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Detailed Solutions

ESE-2024 Mains Test Series

Civil Engineering Test No : 6

Section A : Structural Analysis + CPM PERT

Q.1 (a) Solution:

Let H is the horizontal thrust at A and D when the arch AB is loaded. When AB is loaded, roller moves towards right. Also, when BD is loaded with same udl, roller moves by same amount towards left (due to symmetricity).

Hence if both the arches are loaded, the horizontal thrust at A and D will be 2H and roller support will not move horizontally.

Hence, for this case, horizontal thrust,

$$2H = \frac{wl^2}{8h}$$

$$\Rightarrow \quad H = \frac{wl^2}{16h} \quad \text{(when one of the arch is loaded)}$$

Bending moment diagram

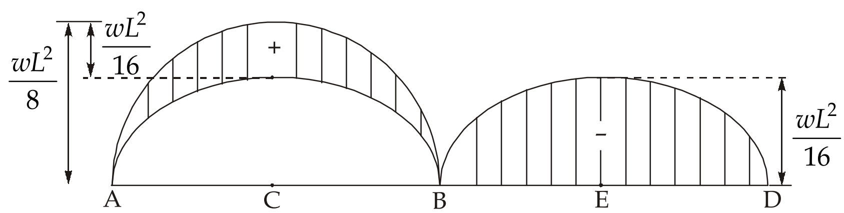
The BMD for the arch AB can be drawn by superimposing the beam moment and the H-moment diagram.

$$\text{Beam moment at } C = +\frac{wL^2}{8}$$

$$\text{H- moment at } C = -\text{H.h} = -\frac{wL^2}{16}$$

$$\therefore \qquad \text{Net moment at } C = \frac{+wL^2}{8} - \frac{wL^2}{16} = +\frac{wL^2}{16}$$

For arch BD, $$B.M \text{ at } E = -H.h = -\frac{wL^2}{16}$$



Q.1 (b) Solution:

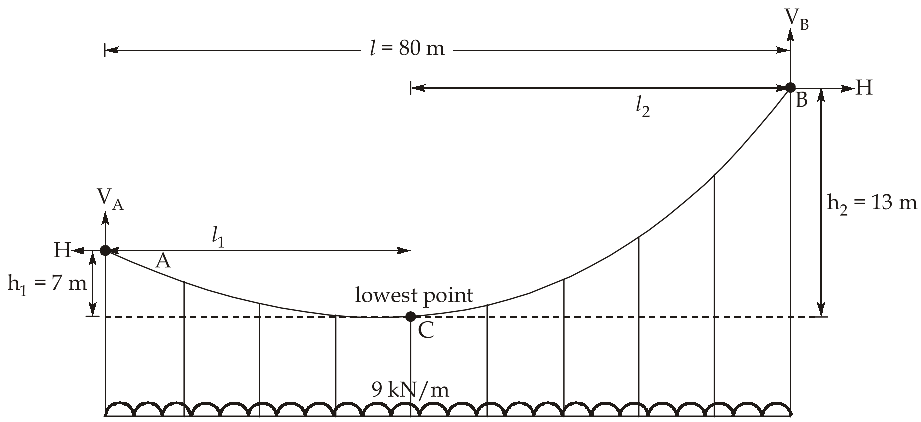
(i) Liquidated damage: It is a fixed stipulated sum payable by the contractor on account of penalty for delays and does not bear any relationship with the real or actual damage to the owner. It is generally high and is fixed on per day basis for the excess period over the specified period in the tender.

(ii) Unliquidated damage: This is known as ordinary damage having relation with the actual damage done resulting by breach of the contract.

(iii) Debitable agency: When the contractor fails to fulfill his contractual obligation in respect of progress or contractual obligation or quality of work even after being given due notice by the owner, a debitable agency is appointed. The expenses incurred will be charged from the bills of contractor or realising money deposited earlier as security deposit.

(iv) Measurement book (MB): Measurement of all works and supplies are recorded in a special type of note book (usually 15 cm × 10 cm) is known as measurement book (MB). Now a days all measurements are recorded in electronic form known as e-MB.

Q.1 (c) Solution:



For cable with ends at different levels.

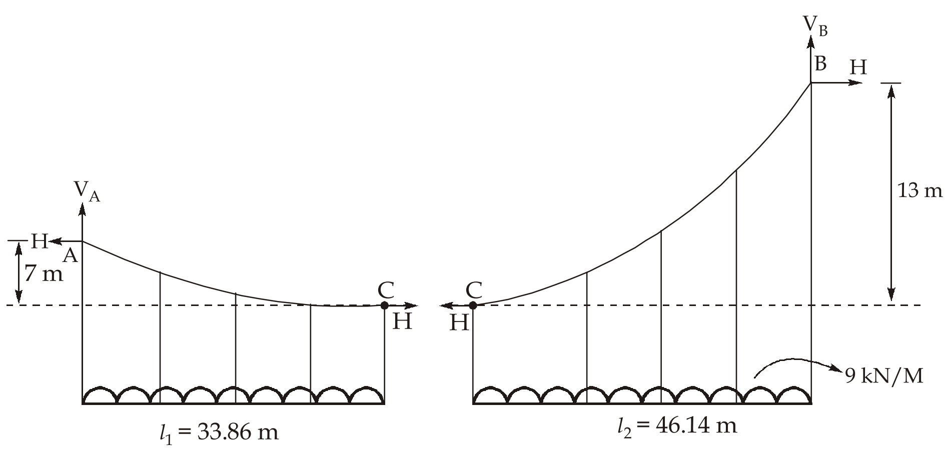
$$l_1 = l\left(\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right)$$

$$= \frac{80\times\sqrt{7}}{\sqrt{7}+\sqrt{13}}$$

$$= 33.86 \text{ m}$$

$\therefore$ $$l_2 = 80 - l_1$$

$$= 80 - 33.86 = 46.14 \text{ m}$$



Using free body diagram of cable,

For AC $$M_A = 0,$$

$$\Rightarrow \quad H(7) - 9 \times 33.86 \times \frac{33.86}{2} = 0$$

$$\Rightarrow \qquad H = 737.04 \text{ kN}$$

$$\sum F_y = 0$$

$$\Rightarrow \qquad V_A = 9 \times 33.86$$

$$= 304.74 \text{ kN}$$

For BC, $\sum F_y = 0$

$$\Rightarrow \qquad V_B = 9 \times 46.14$$

$$= 415.26 \text{ kN}$$

Length of the cable,

$$L = l + \frac{2h_1^2}{3l_1} + \frac{2h_2^2}{3l_2}$$

$$= 80 + \frac{2 \times 7^2}{3 \times 33.86} + \frac{2 \times 13^2}{3 \times 46.14}$$

$$= 83.41 \text{ m}$$

Q.1 (d) Solution

Following are the different types of hosting equipments:

(i) Pulley: Pulley and sheave are used for lifting rough surfaced and heavy objects. Both chains and wire ropes are used for this purpose.

(ii) Chain hoists: It is used for lifting loads upto 50 tonnes. The system consists of hand chain and the load chain. The pull applied through the hand chain is transmitted to the load chain with a multiplication factor of over 20.

(iii) Jacks: It is based on the principle of inclined plane. It is the short name of screw jack. The smallest jack may have capacity of 5 tonnes and is generally used for lifting an automobile wheel, while the bigger variety may be of 100 tonnes capacity.

(iv) Winch: A winch is a combination of gears (spur and pinion), clutches and brakes. The operation is controlled through a series of levers. It is commonly used in lifting the railway gates.

(v) Cranes: Cranes are most widely used equipment as an independent unit. Lifting capacity varies from 1/2 tonnes to 500 tonnes.

Important types of cranes are listed below:

(a) Derrick crane: The power is supplied by a diesel engine or by an electric motor. It consists of a mast supported by a number of guys.

(b) **Mobile crane:** Crane mounted on pneumatic type truck on crawler tractor chasis is termed as mobile crane.

(c) **Whirler crane:** It combines the advantages of long boom of derrick crane and mobility of the mobile crane.

(d) **Tower crane:** The crane is used for erection of very tall industrial and residential buildings.

(e) **Hydraulic crane:** It is very popular in present day because length of the boom can be changed during the working the crane itself.

(f) **Gantry crane:** A gantry crane or overhead crane is a must in factories and workshops. It consists of a bridge and a crab. The crab run on rails.

Q.1 (e) Solution:

I_{column} (moment of inertia of each column)

$$= \frac{300^4}{12} = 6.75 \times 10^8 \text{ mm}^4$$

$$\text{Flexural rigidity, } EI = 23 \times 10^3 \times 6.75 \times 10^8$$

$$= 1.55 \times 10^{13} \text{ N-mm}^2$$

Equivalent lateral stiffness of columns (in parallel) is,

$$k = \frac{12EI}{L_1^3} + \frac{12EI}{L_2^3} + \frac{3EI}{L_3^3}$$

$$= \frac{12 \times 1.55 \times 10^{13}}{(5000)^3} + \frac{12 \times 1.55 \times 10^{13}}{(3000)^3} + \frac{3 \times 1.55 \times 10^{13}}{(5000)^3}$$

$$= 8748.89 \text{ N/mm}$$

$$= 8748.89 \times 10^3 \text{ N/m}$$

$$\text{Natural frequency, } \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{8748.89 \times 10^3}{\frac{40 \times 8 \times 10^3}{9.81}}} = 16.38 \text{ rad/sec}$$

∴ Damped natural frequency,

$$\omega_D = \omega_n \sqrt{1 - \xi^2}$$

$$= 16.38 \sqrt{1 - 0.12^2}$$

$$= 16.26 \text{ rad/sec}$$

Amplitude,
$$A = \sqrt{y_0^2 + \left(\frac{\dot{y}_0 + \omega_n \xi y_0}{\omega_D}\right)^2}$$

$$= \sqrt{20^2 + \left(\frac{15 + 16.38 \times 0.12 \times 20}{16.26}\right)^2}$$

$$= 20.28 \text{ mm}$$

Expression for displacement,

$$y = e^{-\omega_n \xi t}\left[y_0 \cos \omega_D t + \frac{\dot{y}_0 + \omega_n \xi y_0}{\omega_D} \sin \omega_D t\right]$$

$$= e^{-16.380.12 \times t}\left[20 \cos 16.26t + \left(\frac{15 + 16.38 \times 0.12 \times 20}{16.26}\right) \times \sin 16.26t\right]$$

$$= e^{-1.97t}[20 \cos 16.26t + 3.34 \sin 16.26t]$$

Q.2 (a) Solution:

Let V_E and V_F be the vertical reactions at E and F respectively.

$$\sum M_E = 0$$

$$V_F \times 4 = 180(4 + 4\sqrt{3}) + 100(4\sqrt{3} + 4)$$

$$\Rightarrow \quad V_F = 764.97 \text{ kN}$$

$$\sum F_x = 0$$

$$\Rightarrow \quad H_E + 100 = 0$$

$$\Rightarrow \quad H_E = -100 \text{ kN} \quad \text{i.e. } 100 \text{ kN } (\leftarrow)$$

$$\sum F_y = 0$$

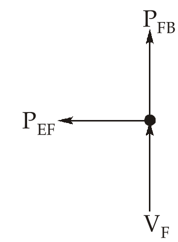
$$\Rightarrow \quad V_E + V_F = 180 \text{ kN}$$

$$\Rightarrow \quad V_E = -584.97 \text{ kN} \quad \text{i.e. } 584.97 \text{ kN } (\downarrow)$$

At joint F,
$$\sum F_y = 0$$

$$P_{FB} + V_F = 0$$

$\Rightarrow$ $$P_{FB} = -V_F$$
$$= -764.97 \text{ kN}$$
$$= 764.97 \text{ kN (Comp.)}$$



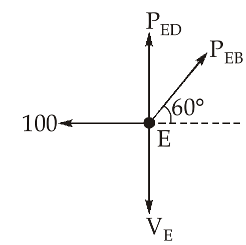
Also, $$\sum F_x = 0$$

$\Rightarrow$ $$P_{FE} = 0$$

At joint E, $$\sum F_y = 0$$

$\Rightarrow$ $$P_{ED} + P_{EB}\sin 60° = 584.97$$

$$\sum F_x = 0$$



$\Rightarrow$ $$P_{EB}\cos 60° = 100$$

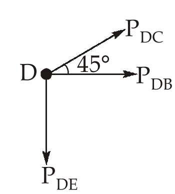
$\Rightarrow$ $$P_{EB} = \frac{100}{\cos 60°} = 200 \text{ kN (Tensile)}$$

$\therefore$ $$P_{ED} = 411.76 \text{ kN (Tensile)}$$

At joint D, $$\sum F_y = 0$$

$$P_{DC}\sin 45° = P_{DE}$$

$\Rightarrow$ $$P_{DC} = 582.32 \text{ kN (Tensile)}$$



$$\sum F_x = 0$$

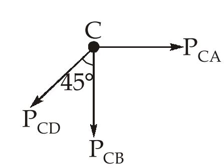
$$P_{DB} + P_{DC}\cos 45° = 0$$

$\Rightarrow$ $$P_{DB} = 411.76 \text{ kN (Compressive)}$$

At joint C, $$\sum F_y = 0$$

$$P_{CD}\cos 45° + P_{CB} = 0$$

$\Rightarrow$ $$P_{CB} = 411.76 \text{ kN (compressive)}$$



$$\sum F_x = 0$$

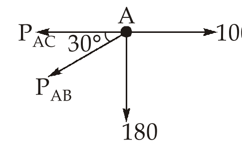
$\Rightarrow$ $$P_{CD}\sin 45° = F_{CA}$$

$\Rightarrow$ $$P_{CA} = 411.76 \text{ kN (Tensile)}$$

At joint A, $$\sum F_y = 0$$

$\Rightarrow$ $P_{AB} \sin 30° + 180 = 0$

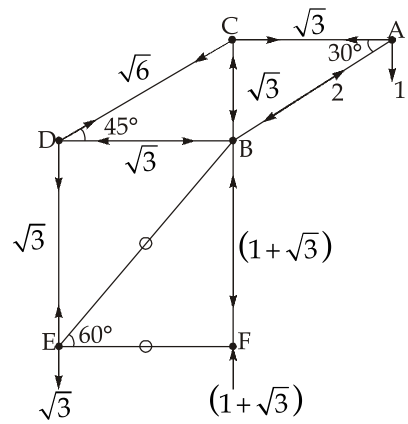
$\Rightarrow$ $P_{AB} = 360$ kN (Compressive)



K-system of forces:

Apply a unit load vertically at A,

The forces in the various members due to the application of unit load at A are shown in figure below.



Vertical deflection at D, $\delta = \sum \frac{PKL}{AE}$

Member	P(kN)	K	L(m)	PKL (kN - m)
AC	411.76	$\sqrt{3}$	$4\sqrt{3}$	4941.12
AB	–360	–2	8	5760
CB	–411.76	$-\sqrt{3}$	4	2852.76
CD	582.32	$\sqrt{6}$	$4\sqrt{2}$	8068.86
BE	200	0	8	0
BF	–764.97	$-(1+\sqrt{3})$	$4\sqrt{3}$	14479.51
BD	–411.76	$-\sqrt{3}$	4	2852.76
DE	411.76	$\sqrt{3}$	$4\sqrt{3}$	4941.12
EF	0	0	4	0

$$\sum PKL = 43896.13$$

$\therefore$ $$\delta = \frac{43896.13}{9000 \times 180} = 0.0270 \text{ m} = 27 \text{ mm}$$

Q.2(b) Solution:

(i)

Total direct cost (normal) of the project

$$= 350 + 450 + 400 + 800 = \text{Rs. } 2000$$

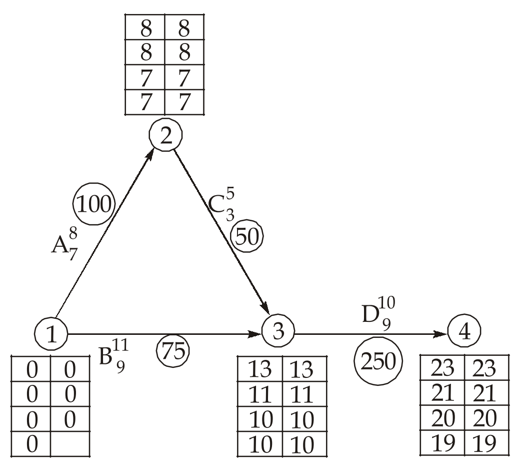
$$\text{Cost slope of } A = \frac{450-350}{8-7} = \text{Rs. } 100/\text{day}$$

$$\text{Cost slope of } B = \frac{600-450}{11-9} = \text{Rs. } 75/\text{day}$$

$$\text{Cost slope of } C = \frac{500-400}{5-3} = \text{Rs. } 50/\text{day}$$

$$\text{Cost slope of } D = \frac{1050-800}{10-9} = \text{Rs. } 250/\text{day}$$

Normal duration, crash duration and cost slope of each activity are shown below.



S. No.	Description	Duration (days)	Indirect Cost (Rs.)	Direct Cost (Rs)	Total Project Cost (Rs.)	Remarks
1	All normal	23	23 × 200 = 4600	2000	6600	Normal Duration
2	Crashing C by 2 days	21	21 × 200 = 4200	2000 + 50 × 2 = 2100	6300	
3	Crashing A and B each 1 day simultaneously	20	20 × 200 = 4000	2100 + 100 + 75 = 2275	6275	Optimum duration
4	Crashing D by 1 day	19	19 × 200 = 3800	2275 + 250 = 2525	6325	Minimum duration

Optimum duration of the project = 20 days
Minimum (all crash) duration of the project = 19 days

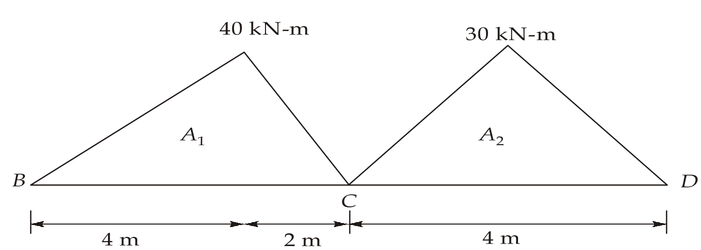
(ii)

S. No.	CPM	PERT
1	It is based on deterministic approach in design of network.	It is based on probabilistic approach in the design of network.
2	Only one time estimate is required for each activity.	Three time estimates (in the form of pessimistic time, optimistic time and most likely time) for each activity are required.
3	It is build-up of activity-oriented diagram.	It is build-up of event oriented diagram.
4	Time and cost both must have zero slack.	Time only is the controlling factors. Cost is assumed proportional to the project duration
5	Critical events must have zero slack	Critical events may be positive/zero/negative depending upon the project scheduled completion time.
6	It is recommended for repetitive nature of work.	It is recommended for for research and development project.

Q.2 (c) Solution:

(i)

Free-moment diagrams:



Span *BC* Maximum ordinate $= \dfrac{Wab}{L} = \dfrac{30\times4\times2}{6}$

$\therefore$ Area, $A_1 = \dfrac{1}{2}\times40\times6 = 120$ kN-m^2

Distance of cg of A_1 from C = $\frac{6+2}{3} = \frac{8}{3} = 2.67$ m

∴ Distance of cg of A_1 from A = 6 – 2.67 = 3.33 m

Span *CD*, Maximum ordinate = $\frac{WL}{4}$

$$= \frac{30 \times 4}{4} = 30\,\text{kN-m}$$

Area, $A_2 = \frac{1}{2} \times 30 \times 4 = 60$ kN-m²

∴ Distance of cg of A_2 from D = 2 m

Applying the three moment equation to *BC* and *CD*,

$$M_B\left(\frac{L_{BC}}{I_{BC}}\right) + 2M_C\left(\frac{L_{BC}}{I_{BC}} + \frac{L_{CD}}{I_{CD}}\right) + MD\left(\frac{L_{CD}}{I_{CD}}\right)$$

$$= \frac{-6A_1a_1}{I_{BC}L_{BC}} - \frac{6A_2a_2}{I_{CD}L_{CD}} + \frac{6Eh_B}{L_{BC}} + \frac{6Eh_D}{L_{CD}}$$

$$M_B = -\left(\frac{20 \times 2^2}{2}\right) = -40 \text{ kN–m}$$

$$M_D = -(30 \times 2) = -60 \text{ kN–m}$$

$h_B = h_0 = 8$ mm [∵ support *C* settles down by 8 mm]

$$-40\left(\frac{6}{2I}\right) + 2M_C\left(\frac{6}{2I} + \frac{4}{I}\right) - 60\left(\frac{4}{I}\right) = \frac{-6 \times 120 \times 3.33}{2I \times 6} - \frac{6 \times 60 \times 20}{I \times 4} + \frac{6 \times 200 \times 10^6 \times 0.008}{6}$$

$$+ \frac{6 \times 200 \times 10^6 \times 0.008}{4}$$

$$\frac{14M_C}{I} - \frac{360}{I} = \frac{-1899}{5I} + 4 \times 10^6$$

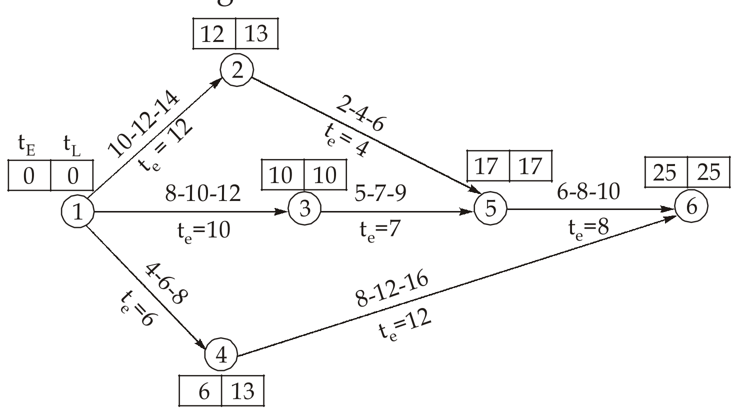
$M_C = 21.44$ *kN* [∵ support C settles down by 8 mm]

(ii)

Force/Compatibility method	Displacement/Stiffness method
1. The unknowns are taken as internal member forces or reactions.	1. The unknowns are taken as joint displacements.
2. To find unknowns, compatibility conditions are written. Number of compatibility conditions needed = Number of redundant forces = Degree of static indeterminacy, D_s.	2. To find unknown displacements, joint equilibrium conditions are written. Joint equilibrium conditions needed = degree of kinematic indeterminacy, D_k.
3. This method is suitable when $D_S < D_K$	3. This method is suitable when $D_K < D_S$
4. **Examples:** Strain energy method or minimum potential energy method, Castingliano's theorem, unit load method, virtual work method, column analogy method, three moment method, flexibility method.	4. **Examples:** Moment distribution method, slope deflection method, Kani's method, stiffness matrix method.

Q.3(a) Solution:

The PERT network is shown in figure below.



- Time box with two compartments at each event is showing left compartment, *TE* = Earliest expected occurrence time and right compartment, *TL* = Latest allowable occurrence time.
- Expected duration of each activity, s given by $t_e = \dfrac{t_0 + 4t_m + t_p}{6}$
- Standard deviation of each activity is given by $\delta_t = \left(\dfrac{t_p - t_0}{6}\right)$
- Variance of each activity is given by $\mu_t = (\delta_t)^2 = \left(\dfrac{t_p - t_0}{6}\right)^2$

Activity	Time estimates (days)			Expected duration (days) t_e	Standard deviation (days) δ_t	Variance $(\text{days})^2$ μ_t	Remarks
	t_o	t_m	t_p				
1 - 2	10	12	14	12	2/3	4/9	
1 - 3	8	10	12	10	2/3	4/9	Critical
1 - 4	4	6	8	6	2/3	4/9	
2 - 5	2	4	6	4	2/3	4/9	
3 - 5	5	7	9	7	2/3	4/9	Critical
4 - 6	8	12	16	12	4/3	16/9	
5 - 6	6	8	10	8	2/3	4/9	Critical

- All events with zero slack are critical hence critical path is 1 – 3 – 5 – 6
- Expected project length = Expected time taken by critical path = 25 days
- Variance of the project = Sum of Variance of critical activities lying on critical path

$$= \frac{4}{9}+\frac{4}{9}+\frac{4}{9}=\frac{4}{3}$$

- Standard duration of the project, σ

$$\sigma = \sqrt{\frac{4}{3}} = 1.155$$

Now probability factor Z is given by

$$Z = \frac{T_S - T_E}{\sigma}$$

Given, $Z = 1.647$ for 95% probability

$$\therefore \quad 1.647 = \frac{T_S - 25}{1.155}$$

$$\Rightarrow \quad T_S = 26.902 \text{ days} \simeq 27 \text{ days} \quad \text{(say)}$$

Q.3 (b) Solution:

Since there is no loading on spans, therefore fixed end moments are zero. There will be sway of the frame (towards right) and moments will be only due to sway.

$$\bar{M}_{AB} = \bar{M}_{BA} = \frac{-6EI\delta}{4^2}$$

$$\bar{M}_{DC} = \bar{M}_{CD} = \frac{-6EI\delta}{2^2}$$

$\therefore \qquad \dfrac{\bar{M}_{AB}}{\bar{M}_{DC}} = \dfrac{1}{4}$

Assume, $\qquad \bar{M}_{AB} = \bar{M}_{BA}$

$= -100 \text{ N- m}$

$\therefore \qquad \bar{M}_{DC} = \bar{M}_{CD} = -400 \text{ N- m}$

Distribution factors at 'B' are:

$$r_{BC} = \frac{\frac{I_{BC}}{l_{BC}}}{\frac{I_{BC}}{l_{BC}} + \frac{I_{BA}}{l_{BA}}} = \frac{\frac{2I}{4}}{\frac{2I}{4} + \frac{I}{4}} = \frac{2}{3}$$

$\therefore \qquad r_{BA} = 1 - r_{BC} = \dfrac{1}{3}$

Distribution factors at 'C' are:

$$r_{CB} = \frac{\frac{2I}{4}}{\frac{2I}{4} + \frac{I}{2}} = \frac{1}{2}$$

$\therefore \qquad r_{CD} = 1 - r_{CB} = \dfrac{1}{2}$

Distribution of assumed sway moments:

Joints	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
D. F.	–	1/3	2/3	1/2	1/2	-
F.E.M.	– 100	– 100			– 400	– 400
Balance		+ 33.33	+ 66.7	+ 200	+ 200	
C.O.	+ 16.7		+100	+33.4		+100
Balance		– 33.3	– 66.7	– 16.7	– 16.7	
C.O.	–16.7		– 8.4	– 33.4		– 8.4
Balance		+2.8	+ 5.6	+ 16.7	+ 16.7	
C.O.	+1.4		+8.4	+2.8		+ 8.4
Balance		– 2.8	– 5.6	– 1.4	– 1.4	
C.O.	–1.4		– 0.7	– 2.8		– 0.7
Balance		+ 0.2	+ 0.5	+ 1.4	+ 1.4	
C.O.	+ 0.1		+ 0.7	+ 0.2		+ 0.7
Balance		– 0.2	– 0.5	– 0.1	– 0.1	
C.O.	–0.1		– 0.05	– 0.25		– 0.05
Final Moments	–100	–100	+100	+200	–200	–300

Let actual sway moments be 'x' times the assumed moments,

$$M_{AB} = -100x,\ M_{BA} = -100x,\ M_{CD} = -200x$$
$$M_{DC} = -300x$$

Using horizontal shear equation,

$$H_A + H_D + 850 = 0$$

$$\Rightarrow \quad \frac{M_{AB} + M_{BA}}{4} + \frac{M_{DC} + M_{CD}}{2} + 850 = 0$$

$$\Rightarrow \quad \frac{-100x - 100x}{4} + \frac{-200x - 300x}{2} + 850 = 0$$

$$\Rightarrow \quad -50x - 250x + 850 = 0$$

$$x = 2.833$$

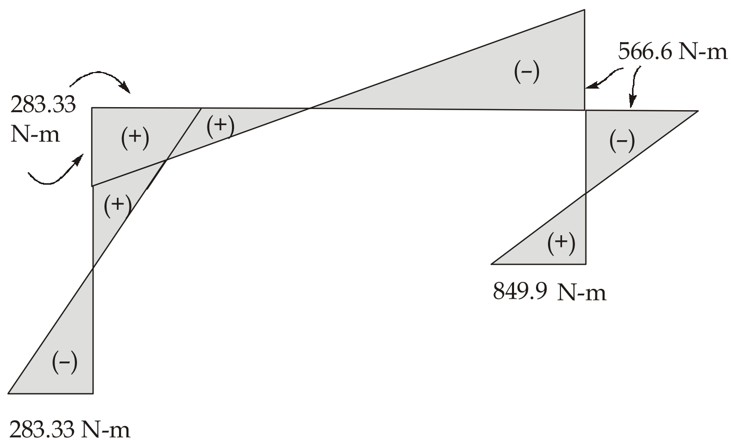
$\therefore$ Final moments are:

$M_{AB} = -283.3$ N–m, $M_{BA} = -283.3$ N–m

$M_{BC} = 283.3$ N–m, $M_{CB} = 566.6$ N–m

$M_{CD} = -566.6$ N–m, $M_{DC} = -849.9$ N–m

Bending moment diagram is as shown below.



Q.3(c) Solution:

(i)

- Maximum possible rimpull prior to slippage of driving tires = 0.50 × 16500 = 8250 kg
- Total weight of the unit = 28,000 kg 28 t
- Rolling resistance of the haul road 50 kg/t of the gross weight = 50 × 28 = 1400 kg

∴ Available rimpull to negotiate the slope = 8250 – 1400 = 6850 kg

The pull required per 1 tonne of gross weight per 1% slope = 10 kg.

∴ The pull required for 28 tonne per 1% slope = 280 kg

∵ 280 kg pull is required per 1% slope

∴ 6850 kg pull is required $\frac{6850}{280} = 24.46\%$

(ii)

(a) Undamped natural frequency (f):

$$K = 39 \text{ N/mm} = 3900 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{39000}{25}}$$

$$= 39.50 \text{ rad/sec}$$

$$f = \frac{\omega_n}{2\pi} = \frac{39.50}{2\pi} = 6.28 \text{ cycles/sec}$$

(b) The logarithmic decrement

$$\delta = ln\left(\frac{y_1}{y_2}\right)$$

$$= ln\left(\frac{1}{0.9}\right)$$
$$= 0.105$$

(c) The damping ratio

$$\xi = \frac{\delta}{2\pi} = \frac{0.105}{2\pi}$$
$$= 0.0167 \simeq 0.017$$

(d) The damped coefficient

$$C = \xi C_{cr}$$
$$= 0.017 \times 2\sqrt{Km}$$
$$= 0.017 \times 2\sqrt{39000 \times 25}$$
$$= 33.6 \text{ Ns/m}$$

(e) The natural frequency of the damped system

$$\omega_D = w_n\sqrt{1-\xi^2}$$
$$= 39.50 \times \sqrt{1-0.017^2}$$
$$= 39.49 \text{ rad/sec}$$

Q.4 (a) Solution:

Step - 1 Fixed end moments:

$$\bar{M}_{AB} = \bar{M}_{BA} = 0$$

$$\bar{M}_{BC} = \frac{-wL^2}{12} = \frac{-50\times 5^2}{12} = -104.17 \text{ kN-m}$$

$$\bar{M}_{CB} = +\frac{wL^2}{12} = +104.17 \text{ kN-m}$$

$$\bar{M}_{BD} = 50\times 2\times\frac{2}{2} = 100 \text{ kN-m}$$

Step - 2 Slope deflection equations:

Member AB:

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= 0 + \frac{2EI}{5}(\theta_B) \qquad (\because \theta_A = 0)$$

$$= 0.4\, EI\theta_B$$

$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$= 0 + \frac{4EI\theta_B}{5} = 0.8EI\,\theta_B$$

Member BC:

$$M_{BC} = \bar{M}_{BC} + \frac{2E(2I)}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$= -104.17 + \frac{4EI}{5}(2\theta_B) \qquad (\because \theta_C = 0)$$

$$= -104.17 + 1.6EI\theta_B$$

$$M_{CB} = \bar{M}_{CB} + \frac{2E(2I)}{5}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$= 104.17 + 0.8\,EI\,\theta_B$$

Member BD:

$$M_{BD} = \bar{M}_{BD}$$

$$= 100 \text{ kN–m}$$

Step - 3 Equilibrium equations:

Here the only unknown is θ_B, therefore only one equilibrium condition is required.

$$\Sigma M_B = 0$$

$$\Rightarrow \quad M_{BA} + M_{BC} + M_{BD} = 0$$

$$\Rightarrow \quad 0.8EI\theta_B - 104.17 + 1.6\,EI\theta_B + 100 = 0$$

$$\Rightarrow \quad 2.4\,EI\theta_B = 4.17$$

$$\Rightarrow \quad EI\theta_B = 1.7375$$

Step 4 Final end moments:

$$M_{AB} = 0.4 \times 1.7375$$

$$= 0.695 \text{ kN–m}$$

$$M_{BA} = 0.8 \times 1.7375$$

$$= 1.39 \text{ kN-m}$$

$$M_{BC} = -104.17 + 1.6 \times 1.7375$$

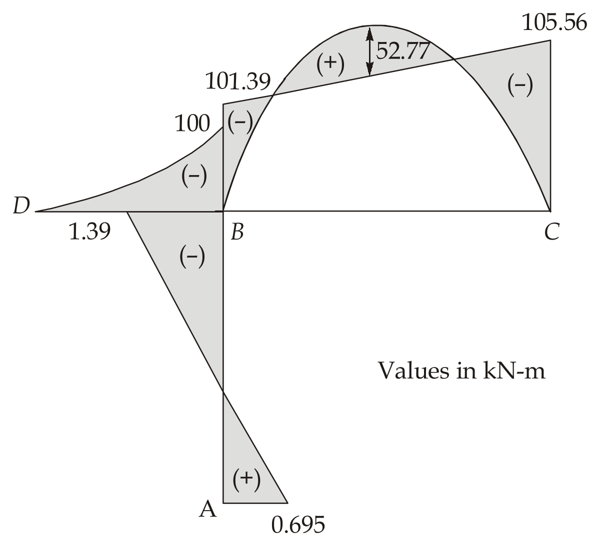
$$= -101.39 \text{ kN-m}$$

$$M_{CB} = 104.17 + 0.8 \times 1.7375$$

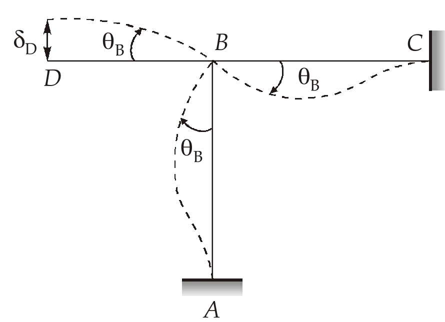
$$= 105.56 \text{ kN-m}$$

$$M_{BD} = 100 \text{ kN-m}$$

Step 5 Bending moment diagram:



Step 6 Deflection of free end D:



$$\delta_D = +(\theta_B \times L_{BD}) - \frac{wL_{BD}^4}{8EI}$$

$$= \frac{1.7375}{EI} \times 2 - \frac{50 \times 5^4}{8EI} = \frac{-3902.77}{EI} (\text{upwards})$$

Q.4 (b) Solution:

(i)

Declining balance method:

$$\text{FDB} = \text{Depreciation factor}$$

$$= 1 - \left(\frac{C_s}{C_i}\right)^{\frac{1}{n}}$$

$$= 1 - \left(\frac{200}{2000}\right)^{\frac{1}{5}}$$

$$= 0.369$$

- Depreciation at the end of 1st year is given by:

$$D_1 = C_i \times FDB = 2000 \times 0.369 = \text{Rs. } 738$$

- Book value at the end of 2nd year, B_1 is given by -

$$B_1 = C_i - D_1 = 2000 - 738 = \text{Rs. } 1262$$

Similarly,

$$D_2 = 1262 \times 0.369 = \text{Rs. } 465.68$$
$$B_2 = 1262 - 465.68 = \text{Rs. } 796.32$$
$$D_3 = 796.32 \times 0.369 = \text{Rs. } 293.84$$
$$B_3 = 796.32 - 293.84 = \text{Rs. } 502.48$$
$$D_4 = 502.48 \times 0.369 = \text{Rs. } 185.42$$
$$B_4 = 502.48 - 185.42 = \text{Rs. } 317.06$$
$$D_5 = 317.07 \times 0.369 = \text{Rs. } 117$$
$$B_5 = 317.07 - 117 = \text{Rs. } 200.06 \simeq \text{Rs. } 200 \text{ (say)}$$

(ii)

(1) **Salvage value:** It is the value of the property at the end of its utility period without being dismantled. Salvage value implies that the property has still further utility. It is affected by several factors. The reason of the present owner for selling may influence the salvage value. If the owner is selling because there is very little commercial need for the property, this will affect the resale/salvage value. It is affected by the location of the property. The physical condition of property will also have a great influence on the resale price that can be obtained.

(2) **Scrap value:** The value of a property realised when it becomes absolutely useless except for sale as junk as its scrap value. The utility of the article or property is assumed to be zero.

(3) **Book value:** It is defined as the value of the property shown in the account books in that particular year i.e. the original cost less total depreciation till that year.

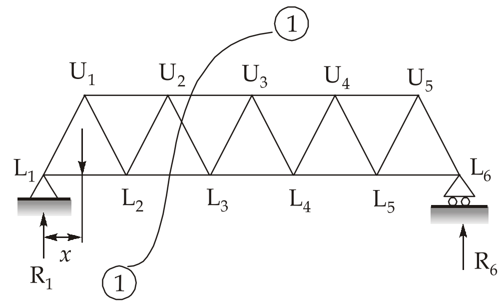
Q.4 (c) Solution:

(i) **ILD for force in number L_3U_3,**

Cut a sectino 1-1 passing through $L_3 L_4$, $L_3 U_3$ and $U_2 U_3$

1. When unit load is at x distance from L_1 and lies between L_1 and $L_3 \left[0 \le x < \overline{8}\text{m}\right]$

Considering right side of section 1 - 1



$$\sum F_y = 0$$

$$F_{L_3U_3}\sin 60° - R_6 = 0$$

$$\Rightarrow \quad F_{L_3U_3} = \frac{R_6}{\sin 60°}$$

where $\quad R_6 = \frac{x}{20}$

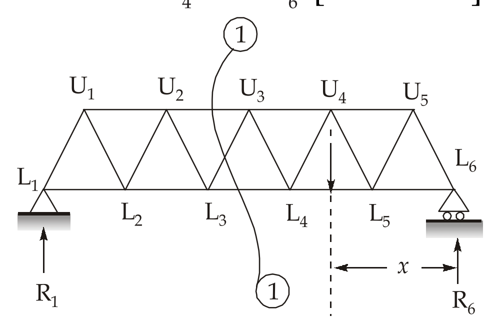
So, $\quad F_{L_3U_3} = \frac{x}{20\sin 60°} = \frac{x}{10\sqrt{3}}$

So when load is at L_1, $F_{L_3U_3} = 0 \quad [\because x = 0]$

So when load is at L_2, $F_{L_3U_3} = \frac{4}{10\sqrt{3}} = \frac{2}{5\sqrt{3}} \quad [\because x = 4\text{ m}]$

When load is at L_3, $F_{L_3U_3} = \frac{8}{10\sqrt{3}} = \frac{4}{5\sqrt{3}} \quad [\because x = 8\text{ m}]$

2. When unit load is at x distance from L_6 and lies between L_4 and L_6 $[0 \le x < 8\text{m}]$ considering left side of section 1 - 1.



$$\sum F_y = 0$$

$$F_{L_3U_3}\sin 60° + R_1 = 0$$

$$\Rightarrow \quad F_{L_3U_3} = \frac{-R_1}{\sin 60°}$$

where $\quad R_1 = \frac{x}{20}$

So, $\quad F_{L_3U_3} = \frac{-R_1}{\sin 60°} = \frac{-x\times 2}{20\times\sqrt{3}} = \frac{-x}{10\sqrt{3}}$

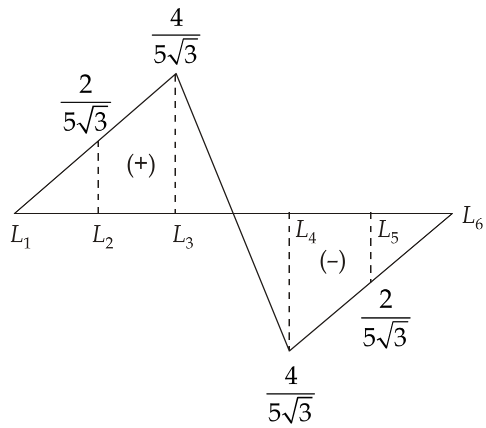
Now,

When load is at L_6, $F_{L_3U_3} = 0 \quad [\because x = 0]$

When load is at L_5, $F_{L_3U_3} = \frac{-4}{10\sqrt{3}} = \frac{-2}{5\sqrt{3}} \quad [\because x = 4\text{ m}]$

When load is at L_4, $F_{L_3U_3} = \frac{-8}{10\sqrt{3}} = \frac{-4}{5\sqrt{3}} \quad [\because x = 8\text{ m}]$

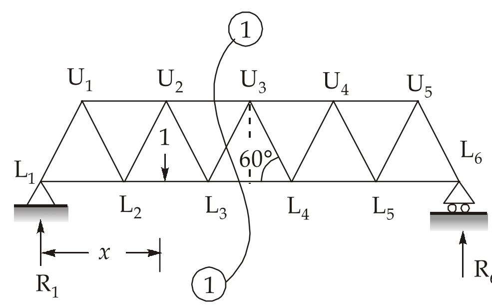
ILD for $F_{L_3U_3}$ for portion L_3L_4 can be drawn by joining the ordinates at L_3 and L_4 as shown below



(ii) ILD for force in member L_3L_4,

(1) When unit load is at x distance from L_1 and lies between L_1 and L_3 $[0 \le x < 8\,\text{m}]$

Considering right side of section 1 - 1.



$$\sum M_{U_3} = 0$$

$$\Rightarrow \quad F_{L_3L_4} \times 4 \sin 60° - R_6 \times (8 + 2) = 0$$

where $\quad R_6 = \dfrac{x}{20}$

$$\Rightarrow \quad F_{L_3L_4} = R_6 \times \frac{10}{4\sin 60°} = \frac{x}{20} \times \frac{10}{4\sin 60°} = \frac{x}{4\sqrt{3}}$$

Now,

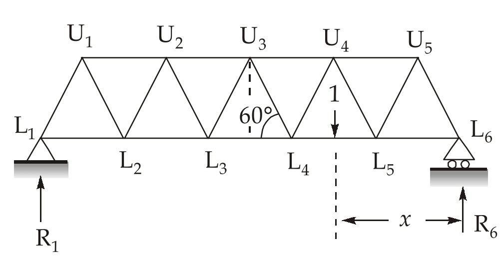
When load is at L_1, $\quad F_{L_3L_4} = 0 \quad [\because x = 0]$

When load is at L_2, $\quad F_{L_3L_4} = \dfrac{1}{\sqrt{3}} \quad [\because x = 4\text{ m}]$

When load is at L_3, $\quad F_{L_3L_4} = \dfrac{2}{\sqrt{3}} \quad [\because x = 8\text{ m}]$

(2) When unit load is at x distance from L_6 and lies between L_4 and L_6 $[0 \le x < 8\,\text{m}]$

Considering left side of section 1 - 1



$$\sum M_{U_3} = 0$$

$$\Rightarrow \quad -F_{L_3L_4} \times 4\sin 60° + R_1 \times (8+2) = 0$$

where $\quad R_1 = \dfrac{x}{20}$

$$\Rightarrow \qquad F_{L_3L_4} = R_1 \times \frac{10}{4\sin 60^\circ} = \frac{x}{20} \times \frac{10}{4\sin 60^\circ} = \frac{x}{4\sqrt{3}}$$

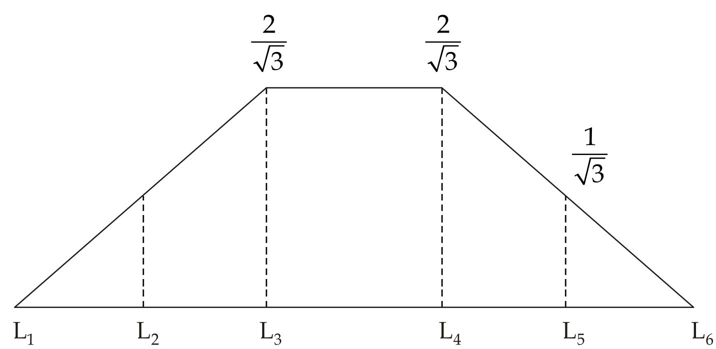
Now,

When load is at L_6, $F_{L_3L_4} = 0$ $[\because x = 0]$

When load is at L_5, $F_{L_3L_4} = \frac{1}{\sqrt{3}}$ $[\because x = 4 \text{ m}]$

When load is at L_3, $F_{L_3L_4} = \frac{2}{\sqrt{3}}$ $[\because x = 8 \text{ m}]$

ILD for $F_{L_3L_4}$ for portion L_3L_4 can be drawn by joining the ordinates at L_3 and L_4 as shown below.



Section B : Flow of Fluids, Hydraulic Machines and Hydro Power-1 + Design of concrete and Masonry Structures-2

Q.5 (a) Solution:

(i) Dead load, $W_d = 0.3 \times 0.6 \times 24 = 4.32$ kN/m.

Initial prestressing force, $P_0 = \sigma \times A_s$

$= 1400 \times 3 \times 500 \times 10^{-3}$ kN

$= 2100$ kN

$$\text{Moment of inertia, } I = \frac{300 \times 600^3}{12} = 54 \times 10^8 \text{ mm}^4$$

Downward deflection due to dead load is given by :

$$\delta_1 = \frac{5}{384} \times \frac{wL^4}{E_c.I}$$

$$= \frac{5}{384} \times \frac{(4.32) \times (10{,}000)^4}{40 \times 10^3 \times 54 \times 10^8}$$

$= 2.604$ mm

Upward deflection of beam due to P_1 is given by :

$$\delta_2 = \frac{P_0 e.L^2}{8E_c.I}$$

$$= \frac{2100\times10^3\times150\times(10,000)^2}{8\times40\times10^3\times54\times10^8}\text{mm}$$

$= 18.23$ mm.

∴ Net upward deflection, $\delta = \delta_2 - \delta_1$

$= 18.23 - 2.604$

$= 15.626$ mm

(ii) Final prestressing force, P_f = K.P.

where $K = 1 - 0.2 = 0.8$

Hence, $P_f = 0.8 \times 2100$ kN

$= 1680$ kN

Live load = 30 kN/m

Total load, $w_T = w_{\text{live}} + w_{\text{dead}}$

$= 30 + 4.32 = 34.32$ kN/m

Downward deflection due to total load is given by:

$$\delta_1 = \frac{5}{384}\cdot\frac{w_T L^4}{E_c I}$$

$$= \frac{5\times(34.32)\times(10,000)^4}{384\times40\times10^3\times54\times10^8}\text{mm}$$

$= 20.69$ mm

Upward deflection of beam due to final P force is given by:

$$\delta_2 = \frac{P_f.e.L^2}{8.E_c I} = \frac{1680\times10^3\times150\times(10,000)^2}{8\times40\times10^3\times54\times10^8}\text{mm}$$

$= 14.58$ mm

Net downward deflection, $\delta = \delta_1 - \delta_2$

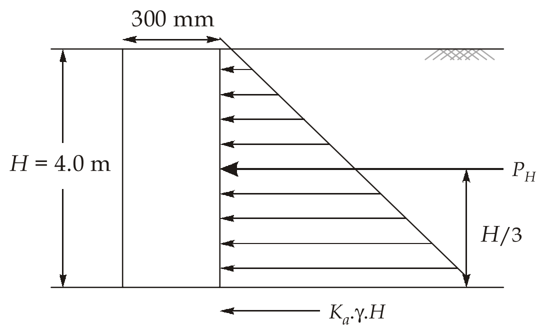
$= 20.69 - 14.58 = 6.11$ mm

Q.5 (b) Solution:

Given, angle of repose, $\phi = 30°$

Density of earth, $\gamma = 19$ kN/m^3

M25 concrete and Fe500 steel are to be used



$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1-\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3}$$

So,
$$P_H = \frac{1}{2}k_a.\gamma.H^2 = \frac{1}{2}\times\frac{1}{3}\times 19\times 4^2 = 50.67 \text{ kN.}$$

$$\text{Bending moment at base} = P_H \times \frac{H}{3} = 50.67\times\frac{4}{3} = 67.56 \text{ kN-m}$$

Factored bending moment, M_u = 1.5 × 67.56 = 101.34 kN-m

$$\text{Required depth of stem, } d = \sqrt{\frac{M_u}{0.133.f_{ck}.B}}$$

$$= \sqrt{\frac{101.34\times 10^6}{0.133\times 25\times 1000}} = 174.58 \text{ mm} \simeq 180 \text{ mm (say)}$$

Given, Depth D = 300 mm

Nominal cover = 50 mm

Assuming 16 mm diameter bars,

$$\text{Effective depth, } d = D - \text{Nominal cover} - \frac{\phi}{2}$$

$$\Rightarrow \quad d = 300 - 50 - \frac{16}{2}$$

$$= 242 \text{ mm} > 180 \text{ mm} \quad \text{(OK)}$$

Now,
$$\frac{M_u}{bd^2} = \frac{101.34\times 10^6}{1000\times(242)^2} = 1.73$$

Now p_t can be calculated from the table as

$$p_t = \left(\frac{0.455-0.427}{1.8-1.7}\right)(1.73-1.7)+0.427$$

$\Rightarrow$ $p_t = 0.4354\%$

$\therefore$ Area of steel, $A_{st} = \dfrac{0.4354}{100} \times 1000 \times 242 = 1053.7 \text{ mm}^2$

Spacing of 16 mm diameter bars $= \dfrac{1000}{1053.7} \times \dfrac{\pi}{4} \times 16^2$

$= 190.8 \text{ mm} \simeq 180 \text{ mm c/c}$ (say)

$\therefore$ Provide 16 mm diameter bars at a spacing of 180 mm c/c.
Minimum reinforcement @ 0.12%

$$= \frac{0.12}{100} \times 1000 \times 300$$

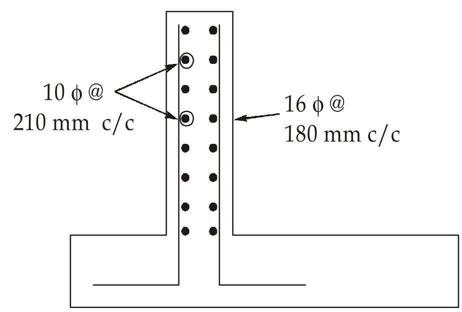
$$= 360 \text{ mm}^2$$

Spacing of 10 mm diameter bars $= \dfrac{1000}{360} \times \dfrac{\pi}{4} \times 10^2$

$= 218.17$ mm

$\simeq 210$ mm (say)

Provide 10 mm diameter bars at 210 mm c/c spacing.



Q.5 (c) Solution:

By using Newton's law of viscosity,

$$\mu\left(\frac{\partial u}{\partial y}\right) = \frac{-r}{2}\left(\frac{\partial p}{\partial x}\right)$$

where y is distance from pipe wall.

Since, in the present case, $y = (R - r)$

$\therefore$ $dy = -dr$

$\therefore$ $$-\mu.\frac{\partial u}{dr} = \frac{-r}{2}\left(\frac{\partial p}{\partial x}\right)$$

$\Rightarrow$ $$\frac{\partial u}{\partial r} = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right).r$$

Integrating both sides, we get

$$\int \partial u = \int \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right).r.dr$$

$$\Rightarrow \qquad u = \frac{r^2}{4\mu}\left(\frac{\partial p}{\partial x}\right) + C \qquad \ldots(i)$$

Applying boundary conditions,

At $r = R, u = 0$

From equation (i),

$$C = \frac{-1}{4\mu}\left(\frac{\partial p}{\partial x}\right).R^2$$

Putting value of C in equation (i),

$$u = \frac{-1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left(R^2 - r^2\right) \qquad \ldots(ii)$$

∴ Equation of velocity distribution is parabolic in nature.

- Maximum velocity, u_{max} occurs at pipe axis i.e. $r = 0$

$$\therefore \qquad u = \frac{-1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left(R^2 - r^2\right)$$

$$\Rightarrow \qquad u_{max} = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)R^2 \qquad \ldots(iii)$$

Combining equation (ii) and (iii), the expression for velocity distribution can also be written in terms of maximum velocity as,

$$u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

- Now, mean velocity:

  Mass flow rate $\dot{m}$ passing through any cross section of a circular pipe can be obtained by integrating a small flow rate passing through an elementary ring of thickness dr,

$$\dot{m} = \int_0^R \rho(2\pi r)\,dr.u \qquad \ldots(iv)$$

Using mean velocity, $$\dot{m} = \rho\left(\pi R^2\right)\bar{u}_{mean} \qquad \ldots(v)$$

and $$u = u_{max}\left(1-\frac{r^2}{R^2}\right)$$

From equation (iv) and (v), we get

$$\rho\pi R^2\bar{u}_{mean} = \int_0^R \rho(2\pi r).dr.u_{max}\left(1-\frac{r^2}{R^2}\right)$$

$$\Rightarrow \quad R^2\bar{u}_{mean} = 2u_{max}\int_0^R\left(1-\frac{r^2}{R^2}\right)r.dr$$

$$\Rightarrow \quad R^2\bar{u}_{mean} = 2u_{max}\times\frac{R^2}{4}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R$$

$$\Rightarrow \quad \bar{u}_{mean} = \frac{u_{max}}{2}$$

$$\therefore \quad u_{mean} = \frac{-1}{8\mu}\left(\frac{\partial P}{\partial x}\right).R^2$$

Q.5(d) Solution:

(i) Panel Wall: It is the exterior non load bearing wall in framed construction wholly supported at each storey and subjected to lateral loads in aN out plane direction like wind loads, earthquake etc.

(ii) Shear Wall : These are vertical structural elements designed to resist lateral loads parallel to the plane of the wall. They are typically made of reinforced concrete or masonry and are strategically placed in a building to resist forces that could cause it to sway or deform during an earthquake or high wind event.

(iii) Cross Wall: It is also known as partition walls. These are non-structural walls that divide spaces within a building. While they may provide some minimal lateral stability, their primary function is to define rooms or areas within a structure. Cross walls are usually made of material such as wood, steel studs, or lightweight concrete blocks and are not specifically designed to resist lateral loads like shear walls.

(iv) Free standing wall: It is compound wall or parapet wall. It is acted upon by wind force which tends to over turn it. This tendency to overturning is resisted by gravity force due to self weight of wall, and also by flexural moment of resistance on account of tensile strength of masonry.

(v) Cavity Wall: It is the wall consisting of two leaves with each leaf separated by a cavity and ties together with metal ties or the bonding units in order to ensure that the two leaves act as one unit. The space between the two leaves is either left free as a continues cavity or filled with non load bearing insulting or water proofing material.

(vi) Curtain Wall: It is non load bearing wall subjected to lateral loads only. It may be laterally supported by vertical or horizontal structural members.

Q.5 (e) Solution:

Let, Q_n = discharge from each nozzle

$$= \frac{1.5}{2} = 0.75\ l/\text{sec}$$

$$v = \frac{1.5 \times 100}{0.8 \times 2} = 937.5 \text{ cm/sec.} = 9.375 \text{ m/sec.}$$

Let ω = angular velocity of the arm. Jet at the shorter arm is designated by 2 and at the longer arm by 3.

∴ Relative velocities, $v_2 = v_3 = 9.375$ m/sec

Tangential velocity,

$$u_2 = \omega.r_2 = 0.3\ \omega$$

$$u_3 = \omega.r_3 = 0.4\ \omega$$

Absolute velocities,

$$V_2 = v_2 + \omega.r_2 = 9.375 + 0.3\ \omega$$

$$V_3 = v_3 + \omega.r_3 = 9.375 - 0.4\ \omega$$

[Note that V_2 and V_3 are in the same direction]

(i) Torque on the arm,

$$T_0 = \rho.Q_n\left[r_3V_3 - r_2V_2\right]$$

For zero frictional resistance, $T = 0$ and hence

$$r_3V_3 = r_2V_2$$

$$\Rightarrow \quad 0.4 \times [9.375 - 0.4\ \omega] = 0.3[9.375 + 0.3\ \omega]$$

$$\Rightarrow \quad \omega = \frac{0.9375}{0.25} = 3.75 \text{ rad/sec}$$

$$\therefore \quad N = \frac{3.75 \times 60}{2\pi} = 35.81 \text{ rpm.}$$

(ii) When the arm is stationary, ω = 0

$$\text{Torque, } T_0 = \rho.Q.\left[r_3V_3 - r_2V_2\right]$$

$\Rightarrow$ $$T_0 = 10^3 \times \frac{0.75}{1000} \times 9.375 \times [0.4 - 0.3]$$

$[V_3 = V_2 = 9.375 \text{ m/s}]$

$$= 0.703 \text{ N-m}$$

Q.6 (a) Solution:

- Gross cross-sectional area of beam,

$$A = 300 \times 500 = 15 \times 10^4 \text{ mm}^2$$

- Moment of inertia of the beam section,

$$I = \frac{B.D^3}{12} = \frac{300 \times 500^3}{12} = 31.25 \times 10^8 \text{ mm}^4$$

- Initial prestress, $p_o = 1500 \text{ N/mm}^2$
- Initial prestressing force, $P_o = 400 \times 1500 \times 10^{-3}$ kN $= 600$ kN

- Modular ratio, $m = \frac{E_s}{E_c} = \frac{210}{35} = 6$

- Stress in concrete at the level of steel is given by

$$f_c = \frac{P}{A} + \frac{P.e}{I/e}$$

$$= \frac{600 \times 10^3}{15 \times 10^4} + \frac{600 \times 10^3 \times 75}{\frac{31.25 \times 10^8}{75}}$$

$$= 4 + 1.08$$

$$= 5.08 \text{ MPa}$$

(i) Loss of stress in pre-tensioned beam

1. Loss of stress due to elastic shortening of concrete

$$= m.f_c$$
$$= 6 \times 5.08$$
$$= 30.48 \text{ MPa}$$

2. Loss of stress due to creep of concrete

$$= 50 \times 10^{-6} \times 5.08 \times 210 \times 10^3$$
$$= 53.34 \text{ MPa}$$

3. Loss of stress due to shrinkage of concrete

$$= 3 \times 10^{-4} \times 210 \times 10^3$$
$$= 63 \text{ MPa}$$

4. Loss of stress due to relaxation of steel

$$= 0.05 \times 1500$$
$$= 75 \text{ MPa}$$

5. Loss of stress due to anchorage = 0
6. Loss of stress due to friction = 0

$\therefore$ Total loss of stress $= 30.48 + 53.34 + 63 + 75$

$$= 221.82 \text{ MPa}$$

$\therefore$ Percentage loss of stress $= \dfrac{221.82}{1500} \times 100$

$$= 14.788\%$$

(ii) Loss of stress in post-tensioned beam

1. Loss of stress due to elastic shortening of concrete = 0
2. Loss of stress due to creep of concrete

$$= 20 \times 10^{-6} \times 5.08 \times 210 \times 10^3$$
$$= 21.336 \text{ MPa}$$

3 Loss of stress due to shrinkage of concrete

$$= 2 \times 10^{-4} \times 210 \times 10^3$$
$$= 42 \text{ MPa}$$

4. Loss of stress due to relaxation

$$= 0.05 \times 1500$$
$$= 75 \text{ MPa}$$

5. Loss of stress due to anchorage slip

$$= \frac{1.25}{10 \times 10^3} \times 210 \times 10^3$$
$$= 26.25 \text{ MPa}$$

6. Loss of stress due to friction

$$= 1500 \times 0.0013 \times 10 = 19.5 \text{ MPa}$$

Total loss of stress $= 21.336 + 42 + 75 + 26.25 + 19.5$

$$= 184.086 \text{ MPa}$$

$\therefore$ Percentage loss of stress $= \dfrac{184.086}{1500}\times 100 = 12.2724\% \simeq 12.3\%$ (say)

Ignore self weight of beam.

Q.6 (b) Solution:

The total drag is the sum of the wave resistance and the skin friction drag. The Froude's criterion of similarity is used for the gravity affected component of the drag, namely the wave resistance. The skin friction is estimated by another formula separately.

For the model

$$\text{Reynolds number, } Re = \frac{\rho VL}{\mu}$$

$$= \frac{1030\times 1.2\times 5}{1\times 10^{-3}}$$

$$= 6.18 \times 10^6$$

Now,
$$C_f = \frac{0.0735}{\left(6.18\times 10^6\right)^{1/5}} = 3.222\times 10^{-3}$$

$$\text{Skin friction resistance, } F_s = \frac{1}{2}.C_f.A.\rho.V^2$$

$$= \left(3.222\times 10^{-3}\right)\times 6\times 1030\times \frac{(1.2)^2}{2}$$

$$= 14.34 \text{ N}$$

Given Total model drag = 25.00 N

Hence, the model wave resistance = 25.00 – 14.34

$$(F_w)_m = 10.66 \text{ N}$$

The corresponding wave resistance in the prototype is calculated by Froude's law of similarity.

For the prototype, by Froude's law, the wave resistance $(F_w)_p$ is given by,

$$\frac{(F_w)_p}{(F_w)_m} = (F_w)_r = \rho_r.V_r^2.L_r^2$$

Now, Froude's number of model, $(F_r)_m = \frac{V_m}{\sqrt{gL_m}}$

Similarly, Froude's number of prototype, $(F_r)_p = \frac{V_p}{\sqrt{gL_p}}$

Using model laws,

$$(F_r)_m = (F_r)_p$$

$$\Rightarrow \quad \frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}}$$

$$\Rightarrow \quad \frac{V_m}{V_p} = V_r = \sqrt{\frac{L_m}{L_p}} = \sqrt{L_r}$$

Froude number of model, $(F_r)_m = \frac{V_m}{\sqrt{g.L_m}}$

$$\therefore \quad V_r = \sqrt{L_r}$$

$$(F_w)_r = \rho_r.L_r^3$$

In the present case, $\rho_r = 1.0$

$$\therefore \quad (F_w)_r = L_r^3$$

$$\therefore \quad (F_w)_p = \frac{(F_w)_m}{L_r^3} = \frac{10.66}{(1/25)^3} = 166.6\times 10^3\,N$$

$$= 166.6 \text{ kN}$$

Skin friction for the prototype:

Prototype Reynolds number, $(R_e)_p = \frac{\rho_P.V_P.L_P}{\mu_P}$

Since $V_p = \frac{V_m}{\sqrt{L_r}}$ and $L_p = \frac{L_m}{L_r}$

Also, $\rho_P = \rho_m$ and $\mu_P = \mu_m$

$$\therefore \quad (R_e)_p = (\text{R}_\text{e})_m \times \frac{1}{(L_r)^{3/2}}$$

$$= \frac{1}{\left(\frac{1}{25}\right)^{3/2}} \times 6.18 \times 10^6 = 7.73 \times 10^8$$

$$C_f = \frac{0.0735}{\left(7.73 \times 10^8\right)^{1/5}} = 1.2265 \times 10^{-3}$$

Skin friction resistance, $(F_s)_p = \frac{1}{2} C_f A.\rho.V^2$

$$\Rightarrow \quad (F_s)_p = \frac{1}{2}\left[1.2265 \times 10^{-3}\right] \times \left(6 \times (25)^2\right) \times 1030 \times \left(1.2 \times \sqrt{25}\right)^2$$

$$= 85272.4 \text{ N} = 85.3 \text{ kN}$$

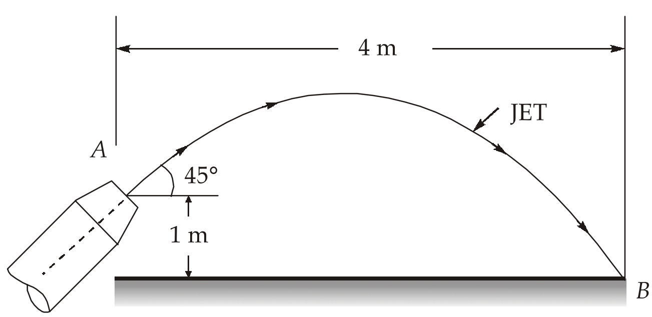
Total prototype resistance $= (F_W)_P + (F_S)_P$

$$= 166.6 + 85.3$$

$$= 251.9 \text{ kN} \simeq 252 \text{ kN} \quad \text{(say)}$$

Q.6 (c) Solution:

(i)



Given:

Distance of nozzle above ground = 1 m

Angle of inclination, $\theta = 45°$

Dia. of nozzle, d = 50 mm = 0.05 m

$\therefore$ Area of jet, $A = \frac{\pi}{4}(0.05)^2 = 0.001963 \text{ m}^2$

Given horizontal distance, x = 4 m

The co-ordinates of the point B, which is on the centre-line of the jet of water and is situated on the ground with respect to A (origin) are

$$x = 4 \text{ m and } y = -1.0 \text{ m}$$

(From A, point B is vertically down by 1 m)

The equation of the jet is given as, $y = x\tan\theta - \dfrac{gx^2}{2U^2}\sec^2\theta$

Substituting the known values we have,

$$-1.0 = 4\tan 45° - \frac{9.81\times 4^2}{2U^2}\times \sec^2 45°$$

$$\Rightarrow \quad -1.0 = 4 - \frac{78.48}{U^2}\times(\sqrt{2})^2$$

$$\Rightarrow \quad -1.0 = 4 - \frac{78.48\times 2}{U^2} \Rightarrow \frac{78.48\times 2}{U^2} = 4.0 + 1.0 = 5.0$$

$$\Rightarrow \quad U^2 = \frac{78.48\times 2.0}{5.0} = 31.39$$

$$\Rightarrow \quad U = \sqrt{31.392} = 5.60 \text{ m/s}$$

Now the rate of flow of fluid = Area × Velocity of jet

$= A \times U = 0.001963 \times 5.6 \text{ m}^3\text{/sec}$

$= 0.01099 \simeq 0.011 \text{ m}^3\text{/s} \simeq 11\ l\text{/sec}$

(ii)

Methods of post tensioning:

1. Mechanical methods
   - Freyssinet system
   - Magnel Balton System
   - Lee-Mccall system
   - Giffard-Udall system
2. Electrical Prestressing
3. Chemical Prestressing

- **Giffard-Udall System:** This system was originated in Britain and is widely used in India. This is a single wire system. Each wire is stressed independently using a double acting jack. Any number of wires can be grouped together to form a cable in this system. There are two types of anchorage device in the system viz.:

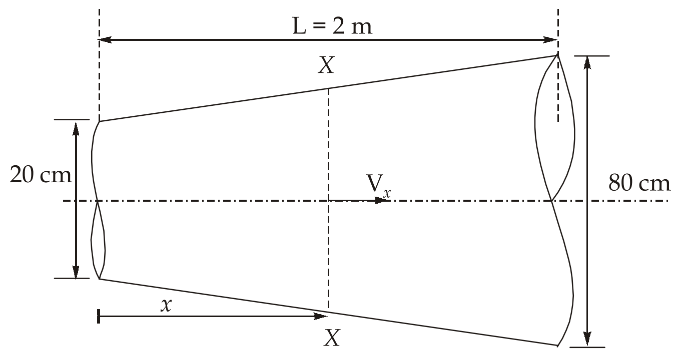
  (a) Tube anchorages

  (b) Plate anchorages

Tube anchorages consist of a bearing plate, achor wedges and anchor grips. Anchor plate may be square or circular and have 8 or 12 tapered holes to accommodate the individual prestressing wires. These holes are plugged by means of anchor wedges.

Q.7 (a) Solution:

(i)



Diameter at section *X-X*,

$$D_x = \left[\frac{D_2 - D_1}{L}\right].x + D_1$$

$$= \left[\frac{0.8 - 0.2}{2}\right] \times x + 0.2$$

$$= 0.3\,x + 0.2$$

So, at $x = 1.5$ m, $D = 0.65$ m

So, area at $x = 0.65$ m, $A_x = \frac{\pi}{4} \times (0.65)^2 = 0.3318\,\text{m}^2$

Velocity through sectionX-X, $V = \frac{Q}{A_x} = \frac{0.2}{0.3318} = 0.6028\ \text{m/sec}$

(i) Local acceleration at section, $x = 1.5$ m

Rate of increase of velocity, $\frac{\delta V}{\delta t} = \frac{\delta(Q/A_x)}{\delta t} = \frac{1}{A_x}\frac{\delta Q}{\delta t}$

$$= \frac{1}{0.3318} \times 0.05 = 0.1507\ \text{m/sec}^2$$

(ii) Convective acceleration,

$$a_s = V_x . \frac{\delta V_x}{\delta x}$$

where,
$$V_x = \frac{Q}{A_x} = \frac{Q}{\frac{\pi}{4}(0.3x + 0.2)^2}$$

$$= \frac{0.2}{\frac{\pi}{4} \times (0.3x + 0.2)^2}$$

$$= \frac{0.2546}{(0.3x + 0.2)^2}$$

$$\frac{\delta V_x}{\delta x} = (0.2546) \times (-2) \times (0.3x + 0.2)^{-3} \times 0.3$$

$$= \frac{-0.15276}{(0.3x + 0.2)^3}$$

Hence convection acceleration,

$$a_s = V_x . \frac{\delta V_x}{\delta x}$$

$$= \frac{0.2546}{(0.3x + 0.2)^2} \times \left[\frac{-0.15276}{(0.3x + 0.2)^3}\right] = \frac{-0.03889}{(0.3x + 0.2)^5}$$

At x = 1.5 m, convective acceleration

$$a_s = \frac{-0.03889}{(0.3 \times 1.5 + 0.2)^5} = -0.3352 \text{ m/s}^2$$

(iii) Total acceleration = local acceleration + Convection acceleration

$$= \frac{\delta V_x}{\delta t} + V_x . \frac{\delta V_x}{\delta x}$$

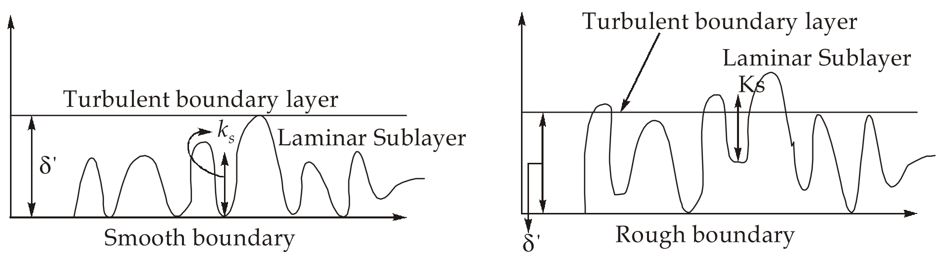
At x = 1.5 m, total acceleration

$$= 0.1507 - 0.3352$$

$$= -0.1845 \text{ m/sec}^2$$

(ii)

- In case of turbulent flow in pipes, very close to the wall, effect of viscosity is maximum. Hence, it is said that a laminar sublayer exists near the boundary.
- The thickness of laminar sublayer is directly proportional to the kinematic viscosity and inversely proportional to flow velocity. Thus, thickness of laminar sublayer decreases with increase in Reynolds number.



- If thickness of laminar sublayer is large and eddies are not able to penetrate upto the boundary, then boundary acts as hydro dynamically smooth boundary where as if thickness of laminar sublayer is small and eddies penetrate upto the boundary it is said to be hydrodynamically rough.
- In rough boundaries, the irregularities will project through laminar sublayer and laminar sublayer gets destroyed. The contact of eddies with irregularity will result in large energy losses.
- A pipe boundary will behave as hydrodynamically smooth or rough depending upon the relative magnitude of average height of surface protrusions (k_s) and the thickness of the laminar sublayer (δ'). Hence, (k_s/δ') is the requisite parameter which will determine whether a pipe boundary is hydrodynamically smooth or rough.

Since, $$\delta' = \frac{11.6\nu}{V^*}$$ where V^* is shear velocity.

or $$\frac{k_s}{\delta'} = \frac{V^* k_s}{\nu} \times \frac{1}{11.6}$$

- As per Nikuradse's experiment,

If $\frac{k_s}{\delta'} < 0.25$ or $\frac{V^* k_s}{v} < 3 \rightarrow$ Hydro-dynamically smooth boundary.

$\frac{k_s}{\delta'} >$ or $\frac{V.^* k_s}{v} > 70 \rightarrow$ Hydrodynamically rough boundary.

$0.25 < \frac{k_s}{\delta'} < 6$ or $3 < \frac{V.^* k_s}{\gamma} < 70 \rightarrow$ boundary in transition.

Q.7 (b) Solution:

$$l_{xo} = 3 \text{ m}, l_{yo} = 8 \text{ m}$$

Approximate depth of slab based on deflection criterion,

$$d = \frac{l_{x0}}{20} = \frac{3000}{20} = 150 \text{ mm}$$

Assuming effective cover as 30 mm

$\therefore$ Total depth of slab, $D = d + 30$

$= 150 + 30$

$= 180$ mm

$$\text{Effective short span, } l_x = \text{minimum}\begin{Bmatrix} l_{x0} + d \\ l_{x0} + w \end{Bmatrix}$$

$$= \text{minimum}\begin{Bmatrix} 3 + 0.15 \\ 3 + 0.25 \end{Bmatrix}$$

$= 3.15$ m

Similarly, effective long span, $l_y = 8.15$ m

Now, $$\frac{l_y}{l_x} = \frac{8.15}{3.15} = 2.59 > 2 \Rightarrow \text{One-way slab}$$

Dead load $= \text{B} \times \text{D} \times 1 \times \gamma_{RCC}$

$= 1 \times 0.18 \times 1 \times 25 = 4.5 \text{ kN/m}^2$

Live load $= 3 + 1 = 4 \text{ kN/m}^2$

Total factored load, $w_u = 1.5(4.5 + 4)$

$= 12.75 \text{ kN/m}^2$

$$\text{Factored bending moment, } M_u = \frac{w_u . l_x^2}{8}$$

$$= \frac{12.75 \times (3.15)^2}{8}$$

$= 15.81$ kN–m

$$\text{Depth required, } d = \sqrt{\frac{M_u}{0.138. f_{ck} .B}}$$

$$= \sqrt{\frac{15.81\times10^6}{0.138\times20\times1000}}$$

$$= 75.69 \text{ mm} < 150 \text{ mm}$$

Area of steel required, $A_{st,x} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}Bd^2}}\right]B.d$

$$= \frac{0.5\times20}{415}\left[1-\sqrt{1-\frac{4.6\times15.81\times10^6}{20\times1000\times150^2}}\right]\times1000\times150$$

$$= 304.93 \text{ mm}^2$$

Using 10 mm ϕ bars, spacing $= \frac{1000}{304.93}\times\frac{\pi}{4}\times10^2$

$= 257.57 \text{ mm} \simeq 250 \text{ mm c/c}$ (say)

$\therefore$ Provide 10 mm ϕ bars @ 250 mm c/c spacing along x-direction.

$\therefore$ $A_{\text{st prov.}} = \frac{1000}{250}\times\frac{\pi}{4}\times10^2 = 314.16 \text{ mm}^2$

Distribution bars:

$$A_{st\,y} = \frac{0.12}{100}\times B\times D$$

$$= \frac{0.12}{100}\times1000\times180$$

$$= 216 \text{ mm}^2$$

Using 8 mm ϕ bars, spacing $= \frac{1000}{216}\times\frac{\pi}{4}\times8^2$

$= 232.7 \text{ mm} \simeq 230 \text{ mm c/c}$ (say)

Check for shear:

Factored shear force, $V_u = \frac{w_u.l_{xo}}{2} = \frac{12.75\times3}{2} = 19.125 \text{ kN}$

Nominal shear stress, $\tau_v = \frac{V_u}{B.d}$

$$= \frac{19.125 \times 10^3}{1000 \times 150}$$

$$= 0.1275 \text{ MPa}$$

$$\therefore \quad \tau_v (= 0.1275 \text{ MPa}) < \tau_c (= 0.28 \text{ MPa})$$

$\therefore$ Slab is safe in shear.

Check for development length:

$$L_d = \frac{0.87 f_y \phi}{4.\tau_{bd}}$$

$$= \frac{0.87 \times 415 \times 10}{4 \times (1.6 \times 1.2)} = 470.12 \text{ mm} \simeq 470 \text{ mm} \quad \text{(say)}$$

M_1 (with 50% curtailment near support)

$$= 0.87 f_y . \left(\frac{A_{stx}}{2}\right) j.d$$

$$= 0.87 f_y . \left(\frac{A_{stx}}{2}\right) (1 - 0.42 K_o).d$$

$$= \left[0.87 \times 415 \times \left(\frac{314.16}{2}\right)(1 - 0.42 \times 0.48) 150\right] \times 10^{-6} \text{kN-m}$$

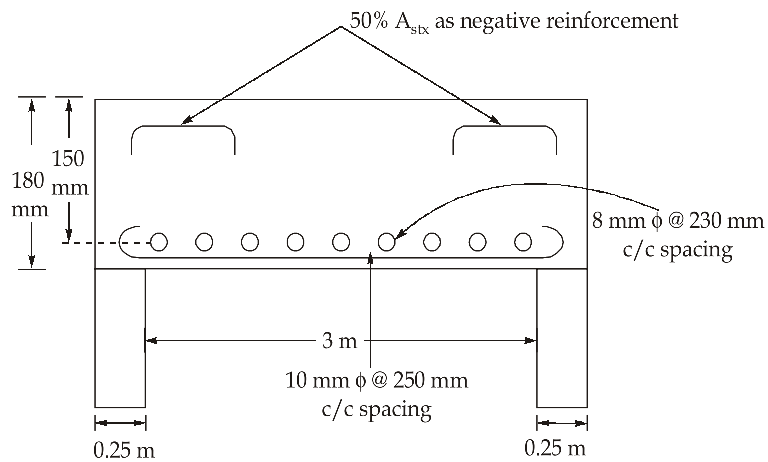
$$= 6.792 \text{ kN-m} \simeq 6.8 \text{ kN-m} \quad \text{(say)}$$

$$L_0 = \frac{250}{2} - 30 = 95 \text{ mm}$$

Now, $$1.3 \frac{M_1}{V} + L_0 = \frac{1.3 \times 6.8 \times 10^6}{19.125 \times 10^3} + 95$$

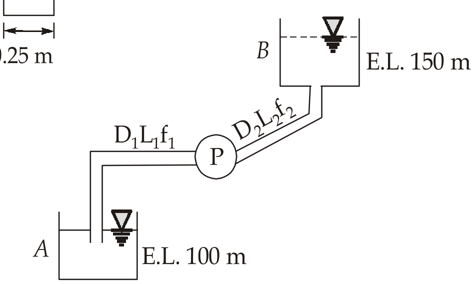
$$= 557.22 \text{ mm} \simeq 550 \text{ mm (say)} > L_d \text{ (= 470 m)}$$

Reinforcement detailing:



Q.7 (c) Solution:

(i)



For suction pipe:

$$\text{Head loss, } h_{L1} = \frac{f_1 L_1 V_1^2}{2gD_1} = \frac{0.025 \times 50 \times V_1^2}{2g \times 0.3}$$

$$= \left(4.167 \frac{V_1^2}{2g}\right) \text{m}$$

For delivery pipe:

$$\text{Head loss, } h_{L2} = \frac{f_2 L_2 V_2^2}{2gD_2} = \frac{0.020 \times 900 \times V_2^2}{2g \times 0.2} = \left(90 \frac{V_2^2}{2g}\right) \text{m}$$

$$\text{Total head loss, } H_L = 4.167 \frac{V_1^2}{2g} + 90 \frac{V_2^2}{2g}$$

By continuity equation, $A_1 V_1 = A_2 V_2$

$$\Rightarrow \quad \frac{\pi}{4} \times (0.3)^2 . V_1 = \frac{\pi}{4} \times (0.2)^2 . V_2$$

$$V_1 = V_2 \left[\frac{0.2}{0.3}\right]^2 = 0.444 V_2$$

$$\therefore \quad \frac{V_1^2}{2g} = 0.1971 \times \frac{V_2^2}{2g}$$

$\therefore$ $$H_L = [(4.167 \times 0.1971) + 90] \times \frac{V_2^2}{2g}$$

$$= \left(90.82 \frac{V_2^2}{2g}\right) \text{m}$$

Static head = 150 – 100 = 50 m

Let, H_p = Head developed by pump

$\Rightarrow$ H_p = Static head + Friction head

$\Rightarrow$ $$H_p = 50 + 90.82 \frac{V_2^2}{2g}$$

$\Rightarrow$ $$H_p = 50 + 90.82 \frac{Q^2}{\left[\frac{\pi}{4} \times (0.2)^2\right]^2} \times \frac{1}{2 \times 9.81}$$

$$H_p = 50 + 4690.107\, Q^2$$

But the head-discharge relationship of the pump is:

$$H_p = 80 - 7000\, Q^2$$

On comparing, $80 - 7000\, Q^2 = 50 + 4690.107\, Q^2$

$\Rightarrow$ $$Q^2 = \frac{30}{11690.107} = 2.566 \times 10^{-3}$$

$\Rightarrow$ $$Q = 0.0507 \text{ m}^3/\text{sec.}$$

$$= 50.7\ l/\text{sec}$$

$\therefore$ $$H_p = 50 + 4690.107 \times (0.0507)^2$$

$$= 62.06 \text{ m}$$

Power delivered by the pump, $P = \gamma.Q.H_p$

$$= 9.81 \times 0.0507 \times 62.046$$

$$= 30.87 \text{ kW}$$

(ii)

For a contracted weir, by neglecting the velocity of approach,

$$Q = \frac{2}{3} C_d \sqrt{2.g}.L_e.H_1^{3/2}$$

where, L_e = Effective length of weir

Considering the accuracies desired i.e. 0.5% throughout the entire range,

$$0.005 = 1.5 \times \frac{0.0005}{H_m}$$

where, H_m = minimum desired head

$$\therefore \quad H_m = \frac{0.0005 \times 1.5}{0.005} \text{m} = 0.15 \text{ m}$$

$$= 15 \text{ cm}$$

Thus H_1 must be greater than or equal to 15 cm. By the discharge equation for the smallest discharge at the maximum L_e,

$$Q = 0.100 = \frac{2}{3} \times 0.62 \times \sqrt{2 \times 9.81} \times L_e \times (0.15)^{3/2}$$

$$\Rightarrow \quad L_e = \frac{0.1}{0.106} = 0.94 \text{ m}$$

L_e should be less than or equal to 0.94 m

Since, $L_e = L - 0.2H_1$

$\Rightarrow \quad L = L_e + 0.2 \times H_1$

$\Rightarrow \quad L = 0.94 + 0.2 \times 0.15 = 0.97$ m

$\therefore$ Maximum length of weir, $L = 0.97$ m

Q.8 (a) Solution:

(i)

$$Q = f_n[H, P, g, L, \rho, \mu, \sigma]$$

In this case, total number of variables, $n = 8$

Number of primary variables, $m = 3$

Hence, number of dimensionless terms $= 8 - 3 = 5$

Selecting ρ, H and g as the repeating variables

Now,

π_1 term : $\pi_1 = [H^a.g^b.\rho^c]Q$

$\therefore \quad M^0L^0T^0 = [L^a][LT^{-2}]^b[ML^{-3}]^c[L^3T^{-1}]$

Equating exponents of M, L, T on both sides

$$c = 0$$

$$a + b - 3c + 3 = 0$$

$$-1 - 2b = 0$$

$\therefore$ $c = 0, b = -1/2$ and $a = -5/2$

Hence, $$\pi_1 = \frac{Q}{H^{5/2}.g^{1/2}}$$

π_2 term: $$\pi_2 = [H^a.g^b.\rho^c].P$$

$$M^0L^0T^0 = [L]^a\ [LT^{-2}]^b\ [ML^{-3}]^c\ [L]$$

Equating exponents of M, L, T on both sides,

$$c = 0$$

$$a + b - 3c + 1 = 0$$

$$-2b = 0$$

$\Rightarrow$ $c = 0, b = 0, a = -1$

$\therefore$ $$\pi_2 = \frac{P}{H}$$

π_3 term: $$\pi_3 = [H^a.g^b.\rho^c].L$$

$$M^0L^0T^0 = [[L]^a,[LT^{-2}]^b,[ML^{-3}]^c].[L]$$

By Inspection,

$b = 0, c = 0$ and $a = -1$

$\therefore$ $$\pi_3 = L/H$$

π_4 term : $$\pi_4 = [H^a.g^b.\rho^c]\mu$$

$$M^0L^0T^0 = [L]^a\ [LT^{-2}]^b\ [ML^{-3}]^c\ [ML^{-1}T^{-1}]$$

By Inspection, We have

$$1 + c = 0$$

$$-1 + a + b - 3c = 0$$

$$-1 - 2b = 0$$

$\therefore$ $c = -1, b = -1/2, a = -3/2$

$\therefore$ $$\pi_4 = \frac{\mu}{H^{3/2}.g^{1/2}.\rho}$$

π_5 term : $$\pi_5 = [H^a.g^b.\rho^c]\sigma$$

$$M^0L^0T^0 = [L]^a\ [LT^{-2}]^b\ [ML^{-3}]^c\ [MT^{-2}]$$

$$1 + c = 0$$
$$a + b - 3c = 0$$
$$-2 - 2b = 0$$

$\therefore$ $c = -1, b = -1, a = -2$

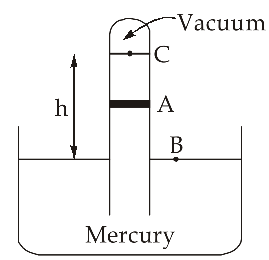
Thus, $$\pi_5 = \frac{\sigma}{H^2 . g . \rho}$$

$\therefore$ $$\frac{Q}{gH^{5/2}} = f_n\left[\frac{P}{H}, \frac{L}{H}, \frac{\mu}{H^{3/2}.g^{1/2}.\rho}, \frac{\sigma}{\rho.g.H^2}\right]$$

(ii)

1. Barometer:

- Atmospheric pressure is measured by a device called barometer. Thus, the atmospheric pressure is often referred to as the barometric pressure.
- The barometer consists of an inverted mercury-filled tube into a mercury container that is open to the atmosphere.



- The pressure at point B is equal to the atmospheric pressure and the pressure at point C can be taken to be zero since there is only mercury vapour point 'C' and the pressure is very low relative to P_{atm} and can be neglected.
- Writing a force balance equation in the vertical direction gives,

  $P_{atm} = \rho gh$
- In barometer, mercury is used because of its two important properties:
  (i) Mercury is a high density fluid. Therefore, height of fluid column required will be less as compared to water.
  (ii) Mercury has very low vapour pressure.
- Diameter of the tube should be sufficiently large to neglect the capillary effect.

2. Single column manometer:

- It is a modified form of U-tube manometer in which a shallow reservoir having a large cross-sectional area as compared to the area of the tube is introduced into one limb of the manometer.
- For any change in pressure, the change in liquid level in the reservoir will be so small that it may be neglected and the pressure is indicated approximately by the height of the liquid in the other limb.

- Only one reading in the narrow limb of the manometer needs to be taken for pressure measurement.
- Narrow limb may be straight or inclined.

Q.8 (b) Solution:

- Axial load (P) = 1000 kN

Assuming 10% of column load as weight of footing and soil backfill.

∴ Weight of footing and soil backfill

$$= 15\% \text{ of } 1000$$
$$= 150 \text{ kN}$$

∴ Area of footing required $= \dfrac{(1000+150)\,\text{kN}}{110\ \text{kN/m}^2} = 10.45\ \text{m}^2$

- Provide square footing of side

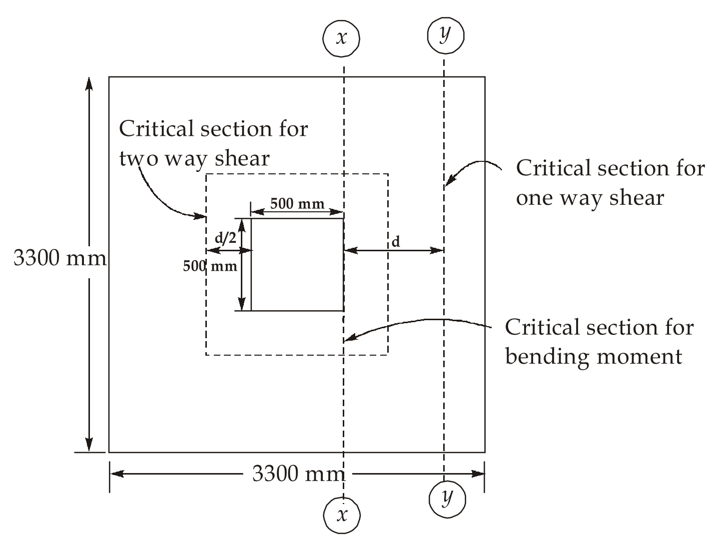
$$= \sqrt{10.45} = 3.23\ \text{m} \simeq 3.3\ \text{m (say)}$$

Provide square footing of size 3.3 m × 3.3 m

Calculation of bending moment:

- Factored soil pressure acting upwards,

$$q_0 = \frac{1.5\times 1000}{3.3\times 3.3} = 137.75\ \text{kN/m}^2$$



Section x -x is at the face of column, which is critical section for bending moment.

$\therefore$
$$M_u = \frac{(q_0.B)\left[\frac{L-a}{2}\right]^2}{2}$$

$$= \frac{137.75 \times 3.3 \times \left[\frac{3.3-0.5}{2}\right]^2}{2}$$

$$= 445.48 \text{ kN-m}$$

- **Depth of footing required:**

$$M_u = M_{u\ \lim}$$

$\Rightarrow$ $$M_u = 0.138\, f_{ck}.b.d^2 \qquad \text{(For Fe415)}$$

$\Rightarrow$ $$445.48 \times 10^6 = 0.138 \times 25 \times 3300 \times d^2$$

$\Rightarrow$ $$d = 197.81 \text{ mm}$$

- Adopt overall depth of footing as 500 mm so that effective depth of footing is,

$$d = 500 - 75 - \frac{20}{2} = 415 \text{ mm}$$

[Assuming 20 mm diameter bars and 75 mm clear cover.]

- Reinforcement required for footing, A_{st} is given by

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck}.Bd^2}}\right]B.d$$

$$= \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 445.48 \times 10^6}{25 \times 3300 \times (415)^2}}\right]3300 \times 415$$

$$= 3090.4 \text{ mm}^2$$

$$A_{st\ \min} = \frac{0.12}{100} \times 3300 \times 415 = 1643.4 \text{ mm}^2$$

As $A_{st} > A_{st\ \min}$. Hence OK.

Using 20 mm diameter bars,

Number of bars required for 1 m length of footng $= \dfrac{3090.4}{\frac{\pi}{4} \times (20)^2} = 9.84$

So, number of bars required for 3.3 m width = 3.3 × 9.84 = 32.47 ≃ 33 bars (say)

- Spacing of bars $= \dfrac{3300 - 2\times 75 - 20\times 33}{(33-1)}$

$= 77.81 \text{ mm} \simeq 75 \text{ mm c/s}$

Provide 20 mm ϕ @ 75 mm c/c spacing

Check for one-way shear:

- Critical section for one way shear is at a distance '*d*' from the face of column i.e. at section y-y.

$$V_{u1} = 137.75\times 3.3\times\left[\left(\frac{3.3-0.5}{2}\right)-0.415\right]\text{kN}$$

$$= 447.76 \text{ kN}$$

Nominal shear stress, $\tau_v = \dfrac{V_{u1}}{B.d} = \dfrac{447.76\times 10^3}{3300\times 415}$

$$= 0.33 \text{ N/mm}^2$$

Shear strength of M25 concrete, is computed as below:

$$p_t = \frac{33\times\frac{\pi}{4}\times 20^2}{3300\times 415}\times 100 = 0.75\%$$

From table, for $p_t = 0.75\%,\ \tau_c = 0.57 \text{ N/mm}^2$

∴ $\tau_c > \tau_V (= 0.33 \text{ N/mm}^2)$ Hence OK

So, the depth provided is safe in one-way shear.

Check for two-way shear:

- Critical section for two way shear is at a distance of $d/2$ from the column face.

Shear force at critical section is given by

$$V_{u2} = q_0[B^2 - (a+d)^2]$$

$$= 137.75[(3.3)^2 - (0.5 + 0.415)^2]$$

$$= 1384.77 \text{ kN}$$

Two-way shear stress, $\tau_V = \dfrac{1384.77\times 10^3}{4(500+415)\times 415}\text{N/mm}^2$

$$= 0.91 \text{ N/mm}^2$$

Design shear strength of M25 concrete in two way shear is computed as below:

$$\tau_c = k_s.\tau_v$$

where, $k_s = \frac{1}{2}+\beta_c$

where $\beta_C = \frac{\text{Shorter side of column}}{\text{Longer side of column}}$

$= \frac{500}{500} = 1 \text{ mm}$

So, $k_s = 0.5 + 1 = 1.5 \not> 1$

$\therefore$ $k_s = 1$

Now, $\tau_c = k_s.\tau_v'$

$= 1\times 0.25\sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25 \text{ N/mm}^2$

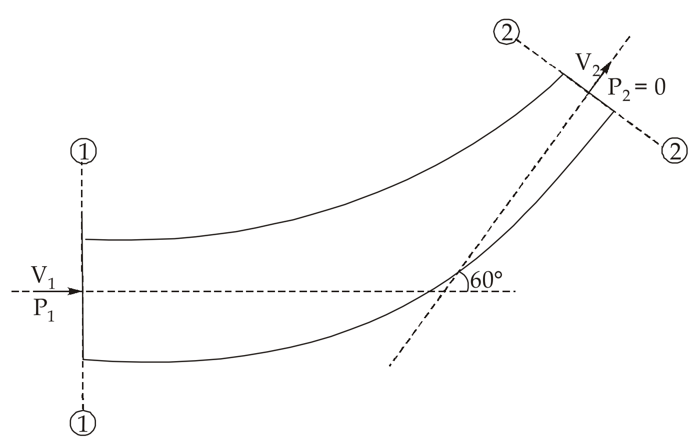
$\therefore$ $\tau_c\left(=1.25 \text{ N/mm}^2\right) > \tau_V\left(=0.91 \text{ N/mm}^2\right)$

Thus, depth of footing is safe in two-way shear.

Q.8 (c) Solution:

(i)

Consider the control volume as shown below.



At section 2 : $P_2 = 0$ [atmospheric pressure]

Velocity, $V_2 = \frac{0.3}{\frac{\pi}{4}\times(0.15)^2} = 16.98 \text{ m/sec.}$

At section 1: $V_1 = V_2\left[\frac{D_2}{D_1}\right]^2 = 16.98\times\left[\frac{15}{20}\right]^2 = 9.55 \text{ m/sec.}$

By applying Bernoulli's equation to sections (1) and (2),

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g} = \frac{P_2}{\gamma}+\frac{V_2^2}{2g} \qquad [\because Z_1 = Z_2]$$

$$\Rightarrow \quad \frac{P_1}{\gamma}+\frac{(9.55)^2}{2\times 9.81} = 0+\frac{(16.98)^2}{2\times 9.81}$$

$$\Rightarrow \quad \frac{P_1}{\gamma} = 14.695 - 4.648 = 10.047 \text{ m} \simeq 10.05 \text{ m}$$

$$\Rightarrow \quad P_1 = 10.05 \times 9810 = 98590.5 \text{ Pa}$$

Let R_x and R_y be the reactions of the pipe on the fluid in the control volume in $(-x)$ and (y)directions respectively.

By applying momentum equation in x-direction,

$$P_1A_1 - R_x = \rho.Q.[V_2\cos 60° - V_1]$$

$$\Rightarrow \quad 98590.5\times\frac{\pi}{4}\times(0.2)^2 - R_x = 10^3 \times 0.3\,[16.98 \times \cos 60° - 9.55]$$

$$\Rightarrow \quad R_x = 3097.31 = 3415.31 \text{ N}$$

By momentum equation in y-direction,

$$0 + R_y = \rho Q(V_2 \sin 60° - 0)$$

$$\Rightarrow \quad R_y = 10^3 \times 0.3\,(16.98 \times \sin 60° - 0)$$

$$\Rightarrow \quad R_y = 4411.53 \text{ N}$$

$$\text{Resultant reaction, } R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{[3415.31]^2 + [4411.53]^2}$$

$$= 5579.06 \text{ N}$$

Resultant reaction R is inclined at an angle θ such that $\tan\theta = \dfrac{R_y}{R_x}$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{4411.53}{3415.31}\right] = 52.25°$$

The force F on the pipe is equal and opposite to R and hence F = 5579.06 N inclined at $(360° - \theta) = [360° - 52.25°] = 307.75°$ to positive x-axis.

(ii)

Using Karman Prandtl velocity distribution equation,

$$\frac{u}{V^*} = 5.75\log.\frac{y}{y'}$$

where y' is distance from pipe wall at which $u = 0$ and V* is shear velocity.

At $y_1 = 5.0$ m, $u_1 = 5$ m/sec.

$$\frac{5}{V^*} = 5.75\log\frac{5}{y'} \quad \text{...(i)}$$

Similarly,

At $y_2 = 10$ m, $u_2 = 6$ m/sec.

$$\therefore \quad \frac{6}{V^*} = 5.75\log\frac{10}{y'} \quad \text{...(ii)}$$

Subtracting (i) from (ii), we get

$$\Rightarrow \quad \frac{6-5}{V^*} = 5.75\log\frac{10}{y'} - 5.75\log\frac{5}{y}$$

$$\Rightarrow \quad \frac{1}{V^*} = 5.75\log\frac{10}{5}$$

$$\Rightarrow \quad V^* = 0.58 \text{ m/s}$$

At $y_3 = 30$ m, the velocity u_3 is given by,

$$\Rightarrow \quad \frac{u_3 - u_1}{V^*} = 5.75\log\frac{y_3}{y_1}$$

$$\Rightarrow \quad \frac{u_3 - 5}{0.58} = 5.75\log\left(\frac{30}{5}\right)$$

$$\Rightarrow \quad u_3 = 2.59 + 5 = 7.595 \text{ m/sec.} \simeq 7.6 \text{ m/sec.}$$

■■■■