

Net present value of alternative, 'A' is given

$$= B_A \cdot F_A - C_A - C_A' F_A$$

$$= 40 \times 18.20 - 350 - 10 \times 18.20$$

= Rs. 196 Lacs

For alternative 'B'

Net present value of alternative 'B' = $B_B \cdot F_B - C_B - C_B' F_B$

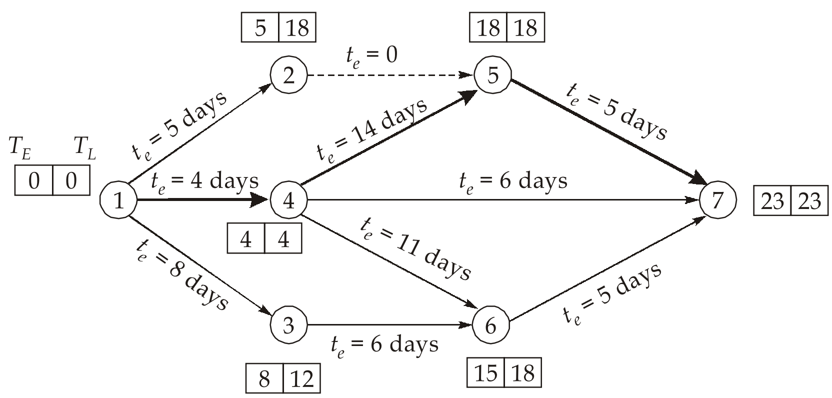
$$= 60 \times 15.01 - 300 - 20 \times 15.01$$

= Rs. 300.4 Lacs

As net present value of alternative 'B' is more than 'A'. Alternative 'B' is more economical than alternative 'A'.

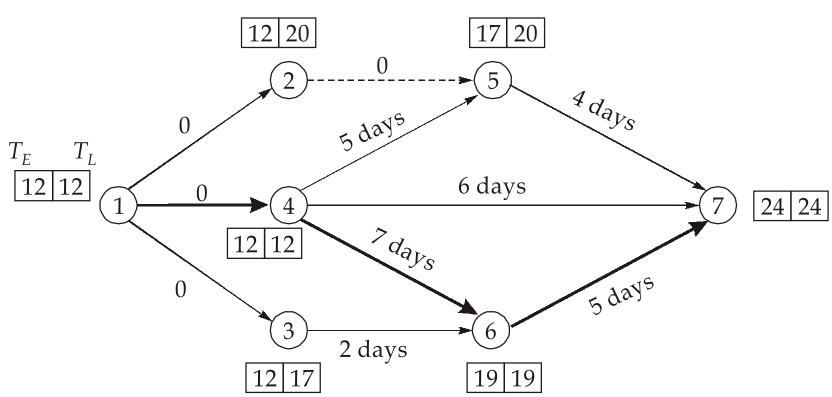
(ii)

Original network:



Original critical path is 1 – 4 – 5 – 7 and project duration is 23 days.

Updated network:



New critical path is 1 – 4 – 6 – 7 and new project duration is 24 days.

Increase in project duration = 24 – 23 = 1 day.

■■■■