• Try to improve presentation

• Try to avoid calculation mistake

Try to attempt all five question completely

Write all Steps

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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electrical Circuits [All Topics]

Control Systems [All Topics]

Name : ....

Roll No :

Test Centres | **Student's Signature**

Delhi ☑ Bhopal ☐ Jaipur ☐

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	13
Q.2	
Q.3	
Q.4	28
Section-B	
Q.5	48
Q.6	53
Q.7	53
Q.8	
Total Marks Obtained	195

Signature of Evaluator Cross Checked by

Sourabh kumar ....

Corp. office : 44 - A/1, Kalu Sarai, New Delhi-110016 | **Ph:** 9021300500 | **Web:** www.madeeasy.in

IMPORTANT INSTRUCTIONS

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DONT'S

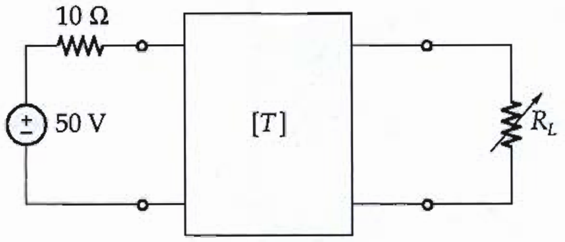
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Electrical Circuits

1 (a) The *ABCD* parameter of the two-port network in figure are $\begin{bmatrix} 4 & 20\,\Omega \\ 0.1S & 2 \end{bmatrix}$.



The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.

[12 marks]

Solution-

ABCD parameter of 2-port network are -

$$[T] = \begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

on comparing → $A = 4$, $B = 20$

$C = 0.1$, $D = 2$

writing the equations -

$V_1 = AV_2 - BI_2$ ——— (1)

$I_1 = CV_2 - DI_2$ ——— (2)

on putting the values of ABCD ——

$V_1 = 4V_2 - 20I_2$ ——— (3)

$I_1 = 0.1V_2 - 2I_2$ ——— (4)

on rearranging the equation - (4) -

$0.1V_2 = I_1 + 2I_2$

$V_2 = 10I_1 + 20I_2$ ——— (5)

on putting the V_2 in eq^n ③ –

$$V_1 = 4\left[10I_1 + 20I_2\right] - 20I_2$$

$$V_1 = 40I_1 + 60I_2 \quad \text{——} ⑥$$

on comparing equation ⑤ and ⑥ with standard Z-parameter.

$Z_{11} = 40 \quad Z_{12} = 60 \quad Z_{21} = 10, \quad Z_{22} = 20$

2

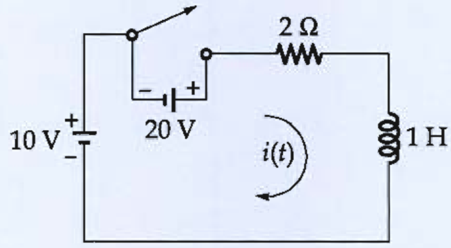
In complete Solution

$P_{max} = ?$

$R_{th} = ?$

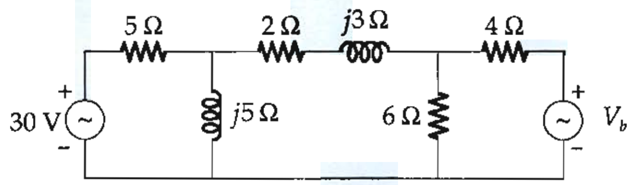
$V_{th} = ?$

1 (b) Determine the current $i(t)$ in the circuit shown in figure at an instant t, after opening the switch at $t = 0$, if a current of 1 A had been passed through the circuit at the instant of opening.



[12 marks]

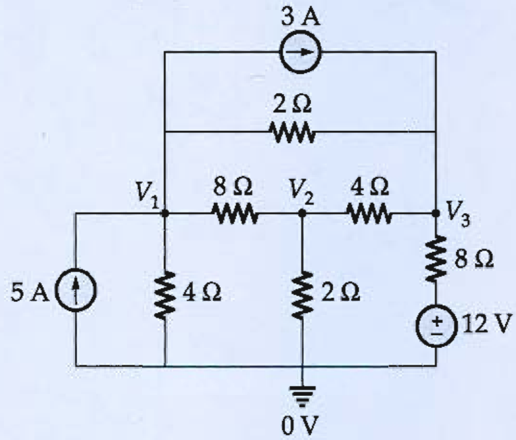
Q.1 (c) For the circuit shown below:



Determine the voltage V_b which results in a zero current through the $(2+j3)\Omega$ impedance branch. Using superposition theorem.

[12 marks]

Q.1 (d) Use nodal analysis to find V_1, V_2 and V_3 in the circuit of figure.



[12 marks]

Applying KCL at V_1 Node one

$$-5+\frac{V_1}{4}+\frac{V_1-V_2}{8}+\frac{V_1-V_3}{2}+3=0$$

$$\Rightarrow \frac{V_1}{4}+\frac{V_1}{8}+\frac{V_1}{2}-\frac{V_2}{8}-\frac{V_3}{2}=2$$

$$\Rightarrow \frac{2V_1+V_1+4V_1-V_2-4V_3}{8}=2$$

$$7V_1-V_2-4V_3=16 \quad \text{——— (1)}$$

Applying KCL at V_2 –

$$\frac{V_2-V_1}{8}+\frac{V_2}{2}+\frac{V_2-V_3}{4}=0$$

$$\Rightarrow -\frac{V_1}{8}+\frac{V_2}{8}+\frac{V_2}{2}+\frac{V_2}{4}-\frac{V_3}{4}=0$$

$$\Rightarrow \frac{-V_1+V_2+4V_2+2V_2-2V_3}{8}=0$$

$$-V_1+7V_2-2V_3=0 \quad \text{——— (2)}$$

Do not write in this margin

applying KCL at V_3 -

$$\frac{V_3 - V_2}{4} + \frac{V_3 - 12}{8} + \frac{V_3 - V_1}{2} - 3 = 0$$

$$-\frac{V_1}{2} - \frac{V_2}{4} + \frac{V_3}{4} + \frac{V_3}{8} + \frac{V_3}{2} = 3 + \frac{12}{8}$$

$$\frac{-4V_1 - 2V_2 + 2V_3 + V_3 + 4V_3}{8} = 3 + 12/8$$

$$-4V_1 - 2V_2 + 7V_3 = 36 \quad \text{— (3)}$$

on solving eq^n (1), (2) and (3)

11

$V_1 = 10$ volts

$V_2 = 4.93$ volts

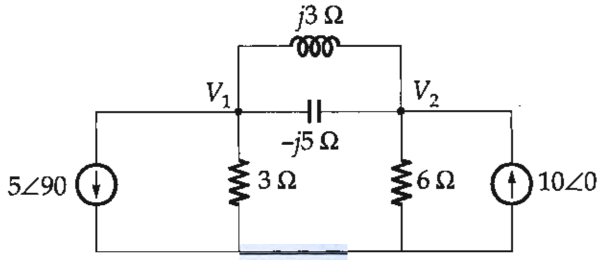
$V_3 = 12.26$ volts.

Try to Improve

Presentation

. write all steps

Q.1 (e) Use nodal analysis on the circuit to find V_2.



[12 marks]

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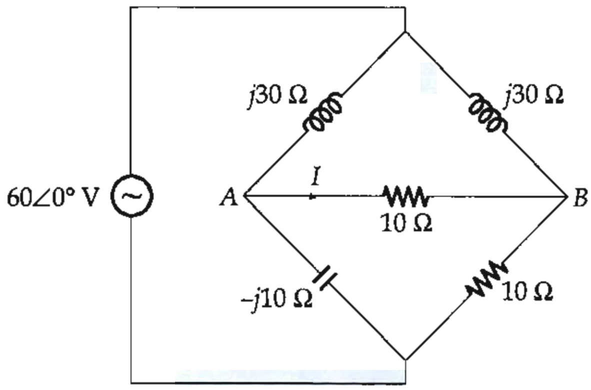
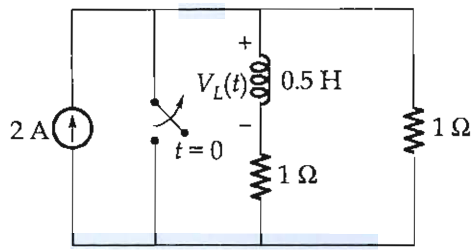
2 (a) Determine the current I through the terminal AB of the network shown below:



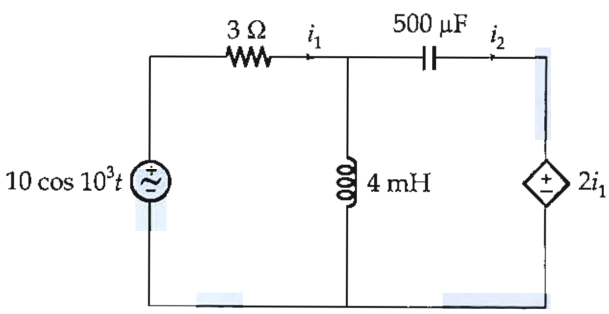
[20 marks]

Q.2 (b) **(i)** For the network shown in figure below, the switch is closed for a long time and at $t = 0$, the switch is opened.



Determine the voltage across inductor for $t > 0$.

(ii) Obtain expressions for the time domain currents i_1 and i_2 in the circuit given as figure.



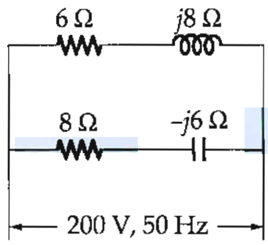
[10 + 10 marks]

Q.2 (c) For the circuit shown below, calculate,

(i) Total admittance, total conductance and total susceptance.

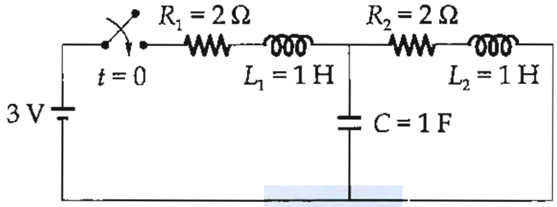
(ii) Total current and total power factor (pf).

(iii) The value of pure capacitance to be connected in parallel with the above combination to make the total power factor (pf) unity.



[20 marks]

Q.3 (a) In the network shown in figure the switch is closed at time $t = 0$. Assuming all the initial currents and voltages as zero, find the current through the inductor L_2 by the use of Norton's theorem.

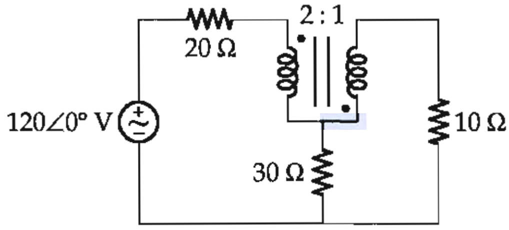


[20 marks]

Q.3 (b) Show that the resonant frequency ω_0 of a series *R-L-C* circuit is geometric mean of ω_1 and ω_2, i.e., the upper and lower half power frequencies respectively.

[20 marks]

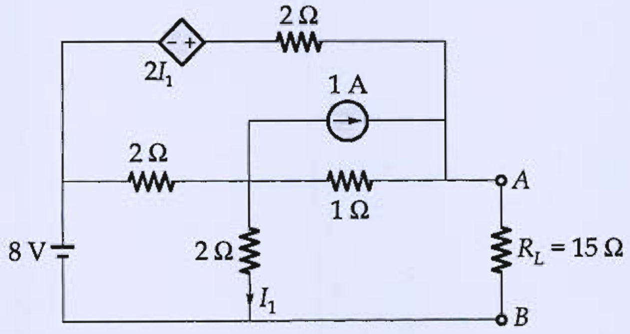
Q.3 (c) Calculate the power supplied to the 10 Ω resistor in the ideal transformer circuit given in the figure below.



[20 marks]

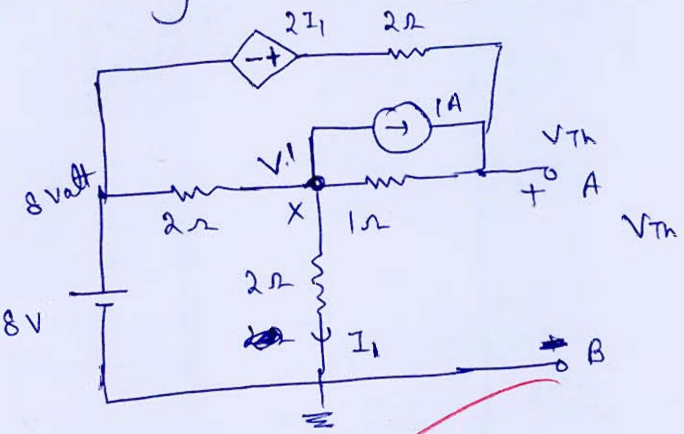
Do not write in this margin

Q.4 (a) Determine the current through the load resistance $R_L = 15\ \Omega$ across the terminal A-B of the circuit shown in figure below, using Thevenin's theorem. Also find the maximum power that can be transferred to the load resistance R_L.



[20 marks]

Sol^n finding thevenin voltage across terminal A & B.



applying nodal at X -

$$\frac{V'-8}{2} + \frac{V'}{2} + \frac{V'-V_{Th}}{1} + 1 = 0$$

$$\frac{V'}{2} - 4 + \frac{V'}{2} + V' - V_{Th} + 1 = 0$$

$$2V' - V_{Th} = 3 \quad \text{——— (1)}$$

applying nodal at A -

$$-1 + \frac{V_{Th} - V'}{1} + \frac{V_{Th} - 8 - 2I_1}{2} = 0$$

$$-1 + V_{Th} - V' + \frac{V_{Th}}{2} - 4 - I_1 = 0 \qquad \left\{ I_1 = V'/2 \right\}$$

$$-1 + V_{Th} - V' + \frac{V_{Th}}{2} - 4 - \frac{V'}{2} = 0$$

$$\frac{3V_{Th}}{2} - \frac{3V'}{2} = 5$$

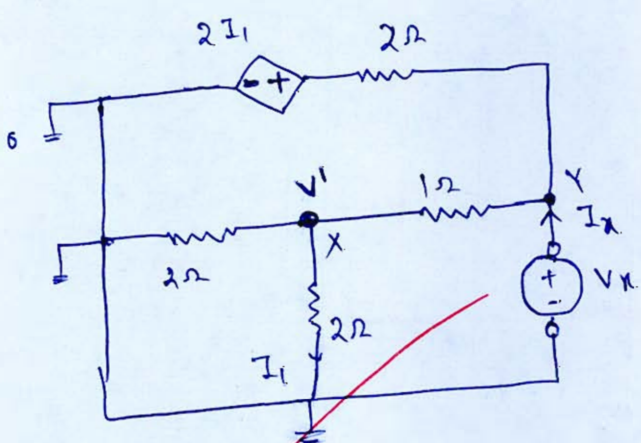
$$3V_{Th} - 3V' = 10 \quad \text{——— (2)}$$

on solving eqn (1) and eqn (2)

$$\boxed{V_{Th} = 9.67 \text{ volts}}$$

⇒ Finding R_{Th} -

By making independent voltage source → S.C
" " Current source → O.C



$$R_{Th} = \frac{V_x}{I_x}$$

applying KCL at node X -

$$\frac{V'}{2} + \frac{V'}{2} + \frac{V' - V_x}{1} = 0$$

$$2V' - V_x = 0$$

$$\boxed{V_x = 2V'} \quad \text{——— (3)}$$

apply KCL at Y –

$$V_x - V' - I_x + \frac{V_x - 2I_1}{2}$$

$$V_x - V' - I_x + \frac{V_x}{2} - I_1 = 0 \qquad \left\{ I_1 = \frac{V'}{2} \right\}$$

$$V_x - \cancel{\frac{V_x}{2}} - I_x + \cancel{\frac{V_x}{2}} - \frac{V'}{2} = 0 \qquad \left\{ V' = \frac{V_x}{2} \right\}$$

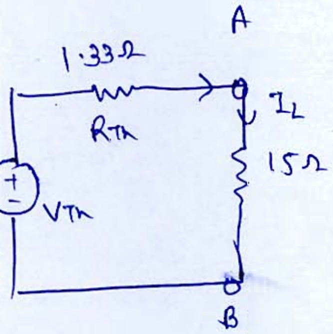
$$V_x - \frac{V_x}{4} - I_x = 0$$

$$\frac{3V_x}{4} - I_x = 0$$

$$\boxed{R_{Th} = \frac{V_x}{I_x} = \frac{4}{3} = 1.33\,\Omega}$$

Current through R_L –

$$I_L = \frac{9.67}{1.33 + 15} = 0.592\text{ A}$$



⇒ and –

maximum power through R_L –

$$P_{max} = \frac{V_{Th}^2}{4 \times R_{Th}} =$$

$$\boxed{P_{max} = 17.57\text{ W}}$$

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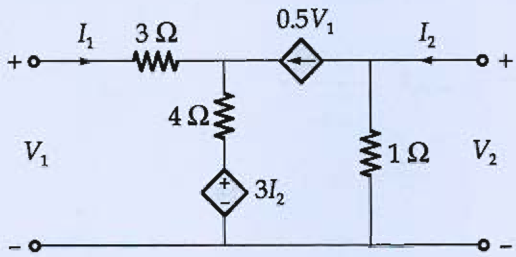
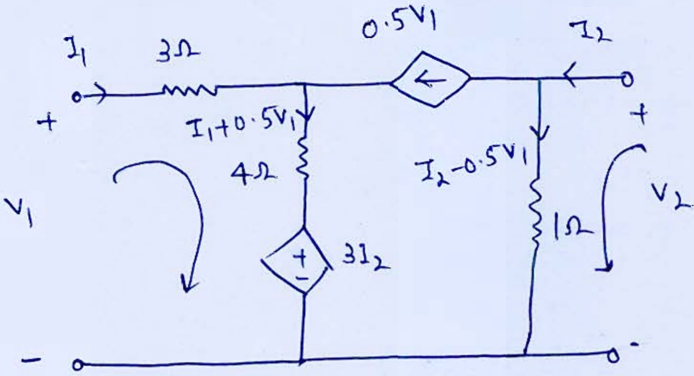
Good Approach

Q.4 (b) Find the *h*-parameters for the two-port network shown



[20 marks]

Sol^n:



~~h-parameters are defined as -~~

$$\cancel{V_1 = h_{11} I_1 + h_{12} V_2} \quad \text{--- (1)}$$

$$\cancel{I_2 = h_{21} I_1 + h_{22} V_2} \quad \text{--- (2)}$$

5

from the above circuit -

$$V_1 = 3(I_1) + 4(I_1 + 0.5V_1) + 3I_2$$

$$V_1 = 3I_1 + 4I_1 + 2V_1 + 3I_2$$

$$V_1 = 7I_1 + 3I_2 + 2V_1$$

$$-V_1 = 7I_1 + 3I_2 \quad \text{--- (3)}$$

from the output circuit -

$$V_2 = I_2 - 0.5V_1$$

$$I_2 = V_2 + 0.5V_1 \quad \text{--- (4)}$$

putting eqⁿ (4) into eqⁿ (3) –

$$-V_1 = 7I_1 + 3(V_2 + 0.5V_1)$$

$$-V_1 = 7I_1 + 3V_2 + 1.5V_1$$

$$-2V_1 = 7I_1 + 3V_2$$

$$V_1 = -3.5I_1 - 1.5V_2 \quad \text{———} \quad (5)$$

on comparing eqⁿ (5) from eqⁿ (1) –

$$\boxed{h_{11} = -3.5 \quad \text{and} \quad h_{12} = -1.5}$$

~~Ans.~~

~~from eqⁿ (3) –~~

~~$V_1 = -7I_1 - 3I_2$ ——— (6)~~

~~put eqⁿ (6) into eqⁿ~~ ~~put eqⁿ (6) in eqⁿ (4)~~

~~$I_2 = V_2 + 0.5(-7I_1 - 3I_2)$~~

~~= 3.5 p.~~

~~$V_2 =$~~

~~$I_2 =$~~

mention unit

put eqⁿ (5) into eqⁿ (4)

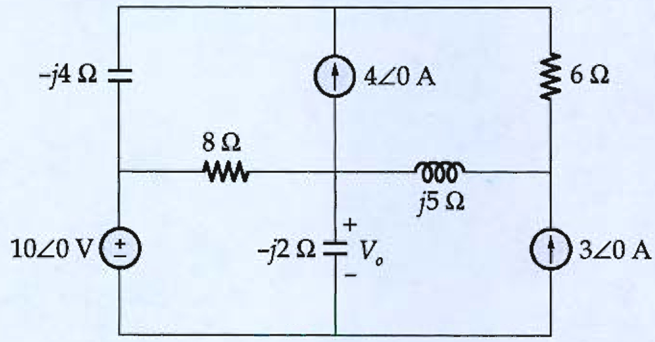
$$I_2 = V_2 + 0.5(-3.5I_1 - 1.5V_2)$$

$$I_2 = -1.75I_1 + 0.25V_2 \quad \text{———} \quad (6)$$

on comparing with eqⁿ (2)

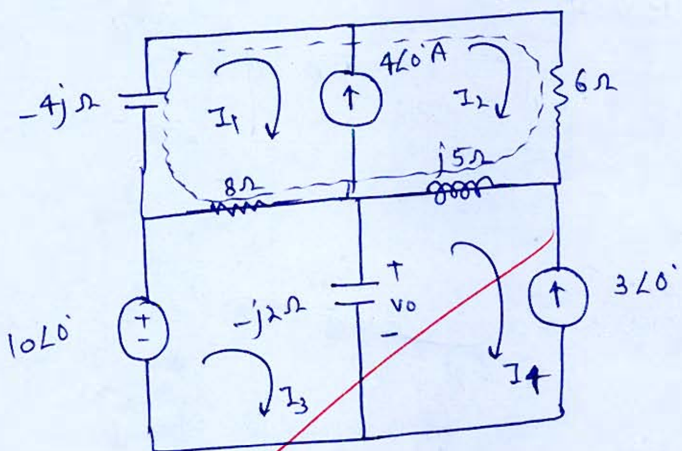
$$\boxed{h_{21} = -1.75} \quad \text{and} \quad \boxed{h_{22} = 0.25}$$

Q.4 (c) Solve for V_o in the circuit of figure using mesh analysis.



[20 marks]

Sol^n ⇒



from the above $\quad I_4 = -3\angle 0° = 3\angle 180°$ —— (1)

and $\quad I_2 - I_1 = 4\angle 0°$ A —— (2)

5

applying KVL in dashed loop -

$$6I_2 + j5(I_2 - I_4) + 8(I_1 - I_3) = 0$$

$$6I_2 + j5I_2 - j5(3\angle 180°) + 8I_1 - 8I_3 = 0$$

$$8I_1 + I_2(6+j5) - 8I_3 + 15\angle -90° = 0$$

$$8I_1 + I_2(6+j5) - 8I_3 = 15j \quad \text{—— (3)}$$

applying KVL in loop I_3 —

$$-10 + 8(I_3 - I_1) + (I_3 - I_4)(-j2) .$$

$$-8I_1 + 8I_3 - j2I_3 + j2(3\angle 180^\circ) = 10\angle 0^\circ$$

$$-8I_1 + I_3(8-j2) = 10+6j$$

$$\cancel{8I_1 = (10+6j) + I_3(8-j2)} \text{ --- } (4)$$

~~put value of $8I_1$ from eq^n (4) into eq^n (3)~~

$$\cancel{(10+6j) + I_3(8-j2) - 8I_3 + I_2}$$

$$I_3(8-j2) = 10+6j + 8I_1$$

$$I_3 = (1+j) + 0.97\angle 14 \cdot I_1 \text{ --- } (4)$$

put value of I_3 eq^n (3)

$$8I_1 + I_2(6+j5) - 8\left[(1+j) + 0.97\angle 14^\circ I_1\right] = 15j$$

$$8I_1 + I_2(6+j5) - (8+8j) - 7.76\angle 14\, I_1 = 15j$$

$$I_1(15.53 + 1.87j) + (6+j5)I_2 = 8+23j \text{ --- } (5)$$

from eq^n (2) $\quad I_2 = 4 + I_1$

$$I_1(15.53+1.87j) + 4(6+j5) + (6+j5)I_1 = 8+23j$$

$$I_1(39.53 + 21.87j) = -16+3j$$

$$\boxed{I_1 = 0.36\angle 140.42}$$

from eq^n (4) $\quad I_3 = (1+j) + (0.97\angle 14)(0.36\angle 140.42)$

$$\boxed{I_3 = 1.34\angle 59.24}$$

$$I_1 = 3.618\angle 274.5\,A$$

Now. $\quad V_0 = -j2(I_3 - I_4)$

$$= -j2\left[1.34\angle 59.24^\circ + 3\right]$$

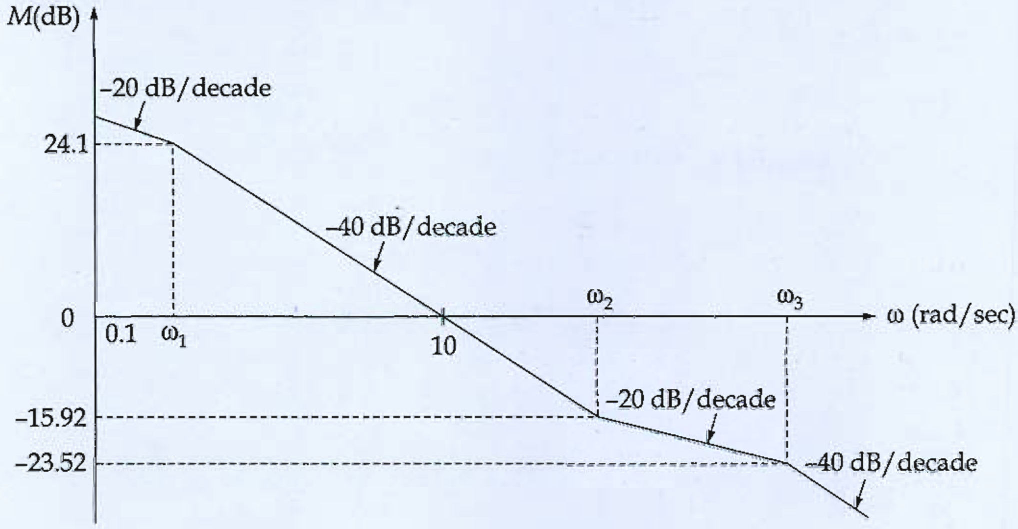
$$\boxed{V_0 = 7.72\angle -72.65^\circ \text{ volts}}$$

Ans

$$V_0 = 9.756\angle 222.3$$

Section B : Control System

Q.5 (a) Obtain the open loop transfer function for a unity negative feedback system whose bode magnitude plot is shown below:



[12 marks]

Solⁿ – let the open loop transfer function of the system –

~~at $\omega = \omega_1$~~

Initial slope $= -20$ dB/dec $\Rightarrow$ simple pole at origin

at $\omega = \omega_1 \Longrightarrow$ slope changes to -40 dB/dec $\Rightarrow$ pole at $\omega = \omega_1$

at $\omega = \omega_2 \Rightarrow$ slope changes to -20 dB/dec $\Rightarrow$ Zero at $\omega = \omega_2$

at $\omega = \omega_3 \Rightarrow$ slope changes to -40 dB/dec $\Rightarrow$ pole at $\omega = \omega_3$

Let the transfer function –

$$G(s)H(s) = \frac{K\left(1+\frac{s}{\omega_2}\right)}{s\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_3}\right)}$$

finding ~~[illegible]~~ $\omega_1 \Rightarrow$

initial point $\Rightarrow ((\log_{10}\omega_1), 24.1) \xrightarrow{-40\text{ dB/dec}} (\log_{10}10, 0\text{ dB})$

$$-40 = \frac{0 - 24.1}{\log_{10}10 - \log_{10}\omega_1}$$

$$\boxed{\omega_1 = 2.5 \text{ rad/sec}}$$

finding $\omega_2 \Rightarrow$

point $\Rightarrow$ $(\log_{10} 10, 0) \xrightarrow{-40\,dB/dec} (\log_{10}\omega_2, -15.92\,dB)$

$$-40 = \frac{-15.92 - 0}{\log_{10}\omega_2 - \log_{10} 10}$$

$$\boxed{\omega_2 = 25 \text{ rad/sec}}$$

finding $\omega_3 \rightarrow$

points $\rightarrow$ $(\log_{10}\omega_2, -15.92\,dB) \xrightarrow{-20\,dB/dec} (\log_{10}\omega_3, -23.52\,dB)$

$$-20 = \frac{-23.52 + 15.92}{\log_{10}\omega_3 - \log_{10} 25}$$

$$\boxed{\omega_3 = 60 \text{ rad/sec}}$$

approximated transfer function at $\omega = \omega_1$ -

$$G(j\omega)(H(j\omega)) = \frac{K}{s j\omega}$$

$$|G(j\omega)(H(j\omega))| = \frac{K}{\omega} \quad \text{---} (1)$$

$$20\log_{10}|G(j\omega)H(j\omega)| = 20\log_{10} K - 20\log_{10}\omega$$

at $\omega = \omega_1 \Rightarrow$ $|G(j\omega)H(j\omega)| = 24.1$ dB

from eq^n (1)

$$24.1 = 20\log_{10} K - 20\log_{10} 2.5$$

$$\boxed{K = 40}$$

Now transfer function-

$$G(s)H(s) = \frac{40\left(1 + \frac{s}{25}\right)}{s\left(1 + \frac{s}{2.5}\right)\left(1 + \frac{s}{60}\right)}$$

$$G(s)H(s) = \frac{240(s+25)}{s(s+2.5)(s+60)} \quad \underline{\underline{Ans}}$$

11

Good Approach

Q.5 (b) A servo mechanism is represented by the equation :

$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144E$$

where $E = C - 0.5y$ is the actuating signal. Find the value of damping ratio, damped and undamped frequency of oscillation. Draw the block diagram of the system described by the above equation.

[12 marks]

Sol^n Given equation -

$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144E$$

Taking laplace transform -

$$s^2Y(s) + 4.8\,sY(s) = 144E$$

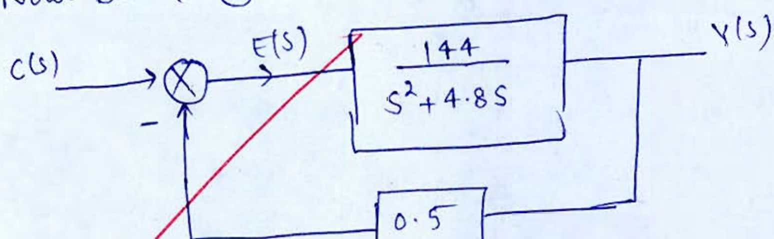
$$\frac{Y(s)}{E(s)} = \frac{144}{s^2+4.8s} \quad \text{——— (1)}$$

and Given - $E = C - 0.5y$

taking laplace transform -

$$E(s) = C(s) - 0.5\,Y(s) \quad \text{——— (2)}$$

Now Block diagram - will be using eq^n (1) and (2)



11

Good Approach

characteristic equation -

$$1+G(s)H(s)=0 \Rightarrow 1+\frac{144}{s^2+4.8s}\times 0.5 = 0$$

$$s^2+4.8s+72=0$$

on comparing with standard 2nd order equation $s^2+2\zeta\omega_n s+\omega_n^2=0$

$\omega_n^2 = 72 \Rightarrow \omega_n = \sqrt{72} = 8.48$ rad/sec (undamped freq.)

$2\zeta\omega_n = 4.8 \Rightarrow \zeta = 0.283$

damped freq. of oscillation -

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 8.48\sqrt{1-0.283^2} = 8.133 \text{ rad/sec}$$

Q.5 (c) Closed loop system with unity feedback has the forward loop transfer function as :

$$G(s) = \frac{28.8}{s(1+0.2s)}$$

Modify the design using cascaded compensation to satisfy the optimum performance criterion, so that the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot. Take gain of proportional controller equal to 5.

[12 marks]

Solⁿ ⇒ Transient response of system is improved by lead compensator or proportional and derivative controller.

So the transfer function of controller will be –

$$= (K_P + S K_D)$$

So overall open loop transfer function –

$$G'(s) = \frac{28.8 (K_P + S K_D)}{S(1+0.2S)}$$

Now: characteristic equation –

$$1 + G'(s) = 0$$

$$S(1+0.2S) + 28.8(K_P + S K_D) = 0 \quad \text{——— (1)}$$

$\cancel{0.2S^2 + S(K_D +}$

Given $K_P = 5$

from eqⁿ (1) –

$$0.2S^2 + S(28.8 K_D + 1) + 28.8 K_P = 0$$

or

$$S^2 + S(144 K_D + 5) + 144 K_P = 0$$

$$S^2 + S(144 K_D + 5) + 720 = 0 \quad \text{——— (2)}$$

on comparing eqⁿ (2) with standard 2nd order equation

$$S^2 + 2\xi\omega_n S + \omega_n^2 = 0$$

$\omega_n^2 = 720$

$\omega_n = 26.832$ rad/sec

and

$2\xi\omega_n = 144K_D + 5$ —— (3)

Given that response with unit input reaches to steady state value in minimum time ⇒

so ⇒ $\xi = 1$

from eq^n (3) –

$2 \times 1 \times 26.832 = 144K_D + 5$

$$\boxed{K_D = 0.338}$$

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So the transfer function of controller –

$= (K_P + sK_D)$

$= (5 + 0.338s)$ Ans.

Good Approach

Q.5 (d) A unity negative feedback system has open loop transfer fucntion, $G(s) = \dfrac{K}{s(1+sT)}$, where K and T are positive constants. Determine the factor by which the amplifier gain K be reduced so that peak overshoot of the unit step response is reduced from 80% to 50%?

[12 marks]

Solⁿ. ⇒ Given open loop transfer function -

$$G(s)H(s) = \frac{K}{s(1+sT)}$$

characteristic eqⁿ ⇒ $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(1+sT)} = 0$$

$$s^2T + s + K = 0$$

$$s^2 + \frac{1}{T}s + \frac{K}{T} = 0 \quad \text{——— (1)}$$

on comparing eqⁿ (1) with standard 2ⁿᵈ order eqⁿ -

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

on comparing ⇒ $\omega_n = \sqrt{\dfrac{K}{T}}$

and $2\xi\omega_n = \dfrac{1}{T}$ ⇒ $2 \times \xi \times \sqrt{\dfrac{K}{T}} = \dfrac{1}{T}$

so ⇒ $\xi = \dfrac{1}{2\sqrt{KT}}$

Thus ⇒ $\xi \propto \dfrac{1}{\sqrt{K}}$ ——— (2)

for peak overshoot $M_{p_1} = 80\%$

Let $\xi = \xi_1$ & $K = K_1$

so ⇒ $M_{p_1} = e^{-\xi_1 \pi / \sqrt{1-\xi_1^2}} = 0.8$

on solving ⇒ $\xi_1 = 0.0708$ —— (2)

Let, for $M_{p_2} = 50\%$

$\xi = \xi_2$ and $K = K_2$

so ⇒ $M_{p_2} = e^{-\xi_2 \pi / \sqrt{1-\xi_2^2}} = 0.5$

on solving $\xi_2 = 0.2154$ —— (3)

Now from eq^n (1) –

$$\frac{\xi_1}{\xi_2} = \frac{\sqrt{K_2}}{\sqrt{K_1}}$$

or. $$\frac{K_2}{K_1} = \frac{\xi_1^2}{\xi_2^2}$$

on putting the values of ξ_1 and ξ_2

$$\frac{K_1}{K_2} = \frac{0.2154^2}{0.0708^2}$$

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Good Approach

so $$\frac{K_2}{K_1} = \frac{0.0708^2}{0.2154^2}$$

$$\boxed{K_2 = \frac{K_1}{9.25}}$$

so gain is reduced by a factor of 9.25 times.

Do not write in this margin

Q.5 (e) The open loop transfer function of a unity negative feedback system is given as, $G(s) = \dfrac{K}{2s(1+0.1s)(1+s)}$. Determine the value of 'K' for which the gain margin of the system is 14 dB.

[12 marks]

Solution ⇒

Given open loop transfer function -

$$G(s)H(s) = \frac{K}{2s(1+0.1s)(1+s)}$$

$$G(j\omega)H(j\omega) = \frac{K}{j2\omega(1+0.1j\omega)\cdot(1+j\omega)}$$

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$$|G(j\omega)H(j\omega)| = m = \frac{K}{\omega\sqrt{1+(0.1\omega)^2}\sqrt{1+\omega^2}} \quad \text{——— (1)}$$

and

$$\angle G(j\omega)H(j\omega) = \phi = -90^\circ - \tan^{-1}(0.1\omega) - \tan^{-1}(\omega) \quad \text{——— (2)}$$

The gain margin of the system is to be found at phase cross over frequency (ω_{pc}).

and $\angle G(j\omega)H(j\omega)\big|_{\omega=\omega_{pc}} = -180^\circ$

from eq^n (2) -

$$-90^\circ - \tan^{-1}(0.1\omega_{pc}) - \tan^{-1}(\omega_{pc}) = -180^\circ$$

$$\tan^{-1}(0.1\omega_{pc}) + \tan^{-1}(\omega_{pc}) = 90^\circ$$

$$\tan^{-1}\left(\frac{0.1\omega_{pc} + \omega_{pc}}{1-0.1\omega_{pc}^2}\right) = 90^\circ$$

on solving —

$$1 - 0.1\omega_{pc}^2 = 0$$

$$\omega_{pc} = \sqrt{10} = 3.162 \text{ rad/sec} \quad \text{——— (3)}$$

Now at $\omega = \omega_{pc}$

$$M\Big|_{\omega=\omega_{pc}} = \frac{K}{\omega_{pc}\sqrt{1+(0.1\omega_{pc})^2}\sqrt{(1+\omega_{pc}^2)}}$$

Given that

$$20\log_{10}\left(\frac{K}{\omega_{pc}\sqrt{1+(0.1\omega_{pc})^2}\sqrt{1+\omega_{pc}^2}}\right) = 14 \text{ dB}$$

on solving with $\omega_{pc} = 3.162$ rad/sec

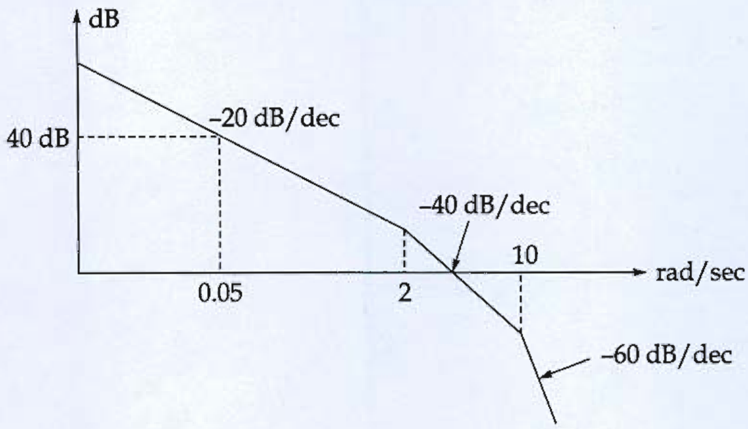
$$\frac{K}{3.162\sqrt{1+(0.1\times 3.162)^2}\sqrt{1+3.162^2}} = 5.011$$

on solving —

$$\boxed{K = 55}$$

So the value of Gain $K = \cancel{2}55$ Ans

Q.6 (a) The open loop transfer function of a unity feedback system is given by $G(s)H(s) = e^{-Ts}G_1(s)$, where $G_1(s)$ is minimum phase system. The approximate bode magnitude plot of the open loop transfer function is shown in the figure below. If the phase margin of the system is $-18.19°$, determine the transportation lag T.



[20 marks]

$Sol^n.$ Given —

$$G(s)H(s) = e^{-Ts} \cdot G_1(s)$$

Since magnitude of e^{-Ts} is a straight line —

so the given bode plot is of ⇒ $G_1(s)$ —

Now, Finding $G_1(s)$ ⇒

from bode plot ⇒

initial slope = -20 dB/dec ⇒ simple pole at origin

at $\omega = 2$ ⇒ slope changes to -40 dB/dec ⇒ pole at $\omega = 2$ rad/sec

at $\omega = 10$ ⇒ slope changes to -60 dB/dec ⇒ pole at $\omega = 10$ rad/sec

so the transfer function $G_1(s)$ —

$$G_1(s) = \frac{K}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)} \quad \text{—— (1)}$$

Now approximate the transfer function at $\omega = 0.05$

$$G_1(s) = K/s$$

$$G_1(j\omega) = \frac{K}{j\omega}$$

at $\omega = 0.05$ $|G_1(j\omega)| = 40\text{ dB}$ — given —

$$20\log_{10}|G_1(\omega)|_{\omega=0.05} = 20\log_{10}\left(\frac{K}{0.05}\right) = 40\text{dB}$$

$$20\log_{10}K - 20\log_{10}0.05 = 40$$

on solving ⇒ $\boxed{K = 5}$

from eq^n (1) —

$$G_1(s) = \frac{5}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)} = \frac{100}{s(s+2)(s+10)} \quad \text{—— (2)}$$

So the transfer function —

$$G(s)H(s) = e^{-Ts}.G_1(s) = \frac{100e^{-Ts}}{s(s+2)(s+10)} \quad \text{—— (3)}$$

Now.

$$G(j\omega)H(j\omega) = \frac{100e^{-j\omega T}}{j\omega(j\omega+2)(j\omega+10)} \quad \text{—— (4)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{100}{\omega\sqrt{\omega^2+4}\sqrt{\omega^2+100}} \quad \text{—— (5)}$$

and

$$\phi = \angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) - \left(\omega \times \frac{180}{\pi} \times T\right) \quad \text{—— (6)}$$

Now finding Gain cross over frequency of $G(s)\cdot H(s)$ —

at $\omega = \omega_{gc} \Rightarrow M = 1$

from eqn (5) -

$$\frac{100}{\omega_{gc}\sqrt{4+\omega_{gc}^2}\sqrt{\omega_{gc}^2+100}} = 1$$

on solving = $\omega_{gc} = 2.8$ rad/sec

Now $\angle G(j\omega)H(j\omega)\big|_{\omega=\omega_{gc}} = \phi\big|_{\omega=\omega_{gc}}$

from eqn (6) -

$$\phi\big|_{\omega=\omega_{gc}} = -90° - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{gc}}{10}\right) - \left(\frac{\omega_{gc}\times 180}{\pi}\times T\right) \quad \text{— (7)}$$

Given phase margin of system = $-18.19°$

so $PM = 180 + \phi\big|_{\omega=\omega_{gc}} = -18.19°$

$$\phi\big|_{\omega=\omega_{gc}} = -198.19° \quad \text{— (8)}$$

from eqn (7) put $\omega_{gc} = 2.8$

$$-90° - \tan^{-1}\left(\frac{2.8}{2}\right) - \tan^{-1}\left(\frac{2.8}{10}\right) - \left(\frac{2.8\times 180}{\pi}\times T\right) = -198.19°$$

$$-\left(\frac{2.8\times 180\times T}{\pi}\right) = -38.085$$

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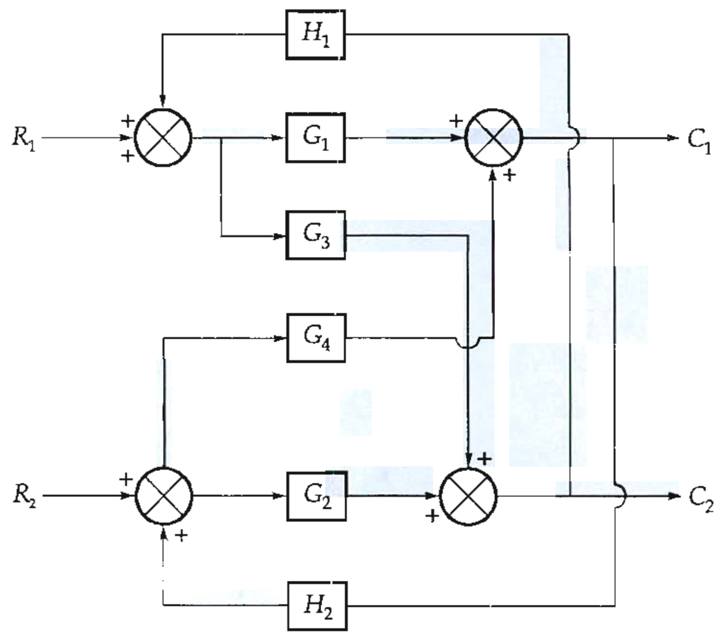
on solving —

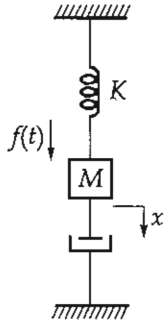
$T = 0.23739$

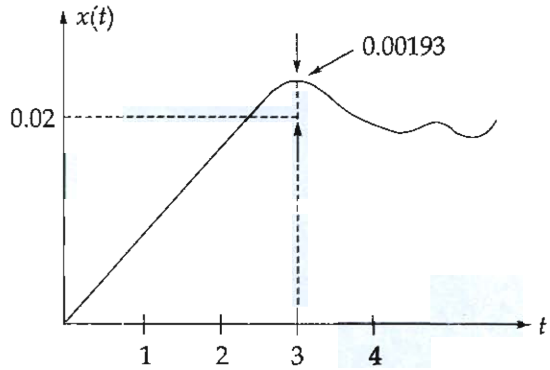
$\boxed{T = 237.39 \text{ msec}}$ Ans

Q.6 (b) (i) Evaluate $\frac{C_2}{R_1}$ for the system whose block diagram representation is shown in figure below. (Use block diagram reduction technique to solve).



(ii) Figure below shows a mechanical system and the response when 10 N of force is applied to the system. Determine the values of M, F, K. The dimension 'x' is in meter.



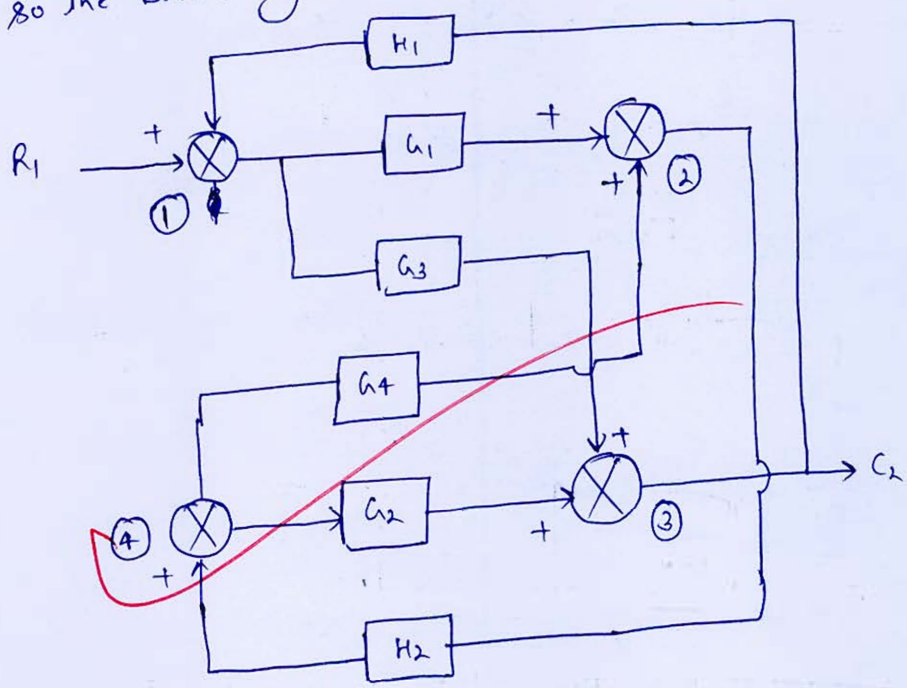


[10 + 10 marks]

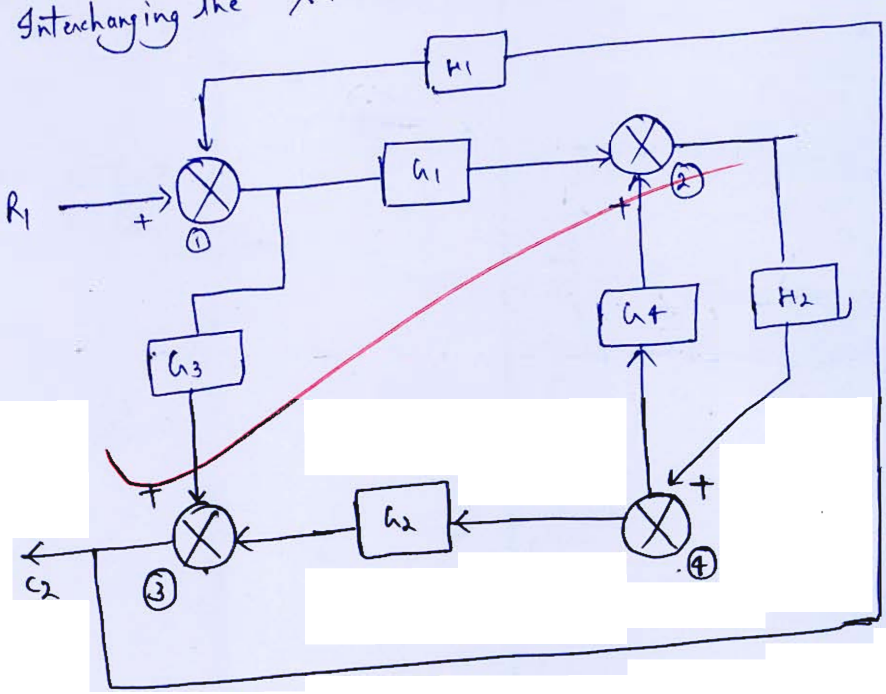
Solⁿ ⇒

6 (b). (i) for evaluating the C_2/R_1 {make $C_1 = 0$ and $R_2 = 0$}

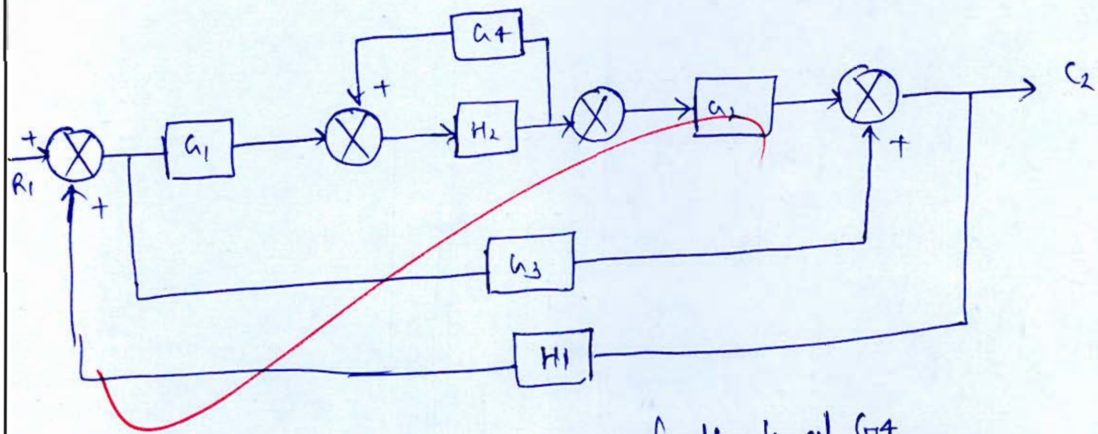
so the Block diagram will be –



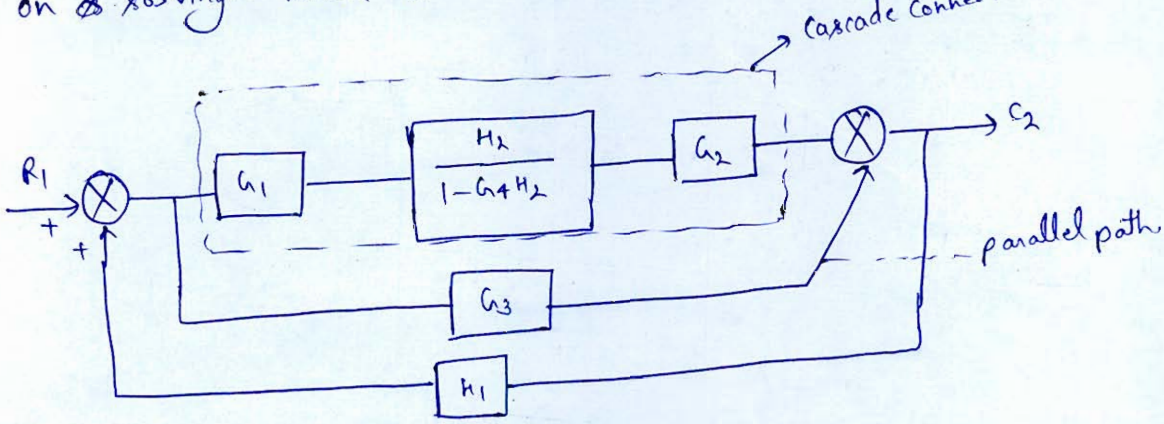
Interchanging the summer ③ and ④ –



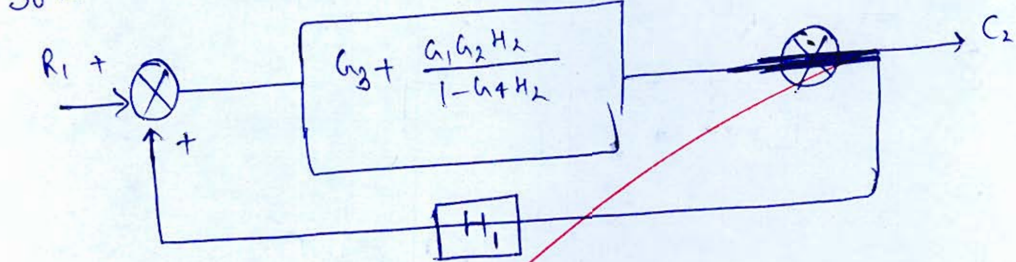
on rearranging the blocks in a line -



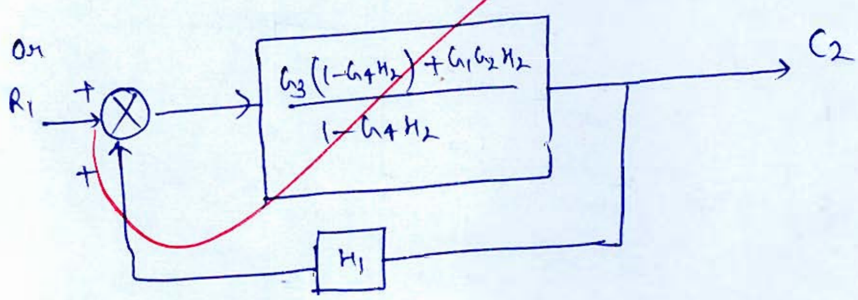
on solving - Block H_2 with +ve feedback of G4

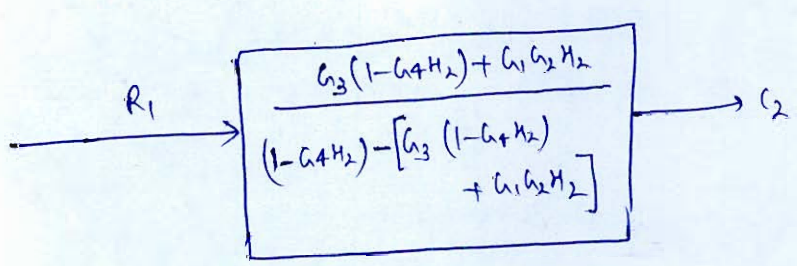


So -



or





so –
$$\frac{C_2}{R_1} = \frac{G_3(1-G_4H_2) + G_1G_2H_2}{(1-G_4H_2) - G_3(1-G_4H_2) - G_1G_2H_2}$$ Ans.

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Solution. 6(b).(ii)

from the mechanical system –

force equation will be –

$$f(t) = m.\frac{d^2x}{dt^2} + F.\frac{dx}{dt} + Kx$$

Taking laplace transform –

$$F(s) = (s^2m + Fs. + K)X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + F/m + K/m}$$

$$X(s) = \frac{F(s).\,1/m}{s^2 + F/m + K/m}$$ { Given $F(s) = 10/s$ }

$$X(s) = \frac{10}{s}\left(\frac{1/m}{s^2 + F/m\, s + K/m}\right) \quad \text{———— } (1)$$

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from the response of $x(t)$ —

$\lim_{s\to} sX(s) = 0.02 = \frac{10}{K} \Rightarrow \boxed{K = 500}$ Ans.

peak overshoot $M_p = \frac{0.00193}{0.02} = 0.0965 = e^{-\xi\pi/\sqrt{1-\xi^2}}$

on solving $\Rightarrow$ $\xi = 0.597 \approx 0.6$

and given peak time $t_p = \pi/\omega_d = 3 \Rightarrow \omega_n\sqrt{1-\xi^2} = \pi/3$

$\omega_n = 1.31$ rad/sec

on comparing eq^n (1) – with 2nd order

$\sqrt{K/m} = \omega_n \Rightarrow \boxed{m = \frac{500}{1.31^2} = 291.35 \text{ kg.}}$ ans.

and $2\xi\omega_n = F/m \Rightarrow \boxed{F = 458}$ N/m/sec.

4

Q.6 (c) Derive the expression for the transfer function of an ac servomotor and obtain the same in respect of a servomotor having following data :

(i) Starting torque = 0.166 N-m

(ii) Moment of inertia, $J = 1 \times 10^{-5}$ kgm^2

(iii) Supply voltage = 115 Volts

(iv) No load speed = 2904 rpm

(Assume friction to be zero)

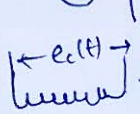
[15 + 5 = 20 marks]

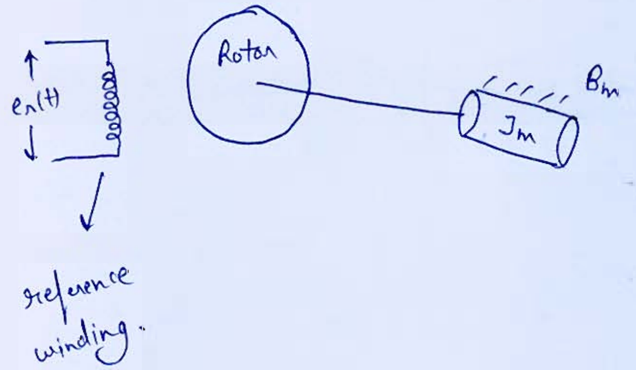
Sol^n ⇒

AC servomotor is ~~equd~~ equivalent to 2-phase induction motor with high rotor resistance.

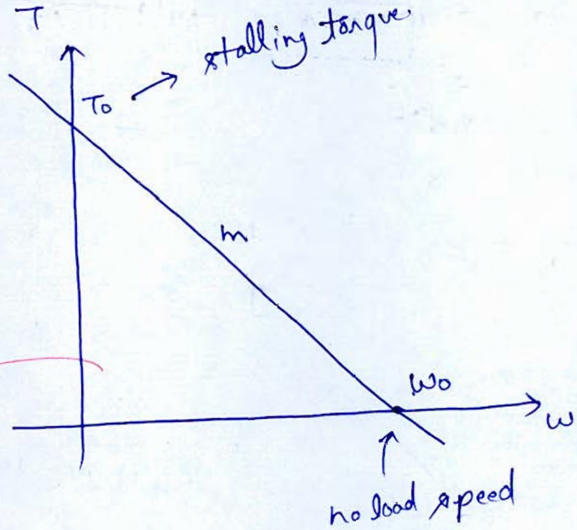
In AC servomotor, two stator winding named as control winding and reference windings are there. Control winding is excited by voltage of variable magnitude and reference winding winding with voltage of constant magnitude.

schematic diagram of ac servomotor –





Torque speed characteristic of AC servo motor.



so the torque developed by motor →

$T_D(t) = m e_c(t) + \omega_m$ —— ①

where $m \Rightarrow$ slope of curve

$m = -T_0/\omega_0$

from eq^n ①

$T_D(t) = \left(-T_0/\omega_0\right) e_c(t) + \omega_m$ —— ②

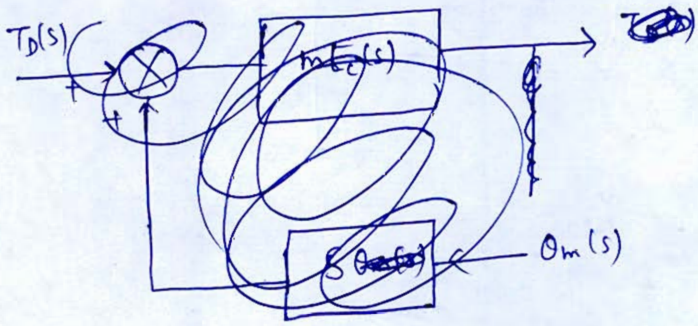
ω_m can be written as → $\omega_m = \frac{d\theta_m}{dt} = \cancel{s\theta_m}$ —— ③

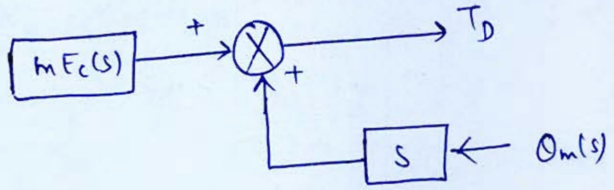
taking laplace transform of eq^n ① -

$T_D(s) = m E_c(s) + s \cdot \theta_m(s)$

$T_D(s) = m E_c(s) + s\,\theta_m(s)$ —— ④

Block diagram of eq^n ④





—(a)

Now motor torque eq^n –

$$T_d(t) = J_m \cdot \frac{d^2\theta_m}{dt^2} + B_m \cdot \frac{d\theta_m}{dt}$$

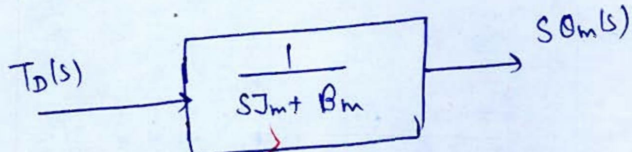
taking laplace transform

$$T_D(s) = s^2 J_m \theta_m(s) + s B_m \theta_m(s) \quad \text{—— (4)}$$

$$\frac{\theta_m(s)}{T_D(s)} = \frac{1}{(s^2 J_m + s B_m)}$$

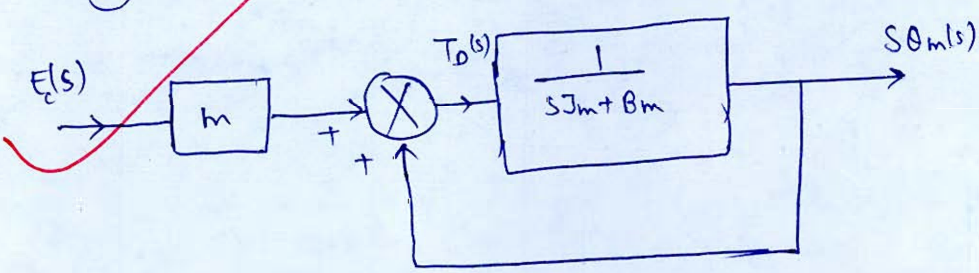
$$\frac{s\theta_m(s)}{T_D(s)} = \frac{1}{(sJ_m + B_m)} \quad \text{—— (5)}$$

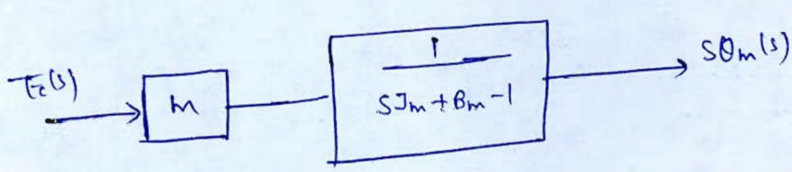
Block diagram eq^n (5)



—(b)

Combining the block diagram (a) & (b) –





Transfer function → $\boxed{\frac{\theta_m(s)}{E_c(s)} = \frac{m}{s(sJ_m + B_m - 1)}}$ Ans —(6)

Numerical part -

starting torque $T_0 = 0.166$ N-m

moment of inertia $J_m = 1 \times 10^{-5}$ Kg m^2

Supply voltage $E_c(t) = 115$ V

no load speed $(N_0) = 2904$ rpm

$B_m = 0$

$\omega_0 = \frac{2\pi N_0}{60} = 304.106$ rad/sec

slope $m = -T_0/\omega_0 = -\frac{0.166}{304.106} = -5.45 \times 10^{-4}$

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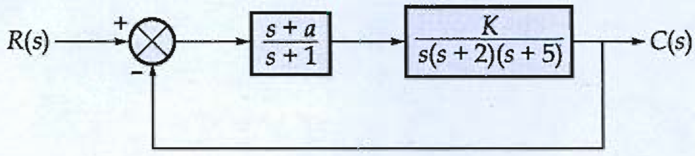
putting the values in eq^n ⑥ -

$$\theta_m(s) = \frac{-5.45 \times 10^{-4} \times 115}{s(s \times 1 \times 10^{-5} - 1)} \text{ rad.}$$

Ans

Q.7 (a) (i) A position control system is shown in figure below :



K and *a* are the parameters of the system. Determine the range of *K* and *a* for which system is stable.

(ii) Sketch the root-locus of $G(s) = \dfrac{K(s+1)}{s^2(s+2)}$.

[10 + 10 marks]

Sol^n 7.(a).(i)

the closed loop transfer function of system will be—

$$\frac{C(s)}{R(s)} = \frac{\dfrac{K(s+a)}{s(s+1)(s+2)(s+5)}}{1+\dfrac{K(s+a)}{s(s+1)(s+2)(s+5)}}$$

$$\frac{C(s)}{R(s)} = \frac{K(s+a)}{s(s+1)(s+2)(s+5)+K(s+a)} \quad \text{——— (1)}$$

characteristic ~~total~~ equation—

$$\cancel{1+G(s)H(s)=0 \Rightarrow 1+\frac{K(s+a)}{s(s+1)(s+2)(s+5)+K(s+a)}=0}$$

$$(s^2+s)(s^2+7s+10)+K(s+a)=0$$

$$s^4+7s^3+10s^2+s^3+7s^2+10s+Ks+Ka=0$$

$$s^4+8s^3+17s^2+(10+K)s+Ka=0 \quad \text{——— (2)}$$

Do not write in this margin

making Routh table -

s^4	1	17	Ka
s^3	8	10+K	0
s^2	$\frac{126-K}{8}$	Ka	0
s^1	$\frac{\frac{(10+K)(126-K)}{8}-8Ka}{126-K/8}$	0	0
s^0	Ka		

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For the system to be stable -
the routh hurwitz table should have no first column of the sign change -

means -

$\rightarrow$ $\frac{126-K}{8} > 0 \Rightarrow K < 126$ ——— ③

$\rightarrow$ $(10+K)(126-K) - 64Ka > 0$ ——— ④

and $\rightarrow$ $Ka > 0$ ——— ⑤

Ans.

Solⁿ. 7(a).(ii)

Given open loop transfer function -

$$G(s)H(s) = \frac{K(s+1)}{s^2(s+2)}$$

(i) poles $s = 0, 0, -2$ $(P=3)$ $P - Z = 2$

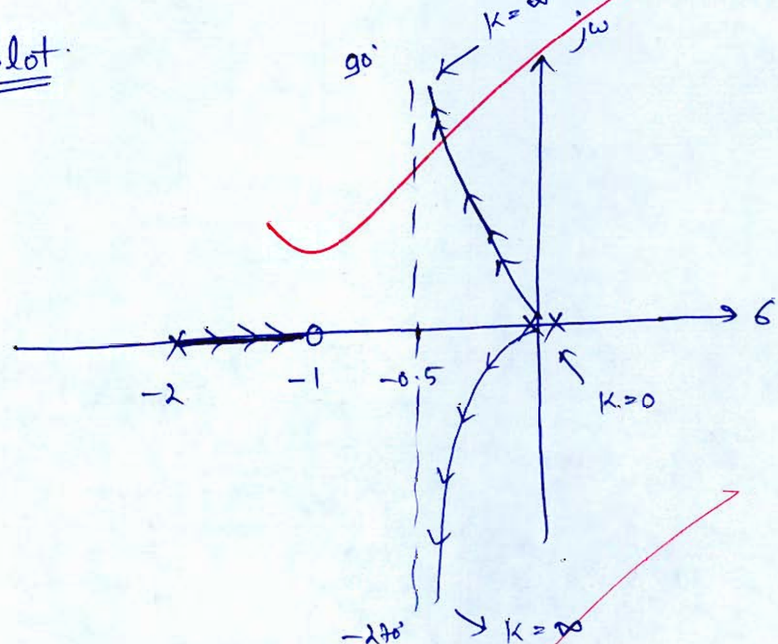
(ii) Zero $s = -1$ $(Z=1)$

(iii) asymptotes -

$$\theta = \frac{(2q+1)\,180^\circ}{(P-Z)} \Rightarrow \begin{cases} \theta_1 = \frac{180^\circ}{2} = 90^\circ \\ \theta_2 = \frac{180^\circ \times 3}{2} = 270^\circ \end{cases}$$

(iv) centroid $= \frac{\Sigma P - \Sigma Z}{P-Z} = \frac{0+0-2-(-1)}{2} = -0.5$

plot.



8

No. of branches terminating to zero = no. of zero = 1

no. of branches terminating to infinity = P - Z = 2

Q.7 (b) Sketch the polar plot of the transfer function given below. Determine whether the plot crosses the real axis. If so, determine the frequency at which the plot cross the real axis and the corresponding magnitude $|G(j\omega)|$.

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

[20 marks]

Sol^n:

Given open loop transfer function -

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$$G(j\omega) = \frac{1}{-\omega^2(1+j\omega)(1+j2\omega)} \quad \text{———} \ (1)$$

$$M = |G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \quad \text{———} \ (2)$$

$$\phi = \angle G(j\omega)H(j\omega) = -180^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \quad \text{———} \ (3)$$

	$\lvert G(j\omega)\rvert$	$\phi = \angle G(j\omega)$
$\omega \to 0$	∞	-180°
$\omega \to \infty$	0	$-180^\circ - 90^\circ - 90^\circ = -360^\circ$

Now from the eq^n (1) -

$$G(j\omega) = \frac{1}{-\omega^2(1+j\omega)(1+j2\omega)}$$

on rationalizing the transfer function -

$$G(j\omega) = \frac{(1-j\omega)(1-j2\omega)}{-\omega^2(1+\omega^2)(1+4\omega^2)}$$

$$G(j\omega) = \frac{[1-j\omega-j2\omega-2\omega^2]}{-\omega^2(1+\omega^2)(1+4\omega^2)}$$

on repeating the real and imaiginary part -

$$G(j\omega) = \frac{1-2\omega^2}{-\omega^2(1+\omega^2)(1+4\omega^2)} + j\frac{(-3\omega)}{-\omega^2(1+\omega^2)(1+4\omega^2)}$$

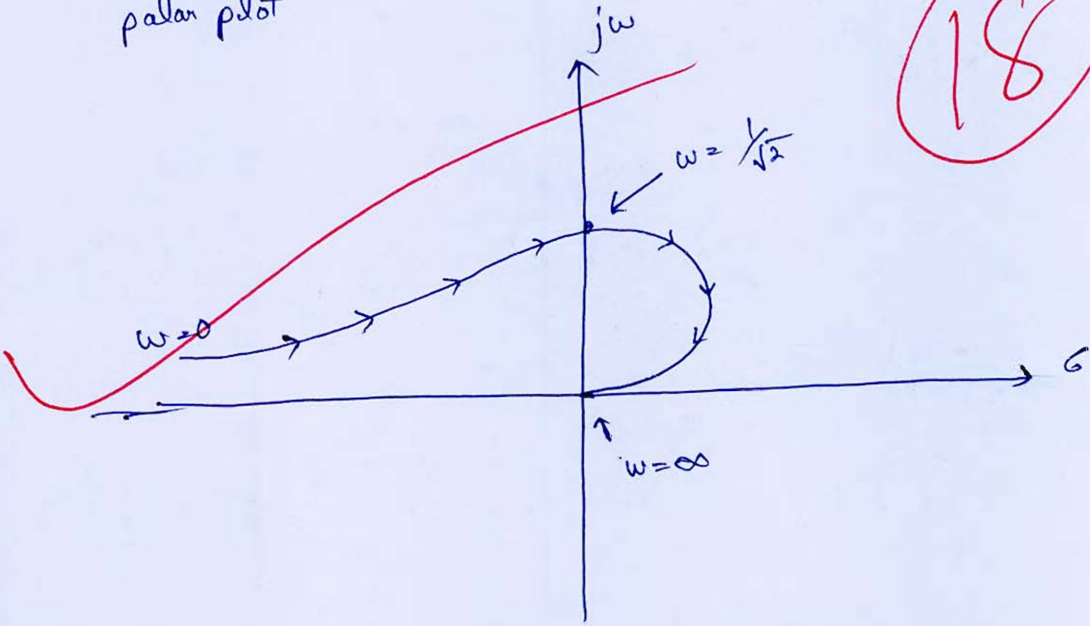
on making the imaiginary part = 0 of $G(j\omega)$, no value of ω we are getting.

So the polar plot does not cross the real axis. hence no magnitude of $|G(j\omega)|$

But it crosses the imaiginary axis at $\omega = 1/\sqrt{2}$

~~polar plot~~

polar plot



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Q.7 (c) Construct the state model for a system characterised by the differential equation :

$$\frac{d^3y}{dt^3}+\frac{6d^2y}{dt^2}+\frac{11dy}{dt}+6y=u$$

Give the block diagram representation of the state model.

[15 + 5 = 20 marks]

Solⁿ ⇒

Given differential equation ⇒

$$\frac{d^3y}{dt^3}+6\frac{d^2y}{dt^2}+11\frac{dy}{dt}+6y=u$$

on taking the laplace transform -

$$s^3Y(s)+6s^2Y(s)+11sY(s)+6Y(s)=U(s)$$

$$\frac{Y(s)}{U(s)}=\frac{1}{s^3+6s^2+11s+6} \quad \text{——— (1)}$$

$$\frac{Y(s)}{U(s)}=\frac{1}{(s+1)(s+2)(s+3)}$$

on doing the partial fraction -

$$\frac{Y(s)}{U(s)}=\frac{1}{2(s+1)}-\frac{1}{(s+2)}+\frac{1}{2(s+3)}$$

$$Y(s)=\frac{1}{2(s+1)}U(s)-\frac{1}{(s+2)}U(s)+\frac{1}{2(s+3)}U(s) \quad \text{——— (2)}$$

$$Y(s)=X_1(s)+X_2(s)+X_3(s) \quad \text{——— (3)}$$

where —

$$X_1(s) = \frac{1}{2(s+1)} U(s)$$

$$2sX_1(s) + 2X_1(s) = U(s) \qquad \text{——— (4)}$$

and

$$2\dot{x}_1 + 2x_1 - u = 0 \qquad \text{——— (5)}$$

and $$X_2(s) = \frac{-1}{(s+2)} U(s)$$

$$sX_2(s) + 2X_2(s) + U(s) = 0$$

$$\dot{x}_2 + 2x_2 + u = 0 \qquad \text{——— (6)}$$

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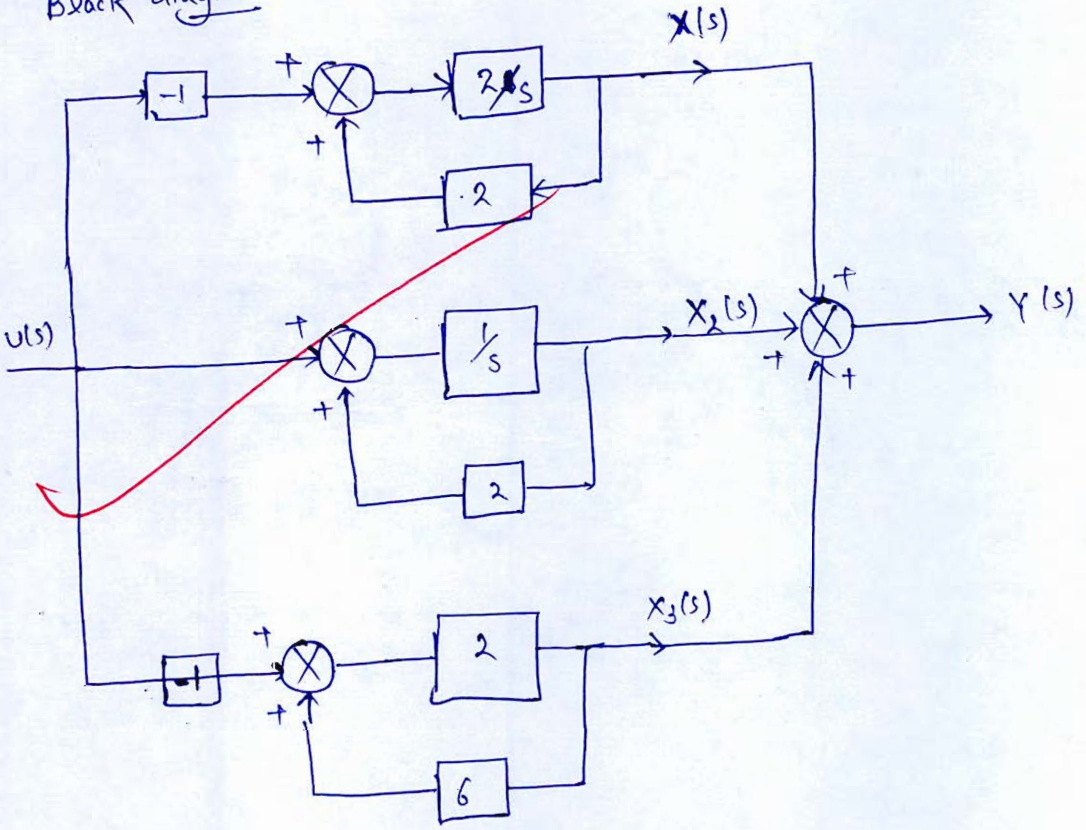
and $$X_3(s) = \frac{1}{2(s+3)} U(s)$$

$$2sX_3(s) + 6X_3(s) - U(s) = 0$$

$$2\dot{x}_3 + 6x_3 - u = 0 \qquad \text{——— (7)}$$

Block diagram



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Q.8 (a) The open-loop transfer function of a unity feedback control system is given below:

$$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}.$$

Plot the root locus and determine the value of K at the breakaway point.

[20 marks]

8 (b) The open loop transfer function of a feedback control system is

$$G(s)H(s) = \frac{K(1+2s)}{s(1+s)(1+s+s^2)}$$

Find the restriction on K for stability. Find the value of K for the system to have a gain margin of 3 dB. With this value of K, find the gain cross over frequency and phase margin. Use Nyquist Approach.

[20 marks]

8 (c) The state space model of a second order system given below is designed using feedback control system.

$$\dot{x} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

(i) What are the conditions for the desired response? Also check whether desired response is possible or not.

(ii) Design an observer system such that the above system has settling time of 0.5 sec and damping frequency of 6 rad/sec.

[8 + 12 marks]

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OOOO

Space for Rough Work

Space for Rough Work

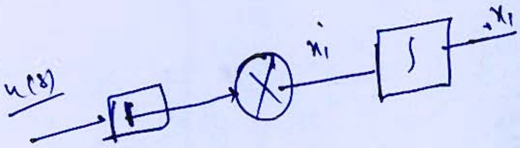
Space for Rough Work

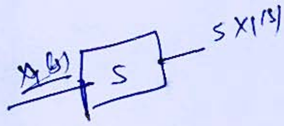
$$\frac{1}{(s+1)(s+2)(s+3)} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+3)}$$

$$1 = A(s^2+5s+6) + B(s^2+4s+3) + C(s^2+3s+2)$$

$$1 = s^2(A+B+C) + s(5A+4B+3C) + (6A+3B+2C)$$

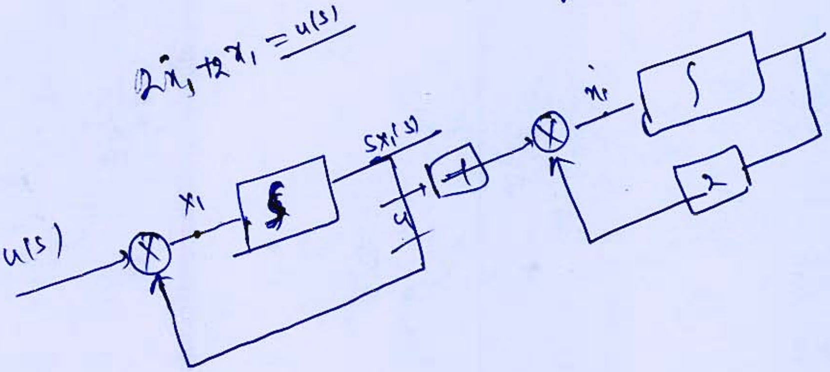
$$\frac{U(s)}{X_1(s)} = \frac{1}{2s+2}$$

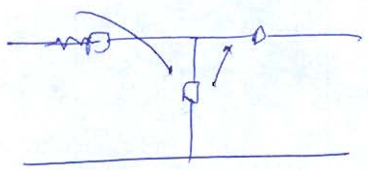




$$2\dot{x}_1 + 2x_1 - u = 0$$

$$2\dot{x}_1 + 2x_1 = u(s)$$





$Z_1 + Z_2 = 40$

$Z_2 =$