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UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-2 : Signals and Systems + Microprocessors and Microcontroller [All topics]
Network Theory-1 + Control Systems-1 [Part Syllabus]

Name :

Roll No :

Test Centres	Student's Signature
Delhi ☑ Bhopal ☐ Jaipur ☐ Pune ☐ Kolkata ☐ Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	10
Q.2	21
Q.3	–
Q.4	
Section-B	
Q.5	23
Q.6	–
Q.7	34
Q.8	37
Total Marks Obtained	125

Signature of Evaluator: Vivek

Cross Checked by ....

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◦ Read Question Carefully.

◦ Attempt more Questions

◦ Improve your accuracy

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Signals and Systems + Microprocessors and Microcontroller

Q.1 (a) Determine the order of a low-pass Butterworth filter that is to provide 40 dB attenuation at $\omega = 2\omega_0$. (Here, ω_o is the cut-off frequency)

[12 marks]

Do not write in this margin

Q.1 (b) Write a 8085 program to generate continuous square wave with a period of 560 μs. Assume the system clock period is 350 ns and use bit D_0 to output the square wave. Use register B as delay counter. Display the square wave at PORT 0.

[12 marks]

Q.1 (c) **(i)** Enumerate all internal registers present in 8259 programmable interrupt controller. Write short notes on their individual functionality.

(ii) Draw the timing diagram for 8085 instruction DAD B.

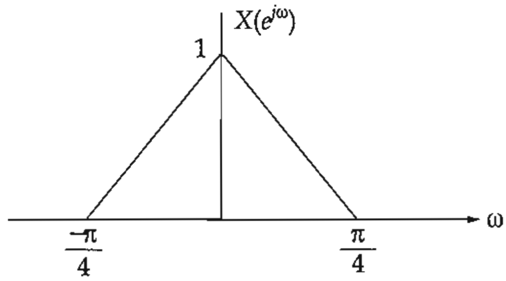
[6 + 6 marks]

Q.1 (d) $X(e^{j\omega})$ is the Discrete time Fourier transform of a discrete time sequence $x(n)$.

Assume $x_1(n) = \begin{cases} x(n/2); & n\text{-even} \\ 0; & n\text{-odd} \end{cases}$

$x_2(n) = x(2n)$

The $X(e^{j\omega})$ is shown in below figure,



Sketch $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$

[12 marks]

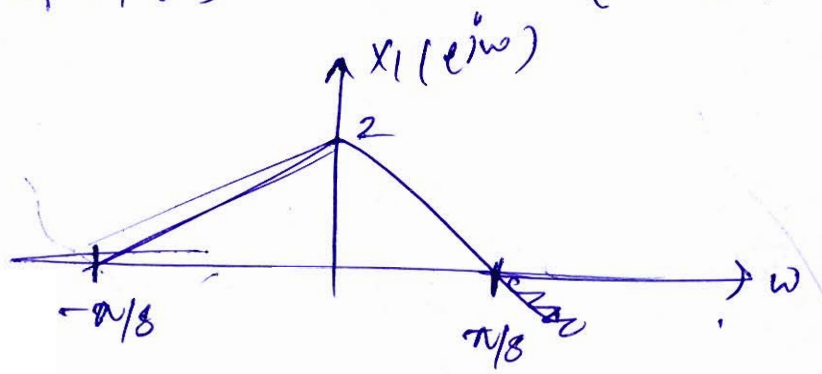
Ans → $x_1[n] = x\left[\frac{n}{2}\right]$ [n = even].

∵ $x[n] \xleftrightarrow{DTFT} X(e^{j\omega})$.

→ $x[an] \xleftrightarrow{DTFT} \frac{1}{|a|} X(e^{j\omega/a})$ [Time scaling property of DTFT]

→ $x\left[\frac{n}{2}\right] \xleftrightarrow{DTFT} 2X(e^{j2\omega})$

∴ $x_1[n] \xleftrightarrow{DTFT} 2X(e^{j2\omega}) = X_1(e^{j\omega})$.

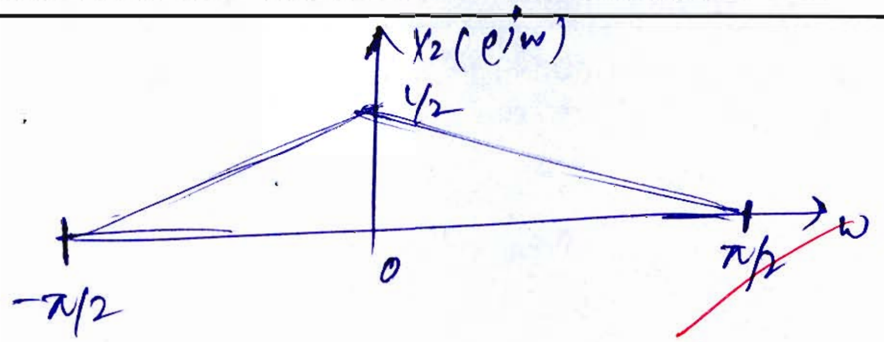


10

for $x_2[n] = x[2n]$

$x[2n] \xleftrightarrow{DTFT} \frac{1}{2} X(e^{j\omega/2})$.

→ $x_2[n] = x[2n] \xleftrightarrow{DTFT} \frac{1}{2} X(e^{j\omega/2}) = X_2(e^{j\omega})$



Draw graph for atleast one time period.

Q.1 (e) Write a 8086 program to find the number of positive and negative data items in an array of 100 bytes of data stored from the memory location 3000 H: 4000 H. Store the result in the offset addresses 1000 H and 1001 H in the same segment. Assume that the negative numbers are represented in 2's complement form.

[12 marks]

Q.2 (a) **(i)** Find the convolution of two sequences:

$y[n] = x[n] * h[n]$

where $x[n] = (0.8)^n u[n]$ and $h[n] = (0.2)^n u[n]$. Find the value of $Y(e^{j\pi})$.

(ii) The differential equation of a stable system with zero initial conditions is given as

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y(t) = x(t) - 2\frac{dx}{dt}$$

Find the impulse response of the system and the initial value of impulse response.

[10 + 10 marks]

Ans (a) Given signals are:-

$x[n] = (0.8)^n u[n]$; $h[n] = (0.2)^n u[n]$.

o/p $y[n] = x[n] * h[n]$.

By time domain convolution property of DTFT.

$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$ — ①

and, $(0.8)^n u[n] \xleftrightarrow{DTFT} \dfrac{1}{1-0.8e^{-j\omega}}$.

$(0.2)^n u[n] \xleftrightarrow{DTFT} \dfrac{1}{1-0.2e^{-j\omega}}$.

$\therefore Y(j\omega) = \dfrac{1}{(1-0.8e^{-j\omega})(1-0.2e^{-j\omega})}$ ②

By partial fractions:-

$Y(e^{j\omega}) = \dfrac{A}{1-0.8e^{-j\omega}} + \dfrac{B}{1-0.2e^{-j\omega}}$

$$\Rightarrow \frac{A(1-0.2e^{-jw}) + B(1-0.8e^{-jw})}{(1-0.8e^{-jw})(1-0.2e^{-jw})} = Y(e^{jw}).$$

$$\Rightarrow Y(e^{jw}) = \frac{4/3}{(1-0.8e^{-jw})} + \frac{-1/3}{(1-0.2e^{-jw})}.$$

(where, $A = \frac{4}{3}$ and $B = -1/3$).

Good

10

$$\Rightarrow Y(e^{j\pi}) = \frac{4/3}{(1-0.8e^{-j\pi})} + \frac{-1/3}{(1-0.2e^{-j\pi})} \quad [\because e^{-j\pi} = -1]$$

$$\Rightarrow Y(e^{j\pi}) = \frac{4/3}{1+0.8} - \frac{1/3}{1+0.2} \Rightarrow \boxed{Y(e^{j\pi}) = 0.463}$$

(ii) Given differential eqn is :-

$$\frac{d^2y(t)}{dt^2} + \frac{dy}{dt} - 2y(t) = x(t) - 2\frac{dx(t)}{dt} \quad \text{--- (1)}.$$

Apply Laplace transform on (1) :-

$$s^2Y(s) + sY(s) - 2Y(s) = X(s) - 2sX(s).$$

$[\because \frac{dx(t)}{dt} \overset{L}{\longleftrightarrow} s.X(s)]$ [also, initial values are 0].

$$\Rightarrow Y(s)[s^2+s-2] = X(s)[1-2s].$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{1-2s}{s^2+s-2} \quad \left[\because \frac{Y(s)}{X(s)} = H(s)\right]$$

$$\Rightarrow H(s) = \frac{1-2s}{s^2+s-2} \Rightarrow ~~H(s) = \frac{1-2s}{(s+1)(s-2)}~~$$

$$H(s) = \frac{1-2s}{(s+2)(s-1)} \quad \text{--- (2)}$$

Good 10

$\Rightarrow H(s) = \frac{A}{s+2} + \frac{B}{s-1}$ (By partial fractions).

$\Rightarrow H(s) = \frac{A(s-1) + B(s+2)}{(s+2)(s-1)}$ (3) $\Rightarrow A = \frac{-5}{3}$; $B = -1/3$.

(comparing (3) with (2))

$\Rightarrow H(s) = \frac{-5/3}{(s+2)} - \frac{1/3}{(s-1)}$.

Apply inverse Laplace Transform.

~~$h(t) = \left(\frac{-5}{3}e^{-2t} - \frac{1}{3}e^{t}\right)$~~ $\because$ System is stable.

$\therefore$ ROC : $-2 < \sigma < 1$. $\rightarrow h(t) = \frac{-5}{3}e^{-2t}u(t) + \frac{1}{3}e^{t}u(-t)$

$\therefore h(0) =$ initial value of $h(t) = \frac{-5}{3} + \frac{1}{3} = -4/3$

Q.2 (b) (i) Explain the concept of direct memory access with reference to 8085 microprocessor.

(ii) Describe briefly microprocessor instructions used for memory location called stack.

[10 + 10 marks]

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Q.2 (c) Determine the 8-point DFT $X(k)$ of a discrete sequence $x(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 4, 5, 6, 7\}$ using the radix-2 DIT-FFT algorithm.

[20 marks]

Ans → $DFT = X[k] = \sum_{k=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$.

Here, $N = 8$.

(T)

$\Rightarrow DFT = X[k] = \sum_{k=0}^{N-1} x[n] e^{-j\frac{2\pi}{8}kn}$

$\Rightarrow DFT = X[k] = \sum_{k=0}^{N-1} x[n] e^{-j\frac{\pi}{4}kn}$.

Q.3 (a) Let $g_1(t) = \{[\cos(\omega_0 t)]x(t)\} * h(t)$ and $g_2(t) = \{[\sin(\omega_0 t)]x(t)\} * h(t)$ where

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk100t}$ is a real valued periodic signal and $h(t)$ is the impulse response of a stable LTI system.

Find the value of ω_0 and any necessary constraints on $H(j\omega)$ to ensure that

$g_1(t) = \text{Re}\{a_5\}$ and $g_2(t) = \text{Img}\{a_5\}$

[20 marks]

Q.3 (b) **(i)** For an 8085 microprocessor, draw the lower and higher order address bus during the machine cycle.

(ii) Explain the RIM instruction format and how it is executed.

(iii) Write an assembly language program for an 8085 microprocessor to find 2's complement of a 16-bit number. Write comments for selected instruction.

[5 + 5 + 10 marks]

Do not write in this margin

Q.3 (c) Explain the all addressing modes of 8051 microcontroller with example for each addressing mode.

[20 marks]

Q.4 (a) (i) Consider the frequency response of an ideal high pass filter,

$$H(e^{j\omega}) = 1 \text{ for } \frac{\pi}{4} \le |\omega| \le \pi$$

$$= 0 \text{ for } |\omega| \le \frac{\pi}{4}$$

1. Find the value of $h(n)$ $\forall$ length of the filter, $N = 11$.
2. Find $H(z)$.

(ii) Write comparisons between IIR and FIR filters.

[15 + 5 marks]

Q.4 (b) A continuous time system has impulse response $h(t) = e^{2t}u(1 - t)$. If the input to the system is given by, $x(t) = u(t) - 2u(t - 2) + u(t - 5)$, then find the output $y(t)$ using convolution integral.

[20 marks]

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Q.4 (c) (i) Explain the control signals in handshake mode with 8155 I/O.

(ii) Explain the following instructions of 8085 microprocessor giving operand, number of T-states, description and flags affected.

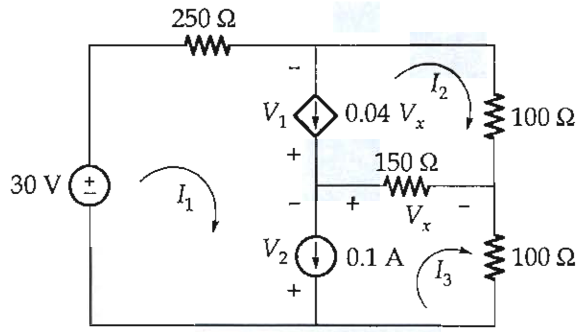
1. XTHL 2. SHLD 3. STAX

4. PCHL 5. SPHL

[10 + 10 marks]

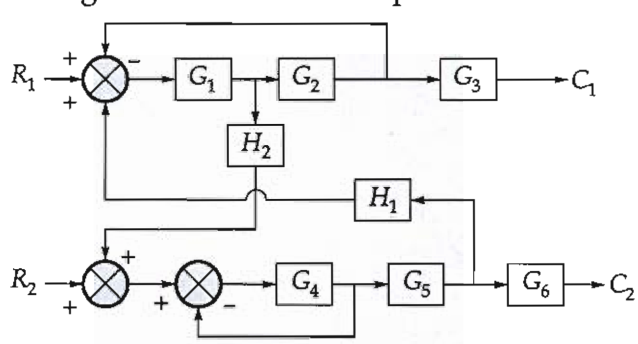
Section B : Network Theory-1 + Control Systems-1

Q.5 (a) Consider the circuit shown below, which contains a 0.1 A independent current source common to loop 1 and 3 as shown in circuit diagram. Find the value of loop currents I_1, I_2, I_3 and the power delivered by each independent and dependent sources.



[12 marks]

Q.5 (b) Evaluate $\frac{C_1}{R_1}$ and $\frac{C_2}{R_1}$ for a system whose block diagram representation is shown in figure. Use block diagram reduction technique.



[12 marks]

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Q.5 (c) **(i)** The open loop transfer function of a feedback system is $G(s)H(s) = \frac{K(1+s)}{(1-s)}$.

Comment on stability of the feedback system using Nyquist plot.

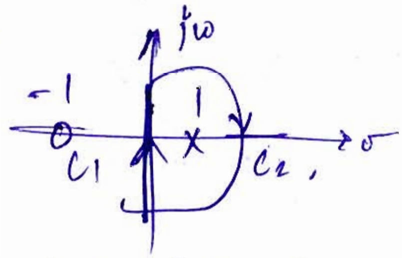
(ii) A unity feedback system has the forward transfer function $G(s) = \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}$.

The input $r(t) = 1 + 6t$ is applied to the system. Determine the minimum value of K_1 if the steady state error is to be less than 0.1.

[6 + 6 marks]

Ans → (i) $G(s)H(s) = \frac{K(1+s)}{(1-s)}$.

No pole at origin:.. we define Nyquist Contour as follows:- and $\boxed{P=1}$.



Nyquist plot is:

Corresponding C_1 : $s = j\omega$ [$\omega = -\infty$ to $+\infty$].

$GH(j\omega) = \frac{K(1+j\omega)}{(1-j\omega)}$.

$M = |GH(j\omega)| = \frac{K\sqrt{\omega^2+1}}{\sqrt{\omega^2+1}} \Rightarrow \boxed{M = K}$.

$\angle GH(j\omega) = \phi = \tan^{-1}(\omega) - [-\tan^{-1}(\omega)] = 2\tan^{-1}(\omega)$.

ω	M	φ
−∞	K	−180°
0	K	0°
∞	K	+180°

Corresponding C_2: $s = Re^{j\theta}$ [$R \to \infty$ and $\theta = +90°$ to $-90°$].

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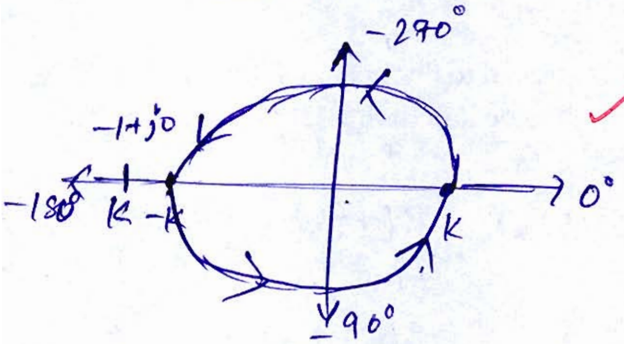
$\Rightarrow GH(Re^{j\theta}) = \lim_{R\to\infty} \frac{K(1+Re^{j\theta})}{(1-Re^{j\theta})} = K.$

∴ Nyquist plot is:-



Based on the Nyquist plot system is Conditionally stable. if $K>1$ then $\boxed{N=1}$.

from $N = P - Z \Rightarrow 1 = 1 - Z \Rightarrow \boxed{Z=0}$.

∴ for $K>1$ System is stable.
for $K=1$ System is marginally stable
for $K<1$ System is unstable

(ii) Given $G(s) = \frac{K(2s+1)}{s(5s+1)(s+1)^2}$ [type '1' system].

input $r(t) = 1+6t$. $e_{ss} < 0.1$ (Given)

for type '1' system, steady state error wrt step input is 0.

∴ $e_{ss\,total} = e_{ss\,ramp}$.

$e_{ss\,ramp} = \frac{A}{K_V}$ [where, A = slope of ramp input]

and $K_V = \lim_{s\to 0} s \cdot G(s)$.

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$\Rightarrow K_{v2} = \lim_{s \to 0} \frac{s \cdot K(2s+1)}{s(5s+1)(s+1)^2} \Rightarrow \boxed{K_v = K}$.

$\therefore e_{ss\,ramp} = \frac{6}{K}$ ($\because$ ramp input $= 6t$).

given $e_{ss} < 0.1 \Rightarrow \frac{6}{K} < 0.1 \Rightarrow 6 < 0.1K$.

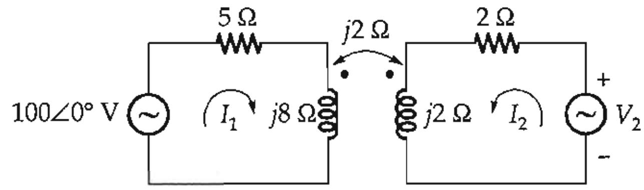
or $\boxed{K > 60}$.

$\therefore$ min. value of $K = 60$.

Good

12

Q.5 (d) (i) In the magnetically coupled circuit shown in figure below, find V_2 for which $I_1 = 0$. What voltage appears at the $j8\Omega$ inductance under this condition?



(ii) In a series LCR circuit, the maximum inductor voltage is twice the maximum capacitor voltage. However, the circuit current lags the applied voltage by 30° and the instantaneous drop across the inductance is given by $V_L = 100 \sin 377t$ V. Assuming the resistance to be 20 Ω, find the values of the inductance and capacitance.

[6 + 6 marks]

(i) By KVL in mesh ①:-

$-100\angle 0° + 5I_1 + j8\,I_1 + j2\,(I_1 + I_2) = 0$. → I_2

$\Rightarrow (5 + j10)\,I_1 + j2I_2 = 100\angle 0°$ ①. → $j8$

3

By KVL in mesh ②:-

$-V_2 + 2I_2 + j2I_2 + j2\,(I_2 + I_1) = 0$.

$\Rightarrow V_2 = (2 + j4)\,I_2 + j2I_1$ ②. → $2 + j2$

$\boxed{I_1 = 0}$ (given).

$\Rightarrow I_2 = \frac{+100}{j2}$ (from ①) $\Rightarrow \boxed{I_2 = -j50\,A}$

Do not write in this margin

substitude in ②:- $V_2 = (2+j4)(-j50)$

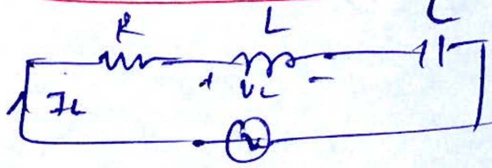
$\Rightarrow V_2 = 200 - j100\ V.$ ~~or $V_2 = 223.606 \angle -26.56° V$~~

~~volt. against j8 inductance $= I_2 \times j8$.~~

~~$\Rightarrow V_{j8} = (-j50 \times j8)$.~~

$\Rightarrow$ Voltage across ~~(j8)~~ inductor $= 400\ V.$

(ii)



V_s, 377 rad s^{-1}.

$\theta = 30°$ (Lag).

$R = 20\ \Omega$

max. inductor volt. $= 2$[Max cap. voltage].

given $V_L = 100 \sin 377t\ V$. [$\because \omega = 377$ rad s^{-1}].

$V_{Lmax} = 100\ V$.

$\Rightarrow$ ~~$V_{Cmax} = 200\ V$.~~

1

$V_L = I_L \times X_L$, and $I_L = \frac{V_s}{Z_{eq}}$,

where, $Z_{eq} = R + j(X_L - X_C)$.

given $\theta = 30°$ Lag.

$\Rightarrow$, $\tan\theta = \frac{X_L - X_C}{R} = \tan 30°$.

$\Rightarrow X_L - X_C = \frac{20}{\sqrt{3}}$.

incomplete

Q.5 (e) The closed loop transfer function of a feedback system is given by

$$T(s) = \frac{1000}{(s+22.5)(s^2+2.45s+44.4)}$$

(i) Determine the resonant peak M_r and resonant frequency ω_r of the system by drawing the frequency response curve.

(ii) Determine the bandwidth of the equivalent second order system.

[6 + 6 marks]

(i) Ans ⟹ $T(s) = \frac{1000}{(s+22.5)(s^2+2.45s+44.4)}$

Comparing with $\frac{K}{(s+a)(s^2+2\xi\omega_n s+\omega_n^2)}$.

⟹ $\omega_n^2 = 44.4$ ⟹ $\boxed{\omega_n = 6.66 \text{ rad s}^{-1}}$

$2\xi\omega_n = 2.45$ ⟹ $2 \times \xi \times 6.66 = 2.45$

⟹ $\boxed{\xi = 0.183}$

Magnetic Resonance peak = $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

⟹ $M_r = \frac{1}{2\times0.183\sqrt{1-0.183^2}} = \frac{2.72}{\sqrt{1-0.183^2}}$

⟹ $\boxed{M_r = 2.766}$ (or) $M_r = 8.837$ dB.

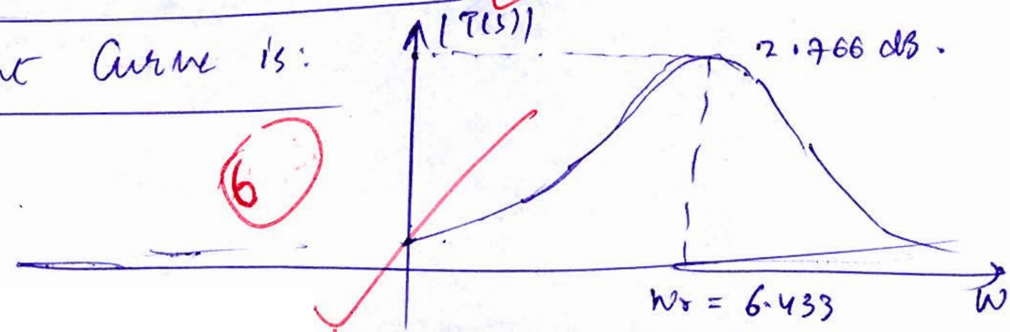
[$M_{dB} = 20\log_{10}(M_r)$]

Resonant frequency = $\omega_r = \omega_n\sqrt{1-2\xi^2}$

⟹ $\omega_r = 6.66\sqrt{1-2\times0.183^2}$

⟹ $\boxed{\omega_r = 6.433 \text{ rad s}^{-1}}$

Resonant Curve is:



Good

6

Don't write unnecessary points, save time for other question

(ii) $T(s) = \dfrac{1000}{(s+22.5)(s^2+2.45s+44.44)}$.

The DC gain of $T(s) = \dfrac{1000}{22.5 * 44.44} = 1$.

∴ DC gain of equivalent second order system is 1.

Here $s = -22.5$ is an insignificant pole.

and poles of $s^2+2.45s+44.44$ are dominant one.

~~∴ equivalent second order~~

By: $s^2+2.45s+44.44 = 0$.

$\Rightarrow (s+1.22-j6.557)(s+1.2+j6.557) = 0$.

∴ equivalent second order system is given by:

$T(s) = \dfrac{K_1}{s^2+2.45s+44.44}$ [∵ DC gain of equivalent system has to be 1].

$\Rightarrow \dfrac{K_1}{44.44} = 1$. $\Rightarrow K_1 = 44.44$.

$\Rightarrow T(s) = \dfrac{44.44}{s^2+2.45s+44.44}$.

1

then Bandwidth of equivalent second order circuit $2\omega_b =$.

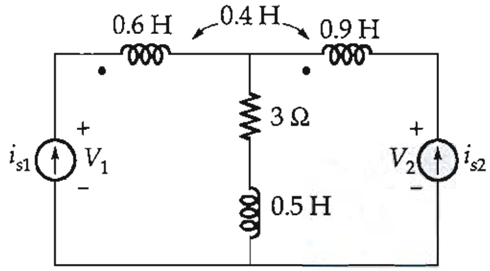
Not required

Revise formula

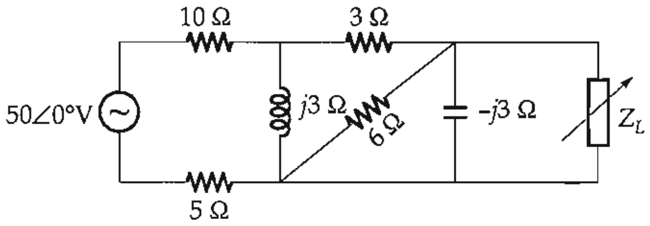
$BW = \omega_n\left[\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}\right]$

Q.6 (a) **(i)** Let $i_{s_1} = 10 \cos 10t$ A and $i_{s2} = 6 \cos 10t$ A in the circuit shown below.

Find: **1.** $V_1(t)$; **2.** $V_2(t)$; **3.** the average power being supplied by each source.



(ii) Find the impedance Z_L so that maximum power can be transferred to it in the network shown below. Also, find the maximum power delivered to load Z_L.



[10 + 10 marks]

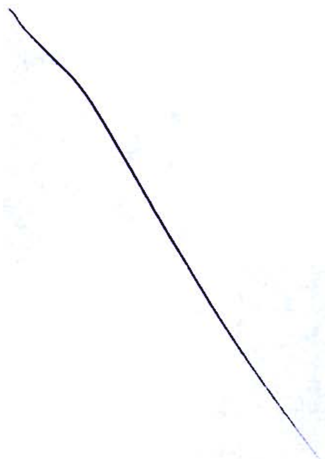
Q.6 (b) A unity negative feedback system has $G(s) = \dfrac{K(s+6)}{s(s+2)}$. When $K = 50$, find change in closed loop pole locations for a 10% change in the value of K.

[20 marks]

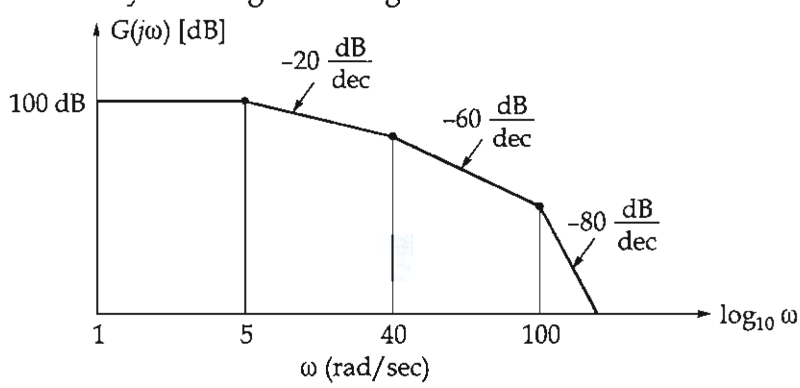
Q.6 (c) (i) Prove that the bandwidth of a series RLC circuit is given as $\frac{R}{L}$ rad/sec.

(ii) A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set to 500 pF, the current has its maximum value while it is reduced to one-half when the capacitor is 600 pF. Find resistance, inductance and Q-factor of inductor.

[8 + 12 marks]



Q.7 (a) The Bode magnitude plot of the open loop transfer function $G(s)$ of a certain unity feedback control system is given in figure.



Estimate the magnitude of transfer function at each of the corner frequencies and also calculate the phase margin.

[20 marks]

Ans → According to given Bode plot, the transfer function G(s) is as follows:-

$$G(s) = \frac{K}{\left(\frac{s}{5}+1\right)\left(\frac{s}{40}+1\right)^2\left(\frac{s}{100}+1\right)}$$

where, K = Gain of G(s).

for K: There are no pole or zero present at origin.

$\therefore$ from: $M(dB) = \pm 20 \log \omega + 20 \log K$.

here $r=0$ (∵ no pole or zero present at origin).

$\Rightarrow 100 = 20\log K \Rightarrow \log K = 5.$

$\Rightarrow \boxed{K = 10^5}$ $\Rightarrow G(s) = \dfrac{10^5}{\left(\frac{s}{5}+1\right)\left(\frac{s}{40}+1\right)^2\left(\frac{s}{100}+1\right)}$ — (a)

Gain of G(s) at $\omega = 5\ rad s^{-1}$:-

∵ until $\omega = 5\ rads^{-1}$, slope in Bode plot $= 0$.

$\Rightarrow |G(j5)| = 100\ dB.$

Gain of G(s) at $\omega = 40\ rads^{-1}$

∵ there is a slope of $-20\ dB/dec$.

from. $\dfrac{|G(j40)| - (100)\ dB}{\log 40 - \log 5} = -20.$

$\Rightarrow \dfrac{|G(j40)| - 100}{\log 8} = -20.$

$\boxed{|G(j40)| = 81.44\ dB.}$

at $\omega = 100\ rads^{-1}$, ; slope $= -60\ dB/dec$.

$\therefore \dfrac{|G(j100)| - 81.94}{\log 100 - \log 40} = -60.$

$\Rightarrow |G(j100)| - 81.94 = -60 \times \log \frac{100}{40}.$

$\Rightarrow \boxed{|G(j100)| = 58.06\ dB.}$

Phase margin of the system = PM.

where $PM = 180^\circ + \angle G(j\omega_{gc})$. — (1)

where, ω_{gc} = freq. at which $|G(j\omega)| = 1$.

or $|G(j\omega)|_{dB} = 0$. $\left[\because |G(j\omega)|_{dB} = 20\log_{10} G(j\omega)\right]$

@ $\omega = 100$ rads^{-1} ; $|G(j100)| = 58.06$ dB.

after $\omega = 100$ rads^{-1}, Slope in Bode plot is -80 dB/dec

and $|G(j\omega_{gc})| = 0$ dB.

$$\Rightarrow \frac{|G(j\omega_{gc})| - |G(j100)|}{\log \omega_{gc} - \log_{10} 100} = -80.$$

$$\Rightarrow \frac{0 \mp 58.06}{\log(\omega_{gc}/100)} = \mp 80 \quad \frac{0 \mp 58.06}{\log_{10}\omega_{gc} - 2} = \mp 80$$

17

$$\Rightarrow 58.06 = 80\log_{10}\omega_{gc} - 160.$$

$$\Rightarrow \frac{218.06}{80} = \log_{10}\omega_{gc} \rightarrow \omega_{gc} = 10^{2.725}.$$

$$\Rightarrow \boxed{\omega_{gc} = 530.88 \text{ rads}^{-1}}.$$

from (a) put $s = j\omega$, hence, $\angle G(j\omega)$ is:-

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{5}\right) - 2\tan^{-1}\left(\frac{\omega}{40}\right) - \tan^{-1}\left(\frac{\omega}{100}\right).$$

$$\angle G(j\omega_{gc}) = -\tan^{-1}\left(\frac{530.88}{5}\right) - 2\tan^{-1}\left(\frac{530.88}{40}\right) - \tan^{-1}\left(\frac{530.88}{100}\right).$$

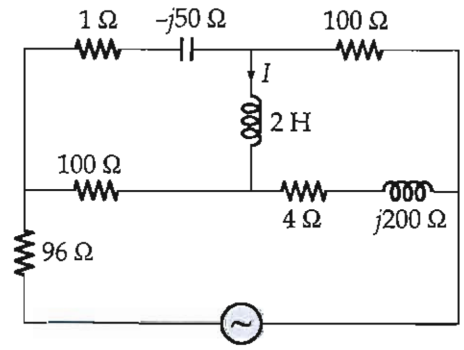
$\Rightarrow \angle G(j\omega_{gc}) =$

Incomplete

Q.7 (b) Find current across 2 Henry inductor as shown in the network below using

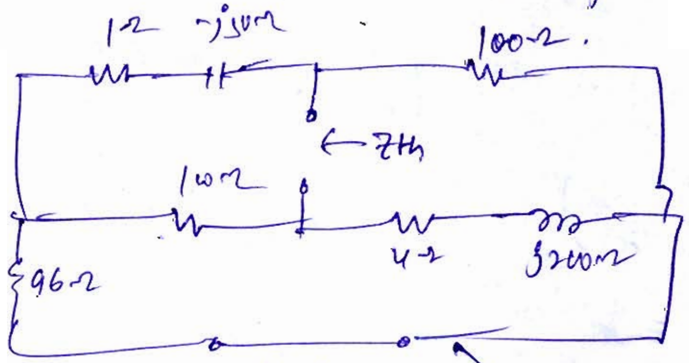
(i) Thevenin's theorem;

(ii) Draw the Norton's equivalent circuit.

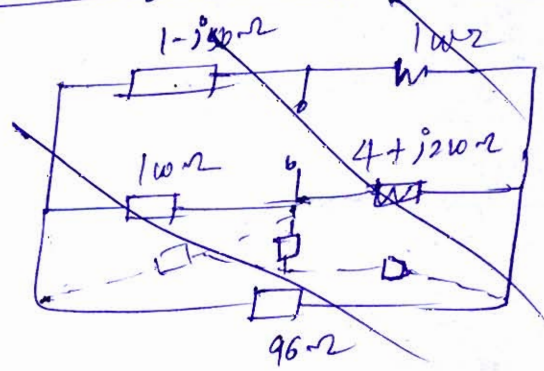


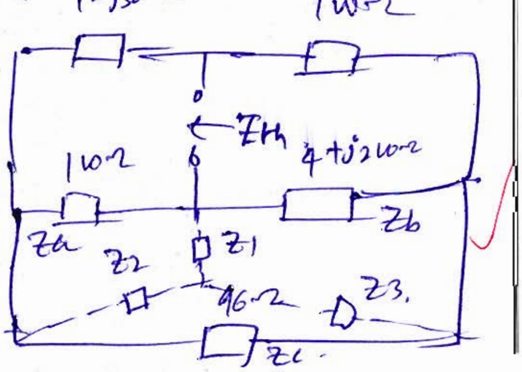
[15 + 5 marks]

Ans-1 Case 1: ~~Zth~~ Zth :- Remove 2H inductor and deactivate all the independent sources :-



Rearranging the circuit as follows:-





By source transformation :-

$Z_2 = \dfrac{Z_b \cdot Z_c}{Z_a + Z_b + Z_c}$ $\Rightarrow$ $Z_2 = \dfrac{96 \times 100}{96 + 100 + 4 + j200}$

[where, $Z_b = 4 + j200$; $Z_c = 96\,\Omega$; $Z_a = 100\,\Omega$]

$\Rightarrow Z_2 = 24 - j24\,\Omega$. $\Rightarrow$ $Z_2 = 24(1-j)\,\Omega$

$Z_1 = \dfrac{Z_a \cdot Z_b}{Z_a + Z_b + Z_c}$ $\Rightarrow$ $Z_1 = \dfrac{100 \times (4 + j200)}{96 + 100 + 4 + j200}$

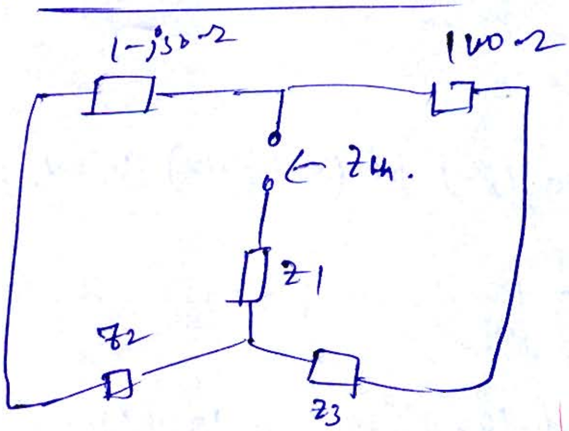
$\Rightarrow Z_1 = 24 + j16\,\Omega$.

Avoid Silly mistake

and $Z_3 = \dfrac{Z_b \cdot Z_c}{Z_a + Z_b + Z_c}$ $\Rightarrow$ $Z_3 = \dfrac{(4 + j200) \times 96}{96 + 100 + 4 + j200}$

$\Rightarrow Z_3 = 48.96 + j47.04\,\Omega$

$\therefore$ Ckt becomes :-



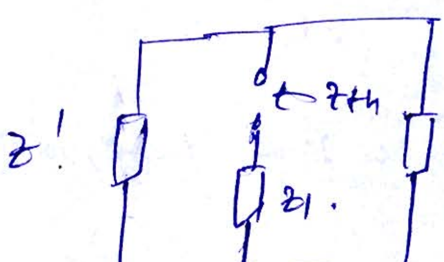
$Z' = 1 - j50 + Z_2$.

$\Rightarrow Z' = 25 - j74\,\Omega$.

$Z'' = 100 + Z_3$.

$Z'' = 148.96 + j47.04\,\Omega$

from circuit diagram

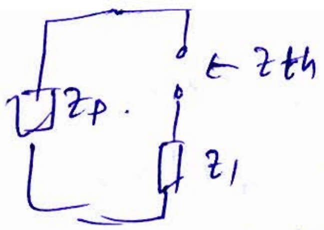


$Z_p = Z' \| Z''$.

$Z_p = \dfrac{Z' \times Z''}{Z' + Z''}$

$\Rightarrow Z_p = 49 - j49\,\Omega$

or $Z_p = 49(1-j)\,\Omega$.



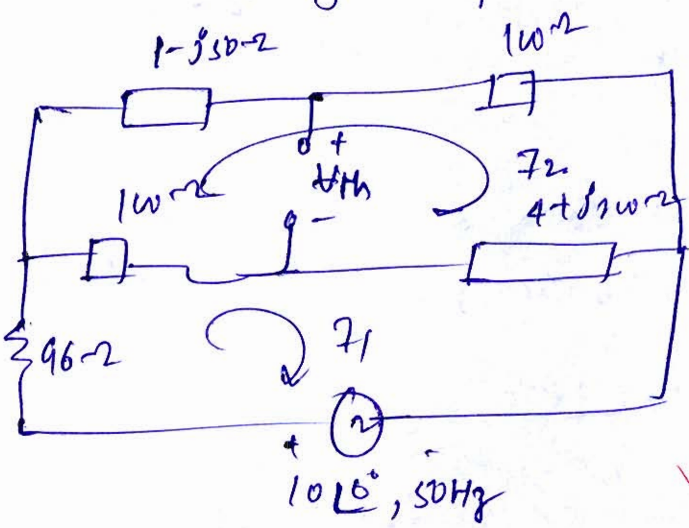
$\Rightarrow Z_{th} = Z_P + Z_1$.

$\Rightarrow Z_{th} = 49(1-j) + 24 + j16\,\Omega$.

$\Rightarrow Z_{th} = 73 - j33\,\Omega$

or $Z_{th} = 80.11\angle -24.37^\circ\,\Omega$

Case 2: Vth : Open circuit voltage: Remove 2H inductor to get Open circuit voltage Vth.



By KVL in mesh ①:-

$-10\angle 0^\circ + 96 I_1 + 100(I_1 - I_2) + (4+j200)(I_1 - I_2) = 0$

$\Rightarrow (196 + 4 + j200) I_1 - 100 I_2 - (4+j200) I_2 = 10$.

$\Rightarrow (200 + j200) I_1 - I_2(104 + j200) = 10$ —(1)

5

By KVL in mesh ②:-

$100(I_2 - I_1) + (1-j50) I_2 + 100 I_2 + (4+j200)(I_2 - I_1) = 0$

$\Rightarrow [100 + 1 - j50 + 4 + j200] I_2 - 100 I_1 - (4+j200) I_1 = 0$.

$\Rightarrow [105 + j150] I_2 = [104 + j200] I_1$. —(2)

Incomplete solution

Q.7 (c) **(i)** Derive the expression for gain margin and phase margin of a unity feedback second order system with transfer function,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

(ii) Sketch the polar plot of the transfer function given below:

$$G(s) = \frac{1+4s}{s(1+s)(1+2s)}$$

Determine whether the polar plot cuts the imaginary axis. If so, determine the frequency at which the plot cross the imaginary axis.

[10 + 10 marks]

Ans → (i). $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

⇒ $G(s)\left[s^2 + 2\xi\omega_n s + \omega_n^2\right] = \omega_n^2\left[1 + G(s)\right]$

⇒ ~~$G(s)\cdot\omega_n^2$~~ $+ G(s)\left[s^2 + 2\xi\omega_n s\right] = \omega_n^2 +$ ~~$\omega_n^2 G(s)$~~

⇒ $G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}$ — (1) or $G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\xi\omega_n)}$

phase margin is calculated at gain cross over freq. = ω_{gc} where $|G(j\omega_{gc})| = 1$.

and Gain margin is calculated at phase cross over freq. = ω_{pc} where $\angle G(j\omega_{pc}) = -180°$

for ω_{pc} :. put $s = j\omega$ in (1) and $\angle G(j\omega_{pc}) = -180°$.

⇒ ~~...~~ ∴ $G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\xi\omega_n)}$.

⇒ $-180° = -90° - \tan^{-1}\left(\frac{\omega_{pc}}{2\xi\omega_n}\right)$

⇒ $\tan^{-1}\left(\frac{\omega_{pc}}{2\xi\omega_n}\right) = 90°$ ⇒ $\boxed{\omega_{pc} = \infty \text{ rads}^{-1}}$.

Do not write in this margin

$\because \omega_{pc} = \infty$. $\therefore$ $\boxed{GM = \infty}$. [Where, GM = Gain margin]

for ω_{gc}: $|G(j\omega_{gc})| = 1$.

$\Rightarrow$ from (1).
$$\frac{\omega_n^2}{\omega_{gc}\left(\sqrt{\omega_{gc}^2 + (2\xi\omega_n)^2}\right)} = 1.$$

$$\Rightarrow \omega_n^2 = \omega_{gc}^2\left[\omega_{gc}^2 + (2\xi\omega_n)^2\right].$$

Very Good

$$\Rightarrow \omega_{gc}^4 + 4\xi^2\omega_n^2\omega_{gc}^2 - \omega_n^2 = 0.$$

$$\Rightarrow \omega_{gc}^2 = \frac{-4\xi^2\omega_n^2 \pm \sqrt{(4\xi^2\omega_n^2)^2 + 4\omega_n^2}}{2}.$$

10

$$\Rightarrow \omega_{gc}^2 = \frac{-4\xi^2\omega_n^2 + \sqrt{16\xi^4\omega_n^4 + 4\omega_n^2}}{2}$$

$$\Rightarrow \omega_{gc}^2 = \frac{-4\xi^2\omega_n^2 + 2\omega_n\sqrt{4\xi^4\omega_n^2 + 1}}{2}.$$

$$\Rightarrow \omega_{gc} = \sqrt{-2\xi^2\omega_n^2 + \omega_n\sqrt{4\xi^4\omega_n^2 + 1}} \quad \text{— (2)}.$$

$$\Rightarrow \angle G(j\omega_{gc}) = -90^\circ - \tan^{-1}\left(\frac{\omega_{gc}}{2\xi\omega_n}\right).$$

$$\Rightarrow PM = 180^\circ + \angle G(j\omega_{gc}).$$

$$\Rightarrow PM = 180^\circ - 90^\circ - \tan^{-1}\left(\frac{\omega_{gc}}{2\xi\omega_n}\right)$$

$$\Rightarrow PM = 180^\circ - \tan^{-1}\left(\frac{\sqrt{-2\xi^2\omega_n^2 + \omega_n\sqrt{4\xi^4\omega_n^2+1}}}{2\xi\omega_n}\right)$$

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(ii) Given $G(s) = \frac{1+4s}{s(s+1)(2s+1)}$; put $s = j\omega$.

$$G(j\omega) = \frac{j4\omega + 1}{j\omega(j\omega+1)(j2\omega+1)}.$$

$$M = |G(j\omega)| = \frac{\sqrt{16\omega^2+1}}{\omega\sqrt{\omega^2+1}\sqrt{4\omega^2+1}};$$

$$\phi = \angle G(j\omega) = +\tan^{-1}(4\omega) - 90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega).$$

ω	M	ϕ
0	∞	-90°
∞	0	-180°

Do Rationalise the TF

Checking the crossing with -180° axis:-

$\Rightarrow \phi = -180^\circ = +\tan^{-1}(4\omega) - 90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$.

$\Rightarrow -90^\circ = \tan^{-1}(4\omega) - \left(\tan^{-1}(\omega) + \tan^{-1}(2\omega)\right)$

$\Rightarrow -90^\circ = \tan^{-1}(4\omega) - \tan^{-1}\left[\frac{3\omega}{1-2\omega^2}\right]$.

$\Rightarrow +90^\circ = \tan^{-1}\left(\frac{3\omega}{1-2\omega^2}\right) - \tan^{-1}(4\omega)$

$\Rightarrow 90^\circ = \tan^{-1}\left(\frac{\frac{3\omega}{1-2\omega^2} - 4\omega}{1 + \frac{3\omega(4\omega)}{(1-2\omega^2)}}\right) \Rightarrow$ ~~$1+(1-2\omega^2)4\omega = 0$~~

$12\omega^2 + 1 - 2\omega^2 = 0$

$\Rightarrow 10\omega^2 + 1 = 0$

2

~~$\Rightarrow 1 + 4\omega - 8\omega^3 = 0 \Rightarrow 8\omega^3 - 4\omega - 1 = 0$.~~

~~$\Rightarrow \omega_1 = 0.809$ rad/s ; $\omega_2 = -0.309$; $\omega_3 = -0.5$.~~

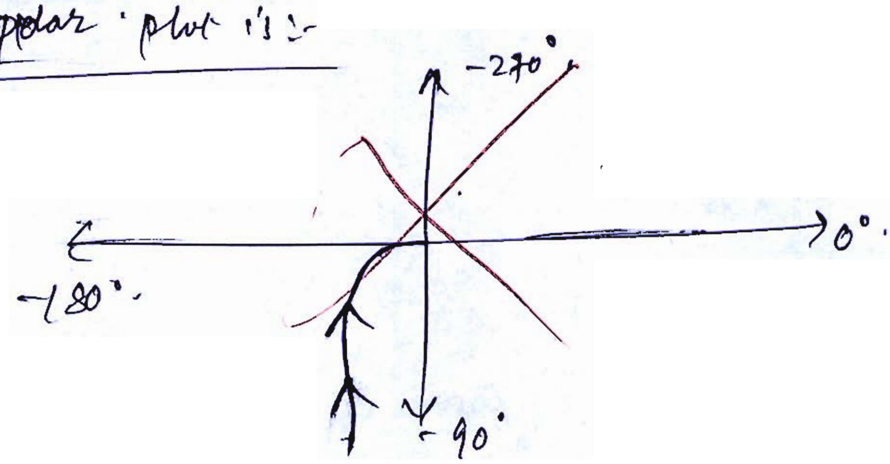
~~$\Rightarrow$ only one valid value of $\omega \Rightarrow \omega = 0.809$ rad/s~~

~~We have a crossing with -180° axis.~~

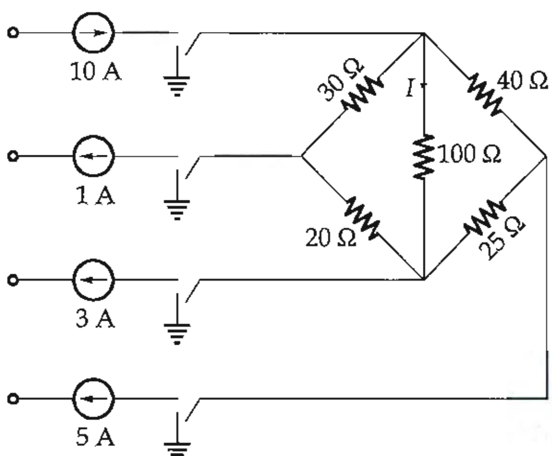
We have no crossing with -180° axis.

$\omega = \frac{1}{2\sqrt{2}}$ rad/s

So, the polar plot is :-



Q.8 (a) (i) Find the value of the current 'I' flowing through the 100 Ω resistor in the bridge shown below using Superposition Theorem. (Assume other sources are grounded, when one is used at a time)

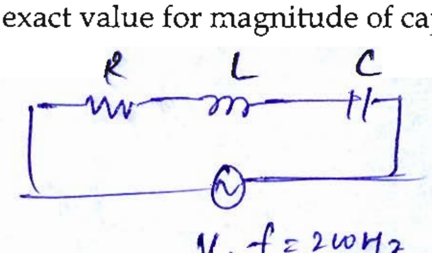


(ii) A certain series RLC resonant circuit has resonant frequency, f_0 = 200 Hz, quality factor, Q_0 = 7.5 and inductive reactance, X_L = 250 Ω at resonance.

1. Find the values of R, L and C
2. If the source voltage, $V_S = 5\angle 45°$ V is connected in series with the circuit, find exact value for magnitude of capacitor voltage, $|V_C|$ at f = 300 Hz.

[10 + 10 marks]

(ii)



$X_L = 250\Omega = \omega L$.

$Q_0 = 7.5$; $f_0 = 200$ Hz.

Under resonance, $|X_L| = |X_C|$ → $\omega_0 L = \dfrac{1}{\omega_0 C}$ —(1)

and $|X_L| = 250\Omega = \omega_0 L$. (Given).

⇒ $2\pi f_0 L = 250$

⇒ $L = \dfrac{250}{2\pi \times 200}$ ⇒ $L = 0.2$ H.

from (1) $\frac{1}{\omega_0 C} = 250 \Rightarrow C = \frac{1}{\omega_0 \times 250}$.

$\Rightarrow C = \frac{1}{2\pi f_0 \times 250} \Rightarrow C = \frac{1}{2\pi \times 200 \times 250}$.

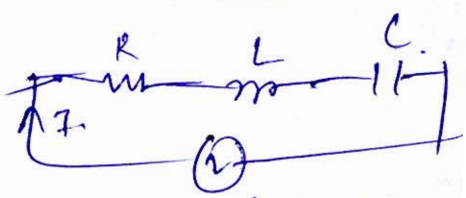
$\Rightarrow \boxed{C = 3.183 \mu F}$

$\because Q_0 = 7.5$ (Given). from $Q_0 = \frac{\omega_0 L}{R}$.

$\Rightarrow 7.5 = \frac{2\pi f_0 L}{R} \Rightarrow R = \frac{2\pi f_0 L}{7.5}$.

$\Rightarrow R = \frac{2\pi \times 200 \times 0.2}{7.5} \Rightarrow \boxed{R = 33.51 \Omega}$.

(2). $V_s = 5 \angle 45^\circ V$.



$5 \angle 45^\circ$, 300 Hz

$Z_{eq} = R + \left(j\omega L - \frac{j}{\omega C}\right)$

$\Rightarrow Z_{eq} = R + j\left(\omega L - \frac{1}{\omega C}\right) = 33.51 + j\left(2\pi f L - \frac{1}{2\pi f C}\right)$.

$\Rightarrow Z_{eq} = 33.51 + j(377 - 166.67)$.

8

$\Rightarrow Z_{eq} = 33.51 + j210.33 \Omega$.

$\therefore I = \frac{V_s}{Z_{eq}} = \frac{5 \angle 45^\circ}{33.51 + j210.33} = \frac{\frac{5}{\sqrt{2}} + j\frac{5}{\sqrt{2}}}{33.51 + j210.33}$

$\Rightarrow I = 0.023 \angle -35.95^\circ A$.

$\therefore V_C = I \times (-jX_C)$.

$\Rightarrow V_C = 0.023 \angle -35.95^\circ \left[\frac{-j}{2\pi f C}\right]$.

Do not write in this margin

$\Rightarrow V_c = \dfrac{0.023 \angle -125.95^\circ}{2\pi \times 300 \times 3.183 \times 10^{-6}}$

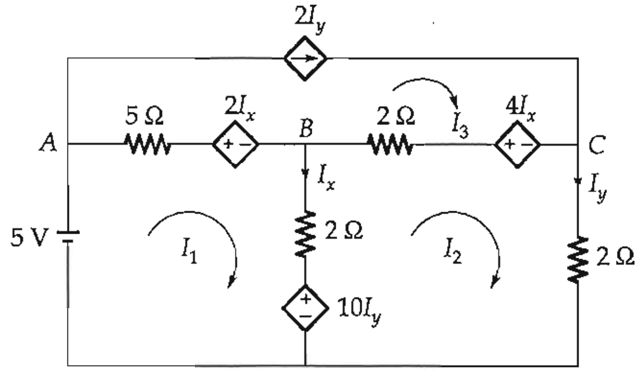
$\Rightarrow V_c = 12.45 \angle -125.95^\circ V.$ → Avoid silly mistakes

$\therefore |V_c| = 12.45 V$

$V_c = 3.91 \angle -125.95^\circ$

(ii)

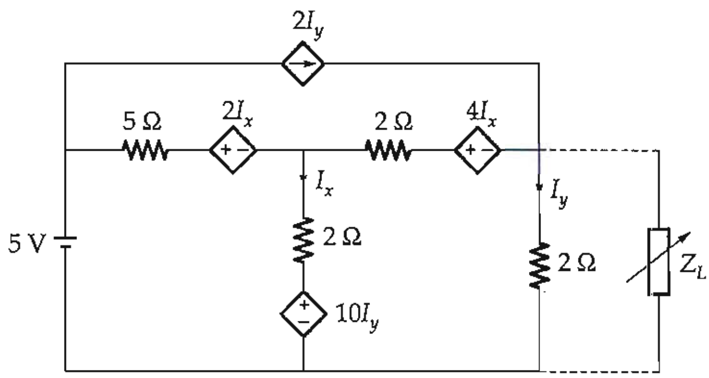
Q.8 (b) Consider the circuit shown below, which contain some dependent and independent sources.



Find

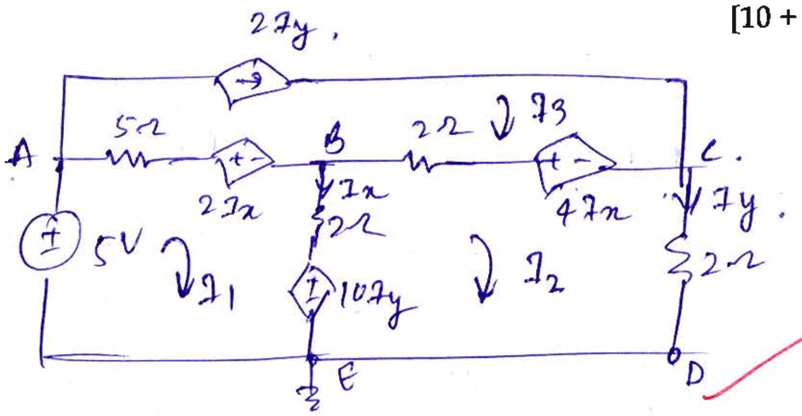
(i) Currents I_1, I_2 and I_3 using mesh analysis.

(ii) The maximum power transferred to the load, connected across 2 Ω as shown below:



[10 + 10 marks]

Ans→ ckt..



By KVL in mesh ①:.

$-5 + 5(I_1 - I_3) + 2I_x + 2(I_1 - I_2) + 10I_y = 0.$

$\Rightarrow 7I_1 - 2I_2 - 5I_3 + 2I_x + 10I_y = 5$ ①

By KCL in mesh ③:- $I_3 = 2I_y$ —②.

By KVL in mesh ②:-

$-10 I_y + 2(I_2 - I_1) + 2(I_2 - I_3) + 4I_x + 2I_2 = 0.$

$\Rightarrow 4I_2 - 2I_1 - 2I_3 + 4I_x + 2I_2 - 10 I_y = 0$

from branch C-D :- $\boxed{I_2 = I_y}$ —(3).

$\Rightarrow 6I_2 - 2I_1 - 2I_3 + 4I_x - 10I_2 = 0$

$\Rightarrow -4I_2 - 2I_1 - 2I_3 + 4I_x = 0.$

from Branch B-E :- $\boxed{I_x = I_1 - I_2}$ —(4).

$\Rightarrow -4I_2 - 2I_1 - 2I_3 + 4(I_1 - I_2) = 0.$

$\Rightarrow 2I_1 - 8I_2 - 2I_3 = 0 \Rightarrow \boxed{I_1 - 4I_2 - I_3 = 0}$ —(5).

put (2), (3), (4) in (1) :-

$7I_1 - 2I_2 - 5I_3 + 2(I_1 - I_2) + 10 I_2 = 5.$

$9I_1 - 4I_2 + 10I_2 - 5I_3 = 5$

$\Rightarrow 9I_1 + 6I_2 - 5I_3 = 5$ —(6).

12

and put (3) in (2). : $\Rightarrow \boxed{I_3 = 2I_2}$ —(7).

put (7) in (5) :- $I_1 - 4I_2 - 2I_2 = 0.$

$\Rightarrow \boxed{I_1 = 6I_2}$ —(8).

put (7), (8) in (6).

$\Rightarrow 9 \times 6I_2 + 6I_2 - 5 \times 2I_2 = 5$

$\Rightarrow (54 + 6 - 10) I_2 = 5. \Rightarrow I_2 = \frac{5}{50}.$

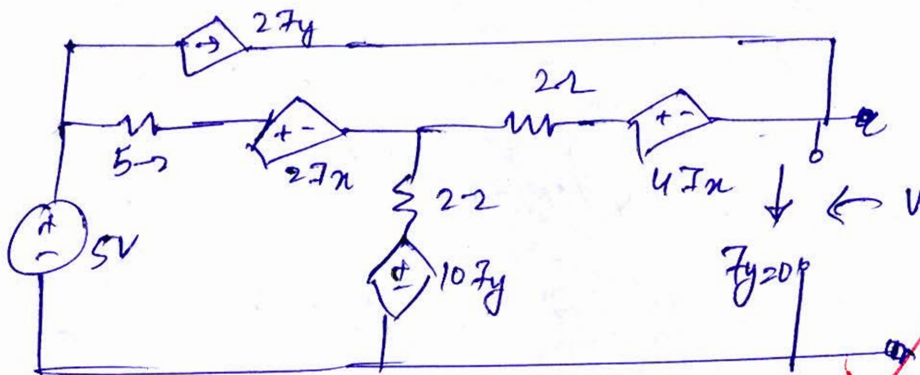
$\Rightarrow \boxed{I_2 = 0.1A}.$

$\therefore I_1 = 6I_2 \Rightarrow \boxed{I_1 = 0.6A}$

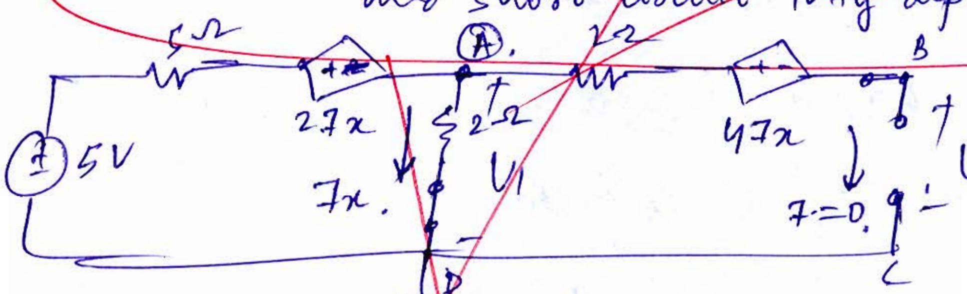
and $I_3 = 2I_2 \Rightarrow \boxed{I_3 = 0.2A}$

For max. power transfer :-

Case 1: V_{th}: Open circuit 2Ω and calculate open circuit voltage V_{th}.



$\because I_y = 0$; we can remove $4I_y$ dependent source and short circuit $10I_y$ dependent source



at node 1 :. $\frac{V_1}{2} + \frac{V_1 + 2I_x - 5}{5} = 0$

$\boxed{\frac{V_1}{2} = I_x} \Rightarrow \boxed{V_1 = 2I_x}$

$\therefore I_x + \frac{4I_x - 5}{5} = 0$

$\Rightarrow 5I_x + 4I_x - 5 = 0 \Rightarrow \boxed{I_x = \frac{5}{9} A}$

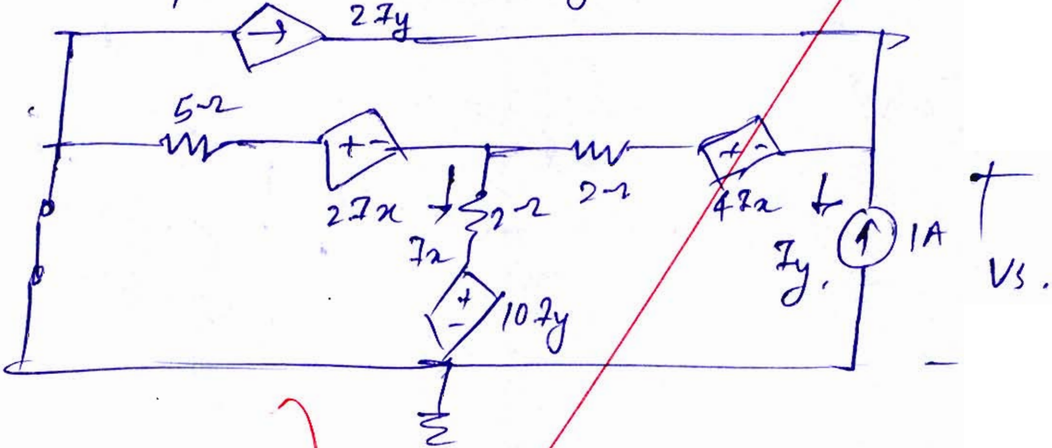
$\Rightarrow V_1 = 2I_x \Rightarrow \boxed{V_1 = \frac{10}{9} V}$

By KVL in loop ABCDA :- $-V_1 + 4I_x + V_{th} = 0$.

$\Rightarrow V_{th} = V_1 - 4I_x \Rightarrow V_{th} = \frac{10}{9} - \frac{4\times5}{9}$.

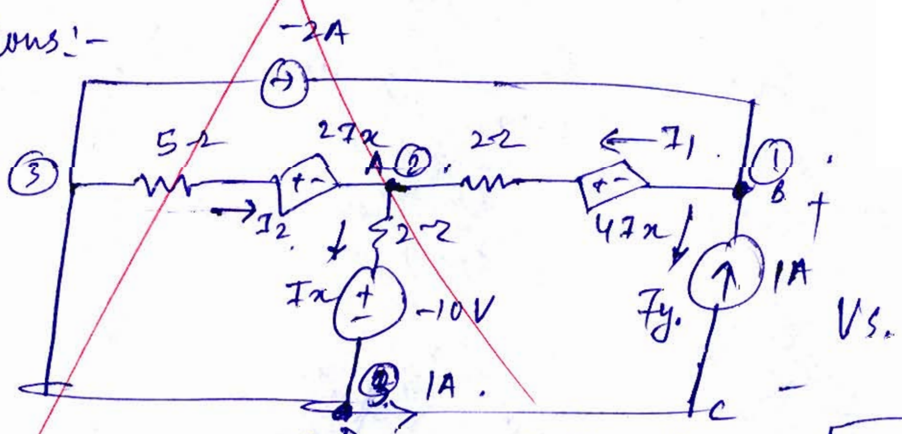
$\Rightarrow \boxed{V_{th} = -\frac{10}{9} V}$

Case 2 :- For R_{th} :- Remove 2Ω and deactivate all the independent sources to get R_{th} :



We have connected 1A source to activate the dependent sources hence, $R_{th} = \frac{V_s}{1}$. $\Rightarrow \boxed{R_{th} = V_s}$

$\Rightarrow \boxed{I_y = -1A}$ ∴ the circuit is simplified as follows:-



By KCL at node ① :- $1 + (-2) = I_1 \Rightarrow \boxed{I_1 = -1A}$

By KCL at node ② :- $I_2 + I_1 = I_x$.

$\Rightarrow I_2 - 1 = I_x$.

By KCL at node ③ :- $I_2 + (-2) + 1 + I_x = 0$.

$\Rightarrow I_2 - 1 + I_x = 0$

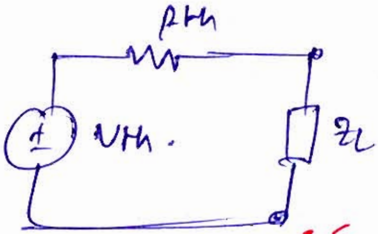
$\Rightarrow \boxed{I_x = 0}$

By KVL in loop ABCDA:-

$-2I_1 + V_S - (-10) = 0$, $\Rightarrow$ ~~$V_S = -10 + 2(-1)$.~~

$\Rightarrow V_S = -12V$ $\Rightarrow$ ~~$R_{th} = -12\Omega$~~

Max. power is transfer:-



Max. power is transferred when $Z_L = R_{th}$.

and max. power $= P_{max} = \frac{V_{th}^2}{4R_{th}}$.

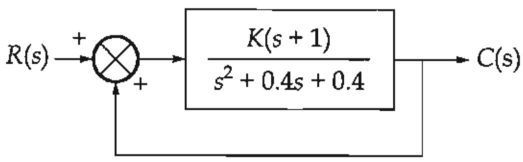
$\Rightarrow P_{max} = \frac{(10/9)^2}{4 \times (-R_{th})} \Rightarrow 0.0257 W$

Your approach is good. ~~but here~~ Avoid silly mistakes.

But Better understanding for see solution,

Q.8 (c) (i) A feedback control system has $G(s) = \frac{10}{s(s+10)}$ and $H(s) = e^{-T_1 s}$. Find T_1 for which system is marginally stable.

(ii) Sketch the root locus for the positive feedback system as drawn below for $0 < K < \infty$.



Also, comment on the stability of the system.

[10 + 10 marks]

(i) Given $G(s) = \frac{10}{s(s+10)}$, $H(s) = e^{-T_1 s}$.

$\Rightarrow G(s)H(s) = OLTF = \frac{10}{s(s+10)} e^{-T_1 s}$. —(1)

9

If PM = 0°, then, we say that system is marginally stable and, when, PM = 0° & GM = 0 dB.

and $\omega_{gc} = \omega_{pc}$.

we find T_1 with help of concept of phase margin (PM).

PM is calculated at gain cross over frequency (w_{gc}).

and, at w_{gc}, $|GH(jw_{gc})| = 1$.

put $s = jw$ in ① :· $\Rightarrow GH(jw) = \dfrac{10}{jw(jw+10)} e^{-jwT_1}$ —②.

$|GH(jw)| = \dfrac{10}{w\sqrt{w^2+100}}$ $\left(\because e^{-jwT} = 1\right)$.

$\Rightarrow |GH(jw_{gc})| = \dfrac{10}{w_{gc}\sqrt{w_{gc}^2+100}} = 1$ · (squaring both sides)

$\Rightarrow 100 = w_{gc}^2[w_{gc}^2+100]$. $\Rightarrow w_{gc}^4 + 100w_{gc}^2 - 100 = 0$.

$\Rightarrow w_{gc}^2 = \dfrac{-100 \pm \sqrt{(100)^2+400}}{2}$.

$\Rightarrow w_{gc}^2 = \dfrac{-100+\sqrt{10400}}{2}$ $[\because w_{gc}^2 > 0]$

$\Rightarrow$ ~~$w_{gc}^2 = -100+100$~~ $w_{gc}^2 = \dfrac{-100+101.98}{2}$.

or $w_{gc}^2 \simeq \dfrac{-100+102}{2}$ $\Rightarrow$ ~~w_{gc}^2~~ $w_{gc}^2 = \dfrac{2}{2}$.

$\Rightarrow \boxed{w_{gc} = 1 \text{ rads}^{-1}}$.

$\therefore \angle G(jw) = -90^\circ - wT_1 - \tan^{-1}\left(\frac{w}{10}\right)$ [from ②]

$\angle G(jw_{gc}) = -90^\circ - w_{gc}T_1 - \tan^{-1}\left(\frac{w_{gc}}{10}\right)$.

$\angle G(jw_{gc}) = -90^\circ - T_1 - \tan^{-1}\left(\frac{1}{10}\right)$.

$\angle G(jw_{gc}) = -95.71^\circ - T_1$. $\Rightarrow$ ~~$\angle G(j$~~ —③.

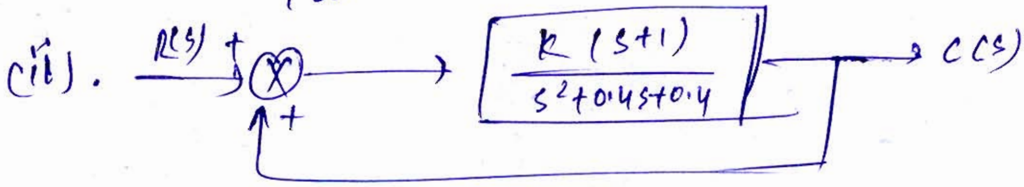
$\therefore PM = 180^\circ + \angle G(jw_{gc})$

$\Rightarrow PM = 180^\circ - 95.71^\circ - T_1$ [from ③].

$\Rightarrow PM = 84.29° - T_1$ [∵ System is marg-stable].

$\Rightarrow T_1 = \frac{84.29° \times \pi}{180°} \Rightarrow \boxed{T_1 = 1.4711}$.

(ii).



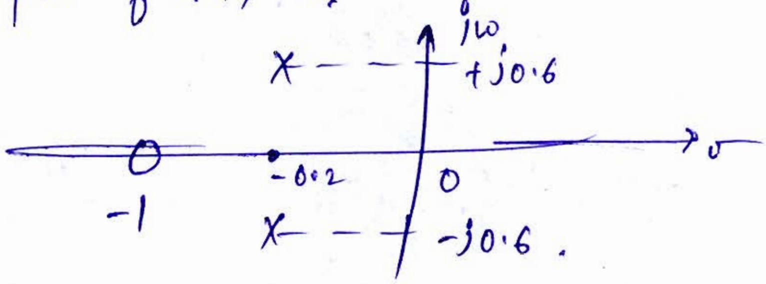
$G(s) = \frac{K(s+1)}{s^2+0.4s+0.4}$ ① and feedback is (+)ve.

∴ q(s) = characteristic eqⁿ = $1 - G(s) = 0$.

$\Rightarrow 1 - \frac{K(s+1)}{s^2+0.4s+0.4} = 0$. ②.

8

pole zero plot of G(s) is:-



~~∴ Root locus always exist on the~~

∵ feed back is (+) ve and K > 0 ⇒ the locus formed ~~with +~~ will be complimentary root locus- here, $G(s) = \frac{K(s+1)}{(s+0.2-j0.6)(s+0.2+j0.6)}$.

P = 2; Z = 1 $\Rightarrow \boxed{P - Z = 1}$.

and no. of root locus branches = 2

ϕ_a = angle of asymptotes = $\frac{2K \times 180°}{P-Z}$ K = 0, 1 --- P-Z-1

$\Rightarrow \boxed{\phi_a = 0°}$.

Centroid = $\sigma_a = \frac{Re\{P\} - Re\{Z\}}{P - Z}$.

$\Rightarrow \sigma_a = \frac{\{-0.2 + (-0.2) - [-1]\}}{1}$.

$\Rightarrow \sigma_a = 1 - 0.4 \Rightarrow \boxed{\sigma_a = 0.6}$

for break points of root locus; we need to solve.

$\frac{dk}{ds} = 0$. $\Rightarrow$ from (2) . $\frac{k(s+1)}{s^2 + 0.4s + 0.4} = 1$.

$\Rightarrow k = \frac{s^2 + 0.4s + 0.4}{s+1}$.

differentiate wrt 's' and put $\frac{dk}{ds} = 0$.

$\Rightarrow \frac{dk}{ds} = \frac{[(2s+0.4)(s+1) - (s^2+0.4s+0.4)]}{(s+1)^2} = 0$.

$\Rightarrow \frac{2s^2 + 2s + 0.4s + 0.4 - s^2 - 0.4s - 0.4}{(s+1)^2} = 0$.

$\Rightarrow s^2 + 2s = 0$. $\Rightarrow s(s+1) = 0$.

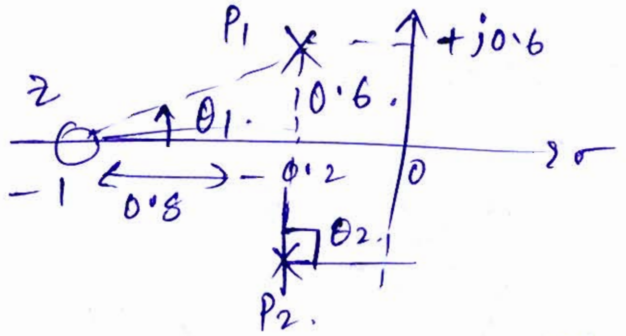
$\Rightarrow s = 0$; $s = -1$. ∵ locus is complimentary root locus ∴ $\boxed{s = 0}$ is valid break point.

from pole-zero plot, we have complex poles.

∴ angle of departure = $\phi d = \pm[180° + \phi]$.

where, ϕ = net angle contribution from the poles and zeroes, to the pole concerned.

∴ for $s = -0.2 + j0.6$ pole, ϕ_d is :-



$\theta_1 = \tan^{-1}\left(\frac{0.6}{0.8}\right)$

$\theta_1 = 36.87°$

$\theta_2 = 90°$

∴ $\phi = +[\theta_1] - [\theta_2]$. [(+) for zero; (-) → for pole]

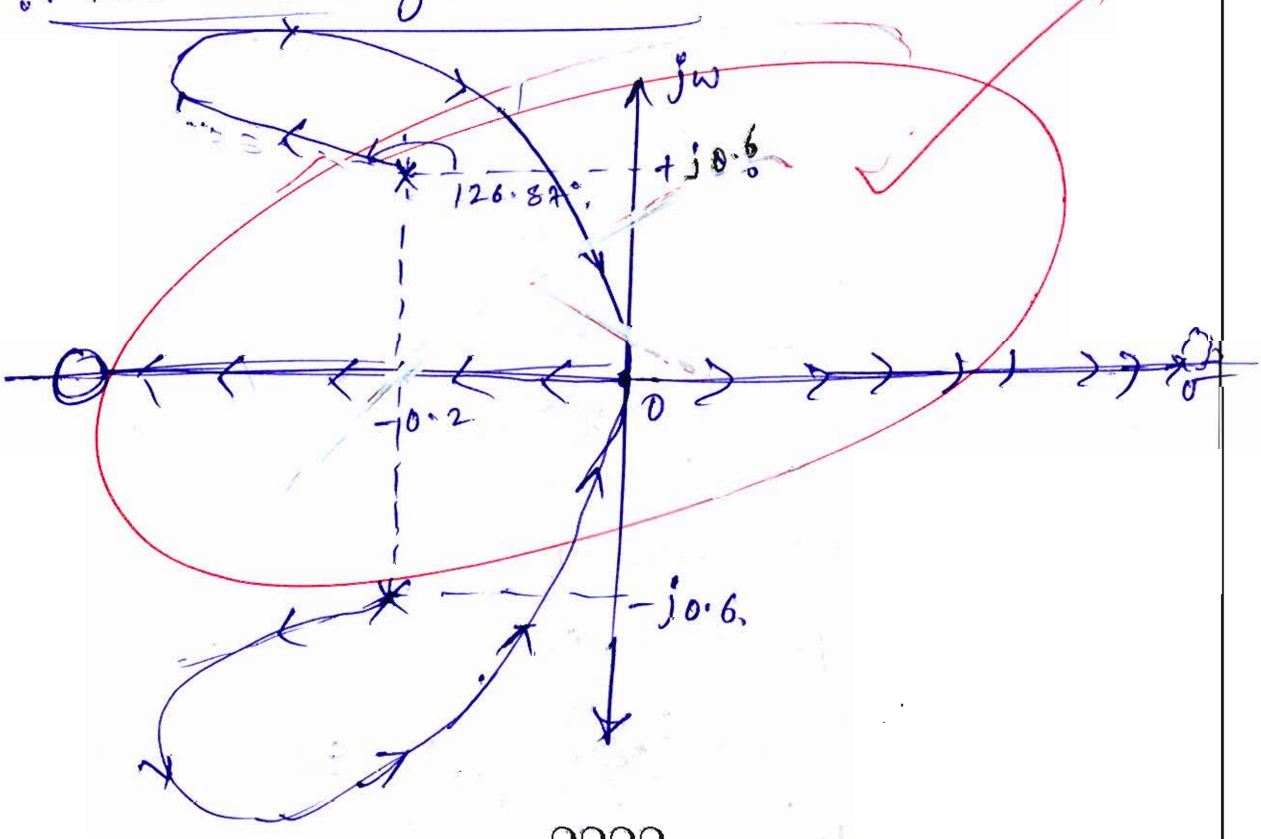
⇒ $\phi = 36.87 - 90°$ ⇒ $\phi = -53.13°$

∴ $\phi_d = \pm[180° + \phi]$

⇒ $\phi_d = \pm[180° - 53.13°]$

⇒ $\phi_d = \pm 126.87°$

∴ Root locus diagram is :-



Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

$\frac{s+1}{1-s}$ ∴ $\phi = 2\tan^{-1}\omega$

$\omega = 0 \rightarrow \phi = 0^\circ$

$\omega = \infty \quad \phi = 180^\circ$

$-e^{+at}u(-t) = \frac{1}{(s+a)} \quad \sigma < +a$