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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-1 : Network Theory + Control Systems [All Topics]

Name : ....

Roll No :

Test Centres | **Student's Signature**

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	34
Q.2	—
Q.3	—
Q.4	32
Section-B	
Q.5	32
Q.6	—
Q.7	17
Q.8	48
Total Marks Obtained	163

Signature of Evaluator: Ch. Rent-1.

Cross Checked by ....

. Can do better .

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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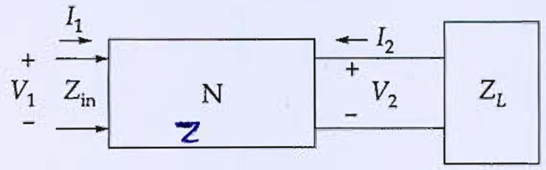
Section A : Network Theory

Q.1 (a) The Z parameter of a two port device N are $Z_{11} = Ks$, $Z_{12} = Z_{21} = 10\,Ks$ and $Z_{22} = 100\,Ks$. A 1 Ω resistor is connected as load across the output port.

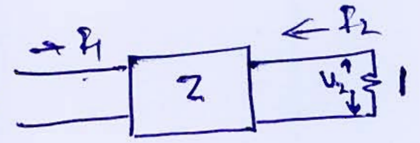
(i) Find the input impedance $Z_{in} = \dfrac{V_1}{I_1}$ and construct its equivalent circuit.

(ii) Give the values of the element for $K = 1$ and $K = 10^6$.



[12 marks]

Sol^n



$Z_{in} = \dfrac{V_1}{I_1}$, z-parameter.

$V_1 = Z_{11} I_1 + Z_{12} I_2$.

$V_2 = Z_{21} I_1 + Z_{22} I_2$.

$\boxed{V_2 = -I_2}$.

$V_1 = KS I_1 + 10KS I_2$ — (1)

↳ put eq (2)

$V_2 = 10KS I_1 + 100KS I_2$ — (II)

$-I_2 = 10KS I_1 + 100KS I_2$.

$-I_2 - 100KS I_2 = 10KS I_1$

$-I_2 (1 + 100KS) = 10KS I_1$

$I_2 = -\left(\dfrac{10KS I_1}{1 + 100KS}\right)$ — put eq (1)

$V_1 = KS I_1 - 10KS \left(\dfrac{-10KS I_1}{1 + 100KS}\right)$

$\dfrac{V_1}{I_1} = Z_{in} = KS + \dfrac{100K^2S^2}{1 + 100KS}$

$\boxed{Z_{in} = \dfrac{V_1}{I_1} \Rightarrow \dfrac{KS + 100K^2S^2 + 100K^2S^2}{1 + 100KS}}$

put $k=1$,

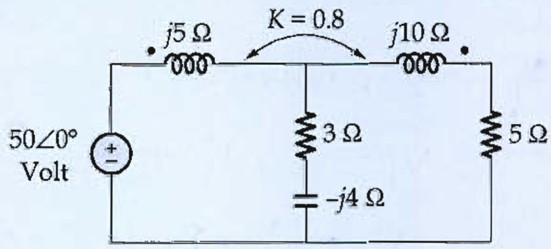
$$\boxed{Z_{in} = \frac{s+200s^2}{1+100s}\ \Omega}$$

7

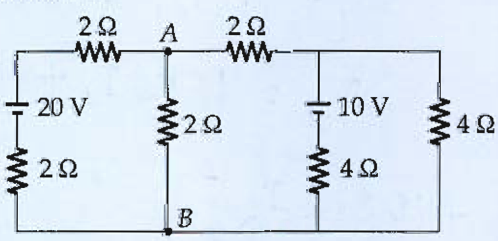
put $k = 10^6$

$$\boxed{Z_{in} = \frac{10^6 s + 2\times10^6 s^2}{1+10^8 s}\ \Omega}$$

Q.1 (b) (i) Find voltage across the 5 Ω resistor using Mesh analysis.



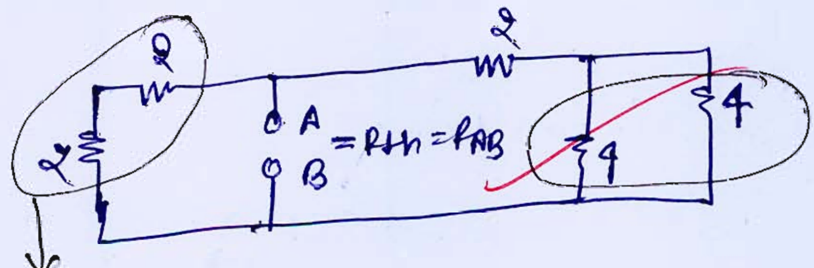
(ii) Determine the current through the branch *AB* of the network shown below using Thevenin's equivalent.



[6 + 6 marks]

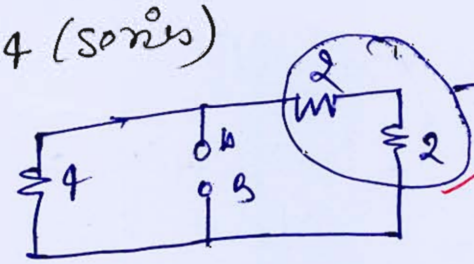
(II) · Determine current AB branch using thevenin's theorem.

Step-1 Calculate Rth. (Disconnect Voltage source → short ckt)

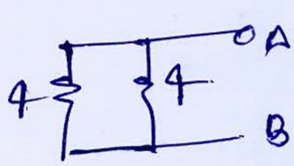


$(4 \| 4) = 2$

$2+2 = 4$ (series)

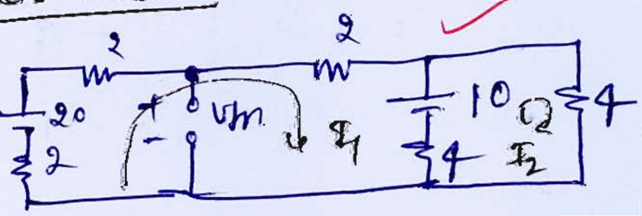


→ series $2+2 = 4$



$\boxed{R_{th} = 2\Omega}$

Step-2 Find out $V_{th} = 2$



using mesh analysis .

loop-1

$20 = 6I_1 + 4I_1 - 4I_2 + 10$.

$10 = 10I_1 - 4I_2$.

$5 = 5I_1 - 2I_2$ — (1)

loop-2

$10 = 8I_2 - 4I_1$

$5 = 4I_2 - 2I_1$ — (2)

$$\boxed{\begin{aligned} I_1 &= 1.875 \text{ Amp} \\ I_2 &= 2.1875 \end{aligned}}$$

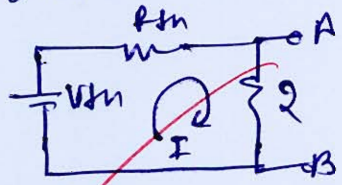
6

$V_{th} = -2I_1 + 20 - 2I_1$

$V_{th} = -4I_1 + 20$

$\boxed{V_{th} = 12.5 \text{ Volt}}$

equivalent cbt

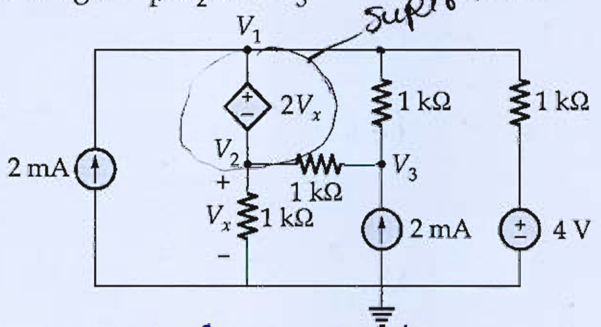


$I = \frac{V_{th}}{2 + R_{th}} = \frac{12.5}{2+2}$

$\boxed{I = 3.125 \text{ Amp}}$

Q.1 (c) Determine node voltages V_1, V_2 and V_3.

super node



solve using nodal analysis . **[12 marks]**

at node 1 $\quad V_1 - V_2 = 2V_x$ → super node

$V_1 - V_2 = 2V_x$ — (I)

$\frac{V_1 - V_3}{1} + \frac{V_1 - 4}{1} + \frac{V_2 - V_3}{1} + \frac{V_2}{1} = 2$.

$2V_1 - 2V_3 + 2V_2 = 6$ — (II) $\quad \therefore V_x = V_2$.

at node 3 $\quad \frac{V_3 - V_2}{1} + \frac{V_3 - V_1}{1} = 2$.

$V_3 - V_2 + V_3 - V_1 = 2$ —

$2V_3 - V_2 - V_1 = 2$ — (III)

Do not write in this margin

$\therefore V_x = V_2$ put eq (1)

$V_1 - V_2 = 2V_2$.

$\boxed{V_1 = 3V_2}$

put V_1 value in eq (2) & (3)

$2\times 3V_2 - 2V_3 + 2V_2 = 6$.

$6V_2 + 2V_2 - 2V_3 = 6$

$8V_2 - 2V_3 = 6$

$4V_2 - V_3 = 3$ — (4)

$2V_3 - V_2 - 3V_2 = 2$.

$2V_3 - 4V_2 = 2$.

$V_3 - 2V_2 = 1$ — (5)

$4V_2 - \cancel{V_3} = 3$ add e (4) + (5)

$-2V_2 + \cancel{V_3} = 1$

$2V_2 = 4$.

$\boxed{V_2 = 2 \text{ Volt}}$

$V_1 = 3V_2$.

$\boxed{V_1 = 6 \text{ Volt}}$

$4V_2 - V_3 = 3$

$8 - V_3 = 3$

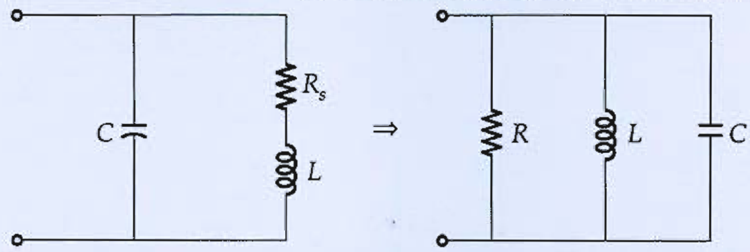
$8 - 3 = V_3$

$V_3 = 5V$

$\boxed{\begin{array}{l} V_1 = 6V \\ V_2 = 2V \\ V_3 = 5V \end{array}}$ Ans

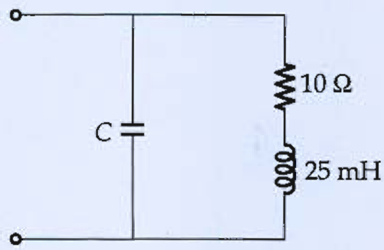
11

Q.1 (d) Show that a high Q-coil resonant circuit can be approximated as shown in figure.

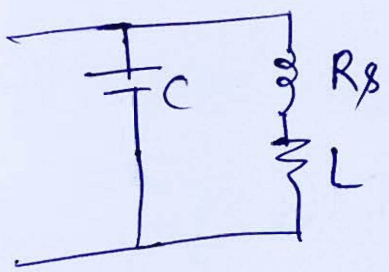


High Q-coil resonant circuit — Approximated equivalent circuit

For a practical tank circuit shown in figure below, the resonance occurs at 1 MHz. Assume a high Q-coil, find out the quality factor of high Q-coil at resonant frequency and the value of capacitance C.



[12 marks]



$$\frac{1}{Z} = \frac{1}{-jX_C} + \frac{1}{R_s + jX_L}$$

$$\Rightarrow \frac{+j}{X_C} + \frac{R_s - jX_L}{R_s^2 + X_L^2}$$

$$j\left(\frac{1}{X_C} - \frac{X_L}{R_s^2 + X_L^2}\right) + \frac{R_s}{R_s^2 + X_L^2}$$

$$\text{Img} = 0 \qquad \frac{1}{X_C} - \frac{X_L}{R_s^2 + X_L^2} = 0$$

$$X_L X_C = R_s^2 + X_L^2$$

$$W_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{LC} - \left(\frac{R_s}{L}\right)^2}$$

$$Q = \frac{1}{WCR_s}.$$

$$Q = \frac{\sqrt{L\cancel{C}}}{\cancel{C} R_s}$$

$$\boxed{Q = \frac{1}{R_s}\sqrt{\frac{L}{C}}.}$$

(ii). $f_0 = 1\,MHz$, $R_s = 10\Omega$, $L = 25\times10^{-3}H$

$$1\times10^6 = \frac{1}{2\times3.14}\frac{1}{\sqrt{0.25}}\sqrt{\frac{1}{0.25} - \frac{10}{25\times10^{-3}}}$$

$$LC = 10\times25\times10^{-3} \Rightarrow \frac{1}{2\times3.14\times0.5}\sqrt{100-400}.$$

$= 0.25$

4

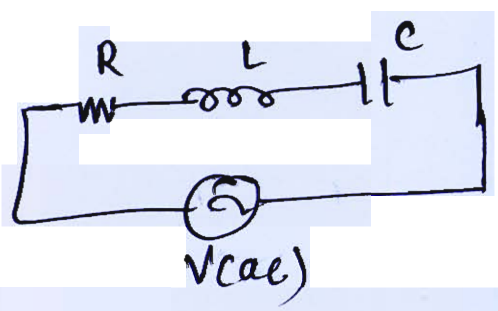
Q.1 (e) A series R-L-C circuit having $R = 25\ \Omega$, $L = 2$ H and $C = 30\ \mu$F is connected across an a.c. variable frequency source. At what frequencies will the phase angle of circuit be

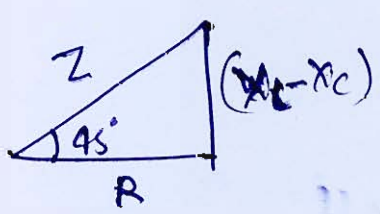
(i) 45° lagging and

(ii) 45° leading, the applied voltage.

[6 + 6 marks]

series R-L-C ckt





$$\tan 45 = \frac{X_L - X_c}{R} = 1$$

$$R = X_L - X_c$$

$$R = \omega L - \frac{1}{\omega C}$$

$$\omega^2 LC - 1 = R\omega C$$

put $R = 25\Omega$, $L = 2$ H, $C = 30\mu F$

$RC = 25 \times 30 \times 10^{-6}$ $= 75 \times 10^{-5}$ $LC = 2 \times 30 \times 10^{-6} = 6 \times 10^{-5}$

$6\times10^{-5}\,w^2 - 1 = 75\times10^{-5}\,w$

$6\times10^{-5}\,w^2 - 75\times10^{-5}\,w - 1 = 0$

$\boxed{W = 135.50 \text{ rad/sec}}$

$\boxed{f = 21.57\ Hz}$

6

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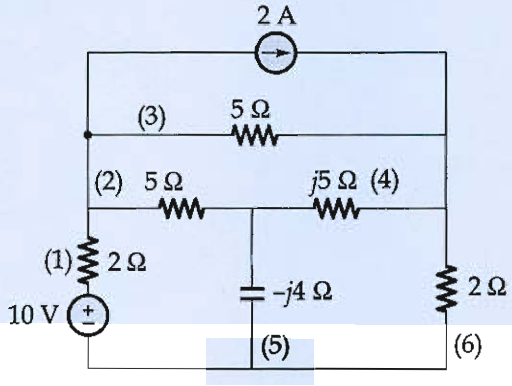
Q.2 (a) For the network shown below, draw its graph and obtain tie set matrix $[B_f]$, taking branches 2, 4, 5 as tree branches. Also, determine the loop impedance matrix and find the loop equations.



[20 marks]

Q.2 (b) **(i)** A heater takes 10 A at 50 V. Calculate the impedance of a choke of 5 Ω resistance to be placed in series with it in order that it may work at 200 V, 50 Hz supply. Find the power factor of the circuit.

(ii) State the maximum power transfer theorem. For the given circuit, what resistance should be connected across *x*-*y*, such that maximum power is developed across this load resistance? What is the amount of this maximum power?

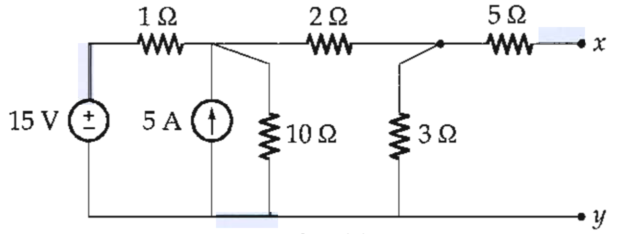


fig. (a)

[10 + 10 marks]

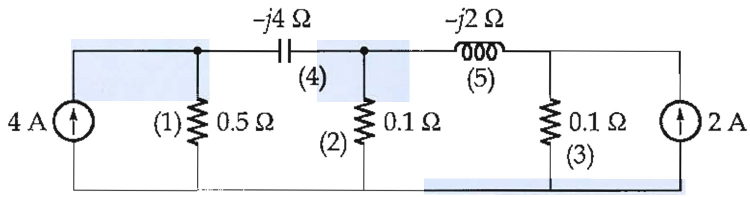
Q.2 (c) (i) A load Z draws 12 kVA at a power factor of 0.856 lagging from a 120 V rms sinusoidal source.

Calculate:

1. the average and reactive power delivered to the load.
2. the peak current and,
3. the load impedance.

(ii) For the circuit diagram shown below, draw its graph and

1. Obtain incidence matrix and cut-set matrix.
2. How many trees are possible for this circuit?



[10 + 10 marks]

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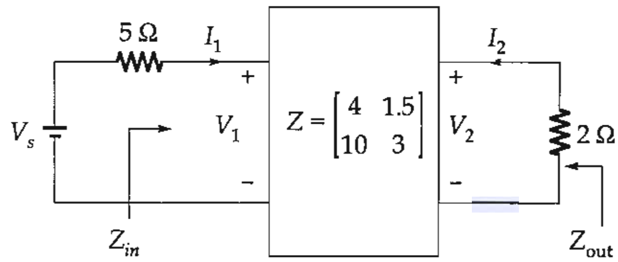
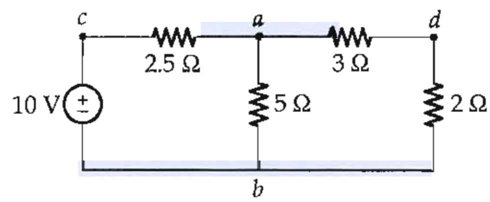
Q.3 (a) For the network shown below, find internal current gain G_I, voltage gain G_V, power gain G_P, input impedance Z_{in} and output impedance Z_{out}.

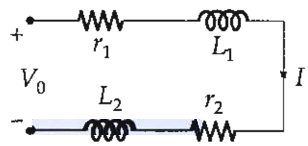


[20 marks]

Q.3 (b) **(i)** Consider the circuit shown below. If the resistance of 5 Ω branch increases to 6 Ω, determine the compensation source and verify the results.



(ii) In the circuit given below, $r_1 = 8.2\ \Omega$, $r_2 = 2.7\ \Omega$, $L_1 = 0.01$ H, $L_2 = 0.03$ **H**, $f = 25$ Hz and the circulating current $I = 10$ A.

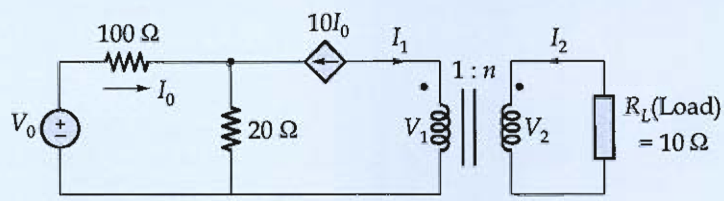


Find:

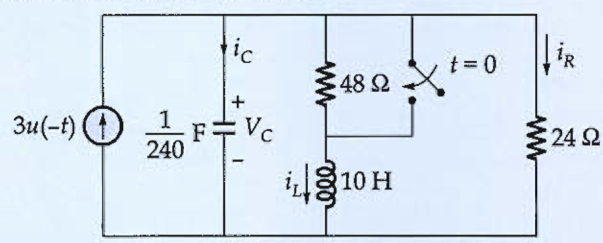
1. Voltage drop across each element.
2. Total resistive and inductive voltage drop.
3. Supply voltage.
4. Impedance angle of each of the R-L branch and power factor of the circuit.

[10 + 3 + 3 + 1 + 3 marks]

Q.3 (c) **(i)** What is the voltage and power gain of the circuit shown in figure? Assume $n = \frac{1}{10}$.



(ii) Consider the circuit shown below:



After being open for a long time, the switch is closed at $t = 0$. Find

1. $i_L(0^-)$ **2.** $V_C(0^-)$ **3.** $i_R(0^+)$

4. $i_C(0^+)$ **5.** $V_C(0.2)$ using Laplace transform approach.

[10 + 10 marks]

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Q.4 (a) Synthesize Cauer-I form and Cauer-II form of the network with driving point immitance function $Y(s) = \dfrac{(s^2+1)(s^2+5)}{s(s^2+3)}$

[20 marks]

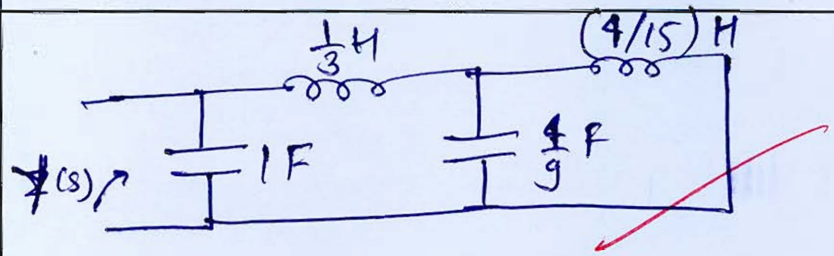
(I) Cauer-I form

$$Y(s) = \frac{s^4 + 5s^2 + s^2 + 5}{s^3 + 3s} = \frac{s^4 + 6s^2 + 5}{s^3 + 3s}$$

$$
\begin{array}{l}
s^3+3s \,)\, s^4 + 6s^2 + 5 \,(\, s \\
\qquad\quad\; s^4 + 3s^2 \\
\qquad\quad\; \overline{\quad 3s^2 + 5 \,)\, s^3 + 3s \,(\, \frac{s}{3}} \\
\qquad\qquad\qquad\qquad s^3 + \frac{5}{3}s \\
\qquad\qquad\qquad\qquad \overline{\frac{4}{3}s \,)\, 3s^2 + 5 \,(\, \frac{9}{4}s} \\
\qquad\qquad\qquad\qquad\qquad 3s^2 \\
\qquad\qquad\qquad\qquad\qquad \overline{5 \,)\, \frac{4}{3}s \,(\, \frac{4s}{3\times 5}} \\
\qquad\qquad\qquad\qquad\qquad\quad \frac{4}{3}s \\
\qquad\qquad\qquad\qquad\qquad\quad \overline{\times}
\end{array}
$$

$3 - \frac{5}{3}$

$\frac{4}{3} \times \frac{9}{4}$



(II) Cauer-II form

$$Y(s) = \frac{s^4 + 5s^2 + s^2 + 5}{s^3 + 3s} = \frac{s^4 + 6s^2 + 5}{s^3 + 3s}$$

$$Y(s) = \frac{5 + 6s^2 + s^4}{3s + s^3}$$

$3s + s^3 \,)\, 5 + 6s^2 + s^4 \,(\, \frac{5}{3s}$

$\quad 5 + \frac{5}{3}s^2$

$\quad -\quad -$

$\frac{13}{3}s^2 + s^4 \,)\, 3s + s^3 \,(\, \frac{9}{13s}$

$\quad 3s + \frac{9}{13}s^3$

$\quad -\quad -$

$\frac{4}{13}s^3 \,)\, \frac{13}{3}s^2 + s^4 \,(\, \frac{13}{3} \times \frac{13}{4s}$

$\quad \frac{13}{3}s^2$

$\quad -$

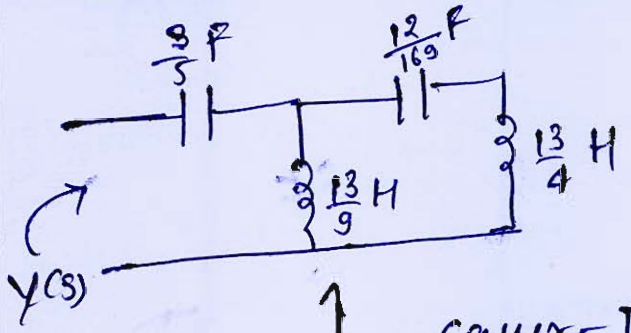
$s^4 \,)\, \frac{4}{13}s^3 \,(\, \frac{4}{13s}$

$\quad \frac{4}{13}s^3$

$\quad \times$

$6 - \frac{5}{3}$

$\frac{9}{13}$



Cauer – II form

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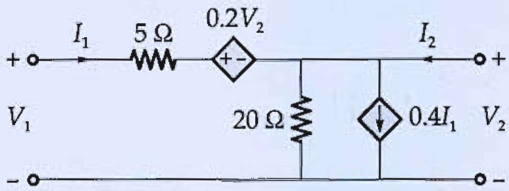
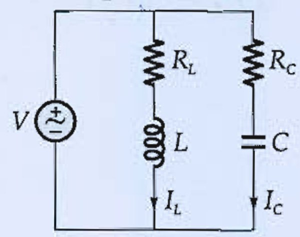
Q.4 (b) (i) Find the Y-parameters for the 2-port network shown below.



(ii) For the circuit shown, draw the phasor diagram. Derive the condition for the two branch currents, I_L and I_C to be in quadrature.



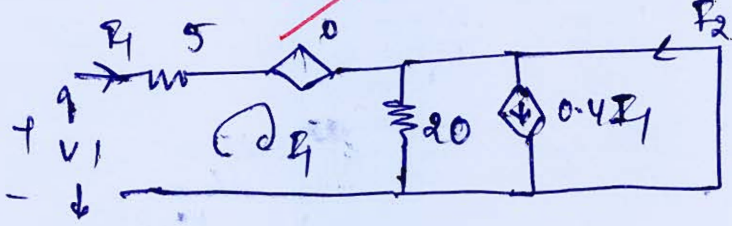
[10 + 10 marks]

<u>Y-parameter</u> $I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$.

Case-1

$\boxed{V_2 = 0}$



$V_1 = 5I_1 + 20(I_1 + I_2 - 0.4I_1)$

$V_1 = 5I_1 + 20(I_2 + 0.6I_1)$

$V_1 = 5I_1 + 20I_2 + 12I_1$.

$V_1 = 17I_1 + 20I_2$ —(1).

$20(I_2 - 0.4I_1 + I_1) = 0$

$20I_2 + 12I_1 = 0$

$20I_2 = -12I_1$

$I_2 = -\frac{3}{5}I_1$

$Y_{11} = \frac{I_1}{V_1}$, $Y_{21} = \frac{I_2}{V_1}$

Do not write in this margin

$Y/P \ \ 1V(1.3\frac{1}{5})$ put I_1 & I_2 value in eq(1)

$V_1 = 17I_1 + 20\left(\frac{-12}{20}\right)I_1$

$V_1 = 17I_1 - 12I_1$

$V_1 = 5I_1$

$Y_s = \frac{I_1}{V_1}$

$\boxed{Y_{11} = \frac{1}{5}\,\mho}$

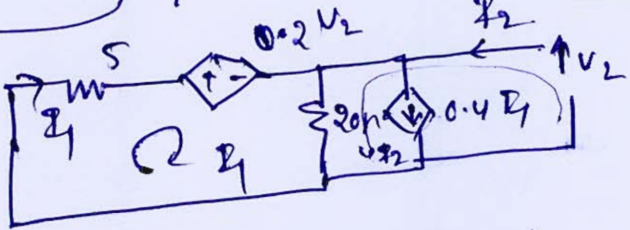
$V_1 = 17\left(\frac{-5}{3}\right)I_2 + 20I_2$.

$V_1 = -\frac{85}{3}I_2 + 20I_2$

$V_1 = -8.33\,I_2$.

$\boxed{Y_{21} = -0.12\,\mho}$

Case-2 $V_1 = 0$



$V_2 = 20(I_1 + I_2 - 0.4I_1)$

$V_2 = 20I_2 + 12I_1$ —(1)

$5I_1 + 0.2V_2 + V_2 = 0$.

$5I_1 = -1.2V_2 \rightarrow \left(I_1 = \frac{-1.2\,V_2}{5}\right)$ - put in eq(1)

$\frac{I_1}{V_2} = -\frac{1.2}{5}$

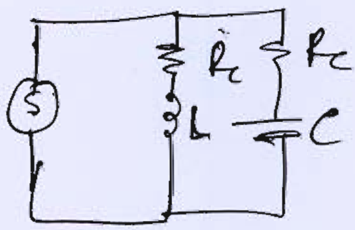
$\boxed{Y_{12} = -0.24\,\mho}$

$V_2 = 20I_2 + \frac{12(-1.2)V_2}{5}$

$V_2 = +17.12\,I_2$

$\boxed{\frac{I_2}{V_2} = 0.0584\,\mho}$

Q (4b)



$Y = Y_1 + Y_2$

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$$\frac{1}{Z} \Rightarrow \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

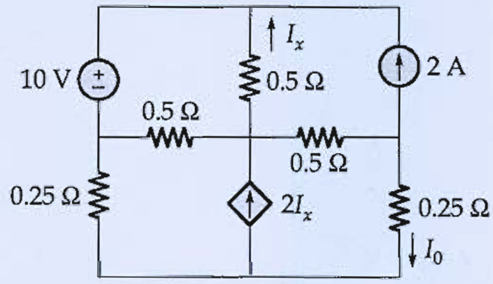
$$\Rightarrow \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C - jX_C}{R_C^2 + X_C^2}$$

$$\Rightarrow \left(\frac{R_L}{R_L^2+X_L^2} + \frac{R_C}{R_C^2+X_C^2}\right) - j\left(\frac{X_L}{R_L^2+X_L^2} + \frac{X_C}{R_C^2+X_C^2}\right)$$

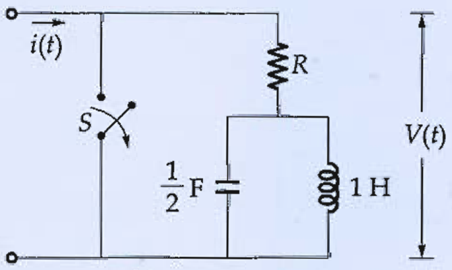
$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}}}$$

Img should be zero. $\frac{X_L}{R_L^2+X_C^2} = \frac{X_C}{R_C^2+X_C^2}$

Q.4 (c) **(i)** Determine I_0 in figure using nodal analysis:



(ii) The circuit shown has zero initial energy. At $t = 0$, the switch 'S' is opened. Find the value of resistor R for the given excitation such that the response is $V(t) = 0.5 \sin \sqrt{2}t\, u(t)$.



The excitation is $i(t) = te^{-\sqrt{2}t}\, u(t)$. **[10 + 10 marks]**

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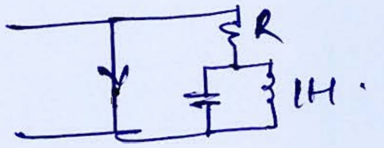
(H) $R = 2$ $\quad v(t) = 0.5 \sin \sqrt{2}t\, u(t)$

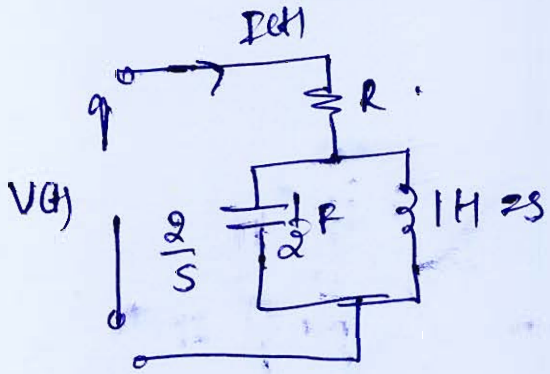
$i(t) = t\, e^{-\sqrt{2}t}\, u(t)$



$V_C(0^-) = 0$

$\cancel{I_L(0^-) = 0}$

S/W open at $t = 0$



2

$Z_{eq} = R + \left(\frac{1}{Cs} \parallel s\right)$

$\Rightarrow R + \dfrac{\frac{1}{Cs}\cancel{s}}{\frac{1}{Cs} + s} \Rightarrow R + \dfrac{(1/C)Cs}{Cs^2 + 1}$

1

$Z_{eq} = \dfrac{1}{s(s^2c+1)} + R \Rightarrow \dfrac{1}{s(s^2 0.5 + 1)} + R.$

$$\boxed{Z_{eq} \Rightarrow \frac{1}{s(0.5s^2+1)} + R.}$$

$$Z_{eq} = \frac{V(s)}{I(s)} = \frac{\frac{\sqrt{2} \times 0.5}{s^2+2}}{\frac{2}{(s+\sqrt{2})^2}}$$

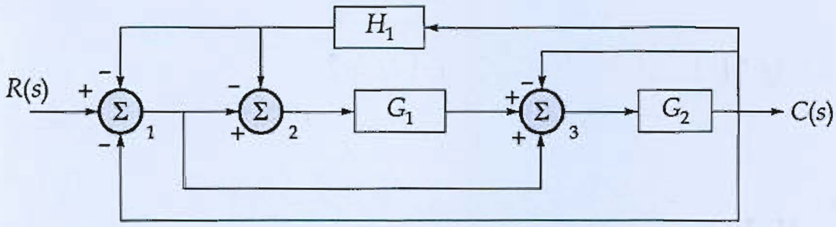
$$I(t) = t\, e^{-\sqrt{2}t}\, u(t)$$

$$I(s) = \frac{2}{(s+\sqrt{2})^2}$$

$$Z_{eq} = R +$$

Section B : Control Systems

Q.5 (a) Using block diagram reduction technique, find the transfer function $\frac{C(s)}{R(s)}$ for the system shown below:



[12 marks]

Q.5 (b) The closed loop transfer function of a control system is given as $\frac{10}{s^3 + 0.1s^2 + 10}$.

Determine the steady state error of the system when input is $5 + 10t + 4t^2$.

[12 marks]

$e_{ss_1} = 2$ for unitstep $e_{ss} = \frac{A}{1+k_p}$

$e_{ss_2} = 2$ for unitramp $e_{ss} = \frac{A}{k_v}$

$e_{ss_3} = 2$ for unit parabol $e_{ss} = \frac{A}{k_a}$.

$e_{ss} = e_{ss_1} + e_{ss_2} + e_{ss_3}$ —(1)

k_p = positional error cont

k_v = velocity "

k_a = acceln "

$k_p = \lim_{s \to 0} GH$

$k_p = \lim_{s \to 0} \frac{10}{s^3 + 0.1s^2}$

$\boxed{GH = \frac{10}{(s^3 + 0.1s^2)}}$

$\boxed{k_p = \infty}$ $\boxed{e_{ss_1} = \frac{5}{1+\infty} = 0}$

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$$k_v = \lim_{s \to 0} s \cdot GH.$$

$$= \lim_{s \to 0} s \cdot \frac{10}{s^3 + 0.1 s^2}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \cdot 10}{\cancel{s}(s^2 + 0.1 s)}$$

$$\boxed{k_v = \infty} \quad e_{ss_2} = \frac{A}{k_v} = 0.$$

$$\boxed{e_{ss_2} = 0}$$

4

$$k_a = \lim_{s \to 0} s^2 \, GH.$$

$$= \lim_{s \to 0} \cancel{s^2} \cdot \frac{10}{\cancel{s^2}(s + 0.1)}$$

$$\Rightarrow \frac{10}{0.1} = 100$$

$$k_a = 100$$

$$e_{ss_3} = \frac{A}{k_a} \Rightarrow \frac{4}{100}$$

$\frac{8}{100}$

$$e_{ss_3} = 4 \times 10^{-2}.$$

$$e_{ss} = 0 + 0 + 4 \times 10^{-2}$$

$$\boxed{e_{ss} = 4 \times 10^{-2}}$$

0

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Q.5 (c) The open loop transfer function of a unity feedback system is given by $G(s) = \frac{K}{s(1+sT)}$, where T and K are constants having positive values. By what factor the amplifier gain be reduced so that

(i) the peak overshoot of unit step response of the system is reduced from 75% to 20%.

(ii) the damping ratio increases from 0.2 to 0.6.

[12 marks]

(I) mp changes 75% to 20%

% Mp = % peak overshoot

$\% mp_1 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

$e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.75$

bothside loge

$\frac{-\pi\xi}{\sqrt{1-\xi^2}} = \log_e(0.75)$

$-3.14\xi = -0.287\sqrt{1-\xi^2}$

squaring bothside

$9.85\xi^2 = 0.082(1-\xi^2)$

$9.85\xi^2 = 0.082 - 0.082\xi^2$

$9.932\xi^2 = 0.082$

$\xi^2 = 8.256\times10^{-3}$

$\boxed{\xi = 0.090}$

let $\xi_1 = 0.090$

$mp_2 = 20\%$

$e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.2$

$\frac{-\pi\xi}{\sqrt{1-\xi^2}} = -1.60$

$-\pi\xi = -1.6\sqrt{1-\xi^2}$

squaring bothside

$9.85\xi^2 = 2.56(1-\xi^2)$

$9.85\xi^2 = 2.56 - 2.56\xi^2$

$12.41\xi^2 = 2.56$

$\xi^2 = 0.206$

$\xi = 0.454$

$mp_2 \rightarrow \xi_2$

$\boxed{\xi_2 = 0.454}$

char. eqn $1+GH=0$, $1+\frac{K}{s(1+sT)}$

$$s^2T+s+K=0$$

$$s^2+\frac{s}{T}+\frac{K}{T}=0 \quad (1)$$

$$s^2+2\xi\omega_n s+\omega_n^2=0 \quad (2)$$ - 2nd order eqn.

compare eq (1) & (2)

$$2\xi\omega_n=\frac{1}{T}, \quad \omega_n=\sqrt{\frac{K}{T}}$$

$$\xi \Rightarrow \frac{1}{2T\omega_n} \qquad \xi_2=\frac{1}{2\sqrt{K_2T}}$$

$$\xi_1 \Rightarrow \frac{1}{2T}\sqrt{\frac{T}{K_1}} \qquad \frac{\xi_1}{\xi_2} \Rightarrow \frac{\sqrt{K_2}}{\sqrt{K_1}}$$

$$\xi_1 \Rightarrow \frac{1}{2\sqrt{K_1T}}$$

$$\frac{0.090}{0.454}=\sqrt{\frac{K_2}{K_1}}$$

$$0.0392=\frac{K_2}{K_1}$$

$$\boxed{K_2=0.0392K_1}$$

SC

(ii) $\boxed{\xi_1 \rightarrow 0.2,\ \xi_2=0.6}$

$$\frac{\xi_1}{\xi_2} \Rightarrow \sqrt{\frac{K_2}{K_1}}$$

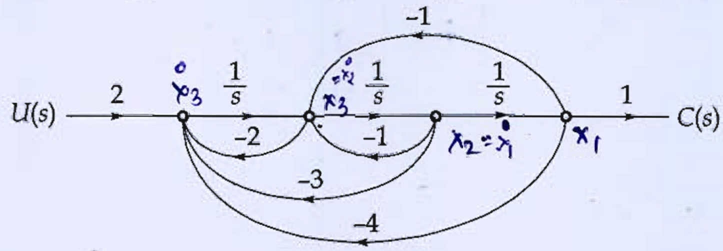
$$\frac{0.2}{0.6}=\sqrt{\frac{K_2}{K_1}}$$

$$0.111=\frac{K_2}{K_1}$$

$$\boxed{K_2=0.111K_1}$$

11

Q.5 (d) A control system is represented using the signal flow graph shown below:



(i) Construct a state model for the above system.

(ii) Using the state model obtained in part (i), find the transfer function $\frac{C(s)}{U(s)}$.

[4 + 8 marks]

(i) $\dot{x}_1\ \dot{x}_2\ \dot{x}_3$ – state variable.

$$\dot{x}_3 = 2u(t) - 2x_3 - 3x_2 - 4x_1 .$$

$$\dot{x}_2 = x_3 - x_2 - x_1 \qquad , \; y = x_1$$

$$\dot{x}_1 = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u(t) \quad \text{— (1)}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\dot{x}(t) = Ax(t) + Bu(t)$

$y(t) = Cx(t) + Du(t)$ — (2)

4

y = o/p , $u(t)$ = input.

compare eqn (1) & (2)

(ii) find out transfer fun.

$$\boxed{T(s) = \left[C(sI-A)^{-1}B + D\right]} \qquad D = 0$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ -4 & -3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

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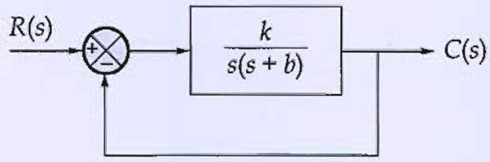
$$SI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 4 & 3 & 2 \end{bmatrix}$$

$$(SI-A) = \begin{bmatrix} s & -1 & 0 \\ 1 & s+1 & -1 \\ 4 & 3 & s+2 \end{bmatrix}, \quad |SI-A| = \begin{vmatrix} s & -1 & 0 \\ 1 & s+1 & -1 \\ 4 & 3 & s+2 \end{vmatrix}$$

$$(SI-A)^{-1} = \frac{\text{adj}\,(SI-A)^{-1}}{|SI-A|}$$

Q.5 (e) (i) Consider the feedback system shown in figure. Find the values of k and b to satisfy the following frequency-domain specifications: $M_r = 1.6$, $\omega_r = 15$ rad/s.

(ii) For the values of k and b determined in part (a), calculate the settling time and bandwidth of the system.



[12 marks]

(i) given that $M_r = 1.6$, $\omega_r = 15$ rad/sec.

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\frac{1}{2\xi\sqrt{1-\xi^2}} = 1.6.$$

$$2\xi\sqrt{1-\xi^2} = 0.625$$

squring both side

$$4\xi^2(1-\xi^2) = 0.3906$$

$$4\xi^2 - 4\xi^4 - 0.3906 = 0$$

let $\xi^2 = x$, $4x - 4x^2 - 0.3906 = 0$

$x_1 = 0.1096$, $\xi = 0.943$

$x_2 = 0.890$, $\xi = 0.3310$

sate.

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$15 = \omega_n\sqrt{1-2\times 0.3310^2}$$

$$\boxed{\omega_n = 16.97 \text{ rad/sec}}$$

Char equation.

$$1 + GH = 0$$

$$1 + \frac{K}{s(s+b)} = 0$$

$$s^2 + bs + K = 0 \quad \text{—(I)}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad \text{—(II)}$$

↳ 2nd order

Comparing eq (1) & (2)

$$2\xi\omega_n = b.$$

$$2\times 0.33\times 16.97 = b$$

$$\boxed{b = 11.23}$$

$$\omega_n^2 = k$$

$$k = (16.97)^2$$

$$\boxed{k = 287.98}$$

$$\boxed{\begin{matrix} \xi = 0.33 \\ \omega_n = 16.97 \text{ rad/Se} \end{matrix} \rightarrow \begin{matrix} b = 11.23 \\ k = 287.98 \end{matrix}}$$

~~at $\xi = 0.943$, $\omega_n = ?$, $15 = \omega_n\sqrt{1-2\times(0.943)^2}$~~

Q 5 e (ii) Settling time.

$t_s = \frac{4}{\xi\omega_n} \longrightarrow 2\%$

$t_s = \frac{3}{\xi\omega_n} \longrightarrow 5\%$

$$\boxed{t_s = \frac{4}{0.33\times16.97} = 0.7142 \text{ sec}}$$

$$\boxed{t_s = \frac{3}{0.33\times16.97} = 0.53570 \text{ sec}}$$

BW = Bandwidth of sys

$$\boxed{BW = \omega_n\sqrt{1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}}}$$

$$= 16.97\sqrt{1-2\times(0.33)^2+\sqrt{4(0.33)^4-4(0.33)^2+2}}$$

$$= 16.97\sqrt{1-2\times0.1089+\sqrt{4\times0.01185-4\times0.1089+2}}$$

$$= 16.97\sqrt{0.7822+\sqrt{1.6118}}$$

$$= 16.97\times1.432$$

$$\boxed{BW = 24.30 \text{ rad/sec.}}$$

11

Q.6 (a) The open loop transfer function of a control system with unity feedback is given by

$$G(s) = \frac{(2K+5)}{s(s-(2+K))}$$

Calculate K for which

(i) The system is stable.

(ii) Both the poles of characteristic equation lies in the left of $s + 1 = 0$ line.

(iii) One pole of the characteristic equation is present in left of $s + 1 = 0$ line.

(iv) Poles are at $-0.125 + 0.7i$ and at $-0.125 - 0.7i$. **[20 marks]**

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Q.6 (b) Write a short note on the following compensators:

(i) Lag compensator

(ii) Lead compensator

(iii) Lead-lag compensator

[6 + 6 + 8 marks]

Q.6 (c) A linear time-invariant system is characterized by the homogeneous state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i) Compute the solution of the homogeneous equation assuming the initial state vector $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(ii) Consider now that the system has a forcing function and is represented by the following non homogeneous state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

where u is a unit-step function. Compute the solution of this equation assuming initial conditions of part (a).

[10 + 10 marks]

Q.7 (a) The block diagram of a unity feedback system is shown in figure (i) and its step response is shown in figure (ii). With the help of the given figures, calculate:

(i) closed loop transfer function.

(ii) the minimum value of 'K' for which the step response of the system would exhibit an overshoot as shown in figure (ii).

(iii) If 'K' is taken twice of the minimum value, then calculate the time period 'T' indicated in figure (ii)

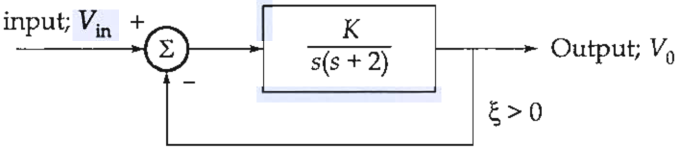


Fig. (i)

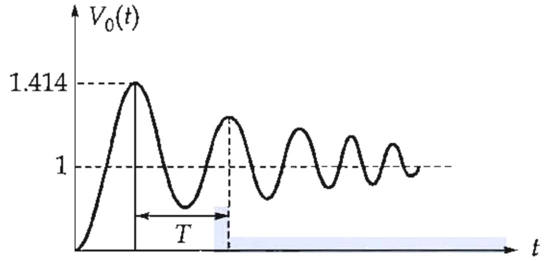
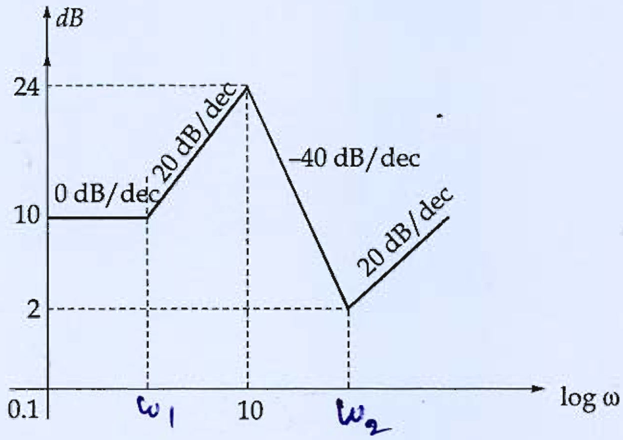


Fig. (ii)

[20 marks]

Q.7 (b) (i) Find the transfer function for the bode plot shown below:



(ii) Sketch the polar plot for the following transfer function:

$$G(s) = \frac{1+5s}{s^2(1+s)(1+2s)}$$

Also, calculate: Phase crossover frequency and corresponding gain margin.

[10 + 10 marks]

Soln 7b(i) $T(s) = \dfrac{K\left(\dfrac{s}{\omega_1}+1\right)\left(\dfrac{s}{\omega_2}+1\right)^3}{\left(\dfrac{s}{10}+1\right)^3}$ —(1)

Find value ω_1 & ω_2

$(\omega_1, 10)$ $(10, 24)$
x_1 y_1 x_2 y_2.

Slop = 20 dB/dec = $\dfrac{y_2-y_1}{x_2-x_1}$ $\Rightarrow 20 = \dfrac{24-10}{\log_{10}10-\log_{10}\omega_1}$

$\dfrac{20}{14} = \dfrac{1}{1-\log_{10}\omega_1}$, $1.428 = \dfrac{1}{1-\log_{10}\omega_1}$

$1-\log_{10}\omega_1 = 0.7$.

$1-0.7 = \log_{10}\omega_1$

$0.3 = \log_{10}\omega_1$

$\boxed{\omega_1 = 1.99 \text{ rad/sec}}$

$(10, 24)$ $(\omega_2, 2)$
$x_1 \quad y_1 \quad x_2 \quad y_2$.

$$\text{slop} = \frac{y_2 - y_1}{x_2 - x_1}$$

slop = -40 dB/dec

$$\Rightarrow -40 = \frac{2-24}{\log_{10}\omega_2 - \log_{10}10}$$

$$+40 = \frac{+22}{\log_{10}\omega_2 - 1}$$

$$\log_{10}\omega_2 - 1 = \frac{22}{40}, \quad \log_{10}\omega_2 - 1 = 0.55$$

$$\log_{10}\omega_2 = 1.55$$

$$\boxed{\omega_2 = 35.48 \text{ rad/sec}}$$

put ω_1 & ω_2 value in eq (1)

$$\text{Trasferfun}^n \; T(s) = \frac{k\left(\frac{s}{1.99}+1\right)\left(\frac{s}{35.48}+1\right)^3}{\left(\frac{s}{10}+1\right)^3}$$

now, find out value of $k = 2$

initial slop $= 0$

$$M = +20 r \log \omega + 20\log_{10} k$$

$$M = 0 + 20\log_{10} k$$

$$10 = 20\log_{10} k$$

$k = 3.16$

$$\boxed{T(s) = \frac{3.16\left(\frac{s}{1.99}+1\right)\left(\frac{s}{35.48}+1\right)^3}{\left(\frac{s}{10}+1\right)^3}}$$

Q.2 b (ii) sketch polar plot

$G(s) = \dfrac{1+5S}{s^2(1+s)(1+2S)}$, put $s = j\omega$.

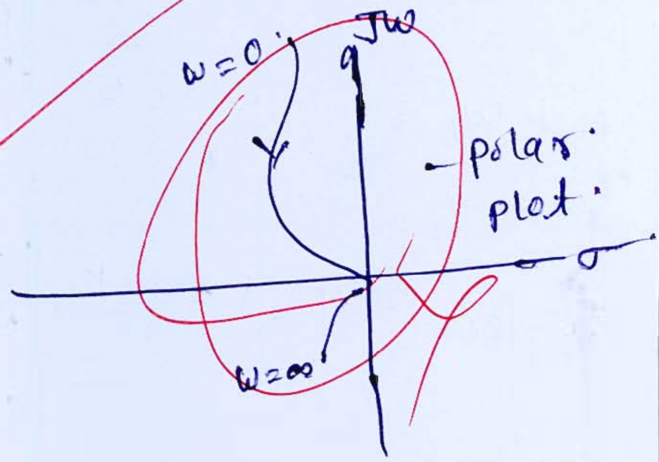
$= \dfrac{1+5j\omega}{-\omega^2(j\omega+1)(1+2j\omega)}$

find out magnitude and Angle

$|M| = \dfrac{\sqrt{1+25\omega^2}}{\omega^2\sqrt{(\omega^2+1)}\sqrt{1+4\omega^2}}$

$\angle\phi = -180 - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}5\omega$

at $\omega = 0$	$\omega = \infty$
$M = \infty$	0
$\phi = -180$	-270



Now calculate $\omega_{pc} = ?$

$\angle\phi = -180$

$-180 - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}5\omega = -180$

$-\tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}5\omega = 0$

$\tan^{-1}\omega + \tan^{-1}2\omega - \tan^{-1}5\omega = 0$

$\tan^{-1}\left(\dfrac{\omega+2\omega}{1-2\omega^2}\right) - \tan^{-1}5\omega = 0$

$\tan^{-1}\left[\dfrac{\dfrac{\omega+2\omega}{1-2\omega^2} - 5\omega}{1 + \dfrac{3\omega}{1-2\omega^2} \times 5\omega}\right] = 0$

$\dfrac{3\omega}{1-2\omega^2} - 5\omega = 0$

$3\omega - 5\omega(1-2\omega^2) = 0$

$3 - 5(1-2\omega^2)$

$3 = 5 - 10\omega^2 = 0$

$+2 = -10\omega^2$

$\omega = \dfrac{1}{\sqrt{5}}$ rad/sec

$\boxed{\omega_{pc} = \dfrac{1}{\sqrt{5}} \text{ rad/sec}}$

Now find out

$GM = \dfrac{1}{x}\Big|_{\omega=\omega_{pc}}$

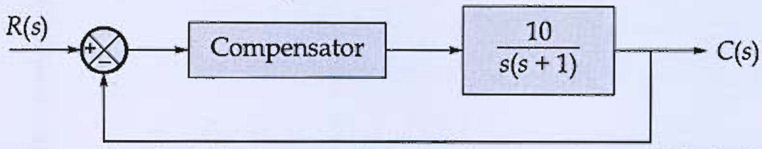
$x = \dfrac{\sqrt{1+25\omega^2}}{\omega^2\sqrt{\omega^2+1}\sqrt{1+4\omega^2}}$ (1.095, 1.34)

$\Rightarrow \dfrac{2.449}{0.2 \times 1.095 \times 1.34}$

$x = 8.345$

$\boxed{GM = 0.1198}$

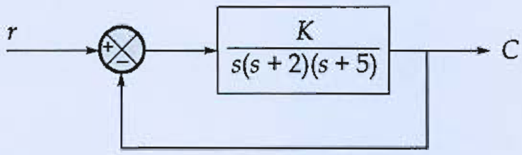
Q.7 (c) Consider the feedback control system shown below:



The compensator block of the system is to be designed, such that the overall system will have a velocity error coefficient of 10 and a minimum phase margin of 43°. Compare the phase margin of the uncompensated system and compensated system.

[20 marks]

Q.8 (a) Consider the following control system.



(i) Sketch the root locus diagram for $0 < K < \infty$

(ii) Without the help of root locus diagram, determine the value of K that gives the system characteristic equation with a damping ratio of 0.5.

[10 + 10 marks]

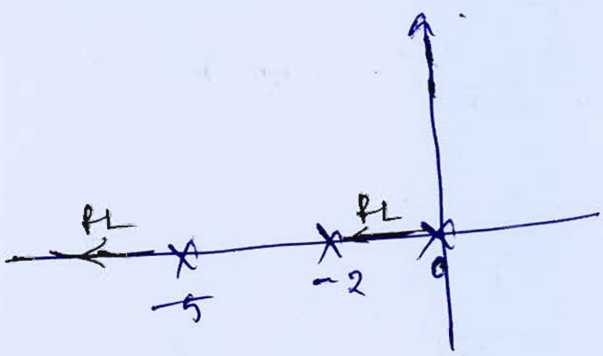
(i)

Sol^n Step-1 $\quad GH(s) = \dfrac{K}{s(s+2)(s+5)}$

No of poles = 3, $s = 0$, $s = -2$, $s = -5$

No of zero ⇒ None.

Step-2 Draw pole zero diagram.



Step-3 Angle of asymptotes.

$$\phi_A = \frac{(2k+1)\,180^\circ}{P-Z} \qquad \therefore k = 0, 1, 2, 3 \ldots$$

$$\phi_A = \frac{(2k+1)\,180}{3} \Rightarrow 60^\circ,\ 180^\circ,\ 300^\circ$$

Step-4 centroid

$$\sigma = \frac{\Sigma \text{ Real value of pole} - \Sigma \text{ Real value of zero}}{P-Z}$$

$$\sigma = \frac{0-2-5-0}{3}$$

$$\sigma = \frac{-7}{3} \quad , \quad \boxed{\sigma = -2.33}$$

<u>Step 5</u> find out breakaway point

Char. equation $1+GH=0$.

$$1+\frac{K}{s(s+2)(s+5)}=0$$

$$(s^2+2s)(s+5)+K=0$$

$$s^3+5s^2+2s^2+10s+K=0$$

$$K=-s^3-5s^2-2s^2-10s$$

$$\frac{dK}{ds}=0 \quad , \quad -3s^2-10s-4s-10=0$$

$$3s^2+10s+4s+10=0.$$

$$3s^2+14s+10=0$$

$$\boxed{s=-0.88 \quad , \quad -3.786 \times}$$

valid break point $= -0.88$, $\boxed{s=-0.88}$

Lie on Root locus.

<u>Step-6</u> Intersection of Img axis.

using RH Criteria.

Char. Eqn $1+GH=0$

$$s^3+7s^2+10s+K=0 \quad -(1)$$

s^3 1 10

s^2 7 K

s^1 $\frac{70-K}{7}$ 0

s^0 K 0

for Intersection on Img $s^1 = 0$, $\boxed{K = 70}$

Auxillary Eqn $\Rightarrow 7s^2 + K = 0$

$K = -7s^2$ $\therefore s = j\omega$

$70 = -7s^2$

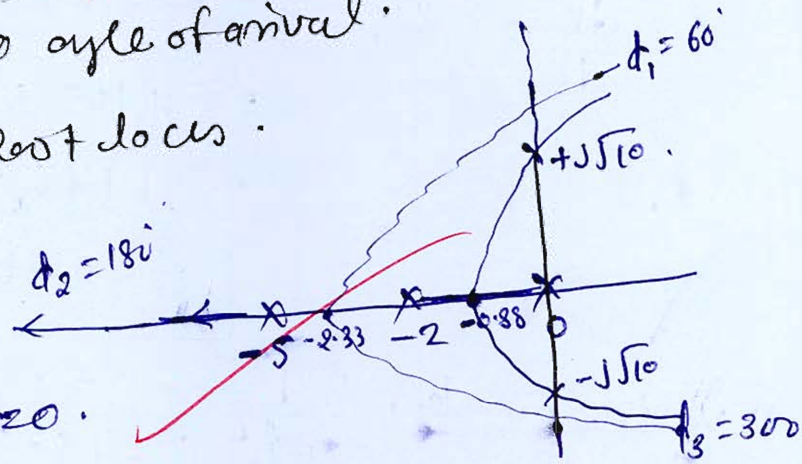
$10 = -(j\omega)^2$

$10 = -(j^2\omega^2)$, $\omega^2 = 10$

$\omega = \pm\sqrt{10}$

step-7- no angle of departure and no angle of arrival.

step 8 draw Root locus.



(ii) char. eqn $1 + GH = 0$.

$1 + \frac{K}{s(s+2)(s+5)} = 0$ $\therefore \xi = 0.5$

$s^3 + 7s^2 + 10s + K = 0$ — (i)

$(s+a)(s^2 + 2\xi\omega_n s + \omega_n^2) = 0$

$s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + as^2 + 2\xi\omega_n s a + \omega_n^2 a = 0$

$s^3 + (2\xi\omega_n + a)s^2 + (\omega_n^2 + 2\xi\omega_n a)s + \omega_n^2 a = 0$ — (ii)

compare eq (i) & (ii) $\omega_n^2 + 2\xi\omega_n a = 10$, $\omega_n^2 a = K$.

$2\xi\omega_n + a = 7$

$2 \times 0.5\omega_n + a = 7$

$\omega_n + a = 7$

$\omega_n^2 + \omega_n a = 10$

$\omega_n^2 + \omega_n(\omega_n - 7) = 10$

$\omega_n^2 + \omega_n^2 - 7\omega_n = 10$

$2\omega_n^2 - 7\omega_n - 10 = 0$

$\omega_n = 4.58$ rad

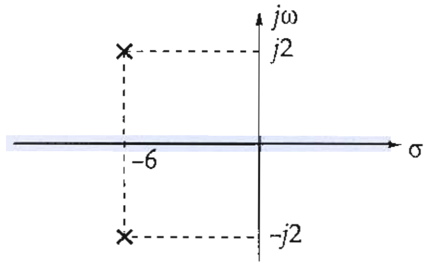
$a = 7 - \omega_n$

$a = 2.42$

$K = (4.58)^2 \times 2.42$

$\boxed{K = 50.76}$

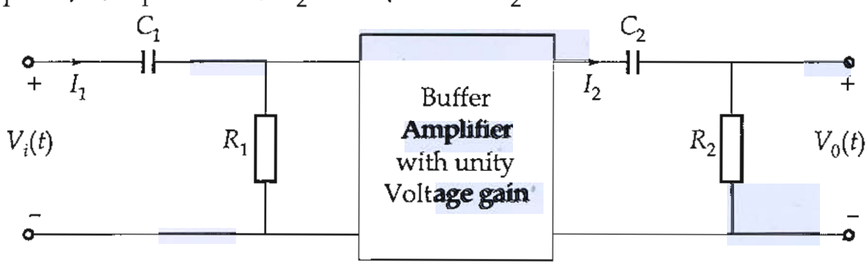
Q.8 (b) (i) The closed-loop poles of a system is shown in figure below:



Find the

1. Transfer function of the system
2. Settling time for 2% tolerance band.
3. Percentage peak overshoot.
4. Rise time
5. Delay time

(ii) Determine the transfer function relating $V_0(s)$ and $V_i(s)$ for network shown in figure below. Calculate output voltage, $t \geq 0$ for a unit step voltage input at $t = 0$ when $C_1 = 1\ \mu F$, $R_1 = 1\ M\Omega$, $C_2 = 0.5\ \mu F$ and $R_2 = 1\ M\Omega$.



[10 + 10 marks]

8 b (i) Sol^n

$$\text{poles} = (s+6 \pm 2j)$$

$$T(s) \Rightarrow \frac{K}{(s+6+2j)(s+6-2j)}$$

$$T(s) \Rightarrow \frac{K}{(s+6)^2+4} \Rightarrow \frac{K}{s^2+36+12s+4}$$

$$T(s) = \frac{K}{s^2+12s+40}$$

$$\boxed{T(s) = \frac{K}{s^2+12s+40}}$$ — transfer fun^n of sys.

char equation $1+GH=0$

$s^2+12s+40=0$ .. ①

$s^2+2\xi\omega_n s+\omega_n^2=0$ — ② — standard 2nd order eqn.

Compare eqn ① & ②

$2\xi\omega_n=\cancel{12}\ 6$

$\boxed{\xi\omega_n=6}$, $\omega_n^2=40$, $\boxed{\omega_n=6.32\text{ rad/sec}}$

② for 2% tolerance band

$T_s=\frac{4}{\xi\omega_n} \Rightarrow \frac{4}{6}=2/3$

$\boxed{T_s=0.66\text{ sec}}$

③ $\xi\omega_n=6$

$\xi=\frac{6}{6.32}$, $\boxed{\xi=0.948}$

% mp = % peak overshoot

$\boxed{\%mp=e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}\times100}$

$=e^{\frac{-\pi 0.948}{\sqrt{1-0.948^2}}}\times100$

$\%mp=8.672\times10^{-3}$.

④ Risetime, $\boxed{T_r=\frac{\pi-\phi}{\omega_d}}$, $\therefore \phi=\cos^{-1}\xi$

$\phi=\cos^{-1}(0.948)$

$\boxed{\phi=18.55^\circ}$

↳ 18.55 → Radian

$\frac{18.55\times3.14}{180}=0.323\text{ Rad}$

$\omega_d=\omega_n\sqrt{1-\xi^2}$

$\omega_d=6.32\sqrt{1-(0.948)^2}$

$\boxed{\omega_d=2.01\text{ rad/sec}}$

$$T_r = \frac{3.14 - 0.323}{2.01} = \frac{2.817}{2.01}$$

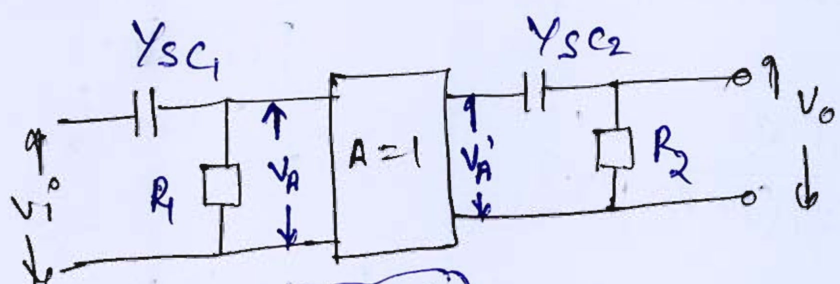
$$\boxed{T_r = 1.4014 \text{ sec}}$$

(5) Delay time $\boxed{T_d = \frac{1 + 0.7\xi}{\omega_n}}$

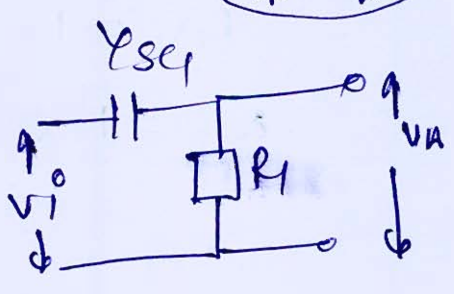
$$T_d = \frac{1 + 0.7 \times 0.948}{6.32}$$

$$\boxed{T_d = 0.263 \text{ sec}}$$

Q 8 a(ii) Determine transfer fun.



$V_A = V_A'$

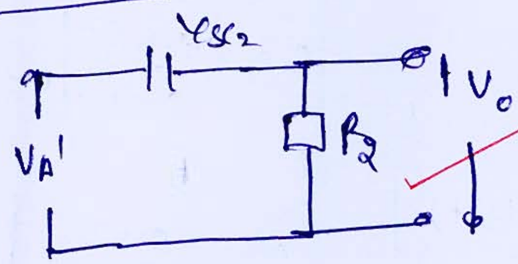


$$\frac{V_A}{V_i} = \frac{R_1}{R_1 + \frac{1}{SC_1}}$$

$$\boxed{\frac{V_A}{V_i} \Rightarrow \frac{R_1 SC_1}{R_1 SC_1 + 1}}$$

$\Rightarrow V_A = V_A'$

$$\boxed{\frac{V_A'}{V_i} = \frac{R_1 SC_1}{R_1 SC_1 + 1}}$$



$$\frac{V_o}{V_A'} = \frac{R_2}{R_2 + \frac{1}{SC_2}}$$

$$\boxed{\frac{V_o}{V_A'} \Rightarrow \frac{R_2 SC_2}{R_2 SC_2 + 1}}$$ (1)

$$V_A' = \frac{R_1 S C_1 V_i}{R_1 S C_1 + 1} \quad \text{put in eq}^n \ (1)$$

$$V_o = \left[\frac{R_2 S C_2}{R_2 S C_2 + 1}\right] \frac{R_1 S C_1 V_i}{R_1 S C_1 + 1}$$

$$\boxed{\frac{V_o}{V_i} = \frac{R_2 S C_2 R_1 S C_1}{(R_2 S C_2 + 1)(R_1 S C_1 + 1)} \Rightarrow \frac{S^2 R_1 R_2 C_1 C_2}{(1 + S R_2 C_2)(1 + S R_1 C_1)}}$$

calculate $V_0 = ?$ $t \geq 0$, input unit step.

$C_1 = 1\,\mu F$, $R_1 = 1\,M\Omega$, $C_2 = 0.5\,\mu F$, $R_2 = 1\,M\Omega$.

$$\boxed{\begin{aligned} V_i &= u(t) \\ V_i(S) &= \frac{1}{S} \end{aligned}}$$

$$V_o(s) = \frac{S^2 R_1 R_2 C_1 C_2}{(1 + S R_2 C_2)(1 + S R_1 C_1)} \frac{1}{S}$$

$$V_o(s) = \frac{S R_1 R_2 C_1 C_2}{(1 + S R_2 C_2)(1 + S R_1 C_1)}$$

$\therefore R_1 C_1 = 10^6 \times 10^{-6}$

$R_1 C_1 = 1$

$R_2 C_2 = 10^6 \times 0.5 \times 10^{-6}$

$R_2 C_2 = 0.5$

$$V_o(s) = \frac{S(1)(0.5)}{(1 + 0.5S)(1 + S)}$$

$$V_o(s) = \frac{0.5 S}{(1 + 0.5S)(S + 1)}$$ — solve using partial fraction

$$\frac{A}{1 + 0.5S} + \frac{B}{S + 1}$$

10

$$A = \left.\frac{0.5S}{S + 1}\right|_{S = -2}$$

$\boxed{A = 1}$

$$V_o(s) = \frac{1}{1 + 0.5S} + \frac{1}{S + 1}$$

taking inverse Laplace form

$$\boxed{V_o(t) \Rightarrow 2 e^{-2t} u(t) - e^{-t} u(t)}$$

$t \geq 0$

$$B = \left.\frac{0.5S}{1 + 0.5S}\right|_{S = -1}$$

$\boxed{B = -1}$

$\frac{-0.5}{+0.5}$

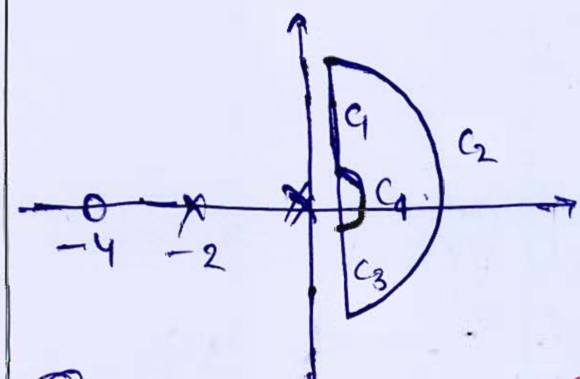
Q.8 (c) Sketch the Nyquist plot and using the plot, assess the stability of the closed loop system whose open-loop transfer function is

$$G(s)H(s) = \frac{K(s+4)}{s^2(s+2)}$$

[20 marks]

<u>Sol^n</u> open loop transfer function,

$$GH = \frac{K(s+4)}{s^2(s+2)}$$



(1) <u>for Region C1</u>, Draw polar plot.

put $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{k(j\omega+4)}{-\omega^2(j\omega+2)}$$

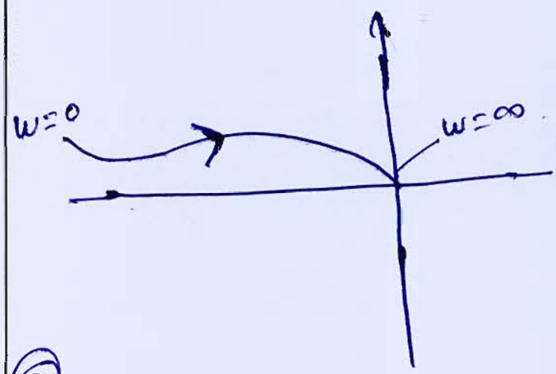
<u>magnitude</u> $|G(j\omega)H(j\omega)|$

$$\Rightarrow \frac{K\sqrt{\omega^2+16}}{\omega^2\sqrt{\omega^2+4}} \quad \text{--- (1)}$$

<u>angle</u> $\angle G(j\omega)H(j\omega) \Rightarrow -180 - \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{4}$ --- (2)

	at $\omega = 0$	$\omega = \infty$.
$\|M\|$.	∞	~~180~~
$\angle\phi$	-180	-180.

polar polar



(2) For Region C_2 put $s = Re^{j\theta}$.

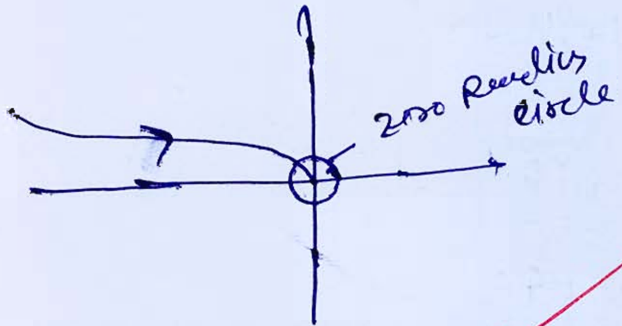
$s = \lim_{R \to \infty} Re^{j\theta}$.

$$GH(Re^{j\theta}) \Rightarrow \frac{K(Re^{j\theta}+4)}{R^2 e^{2j\theta}(Re^{j\theta}+2)}$$

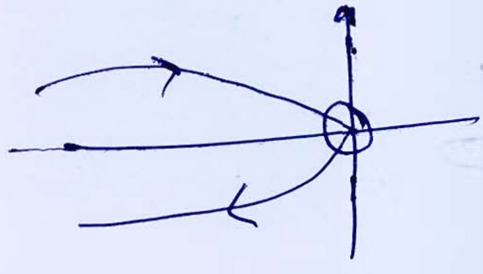
$R \to \infty$, 2 neglected,

$\Rightarrow 0 \cdot e^{j\theta} \cdot e^{-3j\theta} \Rightarrow 0 \angle -3\theta$.

$\theta \to +\pi/2 \to -\pi/2$, $-3\pi/2 \to 3\pi/2$, $-\pi \to \pi$.



(3) For Region C_3 (mirror image of polarplot C_1)



(4) Region C_4 put $s = re^{j\theta}$ in OLTF.

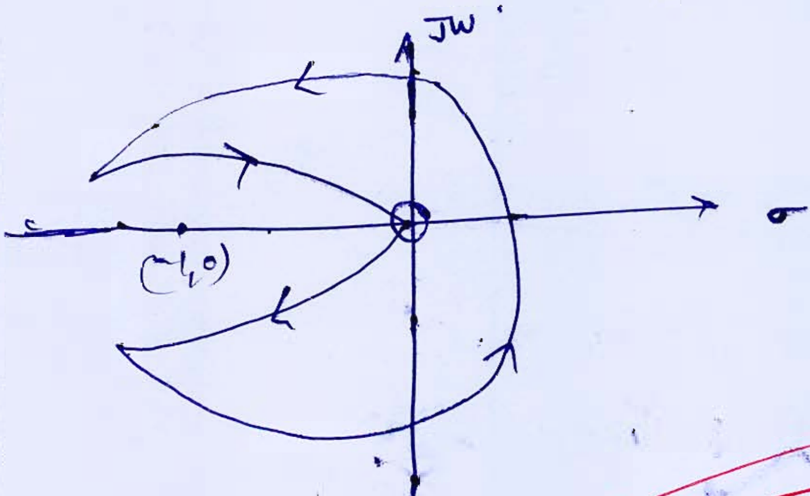
$s = \lim_{r \to 0} re^{j\theta}$.

$$GH(re^{j\theta}) = \frac{k(re^{j\theta}+4)}{r^2 e^{2j\theta}(re^{j\theta}+2)}$$

$r \to 0$, $\infty, \angle +\theta - 2\theta$. neglected.

$\Rightarrow \infty \angle -\theta$. ($\theta \Rightarrow -\pi/2 \to \pi/2$. Now $+\pi/2 \to -\pi/2$)

. ∞ Radius, and Angle $\pi/2 \to -\pi/2$.



for stability

$\boxed{N = P - Z}$

N = no of encirclement of (-1, 0)

P = no of openloop Right pole

Z = no of close loop Right side pole.

$N = 0$, $P = 0$

$Z = 0$

$\boxed{S(s) - \text{is stable}}$

18

Space for Rough Work

Space for Rough Work

Space for Rough Work