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Detailed Solutions

ESE-2024 Mains Test Series

Civil Engineering Test No : 5

Section A : Flow of fluids, hydraulic machines and hydro power

Q.1 (a) Solution:

(i) Boundary layer thickness: The velocity within the boundary layer increases from zero at the boundary surface to the velocity of the main stream asymptotically.

Therefore, the boundary layer thickness represented by 'δ' is arbitrarily defined as that distance from the boundary surface in which the velocity reaches 99% of the velocity of the main stream. In other words, the boundary layer thickness 'δ' may be considered equal to distance 'y' from the boundary surface at which $u = 0.99\ U_\infty$ where U_∞ is main stream velocity.

(ii) Displacement thickness: It is defined as the perpendicular distance by which the boundary surface should be shifted in order to compensate for the reduction in mass flow rate on account of boundary layer formation.

Fig. (a) Ideal Fluid flow | Fig. (b) Real fluid flow | Fig. (c) | Fig. (d) Real fluid flow

If the flow is ideal and there is no formation of boundary layer, then the velocity profile would be as shown in figure (a). But in case of real fluid flow, velocity profile would be as shown in figure (b).

Here, reduction in mass flow rate is equal to shaded area (3). Mass flow rate in (1) will be equal to mass flow rate in (2) or in other words, mass flow rate in (3) is as that in (4).

Hence, $$\rho(\delta^* \times 1)\times U_\infty = \int_0^\delta \rho(U_\infty - u)(dy\times 1) \quad \text{[Taking unit width of flow]}$$

$$\Rightarrow \quad \delta^* = \int_0^\delta \left(1-\frac{u}{U_\infty}\right)dy$$

(iii) Momentum thickness (θ): It is defined as the perpendicular distance from the boundary surface by which boundary should be shifted in order to compensate for the reduction in momentum of the flowing fluid due to the boundary layer formation.

$$\text{Momentum of fluid} = \text{Mass} \times \text{flow velocity}$$

$$= (\rho.u.b.dy)u$$

Loss in momentum due to boundary layer = $\int_0^\delta \rho bu(U_\infty - u)du$

Also loss in momentum through distance, θ = $(\rho\theta b u_\infty).u_\infty$

Assuming unit width i.e. $b = 1$

Thus, $$\rho(\theta\times 1)U_\infty.U_\infty = \int_0^\delta \rho(dy\times 1)(U_\infty - u)u$$

$$\Rightarrow \quad \rho U_\infty^2.\theta = \rho\int_0^\delta (U_\infty - u).u\,dy$$

$$\Rightarrow \quad \theta = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$$

(iv) Energy thickness (δ_E): It is defined as the perpendicular distance from the boundary surface such that the energy flux corresponding to the main stream velocity U_∞ through the distance δ_E is equal to the deficiency or loss of energy due to the boundary layer formation.

Loss of energy as energy flux = $\frac{1}{2}\times$mass flow rate × (flow velocity)^2

Thus, $$\frac{1}{2}.\rho(\delta_E \times 1)U_\infty.U_\infty^2 = \int_0^\delta \frac{1}{2}\times\rho(dy\times 1).u(U_\infty^2 - u^2)$$

$$\Rightarrow \quad \frac{1}{2}\rho U_\infty^3.\delta_E = \frac{1}{2}.\rho\int_0^\delta (U_\infty^2 - u^2)u.dy$$

$$\Rightarrow \quad \delta_E = \int_0^\delta \frac{u}{U_\infty}\left[1-\frac{u^2}{U_\infty}\right]dy$$

Q.1 (b) Solution:

Width of channel, B = 9 m

Depth of channel, y = 2 m

Bed slope, $S = \frac{1}{1000}$

Manning's, n = 0.01

area of channel, A = $By = 9 \times 2 = 18\ m^2$

perimeter of channel, $P = B + 2y$

$= 9 + 2 \times 2 = 13$ m

Now, using Manning's equation,

$$\text{Discharge, } Q = A\times\frac{1}{n}\times\left(\frac{A}{P}\right)^{2/3}\times\sqrt{S}$$

$$= 18\times\frac{1}{0.01}\times\left(\frac{18}{13}\right)^{2/3}\times\sqrt{\frac{1}{1000}}$$

$$= 70.71\ m^3/sec.$$

Now, as the amount of lining is to be same, so perimeter will remain constant.

Now, $A = B.y$ ...(i)

and $P = B + 2y$ ...(ii)

$\Rightarrow$ $B = P - 2y$

Putting value of B in eq. (i), we get

$$A = (P - 2y) \times y$$

Now,
$$Q = A \times \frac{1}{n} \times \left(\frac{A}{P}\right)^{2/3} \times \sqrt{5}$$

$$= (P-2y) \times y \times \frac{1}{n} \times \left[\frac{(P-2y)y}{P}\right]^{2/3} \times \sqrt{S}$$

$$= \frac{\sqrt{S}}{n.P^{2/3}}\left[Py - 2y^2\right]^{5/3}$$

For maximum discharge,
$$\frac{dQ}{dy} = 0$$

$$\Rightarrow \quad \frac{\sqrt{S}}{n.P^{2/3}} \times \frac{5}{3} \times \left[Py - 2y^2\right]^{2/3} \times (P - 4y) = 0$$

$$\therefore \quad \left[Py - 2y^2\right]^{2/3} = 0$$

$$\Rightarrow \quad y = 0, \text{P}/2$$

or
$$(P - 4y) = 0$$

$$\Rightarrow \quad y = P/4$$

When,
$$y = \frac{\text{P}}{4} = \frac{13}{4} = 3.25\,\text{m}$$

$$Q = \frac{\sqrt{1/1000}}{0.01 \times 13^{2/3}} \times \left(13 \times 3.25 - 2 \times 3.25^2\right)^{5/3}$$

$$= 92.33 \text{ m}^3/\text{sec.}$$

So,
$$y = 3.25 \text{ m}$$

and
$$B = 13 - 2 \times 3.25 = 6.5 \text{ m}$$

Now, percent increase in discharge $= \dfrac{92.33 - 70.71}{70.71} \times 100 = 30.58\%$

Q.1 (c) Solution:

$$\text{Dia. of jet, } d = 137 \text{ mm} = 0.137 \text{ m}$$

$$\text{Area of jet, } a = \frac{\pi}{4} \times 0.137 = 0.01474 \text{ m}^2$$

$$\text{Angle of deflection} = 165°$$

$$\text{Angle, } \phi = 180° - 165° = 15°$$

$$\text{Head of water, } H = 400 \text{ m}$$

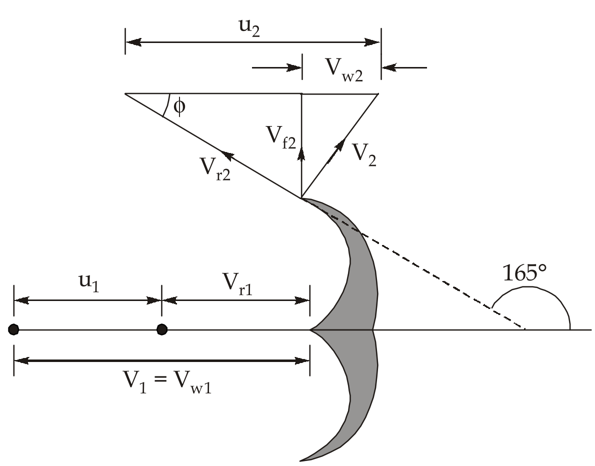
$$\text{Co-efficient of velocity, } C_v = 0.97$$

$$\text{Speed ratio} = 0.46$$

$$\text{Relative velocity at outlet} = 0.85 \times \text{relative velocity at inlet}$$

$$V_{r2} = C_v\sqrt{2gH}$$

$$= 0.97\sqrt{2 \times 9.81 \times 400} = 85.93 \text{ m/s}$$



$$\text{Speed ratio} = \frac{u_1}{\sqrt{2gH}} \text{ or } 0.46 = \frac{u_1}{\sqrt{2 \times 9.81 \times 400}}$$

$\therefore$ $$u_1 = 0.46 \times \sqrt{2 \times 9.81 \times 400} = 40.75 \text{ m/s}$$

Hence, $$V_{r1} = V_1 - u_1 = 85.93 - 40.75 = 45.18 \text{ m/s}$$

$$V_{r2} = 0.85\, V_{r1} = 0.85 \times 45.18 = 38.40 \text{ m/s}$$

$$u_1 = u_2 = u = 40.75 \text{ m/s}$$

$$V_{r2}\cos\theta = 38.40 \times \cos 15° = 37.092$$

Here $V_{r2}\cos\theta$ is less than u_2.

$$V_{w2} = u_2 - V_{r2}\cos\phi = 40.75 - 37.092 = 3.658 \text{ m/s}$$

$$V_{r2} = 0.85\ V_{r1} = 0.85 \times 45.18 = 38.40 \text{ m/s}$$

Force exerted by jet on buckets in tangential direction is given by,

$$F_x = \rho a V_1[V_{w1} - V_{w2}]$$
$$= 1000 \times 0.01474 \times (85.93 - 3.658) \text{ N} = 104206 \text{ N}$$

Power developed is given by,

$$\text{Power} = \frac{F_x \times u}{1000} \text{kW} = \frac{104206 \times 40.75}{1000} = 4246.4 \text{ kW}$$

Q.1 (d) Solution:

Horizontal scale, $L_r = \dfrac{1}{250}$

Vertical scale, $h_r = \dfrac{1}{50}$

For prototype

Discharge, $Q_P = 150 \text{ m}^3/\text{s}$

Width, $B_P = 100 \text{ m}$

Depth, $y_P = 4 \text{ m}$

- **Discharge:**

$$\frac{Q_m}{Q_P} = \frac{A_m V_m}{A_p V_p}$$

$$\text{Froude's number} = \frac{V}{\sqrt{Lg}}$$

$$\Rightarrow \quad \frac{V_m}{\sqrt{y_m g}} = \frac{V_p}{\sqrt{y_p g}}$$

$$\Rightarrow \quad \frac{V_m}{V_p} = \sqrt{\frac{y_m}{y_p}} = \sqrt{h_r}$$

and $$\frac{A_m}{A_p} = \frac{B_m y_m}{B_p y_p} = L_r h_r$$

So, $$\frac{Q_m}{Q_p} = L_r h_r \sqrt{h_r}$$

$$\Rightarrow \quad Q_m = 150 \times \frac{1}{250} \times \left(\frac{1}{50}\right)^{1.5} = 1.697 \times 10^{-3} \text{ m}^3/\text{s}$$

- **Depth**

$$\Rightarrow \quad \frac{y_m}{y_p} = h_r$$

$$\Rightarrow \quad y_m = 4 \times \frac{1}{50} = 0.08 \text{ m}$$

Width

$$\Rightarrow \quad \frac{B_m}{B_p} = L_r$$

$$\Rightarrow \quad B_m = 100 \times \frac{1}{250} = 0.4 \text{ m}$$

- **Manning's n**

$$\frac{V_m}{V_p} = \frac{\frac{1}{n_m} R_m^{2/3} S_m^{1/2}}{\frac{1}{n_p} R_p^{2/3} S_p^{1/2}}$$

$$\Rightarrow \quad \sqrt{h_r} = \frac{n_p}{n_m} \times h_r^{2/3} \times \frac{h_r^{1/2}}{L_r^{1/2}}$$

$$\Rightarrow \quad n_m = \frac{n_p \times h_r^{2/3}}{L_r^{1/2}} = \frac{0.030 \times \left(\frac{1}{50}\right)^{2/3}}{\left(\frac{1}{250}\right)^{1/2}} = 0.0349$$

Q.1 (e) Solution:

Given data:Distance between plates = 8 cm

$$U_{\text{max}} = 1.5 \text{ m/s}$$

Dynamic viscosity of oil,

$$\mu = 2 \text{ Ns/m}^2$$

For flow between two fixed parallel plates

$$\frac{U_{\text{max}}}{U_{avg}} = \frac{3}{2}$$

$$\Rightarrow \quad U_{\text{avg}} = \frac{2}{3} \times 1.5 = 1 \text{ m/s}$$

1. Discharge per unit width,

$$q = U_{avg}(0.08 \times 1)$$
$$= 1 \times 0.08 = 0.08 \text{ m}^3/\text{s/m}$$

2. The pressure gradient in terms of average velocity is given by

$$\frac{-dP}{dx} = \frac{12\mu U_{avg}}{b^2} = \frac{12 \times 2 \times 1}{(0.08)^2} = 3750 \text{ N/m}^2/\text{m}$$

Shear stress at the plates is

$$\tau = \left(-\frac{dP}{dx}\right)\frac{b}{2} = \frac{3750 \times 0.08}{2} = 150 \text{ N/m}^2$$

3. For laminar flow between two parallel plates

$$P_1 - P_2 = \frac{12\mu U_{avg} l}{b^2} = \frac{12 \times 2 \times 1 \times 25}{(0.08)^2} = 93750 \text{ N/m}^2$$

4. The velocity distribution across the passage between two stationary plane surface is

$$u = \frac{1}{2\mu}\left(\frac{-dP}{dx}\right)(by - y^2)$$

At $y = 2 \text{ cm} = 0.02 \text{ m}$ and $b = 8 \text{ cm} = 0.08 \text{ m}$

$$\therefore \quad u = \frac{1}{2 \times 2}(3750)(0.08 \times 0.02 - 0.02^2)$$

$$\Rightarrow \quad u = 1.125 \text{ m/s}$$

Q.2 (a) Solution:

As per Von-karman integral equation,

$$\frac{d\theta}{dx} = \frac{\tau_0}{\rho U^2} \qquad \text{where } \theta \text{ is momentum thickness}$$

Now,

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy$$

$$= \left[\frac{7y^{8/7}}{8\delta^{1/7}} - \frac{7y^{9/7}}{8\delta^{2/7}}\right]_0^\delta$$

$$= \frac{7\delta}{8} - \frac{7\delta}{9} = \frac{7\delta}{72}$$

According to Von-karman equation,

$$\frac{\tau_0}{\rho U^2} = \frac{d\theta}{dx}$$

$$0.0228\left[\frac{\nu}{U\delta}\right]^{1/4} = \frac{\partial\delta}{\partial x}$$

$$\int_0^\delta \delta^{1/4}\partial\delta = \int_0^\delta 0.0228\times\left(\frac{\nu}{U}\right)^{1/4}\times\frac{72}{7}\partial x$$

$$\frac{4\delta^{5/4}}{5} = 0.0228\times\left(\frac{\nu}{U}\right)^{1/4}\times\frac{72x}{7}$$

$$\delta^{5/4} = 0.293\times\left(\frac{\nu}{Ux}\right)^{1/4}\times x^{5/4}$$

$$\left(\frac{\delta}{x}\right)^{5/4} = \frac{0.293}{(\text{Re}_x)^{1/4}}$$

$$\frac{\delta}{x} = \frac{0.375}{(\text{Re}_x)^{1/5}}$$

Now, local skin friction coefficient,

$$C_{f\,x} = \frac{\tau_{o,x}}{\frac{1}{2}\rho U^2} = 0.0456\left[\frac{\nu\times(\text{Re}_x)^{1/5}}{Ux\times 0.375}\right]^{1/4}$$

$$= \frac{0.0456}{(0.375)^{1/4}}\times\frac{1}{(\text{Re}_x)^{1/5}}$$

$$C_{f,\,x} = \frac{0.058}{(\text{Re}_x)^{1/5}}$$

(ii) Drag coefficient,

$$C_D = \frac{\int_0^L \frac{0.058}{(\text{Re}_x)^{1/5}}dx}{L}$$

$$= \frac{\int_0^L \frac{0.058\nu^{1/5}}{(Ux)^{1/5}}dx}{L}$$

$$= \frac{5}{4} \times \frac{0.058}{L} \times \left[\frac{\nu}{U}\right]^{1/5} \times \left[x^{4/5}\right]_0^L$$

$$= \frac{0.0725}{\text{Re}_L^{1/5}}$$

Q.2 (b) Solution:

(i)

Following are the assumptions of hydraulic jump in horizontal rectangular channel:

1. The velocity is nearly constant across the channel and therefore the momentum-flux correction factor is unity across the channel.
2. The pressure in the liquid varies hydrostatically, and we consider gauge pressure only since atmospheric pressure acts on all surfaces and its effect cancels out.
3. The wall shear stress, F_s and the associated losses are negligible relative to the losses that occur during the hydraulic jump due to the intense agitation.
4. The channel is wide and horizontal. The bed slope is so gentle that the weight component of the water comprising the jump is negligibly small.
5. There are no external or body forces other than gravity.

- **Applications of hydraulic jump:** Hydraulic jump primarily serves as an energy dissipator to dissipate the excess energy of flowing water downstream of hydraulic structures such as spillway and sluice gates. Hydraulic jump may also be used in the following situation:

1. To mix chemical used for water purification or wastewater treatment.
2. To aerate the polluted stream.
3. To desalinate sea water.
4. To aid intense mixing and gas transfer in chemical processes.
5. To maintain subcritical flow downstream of hydraulic structures.

(ii)

Discharge per unit width, $q = 11\ \text{m}^3/\text{sec./m}$

At upstream section,

$$\text{Velocity, } V_1 = \frac{q}{y_1} = \frac{11}{0.7} = 15.714 \text{ m/sec.}$$

$$\text{Froude number, } F_1 = \frac{V_1}{\sqrt{g.y_1}} = \frac{15.714}{\sqrt{9.81 \times 0.7}} = 6.0 > 1$$

Let y_2 = Equivalent sequent depth of the hydraulic jump.

$$\text{Sequent depth, } y_2 = \frac{y_1}{2}\left[-1+\sqrt{1+8F_1^2}\right]$$

$$= \frac{0.7}{2}\left[-1+\sqrt{1+8\times(6)^2}\right] = 5.6 \text{ m}$$

Now, sequent depth ratio corresponding to a value of tan θ = 0.15 is,

$$\frac{y_t}{y_2} = 1.0071 \exp[3.2386 \tan\theta]$$

For hydraulic jump inclined floor

$$\Rightarrow \quad \frac{y_t}{y_2} = 1.6369 \simeq 1.64$$

$$\Rightarrow \quad y_t = \text{Sequent depth in the inclined floor jump.}$$

$$= 1.64 \times 5.6 = 9.18 \text{ m}$$

Length of Jump : $L_j = m_s(y_t - y_1)$

$$L_j = 3.8 \times [9.18 - 0.7] = 32.224 \text{ m}$$

Also, $L_j = y_2[6.1 + 4.0 \times \tan\theta]$

$$= 5.6 \times [6.1 + 4 \times 0.15] = 37.52 \text{ m}$$

$$\text{Average value of } L_j = \frac{32.224 + 37.52}{2}$$

= 34.87 m can be taken as the jump length.

Energy loss : Initial specific energy, $E = y_1, \cos\theta + \dfrac{V_1^2}{2g}$

Here, $\cos\theta = 0.98894$

$$\therefore \quad E_1 = 0.7 \times 0.98894 \times \frac{(15.714)^2}{2\times 9.81} = 13.278 \text{ m}$$

Now, H_1 = Total energy at section 1 with bed level at section 2 as datum.

$$= E_1 + L_j \tan\theta$$

$$= 13.278 + 34.87 \times 0.15 = 18.5 \text{ m}$$

$$H_2 = E_2 = y_t \cos\theta + \frac{V_t^2}{2g}$$

$$= 9.18 \times 0.98894 + \frac{(11.0/9.18)^2}{2 \times 9.81}$$

$$= 9.15 \text{ m}$$

Energy loss , $E_L = H_1 - H_2$

$\Rightarrow$ $E_L = 18.5 - 9.15 = 9.35$

Also, $$\frac{E_L}{H_1} = \frac{9.35}{18.5} = 0.505 = 50.5\%$$

Q.2 (c) Solution:

Given, Inlet radius, R_1 = 80 mm = 0.08 m

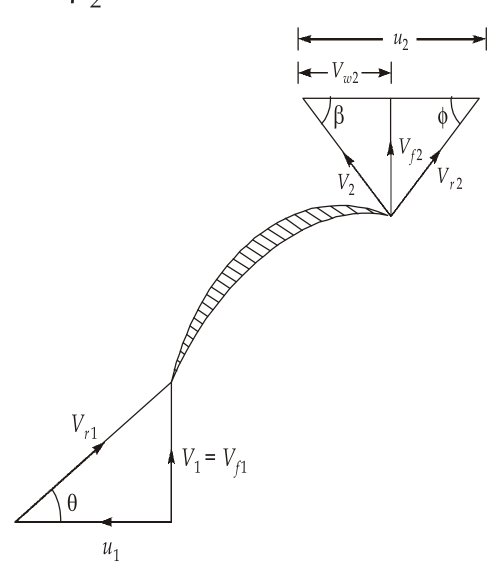
Outlet radius, R_2 = 160 mm = 0.16 m

Width at inlet, B_1 = 50 mm = 0.05 m

Width at outlet, B_2 = 50 mm = 0.05 m

Angles, β_1 = 0.45 radians

and β_2 = 0.25 radians



Hence β_1 is the vane angle at inlet and β_2 is the vane angle at outlet.

$\therefore$ Vane angle at inlet, $\theta = \beta_1 = 0.45$ radians

$\therefore$ Vane angle at outlet, $\phi = \beta_2 = 0.25$ radians

Angular velocity, $\omega = 90$ rad/s

Find:

1. discharge, and 2. head developed

Now tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 \times N}{60} = \frac{2\pi N}{60} \times \frac{D_1}{2} = \omega R_1 = 90 \times 0.08 = 7.2 \text{ m/s}$$

$$u_2 = \omega R_2 = 90 \times 0.16 = 14.4 \text{ m/s}$$

From inlet velocity triangle, $\tan\theta = \dfrac{V_{f1}}{u_1}$

$$V_{f1} = u_1 \times \tan\theta = 7.2 \times \tan(0.45 \text{ rad})$$
$$= 7.2 \times 0.483 = 3.478 \text{ m/s}$$

1. Discharge (Q)

Discharge is given by, $Q = \pi D_1 \times B_1 \times V_{f1} = \pi \times (2R_1) \times B_1 \times V_{f1}$

$$= (\pi \times 2 \times 0.08 \times 0.05 \times 3.478) \text{ m}^3/\text{s}$$
$$= 0.0874 \text{ m}^3/\text{s}$$

2. Head developed (H_m)

For the shockless entry, the losses of the pump will be zero. Hence, the head developed (H_m) will be given by equation,

$$H_m = \frac{V_{w2} \times u_2}{g}$$

where outlet velocity triangle,

$$V_{w2} = (u_2 - V_{f2}) \times \cot\phi$$

The value of V_{f2} is obtained from,

$$Q = \pi D_2 \times B_2 \times V_{f2}$$

$\Rightarrow$ $$V_{f2} = \frac{0.0874}{\pi \times 2 \times 0.16 \times 0.05} = 1.7387 \text{ m/s}$$

$\Rightarrow$ $$V_{f2} = u_2 - V_{f2} \times \cot\phi$$

$= 14.4 - 1.7387 \times \cot(0.25 \text{ radians})$

$= 14.4 - 1.7387 \times 3.9163 = 14.4 - 6.809$

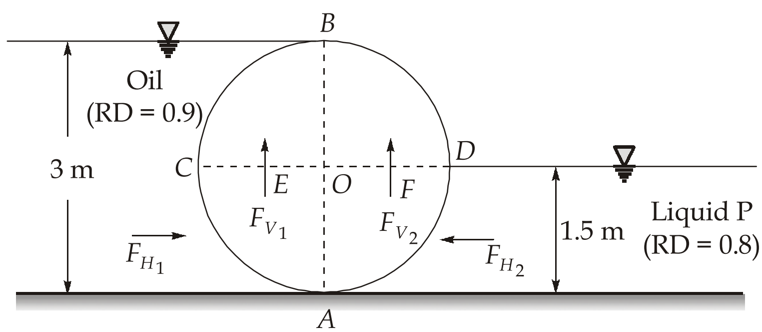
$= 7.591$ m/s

Substituting this value in equation (i) above, we get

$$H_m = \frac{V_{w2} \times u_2}{g} = \frac{7.591 \times 14.4}{9.81} = 11.142 \text{ m}$$

Q.3 (a) Solution:

Considering 1 m length of gate



Force due to oil in the horizontal direction on gate

$$F_{H_1} = \gamma A_p \bar{h}$$

$$= 0.9 \times 1000 \times 9.81 \times (3 \times 1) \times \frac{3}{2} \text{N} = 39.73 \text{kN} (\rightarrow)$$

This force acts at a height of $\frac{3}{3}$ = 1 m above level of A.

Vertical force on gate due to oil,

$$F_{V_1} = \gamma V$$

where V = Volume of portion ABC of cylinder

$$\therefore \quad F_{V_1} = 0.9 \times 1000 \times 9.81 \times \left[\frac{1}{2} \times \frac{\pi}{4} \times 3^2 \times 1\right] \text{N} = 31.188 \text{kN}$$

This force acts at a distance of $\frac{4(D/2)}{3\pi}$ from O at E,

$$\therefore \quad OE = \frac{4 \times (3/2)}{3\pi} = \frac{2}{\pi} = 0.636 \text{m}$$

Force due to liquid P in horizontal direction on gate

$$F_{H_2} = \gamma A_p \bar{h}$$

$$= 0.8 \times 1000 \times 9.81 \times (1.5 \times 1)\left(\frac{1.5}{2}\right) \text{ N}$$

$$= 8.829 \text{ kN } (\leftarrow)$$

This force acts at a distance of $\frac{1.5}{3} = 0.5$ m above level of A.

Vertical force due to liquid, P

$$F_{V_2} = \gamma V$$

where V = Volume of portion AOD

$$\therefore \quad F_{V_2} = 0.8 \times 1000 \times 9.81 \times \left(\frac{1}{4} \times \frac{\pi}{4} \times 3^2 \times 1\right) \text{ N}$$

$$= 13.862 \text{ kN}$$

This force acts at a distance of $\frac{4(D/2)}{3\pi}$ from O at F

$$\therefore \quad OF = \frac{4 \times (3/2)}{3\pi} = 0.636 \text{ m}$$

$\therefore$ Resultant force in the horizontal direction,

$$F_H = |F_{H_1} - F_{H_2}|$$

$$= |39.73 - 8.829|$$

$$= 30.901 \text{ kN}$$

Resultant force in he vertical direction,

$$F_v = |F_{v_1} - F_{v_2}|$$

$$= |31.188 + 13.862|$$

$$= 45.05 \text{ kN}$$

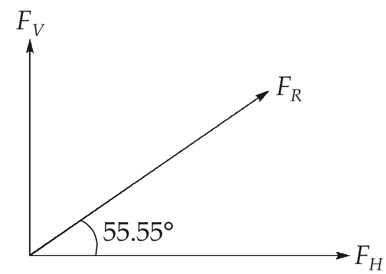
Net resultant force, $F_R = \sqrt{F_H^2 + F_V^2}$

$$= \sqrt{30.901^2 + (45.05)^2} = 54.629 \text{ kN}$$

Let α be the inclination of the resultant to the horizontal then

$$\tan\alpha = \frac{F_V}{F_H} = \frac{45.05}{30.901}$$

$$\therefore \quad \alpha = 55.55°$$



Q.3 (b) Solution:

Let the suffixes 1 and 2 refer to the upstream and downstream sections respectively.

At the upstream section, $V_1 = \frac{16}{4\times 2} = 2$ m/sec.

Froude Number, $F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{2}{\sqrt{9.81\times 2}} = 0.452 < 1$

The upstream flow is subcritical and the transition will cause a drop in the water surface elevation.

Now, $\frac{V_1^2}{2g} = \frac{2^2}{2\times 9.81} = 0.204$ m

Specific energy at section 1, $E_1 = y_1 + \frac{V_1^2}{2g} = 2 + 0.204 = 2.204$ m

Let, q_2 = Discharge intensity at the downstream section

$= \frac{Q}{B_2} = \frac{16}{3.5} = 4.571\ \text{m}^3/\text{sec/m}.$

y_{c2} = Critical depth corresponding to q_2

$\Rightarrow \quad y_{c2} = \left[\frac{q_2^2}{g}\right]^{1/3}$

$\Rightarrow \quad y_{c2} = \left[\frac{(4.571)^2}{9.81}\right]^{1/3} = 1.287$ m

Specific energy corresponding to y_{c2}, $E_{c2} = \frac{3}{2}y_{c2} = \frac{3}{2}\times 1.287 = 1.93$ m

(a) When $\Delta z = 0.2$ m

$$E_2 = \text{Available specific energy at section 2.}$$

$$= E_1 - \Delta z = 2.204 - 0.2 = 2.004 \text{ m} > E_{c2}$$

Hence, the depth $y_2 > y_{c2}$ and the upstream depth will remain unchanged at y_1.

Now,
$$y_2 + \frac{q_2^2}{2gy_2^2} + \Delta z = E_1$$

$$\Rightarrow \quad y_2 + \frac{(4.571)^2}{2 \times 9.81 \times y_2^2} + 0.2 = 2.204$$

$$\Rightarrow \quad y_2 + \frac{1.065}{y_2^2} = 2.004$$

$$\Rightarrow \quad y_2 = 1.574 \text{ m}$$

Hence, when $\Delta z = 0.2$ m, $y_1 = 2.00$ m and $y_2 = 1.574$ m

(b) When $\Delta z = 0.35$ m

$$E_2 = \text{Available specific energy at section (2)}$$

$$= 2.204 - 0.35 = 1.834 \text{ m} < E_{c2}$$

Hence this contraction will be working under choked conditions. The upstream depth must rise to create a higher total head. The depth of flow at section 2 will be critical with $y_2 = y_{c2} = 1.287$ m

Let the new upstream depth is y_1'

$$\therefore \quad y_1' + \frac{Q^2}{2gB_1^2 . y_1'^2} = E_{c2} + \Delta z = 1.93 + 0.35$$

$$\Rightarrow \quad y_1' + \frac{16^2}{2 \times 9.81 \times 4^2 \times y_1'^2} = 2.28$$

$$\Rightarrow \quad y_1' + \frac{0.8155}{y_1'^2} = 2.28$$

$$y_1' = 2.094 \text{ m}$$

The upstream depth will, therefore, rise by 0.094 m due to the choked condition at the contraction. Hence when Δz = 0.35 m. y_1 = 2.094 m and $y_2 = y_{c2}$ = 1.287 m.

Q.3 (c) Solution:

(i)

- Viscosity is the property of fluids by virtue of which they offer resistance to shear or angular deformation. It is primarily due to cohesion and molecular momentum exchange between fluid layers and as flow occurs, these effects appear as shearing stresses between the moving layers.
- Viscosity of fluid is of two types :
  - (i) Dynamic viscosity
  - (ii) Kinematic viscosity [Ratio of dynamic viscosity to mass density of fluid]
- As per Newton's law of viscosity, shear stress, τ is directly proportional to the rate of deformation or viscosity gradient across the flow.

$$\therefore \quad \tau \propto \frac{du}{dy}$$

$$\Rightarrow \quad \tau = \mu.\frac{du}{dy}$$

Where μ = Coefficient of dynamic viscosity

$$\text{Kinematic Viscosity } (\nu) = \frac{\mu}{\rho}$$

Variation of viscosity with temperature :

1. **Dynamic viscosity:** Increase in temperature causes a decrease in dynamic viscosity of a fluid whereas viscosity of gases increase with increase in temperature.

   The reason for above phenomenon is that, in liquids, viscosity is primarily due to molecular cohesion which decreases due to increase in volume due to temperature increment. While in gases, viscosity is due to molecular momentum transfer which increases due to increase in number of collision between gas molecules as temperature increases.

2. **Kinematic viscosity:** Kinematic viscosity is the ratio of dynamic viscosity to the mass density of fluid. In case of liquids, with increase in temperature, the dynamic viscosity as well as density both decrease but decrease in dynamic viscosity is very high as compared to density. So overall kinematic viscosity will decrease for liquids. On the other hand, in case of gases, with increase in temperature, dynamic Viscosity increases and density decreases. So, kinematic viscosity increases for gases.

Variation of viscosity with pressure :

1. **Dynamic viscosity:** In fluids, dynamic viscosity is practically independent of pressure except at extremely high pressure.
2. **Kinematic viscosity :** In liquids, kinematic viscosity is independent of pressure at low to moderate pressure.

   In case of gases, density increases with increase in pressure, therefore kinematic viscosity decreases.

(ii)

As the pipe is rigid

$$\therefore \quad c = \sqrt{\frac{K}{\rho}} = \left(\frac{2.2 \times 10^9}{1000}\right)^{1/2} = 1483.24 \text{ m/s}$$

Critical time,
$$T_0 = \frac{2L}{c} = \frac{2 \times 3500}{1483.24} = 4.72\ s$$

$$\because \quad T = \text{closure time} < \text{critical time}$$

$\therefore$ The pipe closure is rapid.

Hence the water hammer pressure

$$P_h = \rho V_0 c = 1000 \times 1483.24 \times 0.8 \times 10^{-6} \text{ MPa}$$
$$= 1.187 \text{ MPa}$$

Length of the pipe, from the valve end, affected by this peak pressure, during the closure time is,

$$x_0 = L - \frac{cT}{2}$$
$$= 3500 - \frac{1483.24 \times 4}{2} = 533.52 \text{ m}$$

Q.4 (a) Solution:

(i)

Given,

$$U = x^2 - y^2 + x$$
$$V = -(2xy + y)$$

Check :
$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \frac{\partial}{\partial x}\left[x^2 - y^2 + x\right] + \frac{\partial}{\partial y}\left[-(2xy + y)\right]$$

$$= 2x + 1 - 2x - 1$$
$$= 0$$

∴ Flow is possible

Also, $$\omega_z = \frac{1}{2}\left[\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right] = \frac{1}{2}\left[\frac{\partial}{\partial x}(-2xy - y) - \frac{\partial}{\partial y}(x^2 - y^2 + x)\right]$$

$$= \frac{1}{2}[-2y + 2y] = 0$$

∴ Flow is irrotational

As the flow is irrotational, velocity potential function exists.

Now, $$U = \frac{-\partial\phi}{\partial x}$$ where ϕ is velocity potential function.

⇒ $$\partial\phi = -U.\partial x$$

⇒ $$\partial\phi = -(x^2 - y^2 + x)\partial x$$

⇒ $$\int \partial\phi = -\int (x^2 - y^2 + x)\partial x$$

⇒ $$\phi = \frac{-x^3}{3} + xy^2 - \frac{x^2}{2} + f(y) + c \qquad \ldots(i)$$

Also, $$V = -\frac{\partial\phi}{\partial y}$$

⇒ $$\partial\phi = -V.dy = +(2xy + y)dy$$

⇒ $$\int \partial\phi = \int (2xy + y)\partial y$$

⇒ $$\phi = xy^2 + \frac{y^2}{2} + f(x) + c \qquad \ldots(ii)$$

From equation (i) & (ii)

$$\phi = \frac{-x^3}{3} + xy^2 + \frac{y^2}{2} - \frac{x^2}{2} + c$$

Now, $$\partial\psi = -U.\partial y$$ whery ψ is stream function.

$$\partial\psi = -(x^2 - y^2 + x)dy$$

∴ $$\int \partial\psi = -\int (x^2 - y^2 + x)\partial y$$

$$\Rightarrow \qquad \psi = -x^2y+\frac{y^3}{3}-xy+f(x)+c_1 \qquad \ldots\text{(iii)}$$

Also, $\partial\psi = V.\,\partial x = -(2xy+y)\partial x$

$$\Rightarrow \qquad \psi = -x^2y - xy + f(y) + c_1 \qquad \ldots\text{(iv)}$$

From (iii) and (iv),

$$\psi = -x^2y+\frac{y^3}{3}-xy+c_1$$

(ii)

In inlet : Pressure, $P_1 = 1.4\ \text{kg/cm}^2$

Diameter, $D_1 = 40$ cm

At throat : Pressure, $P_2 = 40$ cm of mercury (Vacuum)

$$= \frac{-400}{760}\times 1.0336 = -0.544\ \text{kg/cm}^2$$

Diameter, $D_2 = 15$ cm

Head loss, h_f = 5% of differential head = 0.05 h

Applying equation of continuity,

$$A_1V_1 = A_2V_2$$

$$\Rightarrow \qquad \frac{\pi}{4}\times 40^2\times V_1 = \frac{\pi}{4}\times 15^2\times V_2$$

$$\Rightarrow \qquad V_1 = 0.14\ V_2$$

Coefficient of discharge, $C_d = \sqrt{\dfrac{h-h_f}{h}}$

$$= \sqrt{\frac{h-0.05h}{h}} = 0.9747$$

Applying Bernoulli's equation at the inlet (1) and throat (2) sections,

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

Here, $Z_1 = Z_2$

$$\therefore \quad \frac{1.4\times10^4}{10^3\times9.81}+\frac{(0.14V_2)^2}{2\times9.81} = \frac{-0.544\times10^4}{10^3\times9.81}+\frac{V_2^2}{2\times9.81}$$

$\Rightarrow$ $V_2 = 6.3$ m/sec

Theoretical discharge, $Q_{th} = A_2V_2$

$$= \frac{\pi}{4}\times15^2\times10^{-4}\times6.3$$

$$= 0.111 \text{ m}^3/\text{sec.}$$

Actual discharge, $Q_{act} = C_d \cdot Q_{th}$

$= 0.9747 \times 0.111 = 0.108 \text{ m}^3/\text{sec.} = 108\ l/\text{s}$

Q.4 (b) Solution:

(i)

Classification of turbines on the basis of energy at inlet:

(a) Impulse Turbines

- In impulse turbine, the energy available at inlet is only kinetic energy.
- All the available energy of water is converted into kinetic energy or velocity head by passing it through a contracting nozzle provided at the end of the penstock.
- The water coming out of nozzle in the form of free jet strikes on buckets mounted on the runner.
- The runner revolves in open air i.e. pressure is atmospheric both at inlet and outlet.

For impulse turbine, $P_i = P_0$ and $V_i >> V_0$

- Some examples of impulse turbine are Pelton wheel, Girard turbine, Banki turbine, Jonval turbine, Turgo impulse wheel etc.

(b) Reaction Turbines

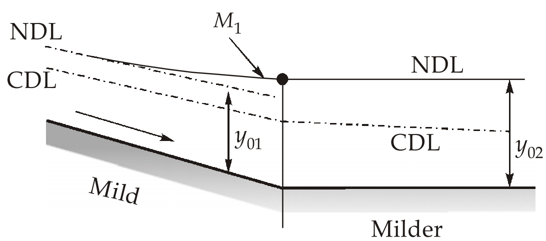
- Reaction turbine operates with its wheel submerged in water.
- At the inlet, water is having both kinetic energy and pressures energy. Water leaving the turbine still have the pressure as well as kinetic energy.
- All pressure energy is not converted into kinetic energy.
- A casing is absolutely essential due to difference of pressures in reaction turbine.

For reaction turbines, $P_i >> P_0$ and $V_i > V_0$

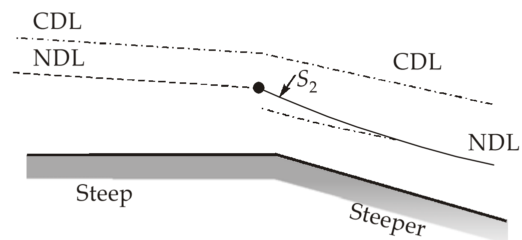
- Some examples of reaction turbine are Francis turbine, Propeller turbine, Kaplan turbine, Thomson turbine, Fourneyron turbine etc.

(ii)

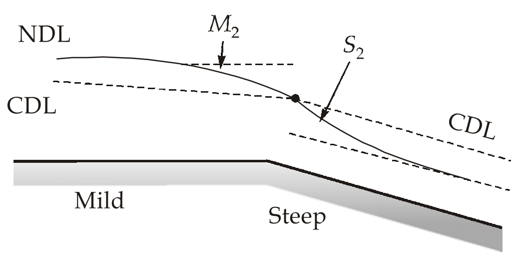
1. Mild to Milder



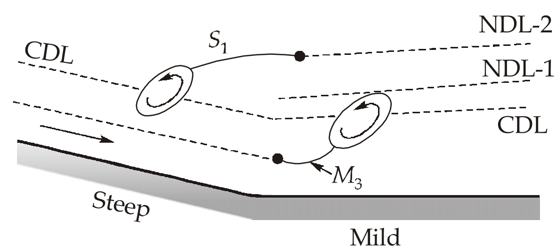
2. Steep to Steeper



3. Mild to Steep

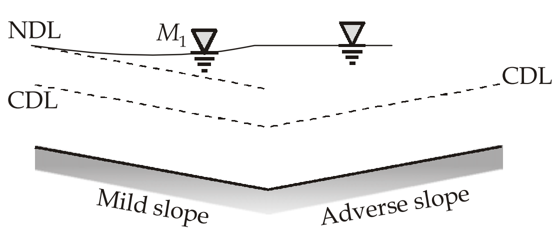


4. Steep to Mild

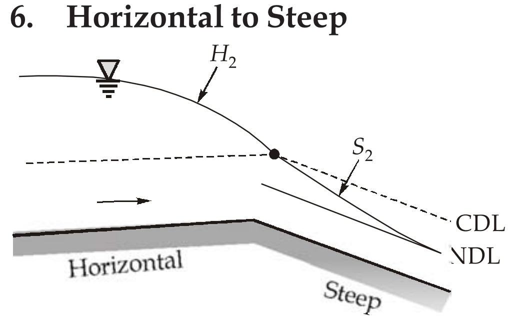


Either S_1 or M_3 jump depending upon sequent depth requirement

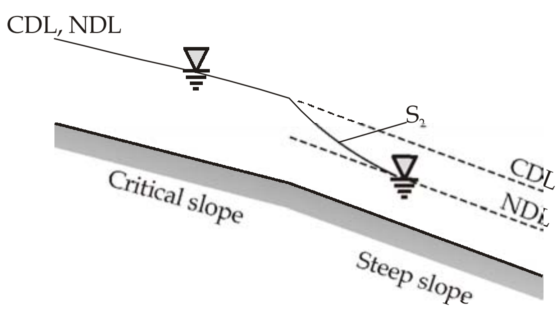
5. Mild to Adverse



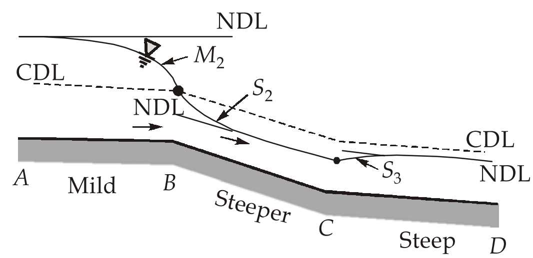
6. Horizontal to Steep



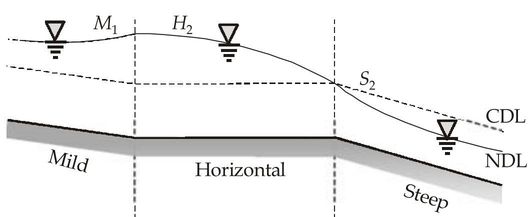
7. **Critical to Steep**



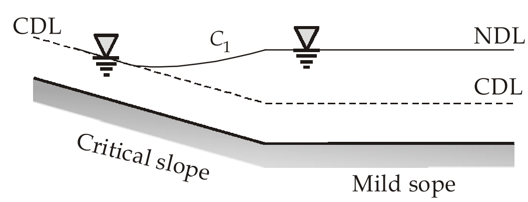
8. **Mild, Steeper and Steep**



9. **Mild, Horizontal and Steep**



10. **Critical to Mild**



Q.4 (c) Solution:

(a) When, Speed, $N = 180$ rpm

$$\text{Angular speed, } \omega = \frac{2\pi\times 180}{60} = 18.85 \text{ rad/sec.}$$

At the wall of the cylinder,

$$r = 15 \text{ cm} = 0.15 \text{ m}$$

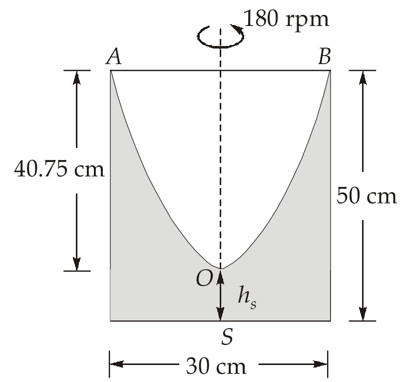
Now,

$$Z = \frac{\omega^2 r^2}{2g}$$

$$= \frac{(18.85)^2\times(0.15)^2}{2\times 9.81}$$

$$= 0.4075 \text{ m} = 40.75 \text{ cm}$$

The position of the water surface in the cylinder will be as shown in the figure. The water surface will start from the rim of the cylinder and will extend downwards as a paraboloid to its vertex at O.



OS = Distance of vertex O from bottom of cylinder

$$\Rightarrow \quad h_s = 50 - 40.75 = 9.25 \text{ cm}$$

Volume of the paraboloid AOB of height 40.75 cm

$$= \frac{1}{2} \times \text{Volume of enclosing cylinder}$$

= Volume of cylinder of height 20.375 cm

Hence, total volume of water inside the cylinder

= Volume corresponding to a depth of (9.25 + 20.375) cm = 29.625 cm

Volume remaining in cylinder,

$$V_{remaining} = \frac{\pi}{4}(0.3)^2 \times 0.29625$$

$$= 20.9 \times 10^{-3} \text{ m}^3 = 20.9\ l$$

$$\text{Original Volume of cylinder, } V = \frac{\pi}{4} \times 0.3^2 \times 0.5$$

$$= 0.0353 \text{ m}^3 = 35.3\ l$$

$$\therefore \quad \text{Volume of water spilled} = 35.3 - 20.9 = 14.4\ l$$

(b) When Speed, N = 240 rpm.

$$\text{Angular speed, } \omega = \frac{2\pi \times 240}{60} = 25.13 \text{ rad./sec.}$$

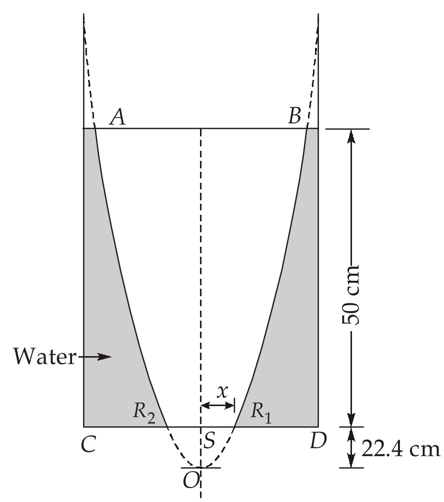
At $r = 0.15$ m

$$Z_{max} = \frac{\omega^2 r^2}{2g} = \frac{(25.13)^2 \times 0.15^2}{2 \times 9.81}$$

$$= 0.724 \text{ m}$$

$$= 72.4 \text{ cm}$$

At this value is larger than 50 cm, it means that the theoretical paraboloid will extend below the base thereby leaving a part of the bottom uncovered by water. The paraboloid will start from the rim and extend upto its vertex O which is 72.4 cm below it.



Let x = radius of the exposed portion of the bottom of the cylinder.

$$x = SR_1 = SR_2$$

Now, $$\frac{z_{max}}{OS} = \frac{r^2}{x^2}$$

$\Rightarrow$ $$x = r\sqrt{\frac{OS}{z_{max}}}$$

$$= 15 \times \sqrt{\frac{22.4}{72.4}} = 8.34 \text{ cm}$$

Volume of water spilled = Volume of the paraboloid AOB

– Volume of the paraboloid R_1OR_2

$$= \frac{\pi}{2} \times r^2 \times Z_{max} - \frac{\pi}{2} \times x^2 \times (OS)$$

$$= \frac{\pi}{2}\times(0.15)^2\times0.724-\frac{\pi}{2}\times(0.0834)^2\times0.224$$

$$= \frac{1}{2}[0.05118-0.001895] = 0.0231 \text{ m}^3 = 23.1\ l$$

Section B : Design of concrete and Masonry Structures - 1 + Strength of Materials - 2

Q.5 (a) Solution:

(i)

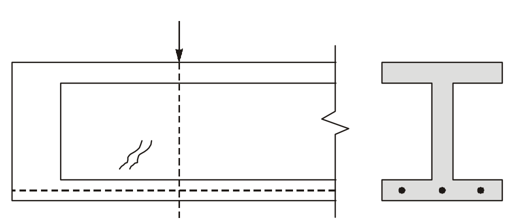
Limit state method as the term suggests design the members for limit state condition which is hypothetical and not going to occur in real circumstances. So to incorporate loading at the state, load factor is taken in this method which is absent in working stress method.

Working stress method emphasises on design of structures on loading upto elastic limit.

1. Limit state method consider limit state of collapse as well as limit service ofserviceability whreas WSM does not consider limit state of collapse.
2. Depth of members designed under LSM is lesser than that of by WSM. Thus, it significantly reduces self weight of members and bring economy in design.
3. Amount of reinforcement required in LSM design is more than that deduced by WSM.
4. Materials in LSM are used upto the values beyond elastic limit. Thus, materials are used more meticulously and economically in LSM.
5. In LSM, capacity of structure can be calculated in different load combinations by changing load factor of safety. In WSM, no load factor of safety is used.

(ii)

For beams having relatively high depth and thin webs, subjected to high shear stresses and low bending stresses, the maximum principle tensile stress is located at the neutral axis level at an inclination of 45° to the longitudinal axis of the beam. The resulting cracks are termed as web shear crack or diagonal tension cracks.

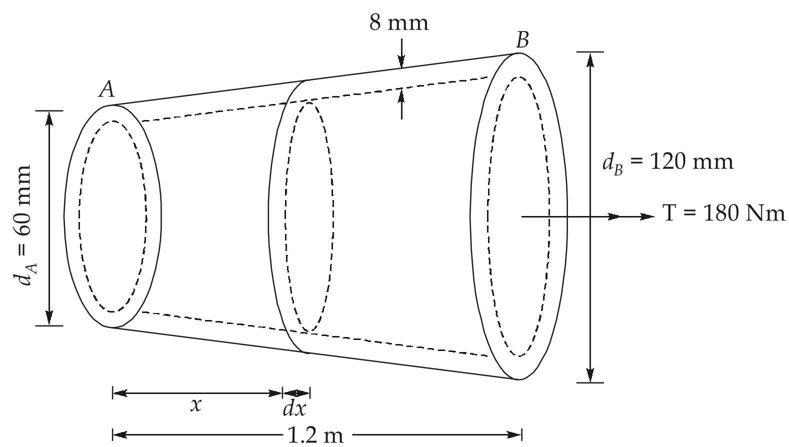


Web-shear crack

In beams subjected to both flexural and shear stresses, a biaxial state of combined tension and compression exists. In such a case, the flexure crack forms first and due to the increased shear stresses at the tip of the cracks, this flexural crack extends into a diagonal tension crack.

Q.5 (b) Solution:

Consider an element of length dx at x distance from left end.



For element of length dx,

$$\text{Angle of twist, } d\phi = \frac{T.dx}{G.I_{P(x)}}$$

Polar moment of hollow circular element,

$$I_{Px} = \frac{\pi d_x^3 t}{4}$$

The diameter of the section at x distance from A,

$$d_x = d_A + \frac{d_B - d_A}{L} \times x$$

$$= 60 + \frac{120 - 60}{1200} \times x = (60 + 0.05\,x) \text{ mm}$$

$$\therefore \quad d\phi = \frac{T.dx}{G.\left(\frac{\pi(60 + 0.05x)^3 \times 8}{4}\right)}$$

So, Angle of twist, $\phi = \int_0^{\phi} d\phi = \frac{T}{2G\pi}\int_0^{1200}\frac{dx}{(60+0.05x)^3}$

$$\Rightarrow \quad \phi = \frac{T}{2G\pi}\left[\frac{1}{(60+0.05x)^2\times 0.1}\right]_{1200}^{0}$$

$$= \frac{180\times 10^3}{2\times 80\times 10^3\times\pi}\times\left[\frac{1}{60^2\times 0.1}-\frac{1}{120^2\times 0.1}\right]$$

$= 7.46 \times 10^{-4}$ radian

$= 0.0427°$

Q.5 (c) Solution:

- Characteristic strength of a material is the probabilistic approach of defining the strength of a material. It is defined as that strength below which not more than a certain percentage of test results conducted on the material are expected to fall. This limiting percentage is generally taken as 5%.

Example:

- For M20 concrete, it implies its characteristic strength is 20 MPa, then which means, if 100 samples of this concrete are tested for compressive strength, then not more than 5 samples [5% of 100] should have a compressive strength less than 20 MPa.
- **Acceptance criteria for concrete:**

1. For all concrete of M15 grade and above, the average strength of four (4) non-overlapping consecutive test results shall be not less than,

   For M15 or higher

$$f_{\text{average}} \leq \begin{cases}\left[f_{ck}+0.825\sigma\right]N/\text{mm}^2 \left(\text{Rounded off to 0.5 N/mm}^2\right)\\ \left(f_{ck}+3\right)\text{N/mm}^2\end{cases}$$

2. Individual test results of any sample, ITR ≥ (f_{ck} – 3) N/mm^2.
3. The test result of the sample shall be average of the strength of three specimens and the individual variation shall not be more than ±15% of average strength. If deviation more then test results of the sample are considered to be invalid.

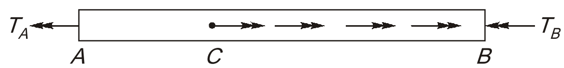
Q.5 (d) Solution:

We know,

$$\text{Modulus of rigidity, } G = \frac{E}{2(1+\mu)} = \frac{200000}{2 \times 1.25} = 80000 \text{ MPa}$$

Uniformly distributed load =60 Nm/m = 60 Nmm/mm

Let the fixed end reactions are T_A and T_B as shown below,



So that, $T_A + T_B = 60 \times 2000 = 120000$ Nmm

Angle of twist at end B due to T_A

$$= \frac{-T_A \times L}{JG} = \frac{-3000 T_A}{JG} \quad \text{...(i)}$$

Angle of twist at end B due to uniformly distributed load in form of torque

$$= \frac{tL^2}{2GJ} = \frac{60 \times 2000^2}{2GJ} = \frac{120000000}{GJ}$$

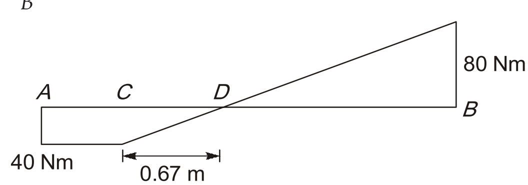
Since the twist at the fixed end B is zero

$$\Rightarrow \frac{-3000T_A}{GJ} + \frac{120000000}{GJ} = 0$$

$\therefore$ $T_A = 40000$ Nmm = 40 Nm

Substituting value in equation (i)

$T_B = 80000$ Nmm = 80 Nm



Variation of torsion

The maximum angle of twist occurs where the torsion is zero.

Polar second moment of area,

$$J = \frac{\pi D^4}{32} = \frac{\pi (20)^4}{32} = 15707.96 \text{ mm}^2$$

$$\text{Maximum angle of twist} = \frac{1}{2} \times \frac{80 \times 10^3 \times (2 - 0.67) \times 10^3}{80000 \times 15707.96} = 0.04234 \text{ radian}$$

Q.5 (e) Solution:

Given, allowable shear force for one screw,

$F = 1.2$ kN

Shear force, $V = 6$ kN

Let, S = longitudinal spacing of screws

Now, Shear flow, $f = \frac{VQ}{I} = \frac{2F}{S}$

$$\Rightarrow \quad S = \frac{2FI}{VQ}$$

$$\text{Moment of inertia, } I = \frac{15\times21^3}{12} - \frac{9\times15^3}{12}$$

$$= 9045 \text{ cm}^4$$

$$Q = Q_{flange} = A_f d_f$$

$$= (15 \times 3) \times \left[\frac{15}{2} + \frac{3}{2}\right]$$

$$= 405 \text{ cm}^3$$

$$\therefore \quad S = \frac{2\times1.2\times9045}{6\times405} = 8.93 \text{ cm}$$

Q.6 (a) Solution:

(i)

- Effective length of column, L_{eff} is given as

$$L_{eff} = 0.65\,(L_0) \qquad \text{[Both ends are fixed]}$$

where L_0 is unsupported length of column

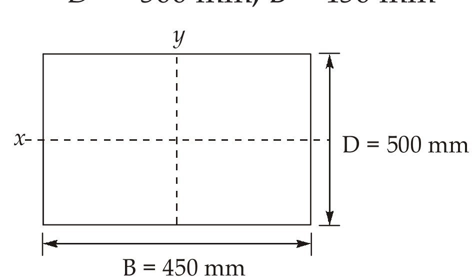
$$= 0.65\,(3.5)$$

$$= 2.275 \text{ m} = 2275 \text{ mm}$$

- Checking the column as short or long:

$$L_{ex} = l_{ey} = 2275 \text{ mm}$$

$$D = 500 \text{ mm, B} = 450 \text{ mm}$$



Slenderness ratio along x-direction, $\lambda_x = \frac{l_{ex}}{D} = \frac{2275}{500}$

$= 4.55 < 12$

Slenderness ratio along y-direction, $\lambda_y = \frac{l_{ey}}{B} = \frac{2275}{450}$

$= 5.06 < 12$

Thus, slenderness ratio in both the directions is less than 12 and so the column is short for both the directions.

- Calculations of minimum eccentricities:

$$e_{x\,\min} = \frac{l_0}{500} + \frac{D}{30}$$

$$= \frac{3500}{500} + \frac{500}{30}$$

$$= 23.67 \text{ mm} < (0.05\,(500) = 25 \text{ mm})$$

$$e_{y\,\min} = \frac{l_0}{500} + \frac{B}{30}$$

$$= \frac{3500}{500} + \frac{450}{30}$$

$$= 22 \text{ mm} < (0.05\,(450) = 22.5 \text{ mm})$$

Since, the minimum eccentricities are less than 0.05 times the lateral dimension in both the directions, the following formula given by IS 456 : 2000 can be used for the design of axially loaded short columns:

Ultimate load capacity, P_u is given by -

$$P_u = 0.4 f_{ck}.A_g + (0.67 f_y - 0.4 f_{ck})\, A_{sc}$$

Now axial service load, $P = 1600$ kN

Factored axial load, $P_u = 1.5 \times 1600 = 2400$ kN

Now, $P_u = 0.4 f_{ck}.\, A_g + (0.67 f_y - 0.4 f_{ck})\, A_{sc}$

$$2400 \times 10^3 = 0.4 \times 20 \times 450 \times 500 + (0.67 \times 415 - 0.4 \times 20)\, A_{sc}$$

$\Rightarrow$ $A_{sc} = 2221.81 \text{ mm}^2$

Check:

Minimum reinforcement, $A_{sc\ min}$ = 0.8% of A_g

$$= \frac{0.8}{100} \times 450 \times 500$$

$$= 1800\ \text{mm}^2$$

Hence, $A_{sc} > A_{sc\ min}$ (OK)

∴ Provide 4 - 25 mm + 4 – 12 mm diameter bars.

Hence, area of steel reinforcement provided,

$$A_{sc\ provided} = 4 \times \frac{\pi}{4} \times 25^2 + 4 \times \frac{\pi}{4} \times 12^2$$

$$= 2415\ \text{mm}^2 > 2221.8\ \text{mm}^2$$

∴ Percentage of reinforcement provided,

$$p_t = \frac{2415}{450 \times 500} \times 100 = 1.073\%$$

$$p_t = 1.073\% > \text{minimum reinforcement } (0.8\%) \quad \text{(OK)}$$

$$< \text{maximum reinforcement } (6\%) \quad \text{(OK)}$$

Lateral ties:

- **Diameter of tie bar:**

$$\phi_t = \max. \begin{cases} \frac{\phi_{max}}{4} = \frac{25}{4} = 6.25\ \text{mm} \\ = 6\ \text{mm} \end{cases}$$

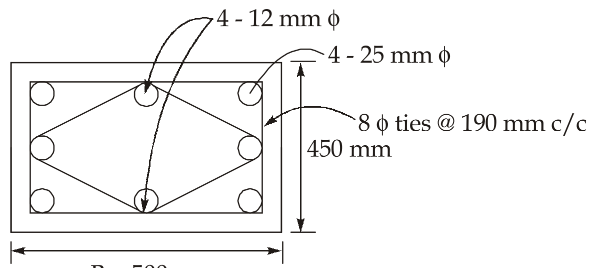
$$= 8\ \text{mm} \quad \text{(say)}$$

- **Spacing of ties,**

$$S_t = \min. \begin{cases} b = 450\ \text{mm} \\ 16\phi_{min} = 16 \times 12 = 192\ \text{mm} \\ 300\ \text{mm} = 300\ \text{mm} \end{cases}$$

$$= 192\ \text{mm}$$

Hence provide 8 mm ϕ @ 190 mm c/c spacing



Reinforcement detailing

(ii)

The term 'limit state of collapse' is used to describe the ultimate limit state of compression members (whether axially loaded or eccentrically loaded). But the actual failure may or may not occur in compression since an eccentrically loaded column is subjected to axial load along with moment and the failure mode depends upon the eccentricity. For small eccentricity, axial compression behavior dominates thereby making the subsequent failure as compression failure whereas for large eccentricity, flexural behavior predominates, there by making the failure as tension failure. Thus depending upon the eccentricity, we can predict in advance that a compression failure or tension failure will take place.

Types of failure:

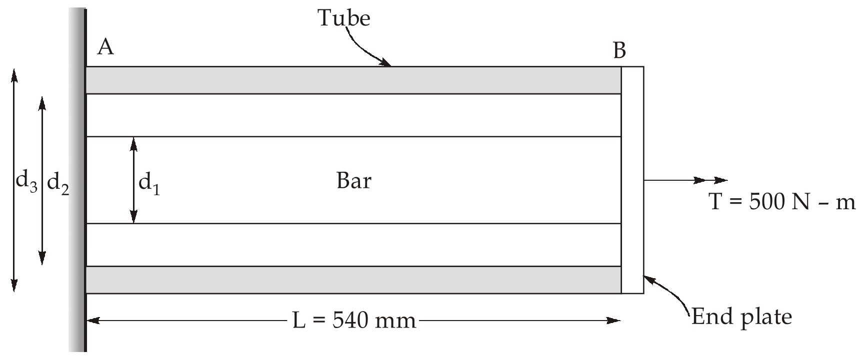
1. **Balanced failure:** In between the two extreme failure conditions, i.e. compression failure & tension failure, there exists a critical failure condition which is called as balanced failure.

   This failure condition is characterized by the yielding of outermost row of reinforcing steel (tension side) and attainment of maximum compressive strain in concrete in flexure which is 0.0035 at the highly compressed edge of the column simultaneously. Thus, both crushing of concrete and yielding of steel occurs simultaneously.

2 **Compression failure:** When the loading eccentricity is less than the eccentricity corresponding to balanced failure condition, then yielding of reinforcing steel does not takes place and failure occurs at ultimate state by crushing of concrete at the highly compressed edge.

3. **Tension failure:** When the eccentricity of loading is more than that corresponding to balanced failure condition, then yielding of tension steel at the outermost row of reinforcing steel takes place. The outermost row of reinforcing bars yield first followed by yielding of inner row of bars with increasing strain.

Q.6 (b) Solution:



(i) Polar moments of inertia of bar,

$$I_{P1} = \frac{\pi}{32} d_1^4$$

$$= \frac{\pi}{32} \times 24^4 = 3.257 \times 10^4 \text{ mm}^4$$

Polar moment of inertia of tube,

$$I_{P2} = \frac{\pi}{32}\left(d_3^4 - d_2^4\right)$$

$$= \frac{\pi}{32}\left(37^4 - 30^4\right) = 10.45 \times 10^4 \text{ mm}^4$$

Since the bar and tube are joined to the end plate.

$\therefore$ $$\phi_{bar} = \phi_{tube}$$

where ϕ_{bar} and ϕ_{tube} are angle of twist of bar and tube respectively due to applied torque.

$\therefore$ $$\frac{T_1 L}{G I_{P_1}} = \frac{T_2 L}{G I_{P_2}}$$

$\Rightarrow$ $$\frac{T_1}{3.257 \times 10^4} = \frac{T_2}{10.45 \times 10^4}$$

$\Rightarrow$ $$T_2 = 3.20\, T_1 \quad \ldots(1)$$

Also, $$T_1 + T_2 = 500 \quad \ldots(2)$$

From (1) and (2)

$$T_1 = 119.05 \text{ N-m} \text{ and } T_2 = 380.96 \text{ N-m}$$

Now, maximum shear stress in bar,

$$\tau_1 = \frac{T_1}{I_{P_1}}\left(\frac{d_1}{2}\right)$$

$$= \frac{119.05\times10^3}{3.257\times10^4}\times\frac{24}{2} = 43.86 \text{ MPa}$$

Maximum shear stress in tube, $\tau_2 = \frac{T_2}{I_{P_2}}\left(\frac{d_3}{2}\right)$

$$= \frac{380.96\times10^3}{10.45\times10^4}\times\frac{37}{2} = 67.44 \text{ MPa}$$

(ii) Angle of rotation of end plate ,

$$\phi = \frac{T_1 L}{GI_{P_1}} = \frac{T_2 L}{GI_{P2}}$$

$$= \frac{119.05\times10^3\times540}{80\times10^3\times3.257\times10^4} = 0.0247 \text{ rad} = 1.42°$$

(iii) Torsional stiffness of composite bar,

$$K_T = \frac{T}{\phi}$$

$$= \frac{500\times10^{-3}}{0.0247} = 20.24 \text{ kN/m}$$

Q.6 (c) Solution:

Effective depth, $d = D - E.C. = 700 - 50 = 650$ mm

Effective length, $l_{eff} = \text{Minimum}\begin{cases} l_{cl}+d = 10+0.65 = 10.65\text{ m} \\ l_{cl}+w = 10+0.3 = 10.3\text{ m}\end{cases}$

∴ $l_{eff} = 10.3$ m

- **Load calculations:**

Dead load, $w_d = B \times D \times 1 \times \gamma_{RCC}$

$= 0.5 \times 0.7 \times 1 \times 25 = 8.75$ kN/m

Live load, $w_e = 60$ kN/m

Total Load, $w_T = 8.75 + 60 = 68.75$ kN/m

Maximum bending moment for simply supported beam is given as:

$$(BM)_{max} = \frac{(w_T).(L_{eff})^2}{8} = \frac{(68.75)(10.3)^2}{8}$$

$$= 911.71 \text{ kNm}$$

- Elastic modules of elasticity of concrete, E_c

$$E_c = 5000\sqrt{f_{ck}}$$

$$= 5000\sqrt{30}$$

$$= 27386.13 \text{ MPa}$$

- Actual depth of neutral axis, x_a is given by:

$$\frac{B}{2}.x_a^2 + m.A_{st}.x_a - m.A_{st}.d = 0$$

where $$m = \frac{E_s}{E_c} = \frac{2\times10^5}{5000\sqrt{30}} = 7.30$$

$$\frac{500}{2}.x_a^2 + 7.30\times\frac{8\times\pi}{4}\times32^2.x_a - 7.30\times\frac{8\times\pi}{4}\times32^2\times650 = 0$$

$\Rightarrow$ $$x_a = 267.92 \text{ mm}$$

- Moment of inertia of cracked section, I_{cr} is given by

$$I_{cr} = \frac{B.x_a^3}{3} + m\left[n\times\frac{\pi}{64}\phi^4 + n\times\frac{\pi}{4}\phi^2.(d-x_a)^2\right]$$

$$= \frac{500\times(267.92)^3}{3} + 7.30\left[\frac{8\times\pi}{64}\times32^4 + \frac{8\times\pi}{4}\times32^2(650-267.92)^2\right]$$

$$= 1.006\times10^{10} \text{ mm}^4$$

- Cracking moment, M_{cr} is given by-

$$M_{cr} = f_{cr}.Z$$

$$= \left(0.7\sqrt{f_{ck}}\right).\left(\frac{BD^2}{6}\right)$$

$$= \left(0.7\sqrt{30}\right).\left(\frac{500\times700^2}{6}\right) \text{Nmm}$$

$$= 156.56 \text{ kNm}$$

- Lever arm, (L.A) is given by

$$\text{L.A.} = \left[d - \frac{x_a}{3}\right]$$

$$= \left[650 - \frac{267.92}{3}\right] = 560.69 \text{ mm}$$

Now, effective moment of inertia, I_{eff} is given by:

$$I_{\text{eff}} = \frac{I_{cr}}{1.2 - \left(\frac{M_{cr}}{BM}\right)\cdot\left(\frac{L.A.}{d}\right)\cdot\left(1 - \frac{x_a}{d}\right)\cdot\left(\frac{b_w}{b}\right)}$$

Now $\quad b_w = b$

So, $\quad I_{\text{eff}} = \dfrac{1.006 \times 10^{10}}{1.2 - \left(\frac{156.56}{911.71}\right)\cdot\left(\frac{560.69}{650}\right)\left(1 - \frac{267.92}{650}\right)\left(\frac{1}{1}\right)}$

$\Rightarrow \quad I_{\text{eff}} = 0.9039 \times 10^{10} \text{ mm}^4$

But, $\quad I_{cr} \le I_{\text{eff}}$

Hence, take $\quad I_{\text{eff}} = I_{cr}$

$\Rightarrow \quad I_{\text{eff}} = 1.18 \times 10^{10} \text{ mm}^4.$

Now short term deflection, δ is given by:

$$\delta_{\text{short}} = \frac{5 \times (68.75).(10.3)^4 (1000)^4}{384 \times (27386.13)(1.006 \times 10^{10})} \text{mm}$$

$$= 36.57 \text{ mm}$$

Now total deflection, δ_{total} is given by -

$$\delta_{\text{total}} = \delta_{\text{short}} + \theta.\delta_{\text{short}}$$

$$= \delta_{\text{short}} (1 + \theta)$$

$$= 36.57 (1 + 1.6) = 95.08 \text{ mm}$$

Q.7 (a) Solution:

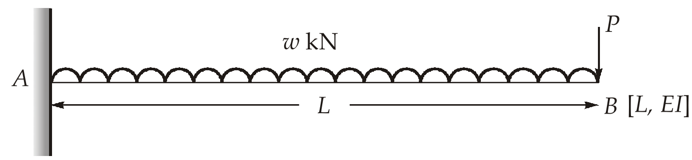
(i)

Principle of superposition:

Principle of superposition states that for a linearly elastic structure, deflection at a given point in a structure produced by several loads acting simultaneously on the structure can be found by superposing deflection at same point produced by loads acting individually.

For example,

Consider a cantilever beam subjected to loads as shown below.



∴ Deflection at free end can be written as

$$\Delta_B = \underbrace{\frac{PL^3}{3EI}}_{\text{due to point load}} + \underbrace{\frac{wL^4}{8EI}}_{\text{due to udl}}$$

Mohr's circle of stress:

It is a graphical representation which is useful in visualizing the relationships between normal and shear stresses acting on various planes at a point in a stressed body.

Some important features of Mohr's circle are:

(1) It is locus of normal and shear stresses at a point in a strained body with the changing value of angle θ at that point.

(2) Radius of Mohr's circle represent maximum value of shear stress.

(3) Centre of Mohr's circle always lies on x-axis.

(4) Points of circle lying on *x*-axis represent principal stresses.

(ii)

Given $B = 500$ mm, $D = 700$ mm

Effective depth, $d = D - 50$

$= 700 - 50 = 650$ mm

Equivalent bending moment, M_{e1} is given as

$$M_{e1} = M_u + M_t$$

Now, $$M_t = \frac{T_u\left(1+\frac{D}{B}\right)}{1.7} = \frac{50\left(1+\frac{700}{500}\right)}{1.7} = 70.588 \text{ kN–m}$$

So, $$M_{e1} = 160 + 70.6 = 230.6 \text{ kN–m}$$

Limiting moment of resistance, $M_{u\,lim}$ is given by

$$M_{u\,lim} = R_o.f_{ck}.B.d^2$$
$$= 0.138 \times 20 \times 500 \times 650^2 \times 10^{-6} \text{ kN-m}$$
$$= 583.05 \text{ kN-m}$$

$\therefore$ $M_u < M_{u\,lim}$. Thus section is under reinforced.

Now, M_{e1} is given by

$$M_{e1} = 0.87\, f_y.A_{st}\,(d - 0.42.x_u) \quad \ldots(i)$$

But $$x_u = \frac{0.87\, f_y.A_{st}}{0.36\, f_{ck}.B}$$

Put this x_u in Equation (i), we get

$$M_{e1} = 0.87\, f_y.A_{st}\left[d - 0.42 \times \frac{(0.87.f_y.A_{st})}{0.36 f_{ck}.B}\right]$$

$$\Rightarrow \quad 230.6\times10^6 = 0.87 \times 415 \times A_{st}\left[650 - \frac{0.42(0.87\times415\times A_{st})}{0.36\times20\times500}\right]$$

$$\Rightarrow \quad 230.6\times10^6 = 361.05\, A_{st}\left[650 - \frac{151.641.A_{st}}{3600}\right]$$

$$A_{st} = 1054.69 \text{ mm}^2$$

Alternatively,

$$R = \frac{M_{e1}}{bd^2} = \frac{230.6\times10^6}{500\times650^2} = 1.091598 \text{ MPa}$$

$$\therefore \quad \frac{p_t}{100} = \frac{20}{2(415)}\left[1 - \sqrt{1 - 4.598\frac{R}{f_{ck}}}\right] = 0.003242$$

$$\therefore \quad A_{st} = \left(\frac{p_t}{100}\right)bd = 0.003242 \times 500 \times 650$$
$$= 1053.53 \text{ mm}^2$$

Q.7 (b) Solution:

Volume of water $= 6 \times 10^5\, l = 600 \text{ m}^3$

Depth of water, $H = 4 - 0.2 = 3.8$ m

Now, $\frac{\pi}{4}.D^2.H = 600$ where D is diameter of tank

$$\Rightarrow \qquad \frac{\pi}{4}.D^2.(3.8) = 600$$

$$\Rightarrow \qquad D = 14.18 \text{ m} \simeq 14.2 \text{ m.} \quad \text{(Say)}$$

(i) Design of steel reinforcement:

1. Hoop tension, $T_H = \frac{P.D}{2} = \frac{\gamma_w.H.D}{2} = \frac{10 \times 3.8 \times 14.2}{2} = 269.8$ kN

2. Area of steel required to resist hoop tension,

$$A_{st \cdot H} = \frac{T_H}{\sigma_s} = \frac{269.8 \times 10^3}{150} = 1798.67 \text{ mm}^2 \simeq 1800 \text{ mm}^2 \quad \text{(say)}$$

If steel is provided in two layers then,

$$A_{st \cdot H} \text{ in each layer} = \frac{1800}{2} = 900 \text{ mm}^2$$

$$\text{Using 10 mm } \phi \text{ bars, spacing} = \frac{1000}{900} \times \frac{\pi}{4} \times 10^2$$

$$= 87.27 \text{ mm} \simeq 85 \text{ mm} \quad \text{(say)}$$

$$\therefore \qquad A_{st.\,H\,\text{provided}} = \frac{1000}{85} \times \frac{\pi}{4} \times 10^2$$

$$= 923.998 \text{ mm}^2 > 900 \text{ mm}^2 \quad \text{(OK)}$$

(ii) Design of concrete:

1. Thickness as per approximate formula,

$$T \text{ (in mm)} = 30\,H + 50 = 30 \times 3.8 + 50 = 164 \text{ mm}$$

2. If $T = 164$ mm, then stress in concrete

$$f_{ct\text{ direct}} = \frac{T_H}{1000T + (m-1)A_{st}}$$

where $$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 8.5} = 10.98$$

$$\therefore \qquad f_{ct\text{ direct}} = \frac{269.8 \times 10^3}{(1000 \times 164) + (10.98 - 1)(923.998 \times 2)}$$

$$= 1.479 \text{ MPa} > 1.3 \text{ MPa} \quad \text{[Failed]}$$

3. Required thickness

$$1.3 \geq \frac{T_H}{1000T + (m-1).A_{st}}$$

$$\Rightarrow \quad T \geq \frac{\left(\frac{T_H}{1.3}\right) - (m-1).A_{st}}{1000}$$

$$\Rightarrow \quad T \geq \frac{\left(\frac{269.8 \times 10^3}{1.3}\right) - (10.98-1).(923.998 \times 2)}{1000}$$

$$\Rightarrow \quad T \geq 189.095 \text{ mm}$$

$\therefore$ Provide thickness, $T = 200$ mm

Now
$$f_{ct} = \frac{269.8 \times 10^3}{(1000 \times 200) + (10.98-1)(923.998 \times 2)}$$
$$= 1.235 \text{ MPa} < 1.3 \text{ MPa} \quad \text{Hence safe}$$

(iii) Vertical reinforcement:

$$A_{st,\ min} = \frac{0.35}{100} \times 1000 \times T_{s1}$$

Now,
$$T_{s1} = T_{s2} = \frac{200}{2} = 100 \text{ mm}$$

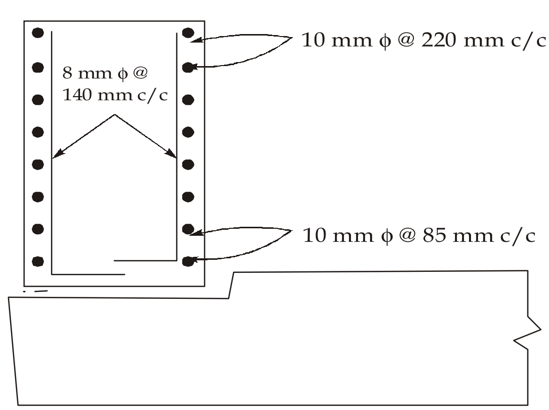
[Should not be greater than 250 mm]

Now,
$$A_{st\ min} = \frac{0.35}{100} \times 1000 \times 100 = 350 \text{ mm}^2$$

Spacing of 8 mm diameter bars $= \frac{1000}{350} \times \frac{\pi}{4} \times 8^2 = 143.61 \text{ mm} \simeq 140 \text{ mm (say)}$

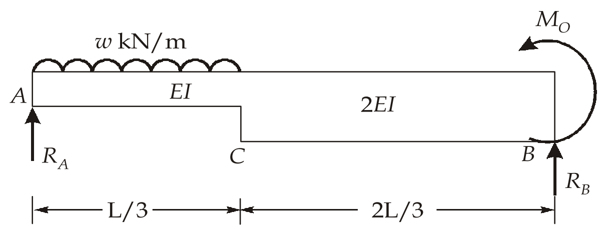
(iv) Minimum Reinforcement at Top:

For 10 mm ϕ bar, spacing $= \frac{1000}{350} \times \frac{\pi}{4} \times 10^2 = 224.4 \text{ mm} \simeq 220 \text{ mm (say)}$



Q.7 (c) Solution:

Apply a imaginary moment M_0 at end B,



Using equilibrium equations,

$$\Sigma F_y = 0$$

$$\Rightarrow \quad R_A + R_B = \frac{wL}{3}$$

$$\Sigma M_A = 0$$

$$\frac{wL}{3}\left(\frac{L/3}{2}\right) - M_0 - R_B(L) = 0$$

$$\Rightarrow \quad R_B = \frac{wL}{18} - \frac{M_0}{L}$$

$$\Rightarrow \quad R_A = \frac{wL}{3} - R_B = \frac{M_0}{L} + \frac{5wL}{18}$$

Now, Bending moment for segment AC,

At a distance x from A = $\left(0 \le x \le \frac{L}{3}\right)$

$$M_x = R_A x - \frac{wx^2}{2} = \left(\frac{M_0}{L} + \frac{5wL}{18}\right)x - \frac{wx^2}{2}$$

Now, $$\frac{\partial M_{AC}}{\partial M_o} = \frac{x}{L}$$

Bending moment for segment BC,

At a distance x from B, $\left(0 \le x \le \frac{2L}{3}\right)$

$$M_{BC} = R_B x + M_0$$

$$= \left(\frac{wL}{18} - \frac{M_0}{L}\right)x + M_0$$

Now, $$\frac{\partial M_{BC}}{\partial M_0} = 1-\frac{x}{L}$$

Using modified castigilano's theorem,

$$\theta_B = \int\left(\frac{M}{EI}\right)\left(\frac{\partial M}{\partial M_0}\right)dx$$

$$= \frac{1}{EI}\left[\int_0^{L/3} M_{AC}\left(\frac{\partial M_{AC}}{\partial M_0}\right) + \int_0^{\frac{2L}{3}} M_{BC}\left(\frac{\partial M_{BC}}{\partial M_0}\right)\right]dx$$

$$= \frac{1}{EI}\left[\int_0^{L/3}\left[\left[\frac{M_0}{L}+\frac{5wL}{18}\right]x-\frac{wx^2}{2}\right]\times\frac{x}{L}\right]dx$$

$$= \frac{2}{2EI}\left[\int_0^{\frac{2L}{3}}\left[\left[\frac{wL}{18}-\frac{M_0}{L}\right]x+M_0\right]\times\left(1-\frac{x}{L}\right)\right]dx$$

Substituting $M_0 = 0$

$$\theta_B = \frac{1}{EI}\int_0^{L/3}\left(\frac{5wLx}{18}-\frac{wx^2}{2}\right)\left(\frac{x}{L}\right)dx\times 2+\frac{1}{EI}\int_0^{\frac{2L}{3}}\left(\frac{wLx}{18}\right)\times\left(1-\frac{x}{L}\right)dx$$

$$= \frac{1}{EI}\left[\frac{5wx^3}{54}-\frac{wx^4}{8L}\right]_0^{L/3} 2+\frac{1}{EI}\left[\frac{wLx^2}{36}-\frac{wx^3}{54}\right]_0^{\frac{2L}{3}}$$

$$= \frac{11}{5832}\frac{wL^3}{EI}+\frac{5}{1458}\frac{wL^3}{EI}$$

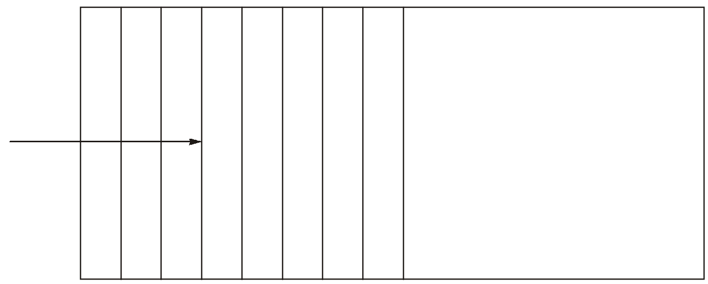
$$= \frac{17}{1944}\frac{wL^3}{EI} \qquad \text{(Counter clockwise)}$$

Q.8 (a) Solution:

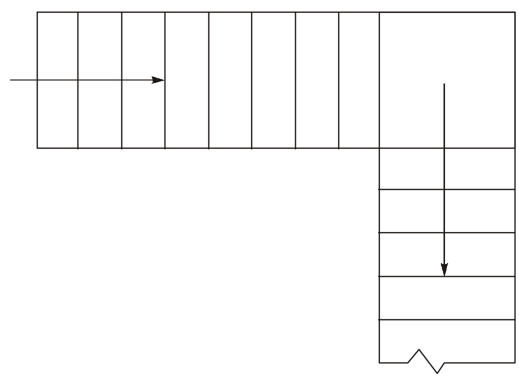
(i)

On the basis of geometrical configuration, following types of staircase are classified:

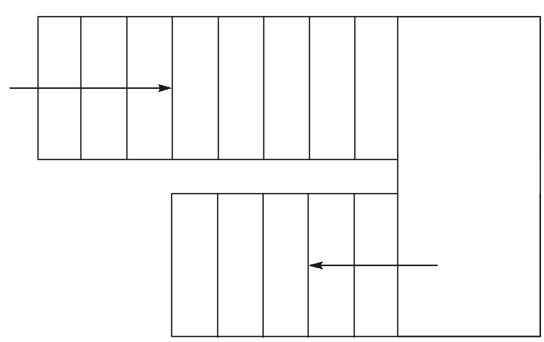
1. **Straight stairs:** This consists of steps leading in the same direction. It is usually provided in a long narrow staircase. It usually consists of one flight but in some circumstances two flights with an intermediate landing can be provided.



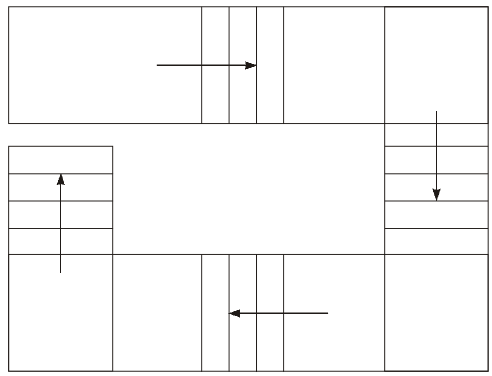
2. **Quarter-turn stairs:** It consists of two straight rows of stairs and one quarter turn of 90°. With this type of staircase from the base to the top, the steps and the person on them turn through 90°.



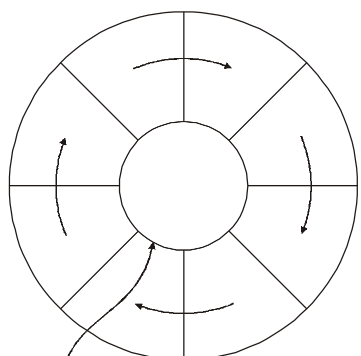
3. **Dog-legged stairs:** In this type of stairs, the succeeding flights rise in opposite direction. A landing is provided corresponding to the level at which the direction of flight changes.



4. **Open-well stairs:** This type of stairs consist of two or more flights arranging in a well or opening between the backward and forward flight.



5. **Spiral/Circular stairs:** In this type of stairs all the steps radiate from a newel or well hole in the form of winders. It is usually adopted at the backside of building to access its various floors.



(ii)

Beam design by WSM:

$$\text{Modular Ratio, } m = \frac{280}{3.\sigma_{cbc}} = \frac{280}{3\times 7} = 13.33$$

$$k_0 = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{13.33\times 7}{13.33\times 7 + 230} = 0.2886$$

$$j = 1 - \frac{k_0}{3} = 1 - \frac{0.2886}{3} = 0.9038 \simeq 0.904$$

Now, for a balanced section,

$$M = Q.b.d^2$$

$$\Rightarrow \quad \frac{wl^2}{8} = \left(0.5\,\sigma_{cbc}.k_0 j\right).bd^2$$

$$\Rightarrow \quad \frac{30\times4^2\times10^6}{8} = (0.5\times7\times0.2886\times0.904)b\times450^2$$

$$\Rightarrow \quad b = 324.48 \text{ mm} \simeq 330 \text{ mm} \quad \text{(say)}$$

Thus, effective size of beam section is 330 mm × 450 mm.

Beam design by LSM: Factored load = 1.5 × 30 = 45 kN/m

$$\text{Factored, } BM = \frac{w_U.L^2}{8} = \frac{45\times4^2}{8} = 90\,\text{kNm}$$

Now for balanced section ultimate moment of resistance,

$$M_u = 0.138 f_{ck}.b.d^2$$

$$90\times10^6 = 0.138\times20\times b\times450^2$$

$$\Rightarrow \quad b = 161.03 \text{ mm} \simeq 165 \text{ mm} \quad \text{(say)}$$

∴ Effective size of beam section is 165 mm × 450 mm

$$\therefore \quad \text{Percent saving in concrete} = \frac{330\times450-165\times450}{330\times450}\times100 = 50\%$$

Q.8 (b) Solution:

(i)

$$\text{Kinetic Energy} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\times\frac{150\times10^3}{9.81}\times\left(8\times\frac{1000}{3600}\right)^2 = 37754.37 \text{ N-m}$$

$$\therefore \quad \text{Energy absorbed per spring} = \frac{37754.37}{4} = 9438.6 \text{ N-m}$$

Now, Work done = Energy absorbed

$$\Rightarrow \quad P\times\frac{\delta}{2} = 9438.6$$

$$\Rightarrow \quad P = \frac{2\times9438.6}{0.14} = 134837.14\,N$$

$$\text{Also, deflection of spring, } \delta = \frac{64PR^3n}{Gd^4}$$

$$\Rightarrow \quad d = \sqrt[4]{\frac{64PR^3n}{G.\delta}}$$

$$= 4\sqrt{\frac{64 \times 134837.14 \times (0.1)^3 \times 9}{80 \times 10^9 \times (0.14)}}$$

$$= 0.05132 \text{ m} = 51.32 \text{ mm}$$

(ii)

Given, Axial pull, $P = 11$ kN

Shear force, $V = 8$ kN

$f_y = 275 \text{ N/mm}^2$

FOS $= 3$ and $\mu = 0.3$

Let 'd' be the diameter of bolt (in mm)

$$\therefore \quad \text{Normal stress, } \sigma = \frac{P}{A} = \frac{11 \times 10^3}{\frac{\pi d^2}{4}} \text{N/mm}^2$$

$$\text{Shear stress, } \tau = \frac{V}{A} = \frac{8 \times 10^3}{\frac{\pi d^2}{4}} \text{N/mm}^2$$

Now principal stresses, $\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

$$= \frac{11 \times 10^3}{2 \times \frac{\pi d^2}{4}} \pm \sqrt{\left(\frac{11 \times 10^3}{\frac{\pi d^2}{4} \times 2}\right)^2 + \left(\frac{8 \times 10^3}{\frac{\pi d^2}{4}}\right)^2}$$

$$= \frac{10^3}{\frac{\pi d^2}{4}} \times \left[5.5 \pm \sqrt{5.5^2 + 8^2}\right]$$

$$\therefore \quad \sigma_1 = \frac{15.21 \times 10^3}{\frac{\pi d^2}{4}} \text{N/mm}^2$$

$$\sigma_2 = \frac{-4.21 \times 10^3}{\frac{\pi d^2}{4}} \text{N/mm}^2$$

1. Maximum principal stress theory

$$\sigma_1 \le \frac{f_y}{FOS}$$

$$\frac{15.21\times10^3}{\frac{\pi d^2}{4}} \le \frac{275}{3}$$

$$\Rightarrow \quad d \ge 14.53 \text{ mm}$$

2. Maximum shear stress theory, $(\tau_{max})_{abs} = \dfrac{f_y}{2(FOS)}$

$$\therefore \quad (\tau_{max})_{abs} \ge \frac{\sigma_1 - \sigma_2}{2}$$

$$\Rightarrow \quad \frac{\sigma_1 - \sigma_2}{2} \ge \frac{f_y}{2(FOS)}$$

$$\Rightarrow \quad \frac{(15.21-(-4.21))\times10^3}{\frac{\pi d^2}{4}} \le \frac{275}{3}$$

$$\Rightarrow \quad d \ge 16.42$$

(iii) Strain energy theory,

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \le \left(\frac{f_y}{FOS}\right)^2$$

$$\left(\frac{15.21\times10^3}{\frac{\pi d^2}{4}}\right)^2 + \left(\frac{-4.21\times10^3}{\frac{\pi d^2}{4}}\right)^2 - \frac{2\times0.3\times15.21\times10^3\times\left(-4.21\times10^3\right)}{\left(\frac{\pi d^2}{4}\right)^2} \le \left(\frac{275}{3}\right)^2$$

$$\Rightarrow \quad \frac{10^6}{\left(\frac{\pi d^2}{4}\right)^2}\left[15.21^2 + 4.21^2 + 38.42\right] \le 8402.78$$

$$d \ge 15.35 \text{ mm}$$

Q.8 (c) Solution:

Effective depth, $d = 600 - 35 = 565$ mm

- Area of steel in compression, $A_{sc} = 2\times\dfrac{\pi}{4}\times12^2 = 226.19 \text{ mm}^2$

$$\simeq 227 \text{ mm}^2 \qquad \text{(say)}$$

- Area of steel in tension, $A_{st} = 3\times\frac{\pi}{4}\times16^2 = 603.18 \text{ mm}^2$

$$\simeq 604 \text{ mm}^2 \qquad \text{(say)}$$

- Limiting depth of neutral axis,

$$x_{u\,\text{lim}} = 0.48.\text{d} \qquad \text{[For Fe415 steel]}$$

$$= 0.48 \times 565 = 271.2 \text{ mm}$$

Actual depth of neutral axis, x_u is given as

$$0.36 f_{ck}.\text{B}.x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y.A_{st}$$

$\Rightarrow$ $0.36 \times 20 \times 300.\ x_u + (f_{sc} - 0.45 \times 20)\ 227 = 0.87 \times 415 \times 604$

$\Rightarrow$ $2160\ x_u + 227 f_{sc} - 2043 = 218074.2$

$\Rightarrow$ $2160\ x_u + 227.f_{sc} = 220117.2$

$\Rightarrow$ $x_u = 101.906 - 0.1051 f_{sc}$ ...(i)

First trial: Let $f_{sc} = 315$ MPa

$\therefore$ $x_u = 101.906 - 0.1051 \times 315$

$= 68.7995$ mm

$\simeq 68.8$ mm (say)

Strain at the level of compression steel,

$$\in_{sc} = \left(\frac{x_u - d_c}{x_u}\right)\times0.0035$$

$$= \left(\frac{68.8-35}{68.8}\right)\times0.0035 = 0.00172$$

Now f_{sc} can be calculated from the table of stress-strain for Fe415.

$$\frac{324.8-306.7}{0.00192-0.00163} = \frac{f_{sc}-306.7}{0.00172-0.00163}$$

$\Rightarrow$ $f_{sc} = 312.32 \text{ MPa} \neq 315 \text{ MPa}$ (as assumed)

Second trial: Let $f_{sc} = 313$ MPa,

Now, x_u from equation (i) can be computed as,

$$x_u = 101.906 - 0.1051 \times 313$$

$$= 69.0097 \text{ mm} \simeq 69.01 \text{ mm} \quad \text{(say)}$$

Now strain at the level of compression steel,

$$\in_{sc} = \left(\frac{x_u - d_c}{x_u}\right) \times 0.0035$$

$$= \left(\frac{69.01 - 35}{69.01}\right) \times 0.0035 = 0.001725$$

Now f_{sc} can be calculated from the table of stres strain for Fe415,

$$\frac{324.8 - 306.7}{0.00192 - 0.00163} = \frac{f_{sc} - 306.7}{0.001725 - 0.00163}$$

$\Rightarrow$ $f_{sc} = 312.63 \text{ MPa} = 313 \text{ MPa}$ (as assumed)

$\therefore$ Solution is converging and thus $x_u = 69.01$ mm.

Now, ultimate moment of resistance of given beam section is given as

$$\text{MOR}_u = 0.36 f_{ck}.B.x_u (\text{d} - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$$

$$= \begin{bmatrix} 0.36 \times 20 \times 300 \times 69.01 (565 - 0.42 \times 69.01) \\ + (313 - 0.45 \times 20) \times 227 \times (565 - 35) \end{bmatrix} \times 10^{-6} \text{ kNm}$$

$$= 116.47 \text{ kNm}$$

Working moment of resistance, M_R is given us

$$M_R = \frac{MOR_u}{1.5} = \frac{116.47}{1.5}$$

$$= 77.65 \text{ kNm}$$

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