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Detailed Solutions

ESE-2024 Mains Test Series

Mechanical Engineering Test No : 5

Section A : Production Engineering & Material Science [All Topics]
Section B : Theory of Machines-1 [Part Syllabus]
Fluid Mechanics & Turbo Machinery-2 [Part Syllabus]

Section : A

1. (a)

Machining allowance :

- Machining or finishing allowance is the extra material provided on certain details of a casting so that the casting may be machined to exact dimensions. The machining allowance depends on the following factors:
  (i) Casting process.
  (ii) Size of the casting.
  (iii) Degree of finish.
  (iv) Machining method.
  (v) Metallic alloy from which the casting is made.
- The amount of this allowance varies from 1.6 mm to 12.5 mm.
- The ferrous metals require more machining allowance than non-ferrous metals.

Rapping or shaking allowance :

- This allowance is provided to compensate for enlargement of the mould cavity because of excessive rapping.
- In small and medium-sized castings, this allowance can be neglected. But in larger casting this allowance is considered by making the part slightly smaller than the

casting (i.e., the allowance is a negative one as the pattern is made smaller to allow rapping operation).

Distortion allowance:

- This allowance is provided on the pattern to compensate for possible distortion of the casting because of the unequal cooling rates of different sections of the casting and uneven internal stresses.
- Such an allowance depends on the judgement and experience of the pattern maker, who understands the shrinkage characteristics of the metal.

1. (b)

Given : d_o = 15 mm; σ_0 = 240 N/mm^2; μ = 0.20; η_{motor} = 95%; RA = 0.3;

$B = \mu \cot\alpha = 0.1 \times \cot 6^\circ = 0.95$

Now,
$$\sigma_d = \frac{\sigma_0(1+B)}{B}\left[1-\left(\frac{r_1}{r_0}\right)^{2B}\right]$$

$$RA = 1-\left(\frac{r_1}{r_0}\right)^2$$

$$\Rightarrow \quad \left(\frac{r_1}{r_0}\right)^2 = 0.7$$

$$\therefore \quad \sigma_d = \sigma_0\frac{(1.95)}{0.95}\left[1-0.7^{0.95}\right] = 141.58$$

Now,
$$r_1 = \sqrt{0.7}\times 7.5 = 6.275 \text{ mm}$$

$$\therefore \quad F_d = 141.58 \times \pi(6.275)^2$$
$$= 17.51 \text{ kN}$$ **Ans.**

$$\text{Power of the motor, } P = \frac{17.51\times 2.5}{0.95} = 46.08 \text{ kW}$$ **Ans.**

1. (c)

Various types of chips produced are:

(i) Continuous chips

(ii) Built-up edge

(iii) Serrated or segmented

(iv) Discontinuous chips

Continuous chips: Continuous chips are usually formed under the following conditions:

1. Machining ductile materials
2. High cutting speeds
3. High rake angles
4. Low depth of cuts
5. Effective cutting fluid

Although these produce good surface finish, continuous chips are not always desirable as they tend to become tangled and interfere with the machining process.

Built-up edge chips: A built-up edge consists of layers of materials from the workpiece that are gradually deposited on the tool, may form at the tip of the tool during cutting. As its becomes larger, the BUE becomes unstable and eventually breaks up. Although BUE is generally undesirable, a thin, stable BUE is regarded as desirable because it reduces wear by protecting rake face. Conditions that promote BUE chips are machining ductile materials at low cutting speeds, high feed and depth of cut and ineffective cutting fluid.

Serrated chips: Serrated chips are semi-continuous chips with zones of low and high shear strain. Metals with low thermal conductivity and strength that decreases with temperature sharply, such as titanium exhibit this behaviour.

Discontinuous chips: These chips are formed under the following conditions:

1. Brittle workpiece materials.
2. Workpiece materials that contain hard inclusions and impurities.
3. Very low or very high cutting speeds.
4. Large depth of cut.
5. Low rake angles.
6. Lack of effective cutting fluid.

1. (d)

Ultrasonic machining (USM) is a mechanical metal-removal process. A schematic of an ultrasonic machining set-up is shown in figure (a). The transducer generates the high frequency vibrations of the order of 20 to 30 kHz with an amplitude of the order of 0.02 mm. This vibration is transmitted to the tool made of soft material, through a mechanical coupler known as tool holder. The tool shape is a close complementary shape of the final surface to be generated.

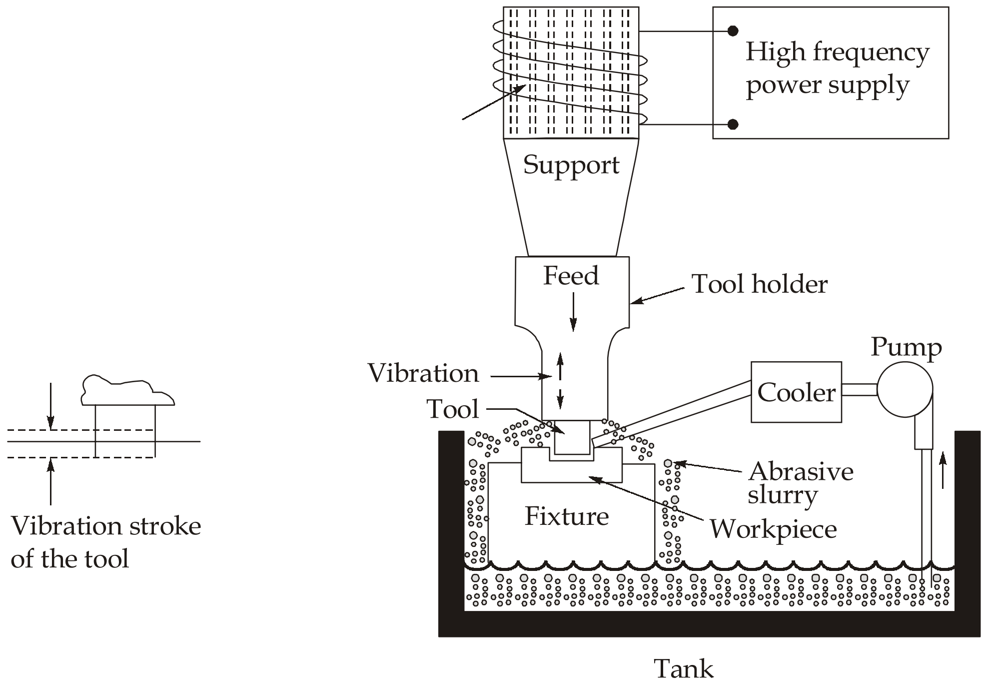


Figure (a)

The tool while oscillating would be pressed against the workpiece and fed continuously. A slurry of abrasive grains suspended in a liquid is fed into the cutting zone under pressure as shown in figure (b). The slurry is about 30% concentration. Abrasive particles are driven into the work surface by the oscillating tool. The force is typically about 150000 times the weight of the individual grains. A small crater will be formed at the impact site of the grain, if the workpiece is brittle. A very large number of such small craters remove sufficiently large material from the workpiece.

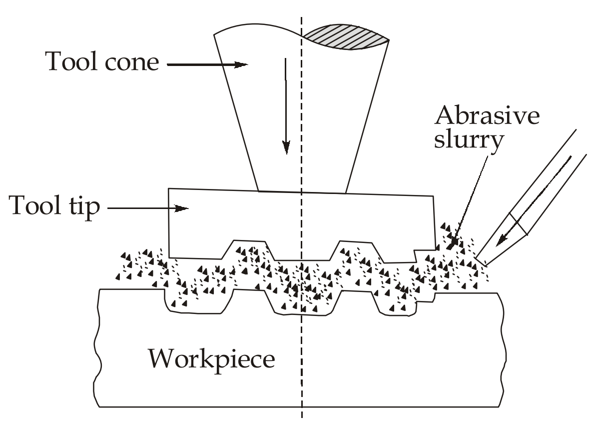


Figure (b) : Schematic of the material-removal process in ultrasonic machining

As the material is removed, the tool is gradually advanced into the workpiece by a servomechanism such that constant gap is maintained between the tool and the workpiece. Finally the shape of the tool is impressed into the workpiece as shown in figure (b).

Advantages of USM

- USM is used for machining hard and brittle materials to complex shapes with good accuracy and reasonable surface finish.
- It is not affected by the electrical or chemical characteristics of the work material.
- Holes of any shape can be produced.

Limitations of USM

- Metal-removal rates are low.
- Depth of hole produced is limited.
- Tool wear is high and sharp corners cannot be produced.
- Flat surfaces cannot be produced at the bottom of the cavity because of the ineffective slurry distribution.

1. (e)

W_{Ag} is the weight percent of Silver = 80%

W_{Cu} is the weight percent of Copper = 20%

C_{Ag} is the atom percent of silver

C_{Cu} is the atom percent of Copper

$$\therefore \quad C_{Ag} = \frac{W_{Ag} \times A_{Cu}}{W_{Ag} \times A_{Cu} + W_{Cu} \times A_{Ag}} \times 100$$

$$C_{Ag} = \frac{80 \times 63.55}{80 \times 63.55 + 20 \times 107.87} \times 100$$

$$\therefore \quad C_{Ag} = 70.2\%$$ **Ans.**

$$C_{Cu} = \frac{W_{Cu} \times A_{Ag}}{W_{Ag} \times A_{Cu} + W_{Cu} \times A_{Ag}} \times 100$$

$$= \frac{20 \times 107.87}{80 \times 63.55 + 20 \times 107.87} \times 100 = 29.8\%$$

2. (a) (i)

Laws of corrosion : During the electrochemical process, the corrosion products form a film on the metal surface, which can be either porous (non-protecting) or non-porous (protecting). There are two types of films: (1) adherent film, in which corrosion products compress upon the surface and (2) expanding film, i.e. in porous corrosion, the film expands to cover up the surface. Based on the nature of films produced on metals, there are three laws of corrosion:

1. Linear law
2. Parabolic law
3. Logarithmic law

If x is the thickness of the film and t is the time for corrosion process, then

Linear law, $x = A_L t,$

where A_L, is the constant for linear corrosion,

Parabolic law, $x = A_P\sqrt{t},$

where A_P, is the parabolic rate constant,

Logarithmic law, $x = A_o \log\left(1+\frac{t}{t_1}\right)$

where A_0 is the logarithmic rate constant and t_1 is any arbitrary time.

Following are few examples of laws of corrosion:

1. Corrosion in plain carbon steels, magnesium, barium and niobium follows linear law.
2. Corrosion in copper, stainless steel, and silicon follows parabolic law.
3. Corrosion in nickel, aluminium, zinc and chromium follows logarithmic law.

2. (a) (ii)

Let M stands for molecular weight, ρ stands for density, c stands for corrosion product and m stands for metal.

Densities :

$$\rho_{Mg} = 1738 \text{ kg/m}^3 = 1.738 \text{ kg/cm}^3$$
$$\rho_{MgO} = 3650 \text{ kg/m}^3 = 3.65 \text{ kg/cm}^3$$
$$M_{Mg} = 24.31$$
$$M_{MgO} = 24.31 + 16 = 40.31 \text{ g}$$

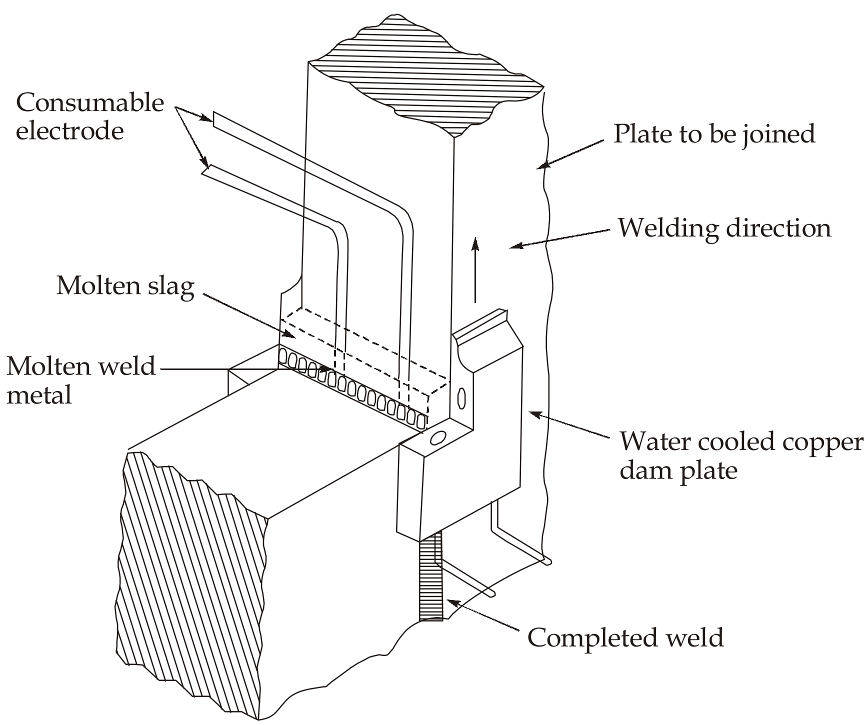
Now, $$\frac{M_c}{\rho_c} = \frac{M_{MgO}}{\rho_{MgO}} = \frac{40.31}{3.65} = 11.0438$$

and $$\frac{M_m}{\rho_m} = \frac{M_{Mg}}{\rho_{Mg}} = \frac{24.31}{1.738} = 13.987$$

$$\because \quad \frac{M_c}{\rho_c} < \frac{M_m}{\rho_m}, \text{ Hence porous film is formed}$$

2. (b) (i)

The electro slag welding (ESW) process is developed essentially to weld very large plates without any edge preparation. This is essentially a single pass process using a consumable electrode for filling the gap between the two heavy plates. The heat required for melting the plates and the electrode is obtained initially by means of an arc so that flux will form the molten slag. Once the molten slag is formed, the arc is extinguished and the heat of welding is obtained by the resistance heating of the slag itself. The typical electro slag welding setup is shown in figure below. For the effective welding it is necessary to maintain a continuous slag pool and therefore the best way to maintain it, is to weld vertically. The slag pool is contained in the groove with the help of two water cooled copper dam plates which move along with the weld, as shown. The size and type of electrodes chosen depending on the width of the joint. In figure, two electrodes for feeding through the feed rollers (not shown) into the weld zone, are shown.



Electro slag welding process

The flux required to maintain a satisfactory amount of slag is fairly small, of the order of 0.2 to 0.3 kg per metre of weld length irrespective of the plate thickness. Thus, the heat utilised for melting the slag is much less. Most of the heat supplied in electro slag welding thus, melts the joint. By this process, a plate of thickness, 200 mm, can easily be welded in a single pass. Because of the vertical welding, any gas present easily bubbles out through the slag and, therefore, better welds can be produced. The heating and cooling of the edge is more gradual. The slag floating at the top would be preheating the joint before the actual melting by the heat liberated from the electrode. Whatever be the thickness of the plate no edge preparation is required. Electro slag welding is useful for welding very thick plates.

2. (b) (ii)

V = 90 mm^3; I = 20000 A; E = 15 J/mm^3; Q_{lost} = 750 J; R = 0.0002 ohms

Total energy supplied during process = 90 × 15 + 750 = 2100 J

Also, $$i^2Rt = 2100 \text{ J}$$

$$\therefore \quad t = \frac{2100}{(20000)^2 \times 0.0002} = 0.02625 \text{ sec} \qquad \textbf{Ans. (i)}$$

$$\text{Thermal efficiency} = \frac{\text{Heat required}}{\text{Heat supplied}} \times 100$$

$$\therefore \quad \eta = \frac{90 \times 15}{2100} \times 100 = 64.28\% \qquad \textbf{Ans. (ii)}$$

2. (c) (i)

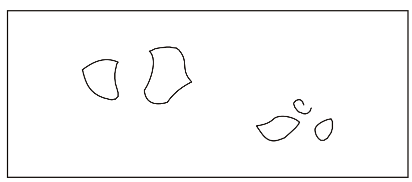
1. **Mis-runs and cold shuts:** Mis-run is caused when the metal is unable to fill the mould cavity completely and thus leaves unfilled cavities. A cold shut is caused when two metal streams while meeting in the mould cavity, do not fuse together properly, thus causing a discontinuity or weak spot in the casting. Sometimes a condition leading to cold shuts can be observed when no sharp corners exist in a casting.

   These defects are caused essentially, by the lower fluidity of the molten metal or when the section thickness of the casting is too small. The latter can be rectified by proper casting design. The remedy available is to increase the fluidity of the metal by changing the composition or raising the pouring temperature. This defect can also be caused when the heat removal capacity is increased such as in the case of green sand moulds. The castings with large surface area to volume ratio are more likely to be prone to these defects. Further cause of this defect is the back pressure due to gases in the mould which is not properly vented. The remedies are basically improving the mould design.

2. **Slag inclusions:** During the melting process, flux is added to remove the undesirable oxides and impurities present in the metal. At the time of tapping, the

slag should be properly removed from the ladle, before the metal is poured into the mould. Otherwise any slag entering the mould cavity will be weakening the casting and also spoil the surface of the casting. This can be eliminated by some of the slag trapping methods such as pouring basin screens or runner extersions.

3. **Blow holes and open blows:** These are the spherical, flattened or elongated cavities present inside the casting or on the surface as shown in figure. On the surface they are called open blows and inside, they are called blow holes. These are caused by the moisture left in the mould and the core. Because of the heat in the molten metal, the moisture is converted into steam, part of which when entrapped in the casting ends up as blow hole or ends up as open blow when it reaches the surface. Apart from the presence of moisture, they occur due to the lower venting and lower permeability of the mould. Thus, in green sand moulds it is very difficult to get rid of the blow holes, unless proper venting is provided. It can be also prevented by converted green sand moulds into dry or hot sand moulds.



Open blows on the surface of the casting

2. (c) (ii)

$$\text{Volume of the casting} = 30 \times 30 \times 10 = 9000 \text{ cm}^3$$

$$\text{Surface area of casting} = 2 \times 30 \times 30 + 4 \times 30 \times 10 = 3000 \text{ cm}^2$$

$$\text{Volume of the riser} = \frac{\pi}{4}D^3, \quad \text{where } D \text{ is the riser diameter and } D = H$$

$$\text{Surface area of the riser} = \pi D^2 + \frac{\pi D^2}{2} = 1.5\pi D^2$$

$$\text{Freezing ratio, } x = \frac{\left(\frac{V}{A}\right)_{riser}}{\left(\frac{V}{A}\right)_{casting}} = \frac{0.25\pi D^3}{1.5\pi D^2} \times \frac{3000}{9000} = 0.056D$$

and

$$y = \frac{V_{riser}}{V_{casting}} = \frac{0.25 \times \pi D^3}{9000}$$

$$y = 8.727 \times 10^{-5} D^3$$

Substituting this in the Caines's equation

$$x = \frac{a}{y-b} + c$$

$$0.056D = \frac{0.1}{8.727 \times 10^{-5} D^3 - 0.03} + 1$$

On solving above equation, we get

$$D = 20.49 \text{ cm} \simeq 21 \text{ cm}$$

$$D = H = 21 \text{ cm}$$ **Ans.**

3. (a)

$$\Delta h = 2R(1 - \cos\alpha)$$

$$5 = 500(1 - \cos\alpha)$$

$\therefore$ $$\alpha = 8.11°$$

$\therefore$ $$\mu = \tan\alpha = \tan 8.11° = 0.142$$ **Ans.**

Now, $$H_0 = 2\sqrt{\frac{R}{h_1}}\tan^{-1}\left(\sqrt{\frac{R}{h_1}}\cdot\alpha\right)$$

$$= 2\sqrt{\frac{250}{25}}\tan^{-1}\left(\sqrt{\frac{250}{25}}\times 0.1415\right) = 2.661$$

$$H_n = \frac{1}{2}\left(H_0 - \frac{1}{\mu}\ln\frac{h_0}{h_1}\right)$$

$$= \frac{1}{2}\left(2.661 - \frac{1}{0.142}\ln\left(\frac{30}{25}\right)\right) = 0.688$$

$$\theta_n = \sqrt{\frac{h_1}{R}}\tan\left(\sqrt{\frac{h_1}{R}}\times\frac{H_n}{2}\right)$$

$$= \sqrt{\frac{25}{250}}\tan\left(\sqrt{\frac{25}{250}}\times\frac{0.688}{2}\right)$$

$\therefore$ $$\theta_n = 0.0345 \text{ radian}$$ **Ans.**

$$h_n = h_1 + 2R(1 - \cos\theta_n)$$

$$= 25 + 500(1 - \cos(0.0345))$$

$$h_n = 25.3 \text{ mm}$$

$$\text{Backward slip} = \frac{V_r - V_0}{V_r} = 1 - \frac{V_0}{V_r}$$

$$= 1 - \frac{h_n}{h_o} = 1 - \frac{25.3}{30} = 0.1566 \text{ or } 15.66\%$$

$$\text{Forward slip} = \frac{V_1 - V_r}{V_r} = \frac{V_1}{V_r} - 1 \qquad \textbf{Ans.}$$

$$= \frac{h_n}{h_1} - 1 = \frac{25.3}{25} - 1$$

$$= 0.012 \text{ or } 1.2\% \qquad \textbf{Ans.}$$

$$\text{Maximum pressure, } P_{max} = \sigma_0' \frac{h_n}{h_1} \times e^{\mu H_n}$$

$$= \frac{2}{\sqrt{3}} \times 120 \times \frac{25.3}{25} \times e^{0.142 \times 0.688}$$

$$= 154.6 \text{ N/mm}^2 \qquad \textbf{Ans.}$$

3. (b) (i)

Pearlite

In pearlite, there are alternate strips (layers) of ferrite and cementite. Ferrite is soft but cementite is harder and more brittle than ferrite. Rather, cementite reinforces the ferrite. If ultimate strength, yield strength and hardness of the steel are plotted with respect to carbon percentage, all three parameters increase with increasing carbon percentage. However, since cementite is more brittle, the ductility and toughness of the steel decrease with increase in carbon percentage.

Moreover, fine pearlite is harder and stronger than coarse pearlite, but coarse pearlite is more ductile than fine pearlite.

For fine pearlite, there are more boundaries through which dislocation must pass during plastic deformation. So with greater reinforcement and restriction of dislocation motion in fine grains, pearlite accounts for its higher strength and hardness.

Spheroidite

In pearlite, there are alternate layers of ferrite and cementite, in which cementite reinforces the ferrite, but in spheroidite structure there are spheres of cementite in ferrite matrix. There is less boundary per unit volume in spheroidite; therefore, plastic deformation is not much constrained, giving rise to a soft and weak material. Moreover, spheroidized steels are extremely ductile, much more than either fine or coarse pearlite.

Bainite

Bainite steels have finer structure (structure contains α-ferrite matrix with cementite particles). They are harder and stronger than pearlite steels, even then they exhibit desirable combination of strength and ductility, i.e. toughness.

Martensite

Among the microstructures developed in steels, martensite is the hardest and strongest, yet most brittle, because of its negligible ductility. The hardness of martensite is dependent on carbon content. The properties of martensite are not due to its microstructure, but due to the effectiveness of interstitial carbon atoms in impeding dislocation motion. BCT structure of martensite has a few slip planes.

On quenching of austenite to BCT structure, there is net increase in volume because austenite is denser than martensite. If carbon content is greater than 0.5 wt%, then there are chances of quenching cracks.

3. (b) (ii)

$\lambda = 1.54\text{Å};\ \theta = 7.5°;\ n = 1$

$$\text{Interplanar distance, } d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

d_{200} = Mean distance for (200) plane,

where $h = 2;\ k = 0;\ l = 0$

$$\therefore \quad d_{200} = \frac{a}{\sqrt{2^2+0^2+0^2}} = 0.5a$$

As per Bragg's law,

$$2d\sin\theta = n\lambda$$

$$\text{or} \quad 2 \times d \times \sin 7.5° = 1 \times \lambda$$

$$\therefore \quad 2 \times 0.5a \times \sin 7.5° = 1.54$$

$$\therefore \quad a = \frac{1.54}{2 \times 0.5 \times \sin 7.5°}$$

$$a = 11.8\text{Å}$$ **Ans.**

3. (c)

From tool designation, $\alpha = 10°;\ \lambda = 90°$

$d = 2$ mm, $t = f = 0.1$ mm, $t_c = 0.4$ mm, $F_c = 1500$ N, $F_t = 900$ N

$$\text{Now,} \quad \tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$

$$r = \frac{t}{t_c} = \frac{0.1}{0.4} = 0.25$$

$$\therefore \quad \phi = 14.43°$$

$$\therefore \quad \text{Shear force, } F_s = F_c\cos\phi - F_t\sin\phi$$

$$F_s = 1500\cos 14.43° - 900\sin 14.43°$$

$$\therefore \quad F_s = 1228.4 \text{ N}$$ **Ans.(i)**

Normal force at shear plane,

$$F_n = F_c \sin\phi + F_t \cos\phi$$
$$= 1500\cos(14.43°) - 900\sin(14.43°)$$
$$= 1245.4 \text{ N}$$ **Ans.(ii)**

Friction force, $F = F_c \sin\alpha + F_t \cos\alpha$
$$= 1500\sin(10°) + 900\cos(10°)$$
$$= 1146.8 \text{ N}$$ **Ans.(iii)**

$$\mu = \frac{F_c \tan\alpha + F_t}{F_c - F_t \tan\alpha}$$

$$\mu = \frac{1500\tan 10° + 900}{1500 - 900\tan 10°} = 0.868$$ **Ans.(iv)**

Specific cutting energy, $e = \frac{F_c}{bt} = \frac{1500}{2 \times 0.1} = 7500 \text{ N/mm}^2$ **Ans. (v)**

4. (a) (i)

Different tool wear causes that are responsible for wearing of tool are:

(i) Sliding of chip along the rake face.

(ii) Sliding of tool along the freshly cut surface (machined surface).

(iii) High localized stresses.

(iv) High temperatures promoting diffusion and adhesion processes at contact areas.

Flank wear: Flank wear occurs on the relief face of the tool and is generally attributed to (a) rubbing of the tool along the machined surface causing adhesive and abrasive wear, (b) high temperatures affect tool-material properties as well as workpiece surface. Tool life can be expressed as the time it takes to develop a certain maximum allowable width of flank wear land.

Crater wear: Crater wear occurs on the rake face of the tool and it changes the chip-tool interface geometry affecting the cutting process. The most significant factors influencing crater wear are (a) temperature at the tool-chip interface and (b) the chemical affinity between the tool and workpiece materials. Crater wear occurs due to mainly diffusion mechanism that is the movement of atoms across the tool-chip interface. The crater wear increases as the temperature increases and the location of maximum depth of crater wear coincides with the location of the maximum temperature at the tool chip interface.

4. (a) (ii)

Tooling cost, c_t = Tool change + Tool regrind + Tool cost depreciation

$$\therefore \quad c_t = \frac{5}{60}\times 10+\frac{5}{60}\times 5+0.3 = ₹1.55\,/\text{-}$$

$$\text{Machining cost, } c_m = ₹\,\frac{5}{60}$$

$$\therefore \quad V_T = c\left[\frac{c_m}{c_t}\cdot\frac{n}{1-n}\right]^n$$

$$= 200\left[\frac{5}{60\times 1.55}\times\frac{0.25}{0.75}\right]^{0.25}$$

$$V_T = 73.17 \text{ m/min}$$ **Ans.**

4. (b) (i)

Jominy bar test is most commonly used to determine the depth to which a piece is hardened by water quenching. The factors that affect the hardenability, i.e. size, shape of the specimen, quenching media and method of cooling are maintained constant during the test. A specimen of cylindrical size of 12.7 mm diameter and 100 mm long, with a collar at one end is held in a fixture as shown in figure (a) below.

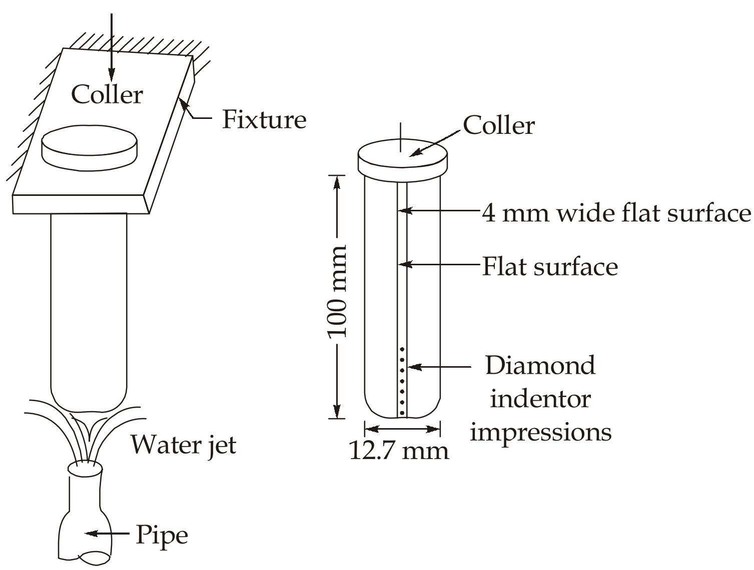


Figure (a) Jominy End Quench Test

The specimen is austenized for a prescribed time period in a furnace, then it is taken out of the furnace and held in a fixture as shown in figure (a). Lower end of the specimen is quenched by a jet of water, at specific flow rate and at room temperature. Cooling rate is maximum at the quenched end. After the specimen is cooled to room temperature, it is taken out from fixture and a flat surface of 4 mm wide and 0.4 mm deep is ground on the surface as shown in Figure (a). Rockwell hardness *C* test is performed on the flat surface at regular interval of 1.6 mm from quench end up to 38 mm and then at an interval of 3.2 mm up to the end. Rockwell hardness *C* readings are plotted along the length of the specimen as shown in figure (b).

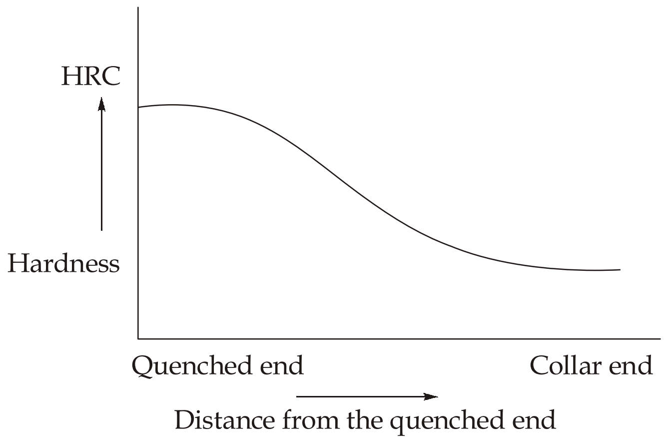


Figure (b) : Jominy Bar Quench Test

Figure (b) shows that the maximum hardness is obtained at quenched end, because quenched end cools most rapidly and 100 per cent martensite is formed. At the collar end, the microstructure of steel contains martensite and troostite. So, hardness is less.

4. (b) (ii)

Spheroidize Annealing

Medium-and high-carbon steels contain coarse pearlite. These steels are difficult to machine and to deform plastically. These steels are heat treated (annealed) to develop spheroidite structure of Fe_3C (cementite) embedded in a matrix of a phase (ferrite) of iron. These steels are heated below the lower critical temperature at about 600°C, soaked at this temperature for about 18-24 h and then slowly cooled at 600°C. This process is costly and time-consuming. The microstructure of spheroidized steel is shown in figure (a). During the annealing process, Fe_3C coalesces to form the spheroid particles.

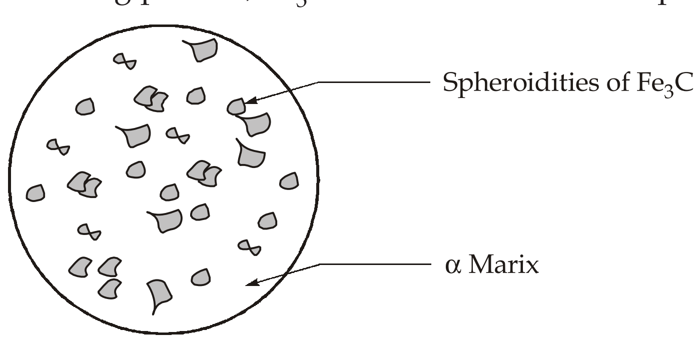


Figure (a) Spheroidite

In spheroidite, there is less boundary area per unit volume and plastic deformation of such steels becomes easier, giving rise to a weak and soft material.

TEMPERING

After quench hardening, steel becomes brittle. Moreover, quench cracks are developed and residual stresses due to warping are induced in steel. To remove all these defects, hardening is always followed by tempering.

Tempering is a subcritical heat treatment process used to improve the toughness of the hardened steels. Tempering temperatures are less than the lower critical temperature, but proper tempering temperature depends on the composition of steels and mechanical properties desired. Figure (b) shows the variation of impact strength and hardness on the variation of tempering temperature.

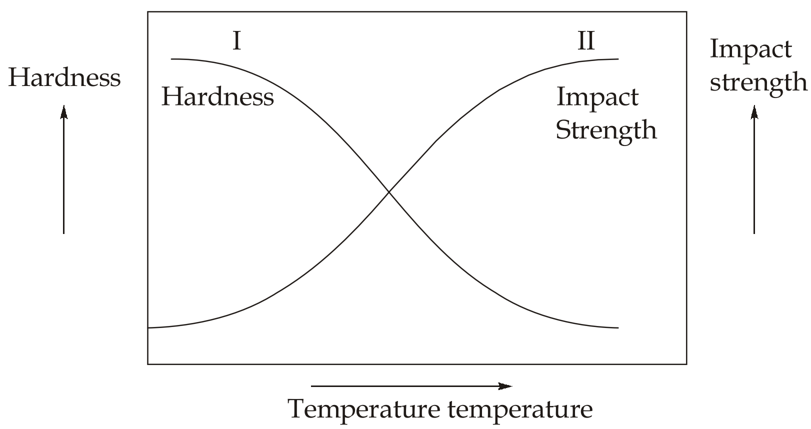


Figure (b) : Variation of Hardness and Impact Strength with Tempering Temperature

In the tempering process, the article is heated in a furnace to a desired temperature-soaked for 2h and slowly cooled in air. During tempering, following microstructural changes take place:

1. At low tempering temperature, a fine transition carbide (Fe_3C) is distributed parallel to the martensite platelets.
2. During tempering, some austenite is retained, which is transferred to cementite and ferrite.
3. At higher temperatures, transition carbide transforms to equilibrium carbide Fe_3C (spheroids) as in the case of spheroidizing process.

If the tempering is performed above 300°C, the residual stresses are completely removed.

Nitriding

Nitrogen in monoatomic form is diffused into the surface of the steel, and very hard nitrides of iron or nitrogen alloy compounds are formed. The resulting nitride case is much harder than carburized case. Subcritical temperatures are used and hardness is achieved without quenching. Hardening is performed in a nitrogen atmosphere that prevents scaling and discolouration.

Source of nitrogen used in diffusion process is ammonia and the nitriding temperature is 500-575°C.

Chemical reaction, $2NH_3 \xrightarrow[\text{Heating}]{} 2N + 3H_2$

Nitrogen is diffused into the steel and hydrogen is exhausted. After nitriding, the parts may be slowly cooled in a retort.

Nitrogen diffusion and formation of hard nitrides can be enhanced by the presence of elements such as Al, Cr, Mo, V and W in the steel. The most common nitrided steels are Mo-Cr steels, i.e. nitralloys. The addition of 1 per cent aluminium greatly enhances the formation of hard nitrides.

4. (c)

It is a well-known fact that the actual surface after machining may look smooth but in reality it is not. All surfaces have some degree of roughness and inaccuracy. When magnified, the surface of a part resembles a series of jagged peaks and valleys. Hence, the surface finish of any given part is described in terms of average heights and depths of these "peaks and valleys" on the surface of the workpiece.

Typical surface-texture characterisation is shown in figure (a). These are some of the parameters that need to be understood to fully characterise the surface texture.

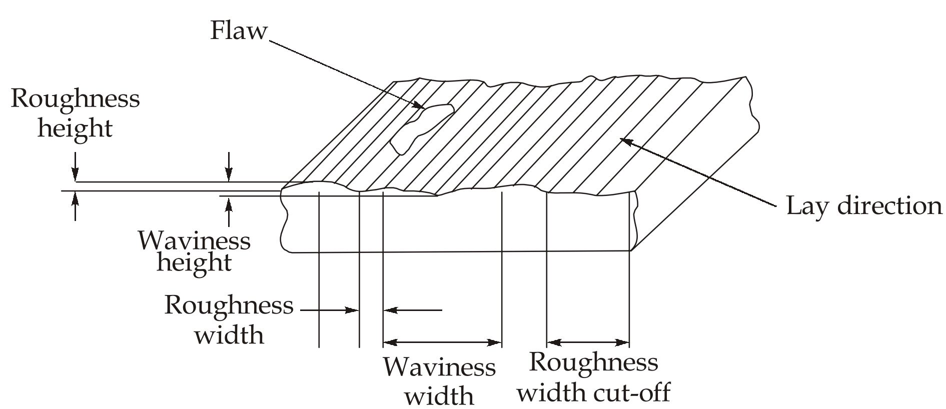


Figure (a)

Roughness Height : This is the parameter with which generally the surface finish is indicated. It is specified either as arithmetic average value or the root-mean-square value.

Roughness Width : It is the distance parallel to the nominal part surface within which lie the peaks and valleys, which constitute the predominant pattern of the roughness.

Roughness Width Cut-off : This is the maximum width of the surface that is included in the calculation of the roughness height.

Arithmetical Average : An imaginary centreline is imposed at a point representing the average midpoint or centre of the distance between the peaks and the valleys of the surface profile as shown in figure (b) below. These are measured for a specified area and the figures are added together and the total is then divided by the number of measurements taken to obtain the mean or arithmetical average (AA). It is also sometimes called the centreline average or CLA value. This in equation form is given by

$$R_a = \frac{1}{L}\int_0^L |y(x)|\,dx \cong \frac{1}{N}\sum |y_i|$$

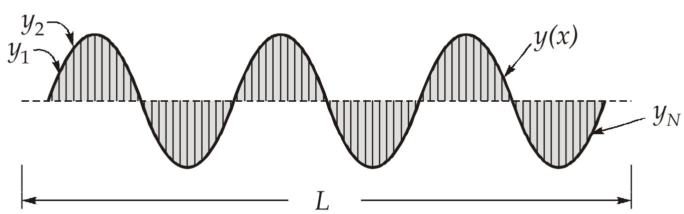


Figure (b) : Surface roughness parameters

The other parameter that is sometimes used is the root-mean-square value of the deviation in place of the arithmetic average, R_{rms}. This in expression form is

$$R_{\text{rms}} \cong \sqrt{\frac{1}{N}\Sigma y_i^2}$$

Waviness : Waviness refers to those surface irregularities that have a greater spacing than that of roughness width. It is determined by the height of the waviness and its width. The greater the width, the smoother is the surface and thus is more desirable. Also the greater the width, the greater is the difference between the size of the measurement units required to measure height (roughness) and those needed to measure the waviness width.

Lay Direction : It is the direction of the predominant surface pattern produced on the workpiece by the tool marks.

Flaw : These are surface irregularities that are present which are random and, therefore, will not be considered.

Generally, the surface roughness is measured by a stylus type of instrument as shown in figure (c). The stylus moves over the sample length of the surface and records the peaks and valleys of the surface as a set of digitised points of the surface. These will then be fed into the computer, where various types of parameters that are relevant for analysing the surface texture can be calculated.

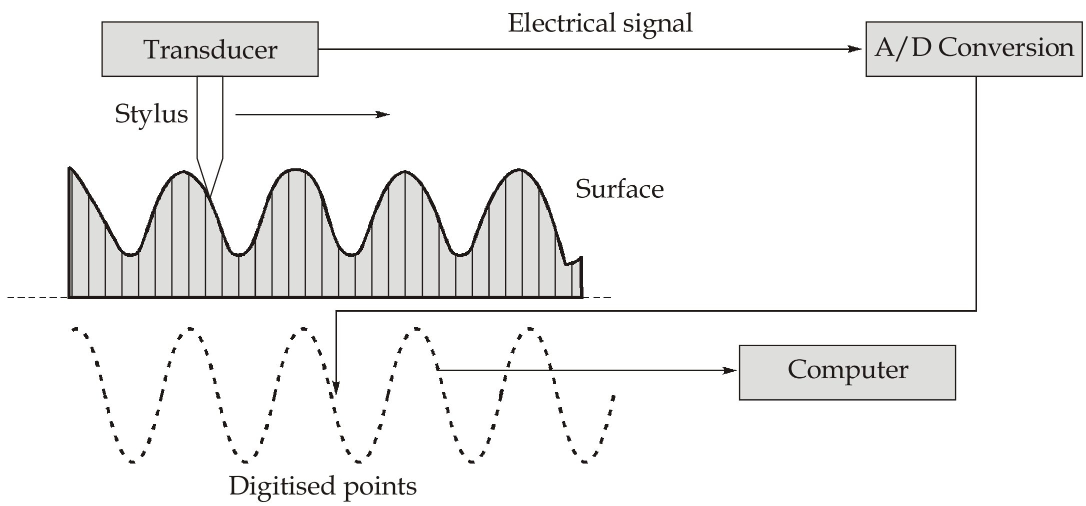


Figure (c) : Principle of surface-roughness measurement

Section : B

5. (a)

Given : $Q_T = 12.6\ l/s$; $D_1 = 0.12$ m; $D_2 = 0.15$ m; $L = 1200$ m; $S_{oil} = 0.97$; $\nu = 9 \times 10^{-4}\ m^2/s$

Assuming the flow in each pipe to be laminar, the head loss is given by,

$$h_f = \frac{32\mu \bar{U} L}{\rho g D^2} = \frac{128\mu Q L}{\pi \rho g D^4}$$

Let Q_1 and Q_2 be the flow rates through pipe of diameter D_1 and D_2, respectively, which being in parallel, we have

$$h_{f_1} = h_{f_2}$$

$$\Rightarrow \quad \frac{128\mu Q_1 L}{\pi \rho g D_1^4} = \frac{128\mu Q_2 L}{\pi \rho g D_2^4}$$

$$\Rightarrow \quad \frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^4$$

$$\Rightarrow \quad Q_1 = \left(\frac{0.12}{0.15}\right)^4 Q_2$$

$$\Rightarrow \quad Q_1 = 0.4096 \times Q_2 \quad \text{...(i)}$$

Also, $$Q_T = Q_1 + Q_2 = 12.6 \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$0.4096 \times Q_2 + Q_2 = 12.6$$

$$Q_2 = 8.9387 \ l/s$$ **Ans.**

$$Q_1 = 0.4096 \times 8.9387 = 3.6613 \ l/s$$ **Ans.**

∴ The head loss through the pipe,

$$h_f = \frac{128\mu Q_1 L}{\pi \rho g D_1^4}$$

$$\frac{128\mu Q_1 L}{\pi \rho g D_1^4} = \frac{128 \times 9 \times 10^{-4} \times 3.6613 \times 10^{-3} \times 1200}{\pi \times 9.81 \times 0.12^4}$$

$$\Rightarrow \quad h_f = 79.2 \text{ m}$$

∴ Power of the pump, $P = \rho Q g h_f$

$$= 970 \times 12.6 \times 10^{-6} \times 9.81 \times 79.2$$

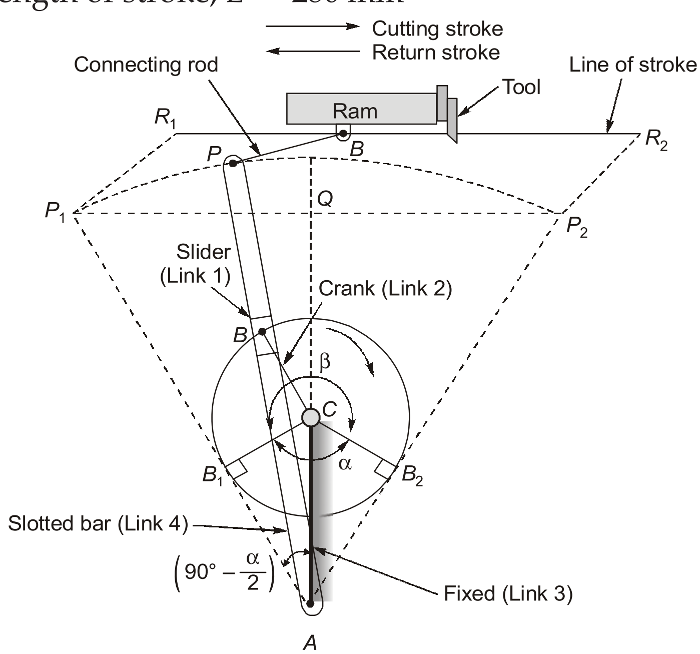
$$= 9495.91 \text{ W}$$

$$= 9.49 \text{ kW}$$ **Ans.**

5. (b)

Given : Quick return ratio, QRR = 2

Length of stroke, L = 280 mm



Crank and slotted lever quick return motion mechanism

$$\text{QRR} = \frac{[V_{s,\max}]_{return}}{[V_{s,\max}]_{forward}} = \frac{c+r}{c-r} = 2$$

where, c = centre distance between fixed centre; r = crank radius

$$\Rightarrow \quad c + r = 2c - 2r$$

$$\Rightarrow \qquad c = 3r \qquad ...(i)$$

$$\therefore \qquad QRR = \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{360° - \alpha}{\alpha}$$

$$\Rightarrow \qquad 2 = \frac{360° - \alpha}{\alpha}$$

$$\Rightarrow \qquad \alpha = 120° \text{ and } \beta = 360° - 120° = 240°$$

$$\text{Length of slotted lever, } L_s = \frac{\left(\frac{L}{2}\right)}{\sin\left(90° - \frac{\alpha}{2}\right)} = \frac{\left(\frac{280}{2}\right)}{\sin\left(90° - \frac{120°}{2}\right)}$$

$$= \frac{140}{\sin 30°} = 280 \text{ mm}$$ **Ans.**

$$\text{Maximum cutting speed, } (V_s)_{max} = \omega\, r \cdot \frac{L_s}{c + r} = \left(\frac{2\pi \times 45}{60}\right) \times \frac{280 \times r}{(3r + r)} \times \frac{1}{1000}$$

$$= 0.33 \text{ m/s}$$ **Ans.**

5. (c)

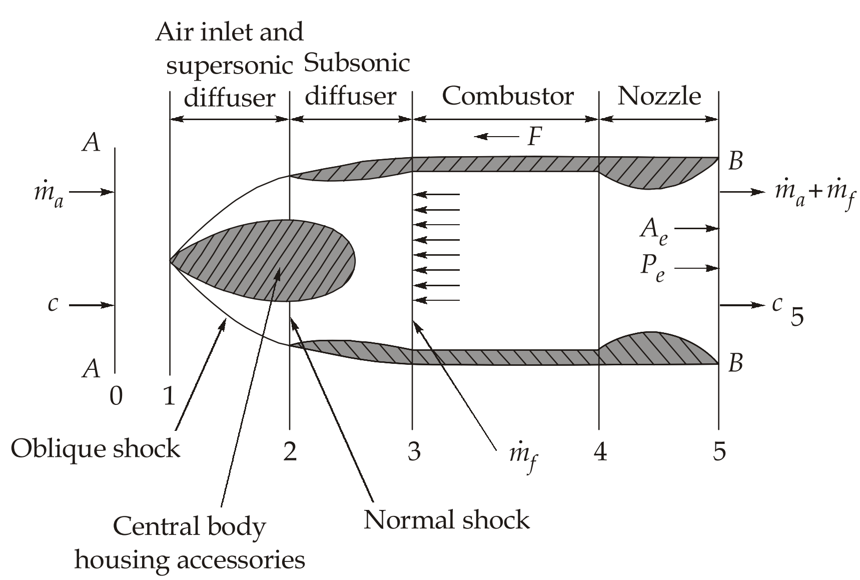
It may be noted that the simplest types of air breathing engine is the ramjet engine and a simplified sketch of the engine is illustrated in figure below.

(i) Supersonic diffuser (1-2).

(ii) Subsonic diffuser section (2-3).

(iii) Combustion chamber (3-4), and

(iv) Discharge nozzle section (4-5)



Ramjet engine

The principle of operation is as follows: Air from the atmosphere enters the engine at a very high speed and its velocity gets reduced first in the supersonic diffuser, thereby its static pressure increases. The air then enters the subsonic diffuser wherein it is compressed further. Afterwards the air flows into the combustion chamber, the fuel is injected by suitable injectors and mixed with the unburnt air. The air is heated to a temperature of the order of 1500 - 2000 K by the continuous combustion of fuel. The fresh supply of air to the diffuser builds up pressure at the diffuser end so that these gases cannot expand towards the diffuser. Instead, the gases are made to expand in the combustion chamber towards the tail pipe. Further, they are allowed to expand in the exhaust nozzle section. The products will leave the engine with a speed exceeding that of the entering air. Because of the rate of increase in the momentum of the working fluid, a thrust, F, is developed in the direction of flight.

Advantages of Ramjet

(i) Ramjet is very simple and does not have any moving part. It is very cheap to produce and requires almost no maintenance.

(ii) Due to the fact that a turbine is not used to drive the mechanical compressor, the maximum temperature which can be allowed in ramjet is very high, about 2000°C as compared to about 900°C in turbojets. This allows a greater thrust to be obtained by burning fuel at air-fuel ratio of about 13:1, which gives higher temperatures.

(iii) The specific fuel consumption is better than other gas turbine power plants at high speed and high altitudes.

(iv) Theoretically there seems to be no upper limit to the flight speed of the ramjet.

Disadvantages of Ramjet

(i) Since the compression of air is obtained by virtue of its speed relative to the engine, the take-off thrust is zero and it is not possible to start a ramjet without an external launching device.

(ii) The engine heavily relies on the diffuser and it is very difficult to design a diffuser which will give good pressure recovery over a wide range of speeds.

(iii) Due to high air speed, the combustion chamber requires flame holder to stabilize the combustion.

(iv) At very high temperatures of about 2000°C dissociation of products of combustion occurs which will reduce the efficiency of the plant if not recovered in nozzle during expansion.

5. (d)

Given : Damping force on the plate in liquid,

$$F_d = 2 \cdot A \cdot \alpha \cdot v \quad \text{...(i)}$$

We know that, $$F_d = c\dot{x} \quad \text{...(ii)}$$

Comparing equation (i) and (ii), we get

$$\Rightarrow \quad c = 2A\alpha \quad \text{...(iii)}$$

$$f_1 = \frac{\omega_n}{2\pi} \quad \text{where } \omega_n = \sqrt{\frac{s}{m}}$$

$$f_2 = \frac{\omega_n}{2\pi}\sqrt{1-\xi^2}$$

$$\therefore \quad \frac{f_1}{f_2} = \frac{1}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \quad (1 - \xi^2) = \left(\frac{f_2}{f_1}\right)^2$$

$$\xi^2 = \frac{f_1^2 - f_2^2}{f_1^2} \quad \text{...(iv)}$$

Also, we know that, damping factor,

$$\xi = \frac{c}{c_c} = \frac{2\alpha A}{2\sqrt{sm}} \quad \text{[from equation (iii)]}$$

$$\xi = \frac{\alpha A}{\sqrt{sm}} \quad \text{...(v)}$$

From equation (iv) and (v), we get

$$\left[\frac{\alpha A}{\sqrt{sm}}\right]^2 = \frac{f_1^2 - f_2^2}{f_1^2}$$

$$\frac{\alpha^2 A^2}{sm} = \frac{f_1^2 - f_2^2}{f_1^2}$$

$$\Rightarrow \quad \alpha = \sqrt{\left(\frac{sm}{A^2}\right) \times \left(\frac{f_1^2 - f_2^2}{f_1^2}\right)}$$

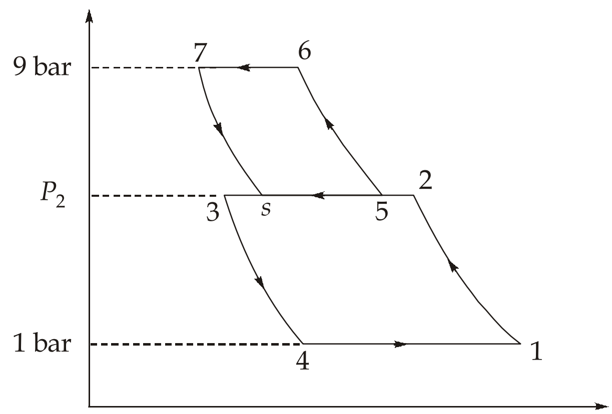
$$\Rightarrow \quad \alpha = \sqrt{\left(\frac{sm}{A^2}\right) \times \left(\frac{2\pi}{\sqrt{s/m}}\right)^2 \times \left(f_1^2 - f_2^2\right)}$$

$$= \sqrt{\frac{m^2}{A^2} \times (2\pi)^2 \times \left(f_1^2 - f_2^2\right)} = 2\pi\frac{m}{A}\sqrt{f_1^2 - f_2^2}$$

$$\alpha = 2\pi \frac{W}{gA}\sqrt{f_1^2 - f_2^2}$$

5. (e)

Given : $\dot{m}$ = 4.8 kg/min; P_1 = 1 bar; T_1 = 17°C = 290 K; r_p = 9; $PV^{1.3} = c$; $V_C = 0.04 \times V_s$; N = 300 rpm; R = 0.287 kJ/kgK



Minimum indicated power,

For perfect intercooling, $P_2 = \sqrt{P_1 P_3} = \sqrt{1 \times 9}$ = 3 bar

The minimum power input,

$$IP_{\min} = 2 \times \left(\frac{n}{n-1}\right) m_a\, RT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 2 \times \left(\frac{1.3}{1.3-1}\right) \times \left(\frac{4.8}{60}\right) \times 0.287 \times 290 \times \left[\left(\frac{3}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= 16.65 \text{ kW}$$ **Ans.**

The mass of air induced per cycle into LP cylinder,

$$m_a = \frac{\dot{m}_a}{Nk} = \frac{4.8}{300 \times 1} = 0.016 \text{ kg/cycle}$$

The volumetric efficiency of LP and HP cylinder

$$\eta_{vol} = 1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = 1 + 0.04 - 0.04\left(\frac{3}{1}\right)^{\frac{1}{1.3}} = 0.9468$$

Volumetric efficiency can be also expressed as

$$\eta_{vol} = \frac{V_1 - V_4}{(V_s)_{LP}} = \frac{V_5 - V_s}{(V_s)_{HP}}$$

Effective swept volume of LP cylinder,

$$(V_1 - V_4) = \frac{m_a RT}{P_1} = \frac{0.016 \times 0.287 \times 290}{100}$$

$$(V_1 - V_4) = 0.01331 \text{ m}^3/\text{cycle}$$

∴ The swept volume of LP cylinder

$$(VS)_{LP} = \frac{V_1 - V_4}{\eta_{vol}} = \frac{0.01331}{0.9468} = 0.0141 \text{ m}^3/\text{cycle}$$

Also, $$\frac{P_1(V_1 - V_4)}{RT_1} = \frac{P_2(V_5 - V_s)}{RT_3}$$

Since $T_1 = T_3$ [Perfect intercooling]

⇒ $$P_1(V_1 - V_4) = P_2(V_5 - V_s)$$

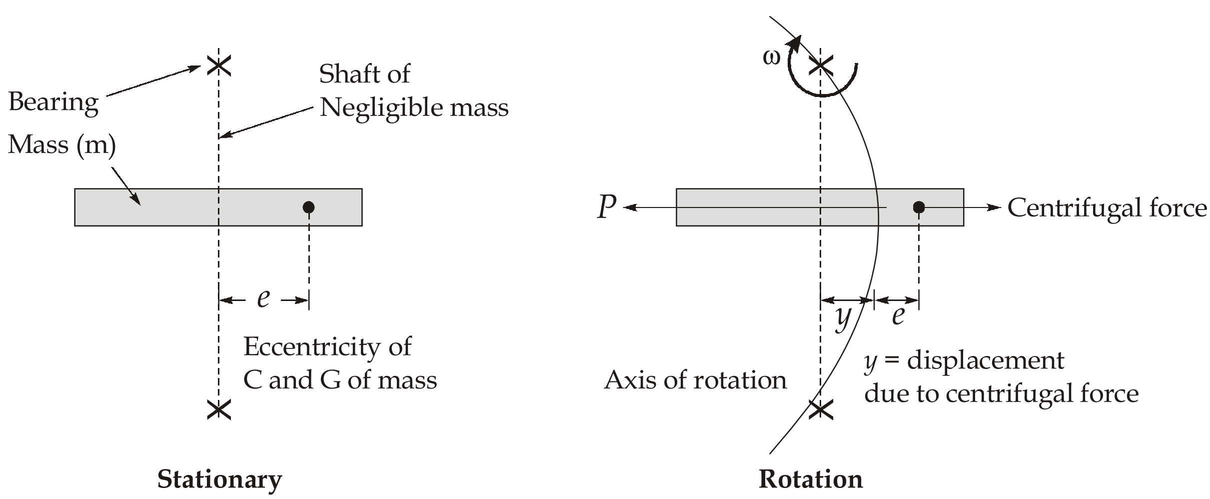
⇒ $$V_5 - V_s = \frac{100 \times 0.01331}{300} = 0.004437 \text{ m}^3/\text{cycle}$$

∴ The swept volume HP cylinder,

$$(V_s)_{HP} = \frac{0.004437}{0.9468} = 0.00468 \text{ m}^3/\text{cycle}$$

6. (a) (i)

The rotor system, the rotating shaft as well as the rotating body is usually assumed to be rigid. However, in many practical applications, such as turbines, compressors, electric motors, and pumps, a heavy rotor is mounted on a lightweight, flexible shaft that is supported in bearings. There will be unbalance in all rotors due to manufacturing errors.



These unbalances as well as other effects, such as the stiffness and damping of the shaft, gyroscopic effects, and fluid friction in bearings, will cause a shaft to bend in a complicated manner at certain rotational speeds, known as the whirling, whipping, or critical speeds.

Whirling is defined as the rotation of the plane made by the line of centers of the bearings and the bent shaft.

6. (a) (ii)

Given : D = 30 mm = 0.03 m; l = 900 mm = 0.9 m; m = 2 kg;
ρ = 50 g/cm^3 = 50 × 10^3 kg/m^3; E = 2 × 10^{11} N/m^2

$$\text{Moment of inertia of shaft, } I = \frac{\pi D^4}{64} = \frac{\pi \times 0.03^4}{64} = 3.976 \times 10^{-8} \text{m}^4$$

Mass of shaft per unit length, m_s = ρ × volume

$$= 50 \times 10^3 \times \frac{\pi}{4} \times 0.03^2 \times 0.9 = 31.808 \text{ kg}$$

Static deflection due to 2 kg of mass at the centre,

$$\Delta_s = \frac{wl^3}{48EI} = \frac{mgl^3}{48EI} = \frac{2 \times 9.81 \times 0.9^3}{48 \times 2 \times 10^{11} \times 3.976 \times 10^{-8}}$$

$$= 3.747 \times 10^{-5} \text{ m}$$

Also, static deflection due to mass of the shaft

$$\Delta_s = \frac{5wL^4}{384EI} = \frac{5 \times (31.808 \times 9.81) \times 0.9^4}{384 \times 2 \times 10^{11} \times 3.976 \times 10^{-8}} = 3.352 \times 10^{-4} \text{m}$$

So, frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\Delta + \frac{\Delta_s}{1.27}}} = \frac{0.4985}{\sqrt{3.747 \times 10^{-5} + \frac{3.352 \times 10^{-4}}{1.27}}}$$

$$= 28.71 \text{ Hz}$$

∴ Whirling speed, N_c = 28.71 × 60 = 1722.6 rpm **Ans.**

6. (b) (i)

CENTRIFUGAL	AXIAL
The flow is radial	The flow is axial i.e., parallel to the direction of the axis of machine.
The pressure ratio per stage is high about 5:1. Thus the unit is compact.	The pressure ratio per stage is low, about 1.2:1. This is due to absence of centrifugal action. To achieve the pressure ratio equal to that of per stage centrifugal, 10 to 12 stages are required. Thus, the unit is less compact and less rugged.
The isentropic efficiency is about 82%.	The isentropic efficiency is about 88%.
Centrifugal compressors have a wide range of operation between surging and chocking limit. The head capacity curve is flat. The part load performance is better.	Axial compressors have a narrow range of operation between surging and chocking limit. The part load performance is poor.
Centrifugal compressors have a larger frontal area and thus produces more drag force for the same mass flow rate and pressure ratio if used in aviation.	Axial flow compressors have a small frontal area for the same mass flow rate and pressure ratio than that of centrifugal. This makes the axial flow compressors more suitable for jet engines. It is invariably used in aviation and power gas turbines.
When working with the contaminating fluids, the accumulation of deposits on the surface of flow passage do not adversely affect the performance.	The accumulation of deposits on the surface of flow passages affect adversely the performance of axial flow compressors.
It needs low starting torque	It needs high starting torque
Its construction is simple, rigid and relatively cheap. At high altitude it is less sensitive to icing trouble	Its construction is complex and costly It is sensitive to icing troubles at high altitude
Multistaging is slightly difficult and upto 400 bar delivery pressure is possible	It is most suitable for multistaging and upto 35 bar delivery pressure in a single casing is possible
It is used in blowing engines in steel mills, low pressure refrigeration, big central air conditioning plants, fertiliser industry, supercharging, gas pumping in long distance pipe line, petrochemical industries. Previously, it was used in jet engines, small air craft and gas turbines.	Due to higher efficiency and smaller frontal area, axial flow compressors are mostly used in jet engines. In power plant gas turbines, axial compressors are invariably used.

6. (b) (ii)

Pressure coefficient (ψ_p) : It is defined as the ratio of isentropic enthalpy increase to the kinetic energy corresponding to tip peripheral velocity. Thus,

$$\psi_p = \frac{\Delta h_{ise}}{u_2^2/2} = \eta_{ise} \cdot \lambda$$

Let ψ_p be the pressure coefficient and

$$\phi_2 = \text{Flow coefficient}$$

$$\beta_2 = \text{Blade angle at outlet}$$

$$\psi_P = \frac{H}{u_2^2/g} = \text{Pressure coefficient}$$

where, Head developed, $H = \dfrac{(V_{w_2}u_2 - V_{w_1}u_1)}{g}$

For centrifugal compressor, as $(V_{w_1} = 0), (V_1 = V_{f_1})$ [Axial inlet]

$$H = \left(\frac{V_{w_2}u_2}{g}\right)$$

$$V_{w_2} = u_2 - V_{f_2}\cot\beta_2$$

So,

$$H = \frac{(u_2 - V_{f_2}\cot\beta_2)u_2}{g}$$

$$\psi_P = \frac{(u_2 - V_{f_2}\cot\beta_2)u_2}{g \times \left(\dfrac{u_2^2}{g}\right)} = \left(\frac{u_2 - V_{f_2}\cot\beta_2}{u_2}\right)$$

$$= 1 - \frac{V_{f_2}}{u_2}\cot\beta_2 \quad \text{...(i)}$$

Let Flow coefficient, $\phi_2 = \dfrac{V_{f_2}}{u_2}$

Putting value of ϕ_2 in equation (i)

$$\psi_P = 1 - \phi_2 \cot\beta_2$$

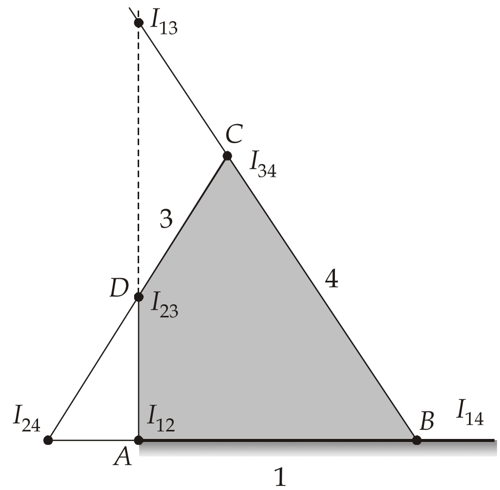
6. (c) (i)

A link or a rigid body as a whole may be considered to be rotating about an imaginary centre or a given centre at a given instant which has zero velocity, then the link is at rest at this point which is known as instantaneous centre of rotation.

Types of instantaneous centres :

1. Primary instantaneous centres
2. Secondary instantaneous Centres

Primary instantaneous centres are further divided into fixed and permanent instantaneous centers.



Example:

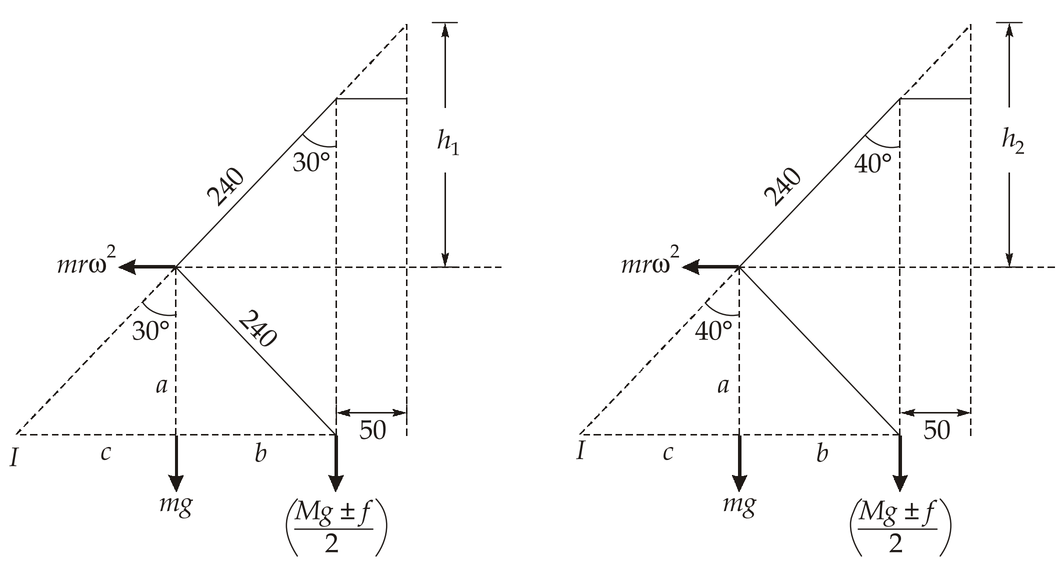
Let us take a 4 bar mechanism with links AB, BC, CD and DA. Here the number of instantaneous centres are

$$N = \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$

In the figure, I_{12} and I_{14} are fixed instantaneous centres of rotation, I_{23} and I_{34} are permanent instantaneous centres. Thus I_{12}, I_{14}, I_{23} and I_{34} are primary instantaneous centres of rotation. Also, I_{13} and I_{24} are secondary instantaneous centres.

6. (c) (ii)

Given : Arm length = 240 mm; m = 2 kg; M = 30 kg; θ_1 = 30°; N = 224 rpm; θ_2 = 40°



From figure (a) r_1 = 240 sin30° + 50 = 170 mm

$$h_1 = \frac{r_1}{\tan 30^\circ} = \frac{170}{\tan 30^\circ} = 294.45 \text{ mm}$$

$\because$ $\theta = \beta$

$$\Rightarrow \quad k = \frac{\tan\beta}{\tan\theta} = \frac{\tan 30^\circ}{\tan 30^\circ} = 1$$

$$\therefore \quad N^2 = \frac{895}{h_1}\left[\frac{2mg + (Mg + f)(1+k)}{2mg}\right]$$

$$224^2 = \frac{895}{0.29445}\left[\frac{(2\times 2\times 9.81) + (30\times 9.81 + f)(1+1)}{2\times 2\times 9.81}\right]$$

$$\Rightarrow \quad f = 9.96 \text{ N}$$

From figure (b), r_2 = 240 sin40 + 50 = 204.27 mm

$$h_2 = \frac{r_2}{\tan 40^\circ} = \frac{204.27}{\tan 40^\circ} = 243.44 \text{ mm}$$

$$N_1^2 = \frac{895}{0.24344}\times\left[1 + \frac{(30\times 9.81) + 9.96}{2\times 9.81}\right]$$

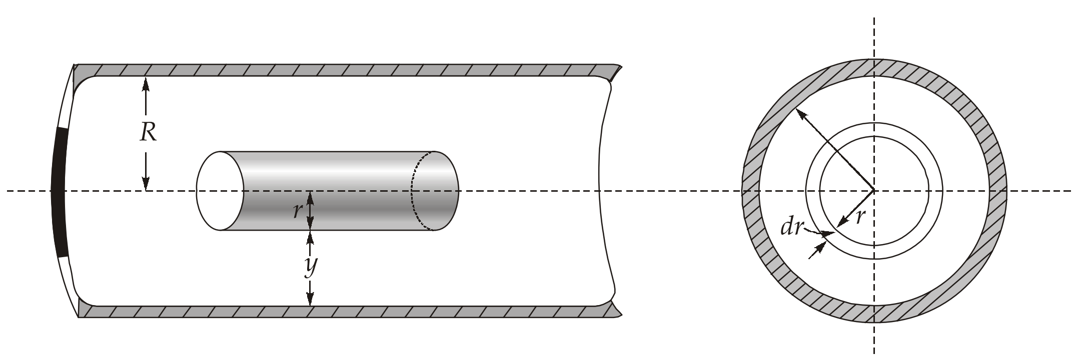
$$N_{max} = N_1 = 246.35 \text{ rpm}$$ **Ans.**

$$N_1^2 = \frac{895}{0.24344}\times\left[1 + \frac{(30\times 9.81) - 9.96}{2\times 9.81}\right]$$

$$N_{min} = N_2 = 238.66 \text{ rpm}$$ **Ans.**

7. (a) (i)

Consider an elementary ring of thickness dr at a radial distance r from the centre of the pipe as shown in figure.



Elementary ring of thickness *dr* at radial distance *r* from pipe centre

The discharge dQ through the elementary ring is given by

$$dQ = v(2\pi r)dr$$

$\therefore$ Total discharge, $Q = \int_0^R dQ = \int_0^R v(2\pi r)\,dr$

$$= \int_0^R v_{max}\left(1-\frac{r}{R}\right)^{1/7} 2\pi r\,dr$$

$$= 2\pi v_{max}\int_0^R r\left(1-\frac{r}{R}\right)^{1/7} dr$$

Let $1-\frac{r}{R} = z$

$\Rightarrow$ $-dr = Rdz$

$\therefore$ $Q = 2\pi v_{max}\int_1^0 -(1-z)z^{1/7}.\,R^2.\,dz$

or $Q = -2\pi v_{max}\times R^2\int_1^0 \left(z^{1/7}-z^{8/7}\right)dz$

$\therefore$ $Q = \frac{49}{60}\times\left(\pi R^2\right)\times v_{max}$

Mean velocity of flow, $V = \frac{Q}{\pi R^2} = \frac{49}{60}v_{max}$

$\Rightarrow$ $\frac{V}{v_{max}} = \frac{49}{60} = 0.817$

For $v = V$

$$V = v_{max}\left(1-\frac{r}{R}\right)^{1/7}$$

or $\frac{V}{v_{max}} = \left(1-\frac{r}{R}\right)^{1/7} = \frac{49}{60}$

$\therefore$ $r = 0.7577R$

i.e., actual velocity is equal to mean velocity at a radius of 0.7577R from the centre of the pipe.

Alternatively if y is the distance from the pipe boundary, then since y = $(R - r)$ and hence

$$y = (1 - 0.7577)R = 0.2422R$$

i.e., actual velocity is equal to mean velocity at a distance of $0.2422R$ from the pipe.

7. (a) (ii)

$$\frac{\tau_0}{\rho V_0^2} = \frac{d\theta}{dx} \quad \text{...(i)}$$

At the boundary,
$$\tau_0 = \mu\left(\frac{\delta v}{\delta y}\right)_{y=0}$$

$$\left(\frac{\delta v}{\delta y}\right)_{y=0} = \frac{\pi}{2}\frac{V_0}{\delta}\left[\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]_{y=0} = \frac{\pi}{2}\frac{V_0}{\delta}$$

$$\tau_0 = \mu\frac{\pi}{2}\frac{V_0}{\delta} \quad \text{...(ii)}$$

Momentum thickness,
$$\theta = \int_0^\delta \frac{u}{V_0}\left(1 - \frac{v}{V_0}\right)dy$$

$$\theta = \int_0^\delta \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\left[1 - \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]dy$$

or
$$\theta = \frac{2\delta}{\pi}\left(1 - \frac{\pi}{4}\right)$$

By substituting the values of τ_0 and θ in equation (i), we get

$$\frac{1}{\rho V_0^2}\left(\mu\frac{\pi}{2}\frac{V_0}{\delta}\right) = \frac{2}{\pi}\left(1 - \frac{\pi}{4}\right)\frac{d\delta}{dx}$$

or
$$\delta d\delta = \frac{11.4976\mu dx}{\rho V_0} \quad \text{...(iii)}$$

Since δ is a function of x only, integration of equation (iii) gives

$$\frac{\delta^2}{2} = \frac{11.4976\mu x}{\rho V_0} + \text{const.}$$

or
$$\delta = 4.795\sqrt{\frac{\mu x}{\rho V_0}}$$

or
$$\delta = \frac{4.795x}{\sqrt{Re_x}}$$

where, $$\text{Re}_x = \frac{\rho V_0 x}{\mu}$$

Substituting the value of δ in equation (ii) yields

$$\tau_0 = \mu \frac{\pi V_0}{2 \times 4.795}\sqrt{\frac{\rho V_0}{\mu x}}$$

or $$\tau_0 = 0.328\sqrt{\frac{\rho\mu V_0^3}{x}} \quad \text{...(iv)}$$

or $$\tau_0 = \frac{0.328\rho V_0^2}{\sqrt{\text{Re}_x}}$$

7. (b)

When AE is vertical, $r' = r = 180$ mm

Neglecting friction ($f = 0$)

$$mr\omega^2 = \frac{a}{e}\tan\theta\left[mg + \frac{Mg}{2}(1+k)\right] \quad \text{...(i)}$$

$$a = \sqrt{300^2 - (180-40)^2} = 265.33 \text{ mm}$$

$$e = 265.33 + 100 = 365.33$$

$$\sin\theta = \frac{180}{300}$$

⇒ $$\theta = \sin^{-1}\left(\frac{180}{300}\right) = 36.87°$$

∴ $$\tan\theta = \tan 36.87° = 0.75$$

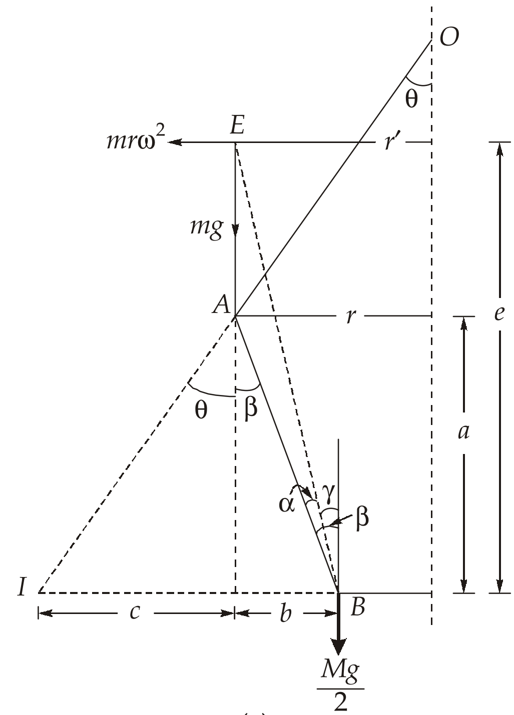
$$\sin\beta = \frac{180-40}{300}$$

⇒ $$\beta = \sin^{-1}\left(\frac{140}{300}\right) = 27.82°$$

∴ $$\tan\beta = \tan 27.82° = 0.528$$

$$k = \frac{\tan\beta}{\tan\theta} = \frac{0.528}{0.75} = 0.704$$

$$b = \sqrt{300^2 - 265.33^2} = 140 \text{ mm}$$



(a)

Putting the known values in equation (i), we get

$$7.2 \times 0.18 \times \omega^2 = \frac{0.26533}{0.36533} \times 0.75 \times \left[7.2 \times 9.81 + \frac{72 \times 9.81}{2}(1 + 0.704)\right]$$

$$\Rightarrow \quad \omega^2 = 282.615$$

$$\Rightarrow \quad \omega = 16.811 \text{ rad/s}$$

$$\Rightarrow \quad \frac{2\pi N}{60} = 16.811$$

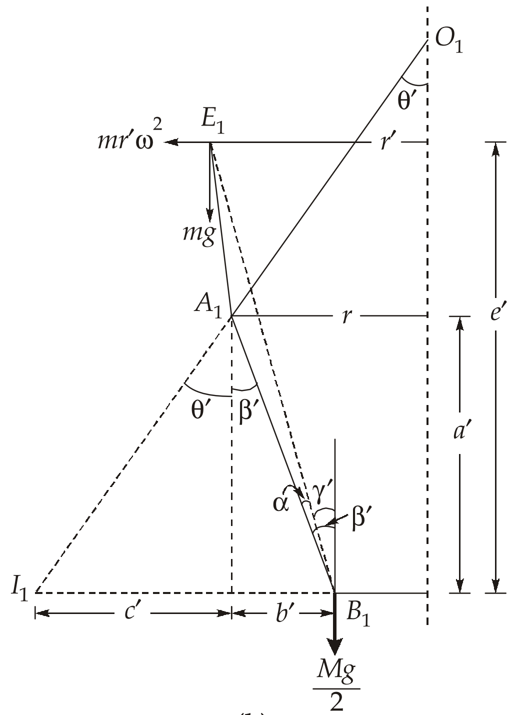
$$\Rightarrow \quad N = 160.53 \text{ rpm}$$ **Ans.**

$$\text{BE} = \sqrt{e^2 + b^2} = \sqrt{365.33^2 + 140^2} = 391.23 \text{ mm}$$

$$\cos\gamma = \frac{e}{BE} = \frac{365.53}{391.23}$$

$$\gamma = \cos^{-1}\left(\frac{365.33}{391.23}\right) = 20.96°$$

$$\alpha = \beta - \gamma = 27.82° - 20.96° = 6.86°$$



(b)

$$\sin\beta' = \frac{b'}{A_1B_1} = \frac{240-40}{300}$$

$\Rightarrow$ $$\beta' = \sin^{-1}\left(\frac{200}{300}\right) = 41.81°$$

$$\gamma' = \beta' - \alpha$$
$$= 41.81° - 6.86° = 34.95°$$
$$e' = B_1E_1 \cos\gamma' = BE\cos\gamma'$$
$$= 391.23 \times \cos 34.95°$$
$$e' = 320.67 \text{ mm}$$
$$r' = B_1E_1 \sin\gamma' + 40$$
$$= 391.23 \times \sin 34.95° + 40 = 264.12 \text{ mm}$$
$$b' = 240 - 40 = 200 \text{ mm}$$
$$a' = A_1B_1 \cos\beta'$$
$$= 300 \cos 41.81° = 223.61 \text{ mm}$$
$$\sin\theta' = \frac{240}{300}$$

$\Rightarrow$ $$\theta' = \sin^{-1}\left(\frac{240}{300}\right) = 53.13°$$

$$c' = a'\tan\theta' = 223.61 \times \tan 53.13° = 298.15 \text{ mm}$$

Taking moments about I', we get

$$mr'(\omega')^2 \times e' = mg(c' + r - r') + \frac{Mg}{2}(c' + b')$$

$$7.2 \times 0.26412 \times (\omega')^2 \times 0.32067 = 7.2 \times 9.81 \times (0.29815 + 0.24 - 0.26412) + \frac{72 \times 9.81}{2} \times (0.29815 + 0.2)$$

$$(\omega')^2 = 320.23$$

$$\Rightarrow \quad \omega' = 17.895 \text{ rad/s}$$

$$\Rightarrow \quad \frac{2\pi N'}{60} = 17.985$$

$$\Rightarrow \quad N' = 170.88 \text{ rpm}$$ **Ans.**

7. (c)

Given : $\alpha_1 = 18°$; $\beta_2 = \beta_4 = 30°$; $\eta_n = 89\%$; k = 0.9; $\eta_{int} = 78\%$; $\alpha_3 = 30°$

For isentropic expansion process 1 to 2s:

$$s_1 = s_{2s}$$

$$\Rightarrow \quad s_1 = [s_f + x_{2s}s_{fg}]_{@0.07 \text{ bar}}$$

$$\Rightarrow \quad 6.5843 = 0.55903 + x_{2s} \times 7.7154$$

$$\Rightarrow \quad x_{2s} = 0.7809$$

$$\Rightarrow \quad h_{2s} = [h_f + x_{2s}\, h_{fg}]_{@0.07 \text{ bar}}$$

$$= 163.35 + 0.7809 \times 2408.4$$

$$= 2044.07 \text{ kJ/kg}$$

Total isentropic enthalpy drop,

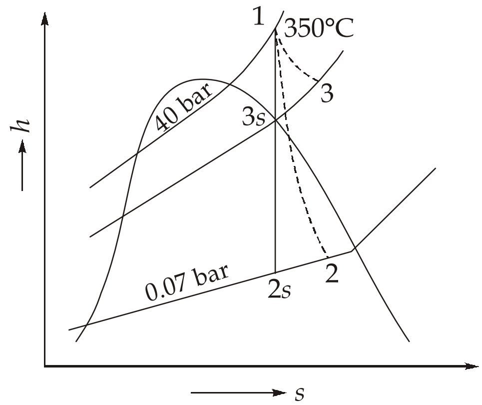
$$(\Delta h_s)_{total} = (h_1 - h_{2s})$$

$$= 3093.3 - 2044.07$$

$$= 1049.23 \text{ kJ/kg}$$

Enthalpy drop in the two-row velocity or custis stage is

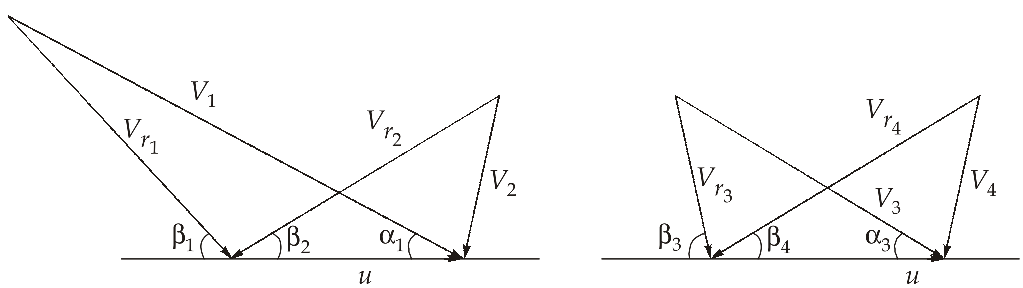
$$h_1 - h_{3s} = \frac{1}{4}(h_1 - h_{2s}) = \frac{1049.23}{4} = 262.3 \text{ kJ/kg}$$



$$\text{Nozzle efficiency, } \eta_n = \frac{(h_1 - h_3)}{(h_1 - 3s)}$$

$$\Rightarrow \quad h_1 - h_3 = 0.89 \times 262.3$$

$$= 233.45 \text{ kJ/kg}$$



∴ Velocity of steam leaving the nozzles,

$$V_1 = \sqrt{2000 \times (h_1 - h_3)} = \sqrt{2000 \times 233.45}$$

$$= 683.3 \text{ kJ/kg}$$

$$u = 0.2 \times V_1 = 0.2 \times 683.3$$

$$= 136.66 \text{ m/s}$$

$$\tan\beta_1 = \frac{V_1 \sin\alpha_1}{V_1 \cos\alpha_1 - u} = \frac{683.3 \times \sin 18°}{683.3 \times \cos 18° - 136.66}$$

$$\beta_1 = 22.36°$$

$$V_{r_1} = \frac{V_1 \sin\alpha_1}{\sin\beta_1} = \frac{683.3 \times \sin 18°}{\sin 22.36°} = 555.04 \text{ m/s}$$

$$V_{r_2} = k \times V_{r_1} = 0.9 \times 555.04 = 499.536 \text{ m/s}$$

$$\Delta V_{w_1} = V_{r_1} \times \cos\beta_1 + V_{r2} \cos\beta_2$$

$$= 55.04 \cos(22.36°) + 499.52 \times \cos 30°$$

$$= 945.92 \text{ m/s}$$

$$V_2^2 = \left(V_{r_2}\sin\beta_2\right)^2 + \left(V_{r_2}\cos\beta_2 - U\right)^2$$
$$= (499.536 \sin(22.36°))^2 + (499.536 \cos(22.36°) - 136.66)^2$$
$$V_2 = 376.75 \text{ m/s}$$
$$V_3 = 0.9 \times V_2 = 0.9 \times 376.75 = 339.07 \text{ m/s}$$
$$\tan\beta_3 = \frac{V_3 \sin 30°}{V_3 \cos 30° - U} = \frac{339.07 \sin 30°}{339.07 \cos 30° - 136.66} = 65.079$$
$$\beta_3 = 47.20°$$
$$V_{r_3} = \frac{V_3 \sin 30°}{\sin 47.20°} = \frac{339.07 \sin 30°}{\sin 47.20°} = 231.06 \text{ m/s}$$
$$V_{r_4} = k \times V_{r_3}$$
$$= 0.9 \times 231.06 = 207.95 \text{ m/s}$$
$$\Delta V_{w_2} = V_{r_3}\cos\beta_3 + V_{r_4}\cos\beta_4$$
$$= 231.06 \times \cos(47.20°) + 207.95 \cos 30°$$
$$= 337.08 \text{ m/s}$$
$$\therefore \quad \Sigma\Delta V_w = \Sigma\Delta V_{w_1} + \Sigma\Delta V_{w_2}$$
$$= 945.92 + 337.08$$
$$= 1283 \text{ m/s}$$

Blade efficiency of the first stage,

$$\eta_{b_1} = \frac{\Sigma V_w \cdot u}{\left(\frac{V_1^2}{2}\right)} = \frac{2 \cdot u \, \Sigma V_w}{V_1^2}$$
$$= \frac{2 \times 136.66 \times 1283}{683.3^2} = 0.7510 \text{ or } 75.10\%$$
$$\eta_{stage_1} = \eta_n \times \eta_{b_1}$$
$$= 0.89 \times 0.7510 = 0.6684 \text{ or } 66.84\%$$ **Ans.**

Total actual enthalpy drop,

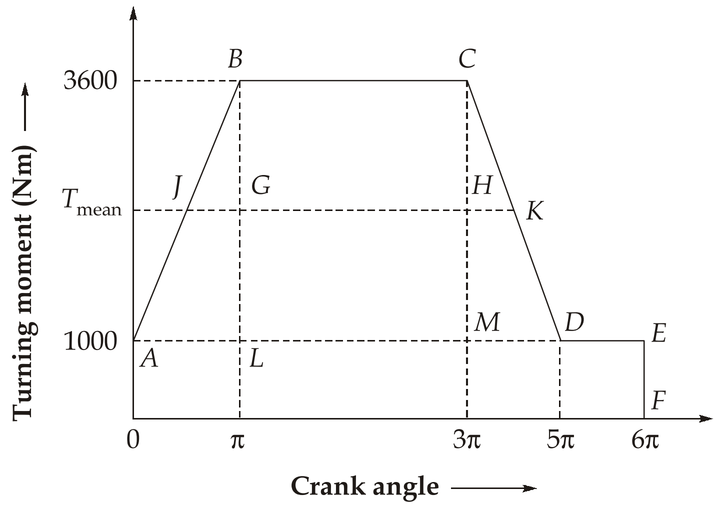
$$(\Delta h)_{\text{total}} = \eta_{\text{int}} \times (h_1 - h_{2s})$$
$$= 0.78 \times 1049.23 = 818.4 \text{ kJ/kg}$$
$$(\Delta h)_{\text{curtis}} = 0.6684 \times (h_1 - h_{3s})$$
$$= 0.6684 \times 262.3 = 175.32 \text{ kJ/kg}$$

∴ The percentage power developed in the curtis stage,

$$= \frac{175.32}{818.4} = 0.2142 \text{ or } 21.42\%$$ **Ans.**

8. (a)

Given : m = 1800 kg; k = 0.5 m; N = 300 rpm; $I = mk^2 = 1800 \times 0.5^2 = 450$ kgm^2



Torque for one complete cycle, $T = OABCDEF$

$$T = \text{Area } OAEF + \text{Area } ABCD$$

$$= (6\pi \times 1000) + \frac{1}{2} \times (3600 - 1000) \times (5\pi + 2\pi)$$

$$= 15100\,\pi \text{ Nm}$$

$$T_{mean} = \frac{T}{6\pi} = \frac{15100\pi}{6\pi} = 2516.67 \text{ Nm}$$

∴ $$\text{Power, } P = T_{mean} \times \omega$$

$$= 2516.67 \times \left(\frac{2\pi \times 300}{60}\right) \times \frac{1}{1000} = 79.06 \text{ kW}$$ **Ans.**

$$JG = AL \times \frac{BG}{BL} = \pi \times \left(\frac{3600 - 2516.67}{3600 - 1000}\right) = 1.309$$

$$HK = MD \times \frac{CH}{CM} = 2\pi \times \left(\frac{3600 - 2516.67}{3600 - 1000}\right) = 2.618$$

The fluctuation of energy,

$$(\Delta E)_{max} = \text{Area } JBCK$$

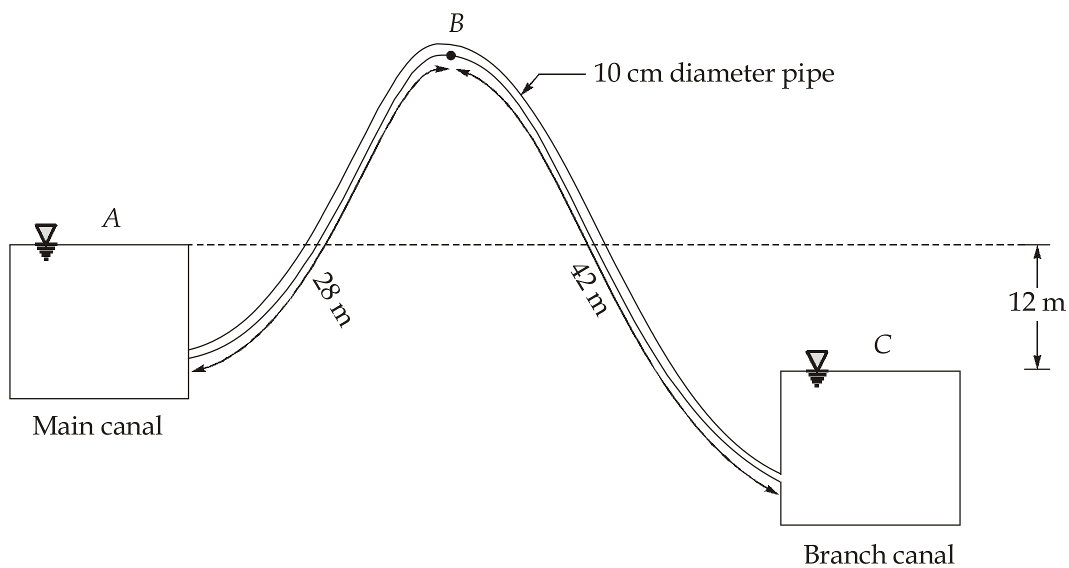
$$= \frac{1}{2} \times (3600 - 2516.67) \times [2\pi + (1.309 + 2.618 + 2\pi)]$$

$$= 8933.8 \text{ Nm}$$

$$(\Delta E)_{max} = I\omega^2 C_s$$

$$C_s = \frac{8933.88}{450 \times \left(\frac{2\pi \times 300}{60}\right)^2} = 0.0201 \text{ or } 2.01\%$$ **Ans.**

8. (b)



The difference in the level of water in the main and branch canals equals the sum of all head losses along the pipeline *ABC*.

$$12 = \frac{0.5V^2}{2g} + \frac{(4f) \times LV^2}{2gD} + \frac{V^2}{2g}$$

$$12 = \frac{0.5V^2}{2g} + \frac{4 \times 0.0078 \times 70 \times V^2}{2g \times 0.1} + \frac{V^2}{2g}$$

$$V = 3.176 \text{ m/s}$$

Discharge through the pipe, $Q = \frac{\pi}{4} \times 0.1^2 \times 3.176$

$= 0.02494 \text{ m}^3\text{/s or } 24.94\ l\text{/s}$

Number of pipes required to convey 64 l/s = $\frac{64}{24.94} = 2.57 \simeq 3$ **Ans.**

Thus, 3 pipes of 10 cm diameter are needed to convey a total discharge of 64 l/s.

Now, applying Bernoulli's equation between point *A* and *B*,

Total energy per unit weight at point *A* = Total energy per unit weight at point *B* + Head loss in the pipe *AB*

$$\Rightarrow \quad \frac{P_a}{w} + y_a + \frac{V^2}{2g} = \frac{P_b}{w} + y_b + \frac{V^2}{2g} + \frac{0.5 \times V^2}{2g} + \frac{4fL_{AB}V^2}{2gD}$$

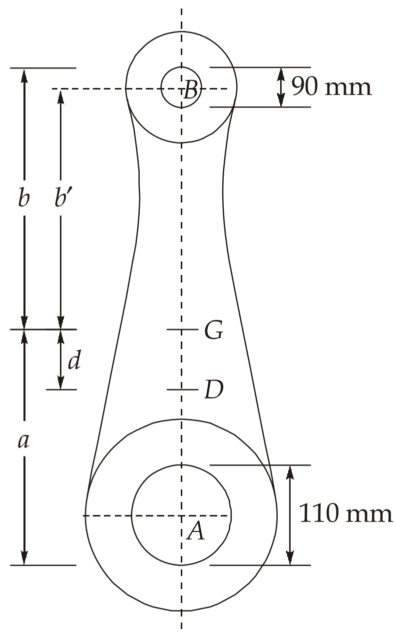
$$\Rightarrow 10.3 + y_a + \frac{3.176^2}{2\times 9.81} = \frac{0.21\times 10^5}{9810} + y_b + \frac{3.176^2}{2\times 9.81} + \frac{0.5\times 3.176^2}{2\times 9.81} + \frac{4\times 0.0078\times 28\times 3.176^2}{2\times 9.81\times 0.1}$$

$$\Rightarrow \quad y_b - y_a = 3.41 \text{ m}$$ **Ans.**

$\therefore$ Maximum permissible height of summit above the water level in the main canal is 3.41.

8. (c)

From big end, time of oscillations



$$t_a = 2\pi\sqrt{\frac{a+\frac{k^2}{a}}{g}}$$

$$\Rightarrow \quad \left(\frac{1.8}{2\pi}\right)^2 \times 9.81 = a + \frac{k^2}{a}$$

$$\Rightarrow \quad a + \frac{k^2}{a} = 0.8051$$

$$\Rightarrow \quad a^2 + k^2 = 0.8051 \times a \qquad \text{...(i)}$$

Also, from small end, $t_b = 2\pi\sqrt{\dfrac{b+\frac{k^2}{b}}{g}} = \left(\dfrac{2}{2\pi}\right)^2 \times 9.81 = b + \dfrac{k^2}{b}$

$$\Rightarrow \quad b + \frac{k^2}{b} = 0.99396 \qquad \text{...(ii)}$$

$$a + b = \left(960 + \frac{90}{2} + \frac{110}{2}\right) \times \frac{1}{1000}$$

$\Rightarrow \quad a + b = 1.06$

$\Rightarrow \quad b = 1.06 - a \quad \text{...(iii)}$

From equation (ii) and (iii), we get

$$(1.06 - a) + \frac{k^2}{(1.06 - a)} = 0.99396$$

$$(1.06 - a)^2 + k^2 = 0.99396 \times (1.06 - a)$$

$$1.1236 - 2.12a + a^2 + k^2 = 1.0536 - 0.99396 \times a$$

$$a^2 + k^2 = 1.12604a - 0.07 \quad \text{...(iv)}$$

From equation (i) and (iv), we get

$$0.8051 \times a = 1.12604a - 0.07$$

$\Rightarrow \quad a = 0.2181$ m

From equation (iii), we get

$$b = 0.8419 \text{ m}$$

From equation (ii), we get

$$0.8419 + \frac{k^2}{0.8419} = 0.99396$$

$k = 0.3578$ m **Ans.**

$I = mk^2 = 54 \times 0.3578^2$

$= 6.913$ kgm^2 **Ans.**

The distance of centre of mass of the connecting rod from the centre of the small end bearing,

$$b' = 0.8419 - \frac{0.09}{2} = 0.7969 \text{ m}$$

Let the second mass placed at D be m_d

$$GD = d = \frac{k^2}{b'} = \frac{0.3578^2}{0.7969} = 0.1606 \text{ m}$$

$$\therefore \quad m_d = \frac{m \cdot b'}{b' + d} = \frac{54 \times 0.7969}{0.7969 + 0.1606}$$

$= 44.94$ kg **Ans.**

Mass at the small end-bearing centre,

$m'_b = 54 - 44.94 = 9.06$ kg **Ans.**

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