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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-2 : Signals and Systems + Microprocessors and Microcontroller [All topics]
Network Theory-1 + Control Systems-1 [Part Syllabus]

Name : ....

Roll No :

Test Centres				Student's Signature
Delhi ☐	Bhopal ☐	Jaipur ☐	Pune ☑	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
Total Marks Obtained	114

Signature of Evaluator .... Cross Checked by ....

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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Section A : Signals and Systems + Microprocessors and Microcontroller

Q.1 (a) Determine the order of a low-pass Butterworth filter that is to provide 40 dB attenuation at $\omega = 2\omega_0$. (Here, ω_o is the cut-off frequency)

[12 marks]

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Q.1 (b) Write a 8085 program to generate continuous square wave with a period of 560 μs. Assume the system clock period is 350 ns and use bit D_0 to output the square wave. Use register B as delay counter. Display the square wave at PORT 0.

[12 marks]

Q.1 (c) (i) Enumerate all internal registers present in 8259 programmable interrupt controller. Write short notes on their individual functionality.

(ii) Draw the timing diagram for 8085 instruction DAD B.

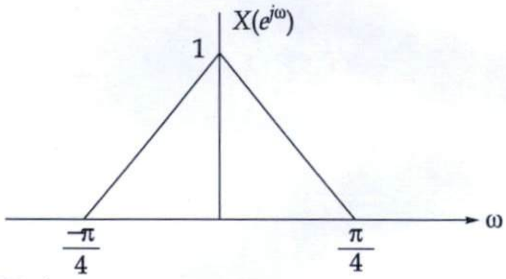
[6 + 6 marks]

Q.1 (d) $X(e^{j\omega})$ is the Discrete time Fourier transform of a discrete time sequence $x(n)$.

Assume $x_1(n) = \begin{cases} x(n/2)\ ; & n\text{-even} \\ 0\ ; & n\text{-odd} \end{cases}$

$x_2(n) = x(2n)$

The $X(e^{j\omega})$ is shown in below figure,



Sketch $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$

[12 marks]

Q.1 (e) Write a 8086 program to find the number of positive and negative data items in an array of 100 bytes of data stored from the memory location 3000 H: 4000 H. Store the result in the offset addresses 1000 H and 1001 H in the same segment. Assume that the negative numbers are represented in 2's complement form.

[12 marks]

Q.2 (a) **(i)** Find the convolution of two sequences:

$y[n] = x[n] * h[n]$

where $x[n] = (0.8)^n u[n]$ and $h[n] = (0.2)^n u[n]$. Find the value of $Y(e^{j\pi})$.

(ii) The differential equation of a stable system with zero initial conditions is given as

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}-2y(t)=x(t)-2\frac{dx}{dt}$$

Find the impulse response of the system and the initial value of impulse response.

[10 + 10 marks]

Q.2 (b) **(i)** Explain the concept of direct memory access with reference to 8085 microprocessor.

(ii) Describe briefly microprocessor instructions used for memory location called stack.

[10 + 10 marks]

Q.2 (c) Determine the 8-point DFT $X(k)$ of a discrete sequence $x(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 4, 5, 6, 7\}$ using the radix-2 DIT-FFT algorithm.

[20 marks]

Q.3 (a) Let $g_1(t) = \{[\cos(\omega_0 t)]x(t)\} * h(t)$ and $g_2(t) = \{[\sin(\omega_0 t)]x(t)\} * h(t)$ where

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk100t}$ is a real valued periodic signal and $h(t)$ is the impulse response of a stable LTI system.

Find the value of ω_0 and any necessary constraints on $H(j\omega)$ to ensure that

$g_1(t) = \text{Re}\{a_5\}$ and $g_2(t) = \text{Img}\{a_5\}$

[20 marks]

Q.3 (b) **(i)** For an 8085 microprocessor, draw the lower and higher order address bus during the machine cycle.

(ii) Explain the RIM instruction format and how it is executed.

(iii) Write an assembly language program for an 8085 microprocessor to find 2's complement of a 16-bit number. Write comments for selected instruction.

[5 + 5 + 10 marks]

Q.3 (c) Explain the all addressing modes of 8051 microcontroller with example for each addressing mode.

[20 marks]

Q.4 (a) (i) Consider the frequency response of an ideal high pass filter,

$H(e^{j\omega}) = 1$ for $\frac{\pi}{4} \leq |\omega| \leq \pi$

$= 0$ for $|\omega| \leq \frac{\pi}{4}$

1. Find the value of $h(n)$ $\forall$ length of the filter, $N = 11$.
2. Find $H(z)$.

(ii) Write comparisons between IIR and FIR filters.

[15 + 5 marks]

i) $H(e^{j\omega}) = 1$ for $\frac{\pi}{4} \leq |\omega| \leq \pi$

0 for $|\omega| \leq \pi/4$.

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{\pi} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{jn}\left\{ e^{j\omega n}\Big|_{-\pi}^{-\pi/4} + e^{j\omega n}\Big|_{\pi/4}^{\pi} \right\}\right]$$

$$h(n) = \frac{1}{2\pi j n}\left[\left(e^{-j\frac{n\pi}{4}} - e^{-jn\pi}\right)^{-1} + \left(e^{jn\pi} - e^{jn\pi/4}\right)\right]$$

$$= \frac{1}{2\pi j n}\left[\cos\left(\frac{n\pi}{4}\right) - j\sin\left(\frac{n\pi}{4}\right) + 1 - 1 - \cos\left(\frac{n\pi}{4}\right) - j\sin\left(\frac{n\pi}{4}\right)\right]$$

$$\boxed{h(n) = -\frac{\sin(n\pi/4)}{n\pi}} \quad n > 0.$$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = -\sum_{n=0}^{\infty} \frac{\sin(n\pi/4)}{n\pi} z^{-n}$$

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Q.4 (b) A continuous time system has impulse response $h(t) = e^{2t}u(1-t)$. If the input to the system is given by, $x(t) = u(t) - 2u(t-2) + u(t-5)$, then find the output $y(t)$ using convolution integral.

[20 marks]

Soln

$h(t) = e^{2t}u(1-t)$ $\quad x(t) = u(t) - 2u(t-2) + u(t-5)$

$y(t) = x(t) * h(t)$

$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$= \int_{-\infty}^{\infty} [u(\tau) - 2u(\tau-2) + u(\tau-5)]\,[e^{2(t-\tau)} u(1-(t-\tau))]\, d\tau$

$y(t) = e^{2t}\Big[\int_{-\infty}^{\infty} e^{-2\tau} u(1+\tau-t)\, u(\tau)\, d\tau - 2\int_{-\infty}^{\infty} e^{-2\tau} u(\tau-2)\, u(1+\tau-t)\, d\tau + \int_{-\infty}^{\infty} e^{-2\tau} u(\tau-5)\, u(1+\tau-t)\, d\tau\Big]$

~~$u(1-t+\tau)u(\tau) \neq 0$ for $\tau>0$, $1-t+\tau>0 \Rightarrow \tau > t-1$~~

$u(\tau) \neq 0$ for $\tau > 0$ $\quad u(\tau-2) \neq 0$ for $\tau > 2$ $\quad u(\tau-5) \neq 0$ for $\tau > 5$

So,

$y(t) = e^{2t}\Big[\int_{0}^{\infty} e^{-2\tau} u(1-t+\tau)\, d\tau - 2\int_{2}^{\infty} e^{-2\tau} u(1-t+\tau)\, d\tau + \int_{5}^{\infty} e^{-2\tau} u(1-t+\tau)\, d\tau\Big]$

$I_1 = \int_{0}^{\infty} e^{-2\tau} u(1-t+\tau)\, d\tau \neq 0$ for $1-t+\tau > 0$ & $\tau > 0$.

$\tau \geq t-1$

$t \leq 1$: $I_1 = 0$

$t > 1$: $I_1 = \int_{t-1}^{\infty} e^{-2\tau}\, d\tau$

So, $I_1 = \int_{t-1}^{\infty} e^{-2\tau} d\tau = \left.\frac{e^{-2\tau}}{-2}\right|_{t-1}^{\infty} = \frac{e^{-2(t-1)}}{2} \quad t \geq 1$

$= \frac{e^{-2(t-1)}}{2} u(t-1)$

Similarly,

$I_2 = 2\int_{2}^{\infty} e^{-2\tau} u(\tau-(t-1)) d\tau.$ → $\tau > t-1$ and $t > 2$

⇓

$I_2 = 0$ for $t<3$ &

$I_2 = 2\int_{t-1}^{\infty} e^{-2\tau} d\tau \quad t \geq 3.$

So,

$I_2 = e^{-2(t-1)} \quad t \geq 3$

$= e^{-2(t-1)} u(t-3)$

$I_3 = \int_{5}^{\infty} e^{-2\tau} u(\tau-(t-1)) d\tau$

$I_5 = 0$ for $t<6$

$I_3 = \int_{t-1}^{\infty} e^{-2\tau} d\tau \quad t \geq 6.$

So,

$I_3 = \frac{e^{-2(t-1)}}{2} u(t-6).$

So, $y(t) = \frac{e^{2t}}{2}\left[e^{-2(t-1)} u(t-1) - 2e^{-2(t-1)} u(t-3) + e^{-2(t-1)} u(t-6)\right]$

$$\boxed{y(t) = \frac{e^{2}}{2}\left[u(t-1) - 2u(t-3)\, u(t-6)\right]}$$

15

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Q.4 (c) (i) Explain the control signals in handshake mode with 8155 I/O.

(ii) Explain the following instructions of 8085 microprocessor giving operand, number of T-states, description and flags affected.

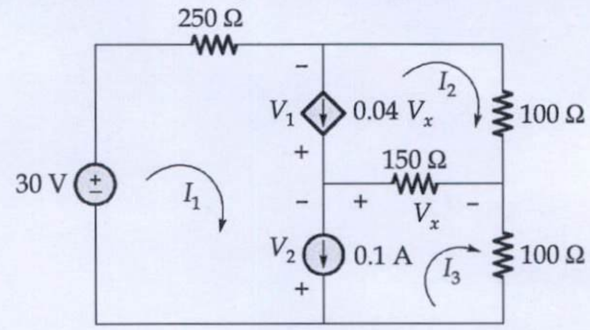
1. XTHL **2.** SHLD **3.** STAX

4. PCHL **5.** SPHL

[10 + 10 marks]

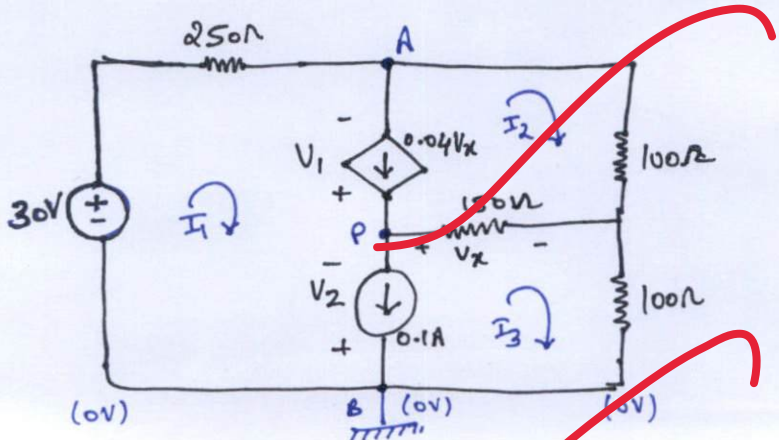
Section B : Network Theory-1 + Control Systems-1

Q.5 (a) Consider the circuit shown below, which contains a 0.1 A independent current source common to loop 1 and 3 as shown in circuit diagram. Find the value of loop currents I_1, I_2, I_3 and the power delivered by each independent and dependent sources.



[12 marks]

Sol^n:



KVL in loop -①.

$$30 - 250I_1 + V_1 + V_2 = 0 \quad -①$$

KCL at P :-

$$0.04V_x = \frac{V_x}{150} + 0.1$$

$$\Rightarrow \boxed{V_x = 3V}$$

KVL in Loop 2 :-

$-V_1 - 100 I_2 + V_x = 0$

$\Rightarrow V_1 + 100 I_2 = 3$ — (2)

KVL in loop 3 :-

$-V_2 - V_x - 100 I_3 = 0.$

$\Rightarrow V_2 + 100 I_3 = -3\,V$ — (3)

B/w A & P $\quad I_1 - I_2 = 0.04 V_x$

$= 0.12\,A.$ — (4)

B/w P & B. $\quad I_1 - I_3 = 0.1\,A.$ — (5)

KVL in outer loop

$30 - 250 I_1 - 100 I_2 - 100 I_3 = 0.$ {From (4) & (5)}

$\Rightarrow 30 = 250 I_1 + 100(I_1 - 0.12) + 100(I_1 - 0.1)$

$\Rightarrow \boxed{I_1 = \frac{52}{450} = 0.115\,A.}$

Using (4) & (5) $\quad \boxed{I_2 = -0.005\,A} \quad \boxed{I_3 = 0.015\,A}$

Power delivered by 30V source $= 30 \times I_1$

$= 30 \times 0.115$

$= \underline{3.45\,W}$

Putting I_2 in (2) $\quad V_1 = 3.5\,V$

Power absorbed by V_1 dependent source $= 3.5 \times 0.12$

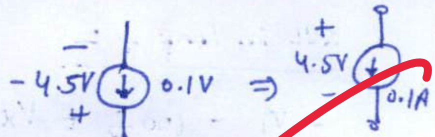
$= 0.42\,W.$

(or)

$P_{\text{delivered by } V_1 \text{ source}} = \underline{-0.42\,W}$

Powser Putting I_3 in ③.

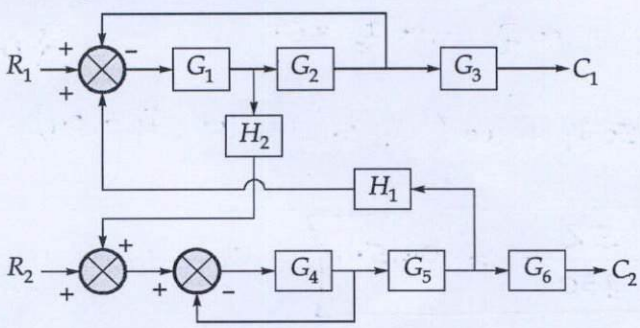
$V_2 = -4.5V$



Power delivered by V_2 source $= 4.5 \times 0.1$

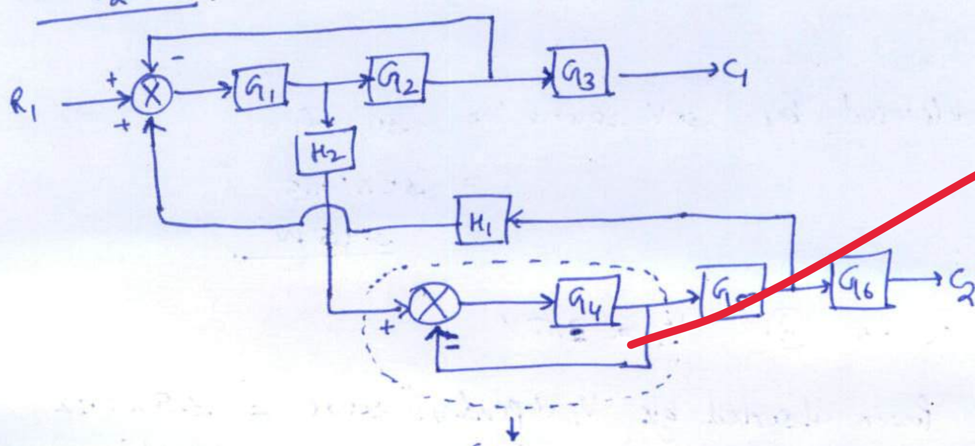
$= 0.45W$

Q.5 (b) Evaluate $\frac{C_1}{R_1}$ and $\frac{C_2}{R_1}$ for a system whose block diagram representation is shown in figure. Use block diagram reduction technique.

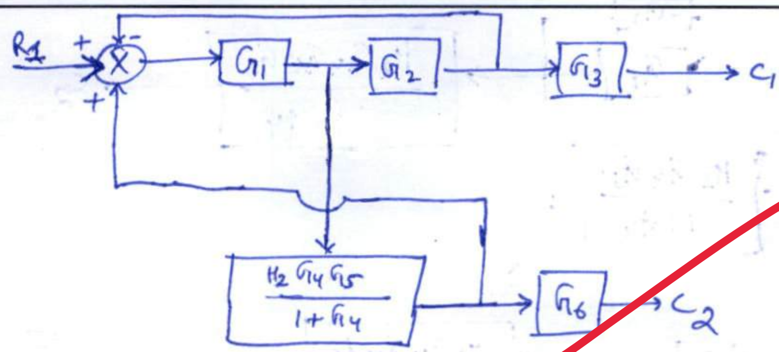


[12 marks]

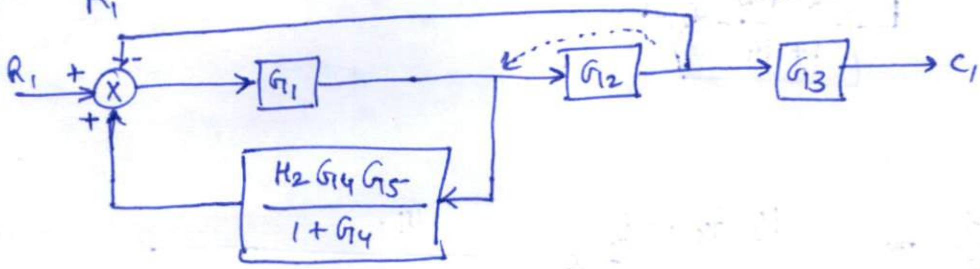
Sol^n: Let $R_2 = 0$.

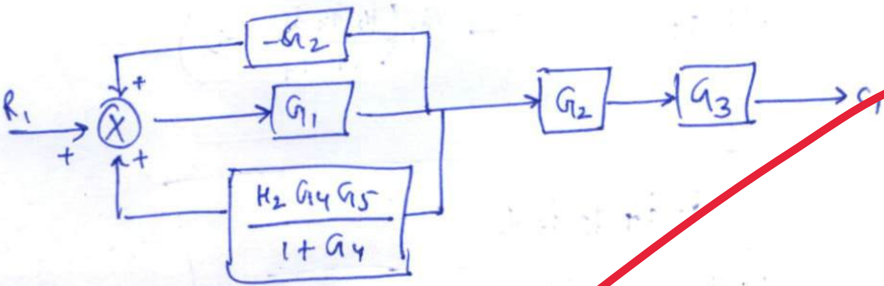


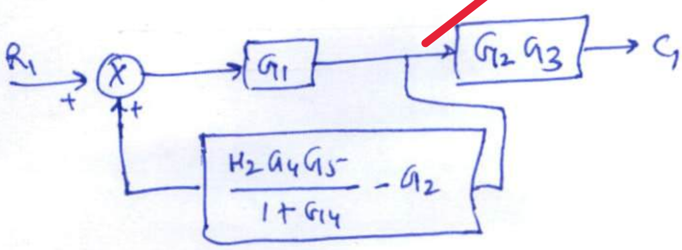
$$\frac{G_4}{1+G_4}$$

⇒ 

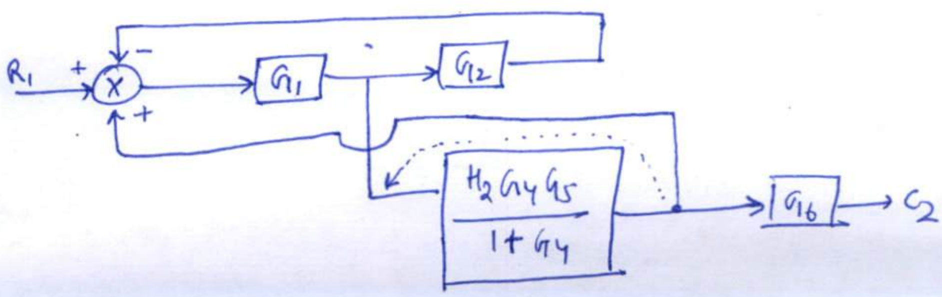
for $\frac{C_1}{R_1}$

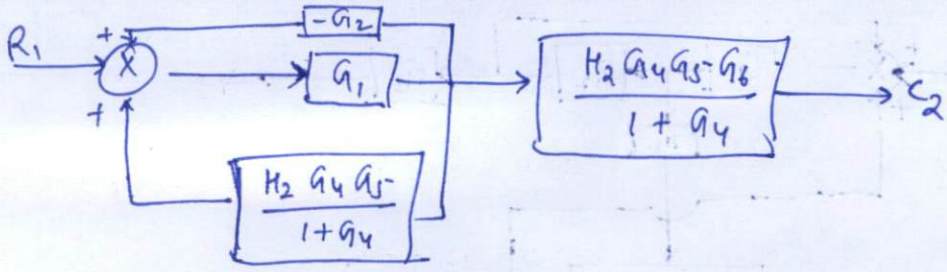


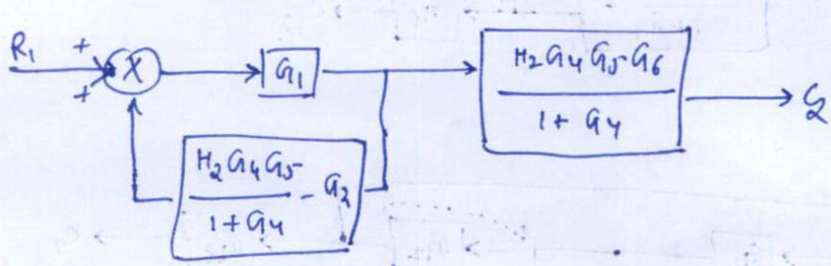


⇒ 

$$\frac{C_1}{R_1} = G_2 G_3 \times \frac{G_1}{1 - G_1\left(\frac{H_2 G_4 G_5}{1+G_4} - G_2\right)}$$

for $\frac{C_2}{R_2}$ 





$$\frac{C_2}{R_1} = \frac{H_2 G_4 G_5 G_6}{1+G_4} \times \frac{G_1}{1 - G_1\left(\frac{H_2 G_4 G_5}{1+G_4} - G_2\right)}$$

$$\boxed{\frac{C_2}{R_1} = \frac{G_1 G_4 G_5 G_6 H_2}{(1+G_4) - G_1\left(H_2 G_4 G_5 - G_2(1+G_4)\right)}}$$

Q.5 (c) (i) The open loop transfer function of a feedback system is $G(s)H(s) = \frac{K(1+s)}{(1-s)}$.

Comment on stability of the feedback system using Nyquist plot.

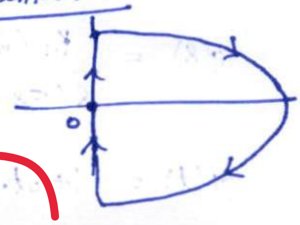
(ii) A unity feedback system has the forward transfer function $G(s) = \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}$.

The input $r(t) = 1 + 6t$ is applied to the system. Determine the minimum value of K_1 if the steady state error is to be less than 0.1.

[6 + 6 marks]

Soln. i) $G(s)H(s) = \frac{K(1+s)}{(1-s)}$

Nyquist Contour.



$GH(j\omega) = \frac{K(1+j\omega)}{(1-j\omega)}$

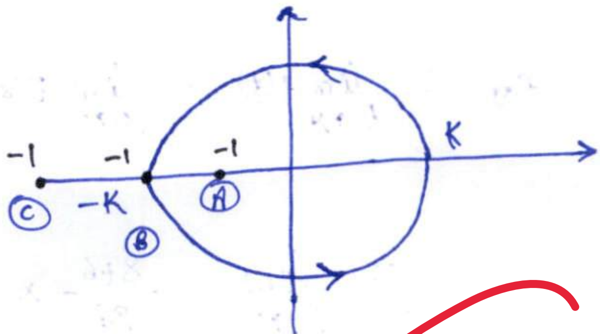
$|GH(j\omega)| = K \quad \forall \omega.$

$\angle GH(j\omega) = 2\tan^{-1}(\omega)$

So, Nyquist plot is.

for $\omega : 0 \longrightarrow \infty$

$\phi : 0 \longrightarrow \pi$



Conditionally stable s/s.

No. of OL s/s pole in RHS $= P = 1$.

for (A) if -1 is inside the circle in GH plot

$N = +1$ $\quad N = P - Z.$

$\Rightarrow Z = 0$ (or) $N = P.$

{Stable s/s}.

So, if -1 is inside the circle then s/s is stable.

$-1 > -K. \quad \Rightarrow$ $\boxed{K > 1}$ stable s/s.

~~$0 < K < 1$ stable s/s.~~

for case (B) $(-1, 0)$ is on Nyquist plot $\Rightarrow$ Marginally stable

$-1 = -K$ or $\boxed{K = 1}$ Marginally stable for ©. $N=0$ $N = P - Z$

$0 = 1 - Z$

$\boxed{Z = 1}$ → 1 pole is in RHS for closed loop s/s

{Unstable s/s}

$-K > -1$ ⇒ $\boxed{0 < K < 1}$ unstable s/s.

ii) $G(s) = \dfrac{K_1(2s+1)}{s(5s+1)(s+1)^2}$ unity f/b s/s.

$r(t) = 1 + 6t$ ⟷ $R(s) = \dfrac{1}{s} + \dfrac{6}{s^2}$

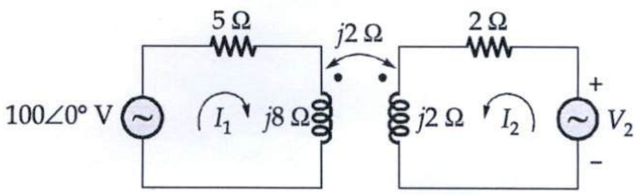
$\dfrac{E(s)}{R(s)} = \dfrac{1}{1 + G(s)H(s)^{\to 1}} = \dfrac{1}{1 + G(s)}$

$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} E(s) = \lim_{s \to 0} \dfrac{\frac{s+6}{s^2}}{1 + \frac{K_1(2s+1)}{s(5s+1)(s+1)^2}}$

$= \lim_{s \to 0} \dfrac{s+6}{s^{2}} \times \dfrac{s(5s+1)(s+1)^2}{s(5s+1)(s+1)^2 + K_1(2s+1)}$

$=$

Q.5 (d) **(i)** In the magnetically coupled circuit shown in figure below, find V_2 for which $I_1 = 0$. What voltage appears at the $j8\Omega$ inductance under this condition?



(ii) In a series LCR circuit, the maximum inductor voltage is twice the maximum capacitor voltage. However, the circuit current lags the applied voltage by 30° and the instantaneous drop across the inductance is given by $V_L = 100 \sin 377t$ V. Assuming the resistance to be 20 Ω, find the values of the inductance and capacitance.

[6 + 6 marks]

Q.5 (e) The closed loop transfer function of a feedback system is given by

$$T(s) = \frac{1000}{(s+22.5)(s^2+2.45s+44.4)}$$

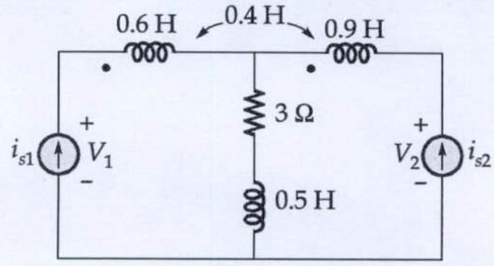
(i) Determine the resonant peak M_r and resonant frequency ω_r of the system by drawing the frequency response curve.

(ii) Determine the bandwidth of the equivalent second order system.

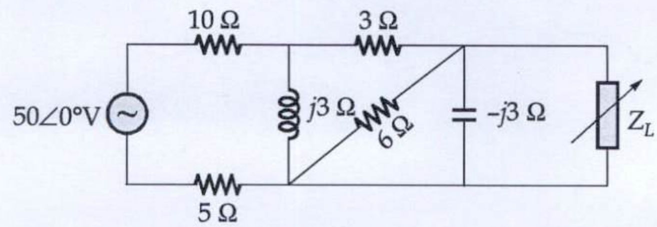
[6 + 6 marks]

Q.6 (a) **(i)** Let $i_{s_1} = 10 \cos 10t$ A and $i_{s2} = 6 \cos 10t$ A in the circuit shown below.

Find: **1.** $V_1(t)$; **2.** $V_2(t)$; **3.** the average power being supplied by each source.



(ii) Find the impedance Z_L so that maximum power can be transferred to it in the network shown below. Also, find the maximum power delivered to load Z_L.



[10 + 10 marks]

Q.6 (b) A unity negative feedback system has $G(s) = \dfrac{K(s+6)}{s(s+2)}$. When $K = 50$, find change in closed loop pole locations for a 10% change in the value of K.

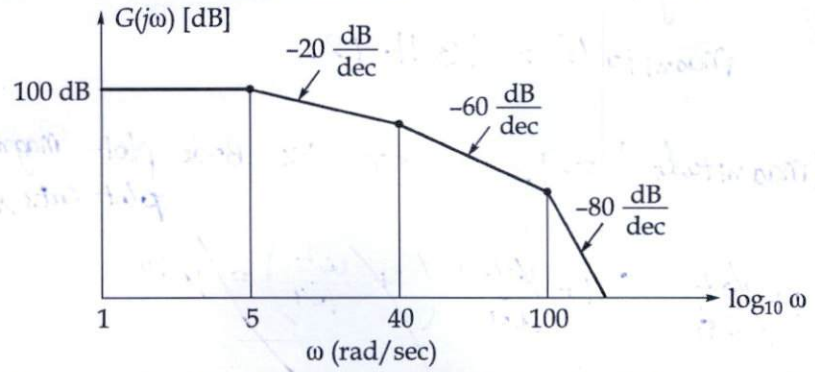
[20 marks]

Q.6 (c) **(i)** Prove that the bandwidth of a series RLC circuit is given as $\frac{R}{L}$ rad/sec.

(ii) A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set to 500 pF, the current has its maximum value while it is reduced to one-half when the capacitor is 600 pF. Find resistance, inductance and Q-factor of inductor.

[8 + 12 marks]

Q.7 (a) The Bode magnitude plot of the open loop transfer function $G(s)$ of a certain unity feedback control system is given in figure.



Estimate the magnitude of transfer function at each of the corner frequencies and also calculate the phase margin.

[20 marks]

Solⁿ:

$20\log K = 100$ {Initially}

$K = 10^5$

1st Corner freq. $\omega = 5$. So, Bode plot decreases by 20dB/dec ⇓ 1 pole at $\omega = 5$.

2nd Corner freq. $\omega = 40$ Bode plot decrease by 40dB/dec. 2 poles at $\omega = 40$.

3rd Corner freq. $\omega = 100$ Bode plot decreases by 20dB/dec ⇒ 1 pole at $\omega = 100$

So, TF of S/S $= \dfrac{K}{\left(1+\frac{s}{\omega_{c_1}}\right)\left(1+\frac{s}{\omega_{c_2}}\right)^2\left(1+\frac{s}{\omega_{c_3}}\right)}$

$= \dfrac{10^5}{\left(1+\frac{s}{5}\right)\left(1+\frac{s}{40}\right)^2\left(1+\frac{s}{100}\right)}$

Magnitude $= \dfrac{10^5}{\sqrt{\left(1+\frac{\omega^2}{25}\right)\left(1+\left(\frac{\omega}{40}\right)^2\right)^2\left(1+\left(\frac{\omega}{100}\right)^2\right)}}$

at $\omega = 1$ magnitude $= 97971.55$.

at $\omega = 5$ Magnitude $= 69535.95$

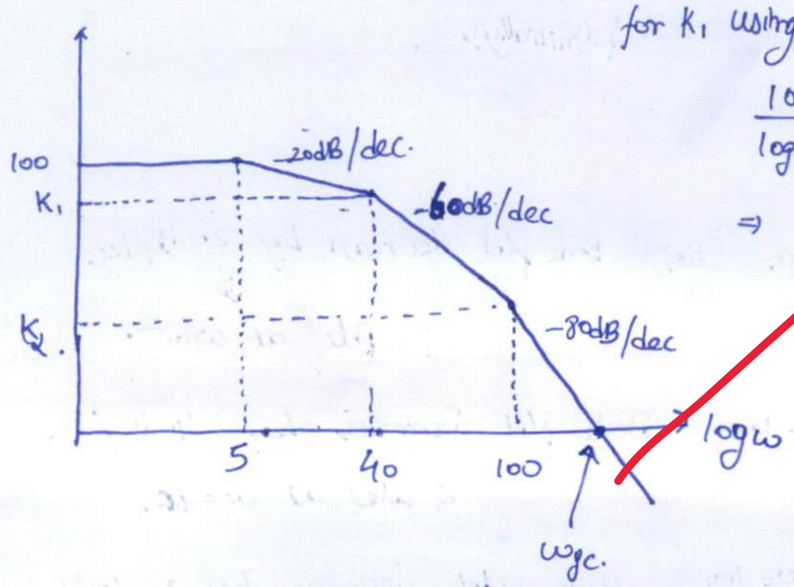
at $\omega = 40$ Magnitude $= 5758.16$

at $\omega = 100$ Magnitude $= 1311.42$.

at ω_{gc} $|$magnitude$| = 1$. or on Bode plot magnitude plot cuts x-axis.

So, $\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{1600}\right)\left(1+\frac{\omega^2}{10^4}\right) = 10^{10}$

$\Rightarrow (\omega^2+10^4)(\omega^2+1600)(\omega^2+25)$



for K_1 using slope of st. line

$\dfrac{100 - K_1}{\log 5 - \log 40} = -20$

$\Rightarrow K_1 = 81.938$ dB

for K_2 using slope of st. line

$$\frac{K_1 - K_2}{\log 40 - \log 100} = -60$$

$K_2 = 58.061$

at ω_{gc}.

$$\frac{K_2 - 0}{\log 100 - \log \omega_{gc}} = -80 \Rightarrow \boxed{\omega_{gc} = 531.82 \text{ rad/sec}}$$

Phase of TF: at ω_{gc}

$$\phi = 0 - \tan^{-1}\left(\frac{\omega_{gc}}{\omega_{c_1}}\right) - 2 \times \tan^{-1}\left(\frac{\omega_{gc}}{\omega_{c_2}}\right) - \tan^{-1}\left(\frac{\omega_{gc}}{\omega_{c_3}}\right)$$

$= -340.2°$.

$-340.2 + [PM] = -180°$

$$\boxed{PM = 160.2°}$$

19

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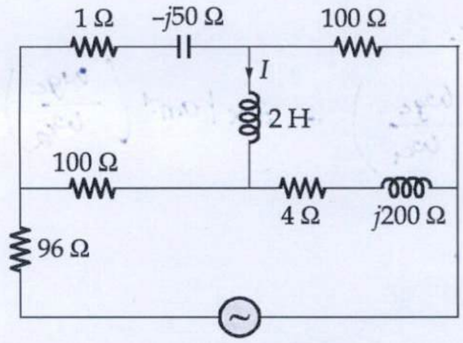
Q.7 (b) Find current across 2 Henry inductor as shown in the network below using

(i) Thevenin's theorem;

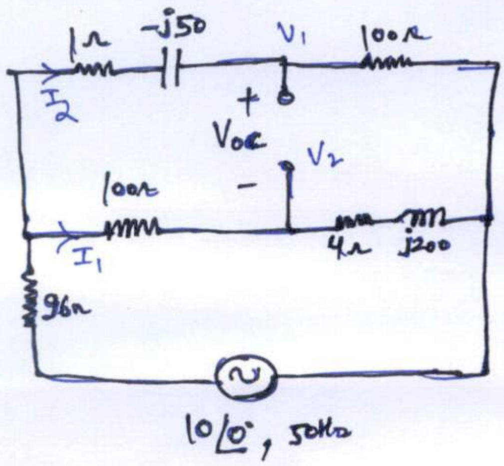
(ii) Draw the Norton's equivalent circuit.



[15 + 5 marks]

Sol:

$f = 50\,Hz \quad \omega = 2\pi f = 100\pi$



KVL in loop -①

$10\angle 0 = 96(I_1+I_2) + 100 I_1 + 4I_1 + j200 I_1$

$\Rightarrow 10\angle 0^\circ = 200 I_1 + j200 I_1 + 96 I_2$ — ①

KVL in loop ②

$10\angle 0 = 96(I_1+I_2) + I_2 + (-j50)I_2 + 100 I_2$

$\Rightarrow 10\angle 0^\circ = 96 I_1 + 197 I_2 - j50 I_2$

$= 96 I_1 + (197 - j50) I_2$ — ②

Equating ① & ②

$$(4I_1 + j200I_1) = (101 - j50)\,I_2.$$

$$I_1 = \frac{(101 - j50)}{(4 + j200)}\,I_2 = 0.56\,\angle -115.2^\circ\,I_2$$

Putting in ①.

$$10\angle 0 = 200\sqrt{2}\,.\,\angle 45^\circ \times 0.56\,\angle -115.2^\circ\,I_2 + 96\,I_2.$$

$$= (158.4\,\angle -70.2 + 96)\,I_2.$$

$$= 211.2\,\angle -44.88^\circ\,I_2.$$

So, $\frac{I}{2} = 0.0473$ $\quad I_2 = 0.0473\,\angle 44.88^\circ$ A

$$I_1 = 0.026\,\angle -70.32\ \text{A.}$$

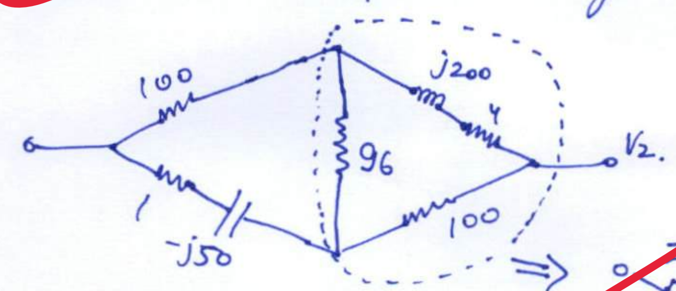
$$V_{oc} = V_1 - V_2 = 100\,I_2 - (4 + j200)\,I_1$$

$$= 4.73\,\angle 44.88 - 5.2\,\angle 18.53^\circ$$

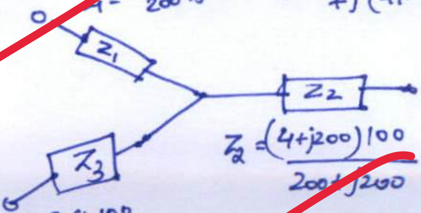
$$= (-1.578 + j1.685)$$

$$= 2.308\,\angle 133.12^\circ.$$

Rth.: Short circuit independent voltage source.



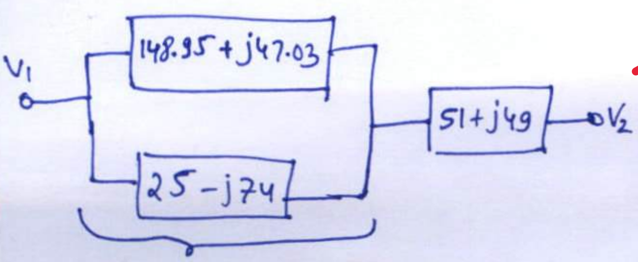
⇒



$Z_1 = \frac{96(4+j200)}{200+j200} = 67.89\,\angle 43.85 = 48.95 + j(47.03)$

$Z_2 = \frac{(4+j200)100}{200+j200} = 70.72\,\angle 43.85 = 51 + j49$

$Z_3 = \frac{96 \times 100}{200+j200} = 33.94\,\angle -45 = 24 - j24$



$Z_{eq} =$

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Q.7 (c) (i) Derive the expression for gain margin and phase margin of a unity feedback second order system with transfer function,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

(ii) Sketch the polar plot of the transfer function given below:

$$G(s) = \frac{1+4s}{s(1+s)(1+2s)}$$

Determine whether the polar plot cuts the imaginary axis. If so, determine the frequency at which the plot cross the imaginary axis.

[10 + 10 marks]

Soln: i)

TF: $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$T(j\omega) = \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + 2\xi\omega_n(j\omega)} = \frac{1}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + j2\xi\left(\frac{\omega}{\omega_n}\right)}$$

$$|T(j\omega)| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

$$|T(j\omega)| = 1 \Rightarrow \left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2 = 1$$

Let $\left(\frac{\omega}{\omega_n}\right)^2 = P$.

$$\Rightarrow (1-P)^2 + 4\xi^2 P = 1$$

$$\Rightarrow P^2 - 2P + 4\xi^2 P + 1 = 1$$

$$P(P - 2 + 4\xi^2) = 0 \quad \Rightarrow P = 0 \text{ (or) } P = 2 - 4\xi^2$$

$\omega = 0$

$$\omega = \omega_n\sqrt{2 - 4\xi^2}$$

↑ gain cross over freq

$$\boxed{\omega_{gc} = \omega_n\sqrt{2 - 4\xi^2}}$$

$$\angle T(j\omega) = -\tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right]$$

for phase crossover freq. ω_{pc} : $\angle T(j\omega) = -180°$.

$$\Rightarrow \frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2} = 0 \qquad \omega = 0$$

⇓

ω_{pc} does not exist

So, for phase Margin

$$\angle T(j\omega)_{\omega_{gc}} = -\tan^{-1}\left[\frac{2\xi\sqrt{2-4\xi^2}}{4\xi^2-1}\right]$$

$$\boxed{PM = 180 - \tan^{-1}\left[\frac{2\xi\sqrt{2-4\xi^2}}{4\xi^2-1}\right]}$$

ii)

$$G(s) = \frac{1+4s}{s(1+s)(1+2s)} \qquad G(j\omega) = \frac{1+j4\omega}{j\omega(1+j\omega)(1+2j\omega)}$$

$$|G(j\omega)| = \frac{\sqrt{1+16\omega^2}}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

at $\omega = 0$: $|G(j\omega)| = \infty$

$\omega = 1$: $|G(j\omega)| = 1.303$

⋮

$\omega = \infty$: $|G(j\omega)| = 0$

$$\angle G(j\omega) = \tan^{-1}(4\omega) - 90° - \tan^{-1}(\omega) - \tan^{-1}(2\omega).$$

$$= -90° - \tan^{-1}(\omega) - \tan^{-1}(2\omega) + \tan^{-1}(4\omega)$$

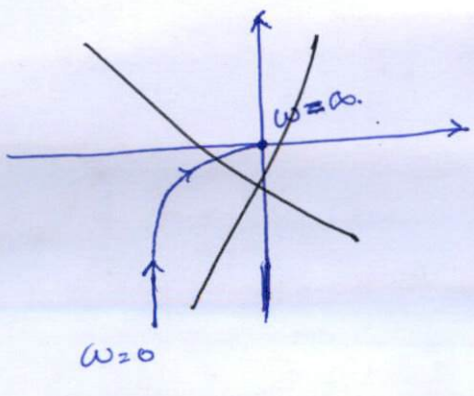
at $\omega = 0$: $\angle G(j\omega) = -90°$

at $\omega = 1$: $\angle G(j\omega) = -122.47°$

⋮

at $\omega = \infty$: $\angle G(j\omega) = -180°$

Polar plot does not cut $j\omega$ axis from $\omega: 0 \to \infty$



~~Polar plot neither cut jω axis nor -180°~~

Mathematically, for -180°

$$-180^\circ = -90 - \tan^{-1}(\omega) - \tan^{-1}(2\omega) + \tan^{-1}(4\omega)$$

$$\Rightarrow \tan^{-1}\left(\frac{\omega + 2\omega}{1-2\omega^2}\right) = 90 + \tan^{-1}(4\omega)$$

$$\Rightarrow \frac{3\omega}{1-2\omega^2} = \frac{-1}{\tan(\tan^{-1}(4\omega))} = \frac{-1}{4\omega}$$

$$\Rightarrow 12\omega^2 = 2\omega^2 - 1$$

$$\Rightarrow \omega^2 = \frac{-1}{10} \times$$ No real root.

for -90°

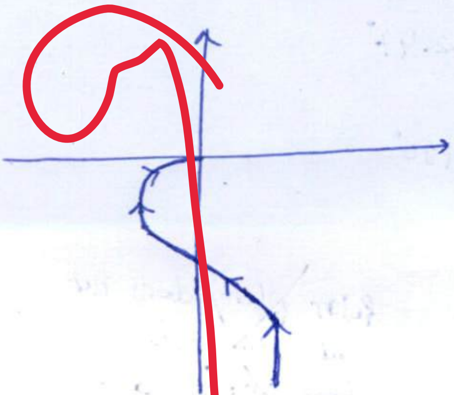
$$-90^\circ = -90 - \tan^{-1}(\omega) - \tan^{-1}(2\omega) + \tan^{-1}(4\omega)$$

$$\tan^{-1}\left(\frac{\omega+2\omega}{1-2\omega^2}\right) = \tan^{-1}(4\omega)$$

$$\Rightarrow 3\omega = 4\omega(1-2\omega^2)$$

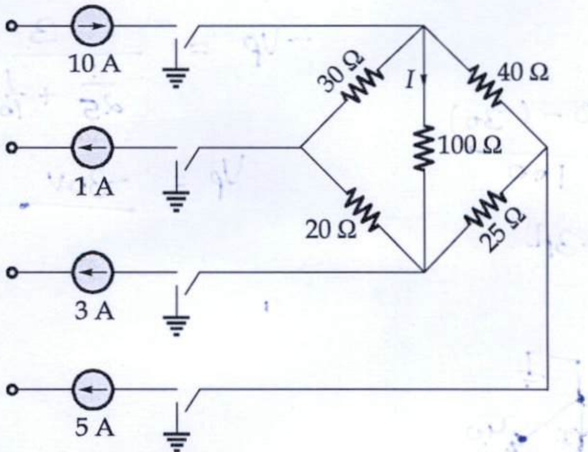
$$\Rightarrow \omega = 0 \text{ (or) } \omega^2 = \frac{1}{8} \Rightarrow \omega = \frac{1}{\sqrt{8}} \text{ rad/sec}$$

Polar plot cuts jω axis for $\omega = \frac{1}{\sqrt{8}}$ rad/sec



Polar plot

Q.8 (a) **(i)** Find the value of the current 'I' flowing through the 100 Ω resistor in the bridge shown below using Superposition Theorem. (Assume other sources are grounded, when one is used at a time).

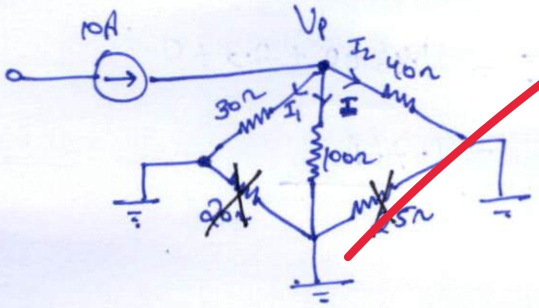


(ii) A certain series RLC resonant circuit has resonant frequency, f_0 = 200 Hz, quality factor, Q_0 = 7.5 and inductive reactance, X_L = 250 Ω at resonance.

1. Find the values of R, L and C
2. If the source voltage, $V_S = 5\angle 45°$ V is connected in series with the circuit, find exact value for magnitude of capacitor voltage, $|V_C|$ at f = 300 Hz.

[10 + 10 marks]

Sol^n: i)



By KCL

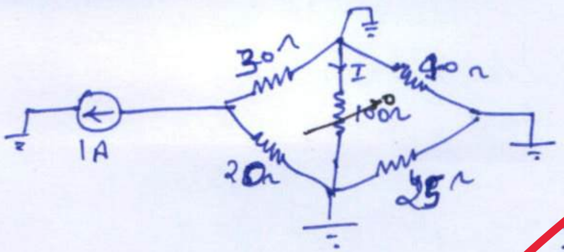
$$\frac{V_P}{30} + \frac{V_P}{100} + \frac{V_P}{40} = 10$$

$$V_P = 10 \times \frac{1}{\frac{1}{30} + \frac{1}{100} + \frac{1}{40}}$$

$$= 146.34\,V.$$

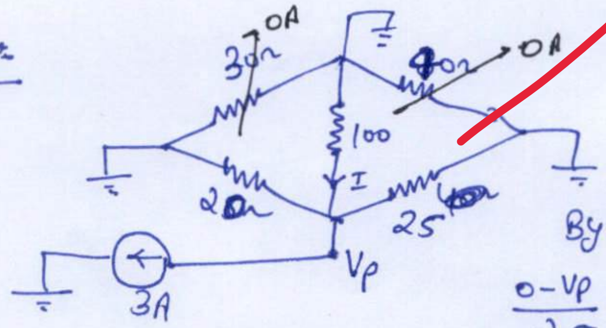
$$I = \frac{V_P}{100} = 1.46A$$

Source 2



I = 0A.

Source 3:



By KCL

$$\frac{0-V_P}{25} + \frac{0-V_P}{100} + \frac{0-V_P}{20} = 3$$

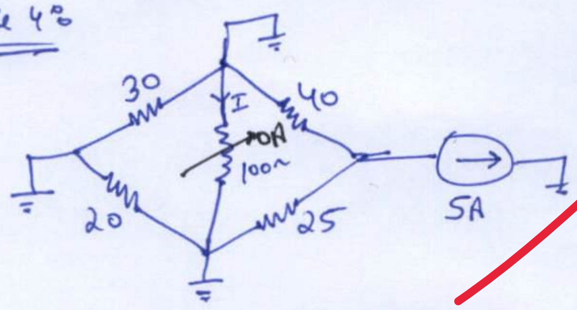
$$-V_P = \frac{3}{\frac{1}{25}+\frac{1}{100}+\frac{1}{20}}$$

$$V_P = -30V$$

So, $I = \frac{0-(-30)}{100}$

$= 0.3A.$

Source 4:



I = 0A.

So using Superposition Thm.

$$I = 1.46 + 0 + 0.3 + 0$$

$$= 1.76A$$

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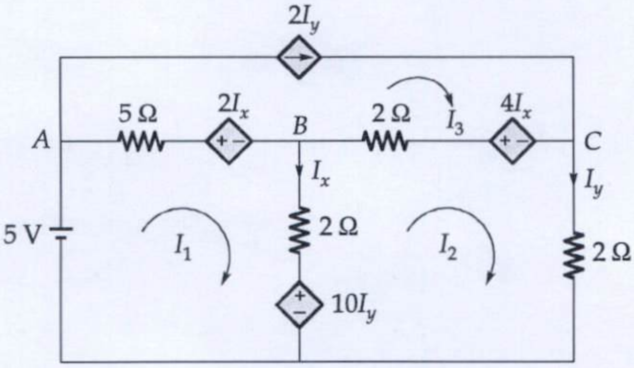
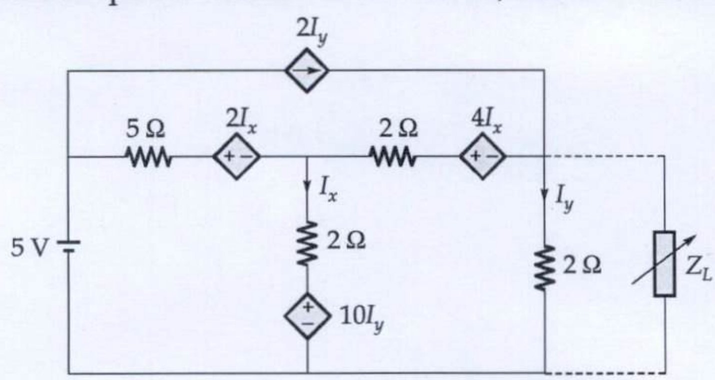
Q.8 (b) Consider the circuit shown below, which contain some dependent and independent sources.



Find

(i) Currents I_1, I_2 and I_3 using mesh analysis.

(ii) The maximum power transferred to the load, connected across 2 Ω as shown below:

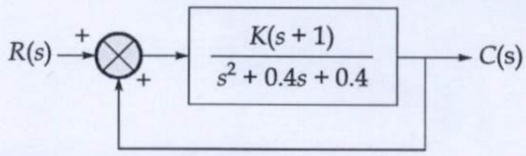


[10 + 10 marks]

Sol^n:

Q.8 (c) **(i)** A feedback control system has $G(s) = \frac{10}{s(s+10)}$ and $H(s) = e^{-T_1 s}$. Find T_1 for which system is marginally stable.

(ii) Sketch the root locus for the positive feedback system as drawn below for $0 < K < \infty$.



Also, comment on the stability of the system.

[10 + 10 marks]

Sol^n:

i) $G(s) = \frac{10}{s(s+10)}$ $\quad H(s) = e^{-T_1 s}$

$GH(j\omega) = \frac{10}{j\omega(j\omega+10)} \cdot e^{-jT_1\omega}$

$|GH(j\omega)| = \frac{10}{\omega\sqrt{100+\omega^2}}$ $\quad \angle GH(j\omega) = -T_1\omega - 90^\circ - \tan^{-1}\left(\frac{\omega}{10}\right)$

By polar plot if plot cuts ~~180° axis at~~ $(-1,0)$ then s/s will be marginally stable

ω for which $\phi = -180^\circ$

$$-180 = -T_1\omega - 90 - \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$90 = T_1\omega + \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$\tan(90 - T_1\omega) = \frac{\omega}{10} \quad -(1)$$

also find ω for which $|GH| = 1$.

$$\Rightarrow \frac{10}{\omega\sqrt{100+\omega^2}} = 1$$

$$\Rightarrow 100 = \omega^2(100+\omega^2)$$

$$\Rightarrow \omega^4 + 100\omega^2 - 100 = 0.$$

$$\omega^2 = \frac{-100 + \sqrt{10^4 + 400}}{2}$$

$$\approx 1.$$

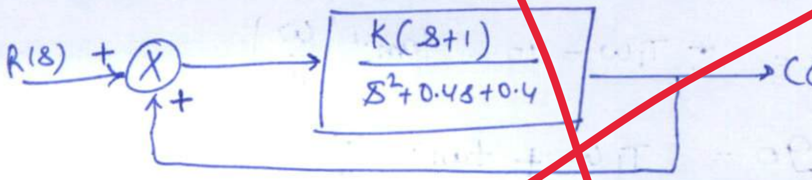
$\omega = 1$ rad/sec.

Putting this ω :-

$$\tan(90 - T_1) = \frac{1}{10}.$$

$$\cot T_1 = \frac{1}{10} \quad \Rightarrow \boxed{T_1 = 84.289}$$

ii)



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Space for Rough Work

Space for Rough Work

Space for Rough Work