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Detailed Solutions

# ESE-2024 Mains Test Series

# Electrical Engineering Test No : 6

## Section A : Power Electronics & Drives + Engineering Mathematics

**Q.1 (a) Solution:**

Let,
$$f(z) = \frac{z-1}{(z+2)(z+3)} = \frac{A}{z+2} + \frac{B}{z+3}$$

$$z - 1 = A(z + 3) + B(z + 2)$$

$$\text{Put } z = -2$$

$$-2 - 1 = A(-2 + 3) + 0$$

$$A = -3$$

Put
$$z = -3$$

$$-3 - 1 = A(0) + B(-3 + 2)$$

$$-4 = -B$$

$$B = 4$$

$\therefore$
$$f(z) = \frac{-3}{z+2} + \frac{4}{z+3}$$

Given region is $2 < |z| < 3$

$2 < |z|$ and $|z| < 3$

$\left|\frac{2}{z}\right| < 1$ and $\left|\frac{z}{3}\right| < 1$

$\therefore$
$$f(z) = \frac{-3}{z\left(1+\frac{2}{z}\right)} + \frac{4}{3\left(1+\frac{z}{3}\right)}$$

$$= \frac{-3}{z}\left(1+\frac{2}{z}\right)^{-1} + \frac{4}{3}\left(1+\frac{z}{3}\right)^{-1}$$

$$= \frac{-3}{z}\left[\left(1-\frac{2}{z}+\left(\frac{2}{z}\right)^2 - \ldots\right)\right] + \frac{4}{3}\left[1-\frac{z}{3}+\left(\frac{z}{3}\right)^2 - \ldots\right]$$

**Q.1 (b) Solution:**

$$(D^2 + 5D + 6)y = e^{-2x} \sec^2 x\,(1 + 2\tan x)$$

Auxiliary equation is

$$m^2 + 5m + 6 = 0$$

$$(m + 2)(m + 3) = 0$$

$$m = -2$$

and $m = -3$

Hence complementary function (C.F.)

$$= C_1 e^{-2x} + C_2 e^{-3x}$$

$$\text{P.I.} = \frac{1}{D^2+5D+6} e^{-2x \sec^2 x(1+2\tan x)}$$

$$= e^{-2x}\frac{1}{(D-2)^2+5(D-2)+6}\sec^2 x(1+2\tan x)$$

$$= e^{-2x}\frac{1}{D^2-4D+4+5D-10+6}\sec^2 x(1+2\tan x)$$

$$= e^{-2x}\frac{1}{D^2+D}\sec^2 x(1+2\tan x)$$

$$= e^{-2x}\left[\frac{\sec^2 x}{D^2+D} + \frac{2\tan x\sec^2 x}{D^2+D}\right]$$

$$= e^{-2x}\frac{1}{D(D+1)}\sec^2 x + \frac{1}{D(D+1)}2\tan x\sec^2 x$$

$$= e^{-2x}\left[\left(\frac{1}{D}-\frac{1}{D+1}\right)\sec^2 x + \frac{1}{D}2\tan x\sec^2 x - \frac{1}{D+1}2\tan x\sec^2 x\right]$$

$$= e^{-2x}\left[\tan x - e^{-x}\int e^x \sec^2 x dx + \tan^2 x - e^{-x}\int 2e^x \tan x \sec^2 x\, dx\right]$$

Now, $\int e^x \sec^2 x\, dx = e^x \sec^2 x - \int e^x 2\sec x \sec^2 x \tan x\, dx$

$$= e^x \sec^2 x - 2\int e^x \sec^2 x \tan x \, dx$$

$$\therefore \quad \text{P.I.} = e^{-2x}\left[\tan x - \sec^2 x + \tan^2 x\right]$$

$$= e^{-2x}\left[\tan x - (\sec^2 x - \tan^2 x)\right]$$

$$= e^{-2x}(\tan x - 1)$$

Complete solution is

$$y = \text{C.F.} + \text{P.I.}$$

$$= c_1 e^{-2x} + c_2 e^{-3x} + e^{-2x}(\tan x - 1)$$

**Q.1 (c) Solution:**

Peak value of resonant current,

$$I_P = V_S\sqrt{\frac{C}{L}} = 230\sqrt{\frac{30\times 10^{-6}}{10\times 10^{-6}}} = 398.37 \text{ A}$$

Resonant frequency, $\omega_0 = \dfrac{1}{\sqrt{LC}}$

$$= \frac{10^6}{\sqrt{300}} = 57.735 \times 10^3 \text{ rad/sec}$$

**(i)** Conduction time for auxiliary thyristor

$$= \frac{\pi}{\omega_0} - \frac{\pi}{57.73\times 10^3} = 54.41 \times 10^{-6}$$

$$= 54.41 \ \mu\text{sec}$$

**(ii)** Since, $\omega_0 \cdot \theta = \sin^{-1}\left(\dfrac{300}{398.37}\right) = 48.857°$

Voltage across the main thyristor, when it gets turned off

$$V' = V_S \cos \omega_0 \cdot \theta$$

$$= 230 \cos 48.857°$$

$$V' = 151.34 \text{ volts}$$

**(iii)** Circuit turn off time for main thyristor

$$t_c = C \cdot \frac{V'}{I_0} = \frac{30\times 10^{-6} \times 151.34}{300} = 15.132 \ \mu\text{sec}$$

**Q.1 (d) Solution:**

Let $d$ be the diameter of the sphere and $x$, $y$, $z$, the three edges of the rectangular solid of volume $V$. Then

$$d^2 = x^2 + y^2 + z^2, \quad \text{...(i)}$$

and $$V = xyz \quad \text{...(ii)}$$

From equations ($i$) and ($ii$),

we have $$V = xy\sqrt{d^2 - x^2 - y^2}$$

$$V^2 = x^2y^2(d^2 - x^2 - y^2)$$

For maximum or minimum of $V^2$ and hence that of $V$

$$\frac{\partial}{\partial x}(V^2) = 2xy^2(d^2 - 2x^2 - y^2) = 0, \quad \text{...(iii)}$$

$$\frac{\partial}{\partial y}(V^2) = 2x^2y(d^2 - x^2 - 2y^2) = 0, \quad \text{...(iv)}$$

From equations ($iii$) and ($iv$), we have

$$d^2 - 2x^2 - y^2 = 0, \quad \text{...(v)}$$

and $$d^2 - x^2 - 2y^2 = 0 \quad \text{...(vi)}$$

$(\because x \neq 0, y \neq 0)$

From equations ($v$) and ($vi$), we get

$$x^2 = y^2 = \frac{d^2}{3}$$

Putting, $x^2 = y^2 = \frac{d^2}{3}$ in equation ($i$), we get

$$z^2 = \frac{d^2}{3}$$

$\therefore$ $$x^2 = y^2 = z^2 = \frac{d^2}{3}$$

Now at $x = y = z = \frac{d}{\sqrt{3}}$, we have

$$r = \frac{\partial^2}{\partial x^2}(V^2) = 2y^2(d^2 - 6x^2 - y^2) = -\frac{8d^4}{9}$$

$$s = \frac{\partial^2}{\partial x \partial y}(V^2) = 4xy(d^2 - 2x^2 - 2y^2) = -\frac{4d^4}{9}$$

$$t = \frac{\partial^2}{\partial y}(V^2) = 2x^2(d^2 - x^2 - 6y^2) = -\frac{8d^4}{9}$$

Hence, $rt - s^2 = \frac{64d^8}{81} - \frac{16d^8}{81} = \frac{48d^8}{81}$, (+ ve).

Since $rt - s^2 > 0$ and $r < 0$,

$V^2$ and hence $V$ is maximum at $x = y = z = \frac{d}{\sqrt{3}}$

Thus the solid is a cube and maximum volume = $\frac{d^3}{3\sqrt{3}}$ cubic units.

**Q.1 (e) Solution:**

A square matrix '$A$' is said to be unitary if $A^\theta A = I$,

Where, $A^\theta = (\bar{A})^T$ and $I$ is an identity matrix

We have, $$N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$$

$$I - N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix} \quad \text{...(i)}$$

Now we have to find $(I + N)^{-1}$

$$I + N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1+2i \\ -1+2i & 1 \end{bmatrix}$$

$$|I + N| = 1 - (-1 - 4) = 6$$

$$\text{Adj. } (I + N) = \begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}$$

$$(I + N)^{-1} = \frac{\text{Adj}(I+N)}{|I+N|} = \frac{1}{6}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix} \quad \text{...(ii)}$$

For unitary matrix, $A^\theta A = I$

From (i) and (ii), we get

$$\therefore \quad (I - N)(I + N)^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} -4 & -2-4i \\ 2-4i & -4 \end{bmatrix} = B$$

Now $$(\bar{B})^T = \frac{1}{6}\begin{bmatrix} -4 & 2+4i \\ -2+4i & -4 \end{bmatrix}$$

$\therefore$ $$(\bar{B})^T B = \frac{1}{36}\begin{bmatrix} -4 & 2+4i \\ -2+4i & -4 \end{bmatrix}\begin{bmatrix} -4 & -2-4i \\ 2-4i & -4 \end{bmatrix}$$

$$= \frac{1}{36}\begin{bmatrix} 36 & 0 \\ 0 & 36 \end{bmatrix} = I$$

**Q.2 (a) Solution:**

**(i)** Impedance angle $= \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$= \tan^{-1}\left(\frac{2\times 50\times 3.14\times 200\times 10^{-3}}{20}\right) = 72.34°$$

Since, $\tan^{-1}\left(\frac{\omega L}{R}\right) > \alpha\ (60°)$

So load current is continuous,

Average output voltage,

$$(V_{0\,avg}) = \frac{2V_m}{\pi}\cos\alpha = \frac{2\times 100\sqrt{2}}{\pi}\cos 60° = 45 \text{ volt}$$

Average output current,

$$(I_{0\,avg}) = \frac{V_{0\,avg}}{R} = \frac{45}{20} = 2.25 \text{ Amp}$$

**(ii)** First dominant harmonic will be 2nd harmonic.

2nd harmonic voltage,

$$(V_2) = \sqrt{a_2^2 + b_2^2}$$

$$a_n = \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \cdot \cos n\omega t \cdot d(\omega t)$$

$$= \frac{V_m}{\pi}\int_{\alpha}^{\pi+\alpha} (\sin(1+n)\omega t + \sin(1-n)\omega t)\cdot d(\omega t)$$

$$a_n = \frac{V_m}{\pi}\left[\frac{-\cos(1+n)\omega t}{(1+n)} + \left(\frac{-\cos(1-n)\omega t}{(1-n)}\right)\right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{-V_m}{\pi}\left[\begin{array}{l}\frac{1}{(1+n)}[\cos(1+n)(\pi+\alpha) - \cos(1+n)\alpha] \\ +\frac{1}{(1-n)}[\cos(1-n)(\pi+\alpha) - \cos(1-n)\alpha]\end{array}\right]$$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(1+n)\alpha}{(1+n)} + \frac{\cos(1-n)\alpha}{(1-n)}\right], \; n = \text{even} \qquad \text{...(i)}$$

$$b_n = \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \cdot \sin n\omega t \cdot d(\omega t)$$

$$= \frac{V_m}{\pi}\int_{\alpha}^{\pi+\alpha} (\cos(1-n)\omega t - \cos(1+n)\omega t)d(\omega t)$$

$$b_n = \frac{V_m}{\pi}\left[\frac{\sin(1-n)\omega t}{(1-n)} - \frac{\sin(1+n)\omega t}{(1+n)}\right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{V_m}{\pi}\left[\begin{array}{l}\left(\frac{\sin(\pi+\alpha)(1-n) - \sin(1-n)\alpha}{(1-n)}\right) \\ -\frac{1}{(1+n)}(\sin(1+n)(\pi+\alpha) - \sin(1+n)\alpha)\end{array}\right]$$

$$= \frac{-V_m}{\pi}\left[\frac{2\sin(1-n)\alpha}{(1-n)} + \frac{2\sin(1+n)\alpha}{(1+n)}\right]; \; n = \text{even}$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(1+n)\alpha}{(1+n)} + \frac{\sin(1-n)\alpha}{(1-n)}\right], \; n = \text{even} \qquad \text{...(ii)}$$

From equation (i),

$$a_2 = \frac{2\times 100\sqrt{2}}{\pi}\left[\frac{\cos\frac{3\pi}{3}}{3} + \frac{\cos\left(\frac{-\pi}{3}\right)}{(-1)}\right]$$

$$= \frac{2V_m}{\pi}\left[\frac{-1}{3} - \frac{1}{2}\right]$$

$$a_2 = \frac{2V_m}{\pi}\left[\frac{-5}{6}\right] \quad \text{...(iii)}$$

From equation (ii),

$$b_2 = \frac{2V_m}{\pi}\left[\frac{\sin\frac{3\pi}{3}}{3} + \frac{\sin\left(\frac{-\pi}{3}\right)}{(-1)}\right] = \frac{2V_m}{\pi}\left[0 + \frac{\sqrt{3}}{2}\right]$$

$$b_2 = \frac{2V_m}{\pi}\left[\frac{\sqrt{3}}{2}\right] \quad \text{...(iv)}$$

Now, $$V_2 = \sqrt{a_2^2 + b_2^2} = \frac{2V_m}{\pi}\sqrt{\frac{25}{36} + \frac{3}{4}}$$

$$= \frac{2 \times 100\sqrt{2}}{\pi}\sqrt{\frac{25}{36} + \frac{3}{4}}$$

$V_2$ = 108.20 V ⇒ Peak value of 2nd harmonic voltage

2nd harmonic impedance,

$$Z = R + j(2\omega)L$$

$$= 20 + j(2 \times 100\pi \times 0.2)$$

$$= 20 + j125.63$$

$$Z_2 = 127.244\angle 80.95°\ \Omega$$

2nd harmonic current,

$$I_2 = \frac{V_2}{Z_2} = \frac{108.20}{127.244} = 0.850 \text{ A}$$

$$I_{rms} = \sqrt{I_{avg}^2 + \frac{1}{2}(I_2)^2} = \sqrt{(2.25)^2 + \frac{1}{2}(0.85)^2}$$

$$= 2.328 \text{ A}$$

Power absorbed by the load,

$$P = I_{rms}^2 \cdot R = 2.328^2 \times 20$$

$$P = 108.475 \text{ W}$$

**Q.2 (b) (i) Solution:**

We have $$\frac{dx}{dt} + y = \sin t \quad \Rightarrow Dx + y = \sin t \quad ...(i)$$

$$\frac{dy}{dt} + x = \cos t \quad \Rightarrow Dy + x = \cos t \quad ...(ii)$$

Multiplying (ii) by $D$, we get

$$D^2y + Dx = D\cos t$$

$$D^2y + Dx = -\sin t \quad ...(iii)$$

Subtracting (i) from (iii), we have

$$D^2y - y = -2\sin t$$

$$(D^2 - 1)y = -2\sin t$$

A.E is $m^2 - 1 = 0$

$$m^2 = 1$$

$$m = \pm 1$$

$$\text{C.F.} = C_1e^t + C_2e^{-t}$$

$$\text{P.I.} = \frac{1}{D^2-1}(-2\sin t) = \frac{1}{-1-1}(-2\sin t) = \sin t$$

$$\text{Complete solution} = \text{C.F.} + \text{P.I.}$$

$$y = C_1e^t + C_2e^{-t} + \sin t \quad ...(iv)$$

Putting $y = 0$ and $t = 0$ in equation (iv), we get

$$0 = C_1 + C_2 \text{ or } C_2 = -C_1$$

On putting, $C_2 = -C_1$ in equation (iv),

$$y = C_1e^t - C_1e^{-t} + \sin t$$

On putting the value of $y$ in equation (ii), we get

$$D(C_1e^t - C_1e^{-t} + \sin t) + x = \cos t$$

$$x = -C_1e^t - C_1e^{-t} \quad ...(v)$$

On putting $x = 2$, $t = 0$ in equation (v), we get

$$2 = -C_1 - C_1$$

$$\Rightarrow \quad C_1 = -1$$

On putting the value of $C_1$ in equation (v) and (iv), we have

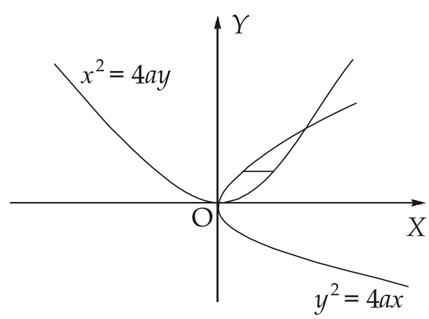
$$x = e^t + e^{-t}$$

$$y = -e^t + e^{-t} + \sin t$$

**Q.2 (b) (ii) Solution:**

Let,
$$I = \int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\, dy\, dx$$

The limits for $y$-varies from $y = \dfrac{x^2}{4a}$ to $y = 2\sqrt{ax}$ and the limits for $x$ varies from $x = 0$ to $x = 4a$. The region of integration is enclosed between the curves (parabolas) $x^2 = 4ay$ and $y^2 = 4ax$ and the lines $x = 0$ and $x = 4a$. The two parabolas intersect at (0, 0) and (4a, 4a). To change the order of integration, first integrate w.r.t $x$ and then w.r.t. $y$. Since first integration is w.r.t $x$, we consider a horizontal strip. The limits for $x$ varies from $x = \dfrac{y^2}{4a}$ to $x = 2\sqrt{ay}$ then $y$ varies from y = 0 to $y = 4a$.




Hence,
$$I = \int_0^{4a} \int_{\frac{y^2}{4a}}^{2\sqrt{ay}} xy\, dx\, dy$$

$$= \int_0^{4a} \left[\frac{x^2}{2} \cdot y\right]_{\frac{y^2}{4a}}^{2\sqrt{ay}} dy$$

$$= \int_0^{4a} \left[\frac{4ay}{2} \cdot y - \frac{y^4}{32a^2} \cdot y\right] dy = \left[2a \cdot \frac{y^3}{3} - \frac{1}{32a^2} \frac{y^6}{6}\right]_0^{4a}$$

$$= \left[2a \cdot \frac{64a^3}{3} - \frac{1}{32a^2} \frac{(4a)^6}{6}\right] = \frac{128a^4}{3} - \frac{64a^4}{3} = \frac{64a^4}{3}$$

**Q.2 (c) Solution:**

The characteristics equation is $\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$

$$(2-\lambda)[(2-\lambda)^2 - 1] + 1[-2+\lambda+1] + 1[1-2+\lambda] = 0$$
$$(2-\lambda)(4-4\lambda+\lambda^2-1) + (\lambda-1) + \lambda - 1 = 0$$
$$8 - 8\lambda + 2\lambda^2 - 2 - 4\lambda + 4\lambda^2 - \lambda^3 + \lambda + 2\lambda - 2 = 0$$
$$-\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$$
$$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0 \qquad \text{...(i)}$$

On putting $\lambda = 1$ in equation (i) is satisfied. So $\lambda - 1$ is one factor of the equation (i)

The other factor $(\lambda^2 - 5\lambda + 4)$ is got on dividing (i) by $\lambda - 1$

$$(\lambda-1)(\lambda^2 - 5\lambda + 4) = 0$$
$$(\lambda-1)(\lambda-1)(\lambda-4) = 0$$
$$\lambda = 1, 1, 4$$

The eigen values are 1, 1, 4

When $\lambda = 4$, $\begin{bmatrix} 2-4 & -1 & 1 \\ -1 & 2-4 & -1 \\ 1 & -1 & 2-4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Applying $R_2 \rightarrow R_2 + R_1$ and $R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} -2 & -1 & 1 \\ -3 & -3 & 0 \\ 3 & 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = x_3$$

Let $x_3 = k_1$

$$x_1 = -x_2$$

$\therefore$ Eigen vector, $X_1 = \begin{bmatrix} k_1 \\ -k_1 \\ k_1 \end{bmatrix}$

But, $k_1 = 1$

$$X_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

when $\lambda = 1$,
$$\begin{bmatrix} 2-1 & -1 & 1 \\ -1 & 2-1 & -1 \\ 1 & -1 & 2-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying $R_2 \to R_2 + R_1$ and $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Let, $x_1 = K_1$ and $x_2 = K_2$

$$K_1 - K_2 + x_3 = 0$$

$$x_3 = K_2 - K_1$$

$$X_2 = \begin{bmatrix} K_1 \\ K_2 \\ K_2 - K_1 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad (K_1 = 1, K_2 = 1)$$

Let
$$X_3 = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

As $X_3$ is orthogonal to $X_1$ since the given matrix is symmetric

$$\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

or $$l - m + n = 0 \qquad \text{...(ii)}$$

As $X_3$ is orthogonal to $X_2$ since the given matrix is symmetric

$$\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

or $$l + m + 0 = 0 \qquad \text{...(iii)}$$

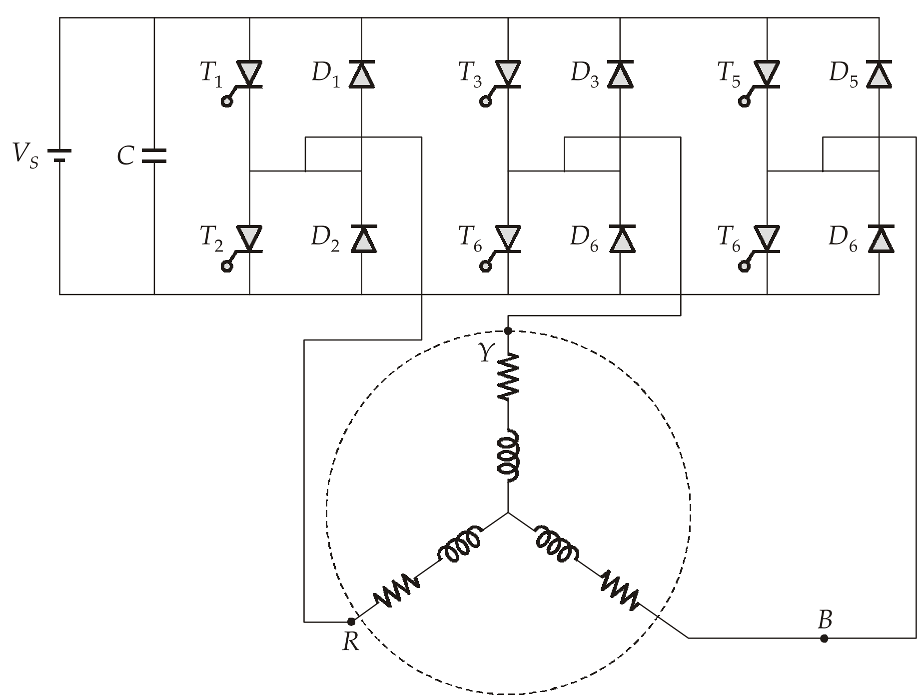
Solving (ii) and (iii), we get

$$\frac{l}{0-1} = \frac{m}{1-0} = \frac{n}{1+1}$$

$$\frac{l}{-1} = \frac{m}{1} = \frac{n}{2}$$

$$X_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

**Q.3 (a) Solution:**




Given, $V_S = 400$ V, $R = 20\ \Omega$,

$L = 50$ mH, $f_0 = 50$ Hz

**(i)** Instantaneous line to line voltage $V_{RY}$ can be written as

$$V_{RY} = \sum_{n=1,5...}^{\infty} \frac{4V_s}{n\pi} \sin\frac{n\pi}{3} \sin\left(n\omega t + \frac{\pi}{6}\right)$$

Even and triplane harmonics are absent

So, $$V_{RY} = \frac{4\times 400}{\pi}\sin\frac{\pi}{3}\sin(\omega t + 30°) + \frac{4\times 400}{5\pi}\sin\frac{5\pi}{3}\sin(5\omega t + 30°)$$

$$+\frac{4\times 400}{7\pi}\sin\frac{7\pi}{3}\sin(7\omega t+30°)+....$$

$$V_{RY} = 441.063\sin(\omega t+30°) - 88.212\sin(5\omega t+30°)$$
$$+ 63\sin(7\omega t+30°) - 40\sin(11\omega t+30°) + ....$$

Now, $$Z_L = \sqrt{R^2+(n\omega L)^2}\angle\tan^{-1}\left(\frac{n\omega L}{R}\right)$$

$$= \sqrt{(20)^2+(15.708n)^2}\angle\tan^{-1}\left(\frac{n(15.708)}{20}\right)$$

Instantaneous line as phase current,

$$I_L = \sum_{n=1,3,5}^{\infty}\left[\frac{4V_s}{\sqrt{3}n\pi Z_n}\sin\frac{n\pi}{3}\right]\sin(n\omega t-\theta_n)$$

where, $$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$$

$$I_L = 10.01\sin(\omega t - 38.146°) - 0.628\sin(5\omega t - 75.71)$$
$$+ 0.325\sin(7\,\omega t - 79.69°)\ ....$$

**(ii)** RMS phase voltage,

$$V_{Ph} = \frac{\sqrt{2}}{3}V_s - \frac{\sqrt{2}}{3}\times 400 = 188.561\text{ volt}$$

**(iii)** Rms phase voltage at fundamental frequency

$$V_{p1} = \frac{V_{L1}}{\sqrt{3}} = \frac{441.063/\sqrt{2}}{\sqrt{3}} = 180.063\text{ V}$$

**(iv)** Total harmonic distortion,

$$\left[\sum_{n=5,7,11}^{\infty}V_{Ln}^2\right]^{1/2} = \left[V_L^2 - V_{L1}^2\right]^{1/2}$$

$$= \sqrt{\left(188.561\times\sqrt{3}\right)^2 - \left(\frac{441.063}{\sqrt{2}}\right)^2} = 96.94\text{ Volts}$$

Now total harmonic distortion $= \dfrac{96.94}{\left(\dfrac{440.063}{\sqrt{2}}\right)} = 0.3115$ or $31.15\%$

**(v)** The lowest harmonic is fifth harmonic,

$$\text{Harmonic factor} = HF_5 = \frac{V_{L5}}{V_{L1}} = \frac{1}{5} = 20\%$$

**(vi)** Load power,

$$P_0 = 3I_L^2 \cdot R$$

$$I_L = \frac{\left[(10.01)^2 + (0.628)^2 + (0.325)^2 + ...\right]^{1/2}}{\sqrt{2}}$$

$$= 7.095 \text{ A}$$

$$P_0 = 3 \times 7.095^2 \times 20$$

$$= 3.021 \text{ kW}$$

**Q.3 (b) (i) Solution:**

Divide the interval (0,6) into six parts each of width $h = 1$. The values of $f(x) = \dfrac{1}{1+x^2}$ are given below:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1<br>$y_0$ | 0.5<br>$y_1$ | 0.2<br>$y_2$ | 0.1<br>$y_3$ | 0.0588<br>$y_4$ | 0.0385<br>$y_5$ | 0.027<br>$y_6$ |

**1.** By Trapezoidal rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2}\,[(y_0 + y_6) + 2\,(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$= \frac{1}{2}\,[(1 + 0.027) + 2(0.5 + 0.2 + 0.1 + 0.0588 + 0.0385)]$$

$$= 1.4108$$

**2.** By Simpson's 1/3 rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}[(y_0 + y_6) + 4\,(y_1 + y_3 + y_5) + 2\,(y_2 + y_4)]$$

$$= \frac{1}{3}[1+0.027) + 4\,(0.5 + 0.1 + 0.0385) + 2\,(0.2 + 0.0588)]$$

$$= 1.3662$$

**3.** By Simpson's 3/8 rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}[(y_0 + y_6) + 3\,(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3}{8}[(1+0.027)+3(0.5+0.2+0.0588+0.0385)+2(0.1)]$$
$$= 1.3570$$

**Q.3 (b) (ii) Solution:**

$$p = 0.001,$$
$$n = 2000,$$
$$m = np = 2000 \times 0.001 = 2$$

$$\therefore \quad P(r) = \frac{e^{-m}m^r}{r\,!} = e^{-2}\frac{2^r}{r\,!} = \frac{1}{e^2} \times \frac{2^r}{r\,!}$$

**1.** $P(\text{Exactly } 3) = P(3) = \frac{1}{e^2} \cdot \frac{2^3}{3\,!} = \frac{1}{(2.718)^2} \times \frac{8}{6} = (0.135) \times \frac{4}{3} = 0.18$

**2.** $P(\text{more than } 2) = P(3) + P(4) + P(5) + ....... + P(2000)$

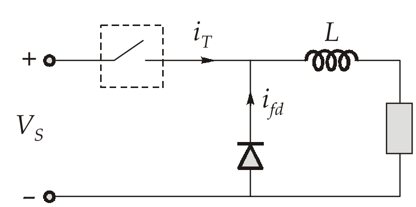
$$= 1 - [P(0) + P(1) + P(2)]$$
$$= 1 - \left[\frac{e^{-2}(2)^0}{0\,!} + \frac{e^{-2}(2)^1}{1\,!} + \frac{e^{-2}(2)^2}{2\,!}\right]$$
$$= 1 - e^{-2}[1+2+2] = 1 - \frac{5}{e^2}$$
$$= 1 - 5 \times 0.135 = 1 - 0.675 = 0.325$$

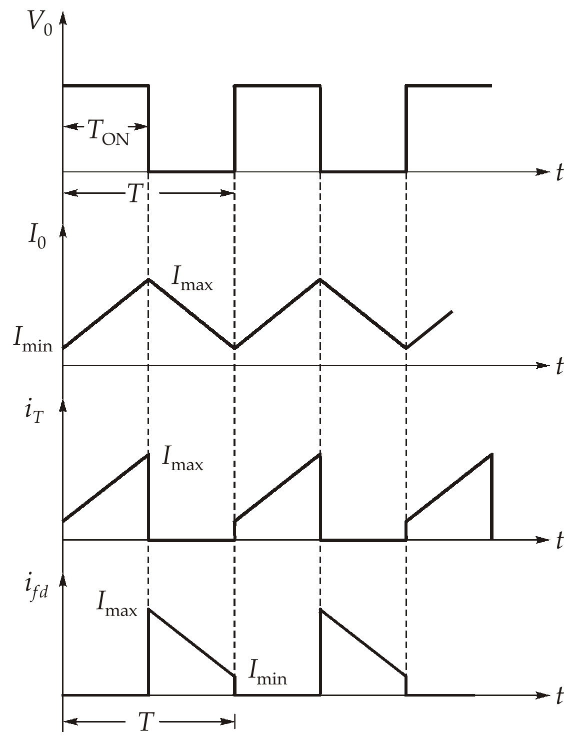
**3.** $P(\text{more than } 1) = P(2) + P(3) + P(4) + ...... + P(2000)$

$$= 1 - [P(0) + P(1)]$$
$$= 1 - \left[\frac{e^{-2}(2)^0}{0\,!} + \frac{e^{-2}(2)^1}{1\,!}\right] = 1 - 3e^{-2} = 1 - 3 \times 0.135$$
$$= 1 - 0.405 = 0.595$$

**Q.3 (c) Solution:**

**(i)** For an A-type chopper, waveforms are drawn for load current $I_0$, input (or thyristor) current $i_T$, freewheeling diode current $i_{fd}$






When chopper is ON, voltage equation for chopper circuit,

$$i_T \cdot R + L \cdot \frac{di_T}{dt} + E = V_S$$

$$Ri_T dt + L \cdot di_T + E \cdot dt = V_S \,.\, dt$$

$$Ri_T dt + L \cdot di_T + E \cdot dt = (V_S - E)dt$$

Taking average on both sides,

$$R \cdot \frac{1}{T}\int_0^{T_{ON}} i_T \cdot dt + \frac{L}{T}\int_0^{T_{ON}} d\,i_T = \frac{V_S - E}{T}\int_0^{T_{ON}} dt$$

$$R\,I_{TAV} + \frac{L}{T}\int_{I_{min}}^{I_{max}} d\,i_T = (V_S - E)\frac{T_{ON}}{T}$$

$$R\,I_{TAV} + \frac{L}{T}[I_{max} - I_{min}] = (V_S - E)\alpha$$

$$I_{TAV} = \frac{\alpha(V_S - E)}{R} - \frac{L}{RT}(I_{max} - I_{min})$$

**(ii)** When the freewheeling diode is conducting, load voltage is zero,

The voltage equation

$$R i_{fd} + L \cdot \frac{di_{fd}}{dt} + E = 0$$

$$R i_{fd} \cdot dt + L \cdot \frac{di_{fd}}{dt} \cdot dt + E \cdot dt = 0$$

Taking average on both sides,

$$\frac{R}{T}\int_{T_{on}}^{T} i_{fd} \cdot dt + \frac{L}{T}\int_{T_{on}}^{T} di_{fd} + \frac{E}{T}\int_{T_{on}}^{T} dt = 0$$

$$R \cdot I_{FD} + \frac{L}{T}\int_{I_{max}}^{I_{min}} di_{fd} + \frac{E}{T}[T - T_{on}] = 0$$

$$R \cdot I_{FD} + \frac{L}{T}(I_{min} - I_{max}) + E(1-\alpha) = 0$$

$$I_{FD} = \frac{L}{RT}(I_{max} - I_{min}) - \frac{E(1-\alpha)}{R}$$

Now average load current over a complete cycle can be obtained by adding $I_{TAV}$ and $I_{FD}$ from above equation,

$$I_{av} = I_{TAV} + I_{FD}$$

$$= \frac{\alpha(V_S - E)}{R} - \frac{L}{RT}(I_{max} - I_{min}) + \frac{L}{RT}(I_{max} - I_{min}) - \frac{E(1-\alpha)}{R}$$

$$I_{av} = \frac{\alpha V_S - \alpha E - E + \alpha E}{R}$$

$$= \frac{\alpha V_S - E}{R} = \frac{V_0 - E}{R}$$

**Q.4 (a) Solution:**

(i) Number of series connected SCR,

$$n_s = \frac{11\text{ kV}}{1.8\text{kV} \times 0.9} = 6.79 \approx 7$$

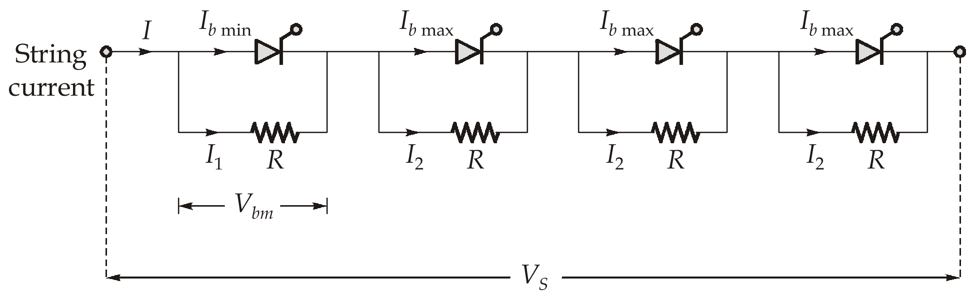
So, series connected SRC = 7

Number of parallel connected SCR

$$n_p = \frac{4\text{ kV}}{1\text{kV} \times 0.9} = 4.44 \approx 5$$

(ii) Static equalization circuit resistance is used in case of series connected SCR only. Let us consider $n$ thyristor connected in series. Let SCR 1 has minimum leakage current $I_{bmn}$ and each of the remaining $(n-1)$ SCR have the same leakage current $I_{b\max} > I_{b\min}$

Note: SCR with lower leakage current blocks more voltage




$$I_1 = I - I_{b\min}$$

and $$I_2 = I - I_{b\max}$$

where, $I$ = total string current

Voltage across SCR-1, $V_{b\max} = I_1 \cdot R$

Voltage across $(n-1)$ $SCR = (n-1)I_2R$

For a string voltage,

$$V_S = I_1R + (n-1)I_2 \cdot R$$
$$= V_{bm} + (n-1)\,R(I - I_{b\max})$$
$$= V_{bm} + (n-1)\,R[I_1 - (I_{b\max} - I_{b\min})]$$
$$= V_{bm} + (n-1)\,RI_1 - (n-1)R\,\Delta I_b$$

where, $$\Delta I_b = I_{b\max} - I_{b\min}$$

as $$RI_1 = V_{bm}$$

$$V_S = n\,V_{bm} - (n-1)R \cdot \Delta I_b$$

So, $$R = \frac{nV_{bm} - V_s}{(n-1)\Delta I_b}$$

The SCR data sheet contains only maximum blocking current $I_{b\max}$ and rarely $\Delta I_b$. In such a case, it is usual to assume $\Delta I_b = I_{b\max}$ with $I_{b\min} = 0$.

When, $$I_{b\max} = 12\text{ mA},$$
$$V_{bm} = 1800\text{ V},$$
$$n_s = 7(\text{SCR}) \text{ and } V_s = 11\text{ kV}$$

$$\Delta I_b = I_{b\,max} = 12 \text{ mA}$$

$$R = \frac{nV_{bm} - V_s}{(n-1)\Delta I_b} = \frac{7 \times 1800 - 11000}{(7-1) \times 12 \times 10^{-3}}$$

$$= \frac{12600 - 11000}{6 \times 12 \times 10^{-3}} = 22.22 \text{ k}\Omega$$

**Q.4 (b) Solution:**

**(i)** The power returned to supply is,

$$P = (1 - \alpha) V_s I_a$$
$$= (1 - 0.5)\, 220 \times 60$$
$$= 6.6 \text{ kW}$$

**(ii)**
$$E = K_a \phi N$$

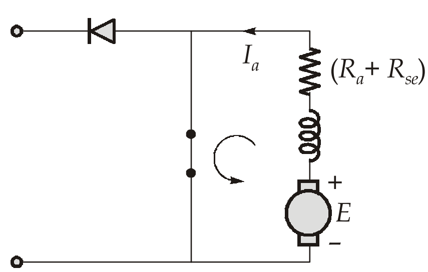
for series motor, $\phi \propto I_a$

$\Rightarrow$
$$\phi = K_{se} I_a$$
$$E = K_a K_{se} I_a\, \omega$$

$$K_a K_{se} = \frac{E}{I_a \omega_m}\left(\frac{V}{A - \text{rad/sec}}\right)$$

Given,
$$K_a K_{se} = 0.05\left(\frac{V - \text{sec}}{A - \text{rad}}\right)$$

In the given circuit when switch is ON,



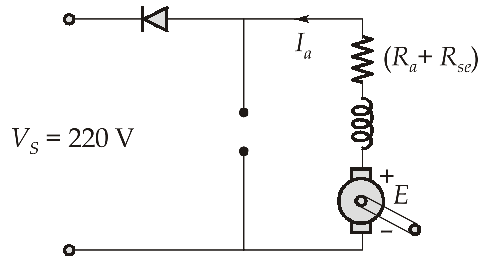

$\therefore$
$$E = I_a(R_a + R_{se}) = 60 \times (0.15) = 9 \text{ V}$$

$$\omega_{min} = \frac{E_{min}}{I_a \times 0.05}$$

$$= \frac{9}{60 \times 0.05} = 3 \text{ rad/sec}$$

Minimum breaking speed = 3 rad/sec

In the given circuit when switch is off,




By applying KVL:

$$-V_S - I_a(R_a + R_{se}) + E = 0$$
$$E = V_S + I_a(R_a + R_{se})$$
$$E_{max} = 220 + 60(0.15) = 229 \text{ V}$$
$$\text{Maximum breaking speed} = \omega_{max} = \frac{229}{0.05 \times 60} = 76.33 \text{ rad/sec}$$

**Q.4 (c) Solution:**

If $\phi(x, y)$, $\psi(x, y)$, $\frac{\partial \phi}{\partial y}$ and $\frac{\partial \psi}{\partial x}$ be continous functions over a region $R$ bounded by simple closed curve $C$ in $x$ - $y$ plane, then

$$\oint_C (\phi\, dx + \psi\, dy) = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\, dy$$

Then by Green's theorem in plane, we have

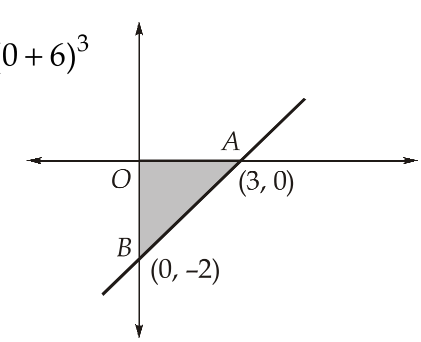
$$\oint\left[(3x^2 - 8y^2)dx + (4y - 6xy)dy\right] = \iint_R (-6y + 16y)dx\, dy = \iint_R 10y\, dx\, dy$$

$$= 10\int_0^3 dx \int_{\frac{1}{3}(2x-6)}^{0} y\, dy = 10\int_0^3 dx \left[\frac{y^2}{2}\right]_{\frac{1}{3}(2x-6)}^{0}$$

$$= \frac{-5}{9}\int_0^3 dx(2x-6)^2$$

$$= \frac{-5}{9}\left[\frac{(2x-6)^3}{3 \times 2}\right]_0^3 = \frac{-5}{54}(0+6)^3$$

$$= \frac{-5}{54} \times 216 = -20$$




Now we evaluate L.H.S of (1),

$$= \int\left[(3x^2 - 8y^2)dx + (4y - 6xy)dy\right]$$

$$= \int_{OB}\left[(3x^2-8y^2)dx+(4y-6xy)dy\right]+\int_{BA}\left[(3x^2-8y^2)dx+(4x-6xy)dy\right]$$

$$+\int_{AO}\left[(3x^2-8y^2)dx+(4x-6xy)dy\right]$$

$$= \int_0^{-2}4y\,dy+\int_{-2}^{0}\left[\frac{3}{4}(6+3y)^2-8y^2\right]\left(\frac{3}{2}dy\right)+\left[4y-3(6+3y)y\right]dy+\int_3^0 3x^2\,dx$$

$$= \left[2y^2\right]_0^{-2}+\int_{-2}^{0}\left[\frac{9}{8}(6+3y)^2-12y^2+4y-18y-9y^2\right]dy+\left(x^3\right)_3^0$$

$$= 2[4]+\int_{-2}^{0}\left[\frac{9}{8}(6+3y)^2-21y^2-14y\right]dy+(0-27)$$

$$= 8+\left[\frac{9}{8}\frac{(6+3y)^3}{3\times 3}-7y^3-7y^2\right]_{-2}^{0}-27$$

$$= -19+\left[\frac{216}{8}+7(-2)^3+7(-2)^2\right]$$

$$= -19+27-56+28$$

$$= -20$$

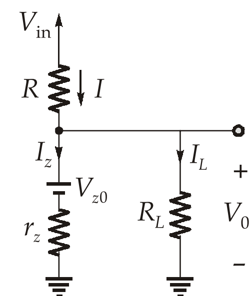
**Section B : Basic Electronics Engineering-1 + Analog Electronics-1 + Electrical Materials-1 + Electrical Machines-2**

**Q.5 (a) Solution:**

| **Plane-A** | $x$ | $y$ | $z$ |
|---|---|---|---|
| Intercepts | $-\frac{L}{2}$ | $\frac{L}{2}$ | $\infty$ |
| Intercepts (coefficients in terms of lattice parameters) | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\infty$ |
| Reciprocals | –2 | 2 | 0 |
| For plane-A, | miller index | = | $(\bar{2}\ \ 2\ \ 0)$ |

| Plane-B | $x$ | $y$ | $z$ |
|---|---|---|---|
| Intercepts | $L$ | $\frac{L}{2}$ | $\frac{L}{2}$ |
| Intercepts (coefficients in terms of lattice parameters) | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| Reciprocals | 1 | 2 | 2 |
| For plane-B, | index | = | (1 2 2) |

**Q.5 (b) Solution:**



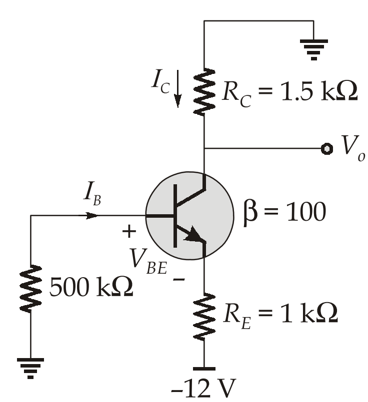

Line regulation,
$$\frac{\Delta V_0}{\Delta V_{in}} = \frac{r_z}{R + r_z}$$
$$= \frac{20}{500 + 20} = 38.5 \text{ mV/V}$$
$$\Delta v_0 = \Delta V_{in} \times 38.5 \text{ mV/V}$$
$$= \pm 38.5 \text{ mV}$$

For a load resistance $R_L$ that draws 1 mA current, the zener current will decrease by 1 mA. The corresponding change in zener voltage would be

$$\Delta V_0 = r_z \Delta I_z$$
$$= 20 \times -1 = -20 \text{ mV}$$

Load regulation, $$\frac{\Delta V_0}{\Delta I_L} = \frac{-20 \text{ mV}}{1 \text{ mA}} = -20 \text{ mV/mA}$$

**Q.5 (c) Solution:**




Applying KVL in the base-emitter loop,

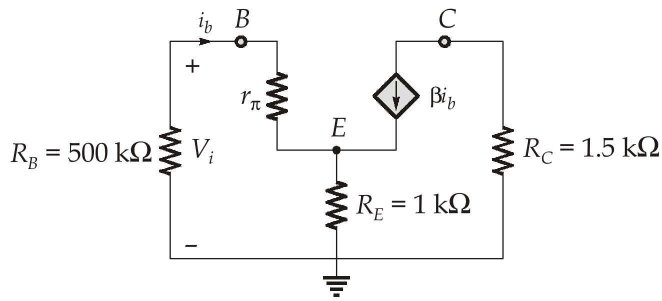
$$500I_B + V_{BE} + (1 + \beta)I_B(1) = 12 \text{ V}$$
$$I_B = \frac{12 - V_{BE}}{500 + (1 + \beta)}$$
$$I_B = \frac{12 - 0.7}{500 + 101} = 18.8 \text{ µA}$$

Hence, $I_C = \beta I_B = 100 \times 18.8 \text{ µA} = 1.88 \text{ mA}$

Calculating the small signal hybrid $\pi$-parameters of BJT,

$$g_m = \frac{I_C}{V_T} = \frac{1.88 \text{ mA}}{26 \text{ mV}} = 72.31 \text{ mA/V}$$
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{72.31 \times 10^{-3}} = 1.383 \text{ k}\Omega$$

Using the hybrid π-model of BJT, the small signal equivalent of the circuit can be drawn as below:



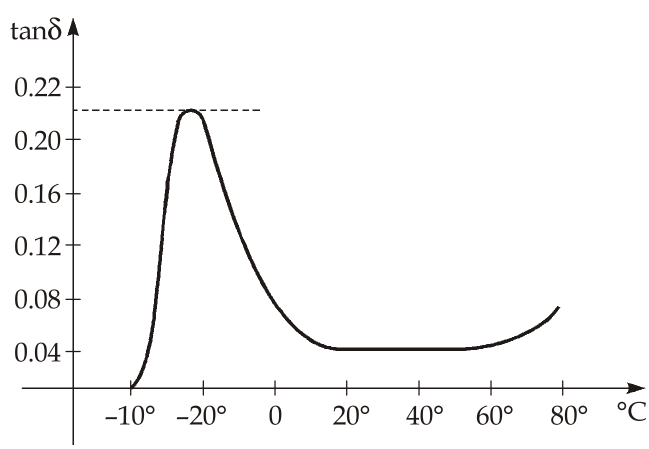

Applying KVL in the input loop,

$$V_i = i_b \times r_\pi + (1+\beta) i_b R_E$$

$$\Rightarrow \qquad R_{ib} = \frac{V_i}{i_b} = r_\pi + (1+\beta) R_E = 1.383 + 101 \times 1 = 102.383 \text{ k}\Omega$$

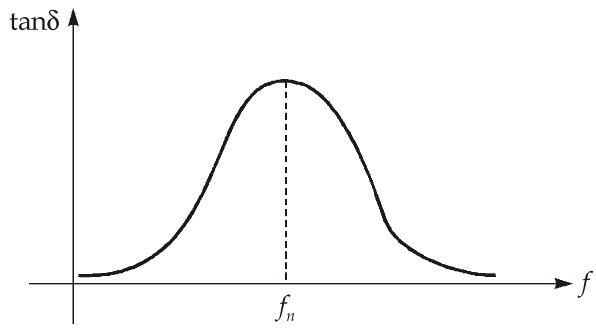
**Q.5 (d) Solution:**

**(i)** The dielectric loss of polar dielectric consists of two components. Those due to leakage current and those resulting from dipole polarization. The dependence of tanδ on temperature is shown below:




At low temperature, the loss due to dipole polarization is greater than that due to the leakage current. At temperature much below 0°C due to less thermal motion, orientation of dipoles is limited. With increase in temperature the dipoles acquire greater mobility, hence increase in temperature causes the loss tangent to drop off due to enhanced thermal agitation. After falling to a minimum, tanδ begins to increase, this time due to an increase in the leakage current.

The loss tangents of polar dielectrics depends on frequency. The variation of loss tangent tanδ with frequency at a constant temperature is shown below.




At low frequency the dipoles make less number of rotations per second resulting in a small amount of power loss. As the frequency is increased beyond a certain limit the dipolar polarization ceases because the molecules will not be able to keep up with increased rate of field reversal. At some mid frequency (depending on the temperature) the loss tangent will be maximum as shown in above figure.

At zero frequency the loss is due to leakage current only and hence tanδ is minimum. At infinite frequency the losses due to both polarization and leakage current become zero.

**(ii)** $\varepsilon_r = 4.94$, $n^2 = 2.69$

From Clausius Mosotti relation,

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\varepsilon_0} \quad ...(1)$$

If measurement are done in optical frequency range,

$$\varepsilon_r = n^2 \text{ and } \alpha_i = 0$$

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\varepsilon_0} \quad ...(2)$$

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} \times \frac{n^2 + 2}{n^2 - 1} = \frac{\alpha_e + \alpha_i}{\alpha_e}$$

$$1 + \frac{\alpha_i}{\alpha_e} = \frac{4.94 - 1}{4.94 + 2} \times \frac{2.69 + 2}{2.69 - 1}$$

$$1 + \frac{\alpha_i}{\alpha_e} = 1.576$$

$$\frac{\alpha_i}{\alpha_e} = 0.576$$

$$\frac{\alpha_e}{\alpha_i} = 1.735$$

**Q.5 (e) Solution:**

Number of poles, $P = \dfrac{120f}{n_s} = \dfrac{120 \times 50}{300} = 20$

Phase current, $I_L = I_P = \dfrac{400 \times 10^3}{\sqrt{3} \times 3300} = 70 \text{ A}$

Maximum value of phase current,

$$I_m = 70\sqrt{2} \text{ A}$$

Slot angle, $\gamma = \dfrac{180° \times 20}{180} = 20°$

Slots per pole per phase, $m = \dfrac{180}{3 \times 20} = 3$

Breadth factor, $k_b = \dfrac{\sin\dfrac{m\gamma}{2}}{m \sin\dfrac{\gamma}{2}} = 0.96$

$$\text{Turns/phase (series)} = \frac{180 \times 8}{2 \times 3} = 240 = N_{ph \text{ (series)}}$$

$$\Rightarrow \quad F_m = \frac{4}{\pi} \times k_b \times \frac{N_{Ph}(\text{series})}{P} \times I_m$$

$$= \frac{4}{\pi} \times 0.96 \times \frac{240}{20} \times 70\sqrt{2}$$

$$= 1452 \text{ AT/pole/phase}$$

$$F_{\text{peak}} = \frac{3}{2} F_m = 2178 \text{ AT/pole}$$

**Q.6 (a) Solution:**

Before an extra resistance is introduced,

$$E = V - I_a R_a$$

$$= 440 - 50 \times 0.2 = 430 \text{ V}$$

$$E = K \phi N = K_1 I_a N \text{ (for a series motor } \phi \propto I_a)$$

$$K_1 = \frac{E}{I_a N} = \frac{430}{50 \times 600} = 0.0143$$

$$T = K_t \phi I_a = K_{1_1} I_a^2$$

When torque is half, say, $T_1$ ,

$$\frac{T}{T_1} = \frac{K_{t_1} I_a^2}{K_{t_1} I_{a_1}^2}$$

or
$$\frac{T}{T/2} = \frac{50 \times 50}{(I_{a_1})^2}$$

$$I_{a_1} = \sqrt{\frac{50 \times 50}{2}} = 35.35 \text{ A}$$

At this armature current $I_a$ and with a resistance $R$ introduced in the circuit, the induced emf $E_1$ is given by

$$E_1 = V - I_{a_1}(R_a + R)$$

But
$$E_1 = K_1 I_{a_1} N_1 = 0.0143 \times 35.35 \times 400 = 202 \text{ V}$$

∴
$$202 = 440 - 35.35\,(0.2 + R)$$

or
$$R = 6.53\ \Omega$$

**Q.6 (b) Solution:**

No load,
$$V_t = 6\text{ V},\quad I_{a0} = 14.5\text{ mA},$$
$$n = 12125 \text{ rpm or } \omega = 1269.7 \text{ rad/s}$$

**(i)**
$$E_a = 6 - 14.5 \times 10^3 \times 4.2 = 5.939 \text{ V}$$
$$5.939 = K_m\omega = K_m \times 1269.7$$

or
$$K_m = 4.677 \times 10^{-3} \text{ V-sec/rad}$$

**(ii)** Rotational loss,
$$P_m = E_a I_a \text{ ; there is no load}$$
$$= 5.939 \times 14.5 \times 10^{-3} = 0.0861 \text{ W}$$

**(iii)** Stalled current,
$$\omega = 0 \text{ so } E_a = 0$$

$$I_a\text{ (stall)} = \frac{6}{4.2} = 1.4285 \text{ A}$$

$$\text{Torque (stall)} = K_m I_a\text{ (stall)} = 4.677 \times 10^{-3} \times 1.428$$
$$= 6.67 \text{ m Nm}$$

**(iv)**
$$P_{out}\text{ (gross)} = 1.6 \text{ W} = E_a I_a$$
$$(6 - 4.2I_a)I_a = 1.6$$
$$4.2I_a^2 - 6I_a + 1.6 = 0$$

Solving we find,

$$I_a = 0.354\text{ A}, 1.074\text{ A}$$

Thus, $$I_a = 0.354\text{ A; higher value rejected}$$

$$E_a = 6 - 0.354 \times 4.2 = 4.513\text{ V} = K_m\omega$$

$$\omega = \frac{4.513 \times 10^3}{4.677} = 965\text{ rad/s}$$

Rotational loss (proportional to square of speed)

$$P_{rot} = 0.0861 \times \left(\frac{965}{1269.7}\right)^2 = 0.05\text{ W}$$

$$P_{out}\text{ (net)} = P_{out}\text{ (gross)} - P_{rot}$$

$$= 1.6 - 0.05 = 1.55\text{ W}$$

Power input, $$P_i = V_t I_a = 6 \times 0.354 = 2.124\text{ W}$$

$$\eta = \frac{1.55}{2.124} \times 100 = 73\%$$

**(v)** Motor speed, $$n = 10250\text{ rpm or } \omega = 1073.4\text{ rad/sec}$$

$$E_a = K_m\,\omega = 4.677 \times 10^{-3} \times 1073.4 = 5.02\text{ V}$$

$$I_a = \frac{6 - 5.02}{4.2} = 0.233\text{ A}$$

$$P_{out}\text{ (gross)} = P_e = E_a I_a$$

$$= 5.02 \times 0.233 = 1.171\text{ W}$$

$$P_{rot} = 0.0861 \times \left(\frac{1073.4}{1269.7}\right)^2 = 0.0615\text{ W}$$

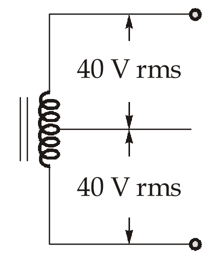
$$P_{out}\text{ (net)} = 1.171 - 0.0615 = 1.1095\text{ W}$$

$$P_{in} = 6 \times 0.233 = 1.398\text{ W}$$

$$\eta = \frac{1.1095}{1.398} \times 100 = 79.36\%$$

**Q.6 (c) Solution:**

$$\frac{V_m}{\sqrt{2}} = 40$$




$$\Rightarrow \quad V_m = 56.56 \text{ V}$$

For FWR with capacitor filter

$$V_{DC} = V_m - \frac{I_{DC}}{4 f_o C}$$

$$\Rightarrow \quad I_{DC} R_L = V_m - \frac{I_{DC}}{4 f_o C}$$

$$\Rightarrow \quad I_{DC}\left(R_L + \frac{1}{4 f_o C}\right) = V_m$$

$$\Rightarrow \quad I_{DC}\left(2000 + \frac{1}{4 \times 50 \times 40 \times 10^{-6}}\right) = 56.56$$

$$\Rightarrow \quad I_{DC} = 26.6 \text{ mA}$$

**(i)** Peak to peak ripple voltage

$$V_r = \frac{I_{DC}}{2 f_o C} = \frac{26.6 \times 10^{-3}}{2 \times 50 \times 40 \times 10^{-6}} = 6.65 \text{ V}$$

RMS value of ripple voltage

$$\Rightarrow \quad V'_{\text{rms}} = \frac{V_r}{2\sqrt{3}} = 1.91 \text{ V}$$

**(ii)** Ripple factor, $r = \dfrac{V'_{\text{rms}}}{V_{DC}}$ or $\dfrac{1}{4\sqrt{3} f_o C R_L}$

$$\Rightarrow \quad r = \frac{1.91}{53.2} = 0.035$$

**(iii)** $\% \text{ regulation} = \dfrac{R}{R_L} \times 100\%$

We have, $V_{DC} = V_m - \dfrac{I_{DC}}{4 f_0 C}$ ...(i)

also $V_{DCFL} = V_{DCNL} - I_{DC} R$ ...(ii)

Comparing equations (i) and (ii),

$$R = \frac{1}{4 f_0 C} = \frac{1}{4 \times 50 \times 40 \times 10^{-6}} = 125\ \Omega$$

$\therefore \quad \% \text{ Regulation} = \frac{R}{R_L} \times 100\% = \frac{125}{2000} \times 100\% = 6.25\%$

**Q.7 (a) Solution:**

$$\vec{I}_a = \frac{800 \times 10^3}{\sqrt{3} \times 3300 \times 0.8} \angle \cos^{-1} 0.8 = 174.95 \angle 36.87° \text{ A}$$

$$\vec{E} = \vec{V} - \vec{I}_a \vec{z}_s$$

$$\vec{E} = \frac{3300}{\sqrt{3}} \angle 0° - 174.95 \angle 36.87° \times (0.8 + j5)$$

$$\vec{E} = 2447.67 \angle -18.68° \; V_{(L-N)}$$

$$E_{L\text{-}L} = 2447.67\sqrt{3} = 4239.488 V_{(L-L)}$$

$$P'_{in} = 12000 \text{ kW}$$

$$P_{in} = \frac{V^2}{z_s} \cos\theta_s - \frac{VE}{z_s} \cos(\theta_s + \delta)$$

$$1200 \times 10^3 = \frac{(3300)^2}{5.063} \cos(80.91) - \frac{3300 \times 4239.488}{5.063} \cos(80.91 + \delta')$$

$$\delta = 27.23°$$

$$\theta_{\text{in}} = \frac{V^2}{z_s} \sin\theta_s - \frac{VE}{z_s} \sin(\theta_s + \delta)$$

$$\theta'_{in} = -502.06 \text{ kVAR}$$

$$\phi' = \tan^{-1}\left(\frac{502.06}{1200}\right)$$

$$\phi' = 22.70°$$

$$pf = \cos 22.70° = 0.9225 \text{ leading}$$

**Q.7 (b) Solution:**

**(i)** Consider an elemental solid dielectric with number density $N$ atoms per $m^3$ and polarizability 'α' Farad $m^2$

$$\vec{P} = N\alpha \vec{E}_{\text{tot}}, \qquad E_{\text{tot}} : \text{Total field} \qquad \text{...(i)}$$

Also,

$$\vec{P} = \vec{D} - \varepsilon_0 \vec{E} = \varepsilon_0 (\varepsilon_r - 1) \vec{E} \qquad \text{...(ii)}$$

$$\vec{E}_{\text{tot}} = \vec{E} + \vec{E}_{\text{int}}$$

From Lorentz's expression,

$$\vec{E}_{\text{tot}} = \vec{E} + \frac{\gamma \vec{P}}{\varepsilon_0}, \qquad \gamma : \text{Internal constant}$$

For atoms surrounded cubically

$$\gamma = \frac{1}{3}$$

$$\Rightarrow \quad \vec{E}_{tot} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0} \quad ...(iii)$$

From (i), (ii) and (iii),

$$\Rightarrow \quad \vec{P} = N\alpha\left(\vec{E} + \frac{1}{3}(\varepsilon_r - 1)\vec{E}\right) = \varepsilon_0(\varepsilon_r - 1)\vec{E}$$

$$\Rightarrow \quad \frac{N\alpha}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \quad \text{(Clausius - Mossotti relation)}$$

**(ii)** Pure copper resistivity, $\rho_{cu} = 1.56 \times 10^{-8}\ \Omega$ W-m

With nickel, $\rho_{cu + ni} = \chi_{ni}\rho_{ni} + (1 - \chi_{ni})\rho_{cu}$, ($\chi_{ni}$ : % nickel)

$$\frac{\Delta\rho_{cu+ni}}{\Delta\chi_{ni}} = \rho_{ni} - \rho_{cu} = 1.25 \times 10^{-8}\ \Omega\text{-m}/\%$$

$$\Rightarrow \quad \rho_{ni} = 2.81 \times 10^{-8}\ \Omega\text{-m}$$

With silver, $\rho_{cu + Ag} = \chi_{Ag}\rho_{Ag} + (1 - \chi_{Ag})\rho_{cu}$, ($\chi_{Ag}$ : % Silver)

$$\frac{\Delta\rho_{cu+Ag}}{\Delta\chi_{Ag}} = \rho_{Ag} - \rho_{cu} = 0.14 \times 10^{-8}\ \Omega\text{-m}\%$$

$$\rho_{Ag} = 1.70 \times 10^{-8}\ \Omega\text{-m}$$

At 300 K,

$$\chi_{ni} = 0.2, \quad \chi_{Ag} = 0.4$$

$$\chi_{cu} = 1 - \chi_{ni} - \chi_{Ag} = 0.4$$

$$\rho_{cu + ni + Ag} = \chi_{ni}\,\rho_{ni} + \chi_{Ag}\,\rho_{Ag} + \chi_{cu}\,\rho_{cu}$$

$$= 0.2 \times 2.81 \times 10^{-8} + 0.4 \times 1.70 \times 10^{-8} + 0.4 \times 1.56 \times 10^{-8}$$

$$= 1.866 \times 10^{-8}\ \Omega\text{-m at 300 K}$$

At 4 K < Debye temperature ($\theta_D$)

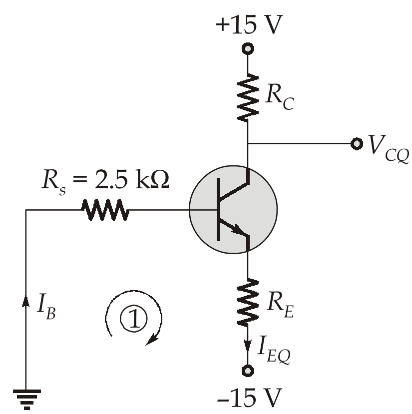
$$\Rightarrow \quad \rho_{Thermal} = 0$$

$$\Rightarrow \quad \rho_{cu + ni + Ag} \approx 0 \text{ (No residual resistivity due to impurity)}$$

**Q.7 (c) Solution:**

Given : $\beta = 99, I_{EQ} = 0.5 \text{ mA}, V_{CQ} = 5 \text{ V}, V_{BE} = 0.7 \text{ V}, V_T = 25 \text{ mV}$

**(i)** DC equivalent circuit




$$I_{CQ} = \frac{\beta}{\beta+1} \times I_{EQ} = \frac{99}{100} \times 0.5 = 0.495 \text{ mA}$$

Also, $$I_{CQ}R_C = V_{CC} - V_{CQ}$$

$$R_C = \frac{15-5}{0.495} = 20.20 \text{ k}\Omega$$

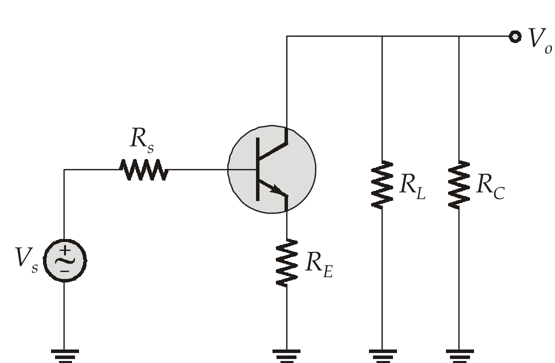
Apply KVL in loop (1)

$$I_B \times 2.5 + V_{BE} + I_{EQ}R_E = 15$$

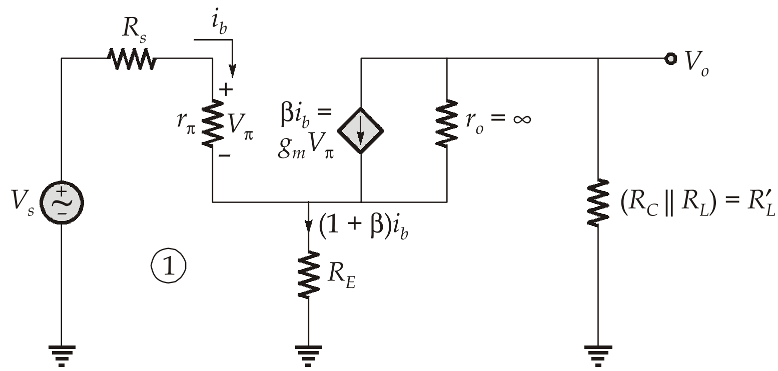
$$R_E = \frac{15 - 0.7 - 2.5 \times \frac{0.495}{99}}{0.5} = 28.57 \text{ k}\Omega$$

**(ii) For AC Analysis :** DC → Grounded Capacitor → Short-circuited

Equivalent AC Circuit :




Replacing the transistor by its equivalent π-model




where $$g_m = \frac{I_{CQ}}{V_T} = \frac{0.495}{25} = 19.8 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{99 \times 10^3}{19.8} = 5 \text{ k}\Omega$$

$$R'_L = 20.20 \,\|\, 10 = 6.68 \text{ k}\Omega$$

From the circuit, $$V_o = -\beta i_b R'_L \quad \text{...(i)}$$

Also, $$V_s = (R_s + r_\pi)i_b + R_E(1 + \beta)i_b$$

$$i_b = \frac{V_s}{R_s + r_\pi + (1+\beta)R_E} \quad \text{...(ii)}$$

On putting eqn. (ii) in eqn. (i)

$$V_o = \frac{-\beta R'_L V_s}{R_s + r_\pi + (1+\beta)R_E}$$

Therefore, Voltage gain, $$\frac{V_o}{V_s} = \frac{-\beta R'_L}{R_s + r_\pi + (1+\beta)R_E}$$

$$A_V = \frac{-99 \times 6.68}{2.5 + 5 + 100 \times 28.57}$$

$$A_V = -0.2308$$

**Q.8 (a) (i) Solution:**

The given values are

$$C = 0.035 \text{ μF} = 0.035 \times 10^{-6} \text{ F}$$

$$\tan \delta = 0.0005 = 5 \times 10^{-4}$$

$$f = 25 \times 10^3 \text{ Hz}$$

and $$I = 250 \text{ A}$$

The voltage across capacitor is given by

$$V = \frac{I}{2\pi fC} = \frac{250}{2\pi \times 25 \times 10^3 \times 0.035 \times 10^{-6}} = 45495 \text{ V}$$

Dielectric loss is given by

$$P = V.I \tan\delta = 45495 \times 250 \times 5 \times 10^{-4} = 5686 \text{ W}$$

**Q.8 (a) (ii) Solution:**

We have,

$$i_1 = \frac{V_S - V}{R_1} = i_2 = \frac{V - V_x}{R_2}$$

or
$$V_X = -\frac{R_2}{R_1}V_S \qquad \text{(since } V = 0\text{)}$$

We also get,
$$i_3 + i_2 = i_4$$

$$\frac{-V_x}{R_3} - \frac{V_x}{R_2} = i_4$$

$$\left(\frac{1}{R_3} + \frac{1}{R_2}\right)\frac{R_2 V_s}{R_1} = i_4$$

Then,
$$V_0 = -i_4 R_4 + V_X$$

$$= \frac{-R_2 V_S}{R_1} - \frac{R_4 R_2 V_S}{R_3 R_1} - \frac{R_4 V_S}{R_1}$$

$$= \frac{-R_2}{R_1}\left(1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}\right)V_S$$

Therefore,
$$\frac{V_0}{V_S} = \frac{-R_2}{R_1}\left(1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}\right)$$

This circuit gives a high-input resistance and high gain without increasing the size of the feedback resistors.

**Q.8 (b) Solution:**

**(i)** 1. When the properties of a material vary with different crystallographics orientations, the material is said be **anisotropic** e.g. composites.

When the properties of a material are the same in all directions, the material is said to be **isotropic**. eg. glass.

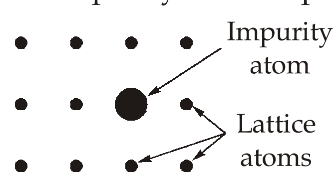
2. **Crystalline** solids are firm, held a definite and fixed shape, are rigid and incompressible. They generally have geometric shapes and flat faces. Eg. diamond, metals, salt crystal etc.

   **Amorphous** solids are rigid structure but they take a well defined shape. They do not have a regular geometric shape, so they are noncrystalline e.g. glass. They lack a systematic and regular arrangement of atoms over large atomic distances relatively.

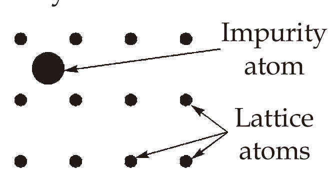
3. **Allotropy** refers to the natural existence of an element in different forms i.e. bonded differently. eg. Diamond-Graphite. Catenation refers to the ability of an element to form long chains eg. Carbon polymers.

**(ii)** Two types of impurity point defects are found in solid solution:

- **Substitutional type :** Solute or impurity atoms replace or substitute the host atoms.




- **Interstitial type :** Solute or impurity atoms occupies a site in crystal structure at which there is no atom usually.




Several factors that determine the degree of impurity are:

- **Atomic size factor :** Appreciable quantities of a solute may be accommodated in the solid solution only when the difference in atomic radii between two atoms is less than ±15%.
- **Crystal structure :** For appreciable solubility the crystal structure for metals of both atoms types must be the same.
- **Electronegativity :** The more electropositive one element and the more electronegative the other, the greater is the likehood that they form a intermetallic compound instead of substitutional solid solution.
- **Valencies :** A metal will have more of a tendency to dissolve another metal of higher valency than one of a lower valency.

**(iii)** Germanium atomic concentration,

$$N_{Ge} = 2.43 \times 10^{21} \text{ atoms/cm}^3$$

Ge density, $\rho_{Ge} = 5.32 \text{ g/cm}^3$

Si density, $\rho_{Si} = 2.33 \text{ g/cm}^3$

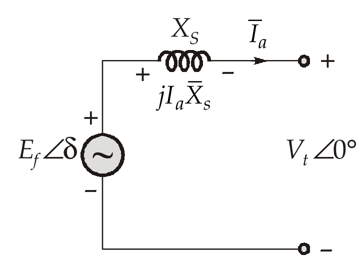
Atomic weight of $Ge$, $A_{Ge} = 72.61$ g/mol

$$\text{Germanium weight\% needed} = \frac{100\%}{1 + \frac{N_A}{N_{Ge}} \cdot \frac{\rho_{si}}{A_{Ge}} - \frac{\rho_{si}}{\rho_{Ge}}}$$

$$= \frac{100\%}{1 + \frac{6.022 \times 10^{23}}{2.43 \times 10^{21}} \times \frac{2.33}{72.61} - \frac{2.33}{5.32}} = 11.745\%$$

**Q.8 (c) Solution:**

The circuit equivalent of the machine is drawn in figure




**(i)** $$I_a = \frac{10 \times 10^3}{\sqrt{3} \times 400} = 14.43 \text{ A}$$

p.f. angle, $\phi = \cos^{-1} 0.8 = 36.9°$ lag

$$\bar{I}_a = 14.43\angle{-36.9°}$$

$$V_t = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

From the circuit equivalent,

$$\bar{E}_f = 231\angle 0° + j14.43\angle{-36.9°} \times 16$$

$$= 231 + 231\angle 53.1° = 369.7 + j184.7$$

or $\bar{E}_f = 413.3\angle 26.5°$

Torque angle, $\delta = 26.5°$, $E_f$ leads $V_t$ (generating action)

**(ii)** Power supplied (source), $P_e = 10 \times 0.8 = 8$ kW (3 phase)

$E_f$ (20% more) $= 413.3 \times 1.2 = 496$ V

$$P_e = \frac{E_f V_t}{X_s} \sin\delta$$

$$\frac{8 \times 10^3}{3} = \frac{496 \times 231}{16} \sin\delta$$

Torque angle, $\delta = 21.9°$

From the circuit equivalent,

$$\bar{I}_a = \frac{E_f\angle\delta - V_t\angle 0°}{jX_s} = \frac{496\angle 21.9° - 231}{j16}$$

$$= \frac{229 + j185}{j16} = 11.6 - j14.3 = 18.4\angle{-50.9°}$$

$$I_a = 18.4 \text{ A, pf} = \cos 50.9° = 0.63 \text{ lagging}$$

**(iii)** $E_f$ = 413 V; field current same as in part (i),

$$P_e(\max) = \frac{E_f V_t}{X_s}; \qquad (\delta = 90°)$$

$$= \frac{413 \times 231}{16} \times 10^{-3}$$

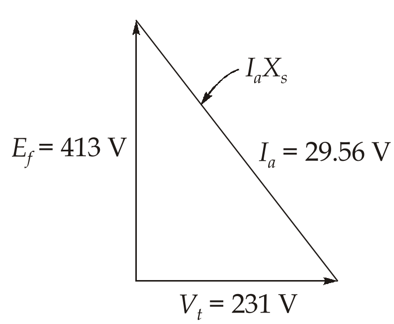
$$= 5.96 \text{ kW/phase or } 17.38 \text{ kW, 3-phase}$$

$$\bar{I}_a = \frac{413\angle 90° - 231}{j16} = 25.8 + j14.43$$

$$= 29.56\angle 29.2° \text{ A}$$

$$I_a = 29.56 \text{ A, pf} = \cos 29.2° = 0.873 \text{ leading}$$

The phasor diagram is drawn




kVAR delivered (negative)

$$\frac{Q_e}{P_e} = \tan(-29.2°)$$

or $$Q_e = 17.38 \times 0.559 = 9.714 \text{ kVAR}$$