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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Systems and Signal Processing + Microprocessors [All topics]

Electrical Circuits-1 + Control Systems-1 [Part Syllabus]

Name :

Roll No

| Test Centres | Student's Signature |
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| Delhi <input type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input checked="" type="checkbox"/> | |
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

| Question No. | Marks Obtained |
|-----------------------------|----------------|
| Section-A | |
| Q.1 | |
| Q.2 | |
| Q.3 | |
| Q.4 | |
| Section-B | |
| Q.5 | |
| Q.6 | |
| Q.7 | |
| Q.8 | |
| Total Marks Obtained | 199 |

Signature of Evaluator

Cross Checked by

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

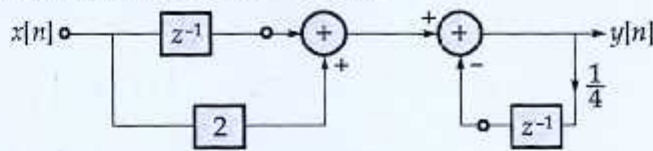
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2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Systems and Signal Processing + Microprocessors

Q.1 (a) Consider the system shown in figure below :



Determine the impulse response of the system?

[12 marks]

Obtaining Transfer function from the given System

$$\text{Transfer function} = \frac{Y(z)}{X(z)} = H(z)$$

$$H(z) = \frac{\text{forward path gain}}{1 - \{ \text{sum of individual loop gain} \}}$$

$$H(z) = \frac{2 + z^{-1}}{1 + \frac{1}{4}z^{-1}}$$

$$\Rightarrow H(z) = \frac{2}{1 + \frac{1}{4}z^{-1}} + \frac{z^{-1}}{1 + \frac{1}{4}z^{-1}} \dots (1)$$

$$\therefore a^n u(n) \xleftrightarrow{ZT} \frac{1}{1 - az^{-1}}, \quad |z| > a$$

$$(-a)^n u(n) \xleftrightarrow{ZT} \frac{1}{1 + az^{-1}}, \quad |z| > a$$

taking inverse Z-Transform of Equation (1)

$$\Rightarrow h(n) = 2 \left(-\frac{1}{4}\right)^n u(n) + \left(-\frac{1}{4}\right)^{n-1} u(n-1)$$

by using time shifting property
 $x(n-n_0) \xrightarrow{ZT} z^{-n} X(z)$

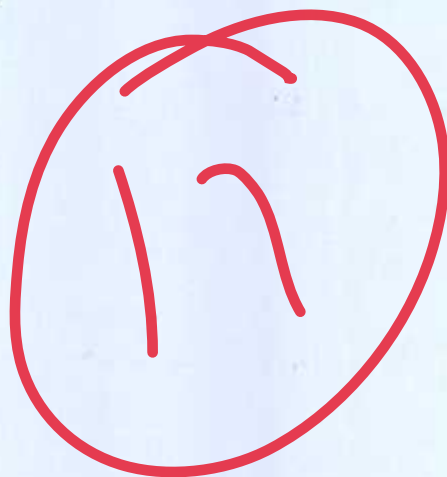
$$\Rightarrow h(n) = 2 \left(-\frac{1}{4}\right)^n \left[\delta(n) + u(n-1) \right] + \left(-\frac{1}{4}\right)^{n-1} u(n-1)$$

$$\Rightarrow h(n) = 3 \left(-\frac{1}{4}\right)^{n-1} u(n-1) + 2 \left(-\frac{1}{4}\right)^n \delta(n)$$

$$x(n] \delta(n-n_0) \rightarrow x(n_0) \delta(n-n_0)$$

then impulse response is:

$$\Rightarrow h(n) = 3 \left(-\frac{1}{4}\right)^{n-1} u(n-1) + 2 \delta(n)$$



Q.1 (b) Find $x(n]$ by using convolution for

$$X(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

[12 marks]

$$d \quad X(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)} \cdot \frac{1}{\left(1 + \frac{1}{4}z^{-1}\right)}$$

$$\Rightarrow X(z) = X_1(z) X_2(z)$$

$$\text{where } X_1(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \Leftrightarrow x_1(n)$$

$$X_2(z) = \frac{1}{\left(1 + \frac{1}{4}z^{-1}\right)} \Leftrightarrow x_2(n)$$

$$x(n) = x_1(n) * x_2(n) \xrightarrow{ZT} X_1(z) X_2(z)$$

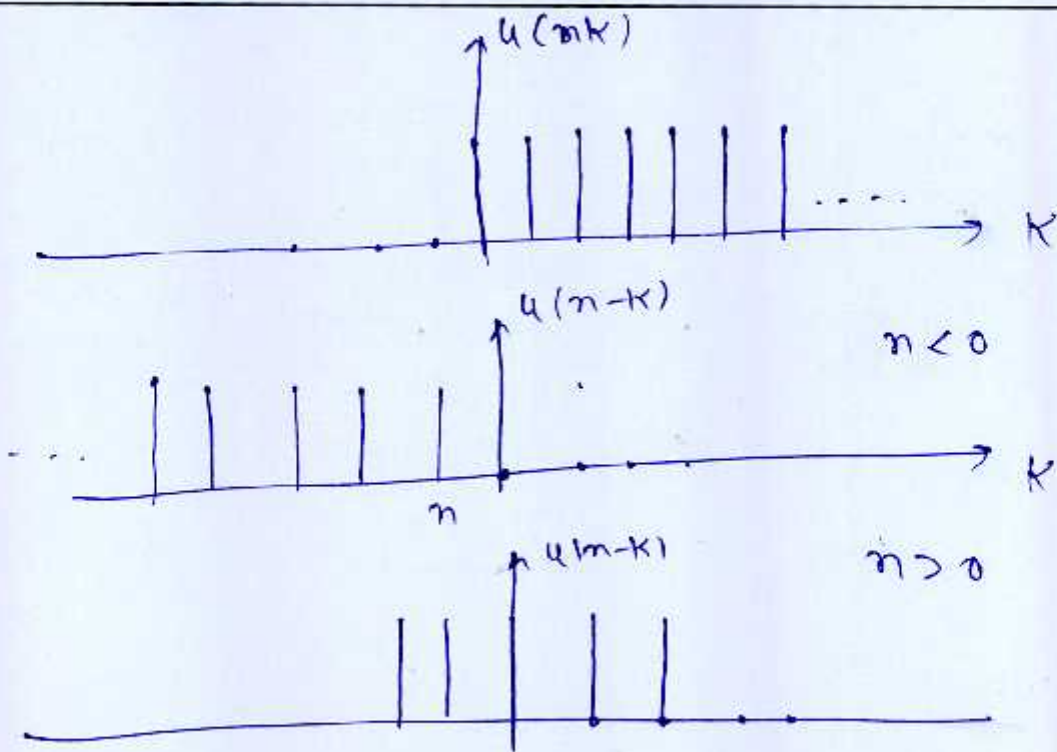
$$x_1(n) = \left(\frac{1}{2}\right)^n u(n) \xrightarrow{ZT} X_1(z)$$

$$\text{and } x_2(n) = \left(-\frac{1}{4}\right)^n u(n) \Leftrightarrow X_2(z)$$

$$\text{then } x(n) = x_1(n) * x_2(n)$$

$$= \left(\frac{1}{2}\right)^n u(n) * \left(-\frac{1}{4}\right)^n u(n)$$

$$= \sum_{k=-\infty}^{\infty} \left(-\frac{1}{4}\right)^k u(k) \left(\frac{1}{2}\right)^{n-k} u(n-k)$$

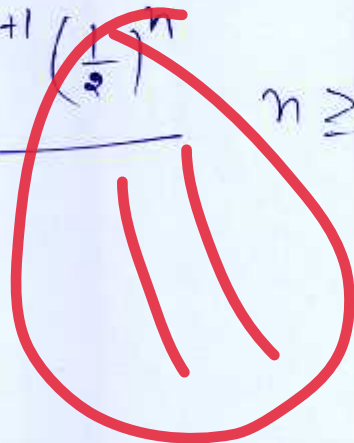


$$x(n) = \left(\frac{1}{2}\right)^n \begin{cases} 0, & n < 0 \\ \sum_{k=0}^n \left(-\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^{-k}, & n \geq 0 \end{cases}$$

$$= \left(\frac{1}{2}\right)^n \left[\sum_{k=0}^n \left(-\frac{1}{2}\right)^k, n \geq 0 \right]$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1 - \left(-\frac{1}{2}\right)^{n+1}}{1 + \frac{1}{2}} \right]$$

$$x(n) = \frac{\left(\frac{1}{2}\right)^n - \left(-\frac{1}{2}\right)^{n+1} \left(\frac{1}{2}\right)^n}{3/2}, n \geq 0$$



Q.1 (c) Find the Inverse DFT of the sequence $Y(K) = \{1, 0, 1, 0\}$.

[12 marks]

by definition of N -point inverse
DFT of discrete sequence

$$x(m) = \frac{1}{N} \sum_{k=0}^{(N-1)} X(k) e^{j \frac{2\pi}{N} km}$$

Here $N = 4$

given $Y(K) = \{1, 0, 1, 0\} \Rightarrow y(m)$

$$y(m) = \frac{1}{4} \sum_{k=0}^3 Y(k) e^{j \frac{2\pi}{4} km}$$

$$\Rightarrow y(m) = \frac{1}{4} \left[Y(0) + Y(1) e^{j \frac{2\pi}{4} m} + Y(2) e^{j \frac{2\pi}{4} (2) m} + Y(3) e^{j \frac{2\pi}{4} (3) m} \right]$$

$$\Rightarrow y(m) = \frac{1}{4} \left[1 + 0 + 1 e^{j \frac{2\pi}{4} (2) m} + 0 \right]$$

$$\Rightarrow y(m) = \frac{1}{4} \left[1 + e^{j \pi m} \right] \quad m = 0, 1, 2, 3$$

put $m = 0$,

$$y(0) = \frac{1}{4} [1 + 1] = \frac{1}{2}$$

put $m = 1$,

$$y(1) = \frac{1}{4} (1 - 1) = 0$$

put $m = 2$

$$y(2) = \frac{1}{4} [1 + 1] = \frac{1}{2}$$

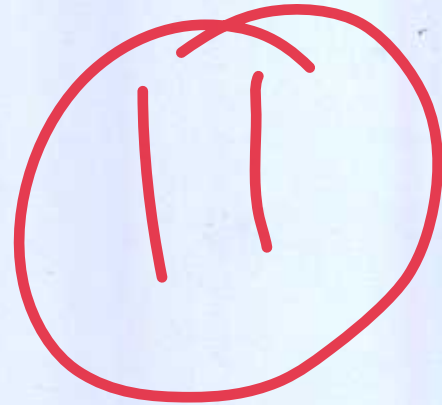
put $n=3$

$$y(n) = \frac{1}{4} [1 + e^{j3\pi n}] = \frac{1}{4} [1 - 1] = 0$$

then inverse DFT:

$$\text{then } y(n) = \left\{ \frac{1}{2}, 0, \frac{1}{2}, 0 \right\}$$

↑



Good Approach

Q.1 (d) Consider an analog filter whose transfer function is :

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Find $H(z)$ by using bilinear transformation. (Assume $T = 1$ sec).

[12 marks]

Given Transfer function for analog filter

$$H(s) = \frac{2}{(s+1)(s+2)} \dots (1) \quad \text{given } T=1 \text{ sec}$$

By bilinear transformation

$$H(z) = H(s) \Big|_{s = \frac{2}{T} \frac{(1-z^{-1})}{(1+z^{-1})}}$$

$$\Rightarrow H(z) = H(s) \Big|_{s = \frac{2(1-z^{-1})}{(1+z^{-1})}} \dots (2) \quad \text{since } T=1 \text{ sec}$$

from Eq. (1)

$$\Rightarrow H(s) = \frac{2}{s^2 + 3s + 2}$$

$$\Rightarrow H(z) = H(s) \Big|_{s = \frac{2(1-z^{-1})}{(1+z^{-1})}}$$

$$\Rightarrow H(z) = \frac{2}{\left(\frac{2(1-z^{-1})}{(1+z^{-1})}\right)^2 + 3 \frac{2(1-z^{-1})}{(1+z^{-1})} + 2}$$

$$\Rightarrow H(z) = \frac{2(1+z^{-1})^2}{4(1-z^{-1})^2 + 6(1-z^{-1})(1+z^{-1}) + 2(1+z^{-1})^2}$$

$$\Rightarrow H(z) = \frac{2(1+z^{-2} + 2z^{-1})}{4(1-z^{-2}) + 6(1-z^{-2}) + 2(1+z^{-2} + 2z^{-1})}$$

then

$$H(z) = \frac{2(1+z^{-2}+2z^{-1})}{4(1+z^{-2}-2z^{-1})+6-6z^{-2}+2+2z^{-2}+4z^{-1}}$$

$$\Rightarrow H(z) = \frac{2(1+z^{-2}+2z^{-1})}{(12-4z^{-1})}$$

$$\Rightarrow H(z) = \frac{1}{2} \frac{(1+z^{-2}+2z^{-1})}{(3-z^{-1})}$$

Good Approach

- Q.1 (e) Write a program in 8085 microprocessor to find the smallest of the two numbers stored at memory location 3025H and 3026H and store the result in the memory location 3027H.

| | |
|-------|-----|
| 3025H | 84H |
| 3026H | 99H |

[12 marks]

- Q.2 (a) The one-sided exponential pulse (i.e., $v(t) = 0$ for $t < 0$), $v(t) = 4e^{-3t}u(t)$ V is applied to the input of an ideal bandpass filter. If the filter passband is defined by $1 < |f| < 2$ Hz, calculate the percentage of output energy w.r.t. to input energy.

[20 marks]

Q.2 (b) Consider an ideal low pass filter with frequency response,

$$H_d(e^{j\omega}) = 1 \quad \text{for } -\frac{\pi}{2} \leq \omega \leq \frac{\pi}{2}$$
$$= 0 \quad \text{for } \frac{\pi}{2} \leq |\omega| \leq \pi$$

(i) Find the value of $h(n)$ for all coefficients of length $N = 11$.

(ii) Find the transfer function of the filter, i.e., $H(z)$.

[12 + 8 = 20 marks]

- Q.2 (c) (i) Let at the memory location 4020 H, the instruction MOV B, A with opcode 47H is stored while the accumulator content is 05H. Draw the timing diagram showing the execution of this instruction in 8085 microprocessor.

[14 marks]

- Q.2 (c) (ii) Define addressing modes of microprocessor system. State and explain addressing mode supported by 8085 microprocessor.

[6 marks]

Q.3 (a) (i) Determine whether each of the following systems are linear, time invariant and static :

(a) $y(t) = x(\cos 3t)$

(b) $y(t) = (t^2 - 1)x(t)$

[6 + 6 = 12 marks]

Sol: (a) $y(t) = x(\cos 3t)$ $x_1(t) \rightarrow \boxed{\phantom{\text{System}}}$ $y_1(t) = x_1(\cos 3t)$

$x_2(t) \rightarrow \boxed{\text{System}} \rightarrow y_2(t) = x_2(\cos 3t)$
 $= x_1(\cos 3t) \rightarrow \boxed{\phantom{\text{System}}} \rightarrow y_2(t) = x_2(\cos 3t)$

Since given system follows ~~Additivity~~
 Additivity and ~~homogeneity~~ ^{doesn't} So

System is ~~Linear~~ Not Linear
~~not~~

for input $x(t-t_0) \rightarrow y'(t) = x(\cos 3t - t_0)$

for output $y(t-t_0) \rightarrow y''(t) = x(\cos 3(t-t_0))$

So $y'(t) \neq y''(t)$

So system is ~~ti~~ Not time invariant
 (or time variant)

• System is not static since
 output depends upon past & future
 value of inputs.

(b) $y(t) = (t^2 - 1)x(t)$

$x_1(t) \rightarrow \boxed{\phantom{\text{System}}} \rightarrow y_1(t) = (t^2 - 1)x_1(t)$

$x_2(t) \rightarrow \boxed{\text{System}} \rightarrow y_2(t) = (t^2 - 1)x_2(t)$

$x_1(t) + x_2(t) \rightarrow \boxed{\phantom{\text{System}}} \rightarrow y'(t) = y_1(t) + y_2(t)$

So given System is Linear.

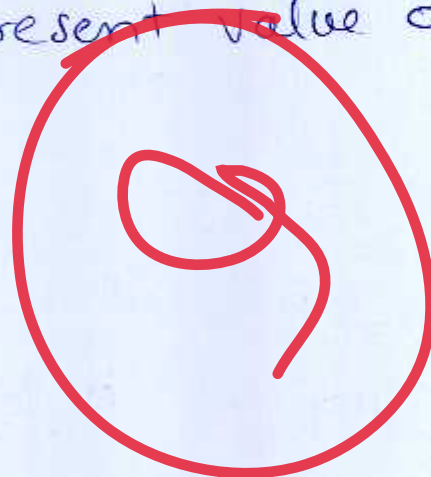
• $x(t-t_0) = x'(t) \xrightarrow{\text{System}} y'(t) = (t^2-1)(x(t-t_0))$

• $y(t-t_0) \longrightarrow y''(t) = ((t-t_0)^2-1)x(t-t_0)$

So $y'(t) \neq y''(t)$

So System is time variant.

• System is static since output depends upon present value of input only



- Q.3 (a) (ii) Compute $x_1(n) * x_2(n)$ using matrix approach, if $x_1(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$ and $x_2(n) = \delta(n) - \delta(n-2) + \delta(n-4)$. (Assume $N = 5$). (* represents circular convolution). [8 marks]

from the given definition of $x_1(n)$

$$x_1(n) = \{1, 1, -1, -1\}$$

and from the given

$$x_2(n) = \{1, 0, -1, 0, 1\}$$

by circular convolution

| | | | | |
|----------|----|----|----|----|
| $x_1(n)$ | 1 | 1 | -1 | -1 |
| $x_2(n)$ | 1 | 0 | -1 | 0 |
| | 1 | 1 | -1 | -1 |
| | 0 | 0 | 0 | 0 |
| | -1 | -1 | 1 | 1 |
| | 0 | 0 | 0 | 0 |
| | 1 | 1 | -1 | -1 |

$$\text{Let } x_3(n) = x_1(n) * x_2(n)$$

then

$$= \{1, 2, -1, -3, 1\}$$

$$\Rightarrow x_3(n) = \{1, 1, -2, -2, 2, 2, -1, -1\}$$

then

$$\begin{aligned} x_3(n) = & f(n) + f(n-1) - 2f(n-2) - 2f(n-3) \\ & + 2f(n-4) + 2f(n-5) \\ & - f(n-6) - f(n-7) \end{aligned}$$

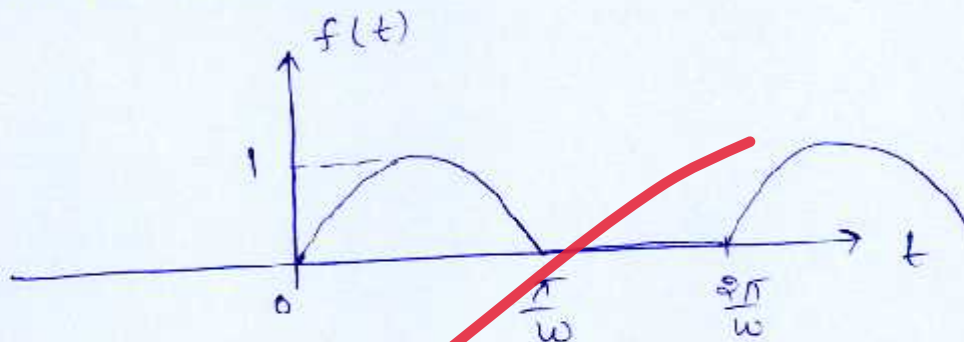
Q.3 (b) (i) Consider the continuous time signal

$$f(t) = \begin{cases} \sin \omega t; & 0 < t < \frac{\pi}{\omega} \\ 0; & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

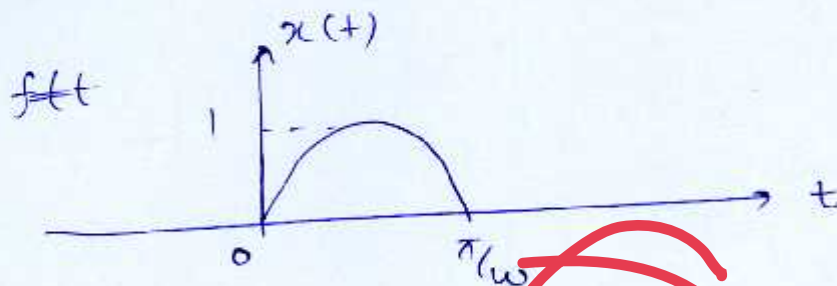
Also, $f\left(t + \frac{2\pi}{\omega}\right) = f(t)$, then find the Laplace transform of $f(t)$.

[12 marks]

from the definition of $f(t)$,



having time period $T = \frac{2\pi}{\omega}$



$$x(t) = \sin \omega t u(t) - \sin \omega \left(t - \frac{\pi}{\omega}\right) u\left(t - \frac{\pi}{\omega}\right)$$

Q.3 (b) (ii) Find $x(n]$ by using convolution property of z-transform for

$$X(z) = \frac{1}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

[8 marks]

by partial

$$X(z) = \frac{1}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$\Rightarrow X(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right) \left(1 + \frac{1}{4}z^{-1}\right)}$$

by partial fraction method

$$X(z) = \frac{A}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1 + \frac{1}{4}z^{-1}\right)}$$

$$A = \frac{2}{3} \quad \text{and} \quad B = \frac{1}{3}$$

$$X(z) = \frac{2/3}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{1/3}{\left(1 + \frac{1}{4}z^{-1}\right)}$$

by applying inverse ZT

$$x(n) = \frac{2}{3} \left(\frac{1}{2}\right)^n u(n) + \frac{1}{3} \left(-\frac{1}{4}\right)^n u(n)$$

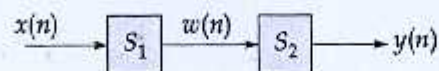
$$|z| > \frac{1}{2}$$

- Q.3 (c) (i) Calculate the time delay in the 8085 assembly language program given below. The system has a clock period of $0.5 \mu\text{s}$.

```
                MVI      B, 00H
NEXT   :        DCR      B
                MVI      C, 11H
DELAY  :        DCR      C
                JNZ      DELAY
                MOV      A, B
                OUT      PORT
                HLT
```

[10 marks]

Q.3 (c) (ii) Consider the cascade of the following two systems S_1 and S_2 as shown in figure.



where, System, S_1 : Causal LTI with the difference equation as below:

$$w(n) = \frac{1}{2}w(n-1) + x(n)$$

System, S_2 : Causal LTI with the difference equation as below:

$$y(n) = \alpha y(n-1) + \beta w(n)$$

If the difference equation relating $x(n)$ and $y(n)$ is

$$y(n) = \frac{-1}{8}y(n-2) + \frac{3}{4}y(n-1) + x(n)$$

Determine α and β .

[10 marks]



$$w(n) = \frac{1}{2}w(n-1) + x(n) \quad \dots (1)$$

$$y(n) = \alpha y(n-1) + \beta w(n) \quad \dots (2)$$

$$y(n) = -\frac{1}{8}y(n-2) + \frac{3}{4}y(n-1) + x(n) \quad \dots (3)$$

by applying Z-Transform on Eq. Difference Equation (1)

$$\Rightarrow W(z) = \frac{1}{2}z^{-1}W(z) + X(z)$$

$$\Rightarrow W(z) \left[1 - \frac{1}{2}z^{-1} \right] = X(z)$$

$$\Rightarrow \frac{W(z)}{X(z)} = \frac{1}{\left(1 - \frac{1}{2}z^{-1} \right)} \quad \dots (4)$$

by applying ZT on Eq. (2)

$$Y(z) [1 - \alpha z^{-1}] = \beta \times W(z)$$

$$\Rightarrow \frac{Y(z)}{W(z)} = \frac{\beta}{(1 - \alpha z^{-1})} \dots (5)$$

then

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{\beta}{(1 - \alpha z^{-1})} \left(\frac{1}{1 - \frac{1}{8} z^{-1}} \right) \dots (6)$$

from the Eq. (3), applying ZT

$$Y(z) \left[1 + \frac{1}{8} z^{-2} + \frac{3}{4} z^{-1} \right] = X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{\left(1 + \frac{1}{8} z^{-2} + \frac{3}{4} z^{-1} \right)}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{\left(1 - \frac{1}{4} z^{-1} \right) \left(1 - \frac{1}{8} z^{-1} \right)} \dots (7)$$

By comparing Eq (6) & (7)

$$\boxed{\alpha = \frac{1}{4}}$$

and

$$\boxed{\beta = 1}$$

Good Approach

- Q.4 (a) (i) Draw direct form-I and direct form-II block diagram for the given transfer function.

$$H(z) = \frac{z^2 - 2z + 4}{\left(z - \frac{1}{2}\right)(2z^2 + z + 1)}$$

- (ii) Draw the cascade-form block diagram for the given transfer function using minimum delay elements.

$$H(z) = \frac{z - 1}{(4z^3 + 2z^2 + 2z + 3)}$$

[12 + 8 marks]



- Q.4 (b) (i) Multiply the 8-bit unsigned number in memory location 4480H by the 8-bit unsigned number in memory location 4481H.

By shift-add routine method and store the 8 least significant bits of the result in memory location 5500H and 8 most significant bits in memory location 5501H. Write comments in selected instruction.

[14 marks]

- Q.4 (b) (ii) The following diagnostic routine can be used to troubleshoot the interfacing circuit of an input port :

| Instruction | Byte | T-states | Machine Cycle | | |
|---------------|------|-------------|----------------|----------------|----------------|
| | | | M ₁ | M ₂ | M ₃ |
| START : IN24H | 2 | 10(4, 3, 3) | | | |
| JMP START | 3 | 10(4, 3, 3) | | | |

1. Identify the machine cycles.
2. If the system clock is 6 MHz, calculate the time required to execute the routine.

[6 marks]

- Q4 (c) (i) Explain the mathematical function that is performed by the following instructions of 8085 processor and find the status of PSW at the end of the program.

LXI H, 2050H

MVI A, 22H

INR A

STA 2050H

INR A

XRA M

HLT

[14 marks]

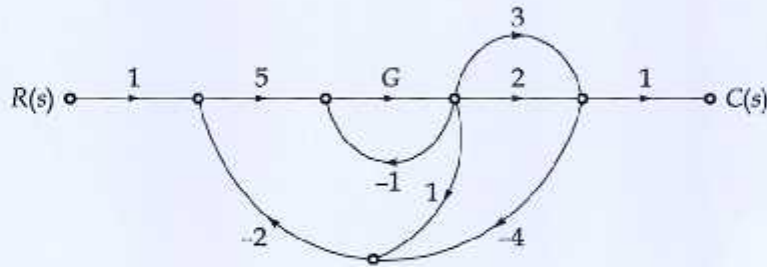
Q.4 (c) (ii) Explain the following in terms of direct memory access (DMA)

1. Cycle Stealing DMA
2. Interleaved DMA
3. Block Transfer DMA

[6 marks]

Section B : Electrical Circuits - 1 + Control Systems - 1

Q.5 (a) Consider the signal flow graph shown below :



Determine the value of gain G if the overall transfer function is given by $\frac{13}{17}$.

[12 marks]

Obtaining Transfer function using Mason's gain formula

$$TF = \frac{\sum_{k=1} P_k \Delta_k}{\Delta}$$

P_k : forward path gain of k^{th} forward path

Δ_k : value of Δ obtained by forming the loop not connected with k^{th} forward path

$$P_1 = 10G \quad \Delta_1 = 1 - 0 = 1$$

$$P_2 = 15G \quad \Delta_2 = 1 - 0 = 1$$

$$\Delta = 1 - \left\{ \text{Sum of individual Loop gain} \right\} + \left\{ \text{sum of product of two non-touching loop gain} \right\}$$

$$L_1 = -G \quad , \quad L_2 = -10G \quad , \quad L_3 = 80G$$

$$L_4 = 120 G$$

then overall Transfer function:

$$TF = \frac{10G + 10G}{1 + G + 10G - 80G - 120G}$$

given $TF = \frac{13}{17}$

then $\frac{13}{17} = \frac{25G}{1 + 11G - 200G}$

$$\Rightarrow \frac{13}{17} = \frac{25G}{1 + 11G - 200G}$$

$$\Rightarrow 13 - 2457G = 425G$$

$$\Rightarrow 13 = 2882G$$

\Rightarrow

$$G = \frac{13}{2882}$$

Good Approach

- Q.5 (b) Using the Routh criterion, check whether the system represented by the following characteristic equation is stable or not. Comment on the location of the roots. Determine the frequency of sustained oscillations if any.

$$s^4 + 2s^3 + 6s^2 + 8s + 8 = 0$$

[12 marks]

by forming the Routh Array
from the given Characteristics Equation

$$CE: s^4 + 2s^3 + 6s^2 + 8s + 8 = 0$$

| | | | |
|-------|---|---|---|
| s^4 | 1 | 6 | 8 |
| s^3 | 2 | 8 | 0 |
| s^2 | 2 | 8 | 0 |
| s^1 | 0 | 0 | 0 |
| s^0 | 8 | | |

s^1 Row is zero so coefficients of
 s^1 Row is formed by difference
polynomial of Auxiliary Equation

$$A(s) = 2s^2 + 8 = 0 \quad \dots (1)$$

$$\frac{dA(s)}{ds} = 4s$$

Since there is no sign change in first
column of Routh-Array so no poles
lies in RHS of s-plane.

Roots of Auxiliary Equation are the roots of characteristic Equation

So by solving Eq. (1)

$$s^2 + \omega = 0$$

$$\Rightarrow \boxed{s = \pm j\omega}$$

• frequency of sustained Oscillations

$$\boxed{\omega = \omega \text{ rad/sec}}$$

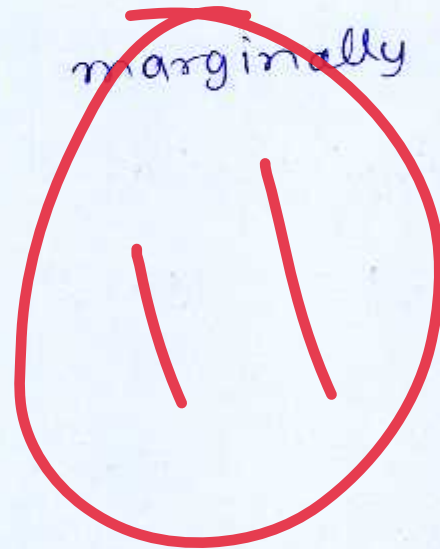
• No. of poles/roots lies on $j\omega$ Axis = ω

• No. of poles/roots lies in LHS of s plane = ω

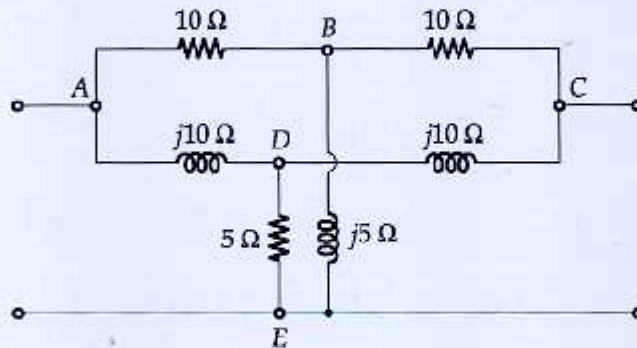
• No. of poles/roots lies in RHS = 0

So, given system is marginally stable.

Good Approach

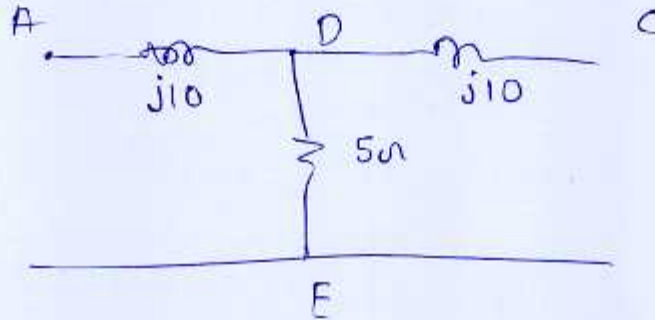


- Q.5 (c) The network shown in figure consists of two star connected circuits in parallel. Obtain the single delta connected equivalent.



[12 marks]

Sol: for

Converting γ to Δ

$$r'_{AE} = \frac{(j10)5 + (j10)5 + (j10)(j10)}{(j10)}$$

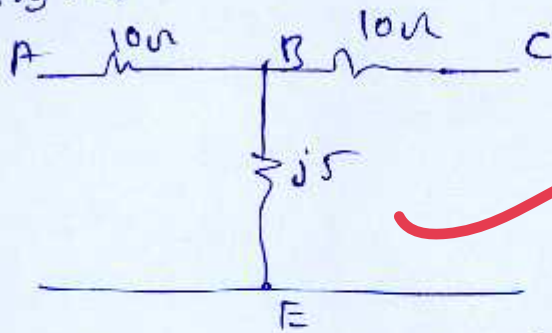
$$= \frac{50j + 50j - 100}{j10} = \frac{100j - 100}{j10}$$

$$r'_{CE} = \frac{100j - 100}{j10}$$

$$r'_{AC} = \frac{100j - 100}{5}$$

2

Similarly for

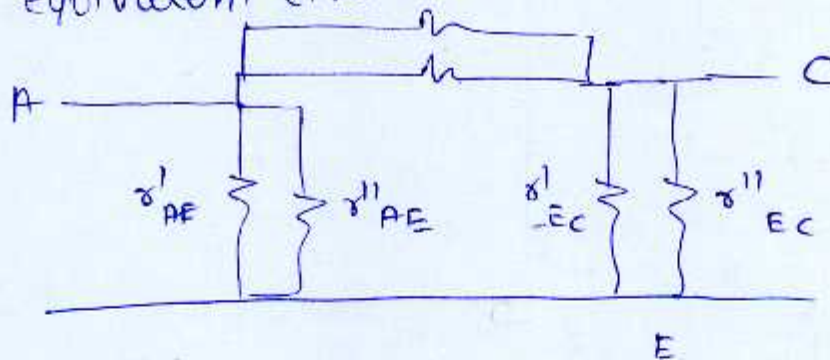


$$Z''_{AE} = \frac{100 + 50j + 50j}{10} = \frac{100 + 100j}{10}$$

$$Z''_{EC} = \frac{100 + 100j}{10}$$

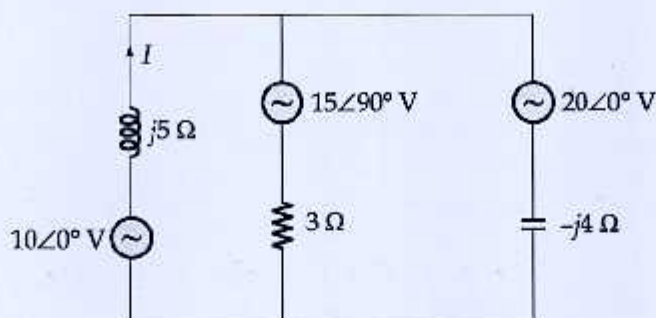
$$Z''_{AC} = \frac{100 + 100j}{j5}$$

the equivalent circ



Incomplete solution

- Q.5 (d) Find current I through $j5 \Omega$ branch using superposition theorem for the network shown in figure.

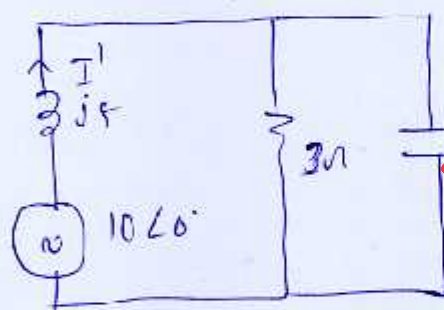


[12 marks]

by Using Superposition Theorem

Step 1: $10\angle 0^\circ$ V source is in circuit and all other voltage source gets disabled (Short circuited)

then circuit is drawn as:

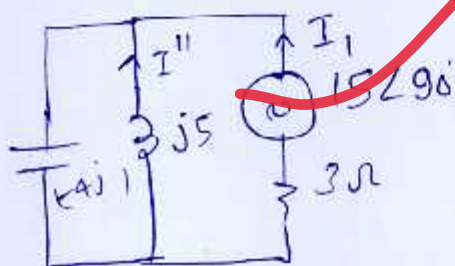


Let, Current through $j5\Omega$ branch is I'

$$\text{then } I' = \frac{10}{j5 + 1.92 - 1.44j}$$

$$\Rightarrow I' = 2.4793 \angle -61.66^\circ \text{ A} \quad (1)$$

Step (2) $15\angle 90^\circ$ V source is in circuit & all other voltage source are short circuit by KVL



$$I_1 = \frac{15\angle 90^\circ}{3 - 20j}$$

$$I_1 = 0.7417 \angle 171.469^\circ$$

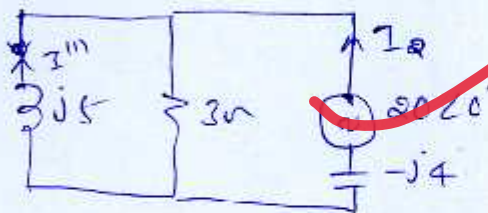
by current division

$$I'' = \frac{-(-4j)}{(5j-4j)} I$$

$$\Rightarrow I'' = \frac{4j}{j} I$$

$$\Rightarrow I'' = 2.9668 \angle 171.47^\circ \text{ A} \dots (1)$$

Step(2) $20 \angle 0^\circ \text{ V}$ Source is in circuit and other sources are short circuited then



$$I_2 = \frac{20}{-j4 + 2 \cdot 3 + 1 \cdot j5}$$

$$I_2 = 5.7664 \angle 50.50^\circ \text{ A}$$

by current division

$$I''' = -I_2 \left(\frac{3}{3+j5} \right)$$

$$\Rightarrow I''' = 2.9668 \angle 171.47^\circ \text{ A} \dots (2)$$

then Current I through $j5 \Omega$ branch, from Eq (1), (2)

$$\Rightarrow I = I' + I'' + I'''$$

$$\Rightarrow \boxed{I = 4.87 \angle -164.568^\circ \text{ A}}$$

Q.5 (e) The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(1+sT)}$$

- (i) By what factor the amplifier gain K should be multiplied so that the damping ratio is increased from 0.2 to 0.8?
- (ii) By what factor the time constant T should be multiplied so that the damping ratio is reduced from 0.9 to 0.3?

[12 marks]

Given open loop Transfer function

$$G(s) = \frac{K}{s(1+sT)}$$

Close Loop Transfer function

$$H(s) = 1$$

$$TF = \frac{G(s)}{1 + G(s)H(s)}$$

$$\Rightarrow TF = \frac{\frac{K}{s(1+sT)}}{1 + \frac{K}{s(1+sT)}}$$

$$\Rightarrow TF = \frac{K}{s^2 T + s + K}$$

$$\Rightarrow TF = \frac{K}{T \left(s^2 + \frac{s}{T} + \frac{K}{T} \right)}$$

by comparing by with standard Transfer function

$$TF = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{\frac{K}{T}} \quad \text{and} \quad 2\zeta\omega_n = \frac{1}{T}$$

$$\Rightarrow 2\pi\omega \sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$\Rightarrow \cancel{2\pi\omega} = \frac{\sqrt{T}}{\sqrt{K}} \quad \Rightarrow T = \frac{1}{\sqrt{KT}} \quad \dots (2)$$

(i) increasing T from 0.2 to 0.8

then

$$\frac{T_1}{T_2} = \sqrt{\frac{K_2}{K_1}}$$

$$\Rightarrow \frac{0.2}{0.8} = \sqrt{\frac{K_2}{K_1}}$$

$$\Rightarrow K_2 = \frac{1}{16} K_1$$

Amplifier gain K should be multiplied by factor $\frac{1}{16}$

(ii) reducing T from 0.9 to 0.3

the from Eq. (2)

$$\frac{T_1}{T_2} = \sqrt{\frac{T_2}{T_1}}$$

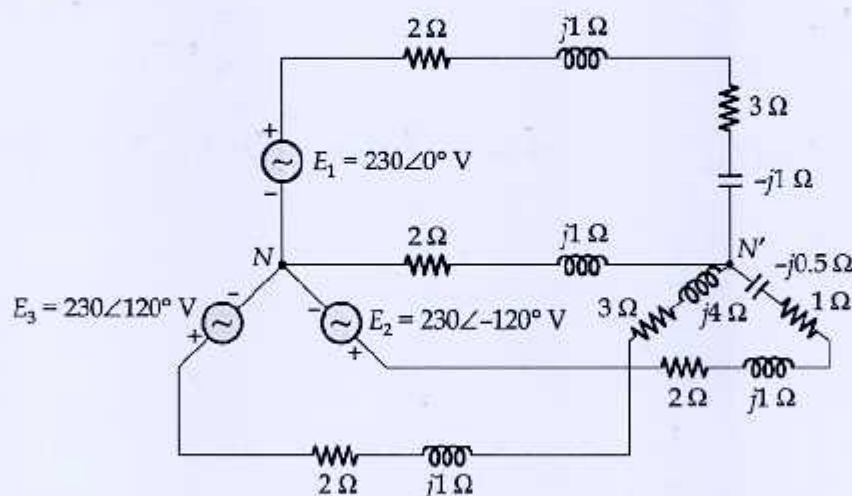
$$\Rightarrow \frac{0.9}{0.3} = \sqrt{\frac{T_2}{T_1}}$$

$$\Rightarrow \boxed{T_2 = 9 T_1}$$

Time constant T should be multiplied by 9

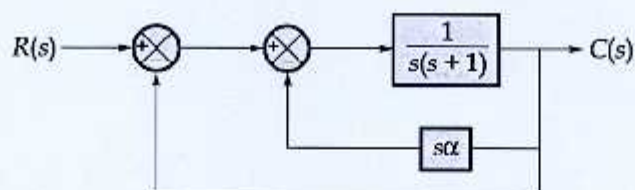
Good Approach

- Q.6 (a) The network shown in figure represents a three phase four wire electrical power system. Use Millman's theorem to determine the potential difference between the two neutral points N and N' .



[20 marks]

Q.6 (b) A control system is shown in the block diagram given below :



Sketch the root locus as the value of the parameter α is varied from 0 to ∞ . Determine the value of α for the transient response to have critical damping.

[20 marks]

Q.6 (c) A unity feedback control system has an open-loop transfer function

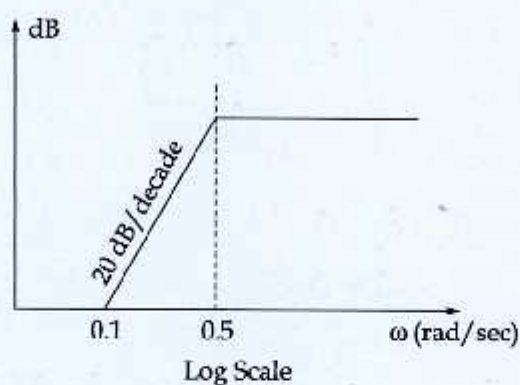
$$G(s) = \frac{5}{s(s+1)}$$

Find the rise time, percentage overshoot, peak time and settling time for a step input of 10 units. Also, determine the peak overshoot.

[20 marks]



- Q.7 (a) (i) The approximate Bode magnitude plot of a lead network with its pole and zero on the left half of the s -plane is shown in the following figure :



Find the frequency at which the phase angle of the network is maximum (in rad/sec).

[10 marks]

Sol: from the given bode - plot :

(i) Initial Slope : 0 dB/decade

(ii) at $\omega = \omega_z = 0.1$ rad/sec, slope changes from 0 dB/decade to 20 dB/decade

So there is a ~~zero~~ zero.

(iii) at $\omega = \omega_p = 0.5$ rad/sec, slope changes from 20 dB/decade to 0 dB/decade

So there is a pole.

Now Transfer function is given as

$$G(s) = \frac{K \left(1 + \frac{s}{0.1}\right)}{\left(1 + \frac{s}{0.5}\right)}$$

To determine K ,

$$0 = 20 \log_{10} K$$

$$\Rightarrow \boxed{K = 10}$$

Now Transfer function is

$$G(s) = \frac{\left(1 + \frac{s}{0.1}\right)}{\left(1 + \frac{s}{0.5}\right)}$$

$$\Rightarrow G(s) = \frac{5(s+0.1)}{(s+0.5)}$$

Comparing with $G(s) = \frac{k(s+a)}{(s+b)}$

$$a = 0.1 \quad \text{and} \quad b = 0.5$$

then frequency at which phase angle is maximum is given by

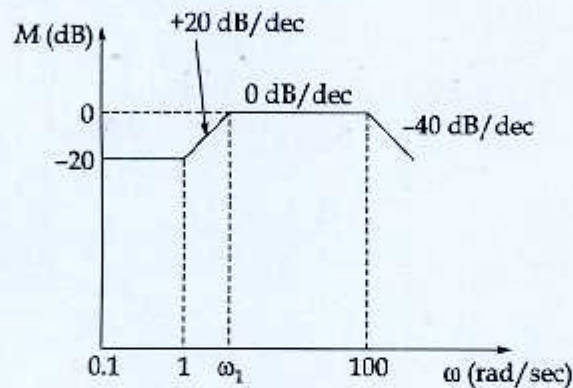
$$\Rightarrow \omega = \sqrt{ab}$$

$$\Rightarrow \omega = \sqrt{0.1(0.5)}$$

$$\Rightarrow \omega = 0.2236 \text{ rad/sec}$$

9

- Q.7 (a) (ii) Obtain the open loop transfer function for a system with unity feedback whose bode plot is shown below :



[10 marks]

Sol: obtaining ω_1 from the given plot

$$20 = \frac{0 - (-20)}{\log \omega_1}$$

$$\Rightarrow \boxed{\omega_1 = 10 \text{ rad/sec}}$$

Now from the plot

- (i) Initial slope is 0 dB/decade
- (ii) at $\omega = 1 \text{ rad/sec}$ slope change from 0 dB/decade to +20 dB/decade so there is a zero.
- (iii) at $\omega = 10 \text{ rad/sec}$ slope change from 20 dB/decade to 0 dB/decade so there is a pole
- (iv) at $\omega = 100 \text{ rad/sec}$ slope change from 0 dB/decade to -40 dB/decade so there is a pole of second order

the transfer function is

$$G(s)H(s) = \frac{K \left(1 + \frac{s}{1}\right)}{\left(1 + \frac{s}{10}\right) \left(1 + \frac{s}{100}\right)^2} \dots (1)$$

finding K:

$$-20 = 20 \log_{10} K$$

$$\Rightarrow \log_{10} K = -1$$

$$\Rightarrow \boxed{K = \frac{1}{10}}$$

Now open loop Transfer function for system is as follows:

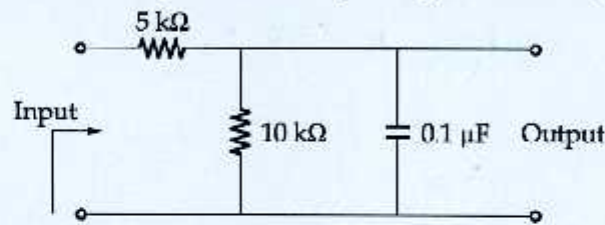
$$G(s)H(s) = \frac{0.1 (s+1)}{\left(1 + \frac{s}{10}\right) \left(1 + \frac{s}{100}\right)^2}$$

$$\Rightarrow G(s)H(s) = \frac{0.1 (s+1)}{\frac{(s+10)}{10} \left(\frac{s+100}{100}\right)^2}$$

$$\Rightarrow G(s)H(s) = \frac{10^4 (s+1)}{(s+10) (s+100)^2}$$

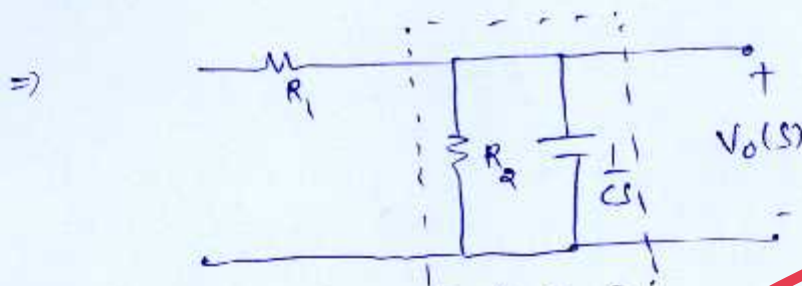
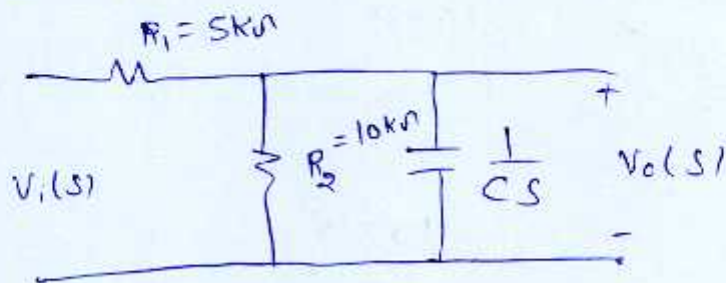
Good Approach

Q.7 (b) (i) Draw the asymptotic magnitude and phase plot on the system shown below :



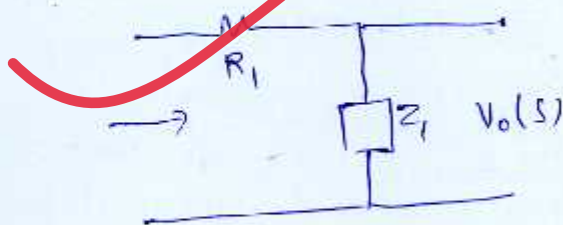
[10 marks]

in S -domain circuit is drawn as



A parallel connected branch

$$\text{So } Z_1 = \frac{R_2 \cdot \frac{1}{CS}}{R_2 + \frac{1}{CS}} = \frac{R_2}{R_2 CS + 1}$$



by voltage division

$$V_o(s) = \frac{Z_1}{Z_1 + R_1} V_i(s)$$

$$\text{Transfer function } \frac{V_o(s)}{V_i(s)} = \frac{\frac{R_2}{R_2 CS + 1}}{\frac{R_2}{R_2 CS + 1} + R_1}$$

$$TF = \frac{R_2}{R_2 + R_1(1 + R_2 C S)}$$

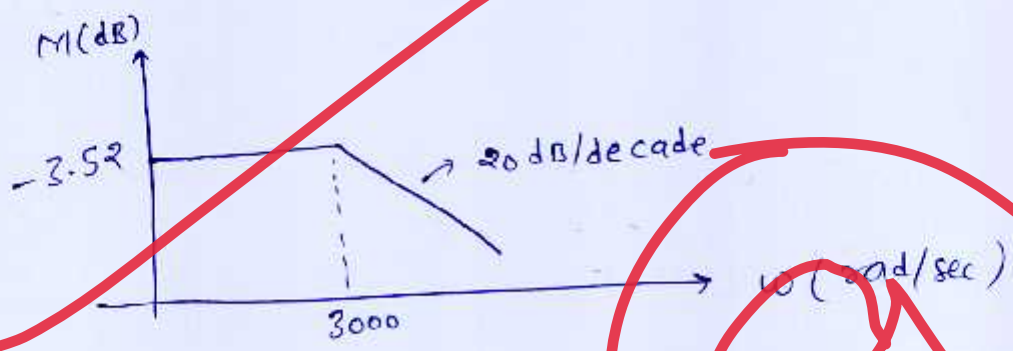
$$\Rightarrow TF = \frac{R_2}{R_1 + R_2 + R_1 R_2 C S}$$

$$\Rightarrow TF = \frac{10 \times 10^3}{15 \times 10^3 + 50 \times 10^6 \times 0.1 \times 10^{-6} S}$$

$$\Rightarrow TF = \frac{10 \times 10^3}{15 \times 10^3 + 5 S}$$

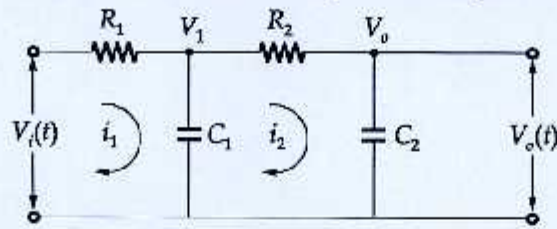
$$\Rightarrow TF = \frac{s^{2/3}}{1 + \frac{5}{15 \times 10^3} S}$$

$$\Rightarrow TF = \frac{s^{2/3}}{1 + \frac{S}{3000}}$$



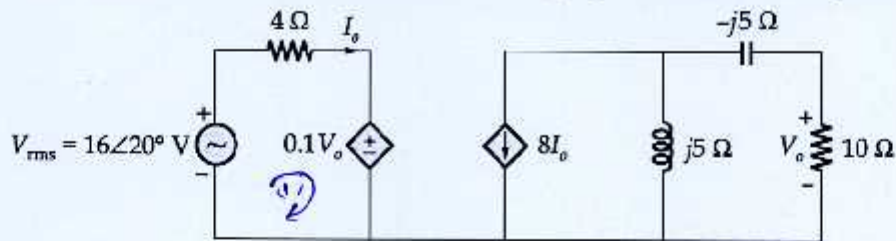
Draw diagram properly

Q.7 (b) (ii) Draw the block diagram for the circuit given in figure below :



[10 marks]

- Q.7 (c) (i) For the circuit shown below, find the average power absorbed by the $10\ \Omega$ resistor



[10 marks]

by applying KVL in Loop (1)

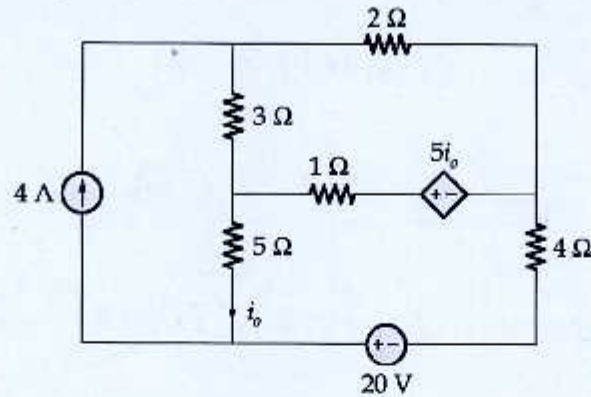
$$4I_0 + 0.1V_0 = 16\angle 20^\circ$$

$$\Rightarrow 4I_0 + 0.1V_0 = 16\angle 20^\circ \quad \dots (1)$$

by Current division

Incomplete solution

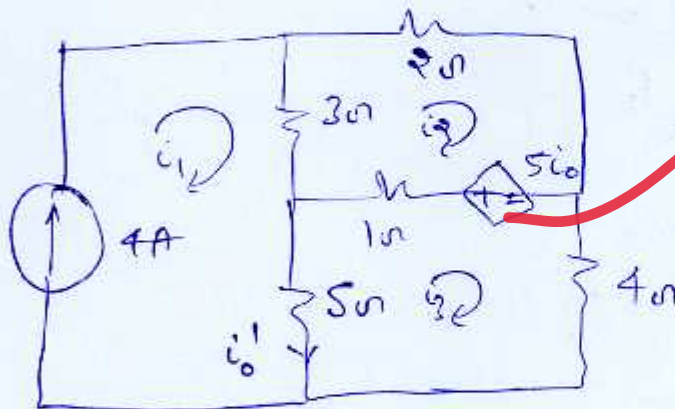
Q.7 (c) (ii) Find the current i_o using super position theorem in the circuit shown below :



[10 marks]

by using Superposition

Step: 1 4A is in circuit and 20V source gets short circuited then



$$i_1 = 4A$$

$$\text{and } i_o' = i_1 - i_3$$

by KVL $6i_2 - 3i_1 - i_3 = 5i_o'$

$$\Rightarrow 6i_2 - 3i_1 - i_3 = 5(i_1 - i_3)$$

$$\Rightarrow 6i_2 - 8i_1 + 4i_3 = 0 \quad \dots (1)$$

$$\Rightarrow 6i_2 + 4i_3 = 8i_1 \quad \dots (2)$$

and $10i_3 - i_2 - 5i_1 + 5i_o' = 0$

$$\Rightarrow 10i_3 - i_2 - 5i_1 + 5i_1 - 5i_3 = 0$$

$$\Rightarrow 5i_3 = i_2 \quad \dots (3)$$

by Eq (1) & (3)

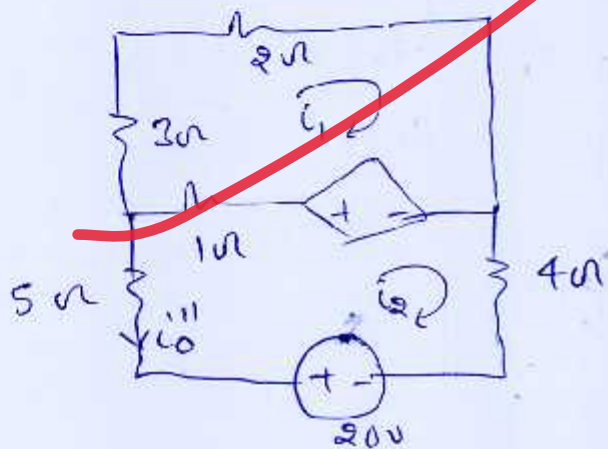
$$34 i_3 = 37$$

$$\Rightarrow i_3 = 0.94117 \text{ A}$$

$$i_0' = i_1 - i_3 = 3.0588 \text{ A}$$

Step 2

Now 20V, source is in circuit and
4 A source gets open circuited.



$$i_0'' = -i_3$$

$$6i_1 - i_2 - 5i_0'' = 0$$

$$\Rightarrow 6i_1 - i_2 - 5(-i_2) = 0$$

$$\Rightarrow 6i_1 + 4i_2 = 0 \quad (3)$$

$$10i_2 - i_1 - 20 + 5i_0'' = 0$$

$$\Rightarrow 10i_2 - i_1 - 5i_2 = 20$$

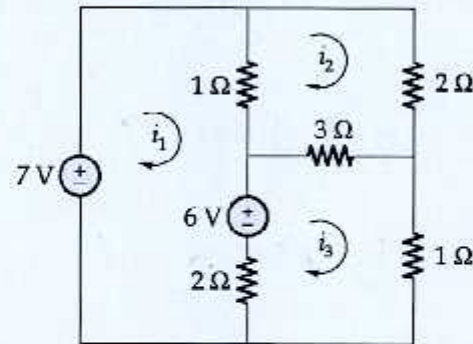
$$\Rightarrow 5i_2 - i_1 = 20 \quad (4)$$

$$\Rightarrow i_2 = \frac{60}{17} \text{ A}$$

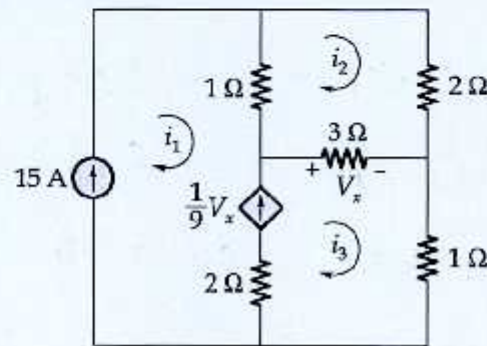
$$\Rightarrow i_0'' = -\frac{60}{17} \text{ A} = -3.53 \text{ A}$$

then i_0 by superposition = $i_0' + i_0''$
 $= -0.4719 \text{ A}$

Q.8 (a) (i) Use mesh analysis to determine mesh currents in the circuit



(ii) Use mesh analysis to determine mesh currents in the circuit



[10 + 10 = 20 marks]

(i) by applying KVL in Loop (1)

$$(i_1 - i_2) + 6 + 2(i_1 - i_2) = 7$$

$$\Rightarrow 3i_1 - i_2 - 2i_3 = 1 \dots (1)$$

by KVL in Loop (2)

$$2i_1 + 3(i_2 - i_3) + i_2 - i_3 = 0$$

$$\Rightarrow 7i_2 - i_1 - 3i_3 = 0 \dots (2)$$

by KVL in Loop (3)

$$3(i_2 - i_1) + i_3 + 2(i_3 - i_2) = 6$$

$$\Rightarrow 6i_3 - 2i_1 - 3i_2 = 6 \dots (3)$$

by solving Eq. (1) (2) and (3)
we get,

$$i_1 = 2.547 \text{ A}$$

$$i_2 = 1.4716 \text{ A}$$

$$i_3 = 2.585 \text{ A}$$

(b) ~~by applying KVL in Loop (1)~~

~~Since Loop (1) & (3) form supermesh~~

~~So writing KVL equation in
Combine form~~

$$\cancel{(i_1 - i_2) + 3(i_3 - i_2)}$$

KVL in Loop (2)

$$6i_2 - 3i_3 - i_1 = 0 \quad \dots (1)$$

$$\Rightarrow 6i_2 - 3i_3 = 15 \quad \dots (2)$$

$$i_1 = 15 \text{ A} \quad (\text{given})$$

and ~~$i_1 - i_3$~~ $i_3 - i_1 = \frac{1}{9} V_x \quad \dots (3)$

and $V_x = 3(i_3 - i_2) \quad \dots (3)$

Substitute in Eq. (3)

$$i_3 - i_1 = \frac{1}{9} 3(i_3 - i_2)$$

$$\Rightarrow i_3 - i_1 = \frac{1}{3} (i_2 - i_1)$$

$$\Rightarrow 3i_3 - 3i_1 = i_2 - i_1$$

$$\Rightarrow 2i_2 + i_2 = 3i_1 \quad \therefore i_1 = 15 \text{ A}$$

$$\Rightarrow 2i_2 + i_2 = 45 \dots (3)$$

by solving eq (2) & (3)
mesh currents are,

$$i_1 = 15 \text{ A}$$

$$i_2 = 11 \text{ A}$$

$$i_3 = 17 \text{ A}$$

9

Q.8 (b) (i) The open-loop transfer function of a unity negative feedback system is given by

$$G(s) = \frac{K(s+1)^2}{(s+2)^2}$$

Without drawing root locus diagram, prove that the root locus (for $K > 0$) of the system lies on a circle.

(ii) The response of a feedback system to a unit step input is

$$C(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$$

(a) Obtain the expression for the closed loop transfer function.

(b) Determine the undamped natural frequency and damping ratio of the system.

[10 + 10 = 20 marks]

Q.9 (i) given that open loop transfer function

$$G(s) = \frac{K(s+1)^2}{(s+2)^2}$$

Let $(\sigma + j\omega)$ point lies on the root locus then phase condition satisfied, so

$$\angle G(s) \Big|_{s=\sigma+j\omega} = 180^\circ$$

$$\angle G(\sigma + j\omega) = 2 \tan^{-1} \left(\frac{\omega}{\sigma+1} \right)$$

$$- 2 \tan^{-1} \left(\frac{\omega}{\sigma+2} \right)$$

$$\angle G(\sigma + j\omega) = 2 \tan^{-1} \left(\frac{\omega}{\sigma+1} \right) - 2 \tan^{-1} \left(\frac{\omega}{\sigma+2} \right)$$

the by using phase condition

$$2 \tan^{-1} \left(\frac{\omega}{\sigma+1} \right) - 2 \tan^{-1} \left(\frac{\omega}{\sigma+2} \right) = 180^\circ$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{\omega}{\sigma+1} - \frac{\omega}{\sigma+2}}{1 + \frac{\omega^2}{(\sigma+1)(\sigma+2)}} \right] = 90^\circ$$

$$\text{So } 1 + \frac{\omega^2}{(s+1)(s+2)} = 0$$

$$\Rightarrow s^2 + 3s + 2 + \omega^2 = 0$$

$$\Rightarrow (s + 1.5)^2 + (\omega - 0)^2 = 0.25$$

$$\Rightarrow (s + 1.5)^2 + (\omega - 0)^2 = (0.5)^2$$

So It is equation of circle having center at $(-1.5, 0)$ and radius = 0.5.

So Root Locus is a circle for $K > 0$

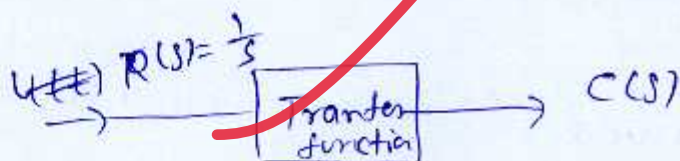
(ii) given response of the system

$$c(t) = 1 + 0.2 e^{-60t} - 1.2 e^{-10t}$$

by applying Laplace transform

$$C(s) = \frac{1}{s} + \frac{0.2}{(s+60)} - \frac{1.2}{(s+10)}$$

$$\Rightarrow C(s) = \frac{(s+60)(s+10) + 0.2s(s+10) - 1.2s(s+60)}{s(s+60)(s+10)}$$



(9)

Transfer function is = $\frac{C(s)}{R(s)}$

$$\Rightarrow \text{TF} = \frac{(s+60)(s+10) + 0.2s(s+10) - 1.2s(s+60)}{(s+60)(s+10)}$$

$$TF = \frac{s^2 + 70s + 600 + 0.2s^2 + 2s - 1.2s^2 - 72s}{(s+60)(s+10)}$$

$$TF = \frac{600}{s^2 + 70s + 600}$$

Comparing with standard 2nd order Transfer function

$$TF = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Now, $\omega_n^2 = 600$

$$\Rightarrow \boxed{\omega_n = \sqrt{600} = 24.4948 \text{ rad/sec}}$$

and $2\zeta\omega_n = 70$

$$\Rightarrow \zeta = \frac{70}{2\sqrt{600}}$$

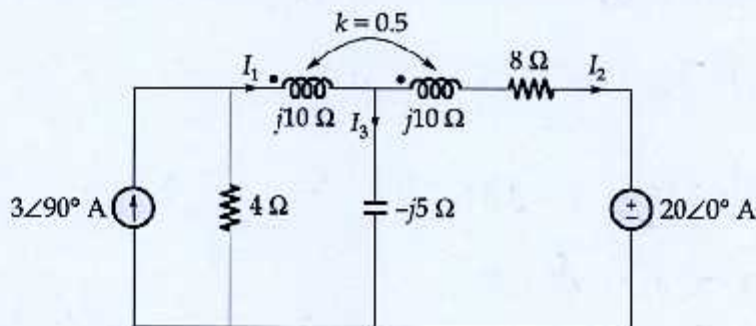
$$\Rightarrow \boxed{\zeta = 1.4288}$$

So Undamped Natural frequency = 24.4948 rad/sec

Damping ratio = 1.4288

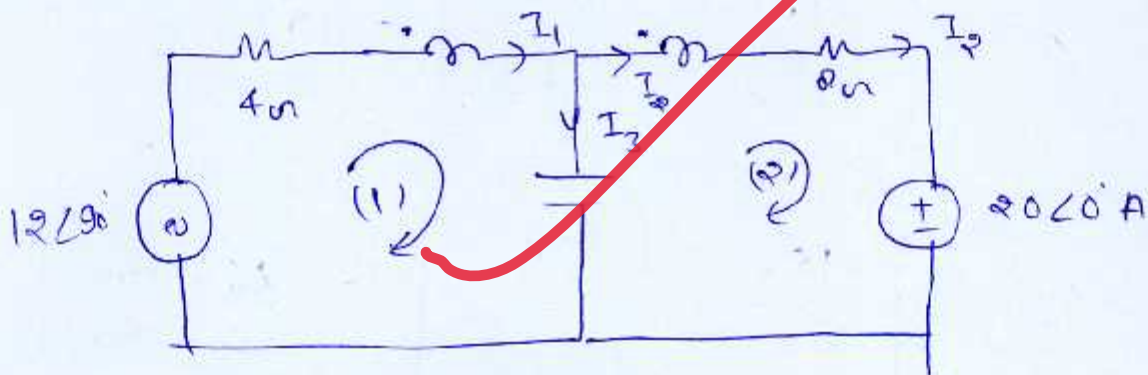
Good Approach

Q.8 (c) Determine the current I_1 , I_2 and I_3 in the circuit shown. Take $\omega = 1000$ rad/sec.



[20 marks]

by Source Transformation
Current Source to voltage source



$$\text{then, } X_M = K \sqrt{X_{L1} X_{L2}}$$

$$\Rightarrow X_M = 0.5 \sqrt{j10 j10}$$

$$\text{Now, } X_M = 5j$$

$$\text{by KCL } I_1 = I_2 + I_3$$

$$\Rightarrow I_2 = I_1 - I_3 \Rightarrow I_3 = I_1 - I_2$$

by KVL in Loop (1)

$$(4 + j10)I_1 - j5(I_1 - I_2) + 5jI_2 = 12\angle 90^\circ$$

$$\Rightarrow (4 + j5)I_1 + 10jI_2 = 12j \dots (1)$$

by KVL in Loop (2)

$$(8+j10) I_2 + 20 + j5 I_1 = -j5(I_1 - I_2)$$

$$\Rightarrow (8+j5) I_2 + j10 I_1 = -20 \quad \dots (2)$$

then in matrix form

$$\Rightarrow \begin{bmatrix} 4+j5 & 10j \\ j10 & (8+5j) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12j \\ -20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 4+j5 & 10j \\ j10 & 8+5j \end{vmatrix} = 107 + 60j$$

$$\Delta_1 = \begin{vmatrix} 12j & 10j \\ -20 & 8+5j \end{vmatrix} = \cancel{96+260j} \\ -60 + 296j$$

$$\Delta_2 = \begin{vmatrix} 4+j5 & 12j \\ j10 & -20 \end{vmatrix} = 40 - 100j$$

Now

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-60 + 296j}{60j + 107} = 2.462 \angle 72.177^\circ \text{ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{40 - 100j}{107 + 60j} = 0.878 \angle -97.48^\circ \text{ A}$$

$$\text{and } I_3 = 3.33 \angle 74.89^\circ \text{ A}$$

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Good Approach

Space for Rough Work

Space for Rough Work
