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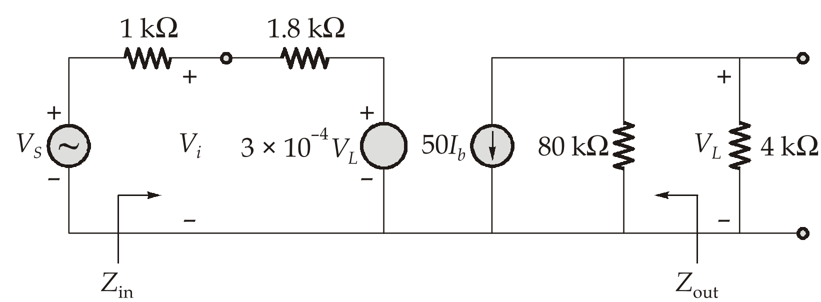
Detailed Solutions

# ESE-2024 Mains Test Series

# Electrical Engineering Test No : 5

## Section A : Basic Electronics Engineering + Analog Electronics + Electrical Materials

**Q.1 (a) Solution:**




Given,

$$R_S = 1\text{ k}\Omega,$$
$$h_{ie} = 1.8\text{ k}\Omega,$$
$$R_L = 4\text{ k}\Omega,$$
$$h_{re} = 3 \times 10^{-4},$$
$$h_{fe} = 50$$

and

$$\frac{1}{h_{0e}} = 80\text{ k}\Omega$$

**(i)** To obtain the current gain,

$$A_i = \frac{-h_{fe}}{1 + h_{0e}R_L} = \frac{-50}{1 + \frac{4}{80}} = -47.62$$

**(ii)** Input impedance ($Z_i$) :

$$Z_i = h_{ie} + h_{re} A_i R_L$$
$$= 1.8\text{ K} + (3 \times 10^{-4} \times (-47.62) \times 4 \times 10^3)$$
$$\therefore \quad Z_{in} = 1742.86\ \Omega$$

**(iii)** To calculate the voltage gain :

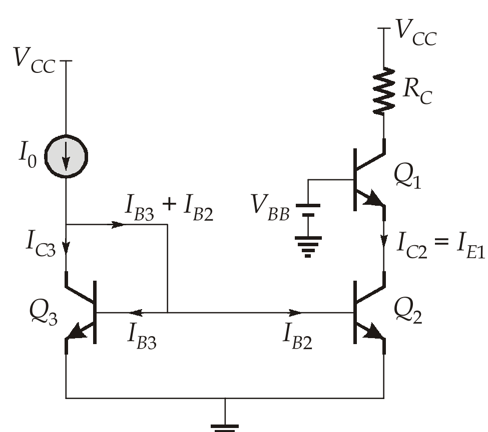
$$A_V = \frac{A_i R_L}{Z_{in}} = \frac{(-47.62)\times 4\text{k}}{1742.86\,\Omega} = -109.29$$

$$\frac{V_L}{V_S} = A_{vs} = \frac{A_V Z_{in}}{Z_{in} + R_S} = \frac{-109.29 \times 1742.86}{1742.86 + 1000} = -69.45$$

**(iv)** To calculate the output resistance ($R_0$) :

$$Y_0 = h_{0e} - \frac{h_{fe} h_{re}}{h_{ie} + R_s}$$
$$= \frac{1}{80\text{k}} - \frac{50 \times 3 \times 10^{-4}}{2.8\text{ k}}$$
$$= 0.0125 \times 10^{-3} - 53.57 \times 10^{-7}$$
$$= 7.143 \times 10^{-6}$$
$$R_0 = \frac{1}{Y_0} = 0.14 \times 10^6 = 0.14\text{ M}\Omega$$

**Q.1 (b) Solution:**




Applying KCL at collector of $Q_3$, we get

$$I_{C3} + I_{B3} + I_{B2} = I_0$$
$$I_0 = I_{C3} + 2I_{B3} (\because \text{All transistors are identical, } I_{B3} = I_{B2})$$

$$I_0 = I_{C3}\left[1+\frac{2}{\beta}\right] = I_{C3}\left[\frac{\beta+2}{\beta}\right]$$

Also, $$I_{C3} = I_{C2} = I_{E1} \qquad (\because V_{BE3} = V_{BE2})$$

$\therefore$ $$I_0 = I_{E1}\left[\frac{\beta+2}{\beta}\right]$$

But, $$I_{E1} = \frac{(1+\beta)I_{C1}}{\beta}$$

$$I_0 = \frac{(1+\beta)(\beta+2)I_{C1}}{\beta^2}$$

$$I_{C1} = \frac{I_0\beta^2}{(1+\beta)(2+\beta)} \qquad \text{...(i)}$$

But, we know,

$$\text{Stability factor } S'' = \frac{\partial I_{C1}}{\partial \beta} \qquad \text{(For collector current of } Q_1\text{)}$$

$\therefore$ Differentiating equation (i) w.r.t. $\beta$

$$S'' = \frac{\partial I_{C1}}{\partial \beta} = I_0\left[\frac{(1+\beta)(2+\beta)2\beta - \beta^2[2\beta+3]}{(1+\beta)^2(2+\beta)^2}\right]$$

$$= I_0\left[\frac{2\beta^2+6\beta+4-2\beta^2-3\beta}{(1+\beta)^2(2+\beta)^2}\right]\beta$$

$$S'' = I_0\left[\frac{(3\beta+4)\beta}{(1+\beta)^2(2+\beta)^2}\right]$$

For $\beta = 100$ and $I_0 = 1$ mA

$$S'' = 1\times 10^{-3}\left[\frac{100[300+4]}{(101)^2\times(102)^2}\right]$$

$$S'' = 2.865 \times 10^{-7}$$

**Q.1 (c) Solution:**

Resistance of copper conductor at 0° C,

$$R_0 = 17.5\ \Omega$$

Temperature coefficient of copper at 0° C,

$$\alpha_0 = 0.00428 \text{ per °C}$$

Percentage conductivity at 16° C

Now, $R_t = R_0(1 + \alpha t)$

$= 17.5(1 + 0.00428 \times 16)$

$= 17.5 \times 1.06848$

$= 18.6984$

Since conductance is reciprocal of resistance

∴ Percentage conductivity is at 16° C

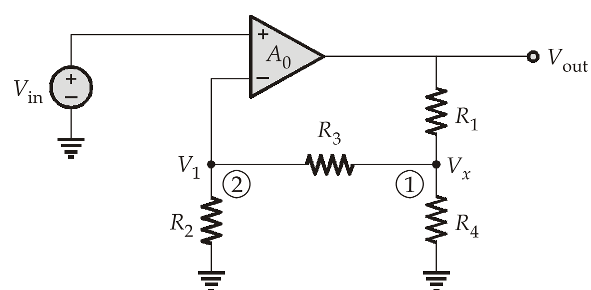
$$= \frac{\text{Resistance at } 0° \text{ C}}{\text{Resistance at } 16° \text{ C}} \times 100$$

$$= \frac{17.5}{18.6984} \times 100 = 93.59\%$$

Hence, percentage conductivity at 16° C = 93.59%

**Q.1 (d) Solution:**

Given non-inverting amplifier




By applying virtual short circuit concept,

$$V_{in} = V_1$$

Applying KCL at node (1)

$$\frac{V_x}{R_4} + \frac{V_x - V_1}{R_3} + \frac{V_x - V_o}{R_1} = 0$$

$$\frac{V_o}{R_1} = V_x\left[\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right] - \frac{V_1}{R_3} \quad \text{...(i)}$$

Applying KCL at node (2)

$$\frac{V_1}{R_2} + \frac{(V_1 - V_x)}{R_3} = 0$$

$$\frac{V_x}{R_3} = V_1\left[\frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$V_x = R_3 V_1\left[\frac{1}{R_2} + \frac{1}{R_3}\right]$$

Substitute this value in equation (i)

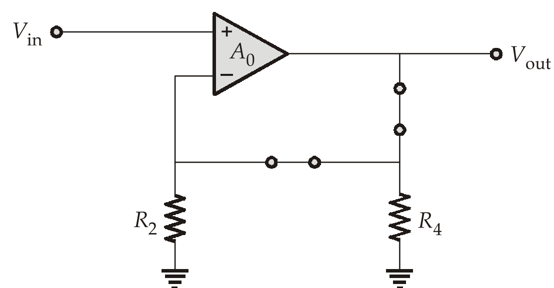
$$\frac{V_o}{R_1} = R_3 V_1\left[\frac{1}{R_2}+\frac{1}{R_3}\right]\left[\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}\right]-\frac{V_1}{R_3}$$

$$\frac{V_o}{R_1} = V_1\left[\left[1+\frac{R_3}{R_2}\right]\left[\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}\right]-\frac{1}{R_3}\right]$$

$$V_o = V_1\left[\left(1+\frac{R_3}{R_2}\right)\left[1+\frac{R_1}{R_3}+\frac{R_1}{R_4}\right]-\frac{R_1}{R_3}\right]$$

Closed loop gain, $$A_v = \frac{V_o}{V_1}=\frac{V_o}{V_{in}}=\left[\left(1+\frac{R_3}{R_2}\right)\left(1+\frac{R_1}{R_3}+\frac{R_1}{R_4}\right)-\frac{R_1}{R_3}\right]$$

when $R_1 \to 0$ and $R_3 \to 0$




$\therefore$ $$V_{out} = V_{in}$$ (Using concept of virtual short circuit)

**Q.1 (e) Solution:**

Ferromagnetic materials are broader class of magnetic materials of which ferrimagnetic materials form a special subdivision. Ferrimagnetic materials exhibit all the properties of ferromagnetic materials like high permeability saturation, magnetization, hysteresis etc. They are different from ferromagnetic materials in the way in spin magnetic moments are aligned in a crystal.

Below given are electric and magnetic characteristics of ferrites:

- They exhibit a high value of resistivity, normally more than $10^5$ ohm-cm.
- They exhibit extremely low dielectric loss.
- Permeability is quite high.
- They exhibit appreciable saturation magnetization, but smaller than ferromagnetic materials.
- They exhibit low coercivity.
- Curie temperature varies from order of hundred degree Celsius to several hundred degree Celsius.

**Q.2 (a) (i) Solution:**

We know that,

$$H_C(T) = H_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right] \quad ...(i)$$

Given, At $T = 14$ K, $H_C(T) = 0.176$ T and at $T = 13$ K, $H_C(T) = 0.528$ T

Therefore,
$$0.176 = H_C(0)\left[1-\left(\frac{14}{T_C}\right)^2\right] \quad ...(ii)$$

and
$$0.528 = H_C(0)\left[1-\left(\frac{13}{T_C}\right)^2\right] \quad ...(iii)$$

Division equation (iii) by equation (ii), we get

$$\frac{1-\left(\frac{13}{T_C}\right)^2}{1-\left(\frac{14}{T_C}\right)^2} = \frac{0.528}{0.176} = 3$$

or
$$1-\frac{169}{T_C^2} = 3-\frac{588}{T_C^2}$$

or
$$\frac{419}{T_C^2} = 2$$

or
$$T_C = \left(\frac{419}{2}\right)^{1/2} = 14.47 \text{ K}$$

Substituting this value of $T_C$ in equation (ii), we get

$$0.176 = H_C(0)\left[1-\left(\frac{14}{14.47}\right)^2\right] = H_C(0)\,[1-(0.9672)^2]$$

or
$$H_C(0) = \frac{0.176}{1-0.9356} = 2.731 \text{ T}$$

For $T = 4.2$ K

$$H_C(T) = 2.731\left[1-\left(\frac{4.2}{14.47}\right)^2\right] = 2.731[1-0.0842]$$

$$H_C(T) = 2.731 \times 0.9158 = 2.5 \text{ T}$$

**Q.2 (a) (ii) Solution:**

We have, $$A_f = 1+\frac{R_1}{R_2} = 1+\frac{99\text{ k}\Omega}{1\text{ k}\Omega} = 100$$

Therefore, $$1 + A\beta = \frac{A}{A_f} = \frac{10^5}{10^2} = 10^3$$

Hence, $$Z_{in\,f} = Z_{in}(1 + A\beta) = 1\text{ M}\Omega \times 10^3 = 1000\text{ M}\Omega$$

and $$Z_{out\,f} = \frac{Z_{out}}{1+A\beta} \,||\, (R_1 + R_2) = \frac{300}{1000} \,||\, (100\text{ k}\Omega) = 0.3\ \Omega$$

When $R_1 = 0$ and $R_2 = \infty$, $$A_f = 1$$

$$1 + A\beta = \frac{A}{A_f} = 10^5$$

$$Z_{in\,f} = Z_{in}(1 + A\beta) = 1\text{ M}\Omega \times 10^5 = 10^5\text{ M}\Omega$$

$$Z_{out\,f} = \frac{Z_{out}}{1+A\beta} = \frac{300}{10^5} = 0.003\ \Omega$$

**Q.2 (b) Solution:**

**(i)** The magnetic field by which the super conducting property of the material gets lost may not necessarily be an externally applied field. It can also be a magnetic field due to current flowing in the super conducting wire itself. It is defined as the maximum value of the current flowing through the super conductor at which the superconducting property ceases, it is denoted by $I_C$. If a superconducting wire of radius $r$ carries a current $I$, then by ampere's law

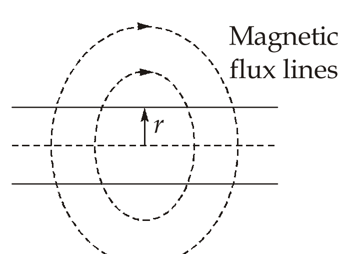
$$\int_C H \cdot dl = I$$

or $$2\pi rH = I$$

At $$H = H_C$$

$$I = I_C$$

$\therefore$ $$I_C = 2\pi rH_C$$



Using above relation, $H_C = \frac{I_C}{2\pi r}$

as $B = \mu_0 H$

or $I_C = \frac{2\pi r B_C(T)}{\mu_0} = \frac{\pi D B_C(T)}{\mu_0}$

$D = 2r = 1 \text{ mm},$

$\mu_0 = 4\pi \times 10^{-7}$

$$I_C = \frac{\pi \times 10^{-3} \times 0.0548}{4\pi \times 10^{-7}}$$

$$= \frac{0.0548 \times 10^{-3}}{4 \times 10^{-7}} = 137 \text{ A}$$

**(ii)** Critical magnetic field at 0 K,

$$H_C(0) = 7 \times 10^5$$

Critical temperature, $T_C = 7.26$ K

We know, $H_C(T) = H_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right]$

$$= 7 \times 10^5 \left[1 - \left(\frac{4}{7.26}\right)^2\right] = 4.87 \times 10^5 \text{ A/m}$$

**Q.2 (c) (i) Solution:**

We have defined the stability factor '$S$' as follows:

$$S = \left.\frac{\Delta I_C}{\Delta I_{CBO}}\right|_{\text{Constant } V_{BE} \text{ and } \beta_{dc}}$$

$S$ gives us the change in $I_C$ due to change in the reverse saturation current $I_{CBO}$. As $I_{CBO}$ changes by $\Delta I_{CBO}$, the base current $I_B$ will change by $\Delta I_B$ and the collector current $I_C$ changes by $\Delta I_C$

For a $CE$ configuration we know that,

$$I_C = \beta_{dc} I_B + I_{CEO}$$

$$= \beta_{dc} I_B + (1 + \beta_{dc}) I_{CBO}$$

Therefore change in $I_C$ is given by

$$\Delta I_C = \beta_{dc} \Delta I_B + (1 + \beta_{dc}) \Delta I_{CBO}$$

Dividing both the sides by $\Delta I_C$ we get,

$$1 = \beta_{dc}\left[\frac{\Delta I_B}{\Delta I_C}\right] + (1+\beta_{dc})\left[\frac{\Delta I_{CBO}}{\Delta I_C}\right]$$

$$\therefore \quad 1-\beta_{dc}\left[\frac{\Delta I_B}{\Delta I_C}\right] = (1+\beta_{dc})\left[\frac{\Delta I_{CBO}}{\Delta I_C}\right]$$

$$\therefore \quad \frac{\Delta I_{CBO}}{\Delta I_C} = \frac{1-\beta_{dc}\left[\frac{\Delta I_B}{\Delta I_C}\right]}{(1+\beta_{dc})}$$

But,

$$S = \frac{\Delta I_C}{\Delta I_{CBO}}$$

$$\therefore \quad S = \frac{(1+\beta_{dc})}{1-\beta_{dc}\left[\frac{\Delta I_B}{\Delta I_C}\right]} \qquad \text{...(i)}$$

But for the fixed bias circuit,

$$I_B = \frac{V_{CC}-V_{BE}}{R_B}$$

In this equation $V_{CC}$, $V_{BE}$ and $R_B$ all are fixed. Thefore $I_B$ cannot change

$$\therefore \quad \Delta I_B = 0$$

Substituting this in equation (i),

We get, $S = 1 + \beta_{dc}$

**Q.2 (c) (ii) Solution:**

The stability factor $S$ for the fixed bias circuit is given by

$$S = 1 + \beta_{dc}$$

Therefore we must find the value of $\beta_{dc}$

But,

$$\beta_{dc} = \frac{I_C}{I_B}$$

So, we have to obtain the values of $I_C$ and $I_B$

We know that,

$$I_C = \frac{V_{CC}-V_{CE}}{R_C} = \frac{12-6}{1\text{ k}\Omega} = 6\text{ mA}$$

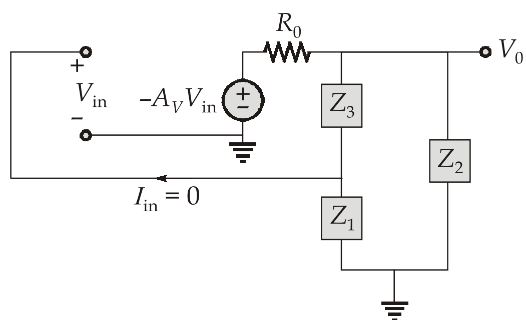
and $$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{100\ \text{k}\Omega} = 113\ \mu\text{A}$$

Therefore $$\beta_{dc} = \frac{I_C}{I_B} = \frac{6 \times 10^{-3}}{113 \times 10^{-6}} = 53.097$$

Hence the stability factor, $$S = 1 + 530.97 = 54.097$$

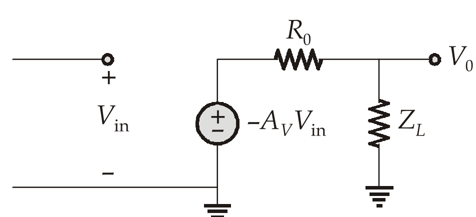
**Q.3 (a) Solution:**

Drawing the small signal model of the amplifier we have,




$\because$ $$I_{in} = 0\ ;$$

The above circuit can be reduced as

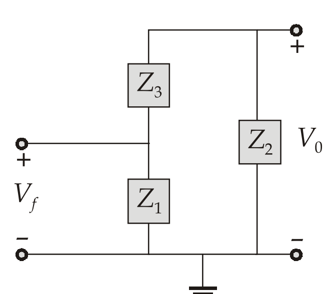



Thus, the overall gain of the amplifier,

$$A = \frac{V_0}{V_{in}} = \frac{-A_V Z_L}{Z_L + R_0}$$

where, $$Z_L = \frac{(Z_1 + Z_3) Z_2}{(Z_1 + Z_2 + Z_3)}$$

For the feedback circuit,



The feedback gain, $\beta = \frac{V_f}{V_0} = \frac{Z_1}{Z_1 + Z_3}$

∵ The phase shift of the feed back circuit is negative.

∴ $$A\beta = \frac{-A_V Z_1 Z_L}{(R_0 + Z_L)(Z_1 + Z_3)} = \frac{-A_V Z_1 \left[\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right]}{\left[R_0 + \frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right](Z_1+Z_3)}$$

$$= \frac{-A_V Z_1 Z_2}{R_0(Z_1+Z_2+Z_3) + Z_2(Z_1+Z_3)}$$

Now, $Z_1 = jX_1$, $Z_2 = jX_2$ and $Z_3 = jX_3$

⇒ $$A\beta = \frac{A_V(X_1X_2)}{jR_0(X_1+X_2+X_3) - X_2(X_1+X_3)}$$

To produce sustained oscillations the phase shift of the loop gain $A\beta$ should be 0°.

Thus, $R_0(X_1 + X_2 + X_3) = 0$

⇒ $X_1 + X_2 + X_3 = 0$

$(X_1 + X_3) = -X_2$

∴ $$A\beta = \frac{-A_V X_1}{(X_1 + X_3)}$$

⇒ $$A\beta = \frac{A_V X_1}{X_2}$$

Hence $X_1$ and $X_2$ should be of the same type of reactance.

**Q.3 (b) Solution:**

**(i)** Assuming transistor is in saturation,

∴ $$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)[V_{GS} - V_T]^2 \quad \text{...(i)}$$

Given: $V_G = 1.8$ V, $\mu_n C_{ox}\left(\frac{W}{L}\right) = 2\ \text{mA/V}^2$

$V_S = 0.5\, I_D$, $V_T = 1$ V ....($I_D$ is in mA)

∴ $V_{GS} = 1.8 - 0.5I_D$

and $$I_D = \frac{1.8 - V_{GS}}{0.5} = 2[1.8 - V_{GS}]$$

$$I_D = 3.6 - 2V_{GS} \quad \text{...(ii)}$$

Now, from equation (i) and (ii)

$$3.6 - 2V_{GS} = \frac{1}{2}\times 2[V_{GS}-1]^2$$

$$3.6 - 2V_{GS} = V_{GS}^2 + 1 - 2V_{GS}$$

$$V_{GS}^2 = 2.6$$

$$V_{GS} = 1.6124 \text{ Volt}$$

Hence, from equation (ii),

$$I_D = 0.375 \text{ mA}$$

$$V_S = 0.1875 \text{ V}$$

Now, $V_D = 3.3 - I_D R_D$

$$= 3.3 - 0.375 \times 10$$

$$V_D = -0.45 \text{ V} \qquad \text{i.e., } V_D < V_T$$

$V_D$ can't be negative. It means conditions for saturation are not being satisfied.

∴ Transistor is in triode region.

∴ $$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_D = 2\left[V_{GS} - V_T - \frac{V_{DS}}{2}\right]V_{DS} \quad \text{....}(I_D \text{ is in mA}) \quad \text{...(iii)}$$

But $V_D = 3.3 - 10I_D,\ V_S = 0.5I_D$

$$I_D = 2\left[V_G - V_S - V_T - \frac{V_D}{2} + \frac{V_S}{2}\right][V_{DS}]$$

$$= 2\left[1.8 - 0.5V_S - 1 - \frac{V_D}{2}\right][3.3 - 10.5I_D]$$

$$= 2[0.8 - 0.5V_S - 0.5V_D][3.3 - 10.5I_D]$$

$$= (1.6 - V_S - V_D)(3.3 - 10.5I_D)$$

$$= (1.6 - 0.5I_D - 3.3 + 10I_D)(3.3 - 10.5I_D)$$

$$= (9.5I_D - 1.7)(3.3 - 10.5I_D)$$

$$I_D = 31.35I_D - 99.75I_D^2 - 5.61 + 17.85I_D$$

$$99.75I_D^2 - 49.2I_D + 5.61 = 0$$

After solving we get,

$$I_D = 0.31428 \text{ mA}, 0.17895 \text{ mA}$$

**Case (I):**

$$I_D = 0.17895 \text{ mA}$$
$$V_S = 0.5 \times I_D = 0.089475 \text{ V}$$
$$V_D = 3.3 - 10I_D = 1.5105 \text{ V}$$
$$V_{DS} = V_D - V_S = 1.4210 \text{ V}$$
$$(V_{GS} - V_T) = (1.8 - 0.089475 - 1) = 0.710 \text{ V}$$

$V_{DS} > V_{GS} - V_T$ which is not valid, as transistor is in triode region.

**Case (2):**

$$I_D = 0.31428 \text{ mA}$$
$$V_S = 0.5 \times 0.31428 = 0.1571 \text{ V}$$
$$V_D = 3.3 - 10I_D = 0.1572 \text{ V}$$
$$V_{DS} = V_D - V_S \approx 0 \text{ V}$$
$$(V_{GS} - V_T) = V_G - V_S - V_T = 1.8 - 0.15714 - 1 = 0.64286 \text{ V}$$

As $V_{DS} < (V_{GS} - V_T)$ $\therefore$ $I_D = 0.31428$ mA is valid value which confirm that transistor is in triode region.

$\therefore$ Drain voltage $V_D = 0.1572$ V

**(ii) Transconductance:** Transconductance is the ratio of change in drain current ($\partial I_D$) to change in the gate to source voltage ($\partial V_{GS}$) at a constant drain to source voltage ($V_{DS}$ = constant)

$$g_m = \frac{\partial I_D}{\partial V_{GS}} \text{ at constant } V_{DS}$$

This value is maximum at $V_{GS} = 0$. This is denoted by $g_{m0}$.

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{2I_{DSS}}{V_{GS(\text{off})}}\left[1 - \frac{V_{GS}}{V_{GS(\text{off})}}\right]$$

where,
$$g_{m0} = \frac{2I_{DSS}}{V_{GS(\text{off})}}$$

$$\therefore \quad g_m = g_{m0}\left[1 - \frac{V_{GS}}{V_{GS\,(\text{off})}}\right]$$

**Dynamic Output Resistance :** This is the ratio of change in drain to source voltage ($\partial V_{DS}$) to the change in drain current ($\partial I_D$) at a constant gate to source voltage ($V_{GS}$ = constant). It is denoted as $r_d$.

$$r_d = \frac{\partial V_{DS}}{\partial I_D} \text{ at constant } V_{GS}$$

**Amplification factor:** It is defined as the ratio of change in drain voltage ($\partial V_{DS}$) to change in gate voltage ($\partial V_{GS}$) at a constant drain current ($I_D$ = constant)

$$\mu = \frac{\partial V_{DS}}{\partial V_{GS}} \text{ at constant } I_D$$

There is a relation between transconductance ($g_m$), dynamic output resistance ($r_d$) and amplification factor ($\mu$) given by

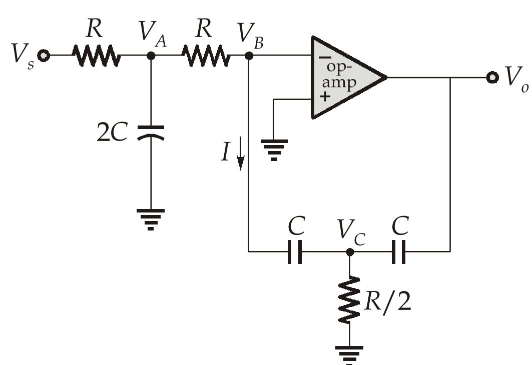
$$\mu = \frac{\partial V_{DS}}{\partial V_{GS}} = \frac{\partial V_{DS}}{\partial I_D} \times \frac{\partial I_D}{\partial V_{GS}}$$

$$\mu = r_d \times g_m$$

**Q.3 (c) Solution:**

By virtual ground,

$$V_B = 0 \text{ V}$$




Apply KCL at node-A,

$$\frac{V_A - V_S}{R} + V_A(2sC) + \frac{V_A}{R} = 0$$

$$V_A - V_S + V_A(2sRC) + V_A = 0$$

$$V_S = V_A(2 + 2sRC) \quad \text{...(i)}$$

Apply KCL at node-C,

$$(V_C - V_B)sC + \frac{2V_C}{R} + (V_C - V_0)sC = 0$$

$$V_C\left[2sC + \frac{2}{R}\right] = V_0 sC$$

$$V_C[2sRC + 2] = V_0 sCR$$

$$V_C = \frac{V_0 sCR}{2(1 + sRC)} \quad \text{...(ii)}$$

Also, $$I = \frac{V_A - V_B}{R} = \frac{V_A}{R}$$

and $$I = (V_B - V_C)sC = -V_C sC$$

Therefore, $$-V_C sC = \frac{V_A}{R}$$

$$V_A = -V_C sRC$$

Now from equation (i),

$$V_A = \frac{V_S}{2(1+sRC)} = -V_C sRC$$

$$\frac{V_S}{2(1+sRC)} = -\left(\frac{V_0 sCR}{2(1+sRC)}\right)(sRC)$$

Therefore, $$\frac{V_0}{V_S} = -\frac{1}{(sRC)^2}$$

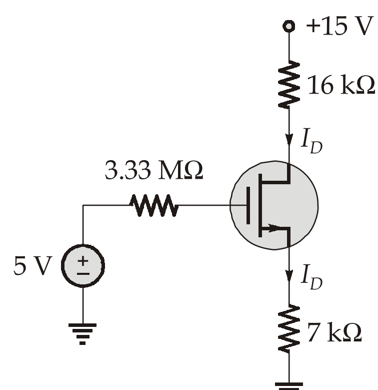
By taking the Laplace transfer of above equation

$$V_0 = -\frac{1}{(RC)^2}\int_{-\infty}^{t}\int_{-\infty}^{t} V_s dt$$

Hence the output expression clearly shows the given op-amp circuit acts as double integer.

**Q.4 (a) Solution:**

The DC equivalent circuit can be drawn as below:




$$V_G = 5 \text{ Volt}, V_T = 1 \text{ Volt}$$

Let's consider MOSFET is in saturation.

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2$$

$$I_D = \frac{1}{2}\times 4\times 10^{-3}(V_{GS} - 1)^2$$

$$I_D = 2(V_{GS} - 1)^2 \text{ mA} \quad \text{...(i)}$$

Also,
$$V_{GS} = V_G - V_S$$
$$V_{GS} = 5 - (I_D \times 7)$$
$$7I_D = 5 - V_{GS}$$
$$I_D = \left(\frac{5 - V_{GS}}{7}\right) \text{ mA}$$

Substitute in eqn. (i),
$$\left(\frac{5 - V_{GS}}{7}\right) = 2(V_{GS} - 1)^2$$
$$5 - V_{GS} = 14[V_{GS}^2 + 1 - 2V_{GS}]$$
$$14V_{GS}^2 - 28V_{GS} + 14 = 5 - V_{GS}$$
$$14V_{GS}^2 - 27V_{GS} + 9 = 0$$
$$V_{GS} = 1.5 \text{ Volt, } 0.428 \text{ Volt}$$

$V_{GS} > V_T$, ∴ Valid $\quad V_{GS} = 1.5$ Volt

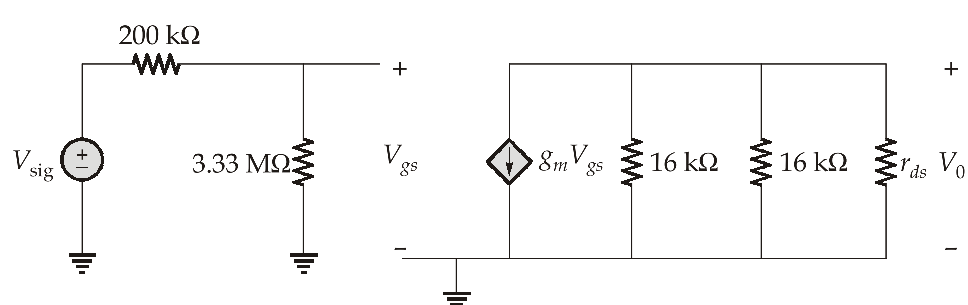
∴ $\quad I_D = 2(1.5 - 1)^2$
$$I_D = 0.5 \text{ mA}$$
$$V_S = 0.5 \times 7 = 3.5 \text{ Volt}$$

Now,
$$V_{DS} = 15 - I_D(16 + 7) = 3.5 \text{ Volt}$$
$$V_{GS} - V_t = 5 - 3.5 - 1$$
$$V_{GS} - V_t = 0.5 \text{ Volt}$$

∴ $\quad V_{DS} > (V_{GS} - V_T)$

Hence, MOSFET is in saturation mode.

Now, drawing small signal AC equivalent circuit.

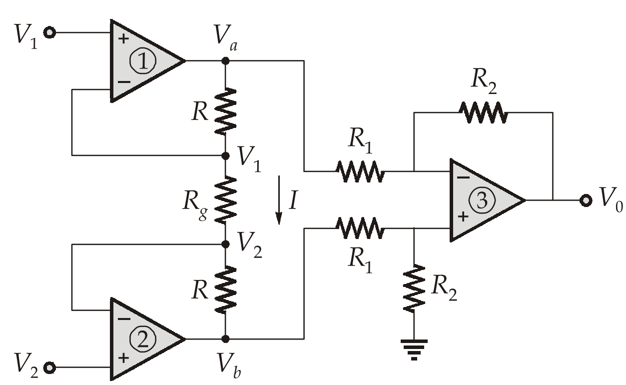



$$r_{ds} = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

Now,
$$V_o = -g_m V_{gs}(16 \,||\, 16 \,||\, 200)$$
$$V_0 = -g_m V_{gs}(7.69) \times 10^3$$
$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)$$
$$= 4 \times 0.5 \times 10^{-3}$$
$$g_m = 2 \text{ m℧}$$
$$V_{gs} = \frac{V_{\text{sig}}(3.33\times10^6)}{(200\times10^3 + 3.33\times10^6)}$$
$$V_{\text{sig}} = \frac{V_{gs}(200\times10^3 + 3.33\times10^6)}{(3.33\times10^6)}$$
$$V_{\text{sig}} = V_{gs} \times 1.06$$

Now,
$$A_V = \frac{V_o}{V_{\text{sig}}} = \frac{-2\times7.69}{1.06}$$
$$A_V = -14.51 \text{ V/V}$$

**Q.4 (b) Solution:**




As all op-amps has –ve feedback so current zero concept is applied

So, we have,
$$I = \frac{V_a - V_b}{R + R + R_g} = \frac{V_1 - V_2}{R_g}$$
$$V_a - V_b = \frac{2R + R_g}{R_g}(V_1 - V_2) \quad \text{...(i)}$$

For op-amp-3,
$$V_0 = V_b \times \frac{R_2}{R_1 + R_2} \times \left[1 + \frac{R_2}{R_1}\right] - V_a \times \frac{R_2}{R_1}$$

$$= \frac{R_2}{R_1}(V_b - V_a) \quad ...(ii)$$

From equation (i),

$$V_0 = \left[\frac{2R + R_g}{R_g}\right]\left[\frac{R_2}{R_1}\right] \times (V_2 - V_1)$$

At $V_1 = 5$ V and $V_2 = 5.05$ V ; $V_0 = 5$ V

So, $$5 = \left[\frac{2R}{R_g} + 1\right]\left[\frac{R_2}{R_1}\right] \times (5.05 - 5)$$

$$\left[\frac{2R}{R_g} + 1\right]\left[\frac{R_2}{R_1}\right] = 100 \quad ...(iii)$$

Gain of I$^{st}$ stage , $$A_1 = \frac{V_a - V_b}{V_1 - V_2} = \frac{2R + R_g}{R_g}$$

Gain of II$^{nd}$ stage, $$A_2 = \frac{V_0}{V_b - V_a} = \frac{R_2}{R_1}$$

$$\frac{A_1}{A_2} = \frac{10}{1}$$

$$\frac{2R + R_g}{R_g} = 10 \times \frac{R_2}{R_1} \quad ...(iv)$$

From equation (iii) and (iv),

$$10 \times \frac{R_2}{R_1} \times \frac{R_2}{R_1} = 100$$

$$\frac{R_2}{R_1} = 3.16$$

From equation (iv),

$$\frac{2R + R_g}{R_g} = 10 \times \sqrt{10}$$

$$\frac{2R}{R_g} + 1 = 10\sqrt{10}$$

$$\frac{R}{R_g} = 15.3$$

**Q.4 (c) (i) Solution:**

The soft magnetic materials are characterized by low hysteresis and eddy current losses. These materials are easy to magnetize and demagnetize. This enables them to reverse magnetize rapidly in response to alternating electric field where they are required to concentrate magnetic flux in transformers and inductances.

Characteristic of soft magnetic materials:

- These are easily magnetized and demagnetized rapidly hence suitable for rapid switching of magnetization needed in AC fields.
- They have low retentively.
- They have low coercivity.
- They have high permeability.
- They have high magnetic saturation.
- They have high curie temperature.
- They have low hysteresis loss (small hysteresis loop area)
- Nature of hysteresis loop is tall and narrow.

Example of soft magnetic materials are:

Fe-Si alloy, also known as soft iron Si-Steel, FeNi alloys like permalloy, Mu-Metal etc. These are used for construction of transformers and inductor cores.

**Q.4 (c) (ii) Solution:**

**Magnetic dipole:**

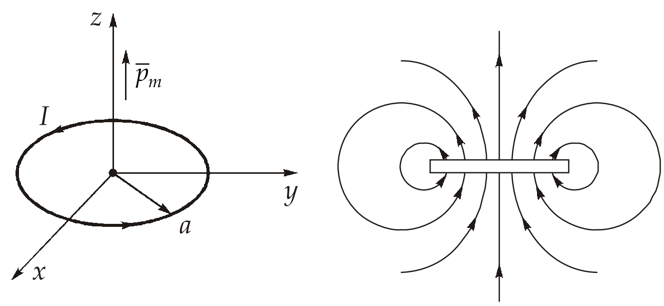
The magnetic field produced by a small current loop is similar to the electric field produced from a small electric dipole. That is why a small current loop is called magnetic dipole. Its dipole moment is defined as equal to the product of the area of the plane loop and the magnitude of circulating current. The vector direction of the dipole moment is perpendicular to the plane of the loop and is along the direction of a right hand screw when moved in the direction of current in the loop.

Dipole moment, $p_m$ = current in the loop × area of loop.

For a circular loop, dipole moment as shown in figure (a) is in z-direction.

$$p_m = \pi a^2 I \hat{a}_z$$

Written as $\hat{a}_z$ is a unit vector in $z$-direction. The field lines produced by a magnetic dipole are shown in figure (b).




(a) (b)

When a magnetic dipole is placed in a magnetic field it has tendency to orient itself in the direction of the magnetic field.

**Magnetization:**

All materials are affected by the presence of a magnetic field. It is found experimentally that they acquire magnetic dipole moments. The magnitude of dipole moment per unit volume, is called the magnetization of the material and is described by a vector quantity $\bar{M}$, A/m. When a magnetic field is applied to a material, the magnetic induction (flux density) is the sum of the effect on vacuum and that on the material, so that,

$$B = \mu_0 H + \mu_0 M = \mu_0 \mu_r H$$

Then, $$M = (\mu_r - 1)H = \chi_m H$$

Where $\chi_m$ is the magnetic susceptibility. Above Equation defines $M$ to have same units as that of $H$, A/m.

The magnetization $M$ of a material may be expressed in terms of its elementary magnetic dipole moment, $p_m$ by

$$M = Np_m$$

where $N$ is the number of magnetic dipoles per unit volume.

## Section B : Electrical Machine-1 + Power Systems-2

**Q.5 (a) Solution:**

Operation of motor on dc,

Back emf, $$E_{bdc} = V - I_a R_a = 250 - 0.8 \times 30 = 226 \text{ V}$$

and speed, $$N_{dc} = 2000 \text{ rpm}$$

Operation of motor on ac,

$$X_L = 2\pi f L$$
$$= 2\pi \times 50 \times 0.5 = 157\ \Omega$$

From the phasor diagram shown in figure below,

$$AF^2 = AG^2 + GF^2$$

$$V^2 = (AB + BG)^2 + GF^2 = (AB + DF)^2 + GF^2$$

$$= (I_a R_a + E_{bac})^2 + (I_a X_L)^2$$

$$E_{bac} + I_a R = \sqrt{V^2 - (I_a X_L)^2}$$

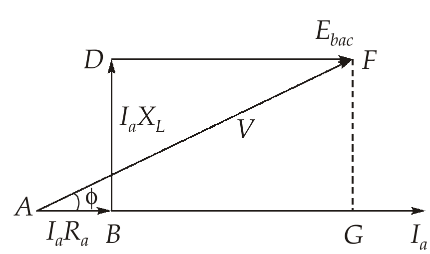
$$E_{bac} = -0.8 \times 30 + \sqrt{(250)^2 - (0.8 \times 157)^2}$$

$$= -24 + 216.15 = 192.15 \text{ V}$$

Since the currents in dc and ac operation are equal, the flux will also be equal ($\Phi_{dc} = \Phi_{ac}$)

$$\frac{E_{bdc}}{E_{bac}} = \frac{K N_{dc}}{K N_{ac}} \frac{\Phi_{dc}}{\Phi ac} = \frac{N_{dc}}{N_{ac}}$$

$$N_{ac} = N_{dc} \frac{E_{bac}}{E_{bdc}} = 2000 \times \frac{192.15}{226} = 1700 \text{ rpm}$$




from above figure,

$$\text{Power factor, } \cos\phi = \frac{AG}{AF} = \frac{E_{bac} + I_a R_a}{V}$$

$$= \frac{192.15 + 0.8 \times 30}{250} = 0.8646 \text{ (lagging)}$$

Mechanical power developed,

$$P_{\text{mech}} = E_{bac} I_a = 192.15 \times 0.8 = 153.7 \text{ W}$$

Torque developed,

$$T = \frac{P_{\text{mech}}}{\omega_m} = \frac{P_{\text{mech}}}{2\pi N_{ac}}$$

$$= \frac{153.7}{2\pi \times (1700/60)} = 0.8633 \text{ Nm}$$

**Q.5 (b) Solution:**

DC transmission possesses many technical and economic advantages over AC transmission. Some of these are given below:

1. **Cheaper in cost:** Bipolar HVDC transmission lines require two-pole conductors while AC system requires 3 conductor to carry power.

   HVDC transmission can utilize earth return and therefore can operate with single pole (conductor) while EHV-AC transmission always requires three conductors.

   The potential stress on the insulation is case of dc system is $\frac{1}{\sqrt{2}}$ times of that in AC system for the same operating voltage. Hence for the same operating voltage less insulation required.

2. **An HVDC line can be built in stages:** The DC line can be built as a monopolar line with ground return in the initial stage and may be converted into a bipolar line on a later date when the load requirement increases.

3. **No skin effect:** There is no skin effect in DC so there is a uniform distribution of current over the section of the conductor.

4. **Lower transmission losses:** HVDC transmission system needs only two conductors and therefore, the power losses in a DC line are lesser than the losses in AC line of the same power transfer capability.

5. **Voltage regulation:** There is no line inductance, hence the voltage drop due to inductive reactance does not exist in DC transmission line. Thus voltage regulation is better incase of DC transmission.

6. **Line loading:** The permissible loading on an EHV-AC line is limited by transient stability limit and line reactance to almost one-third of thermal rating of conductors. No such limit exists in case of HVDC lines.

7. **Surge impedance loading:** Long EHV-AC lines are loaded to less than 80 percent of natural load. No such condition is applicable to HVDC lines.

8. **Greater reliability:** A two conductor bipolar DC line is more reliable than a 3-wire, 3-phase AC line because the DC line may be operated in monopolar mode with ground return when the other line develops a fault.

9. **Rapaid change of energy flow:** The control of converter (thyristor) valve permit rapid changes in magnitude and direction of power flow when the two AC systems are interconnected by a DC line.

10. **Independent control:** The AC systems interconnected by a DC line can be controlled independently. They can be completely independent as regards frequency system control, short-circuit rating, future extension etc.
11. **Lesser dielectric power loss and higher current carrying capacity:** The cables have lesser dielectric power loss with DC in comparison with AC and therefore, have higher current carrying capacity.
12. **Negligible sheath losses:** In case of DC, only leakage current flows in the cable sheath wheareas in case of AC charging, circulating and eddy current flow through the sheath of cable. Thus in case of DC transmission with cables, the sheath losses in the cable are negligible.
13. **Higher natural dielectric strength and longer life of cable insulation:** The natural dielectric strength of cable insulation with DC is substantially greater than that with AC. Phenomenon of dielectric fatigue is also absent, therefore the cable insulation has longer life.
14. **Absence of charging current and limitations of cable lengths:** Because of large charging currents, the use of EHV-AC for underground transmission over long distances is prohibited but because of absence of charging current in DC system there is no limit on the length of the cable.
15. **Lesser corona loss and radio interference**.
16. **Higher operating voltages:** The level of switching surges due to DC is lower than that due to AC and therefore the same size of conductors and string insulators can be employed for higher voltages in case of DC as compared to AC.
17. HVDC lines do not require any reactive power compensation like EHV-AC lines. It is because of absence of charging currents and unity power factor operation.

**Q.5 (c) Solution:**

Case - 1 :

$$\text{Power factor} = 0.8,$$

$$\text{Voltage regulation} = 6\%$$

Case - 2 :

$$\text{Power factor} = 0.6,$$

$$\text{Voltage regulation} = 6.6\%$$

In a transformer,

$$V_{\text{regulation}} = R_{pu} \cos\theta + X_{pu} \sin\theta$$

Where $R_{pu}$ and $X_{pu}$ are per unit resistance and leakage reactance in percent

$$V_{\text{reg }1} = R_{pu}(0.8) + X_{pu}(0.6)$$
$$6 = 0.8R_{pu} + 0.6X_{pu}$$
$$30 = 4R_{pu} + 3X_{pu} \quad \text{...(i)}$$
$$V_{\text{reg }2} = R_{pu}(0.6) + X_{pu}(0.8)$$
$$6.6 = R_{pu}(0.6) + X_{pu}(0.8)$$
$$3R_{pu} + 4X_{pu} = 33 \quad \text{...(ii)}$$

On solving equation (i) and (ii), we get

$$R_{pu} = 3\% \text{ and } X_{pu} = 6\%$$
$$V_{\text{regulation}} = R_{pu}\cos\theta + X_{pu}\sin\theta$$

For maximum regulation,

$$\frac{dV_{\text{regulation}}}{d\theta} = 0$$
$$-R_{pu}\sin\theta + X_{pu}\cos\theta = 0$$
$$\tan\theta = \frac{X_{pu}}{R_{pu}}$$
$$\tan\theta = 2$$
$$\theta = 63.43°$$

Power factor at maximum voltage regulation

$$= \cos(63.43°) = 0.4472$$

Full load efficiency, $P_{cu\,fL} = R_{pu} = 0.03$

$$\eta_{FL} = \frac{1\times1}{1\times1+0.03+0.03}\times100 = 94.37\%$$

**Q.5 (d) Solution:**

For maximum efficiency,

$$\text{Output} = 20\times1.0 = 20 \text{ kW}$$
$$\text{Input} = \frac{\text{Output}}{\eta} = \frac{20}{0.98} = 20.408 \text{ kW}$$
$$\text{Total losses} = \text{Input} - \text{Output}$$
$$= 20.408 - 20 = 0.408 \text{ kW}$$

Full-load copper losses, $P_c$ = Iron loss,

$$P_e = \frac{\text{Total losses}}{2} = \frac{0.408}{2} = 0.204 \text{ kW}$$

$$\text{All-day output} = 2 \times 12 + 10 \times 6 + 20 \times 6$$
$$= 204 \text{ kWh}$$
$$\text{Iron losses for 24 hours} = 0.204 \times 24 = 4.896 \text{ kWh}$$
$$\text{Copper losses for 24 hours} = \left(\frac{2/0.6}{20}\right)^2 \times 0.204 \times 12 + \left(\frac{10/0.8}{20}\right)^2 \times 0.204 \times 6$$
$$+ \left(\frac{20/0.9}{20}\right)^2 \times 0.204 \times 6$$
$$= 0.068 + 0.478 + 1.511 = 2.057 \text{ kWh}$$
$$\text{All day efficiency, } \eta = \frac{\text{Output}}{204 + 4.896 + 2.057} \times 100 = 96.7\%$$

**Q.5 (e) Solution:**

Given, $\dfrac{T_s}{T_{fL}} = 1.6$ and $\dfrac{T_m}{T_f} = 2$

Dividing, $$\frac{\frac{T_s}{T_{fL}}}{\frac{T_m}{T_{fL}}} = \frac{T_s}{T_m} = \frac{1.6}{2} = 0.8$$

Again, $\dfrac{T_s}{T_m} = \dfrac{2a}{1+a^2}$ where $a = \dfrac{R_2}{X_{20}}$

Hence, $$\frac{2a}{1+a^2} = 0.8$$
$$a^2 = -2.5a + 1 = 0$$
$$a = 2, 0.5$$
$a = 0.5$ is permissible value,

because $a = 2$ lies in plugging region

$\therefore$ $a = 0.5$

$$\frac{R_2}{X_{20}} = 0.5$$

Slip at maximum torque, $s_m = 0.5$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$N_r = (1 - s)N_s = (1 - 0.5) \times 1500 = 750 \text{ rpm}$$

$$\frac{T_m}{T_{fL}} = \frac{a^2 + s_{fL}^2}{2as_{fL}} = 2$$

Substituting value of $a$,

$$\frac{(0.5)^2 + s_{fL}^2}{2 \times 0.5 \times s_{fL}} = 2$$

$$0.25 + s_{fL}^2 = 2s_{fL}$$

$$s_{fL} = 1.866, 0.1339$$

or, $s_{fL} = 0.1339$

Full load speed, $N_r = (1 - s)N_s = (1 - 0.1339) \times 1500 = 1300 \text{ rpm}$

**Q.6 (a) Solution:**

$$V_1 = 1.05 \text{ p.u.}$$

$$V_2 = 0.98183 \text{ p.u.}$$

$$V_3 = 1.00125 \text{ p.u.}$$

$$\delta_1 = 0°$$

$$\delta_2 = -3.5035°$$

and $\delta_3 = -2.8624°$

Defining impedances,

$$Z_{12} = (0.02 + j\,0.04) \text{ p.u.}$$

$$Z_{13} = (0.01 + j\,0.03) \text{ p.u.}$$

$$Z_{23} = (0.0125 + j\,0.025) \text{ p.u.}$$

Line admittances,

$$y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.02 + j0.04}$$

$$= 10 - j\,20 = 22.36\angle{-63.435°} \text{ p.u.}$$

$$y_{13} = \frac{1}{Z_{13}} = \frac{1}{0.01 + j0.03}$$

$$= 10 - 30i = 31.62\angle{-71.565°} \text{ p.u.}$$

$$y_{23} = \frac{1}{Z_{23}} = \frac{1}{0.0125 + j0.025}$$

$$= (16 - j32) = 35.78\angle{-63.433} \text{ p.u.}$$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

Elements of $Y_{bus}$ are

$$Y_{11} = y_{12} + y_{13} = (10 - j20) + (10 - j30)$$
$$= (20 - j50) = 53.85\angle{-68.2°} \text{ p.u.}$$
$$Y_{22} = y_{12} + y_{23} = (10 - j20) + (16 - j32)$$
$$= (26 - j52) = 58.13\angle{-63.435°} \text{ p.u.}$$
$$Y_{33} = y_{13} + y_{23} = (10 - j30) + (16 - j32)$$
$$= 26 - j62 = 67.23\angle{-67.25°} \text{ p.u.}$$
$$Y_{12} = Y_{21} = -y_{12} = 22.36\angle(180° - 63.43°)$$
$$= 22.36\angle 116.57° \text{ p.u.}$$
$$Y_{13} = Y_{31} = -y_{13} = 31.62\angle(180° - 71.565°)$$
$$= 31.62\angle 108.435° \text{ p.u.}$$
$$Y_{23} = Y_{32} = -y_{23} = 35.78\angle(180° - 63.435°)$$
$$= 35.78\angle 116.565° \text{ p.u.}$$

Now, power injected at $i^{th}$ bus

$$P_i = V_i \sum_{K=1}^{n} V_k Y_{ik} \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$Q_i = -V_i \sum_{K=1}^{n} V_k Y_{ik} \sin(\theta_{ik} + \delta_k - \delta_i)$$

Slack bus power,

$$P_1 = V_1 [V_1 Y_{11} \cos(\theta_{11} + \delta_1 - \delta_1) + V_2 Y_{12} \cos(\theta_{12} + \delta_2 - \delta_1) + V_3 Y_{13} \cos(\theta_{13} + \delta_3 - \delta_1)]$$
$$= 1.05 [1.05 \times 53.85 \cos(-68.2°) + 0.98183 \times 22.36 \cos(116.57° - 3.5035° - 0) + 1.00125 \times 31.62 \cos(108.435° - 2.8624° - 0)]$$
$$= 1.05[20.998 - 8.597 - 8.499]$$
$$= 4.094 \text{ p.u.}$$

$$Q_1 = -V_1[V_1 Y_{11} \sin(\theta_{11} + \delta_1 - \delta_1) + V_2 Y_{12} \sin(\theta_{12} + \delta_2 - \delta_1) + V_3 Y_{13} \sin(\theta_{13} + \delta_3 - \delta_1)]$$

$$= -1.05[1.05 \times 53.85\sin(-68.2°) + 0.98183 \times 22.36\sin(116.57° - 3.5035°) + 1.00125 \times 31.62\sin(108.435° - 2.8624° - 0°)]$$

$$= 1.874 \text{ p.u.}$$

$$\text{Slack bus power} = S_1 = P_1 + jQ_1 = (4.094 + j1.874) \text{ p.u.}$$

$$\text{MVA}_{\text{base}} = 100 \text{ MVA}$$

So, $S_1 = (409.4 + j187.4)$ MVA

Complex power on line 1 - 2 :

$$S_{12} = V_1 I_{12}^*$$

where, $I_{12} = \dfrac{V_1 - V_2}{Z_{12}} = \dfrac{1.05\angle 0° - 0.98183\angle -3.5035}{0.02 + j0.04}$

$$= 2.062\angle -22.835° \text{ p.u.}$$

$$S_{12} = V_1 I_{12}^* = 1.05\angle 0° \times 2.062\angle 22.835° \text{ p.u.}$$

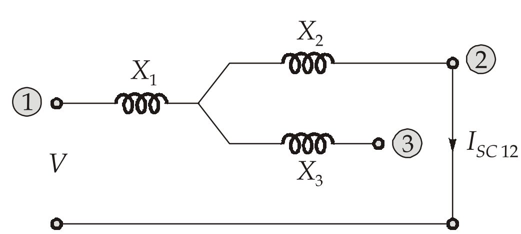
$$= 2.165\angle 22.835° \text{ p.u.} = (1.995 + j0.84) \text{ p.u.}$$

$$(\text{MVA})_{\text{base}} = 100$$

So, $S_{12} = (199.5 + j84)$ MVA

**Q.6 (b) Solution:**

The leakage reactances referred to common base of 12.5 MVA are




$$\frac{V}{I_{SC12}} = X_{12} = X_1 + X_2$$

$$X_{12} = X_1 + X_2$$

$$X_{23} = X_2 + X_3$$

$$X_{13} = X_1 + X_3$$

$$X_{12} + X_{13} - X_{23} = 2X_1$$

$$\therefore \quad X_1 = \frac{1}{2}(X_{12} + X_{13} - X_{23})$$

$$= \frac{1}{2}(0.2 + 0.3 - 0.15) = 0.175 \text{ p.u.}$$

$$X_2 = \frac{1}{2}(X_{12} + X_{23} - X_{13})$$

$$= \frac{1}{2}(0.2 + 0.15 - 0.3) = 0.025 \text{ p.u.}$$

$$X_3 = \frac{1}{2}(X_{13} + X_{23} - X_{12})$$

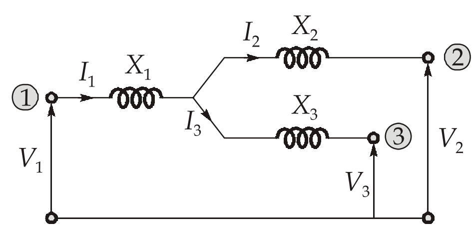
$$= \frac{1}{2}(0.3 + 0.15 - 0.2) = 0.125 \text{ p.u}$$

Both the secondary and tertiary windings operate at rated currents. Let us assume that secondary and tertiary terminal voltages $V_2$ and $V_3$ are in phase. Then with $V_2$ and $V_3$ as reference phasors,

$$\text{Secondary current, } I_2 = 1\angle{-\cos^{-1} 0.8°} = (0.8 - j0.6) \text{ p.u.}$$

$$\text{Tertiary current, } I_3 = 1\angle 0° = 1 + j\,0 \text{ p.u.}$$

$$\text{Primary current, } I_1 = I_2 + I_3 = 0.8 - j\,0.6 + 1 = (1.8 - j\,0.6) \text{ p.u.}$$




**(i)** From figure, $V_1 = V_2 + I_2Z_2 + I_1Z_1$

Since the secondary voltage is equal to the rated voltage

$$V_2 = 1.0$$

$$V_2 = 1\angle 0° = 1 + j0$$

$$I_2 = 0.8 - j0.6 \text{ p.u.}$$

$$I_1 = 1.8 - j0.6 \text{ p.u.}$$

$$Z_1 = j\,X_1 = j\,0.175 \text{ p.u.}$$

$$Z_2 = j\,X_2 = j\,0.025 \text{ p.u.}$$

$$\therefore \quad V_1 = 1 + j0 + (0.8 - j0.6)(j\,0.025) + (1.8 - j\,0.6)(j\,0.175)$$

$$= 1.12 + j\,0.335 = 1.169\angle 16.7° \text{ p.u.}$$

$\therefore$ Primary line-to-line voltage $= 1.169 \times 220 = 257.18$ kV

**(ii)** Again from figure, by KVL in secondary and tertiary circuits,

$$V_2 + I_2Z_2 - I_3Z_3 - V_3 = 0$$

$$V_3 = V_2 + I_2Z_2 - I_3Z_3$$

$$= 1 + j0 + (0.8 - j0.6)\,(j\,0.025) - (1 + j0)\,(j0.125)$$
$$= 1.015 - j0.105 = 1.0204\angle{-5.9°}\ \text{p.u.}$$
$$\therefore \quad V_3 = 1.0204 \times 11 = 11.22\ \text{kV}$$

**(iii)** When the secondary load is reduced to zero,

$$I_2 = 0,$$
and
$$I_1 = I_3$$

By KVL in primary and tertiary circuits

$$V_1 = I_1Z_1 + I_3Z_3 + V_3$$
$$V_1 = I_3Z_1 + I_3Z_3 + V_3$$
$$1.12 + j\,0.335 = 1(j\,0.175) + 1(j\,0.125) + V_3$$
$$V_3 = 1.12 + j0.035 = 1.1205\angle 1.8°\ \text{p.u.}$$
$$V_3 = 1.1205 \times 11 = 12.32\ \text{kV}$$

Therefore with the secondary load reduced to zero, the tertiary voltage rises from 11.22 kV to 12.32 kV.

**Q.6 (c) Solution:**

**(i)** Rotor standstill impedance per phase,

$$E_2 = \frac{V_1}{\sqrt{3}} = \frac{1000}{\sqrt{3}\times 2} = 288.675\ \text{V}$$

Synchronous speed, $$N_s = \frac{120f}{P} = \frac{120\times 50}{24} = 250\ \text{rpm}$$

At full load, Slip, $$S_f = \frac{N_s - N}{N_s} = \frac{250-245}{250} = 0.02$$

Power input to rotor, $$P_2 = \frac{3R_2sE_2^2}{R_2^2 + s^2X_2^2} = \frac{3\times 0.02\times 0.02\times 288.675^2}{0.02^2 + (0.02\times 0.3)^2} = 229358\ \text{W}$$

Torque developed at full-load,

$$T_f = \frac{P_2}{\frac{2\pi N_s}{60}} = \frac{229358\times 60}{2\pi\times 250} = 8761\ \text{N-m}$$

At maximum torque,

Slip corresponding to maximum torque,

$$S_{\max,\,T} = \frac{R_2}{X_{20}} = \frac{0.02}{0.3} = 0.06667$$

**(ii)** Slip corresponding to maximum torque

$$N_{max,\,T} = N_S(1 - S_{max,\,T})$$
$$= 250\,(1 - 0.06667) = 233.3 \text{ rpm}$$

**(iii)** Power input to rotor $= \dfrac{3E_2^2}{2X_2} = \dfrac{3\times 288.675^2}{2\times 0.3} = 416667$ W

$$\text{Maximum torque, } T_{max} = \frac{\text{Power input to rotor}}{\frac{2\pi N_s}{60}} = \frac{416667\times 60}{2\pi\times 250} = 15915 \text{ Nm}$$

**Q.7 (a) Solution:**

**(i)** Current the coil,

$$i = \frac{V}{R} = \frac{120}{6} = 20 \text{ A}$$

Because of reluctance of magnetic core is neglected, the field energy in magnetic core is negligible. All field energy is in the air gaps,

Therefore, $\quad Ni = H_g l_g = \dfrac{B_g}{\mu_0} l_g$

$$B_g = \frac{\mu_0 Ni}{2g} = \frac{4\pi\times 10^{-7}\times 300\times 20}{2\times 5\times 10^{-3}} = 0.754 \text{ Tesla}$$

Therefore the field energy is,

$$W_f = \frac{B_g^2}{2\mu_0}\times \text{Volume of air gap}$$
$$= \frac{0.754^2}{2\times 4\pi\times 10^{-7}}\times 2\times 6\times 6\times 10^{-4}\times 5\times 10^{-3} \text{ J}$$
$$= 8.1434 \text{ J}$$

The lifting force can be given as

$$f_m = \frac{B_g^2}{2\mu_0}\times \text{airgap area}$$
$$= \frac{0.754^2}{2\times 4\pi\times 10^{-7}}\times 2\times 6\times 6\times 10^{-4} \text{ N}$$
$$f_m = 1628.67 \text{ N}$$

**(ii)** For $AC$ excitation, the impedance of the coil is

$$Z = R + j\omega L$$

Inductance of coil,
$$L = \frac{N^2}{\text{Reluctance}} = \frac{N^2\mu_0 A_g}{l_g}$$

$$= \frac{300^2 \times 4\pi \times 10^{-7} \times 6 \times 6 \times 10^{-4}}{2 \times 5 \times 10^{-3}} = 40.71 \text{ mH}$$

Therefore,
$$Z = 6 + j377 \times 40.71 \times 10^{-3}$$
$$= 16.48\angle 68.65° \ \Omega$$

Current in the coil is

$$I_{rms} = \frac{120}{|z|} = \frac{120}{16.48} = 7.29 \text{ A}$$

Airgap flux density,

$$B_g = \frac{\mu_0 Ni}{2g}$$

The flux density, $B_g \,\alpha\, i$, therefore changes sinusoidally with the time. The rms value of flux density is

$$B_{rms} = \frac{\mu_0 N I_{rms}}{2g} = \frac{4\pi \times 10^{-7} \times 300 \times 7.29}{2 \times 5 \times 10^{-3}} = 0.2748 \text{ Tesla}$$

Therefore, the lifting force is

$$f_m = \frac{B_g^2}{2\mu_0} \times 2A_g$$

$$f_m \propto B_g^{\ 2}$$

$$F_m\Big|_{avg} = \frac{B_g^2}{2\mu_0}\Bigg|_{avg} \times 2A_g$$

$$= \frac{B_{rms}^2}{2\mu_0} \times 2A_g = \frac{0.2748^2 \times 2 \times 6 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 216.37 \text{ N}$$

Lifting force with ac excitation is almost one-eight as obtained with dc excitation. Therefore the lifting magnets are normally excited/operated from dc source.

**Q.7 (b) Solution:**

$$\text{Secondary load kVA per phase} = \frac{150}{3} = 50 \text{ kVA}$$

$$\text{Secondary voltage per phase} = 1100 \text{ V}$$

In Δ, $$V_p = V_l = 1.1 \text{ kV}$$

$$\therefore \quad \text{Secondary current per phase} = \frac{\text{Load kVA per phase}}{\text{Voltage per phase in kV}} = \frac{50}{1.1} \text{ A}$$

Phasor secondary current per phase referred to primary,

$$I_2' = \left(\frac{50}{1.1} \times \frac{1100}{33000}\right) \angle -\cos^{-1} 0.8°$$

$$= 1.515(0.8 - j0.6) = (1.212 - j0.909) \text{ A}$$

$$\text{Tertiary load kVA/phase} = \frac{50}{3} \text{ kVA}$$

$$\text{Tertiary voltage/phase} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$\text{Tertiary current per phase} = \frac{\text{Load kVA per phase}}{\text{Voltage per phase in kV}} = \frac{50/3}{0.231} \text{ A}$$

Phasor tertiary current per phase referred to primary,

$$I_3' = \left(\frac{50/3}{0.231}\right)\left(\frac{231}{33000}\right) \angle -\cos^{-1} 0.9°$$

$$= 0.505\,(0.9 - j0.436) = (0.4545 - j0.220) \text{ A}$$

Magnetizing current, $I_\mu = 4\%$ of rated current

$$= \frac{4}{100} \times \left(\frac{200 \times 10^3}{3 \times 33000}\right) = 0.0808 \text{ A}$$

Core loss component of no-load current,

$$I_c = \frac{1 \times 1000}{3 \times 33000} = 0.0101 \text{ A}$$

$\therefore$ Primary no-load current, $$I_0 = I_c - jI_\mu$$

$$= (0.0101 - j0.0808) \text{ A}$$

Total primary current, $$I_1 = I_2' + I_3' + I_0$$

$$= 1.212 - j0.909 + 0.4545 - j0.220 + 0.0101 - j0.0808$$

$$= 1.6766 - j1.2908 = 2.116\angle -37.6° \text{ A}$$

**Alternate Solution:**

By complex power balance in the transformer,

$$\vec{S}_{1(\text{input})} = \vec{S}_0 + \vec{S}_2 + \vec{S}_3$$

$\vec{S}_0^*$ :

$$I_m = 0.04 \times I_{\text{rated}} = 0.04 \times \frac{200 \times 10^3}{\sqrt{3} \times 33000}$$

$$I_{mL} = 0.14 \text{ A}$$

$$I_{CL} = \frac{P_C}{\sqrt{3} \times V_{L1}} = \frac{1000}{\sqrt{3} \times 33000}$$

$$I_{CL} = 0.0175 \text{ A}$$

$$I_0^* = (0.0175 + j0.14) \text{ A}$$

$$\vec{S}_0 = \sqrt{3} \times 33000(0.0175 + j0.14)$$

$$= 8.064\angle 82.88° \text{ kVA}$$

$$\vec{S}_2 = 150\angle 36.87° \text{ kVA}$$

$$\vec{S}_3 = 50\angle 25.84° \text{ kVA}$$

$$\vec{S}_1 = 8.064\angle 82.48° + 150\angle 36.87° + 50\angle 25.84°$$

$$= 204.71\angle 35.82° \text{ kVA}$$

$$\sqrt{3} V_{L1} I_1^* = 204.71\angle 35.82° \text{ kVA}$$

$$I_1^* = \frac{204.71 \times 10^3 \angle 35.82°}{\sqrt{3} \times 33000}$$

$$I_1 = 3.58\angle -35.82° \text{ A (line current)}$$

$$I_{1(\text{ph})} = \frac{3.58}{\sqrt{3}} = 2.07 \text{ A (in each phase of } \Delta)$$

**Q.7 (c) Solution:**

**(i)** Given, $G = 100 \text{ MVA}, \; H = 10 \text{ MJ/MVA}$

Kinetic energy stored in rotor $= G \cdot H \times 100 \times 10 \text{ MJ} = 1000 \text{ MJ}$

**(ii)** $P_a = P_i - P_e = (60 - 50) \text{ MW} = 10 \text{ MW}$

We know, $M = \dfrac{GH}{180 f} = \dfrac{100 \times 10}{180 \times 60} = \dfrac{5}{54}$ MJ-sec/ele-deg.

Now, $$M \cdot \frac{d^2\delta}{dt^2} = P_a$$

$$\Rightarrow \quad \frac{5}{54} \cdot \frac{d^2\delta}{dt^2} = 10$$

$$\therefore \quad \frac{d^2\delta}{dt^2} = \alpha = \frac{10 \times 54}{5} = 108 \text{ elec-deg/sec}^2$$

$$\alpha = 108 \text{ elec-deg/sec}^2$$

$$= 108 \times \frac{2}{P} \text{ mech-deg/sec}^2$$

$$= 108 \times \frac{2}{4} \times \left(\frac{60°}{360°}\right) = 9 \text{ rpm / sec}$$

**(iii)** 12 cycles is equivalent to $\frac{12}{60} = 0.2$ sec

Change in load angle, $$\Delta\delta = \frac{1}{2}\alpha(\Delta t)^2 = \frac{1}{2} \times 108 \times (0.2)^2$$

$$\Delta\delta = 2.16 \text{ elec-degree}$$

Now, $$\alpha = 108 \text{ elec-degree/sec}^2$$

$$\alpha = 60 \times \frac{108}{360} \times \frac{2}{4} = 9 \text{ rpm/sec}$$

Assuming no accelerating torque, before beginning of 12 cycle period

Thus, rotor speed at the end of 12 cycles

$$= \frac{120f}{P} + \alpha\Delta t = \left(\frac{120 \times 60}{4} + 9 \times 0.2\right)\text{rpm} = 1801.8 \text{ rpm}$$

**Q.8 (a) Solution:**

The admittance of the various lines are calculated :

| Line | Admittance (p.u.) |
|---|---|
| 1–2 | $\frac{1}{0.05 + j0.15} = 2 - j6.0$ |
| 1–3 | $\frac{1}{0.1 + j0.3} = 1 - j3.0$ |
| 2–3 | $\frac{1}{0.15 + j0.45} = 0.666 - j2$ |
| 2–4 | $\frac{1}{0.1 + j0.3} = 1 - j3.0$ |

3-4 $\qquad \dfrac{1}{0.05 + j0.15} = 2 - j6.0$

Bus admittance matrix :

$$Y_{\text{Bus}} = \begin{bmatrix} 3 - j9.0 & -2 + j0.6 & -1 + j0.3 & 0.0 \\ -2 + j6.0 & 3.666 - j11.0 & -0.666 + j2.0 & -1 + j3.0 \\ -1 + j3.0 & -0.666 + j2.0 & 3.666 - j11.0 & -2 + j6.0 \\ 0.0 & -1 + j3.0 & -2 + j6.0 & 3 - j9.0 \end{bmatrix}$$

Since all buses except slack bus are $PQ$ buses, we start the iterations with following initial voltages.

$$V_1^\circ = 1.04\angle 0^\circ$$

$$V_3^\circ = V_2^\circ = V_4^\circ = 1.0\angle 0^\circ$$

**1st Iteration :**

$$V_2' = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{0*}} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j11.0}\left[\frac{0.5 + j0.2}{1.0} - (-2 + j6)(1.04) - (-0.666 + j2.0)(1.0) - (-1 + j3)(1.0)\right]$$

$$= \frac{4.246 - j11.04}{3.666 - j11.0} = 1.02014\angle 2.60^\circ$$

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31}V_1^0 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1.0 - j0.5}{1.0} - (-1 + j3.0)(1.04) - (-0.666 + j2.0)(1.01908 + j0.04636) - (-2 + j6.0)(1.0)\right]$$

$$\frac{2.81142 - j11.62}{3.666 - j11} = \frac{11.9552\angle -76.39^\circ}{11.5948\angle -71.568^\circ} = 1.03108\angle -4.83^\circ$$

$$V_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{41}V_1^0 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

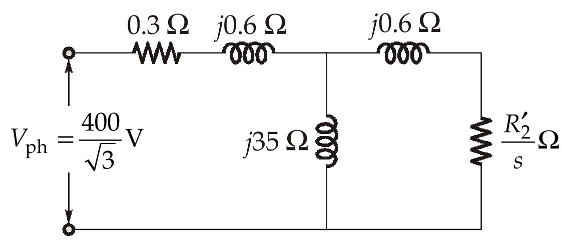
$$= \frac{1}{3 - j9.0}\left[\frac{0.3 + j0.1}{1} - (0.0) - (-1 + j3)(1.01908 + j0.04636)\right.$$

$$\left. - (-2 + j6)(1.02741 - j0.08683)\right]$$

$$= \frac{2.996 - j9.249}{3 - j9.0} = 1.0247 - 0.51$$

$$V_4' = 1.02468\angle{-0.51°}$$

**Q.8 (b) Solution:**




**(i)** At $s = 1$,

$$V_{ph} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$Z_f = j35 \,||\, (0.25 + j0.6) = 0.639\angle 67.78\ \Omega$$

$$= 0.241 + j0.591\ \Omega = R_f + jX_f$$

$$I_{st} = \frac{V_{ph}}{Z_1 + Z_f} = \frac{231}{0.3 + j0.6 + 0.639\angle 67.78°}$$

$$= 176.48\angle{-65.56°} \text{ A}$$

$$T_{st} = \frac{3I_{st}^2 R_f}{\omega_{sm}} = \frac{3 \times 176.48^2 \times 0.241}{2\pi \times 1500} \times 60 = 143.35 \text{ Nm}$$

**(ii)**

$$s_{fl} = \frac{N_S - N}{N_S} = \frac{1500 - 1450}{1500} = \frac{1}{30} = 0.033$$

$$\frac{R_2'}{s} = \frac{0.25}{\frac{1}{30}} = 7.5\ \Omega$$

$$Z_f = j35 \,||\, (7.5 + j0.6) = 7.23\angle 16.47\ \Omega$$

$$Z_f = 6.94 + j2.052\ \Omega$$

$$Z_{in} = Z_f + 0.3 + j0.6 = 7.71\angle 20.12\ \Omega$$

$$Z_{in} = 7.24 + j2.65\ \Omega$$

$$I_1 = \frac{V_{ph}}{Z_{in}} = \frac{231}{7.24 + j2.65} = 29.96\angle{-20.12°} \text{ A}$$

$$\text{Power factor} = \cos 20.12° = 0.938 \text{ lagging}$$

$$\text{Output, } P_{gross} = 3I_1^2 R_f (1 - s)$$

$$P_{gross} = 3 \times 29.96^2 \times 6.94(1 - 0.033) = 18.07 \text{ kW}$$
$$\text{Rotational loss} = 1.5 \text{ kW}$$
$$\text{Net, } P_{out} = 18.07 - 1.5 = 16.57 \text{ kW}$$
$$T_{net} = \frac{P_{out}}{\omega_n} = \frac{16.57 \times 10^3}{2\pi \times 1450} \times 60 = 109.12 \text{ Nm}$$
$$P_{in} = \sqrt{3} V_L I_1 \cos\phi_1$$
$$= \sqrt{3} \times 400 \times 29.96 \times \cos 20.12° = 19.49 \text{ kW}$$

Overall efficiency, $$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{16.57}{19.49} \times 100 = 85.02\%$$

Internal efficiency, $$\eta_{int} = \frac{P_{gross}}{P_{in}} \times 100 = \frac{18.07}{19.49} \times 100 = 92.71\%$$

**(iii)**
$$\frac{R_2'}{s} = \sqrt{R_{th}^2 + (X_{th} + X_2)^2}$$
$$Z_{th} = j35 \parallel (0.3 + j0.6) = 0.29 + j0.592 \ \Omega$$
$$\frac{0.25}{s_{max}} = \sqrt{0.29^2 + (0.592 + 0.6)^2}$$
$$s_{max} = 0.204$$
$$\frac{R_2'}{s_{max}} = \frac{0.25}{0.204} = 1.225 \ \Omega$$
$$Z_f = (j35) \parallel (j0.6 + 1.225)$$
$$Z_f = 1.183 + j0.631 \ \Omega$$
$$I_1 = \frac{V_{ph}}{Z_1 + Z_f} = \frac{231}{1.34\angle 28.1 + (0.3 + j0.6)}$$
$$= 119.89\angle -39.69° \text{ A}$$
$$P_G = 3I_1^2 R_f$$
$$= 3 \times 119.89^2 \times 1.183 = 51.01 \text{ kW}$$
$$\text{Gross } P_{out} = P_G(1 - s) = 51.01(1 - 0.204) = 40.61 \text{ kW}$$
$$\text{Net } P_{out} = P_G(1 - s) - P_{rot}$$
$$= 40.61 - 1.5 = 39.11 \text{ kW}$$
$$T_{max} = \frac{P_{out}}{\omega_m} = \frac{39.11 \times 10^3}{2\pi \times \frac{1500}{60}(1 - 0.204)} = 312.76 \text{ Nm}$$

**Q.8 (c) Solution:**

From the condition for maximum torque we have,

$$R_2 = sX_{20}$$

or
$$s = \frac{R_2}{X_{20}} = \frac{0.02}{0.1} = 0.2$$

and
$$N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{6} = 1000\,\text{rpm}$$

Substituting the values in $s = \dfrac{N_s - N_r}{N_s}$

$$0.2 = \frac{1000 - N_r}{1000}$$

$$N_r = 1000 - 200 = 800 \text{ rpm}$$

Torque,
$$T = \frac{KsR_2}{R_2^2 + s^2X_{20}^2}$$

At starting, $s = 1$,

∴ Starting torque,
$$T_{st} = \frac{KR_2}{R_2^2 + X_{20}^2}$$

Maximum torque,
$$T_m = \frac{K}{2X_{20}}$$

Here,
$$T_{st} = \frac{2}{3}T_m$$

or
$$\frac{KR_2}{R_2^2 + X_{20}^2} = \frac{K}{3X_{20}}$$

Substituting the value of $X_{20} = 0.1$,

We have7

$$\frac{R_2}{R_2^2 + (0.1)^2} = \frac{1}{3 \times 0.1}$$

or
$$0.3\,R_2 = R_2^2 + 0.01$$

or
$$R_2^2 - 0.3R_2 + 0.01 = 0$$

or
$$100R_2^2 - 30R_2 + 1 = 0$$

Calculating, $R_2 = 0.262$ or $0.038$

This $R_2$ is the value of the total rotor circuit resistance,

Rotor winding resistance/phase = 0.02

Therefore, extra resistance = $0.262 - 0.02 = 0.242\ \Omega$

or $0.038 - 0.02 = 0.018\ \Omega$

$R_{ext} = 0.018\ \Omega$ should be chosen to keep rotor copper losses at a low value.

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