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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Systems and Signal Processing + Microprocessors [All topics]

Electrical Circuits-1 + Control Systems-1 [Part Syllabus]

Name :

Roll No :

Test Centres	Student's Signature
Delhi <input type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input checked="" type="checkbox"/>	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
Total Marks Obtained	200

Signature of Evaluator

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

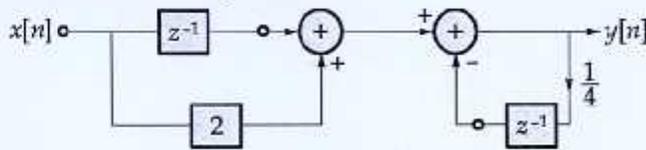
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Systems and Signal Processing + Microprocessors

Q.1 (a) Consider the system shown in figure below :



Determine the impulse response of the system?

[12 marks]

Sol.

By using block diagram reduction technique.

$$Y(z) = (z^{-1} + 2) \left(\frac{1}{1 + \frac{z^{-1}}{4}} \right) X(z)$$

$$H(z) = (z^{-1} + 2) \left(\frac{1}{1 + \frac{z^{-1}}{4}} \right)$$

$$= \frac{z + z^{-1}}{1 + \frac{z^{-1}}{4}} = \frac{z}{1 + \frac{z^{-1}}{4}} + \frac{z^{-1}}{1 + \frac{z^{-1}}{4}}$$

$$H(z) = \frac{z}{1 + (-\frac{1}{4})z^{-1}} + \frac{z^{-1}}{1 + (-\frac{1}{4})z^{-1}}$$

$$\therefore z^{-1}X(z) \Rightarrow X(n-1)$$

Taking Inverse ~~Laplace~~ Z-Transform

$$h(n) = 2 \cdot \left(-\frac{1}{4}\right)^n u(n) + \left(-\frac{1}{4}\right)^{n-1} u(n-1)$$

$$= 2 \left(-\frac{1}{4}\right)^n u(n) + \left(-\frac{1}{4}\right)^n \cdot \left(-\frac{1}{4}\right)^{-1} u(n-1)$$

Impulse response

$$h(n) = \left(-\frac{1}{4}\right)^n [2u(n) - 4u(n-1)]$$

Q.1 (b) Find $x(n]$ by using convolution for

$$X(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

[12 marks]

Sol.

$$X(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

$$= \frac{2/3}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{1/3}{\left(1 + \frac{1}{4}z^{-1}\right)}$$

$$X(z) = \frac{1}{3} \left[\frac{2}{\left(1 - \frac{z^{-1}}{2}\right)} + \frac{1}{\left(1 + \frac{1}{4}z^{-1}\right)} \right]$$

Taking inverse Z -transformation.

$$x(n) = \frac{1}{3} \left[2\left(\frac{1}{2}\right)^n u(n) + \left(\frac{-1}{4}\right)^n u(n) \right]$$

$$x(n) = \left[\frac{2}{3} \left(\frac{1}{2}\right)^n + \left(\frac{-1}{4}\right)^n \right] u(n)$$

Write all steps

Q.1 (c) Find the Inverse DFT of the sequence $Y(K) = \{1, 0, 1, 0\}$.

[12 marks]



Q.1 (d) Consider an analog filter whose transfer function is :

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Find $H(z)$ by using bilinear transformation. (Assume $T = 1$ sec).

[12 marks]

Sol.

Analog filter

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Sampling time $T = 1$ sec

In bilinear transformation $H(s)$ can be converted to $H(z)$ by replacing s with $\frac{2}{T_s} \left(\frac{1-z^{-1}}{1+z^{-1}} \right)$

$$H(z) = H(s) \Big|_{s = \frac{2}{1} \left(\frac{1-z^{-1}}{1+z^{-1}} \right)}$$

$$H(z) = \frac{2}{\left[2 \left(\frac{1-z^{-1}}{1+z^{-1}} \right) + 1 \right] \times \left(\frac{1-z^{-1}}{1+z^{-1}} + 2 \right)}$$

$$H(z) = \frac{2}{\frac{[2(1-z^{-1}) + (1+z^{-1})]}{(1+z^{-1})} \times \frac{[(1-z^{-1}) + 2(1+z^{-1})]}{(1+z^{-1})}}$$

$$H(z) = \frac{2(1+z^{-1})^2}{2 - 2z^{-1} + 1 + z^{-1} + 1 - z^{-1} + 2 + 2z^{-1}}$$

$$H(z) = \frac{2(1+z^{-1})^2}{9}$$

$$H(z) = \frac{1+z^{-1}+2z^{-1}}{2}$$

$$H(z) = \frac{1}{2} + z^{-1} + \frac{1}{2}z^{-2}$$

- Q.1 (e) Write a program in 8085 microprocessor to find the smallest of the two numbers stored at memory location 3025H and 3026H and store the result in the memory location 3027H.

3025H	84H
3026H	99H

[12 marks]

- Q.2 (a) The one-sided exponential pulse (i.e., $v(t) = 0$ for $t < 0$), $v(t) = 4e^{-3t}u(t)$ V is applied to the input of an ideal bandpass filter. If the filter passband is defined by $1 < |f| < 2$ Hz, calculate the percentage of output energy w.r.t. to input energy.

[20 marks]

Q.2 (b) Consider an ideal low pass filter with frequency response,

$$\begin{aligned} H_d(e^{j\omega}) &= 1 && \text{for } \frac{-\pi}{2} \leq \omega \leq \frac{\pi}{2} \\ &= 0 && \text{for } \frac{\pi}{2} \leq |\omega| \leq \pi \end{aligned}$$

(i) Find the value of $h(n)$ for all coefficients of length $N = 11$.

(ii) Find the transfer function of the filter, i.e., $H(z)$.

[12 + 8 = 20 marks]

- Q.2 (c) (i) Let at the memory location 4020 H, the instruction MOV B, A with opcode 47H is stored while the accumulator content is 05H. Draw the timing diagram showing the execution of this instruction in 8085 microprocessor.

[14 marks]

Q.2 (c) (ii) Define addressing modes of microprocessor system. State and explain addressing mode supported by 8085 microprocessor.

[6 marks]

Q.3 (a) (i) Determine whether each of the following systems are linear, time invariant and static

(a) $y(t) = x(\cos 3t)$

(b) $y(t) = (t^2 - 1)x(t)$

[6 + 6 = 12 marks]

Sol.

(i) (a) $y(t) = x(\cos 3t)$

Linearity

for input $x_1(t) \rightarrow y_1(t) = x_1(\cos 3t)$

for input $x_2(t) \rightarrow y_2(t) = x_2(\cos 3t)$

for input $x_1(t) + x_2(t) \rightarrow y'(t) = x_1(\cos 3t) + x_2(\cos 3t)$

$y'(t) = y_1(t) + y_2(t)$

therefore $y(t) = x(\cos 3t)$ is linear.

• Time variance:-

• Providing a delay of ' t_0 ' in output $y(t)$

$$y(t-t_0) = x[\cos(3(t-t_0))] \\ = x[\cos(3t-3t_0)]$$

• Providing a delayed input $x(t-t_0)$

$$y'(t) = x[\cos 3(t-t_0)]$$

$$y'(t) = y(t-t_0)$$

therefore the system $y(t) = x[\cos 3t]$ is time invariant

• $y(t) = x[\cos 3t]$

$$\text{At } t=0 \quad y(0) = x[1]$$

System is not static

Therefore system $y(t) = x[\cos t]$ is Linear, Time invariant and dynamic system.

$$\textcircled{b} \quad y(t) = (t^2 - 1)x(t)$$

$$\text{for input } x_1(t) \rightarrow y_1(t) = (t^2 - 1)x_1(t)$$

$$x_2(t) \rightarrow y_2(t) = (t^2 - 1)x_2(t)$$

$$x_1(t) + x_2(t) \rightarrow y'(t) = (t^2 - 1)[x_1(t) + x_2(t)]$$

$$y'(t) = y_1(t) + y_2(t) \quad \therefore \text{System is Linear.}$$

• Delaying output by t_0 units.

$$y(t - t_0) = [(t - t_0)^2 - 1]x(t - t_0)$$

Delaying input by t_0 units.

$$y'(t) = (t^2 - 1)x(t - t_0)$$

$$y'(t) \neq y(t - t_0) \quad \text{System is time variant.}$$

• At $t=0 \quad y(0) = -x(0)$

$$t=1 \quad y(1) = 0 \cdot x(1)$$

$$t=2 \quad y(2) = 3 \cdot x(2)$$

System is static

System $y(t) = (t^2 - 1)x(t)$ is linear, time variant and static system.

Q.3 (a) (ii) Compute $x_1(n) * x_2(n)$ using matrix approach, if $x_1(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$ and $x_2(n) = \delta(n) - \delta(n-2) + \delta(n-4)$. (Assume $N = 5$). (* represents circular convolution).

[8 marks]

Sol. (ii)

$$x_1(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$$

$$x_1(n) = \{1, 1, -1, -1, 0\}$$

$$x_2(n) = \delta(n) - \delta(n-2) + \delta(n-4)$$

$$x_2(n) = \{1, 0, -1, 0, 1\}$$

Let $y(n) = x_1(n) * x_2(n)$

$x_1(n)$	1	1	-1	-1	0
$x_2(n)$	1	1	-1	-1	0
1	1	1	0	0	0
0	0	0	0	0	0
-1	-1	-1	1	1	0
0	0	0	0	0	0
1	1	1	-1	-1	0

2

~~$$y(n) = \{1, 1, -2, -2, 2, 2, -1, -1, 0\}$$~~

~~$y(n)$ will have 9 terms.~~

~~$$y(n) = \delta(n) + \delta(n-1) - 2\delta(n-2) - 2\delta(n-3) + 2\delta(n-4) + 2\delta(n-5) - \delta(n-6) - \delta(n-7)$$~~

~~$$N = 9$$~~

Q.3 (b) (i) Consider the continuous time signal

$$f(t) = \begin{cases} \sin \omega t; & 0 < t < \frac{\pi}{\omega} \\ 0; & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

Also, $f\left(t + \frac{2\pi}{\omega}\right) = f(t)$, then find the Laplace transform of $f(t)$.

[12 marks]

Sol

$$f(t) = \begin{cases} \sin \omega t & ; 0 < t < \pi/\omega \\ 0 & ; \pi/\omega < t < 2\pi/\omega \end{cases}$$

$$f\left(t + \frac{2\pi}{\omega}\right) = f(t)$$

Time period of $f(t)$ $T_0 = \frac{2\pi}{\omega}$

Laplace transform of $f(t)$ is given by.

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$= \int_0^{\pi/\omega} [\sin \omega t \cdot e^{-st}] dt + \int_{\pi/\omega}^{2\pi/\omega} 0 \cdot e^{-st} dt$$

$$= \int_0^{\pi/\omega} \sin \omega t \cdot e^{-st} dt \quad \sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$= \int_0^{\pi/\omega} \left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right) e^{-st} dt$$

$$= \frac{1}{2j} \left[\int_0^{\pi/\omega} \left(e^{(j\omega - s)t} - e^{(-j\omega - s)t} \right) dt \right]$$



$$= \frac{1}{2j} \left[\frac{e^{(j\omega-s)t}}{j\omega-s} - \frac{e^{-(j\omega+s)t}}{-j\omega+s} \right]_0^{\pi/\omega}$$

$$= \frac{1}{2j} \left[\frac{e^{(j\omega-s)t}}{j\omega-s} + \frac{e^{(-j\omega-s)t}}{(j\omega+s)} \right]_0^{\pi/\omega}$$

$$= \frac{1}{2j} \left[\frac{(s+j\omega)e^{(j\omega-s)t} + (j\omega-s)e^{-(j\omega+s)t}}{s^2+\omega^2} \right]_0^{\pi/\omega}$$

$$= \frac{1}{2j(s^2+\omega^2)} \left[(s+j\omega)e^{(j\omega-s)\frac{\pi}{\omega}} + (j\omega-s)e^{-\frac{(j\omega+s)\pi}{\omega}} - [(s+j\omega) + j\omega-s] \right]$$

$$= \frac{1}{2j(s^2+\omega^2)} \left\{ s e^{j\pi - \frac{s\pi}{\omega}} + j\omega e^{j\pi - \frac{s\pi}{\omega}} + j\omega e^{-\frac{j\pi - s\pi}{\omega}} - s e^{-\frac{j\pi - s\pi}{\omega}} - 2j\omega \right\}$$

$$= \frac{\omega - \omega e^{-\pi/\omega}}{s^2 + \omega^2}$$

$$F(s) = \frac{\omega(1 - e^{-\pi/\omega})}{s^2 + \omega^2}$$

Q.3 (b) (ii) Find $x(n]$ by using convolution property of z-transform for

$$X(z) = \frac{1}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

[8 marks]

Sol.

$$H(z) = \frac{1}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$= \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

$$H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)} = \frac{A}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1 + \frac{1}{4}z^{-1}\right)}$$

$$\frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)} = \frac{A\left[1 + \frac{1}{4}z^{-1}\right] + B\left[1 - \frac{1}{2}z^{-1}\right]}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

$$A + B = 1$$

$$\frac{A}{4} - \frac{B}{2} = 0$$

$$A = 1 - B$$

$$A = 2B$$

$$B = \frac{1}{3}, A = \frac{2}{3}$$

$$H(z) = \frac{2}{3\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{1}{3\left(1 + \frac{1}{4}z^{-1}\right)}$$

Taking inverse z-transform.

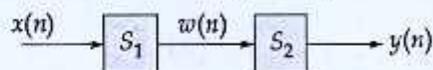
$$h[n] = x[n] = \frac{2}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{3}\left(-\frac{1}{4}\right)^n u[n]$$

- Q.3 (c) (i) Calculate the time delay in the 8085 assembly language program given below. The system has a clock period of $0.5 \mu\text{s}$.

```
                MVI      B, 00H
NEXT   :        DCR      B
                MVI      C, 11H
DELAY  :        DCR      C
                JNZ      DELAY
                MOV      A, B
                OUT      PORT
                HLT
```

[10 marks]

Q.3 (c) (ii) Consider the cascade of the following two systems S_1 and S_2 as shown in figure.



where, System, S_1 : Causal LTI with the difference equation as below:

$$w(n) = \frac{1}{2}w(n-1) + x(n)$$

System, S_2 : Causal LTI with the difference equation as below:

$$y(n) = \alpha y(n-1) + \beta w(n)$$

If the difference equation relating $x(n)$ and $y(n)$ is

$$y(n) = \frac{-1}{8}y(n-2) + \frac{3}{4}y(n-1) + x(n)$$

Determine α and β .

[10 marks]

Sol. For system S_1 :

$$w(n) = \frac{1}{2}w(n-1) + x(n)$$

$$w(n) - \frac{1}{2}w(n-1) = x(n)$$

Taking z -transform

$$W(z) - \frac{1}{2}[z^{-1}W(z) + W(-1)] = X(z)$$

as S_1 is causal $W(-1) = 0$

$$W(z) \left[1 - \frac{1}{2}z^{-1} \right] = X(z)$$

$$S_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1 - \frac{1}{2}z^{-1}} \quad \text{--- (1)}$$

For system S_2 :

$$y(n) = \alpha y(n-1) + \beta w(n)$$

$$y(n) - \alpha y(n-1) = \beta w(n)$$

Taking z -Transformation.

$$Y(z) - \alpha [z^{-1}Y(z) + Y(-1)] = \beta W(z)$$

$$Y(-1) = 0 \quad \because S_2 \text{ is causal.}$$

$$Y(z) [1 - \alpha z^{-1}] = \beta W(z)$$

$$S_2(z) = \frac{Y(z)}{W(z)} = \frac{\beta}{1 - \alpha z^{-1}}$$

Overall system is cascade of S_1 & S_2 therefore

$$\frac{Y(z)}{X(z)} = S_1(z) \cdot S_2(z) = \left(\frac{1}{1 - \frac{1}{2}z^{-1}} \right) \left(\frac{\beta}{1 - \alpha z^{-1}} \right)$$

$$Y(z) \left[\left(1 - \frac{z^{-1}}{2}\right) (1 - \alpha z^{-1}) \right] = \beta X(z) \Rightarrow Y(z) \left[1 - \left(\frac{1}{2} + \alpha\right)z^{-1} + \frac{\alpha}{2}z^{-2} \right] = \beta X(z)$$

Taking inverse z -Transform

$$y(n) - \left(\alpha + \frac{1}{2}\right)y(n-1] + \frac{\alpha}{2}y(n-2) = \beta x(n)$$

$$y(n) = \left(\alpha + \frac{1}{2}\right)y(n-1] + \frac{\alpha}{2}y(n-2) + \beta x(n) \quad \text{--- (2)}$$

Comparing eq (2) with given information.

$$\alpha + \frac{1}{2} = \alpha + \frac{1}{8}$$

$$\boxed{\alpha = \frac{1}{4}}$$

$$\alpha + \frac{1}{2} = \frac{3}{4}$$

$$\boxed{\beta = 1}$$

- Q.4 (a) (i) Draw direct form-I and direct form-II block diagram for the given transfer function.

$$H(z) = \frac{z^2 - 2z + 4}{\left(z - \frac{1}{2}\right)(2z^2 + z + 1)}$$

- (ii) Draw the cascade-form block diagram for the given transfer function using minimum delay elements.

$$H(z) = \frac{z - 1}{(4z^3 + 2z^2 + 2z + 3)}$$

[12 + 8 marks]

- Q.4 (b) (i) Multiply the 8-bit unsigned number in memory location 4480H by the 8-bit unsigned number in memory location 4481H.

By shift-add routine method and store the 8 least significant bits of the result in memory location 5500H and 8 most significant bits in memory location 5501H. Write comments in selected instruction.

[14 marks]

- Q.4 (b) (ii) The following diagnostic routine can be used to troubleshoot the interfacing circuit of an input port :

Instruction	Byte	T-states	Machine Cycle		
			M ₁	M ₂	M ₃
START : IN24H	2	10(4, 3, 3)			
JMP START	3	10(4, 3, 3)			

1. Identify the machine cycles.
2. If the system clock is 6 MHz, calculate the time required to execute the routine.

[6 marks]

- Q.4 (c) (i) Explain the mathematical function that is performed by the following instructions of 8085 processor and find the status of PSW at the end of the program.

LXI H, 2050H

MVI A, 22H

INR A

STA 2050H

INR A

XRA M

HLT

[14 marks]

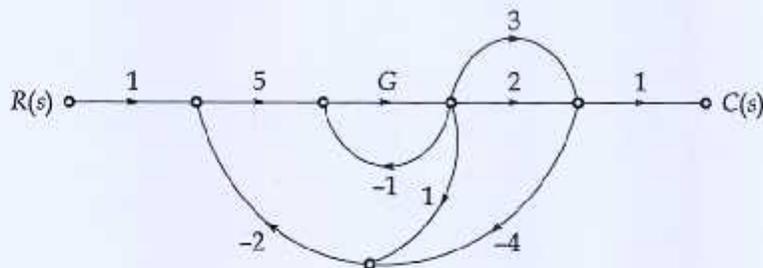
Q.4 (c) (ii) Explain the following in terms of direct memory access (DMA)

1. Cycle Stealing DMA
2. Interleaved DMA
3. Block Transfer DMA

[6 marks]

Section B : Electrical Circuits - 1 + Control Systems - 1

Q.5 (a) Consider the signal flow graph shown below :

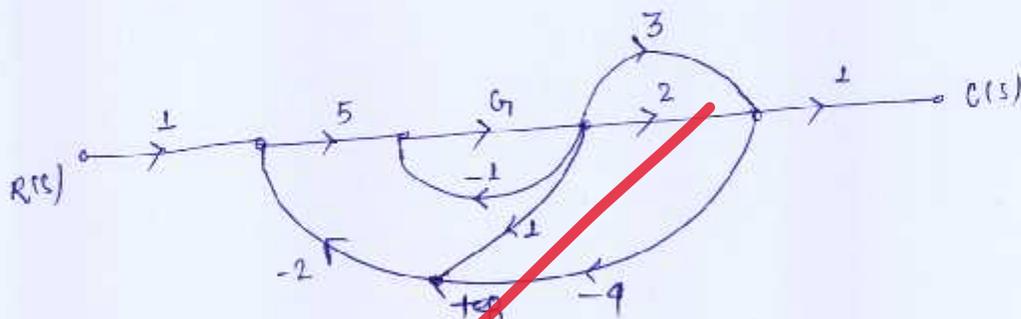


Determine the value of gain G if the overall transfer function is given by $\frac{13}{17}$.

[12 marks]

Sol.

Given signal flow graph is



Given SFG have two forward paths.

$$P_1 = 5G \times 2 = 10G$$

$$\text{Path factor for } P_1 (\Delta_1) = 1 - (0) = 1$$

$$P_2 = 5 \cdot G \cdot 3 = 15G$$

$$\text{Path factor for } P_2 (\Delta_2) = 1 - 0 = 1$$

Given SFG have 4 independent loops.

$$L_1 = -G$$

$$L_2 = -2 \times 5 \times G = -10G$$

$$L_3 = 5 \times G \times 2 \times -4 \times -2 = 80G$$

$$L_4 = 5 \times G \times 3 \times -2 \times -4 = 120G$$

Determinant of given s.c.F is

$$\Delta = 1 - \left\{ \sum \text{individual loop gains} \right\} + \left\{ \text{SOP to non touching loops} \right\} - \left\{ \dots \right\}$$

$$\begin{aligned} \Delta &= 1 - \left\{ -G - 10G + 80G + 120G \right\} \\ &= 1 - \left\{ 200G - 11G \right\} \\ &= 1 - 189G \end{aligned}$$

According Mason's gain formula.

$$\begin{aligned} \text{T.F.} &= \frac{\sum P_i \Delta_i}{\Delta} \\ &= \frac{10G + 15G}{1 - 189G} = \frac{25G}{1 - 189G} = \frac{13}{17} \end{aligned}$$

$$17 \times 25G = 13 - 13 \times 189G$$

$$G = \frac{13}{17 \times 25 + 13 \times 189}$$

$$G = \frac{13}{2882}$$

Good Approach

- Q.5 (b) Using the Routh criterion, check whether the system represented by the following characteristic equation is stable or not. Comment on the location of the roots. Determine the frequency of sustained oscillations if any.

$$s^4 + 2s^3 + 6s^2 + 8s + 8 = 0$$

[12 marks]

Sol.

Given characteristics eq. is

$$q(s) = s^4 + 2s^3 + 6s^2 + 8s + 8 = 0$$

using Routh criterion ($q(s)$ is following the essential condⁿ)

s^4	1	6	8
s^3	2	8	0
s^2	2	8	
s^1	4	0	
s^0	8		

• s^1 Row in R-H table is zero., which implies there exists two pole symmetric to origin. can be obtained by Auxiliary eq.

$$A(s) = 2s^2 + 8 = 0$$

$$s^2 + 4 = 0$$

$$s = \pm j2$$

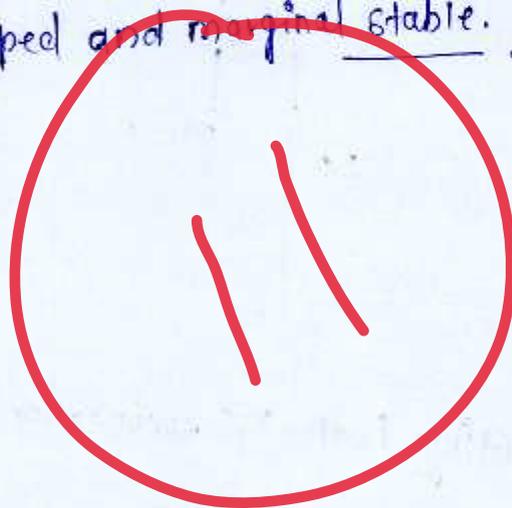
$$\frac{dA(s)}{ds} = 4s$$

There is no sign change in first column of R-H table which implies no root lies in right half of the s-plane.

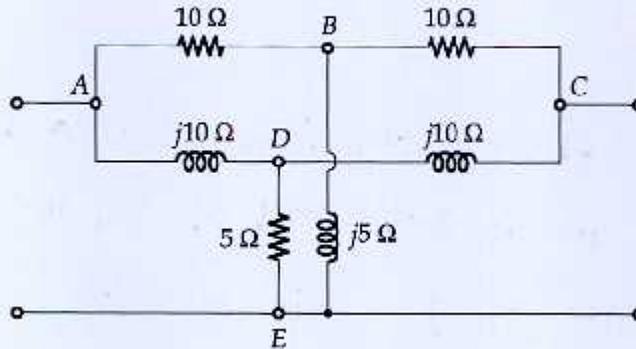
Therefore the above system is marginal stable. whose two roots are in left half of s-plane and two roots are on $j\omega$ axis $s = \pm j2$.

• The system will provide sustained oscillations of $\omega = 2 \text{ rad/sec}$.

as it is critically damped and marginal stable.

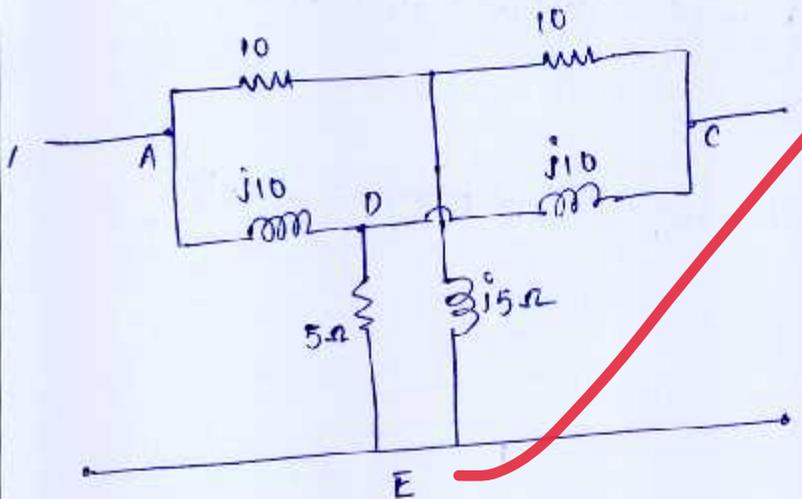


- Q.5 (c) The network shown in figure consists of two star connected circuits in parallel. Obtain the single delta connected equivalent.

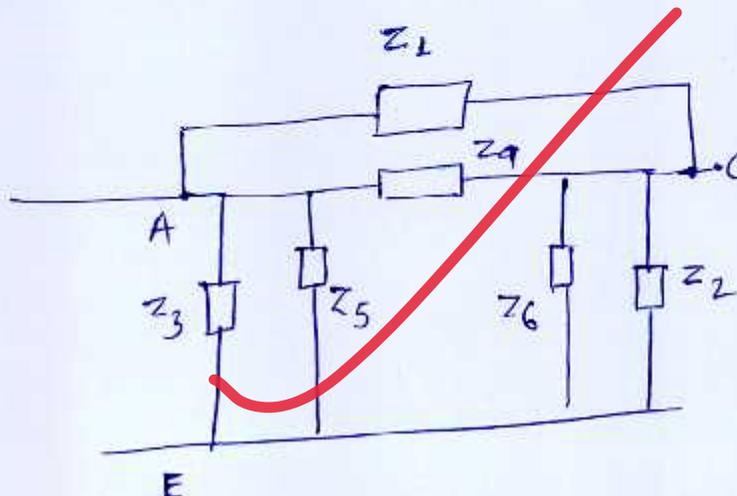


[12 marks]

Sol.



converting both γ -connection to equivalent delta connection.



$$Z_1 = 10 + 10 + \frac{10 \times 10}{j5} = 20 - j20$$

$$Z_2 = 10 + j5 + \frac{j5 \times 10}{10} = 10 + j10$$

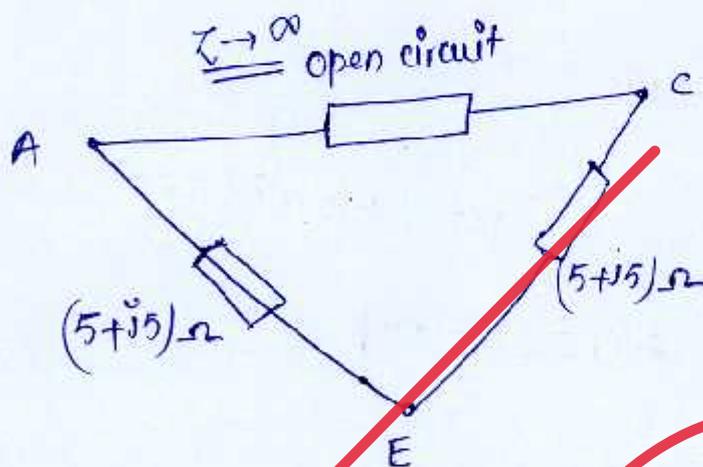
$$Z_3 = 10 + j5 + \frac{j5 \times 10}{10} = 10 + j10$$

$$Z_4 = j10 + j10 + \frac{j10 \times j10}{5} = j20 - 20$$

$$Z_5 = j10 + 5 + \frac{j10 \times 5}{j10} = 10 + j10$$

$$Z_6 = j10 + 5 + \frac{j10 \times 5}{j10} = 10 + j10$$

Both Δ -connections are in parallel
equivalent Δ -connection is



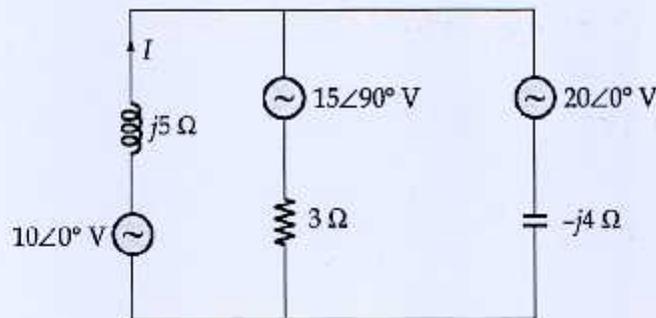
$$Z_{AC} = Z_1 \parallel Z_4 = (20 - j20) \parallel (j20 - 20) = \text{open circuit}$$

$$Z_{AE} = 5 + j5$$

$$Z_{EC} = 5 + j5$$

Good Approach

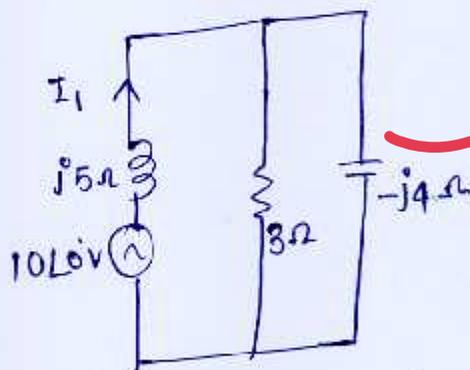
- Q.5 (d) Find current I through $j5 \Omega$ branch using superposition theorem for the network shown in figure.



[12 marks]

Sol. There are three independent sources acting in the network

Case ① considering $V = 10\angle 0^\circ$ V source is acting alone.



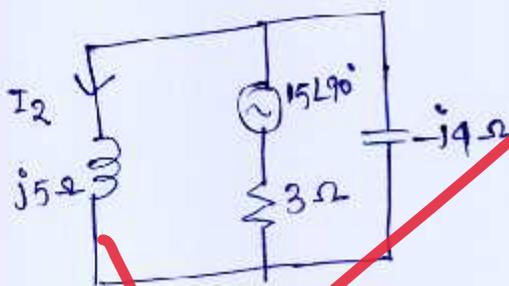
$$Z_{net} = (3 \parallel -j4) + j5$$

$$= \frac{-j12}{3-j4} + j5$$

$$Z_{net} = 1.92 + j3.56 \Omega$$

$$I_1 = \frac{10\angle 0^\circ}{Z_{net}} = 2.07 \angle -61.66^\circ \text{ Amp.}$$

Case ② $V = 15\angle 90^\circ$ V source acting alone



$$Z_{net} = (j5 \parallel -j4) + 3$$

$$= \frac{20}{j} + 3$$

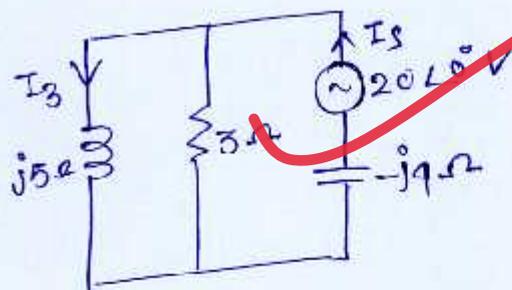
$$= 3 - j20$$

$$I_s = \frac{15\angle 90^\circ}{3-j20}$$

$$I_2 = I_s * \frac{-j4}{j} = -4I_s$$

$$I_2 = -4 \left[\frac{15 \angle 40^\circ}{3 - j20} \right] = 2.966 \angle -8.53 \text{ Amp.}$$

Case ③ $V = 20 \angle 0^\circ \text{ V}$ Voltage source acting alone



$$Z_{net} = (j5 \parallel 3) - j4$$

$$= \frac{j15}{3+j5} - j4$$

$$Z_{net} = 2.20 - j2.67 \Omega$$

$$I_3 = \frac{20}{Z_{net}} = 3.66 + j9.95 \text{ Amp.}$$

$$I_3 = \frac{I_3 \times 3}{3+j5} = 2.966 \angle -8.5307 \text{ Amp.}$$

$$\text{Net current } I = I_1 - I_2 - I_3$$

$$= 2.47 \angle -61.67 - 2 * (2.96 \angle -8.53)$$

$$I = 4.85 \angle -109.52 \text{ Amp.}$$

Good Approach

Q.5 (e) The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(1+sT)}$$

- (i) By what factor the amplifier gain K should be multiplied so that the damping ratio is increased from 0.2 to 0.8?
 (ii) By what factor the time constant T should be multiplied so that the damping ratio is reduced from 0.9 to 0.3?

[12 marks]

Sol.

$$G(s) = \frac{K}{s(1+sT)}$$

$$G(s) \text{ T.F} = \frac{K}{s^2T + s + K}$$

$$= \frac{K/T}{s^2 + \frac{s}{T} + \frac{K}{T}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{\frac{K}{T}}$$

$$\zeta \propto \frac{1}{\sqrt{KT}}$$

$$2\zeta \omega_n \sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$\zeta = \frac{1}{2\sqrt{KT}}$$

$$\frac{\zeta_1}{\zeta_2} = \sqrt{\frac{K_2 T}{K_1 T}}$$

$$\frac{0.2}{0.8} = \sqrt{\frac{K_2}{K_1}} = \frac{1}{4}$$

$$K_2 = \frac{1}{16} K_1$$

~~$K_2 = 0.0625 K_1$~~ K should be amplified by factor 0.0625 to get $\zeta = 0.8$.

(ii) Let K : constant.

$$E \propto \frac{1}{\sqrt{KT}}$$

$$\frac{E_1}{E_2} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{0.9}{0.3} = \sqrt{\frac{T_2}{T_1}}$$

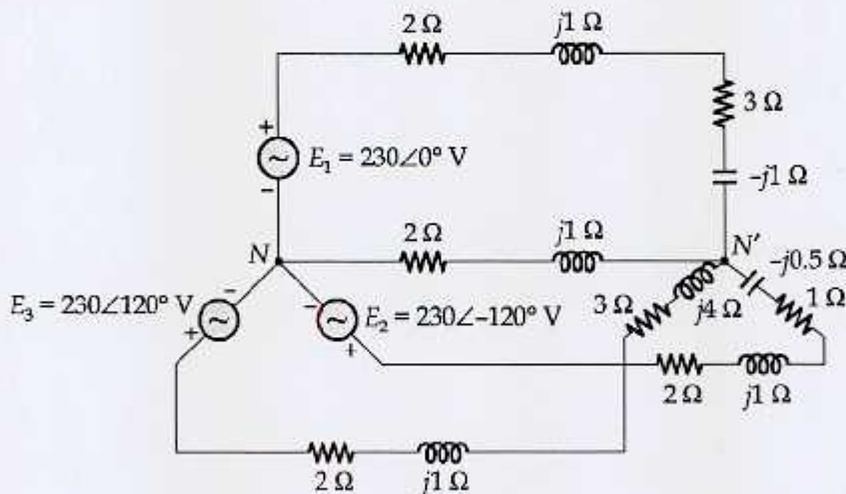
$$(3)^2 = \frac{T_2}{T_1}$$

$$T_2 = 9 \cdot T_1$$

T should be multiplied by 9 to reduce E from 0.9 to 0.3.

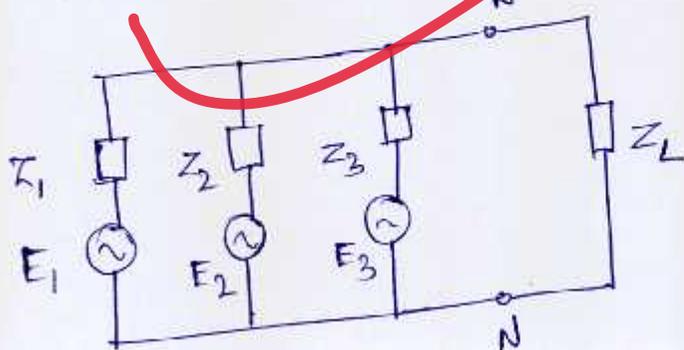
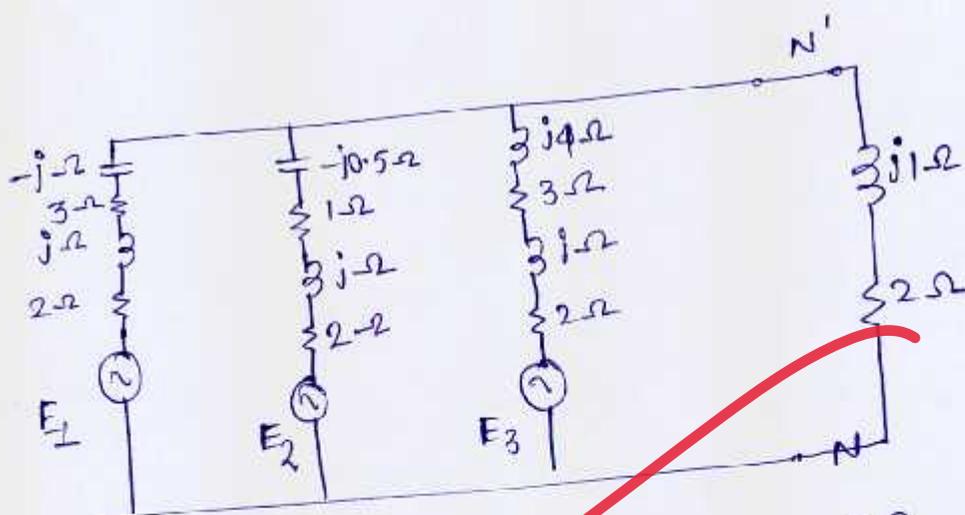
Good Approach

Q.6 (a) The network shown in figure represents a three phase four wire electrical power system. Use Millman's theorem to determine the potential difference between the two neutral points N and N' .



[20 marks]

Sol. Above network can be redrawn as.

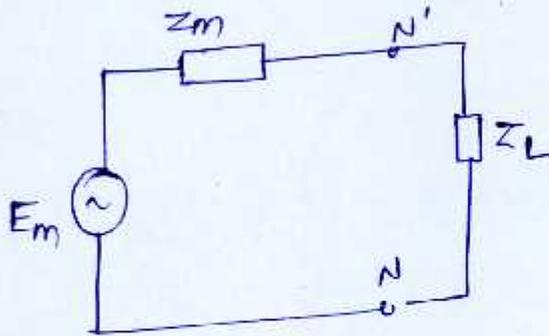


$$Z_1 = 5 \Omega$$

$$Z_2 = 3 + j0.5 \Omega$$

$$Z_3 = 5 + j5 \Omega$$

above n/w can be simplified using Millman's theorem



$$\text{Where } E_m = \frac{V_1}{Z_1} + \frac{V_2}{Z_2} + \frac{V_3}{Z_3} \quad \text{V}$$

$$\frac{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

$$\text{and } Z_m = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} \quad \Omega$$

$$E_m = \frac{\frac{230}{5} + \frac{230 \angle 120^\circ}{3+j0.5} + \frac{230 \angle 120^\circ}{5+j5}}{\frac{1}{5} + \frac{1}{3+j0.5} + \frac{1}{5+j5}} = 43.28 \angle -62.88^\circ \text{ V}$$

$$Z_m = 1.554 \angle 13.85^\circ$$

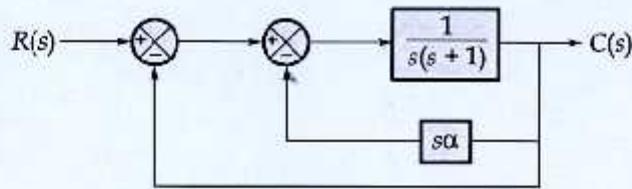
$$V_{NN'} = \frac{E_m * Z_L}{Z_m + Z_L} = \frac{43.28 \angle -62.88^\circ * (2+j)}{(1.554 \angle 13.85^\circ) + (2+j)}$$

$$V_{NN'} = 25.69 \angle -57.67^\circ \text{ V}$$

• Good Approach

18

Q.6 (b) A control system is shown in the block diagram given below :



Sketch the root locus as the value of the parameter α is varied from 0 to ∞ . Determine the value of α for the transient response to have critical damping.

[20 marks]

Sol.

By using block diagram reduction,

$$\frac{C(s)}{R(s)} = \frac{1}{1 + \frac{s\alpha}{s(s+1)}} = \frac{s(s+1+\alpha)}{1 + \frac{1}{s(s+1+\alpha)}} = \frac{1}{s^2 + (1+\alpha)s + 1}$$

$$\text{OLTF} = \frac{\alpha}{s(s+1)}$$

open loop has poles at $s=0, -1$

$$p=2$$

$$z=0$$

$(p-z)=2 \Rightarrow$ two root locus branches tends to infinity.

$$\alpha = -s(s+1)$$

$$\frac{d\alpha}{ds} = -2s-1=0$$

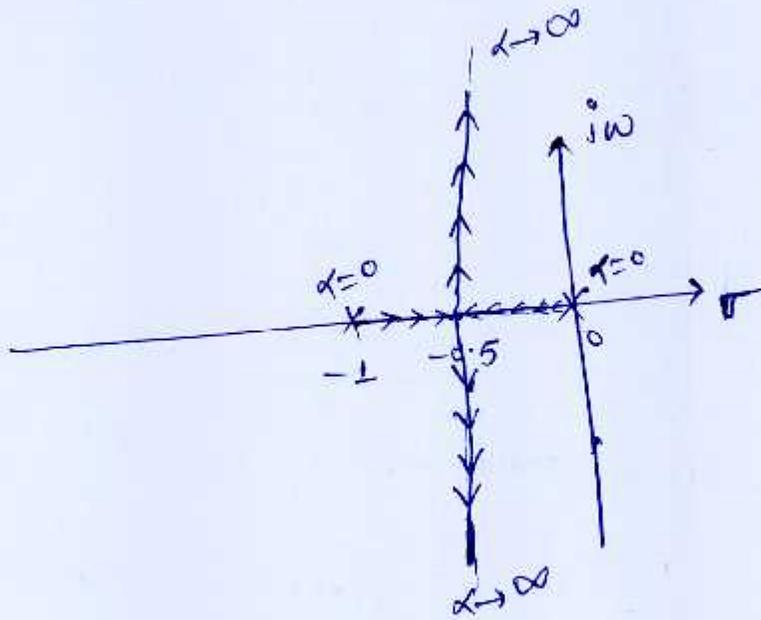
$$s = -1/2 = -0.5$$

break away point $s = -0.5$

$$\text{Centroid } \sigma = \frac{\sum P_{\text{real}} - Z_{\text{real}}}{p-z} = \frac{-1}{2} = -0.5$$

Angle of Asymptots = $90^\circ, 270^\circ$





$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + (1+d)s + 1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = 1 \text{ rad/sec}$$

$$2\zeta\omega_n = (1+d)$$

$$\zeta = \frac{(1+d)}{2}$$

For critical damping $\zeta = 1$

$$\frac{(1+d)}{2} = 1$$

$$\boxed{d = 1}$$

Q.6 (c) A unity feedback control system has an open-loop transfer function

$$G(s) = \frac{5}{s(s+1)}$$

Find the rise time, percentage overshoot, peak time and settling time for a step input of 10 units. Also, determine the peak overshoot.

[20 marks]

Sol.

$$G_1(s) = \frac{5}{s(s+1)}$$

$$CLTF = \frac{5}{s^2 + s + 5} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n =$

$$Y(s) = G_1(s) \cdot R(s) = \frac{5}{(s^2 + s + 5)} \cdot \frac{10}{s}$$

$$= \frac{50}{s(s^2 + s + 5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + s + 5}$$

$$= \frac{As^2 + As + 5A + Bs^2 + Cs}{s^3 + s^2 + 5s}$$

$$A = 10$$

$$A + C = 0$$

$$A + B = 0$$

$$B = C = -10$$

$$Y(s) = \frac{10}{s} - \frac{10(s+1)}{s^2 + s + 5}$$

$$= 10 \left[\frac{1}{s} - \frac{(s+1)}{(s^2 + s + 5)} - \frac{1}{s^2 + s + 5} \right]$$

Taking inverse Laplace

$$y(t) = 10 \left[1 - e^{-0.5t} \cos 2.18t \right]$$

$$\omega_d = 2.18 \text{ rad/sec} = \omega_n \sqrt{1 - \frac{1}{9}} = \omega_n \frac{\sqrt{8}}{2}$$

$$\omega_n = 2.51 \text{ rad/sec}$$

$$\xi \omega_n = 0.5$$

(i) Rise time

Response reaches to 90% of its final value.

$$1 - 0.9 = e^{-0.5t}$$

$$t_R = 2 \times \ln 0.1$$

$$t_R = 4.605 \text{ sec}$$

(ii)

$$\text{overshoot} = y(t)_{\text{peak}} - 10$$

$$= y(t) \Big|_{t=t_p} - 10$$

$$= 14.86 - 10$$

$$= 4.86$$

$$\% \text{ MP} = \underline{48.6\%}$$

(iii) Peak time

$$y(t) = 10 \left[1 - e^{-0.5t} \cos 2.18t \right]$$

$$y'(t) = 10 \left[0.5 e^{-0.5t} \cos 2.18t + e^{-0.5t} \sin 2.18t \right]$$

$$= 0$$

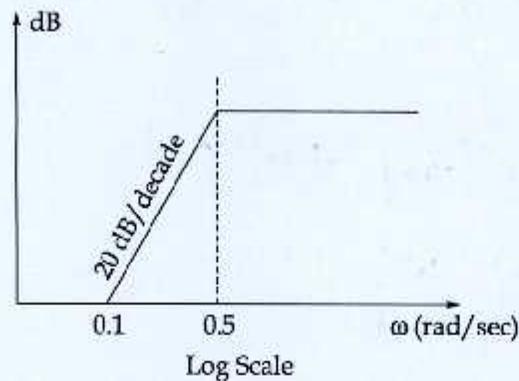
$$t_p = \underline{1.4 \text{ sec}}$$

$$y(t) \Big|_{t=t_p}$$



$$\text{Peak overshoot} = \underline{14.86} .$$

- Q.7 (a) (i) The approximate Bode magnitude plot of a lead network with its pole and zero on the left half of the s -plane is shown in the following figure :



Find the frequency at which the phase angle of the network is maximum (in rad/sec).
[10 marks]

Sol.

$$G_c(s) = \frac{K(s+z_1)}{(s+p_1)}$$

$$= \frac{K(s+0.1)}{(s+0.5)}$$

Phase angle of $G_c(s)$ is

$$\angle G_c(s) = \tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{0.5}\right)$$

$$= 180^\circ + \tan^{-1}(10\omega) - [180^\circ - \tan^{-1}\left(\frac{\omega}{0.5}\right)]$$

$$= -\tan^{-1}(10\omega) + \tan^{-1}(2\omega)$$

$$= \tan^{-1}\left[\frac{2\omega - 10\omega}{1 + 20\omega^2}\right]$$

$$\frac{d\angle G_c(j\omega)}{d\omega} = 0 \Rightarrow \omega = \frac{1}{\sqrt{20}} \text{ rad/sec}$$

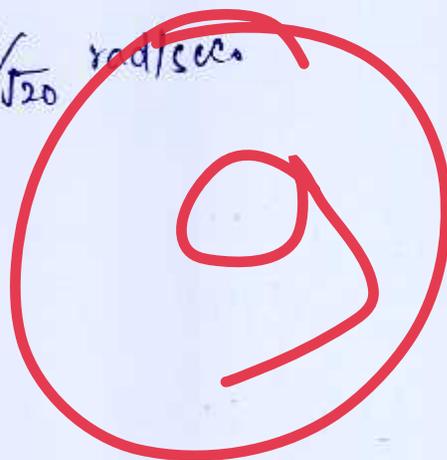
$G_c(s)$ will provide maximum phase shift of 90° at $\omega = \frac{1}{\sqrt{20}}$ rad/sec

$$\angle G_c(j\omega) = \tan^{-1}(10\omega) - \tan^{-1}(2\omega)$$

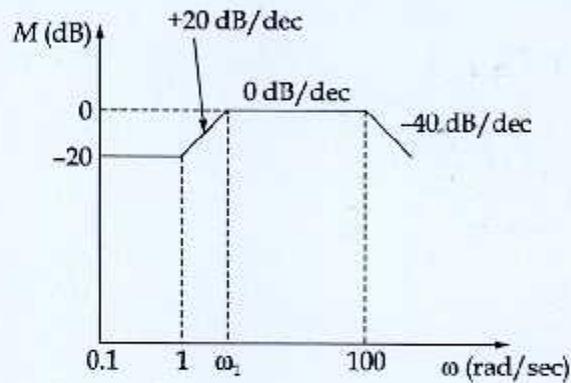
$$= \tan^{-1}\left(\frac{8\omega}{1+20\omega^2}\right)$$

$$\frac{d\angle G_c(j\omega)}{d\omega} = 0 \Rightarrow \omega = \frac{1}{\sqrt{20}} \text{ rad/sec}$$

$$\angle G_c(j\omega) \Big|_{\omega = \frac{1}{\sqrt{20}}} = 90^\circ$$



- Q.7 (a) (ii) Obtain the open loop transfer function for a system with unity feedback whose bode plot is shown below :



[10 marks]

Sol.

$$\text{Let } G(s).H(s) = \frac{K.(s+1)}{(s+\omega_1)(s+100)^2}$$

At $\omega = 1$ mag is -20 dB.

from $\omega = 1$ to $\omega = \omega_1$
slope is 20 dB/dec.

at $\omega = \omega_1$ mag. is 0 dB.

therefore $\omega_1 = 10$ rad/sec.

$$\text{T.F.} = \frac{K(s+1)}{(s+10)(s+100)^2}$$

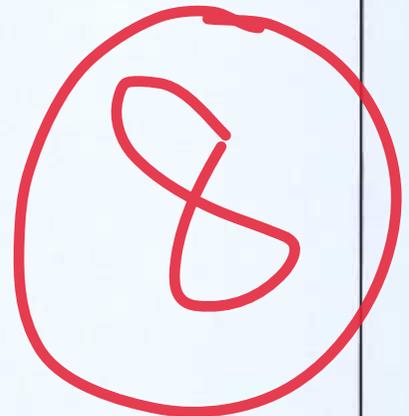
at $\omega = 1$ rad/sec mag. $= -20$ dB.

$$-20 = 20 \log K$$

$$K = 0.1$$

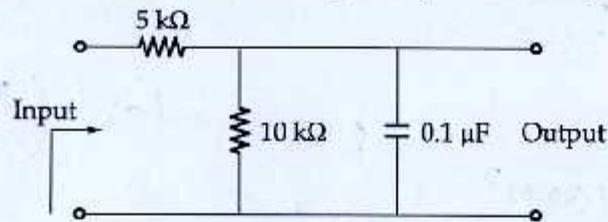
$$\frac{K}{10^5} = 0.1$$

$$\text{T.F.} = \frac{K(s+1)}{10^5 \left(\frac{s}{10} + 1\right) \left(\frac{s}{100} + 1\right)^2}$$



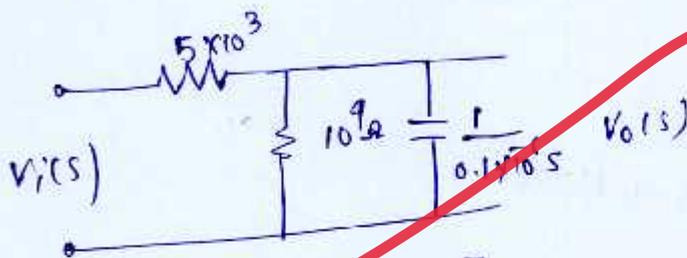
$$K = 10^9$$
$$T.F. = \frac{10^9 (s+1)}{\left(\frac{s}{10} + 1\right) \left(\frac{s}{100} + 1\right)^2}$$

Q.7 (b) (i) Draw the asymptotic magnitude and phase plot on the system shown below :



[10 marks]

Sol. Let input is $V_i(s)$ & output is $V_o(s)$



$$Z(s) = \frac{10^4 \times 10^7}{10^4 + \frac{10^7}{s}} + 5 \times 10^3$$

$$V_o(s) = V_i(s) * \frac{10^{11}}{5 \times 10^9 + 10^7}$$

$$\frac{10^{11}}{5 \times 10^9 + 10^7} + 5 \times 10^3$$

$$\frac{V_o(s)}{V_i(s)} = \frac{10^{11}}{5 \times 10^7 s + 5 \times 10^{10} + 10^{11}}$$

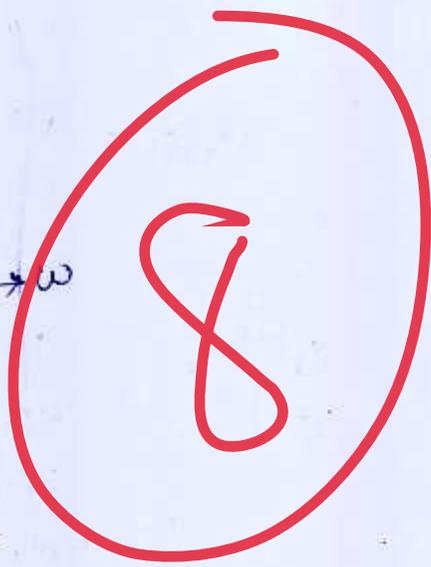
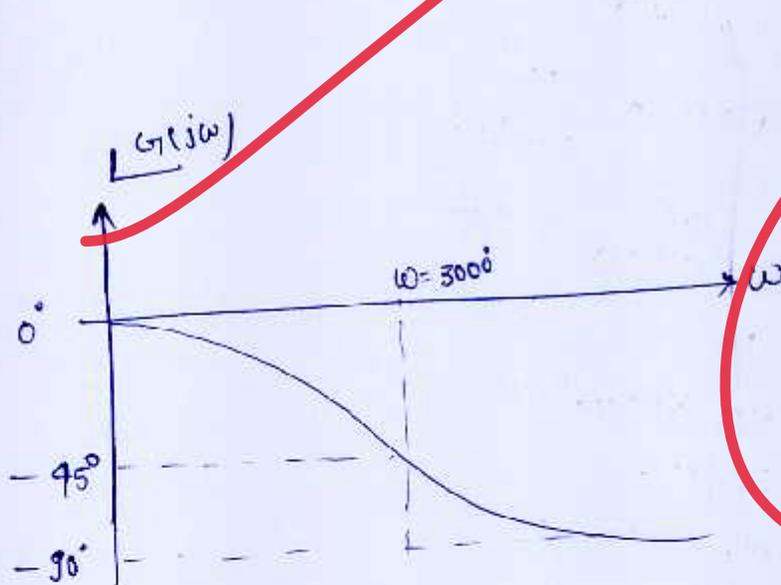
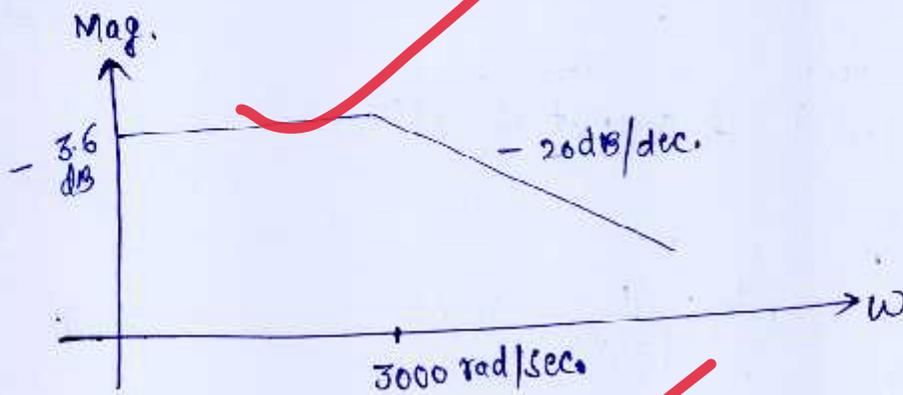
$$= \frac{10^{11}}{5 \times 10^7 s + 15 \times 10^{10}}$$

$$= \frac{10^{11}}{5 \times 10^7 \left(s + \frac{15 \times 10^{10}}{5 \times 10^7} \right)}$$

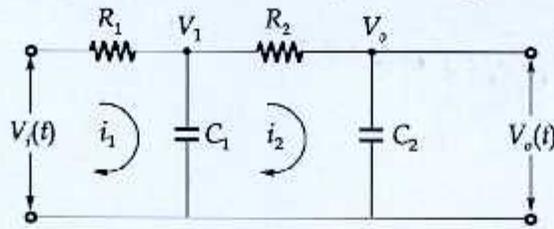
$$= \frac{10^9}{s} \frac{1}{\left(s + 3 \times 10^3 \right)} = \frac{2000}{s(s+3000)}$$

$$G_1(j\omega) = \frac{2000}{j\omega + 3000}$$

$$|G_1(j\omega)| = \frac{2000}{\sqrt{\omega^2 + (3000)^2}} \quad \text{phase} = -\tan^{-1}\left(\frac{\omega}{3000}\right)$$



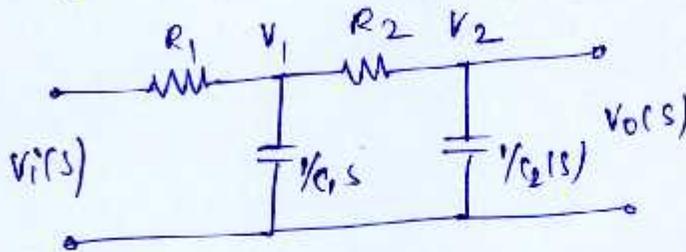
Q.7 (b) (ii) Draw the block diagram for the circuit given in figure below :



[10 marks]

Sol.

Redrawing the circuit in s-domain.



$$Z(s) = \left[\left(R_2 + \frac{1}{C_2 s} \right) \parallel \frac{1}{C_1 s} \right] + R_1$$

$$V_1(s) = V_i(s) \cdot \frac{\left(R_2 + \frac{1}{C_2 s} \right) \parallel \frac{1}{C_1 s}}{Z(s)}$$

$$= V_i(s) \cdot \frac{R_2 C_2 s + 1}{R_2 C_2 s^2 + C_2 s + C_1}$$

$$\frac{R_2 C_2 s + 1}{R_2 C_2 s^2 + C_2 s + C_1} + R_1$$

$$Z(s) = \frac{\frac{R_2}{C_1 s} + \frac{1}{C_1 C_2 s^2}}{R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}} + R_1$$

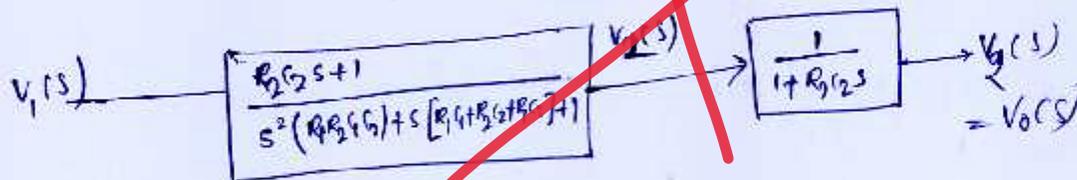
$$= \frac{R_2 C_2 s + 1}{R_2 C_1 C_2 s^2 + C_2 s + C_1} + R_1$$

$$V_1(s) = \frac{V_i(s) (R_2 C_2 s + 1)}{R_2 C_2 s^2 + C_2 s + C_1 + R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + R_1 C_2 s}$$

$$V_2(s) = V_1(s) \cdot \frac{1}{C_2 s} = \frac{V_i(s)}{R_2 C_2 s^2 + 1}$$

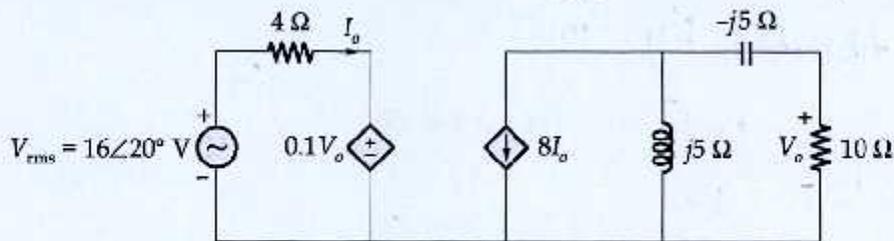
$$V_2(s) = V_o(s) = \frac{V_i(s)}{1 + R_2 C_2 s^2 + R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + R_1 R_2 C_1 C_2 s^2}$$

$$T.F. = \frac{1}{R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + R_1 C_2 s + R_2 C_2 s + 1}$$



Incomplete solution

Q.7 (c) (i) For the circuit shown below, find the average power absorbed by the $10\ \Omega$ resistor



[10 marks]

Sol.

Applying KVL in loop ①

$$-16\angle 20^\circ + 4I_o + 0.1V_o = 0 \quad \text{--- (1)}$$

$$V_o = \frac{16\angle 20^\circ - 4I_o}{0.1}$$

$$V_o = 160\angle 20^\circ - 40I_o \quad \text{--- (2)}$$

In loop ② $-V_o + j5I_2 + j5(I_2 - 8I_o) = 0$

$$-V_o + j5 \cdot \frac{V_o}{10} + j5 \left(\frac{-V_o}{10} - 8I_o \right) = 0$$

$$-V_o + \frac{jV_o}{2} - \frac{jV_o}{2} - j40I_o = 0$$

$$V_o = -j40I_o \quad \text{--- (3)}$$

by eq ② & ③

$$160\angle 20^\circ - 40I_o = -j40I_o$$

$$I_o = \frac{160\angle 20^\circ}{40 + j40} = 2.82\angle 65^\circ \text{ Amp.}$$

$$V_o = -j40 \cdot I_o = 113.13\angle -25^\circ \text{ V}$$

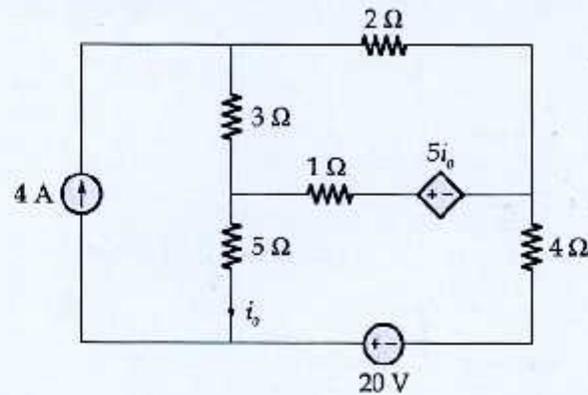
$I_2 = \frac{-V_o}{10}$
2

Power Absorbed by $10\text{-}\Omega$

$$P = \frac{|V|^2}{10} = \frac{[113.13 \angle -25^\circ]^2}{10}$$
$$= \frac{(112.53)^2}{10}$$

$P = 1052.25 \text{ Watts.}$

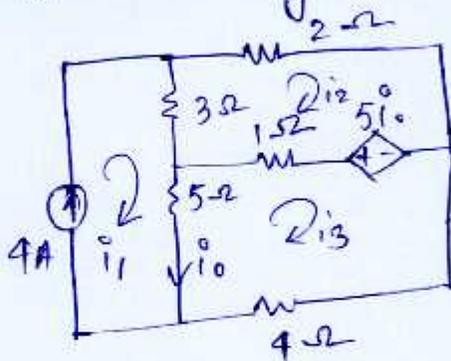
Q.7 (c) (ii) Find the current i_o using super position theorem in the circuit shown below :



[10 marks]

Sol.

4A source acting alone.



$$i_1 = 4 \text{ Amp}$$

$$-5i_o + 6i_2 - i_3 - 3i_1 = 0$$

$$-5i_o + 6i_2 - i_3 - 12 = 0$$

$$-5(i_1 - i_3) + 6i_2 - i_3 = 12$$

$$-5(4 - i_3) + 6i_2 - i_3 = 12$$

$$-20 + 5i_3 + 6i_2 - i_3 = 12$$

$$6i_2 - i_3 = 32 \quad \text{--- (1)}$$

$$10i_3 + 5(i_1 - i_3) - i_2 - 5i_1 = 0$$

$$10i_3 + 5(4 - i_3) - i_2 - 20 = 0$$

$$10i_3 + 20 - 5i_3 - i_2 - 20 = 0$$

$$5i_3 - i_2 = 0$$

$$i_2 = 5i_3$$

$$30i_3 - i_3 = 32$$

$$i_3 = \frac{32}{29} = 1.103 \text{ Amp.}$$

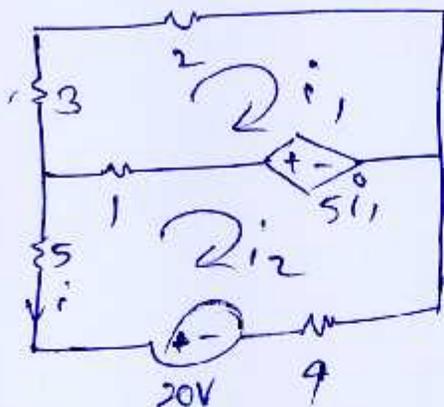
$$i_{o1} = i_1 - i_3$$

$$= 4 - 1.103$$

$$i_{o1} = 2.897 \text{ Amp.}$$

3

20V source acting alone.



$$i_2 = -i_0$$

$$-5i_0 + 6i_1 - i_2 = 0$$

$$+5i_0 + 6i_1 + i_2 = 0$$

$$4i_2 + 6i_1 = 0 \quad \text{--- (1)}$$

$$5i_0 + 10i_2 - i_1 - 20 = 0$$

$$5i_2 - i_1 = 20 \quad \text{--- (2)}$$

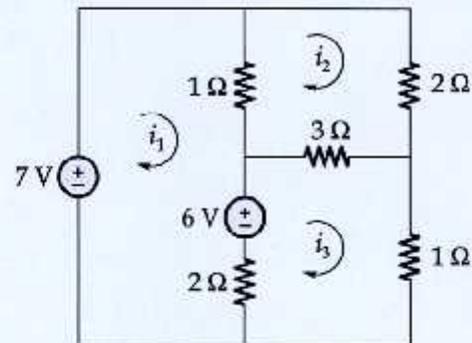
$$i_1 = -2.35$$

$$i_2 = 3.53$$

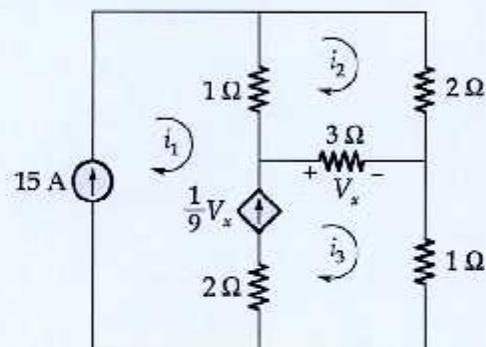
$$i_0 = -3.53 \text{ Amp}$$

$$i = -0.36 \text{ Amp}$$

Q.8 (a) (i) Use mesh analysis to determine mesh currents in the circuit



(ii) Use mesh analysis to determine mesh currents in the circuit



[10 + 10 = 20 marks]

- Q.8 (b) (i) The open-loop transfer function of a unity negative feedback system is given by

$$G(s) = \frac{K(s+1)^2}{(s+2)^2}$$

Without drawing root locus diagram, prove that the root locus (for $K > 0$) of the system lies on a circle.

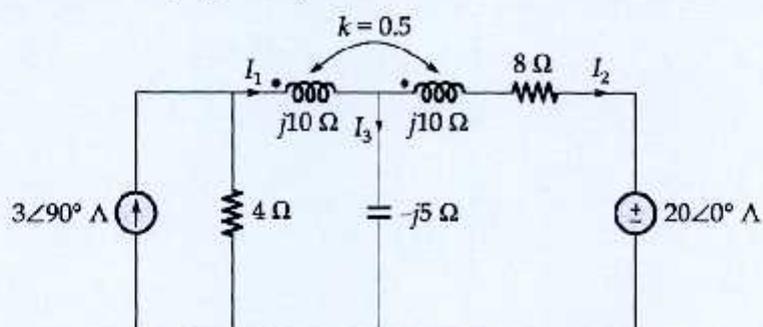
- (ii) The response of a feedback system to a unit step input is

$$C(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$$

- (a) Obtain the expression for the closed loop transfer function.
(b) Determine the undamped natural frequency and damping ratio of the system.

[10 + 10 = 20 marks]

Q.8 (c) Determine the current I_1 , I_2 and I_3 in the circuit shown. Take $\omega = 1000$ rad/sec.



[20 marks]

Space for Rough Work

Space for Rough Work

$$\frac{s+1}{s^2+s+5}$$
$$\frac{s+1}{s(s+1)+5}$$

$$\frac{s+2}{s^2+2s+1+4-5}$$

$$\frac{(s+1)}{(s+1)^2 - s+4}$$