Try to complete all five questions
try to avoid over writing
mention unit

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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electrical Circuits [All Topics]

Control Systems [All Topics]

Name :

Roll No

Test Centres			Student's Signature
Delhi ☐	Bhopal ☐	Jaipur ☑	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE	
Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
Total Marks Obtained	234

Signature of Evaluator ....

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

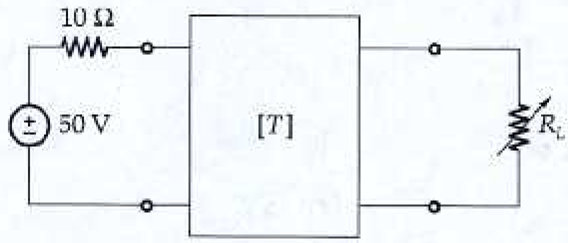
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Electrical Circuits

Q.1 (a) The *ABCD* parameter of the two-port network in figure are $\begin{bmatrix} 4 & 20\Omega \\ 0.1S & 2 \end{bmatrix}$.



The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.

[12 marks]

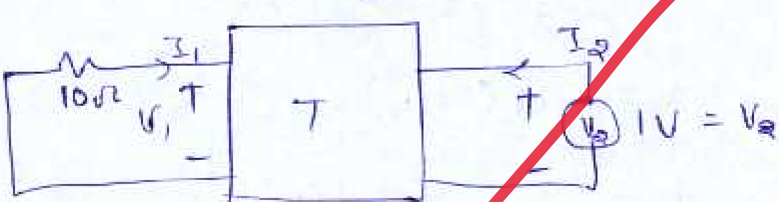
Sol: given two port Network having ABCD parameters as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\Rightarrow \quad V_1 = 4V_2 - 20 I_2 \quad \ldots (1)$$

$$I_1 = 0.1 V_2 - 2 I_2 \quad \ldots (2)$$

To determine R_L, a connected a test source at output as shown



$$V_1 = -10 I_1 \quad \text{and} \quad V_2 = 1V$$

from Eq (1) & (2)

$$-10 I_1 = 4 - 20 I_2 \quad \ldots (3)$$

$$I_1 = 0.1 - 2 I_2 \quad \ldots (4)$$

put I_1 in Eq. (3)

$$-10(0.1 - 2 I_2) = 4 - 20 I_2$$

$$-1 + 20 I_2 = 4 - 20 I_2$$

$$\Rightarrow \quad 40 I_2 = 5$$

$$\Rightarrow \quad I_2 = \frac{5}{40} \text{ A}$$

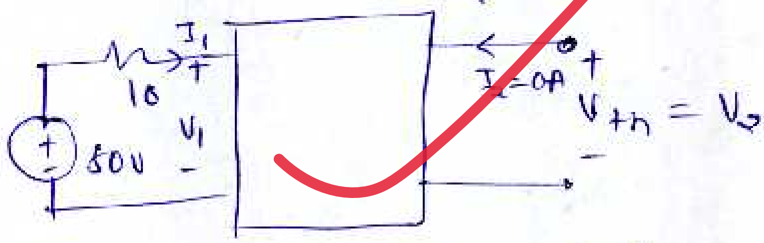
$$R_{th} = \frac{V_2}{I_2} = \frac{1}{5/40} = 8\,\Omega$$

for max. power to R_L, for that

$$\boxed{R_L = R_{th} = 8\,\Omega}$$

find V_{th} across (it is open circuit voltage)

$$I_2 = 0$$



$$V_1 = 50 - 10 I_1 \quad \ldots (5)$$

from Eq. (1) & (2)

$$50 - 10 I_1 = 4 V_2$$

$$I_1 = 0.1 V_2$$

$$\Rightarrow \quad 50 - 10(0.1 V_2) = 4 V_2$$

$$\Rightarrow \quad \boxed{V_{th} = V_2 = 10\text{ V}}$$

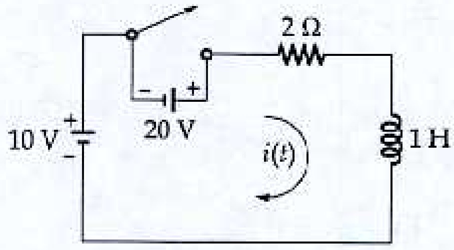
Good Approach

Maximum power delivered to Load

$$P_{max} = \frac{V_{th}^2}{4 R_{th}} = \frac{10 \times 10}{4 \times 8}$$

$$\boxed{P_{max} = 3.125 \text{ watt}}$$

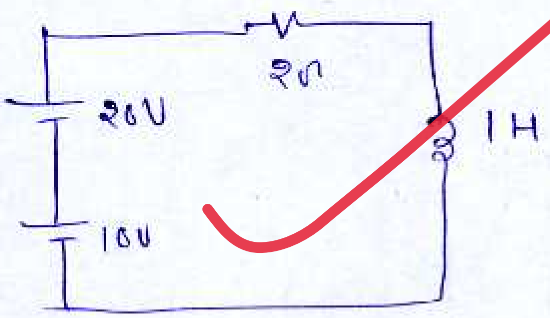
Q.1 (b) Determine the current $i(t)$ in the circuit shown in figure at an instant t, after opening the switch at $t = 0$, if a current of 1 A had been passed through the circuit at the instant of opening.



[12 marks]

$i_L(0^-) = 1\text{ A}$

at $t > 0$ & circuit is drawn as



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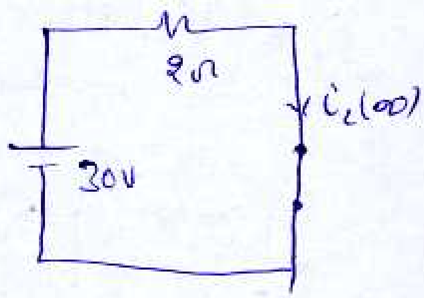

$i_L(\infty) = \frac{30}{2} = 15\text{ A}$

then current through circuit is

$i(t) = i_L(\infty) + (i_0 - i_L(\infty))e^{-Rt/\tau}$

$= 15 + (1 - 15)e^{-2t/1}$

$\boxed{i(t) = (15 - 14e^{-2t})\text{A}}$ $t \geq 0$

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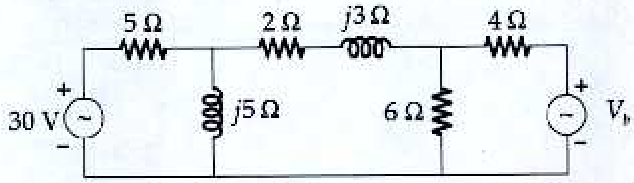
Do not write in this margin

Q.1 (c) For the circuit shown below:

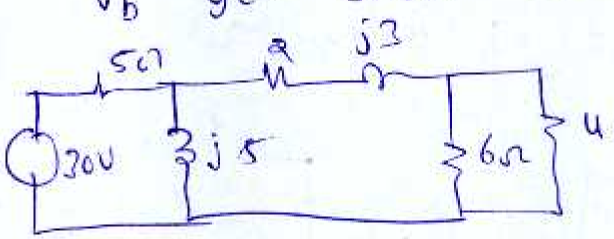


Determine the voltage V_b which results in a zero current through the $(2 + j3)\Omega$ impedance branch. Using superposition theorem.

[12 marks]

Using superposition Theorem:

Step - 1 30 V Source is in Circuit and V_b get short circuit then I'



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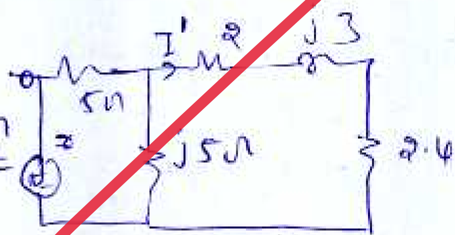
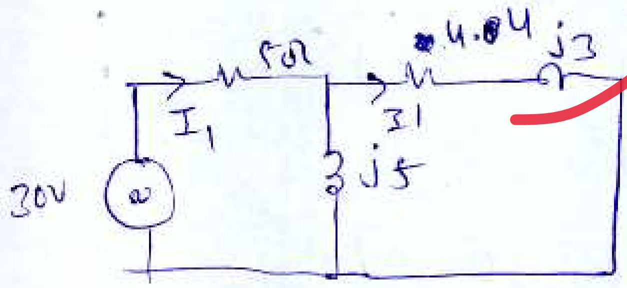




$$I_1 = \frac{30}{Z_T}$$

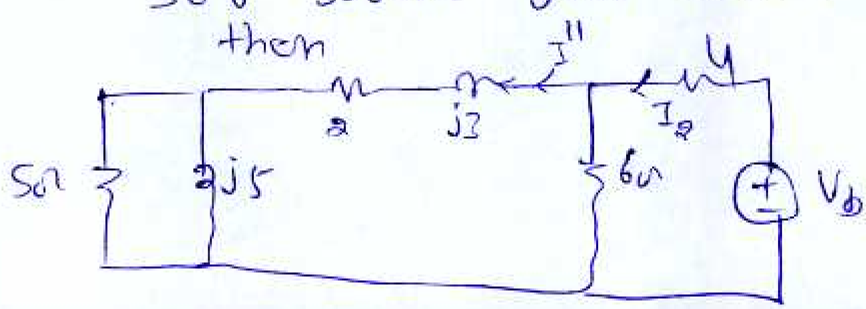
$$\Rightarrow I_1 = 4.39\angle -22.37 \text{ A}$$

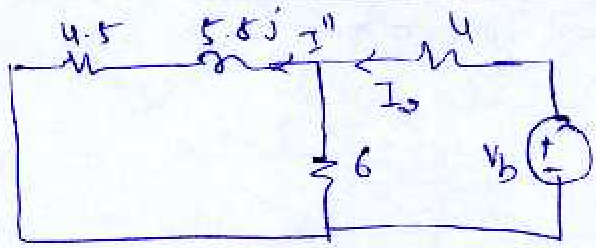
by current division

$$I' = \frac{j5}{4.4 + j8} I_1$$

$$I' = 2.464\angle 6.4416 \text{ A} \quad \ldots (1)$$

Step: 2 V_b source is in Circuit and 30 V source gets ~~cou~~ short circuit then





$$I_2 = \frac{V_b}{Z_T}$$

$$\Rightarrow I_2 = \frac{V_b}{7.31 + j1.41}$$

$$\Rightarrow I_2 = V_b\, 0.1343 \angle -10.912^\circ \text{ A}$$

by current division

$$\Rightarrow I'' = \frac{V_b (0.1343 \angle -10.912^\circ)\, 6}{10.5 + 5.5j}$$

$$\Rightarrow \boxed{I'' = (0.068 \angle -38.5584\, V_b) A}$$

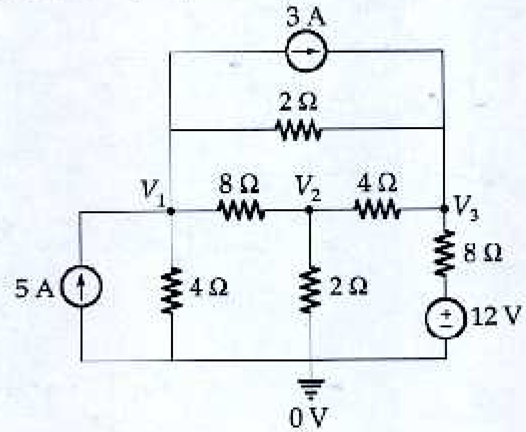
Since current through $(2+j3)\Omega$ is zero

so $I' = I''$

$$\Rightarrow 2.404 \angle 6.4416 = (0.068 \angle -38.5584) V_b$$

$$\Rightarrow \boxed{V_b = 35.354 \angle 45^\circ \text{ V}}$$

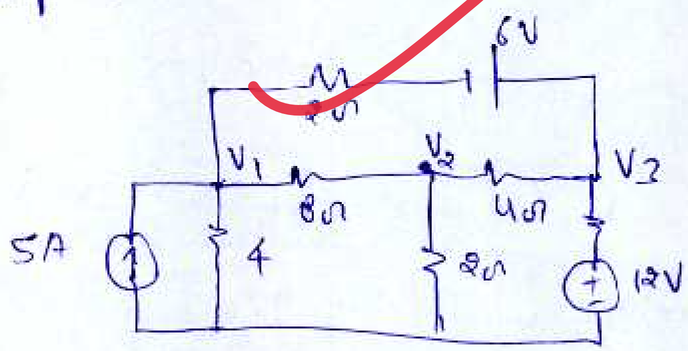
Q.1 (d) Use nodal analysis to find V_1, V_2 and V_3 in the circuit of figure.



[12 marks]

by Source transformation [Current Source to voltage Source]

Simplified Current is



by KCL at Node - 1

$$\frac{V_1}{4} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3 + 6}{2} = 5$$

$$\Rightarrow V_1\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right) - \frac{V_2}{8} - \frac{V_3}{2} = 2$$

$$\Rightarrow V_1(0.875) - 0.125V_2 - 0.5V_3 = 2 \quad ..(1)$$

by KCL at Node - 2

$$\frac{V_2 - V_1}{8} + \frac{V_2 - V_3}{4} + \frac{V_2}{2} = 0$$

$$\Rightarrow V_2(0.875) - 0.125V_1 - 0.25V_3 = 0 \quad ..(2)$$

KCL at Node-3

$$\frac{V_3-V_2}{4} + \frac{V_3-12}{8} + \frac{V_3-V_1}{2} = 3$$

$$\Rightarrow V_3\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{2}\right) - \frac{V_2}{4} - \frac{V_1}{2} = 4.5$$

$$\Rightarrow V_3(0.875) - 0.25V_2 - 0.5V_1 = 4.5 \quad (2)$$

by Solving Eq. (1) (2) and (3)

$$V_1 = 10\ V$$

$$V_2 = 4.932\ V$$

$$V_3 = 12.267\ V$$

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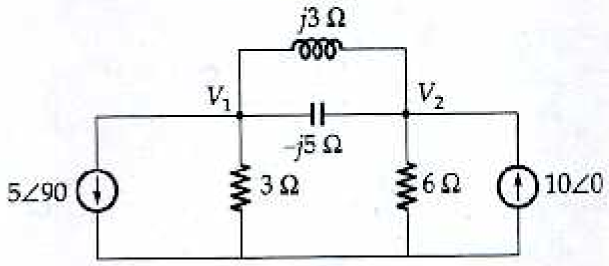
Q.1 (e) Use nodal analysis on the circuit to find V_2.



[12 marks]

by Nodal Analysis

KCL at Node - 1

$$\frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j3} + \frac{V_1}{3} + 5j = 0$$

$$\Rightarrow V_1\left(\frac{1}{-j5} + \frac{1}{j3} + \frac{1}{3}\right) + V_2\left(\frac{1}{j5} - \frac{1}{j3}\right) + 5j = 0$$

$$V_1\left(\frac{1}{3} - \frac{2}{15}j\right) + \frac{2}{15}iV_2 + 5i = 0 \ldots(1)$$

KCL at Node (2)

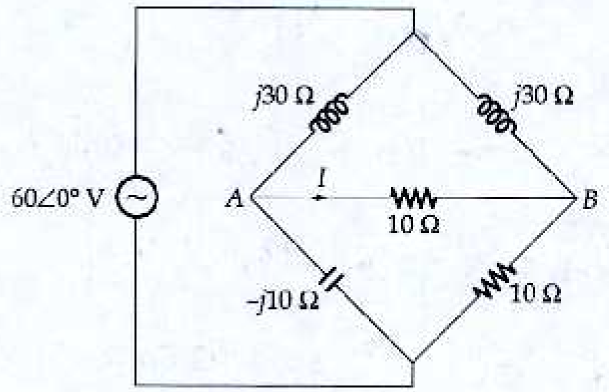
$$\frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j3} + \frac{V_2}{6} = 10$$

$$\Rightarrow V_2\left(\frac{1}{-j5} + \frac{1}{j3} + \frac{1}{6}\right) + V_1\left(\frac{1}{j5} - \frac{1}{j3}\right) = 10$$

$$\Rightarrow V_2\left(\frac{1}{6} - \frac{2}{15}j\right) + \frac{2}{15}iV_1 = 10 \ldots(2)$$

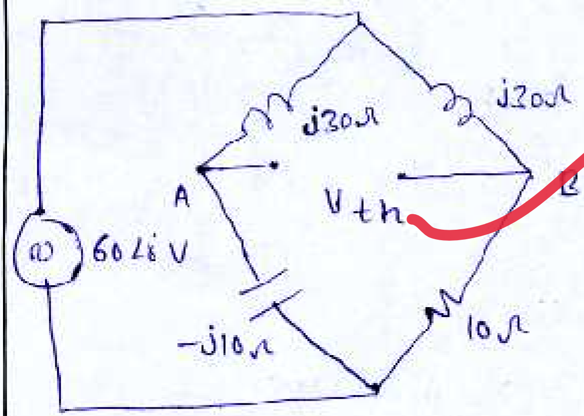
Incomplete solution

Q.2 (a) Determine the current I through the terminal AB of the network shown below:



[20 marks]

Sol: To determine the current through 10 Ω find out the thevenin's Equivalent across terminal A B. then circuit is



Step 1: find out Vth across terminal A-B.

by voltage divison,

$$V_A = \frac{-j10}{(30j - 10j)} 60$$

$$= \frac{-j10}{20j}(60) = -30\,V$$

Similarly by voltage divison,

$$V_B = \frac{10}{10+j30}(60) = (6-18j)\,V$$

then thevenin voltage is given as

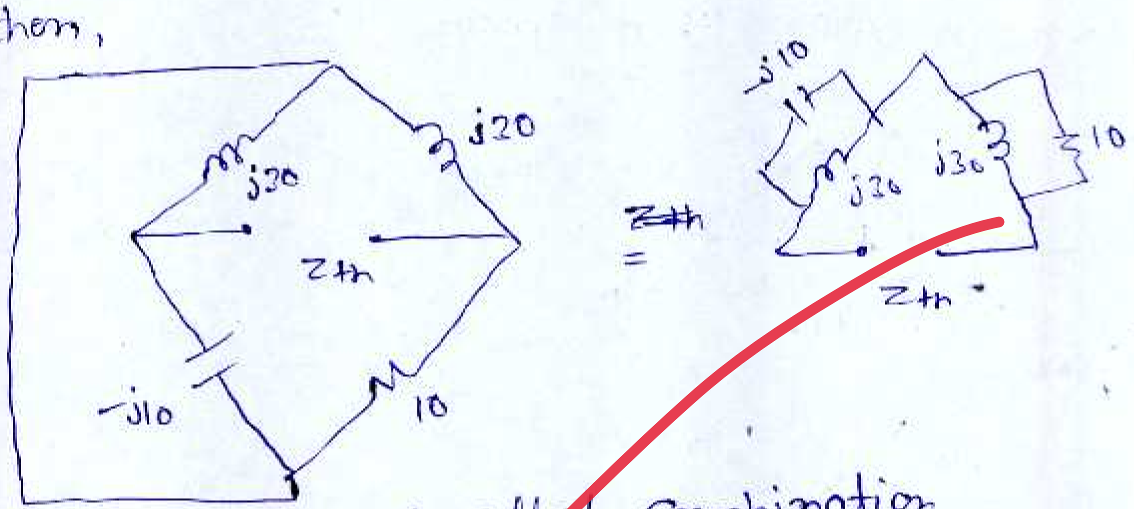
$$V_{th} = V_A - V_B$$

$$= -30 - (6 - 18j)$$

$$\boxed{V_{th} = (-36 + 18j)\ V}$$

Step: 2 find out Z_{th} across terminal A-B, for that disable the AC voltage source (short circuited)

then,



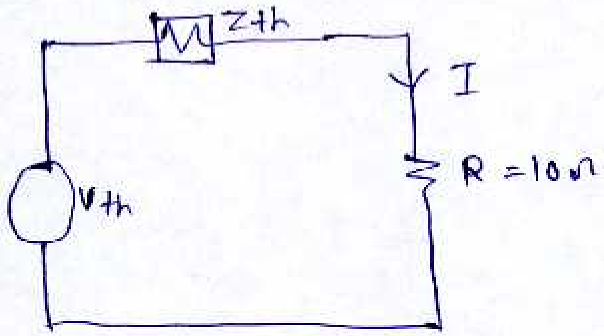
by combine parallel combination

$$Z_{th} = [30j \parallel (-j10)] + (10 \parallel j20)$$

$$= -15i + 9 + 3i$$

$$Z_{th} = (9 - 12i)\ \Omega$$

Step 3: then Equivalent thevenin circuit can be drawn as



where

$V_{th} = \cancel{(6-18j)}\ (-36+18j)\,V$

$Z_{th} = (9-12i)\,\Omega$

then by ohm's Law.

current flowing through $R = 10\Omega$

$$I = \frac{V_{th}}{Z_{th}+R}$$

$$\Rightarrow \quad I = \frac{\cancel{(6-18j)}\ (-36+18j)}{(9-12i)+10}$$

$$\Rightarrow \quad \boxed{I = 1.791\angle -174.289^\circ\ A}$$

then current I through terminal A-B

$$\boxed{I = 1.791\angle -174.289^\circ\ A}$$

or

$$\boxed{I = 1.791\angle 5.711^\circ\ A}$$

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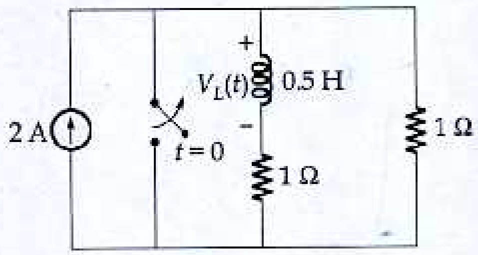
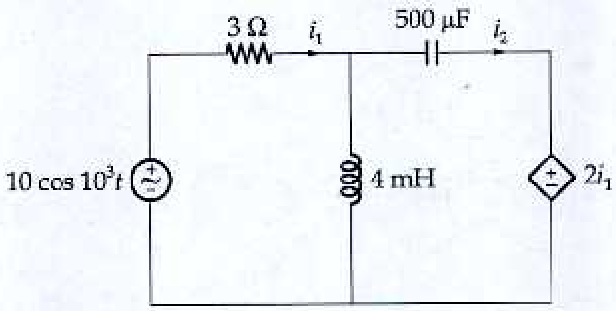
Q.2 (b) **(i)** For the network shown in figure below, the switch is closed for a long time and at $t = 0$, the switch is opened.



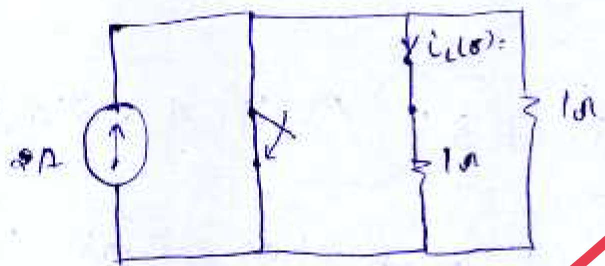
Determine the voltage across inductor for $t > 0$.

(ii) Obtain expressions for the time domain currents i_1 and i_2 in the circuit given as figure.



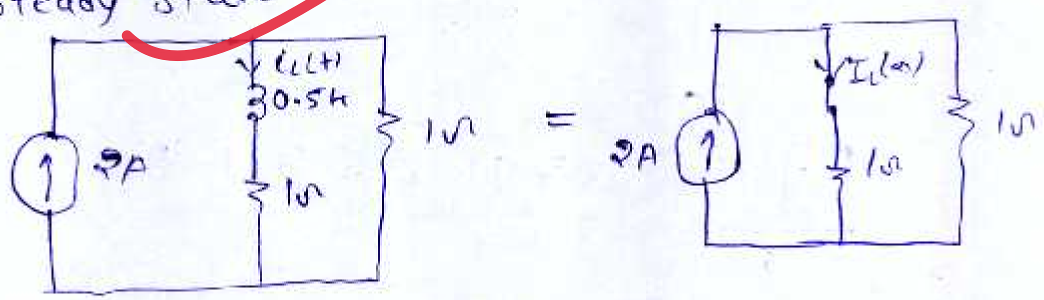
[10 + 10 marks]

(i) Circuit at $t < 0^-$



$i_L(0^-) = 0\ A$

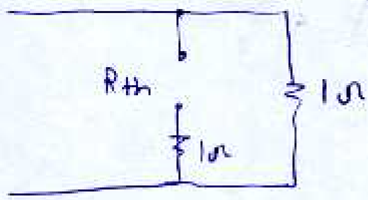
Circuit at $t > 0^+$, determine $I_L(\infty)$ at Steady state



by Current division

$$I_L(\infty) = 2\left(\frac{1}{2}\right) = 1\ A$$

then ~~for~~ find out R_{th} across inductor by open circuited current source



$R_{th} = 1+1$

$\boxed{R_{th} = 2\,\Omega}$

then Current through inductor

$$i_L(t) = I(\infty) + (i(0) - I(\infty))e^{-\frac{t}{\tau}}$$

where τ : time constant

$$\tau = \frac{L}{R} = \frac{0.5}{2} \text{ sec}$$

then

$$i_L(t) = 1 + (0-1)e^{-\frac{2t}{0.5}}$$

$$\boxed{i_L(t) = (1 - e^{-4t})\ A}$$

then Voltage across inductor is given by

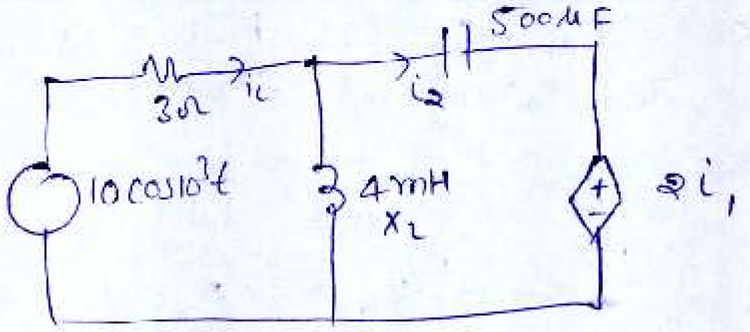
$$V_L(t) = L\frac{di_L}{dt}$$

$$= 0.5\frac{d}{dt}(1 - e^{-4t})$$

$$= 0.5(4e^{-4t})$$

$$\boxed{V_L(t) = 2e^{-4t}\ V \quad \text{at } t>0}$$

(ii)



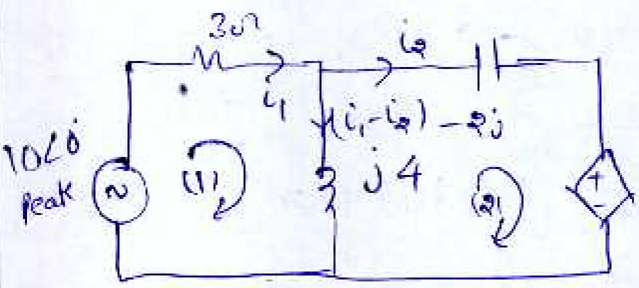
$X_L = j\omega L = j(1000) \times 4 \times 10^{-3}$

$= j4\ \Omega$

$X_c = \dfrac{-j}{1000 \times 500 \times 10^{-6}} = \dfrac{-j}{5 \times 10^{-1}}$

$\boxed{X_c = -2j\Omega}$

then circuit can be drawn as



KVL in Loop (1)

$3i_1 + j4(i_1 - i_2) = 10$

$\Rightarrow (3+j4)i_1 - 4j i_2 = 10 \quad \ldots (1)$

KVL in Loop (2)

$(-2j) i_2 + 2i_1 = j4(i_1 - i_2)$

$\Rightarrow \cancel{2j i_2 = 2j i_1}$ $\Rightarrow (2-4j)i_1 = -2j\, i_2$

$\Rightarrow i_2 = \dfrac{(2-4j)}{-2j} i_1$

$\Rightarrow \cancel{i_1 = i_2} \quad \ldots (2)$

from Equation (1) and (2), sustituing values then

$$\cancel{3+4j\, i_1 - 4j\, i}$$

$$(3+j4)\, i_1 - 4j \left(\frac{2-4j}{-2j}\right) i_1 = 10$$

$$\Rightarrow (3+j4)\, i_1 + 2(2-4j)\, i_1 = 10$$

$$\Rightarrow (7-j4)\, i_1 = 10$$

$$\Rightarrow i_1 = 1.24 \angle 29.744^\circ \text{ A}$$

and $i_2 = 2.7735 \angle 56.31^\circ$ A

then i_1 and i_2 in time domain

$$i_1(t) = 1.24 \cos(10^3 t + 29.744^\circ) \text{ A}$$

$$i_2(t) = 2.7735 \cos(10^3 t + 56.31^\circ) \text{ A}$$

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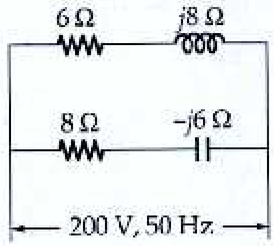
Good Approach

Q.2 (c) For the circuit shown below, calculate,

(i) Total admittance, total conductance and total susceptance.

(ii) Total current and total power factor (pf).

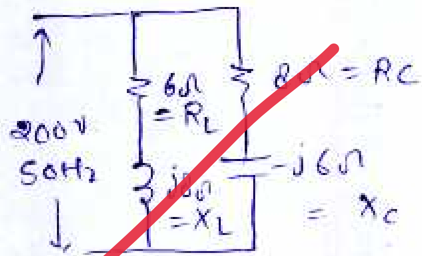
(iii) The value of pure capacitance to be connected in parallel with the above combination to make the total power factor (pf) unity.



[20 marks]

Sol: (i)

Equivalent (Total) admittance is



$$Y_T = Y_1 + Y_2$$

$$Y_T = \frac{1}{R_1 + jX_L} + \frac{1}{R_C - jX_C}$$

$$\Rightarrow Y_T = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$\Rightarrow Y_T = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} + j\left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

by sustituing values,

$$\Rightarrow Y_T = \frac{6}{6^2 + 8^2} + \frac{8}{6^2 + 8^2} + j\left[\frac{6}{6^2 + 8^2} - \frac{8}{6^2 + 8^2}\right]$$

$$\Rightarrow Y_T = (0.06 + 0.08) + j(0.06 - 0.08)$$

$$\Rightarrow Y_T = 0.14 + j(-0.02) \quad \text{.....(1)}$$

$\text{Real}[Y_T] = \text{conductance} = 0.14 \ \text{ohm}^{-1}$

$\text{Img}[Y_T] = \text{total Susptance} = -0.02 \ \text{ohm}^{-1}$

(ii) total Current

$$I_T = V \, Y_T$$ from Eq.(1).

$$\Rightarrow I_T = 200(0.14 - j0.02)$$

$$\Rightarrow I_T = (28 - 4j) \ A$$

$$\Rightarrow \boxed{I_T = 28.2842 \angle -8.13^\circ}$$

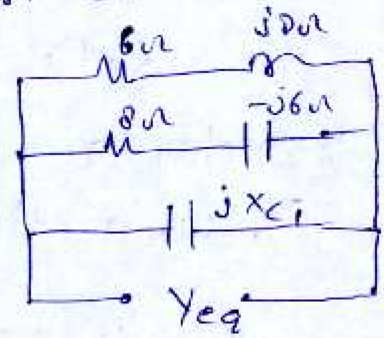
Total power factor $= \cos\phi$

$= \cos(8.13^\circ)$

$= 0.9899 \, [\text{Lagging}]$

(iii) Let value of capacitance connected across is C_1, then circuit

then, sustitute from Eq (1)



$Y_{eq} = 0.14 + j(-0.02) + jX_C$

$\Rightarrow Y_{eq} = 0.14 + j(X_{C_1} - 0.02)$

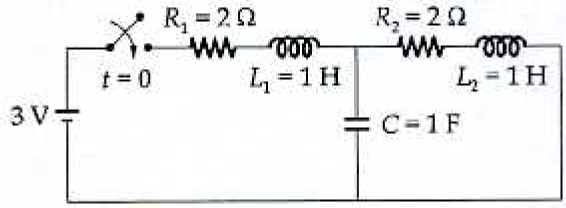
for unity power factor

$\text{Img}[Y_{eq}] = 0 \Rightarrow \boxed{X_{C_1} = 0.02 \ \Omega}$

Now, $\frac{1}{\omega C} = 0.02 \quad [\because f = 50 \text{ Hz}]$

$\Rightarrow \boxed{C = 0.15915 \text{ F} = 159.15 \text{ mF}}$

Q.3 (a) In the network shown in figure the switch is closed at time $t = 0$. Assuming all the initial currents and voltages as zero, find the current through the inductor L_2 by the use of Norton's theorem.



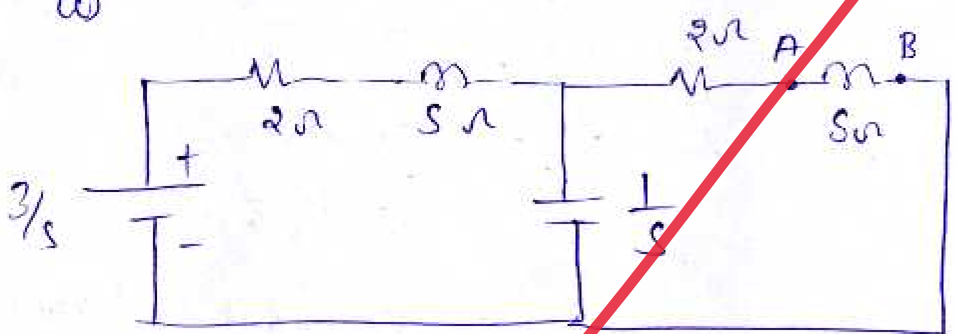
[20 marks]

Sol: given data $V_c(0^-) = 0\,V$

$i_{L_1}(0^-) = 0\,A$

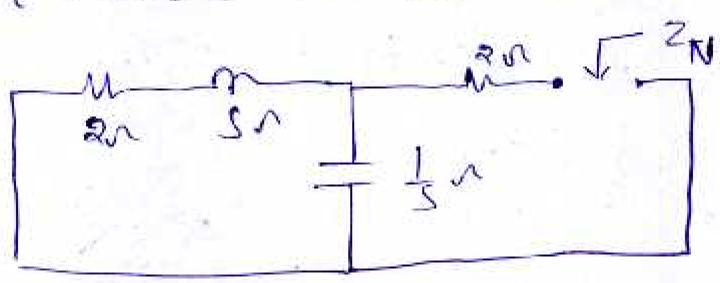
$i_{L_2}(0^-) = 0\,A$

Circuit in S- domain can be drawn as



To determine the Nortor Equivalent across the terminal A - B

Step: (1) determine Norton Equivalent resistance across terminal A-B [disable the dc source]



by combine parallel branch we get,

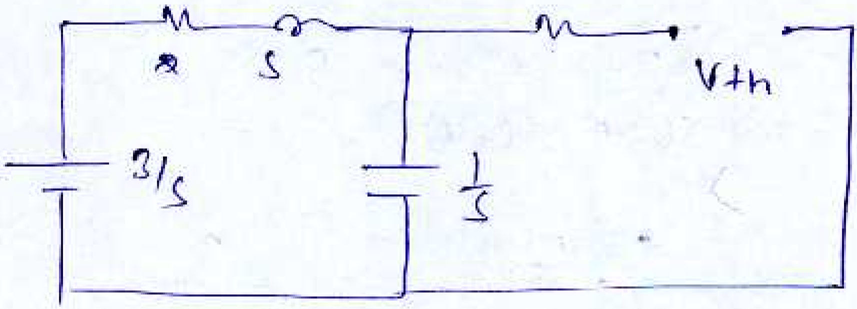
$$\Rightarrow Z_N = \left[(2+s) \,||\, \frac{1}{s}\right] + 2$$

$$\Rightarrow Z_N = \frac{(2+s) \times \frac{1}{s}}{2+s+\frac{1}{s}} + 2$$

$$\Rightarrow Z_N = \frac{(2+s)}{s^2+2s+1} + 2 = \frac{2s^2+4s+2+2+s}{s^2+2s+1}.$$

$$\Rightarrow Z_N = \frac{2s^2+5s+4}{s^2+2s+1} \quad \ldots \quad (1)$$

To determine V_{th} across AB, then



by voltage divison,

$$V_{th} = \frac{1/s}{\left(2+s+\frac{1}{s}\right)} \frac{3}{s}$$

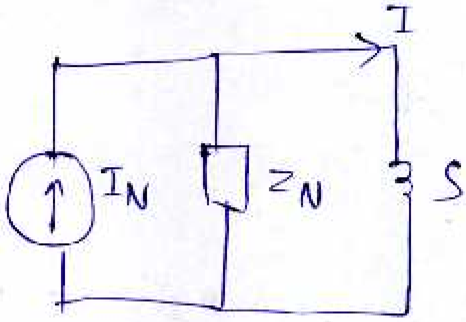
$$\Rightarrow V_{th} = \frac{1}{(s^2+2s+1)} \frac{3}{s} \quad \ldots (2)$$

then Norton Equivalent current is given as

$$I_N = \frac{V_{th}}{Z_N} = \frac{1}{(s^2+2s+1)} \frac{3}{s} \frac{(s^2+2s+1)}{(2s^2+5s+4)}$$

$$\Rightarrow I_N = \frac{3}{s(2s^2+5s+4)} \quad \ldots (3)$$

then Norton Equalent across A-B is drawn as



current through inductor L_2 is given as by current division,

$$I = \frac{Z_N}{Z_N + S} I_N$$

by sustituting values from Eq (1) & (3)

$$\Rightarrow I = \frac{\frac{3}{s(2s^2+5s+4)}}{\frac{3}{s(2s^2+5s+4)} + s}$$

$$\Rightarrow I = \frac{\frac{2s^2+5s+4}{s^2+2s+1}}{\frac{2s^2+5s+4}{s^2+2s+1} + s} \cdot \frac{3}{s(2s^2+5s+4)}$$

$$\Rightarrow I = \frac{2s^2+5s+4}{2s^2+5s+4 + s(s^2+2s+1)} \cdot \frac{3}{s(2s^2+5s+4)}$$

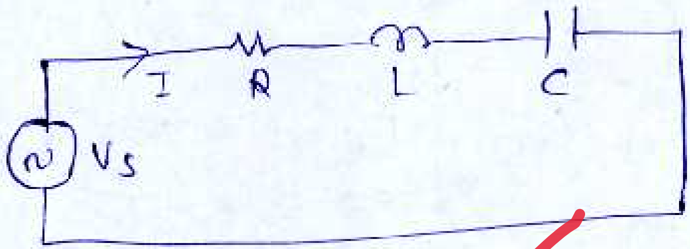
$$\Rightarrow I = \frac{3}{s[s^3+4s^2+6s+4]}$$

$$\Rightarrow I = \frac{3}{s(s+2)(s+1+i)(s+1-i)}$$

Q.3 (b) Show that the resonant frequency ω_0 of a series *R-L-C* circuit is geometric mean of ω_1 and ω_2, i.e., the upper and lower half power frequencies respectively.

[20 marks]

Sol: R-L-C Series circuit can be drawn as



Current I flowing through circuit

$$I = \frac{V_s}{Z_T} \quad \dots (1)$$

$$Z_T = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

then

$$I = \frac{V_s}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\Rightarrow \quad I = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \dots (2)$$

for determing, current at resonance condition, $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

$\mathrm{Img}[Z_T] = 0 \quad \Rightarrow \quad \boxed{Z_T = R}$

Let current at resonance is I_0,

then

$$I_0 = \frac{V_s}{R} \quad \dots (3)$$

$\omega_0 \rightarrow$ resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

$\omega_1, \omega_2 \rightarrow$ half power frequency

then current at half power point

$$I = \frac{I_0}{\sqrt{2}} \quad \ldots (3)$$

sustitute values from Eq (1) and (2)

$$\frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V_s}{\sqrt{2}\, R}$$

by solving we get,

$$\Rightarrow \omega L - \frac{1}{\omega C} = \pm R$$

at $\omega = \omega_1$ [lower half power frequency]

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\Rightarrow \omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad \ldots (4)$$

at $\omega = \omega_2$ [upper half power frenency

$$\omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$\Rightarrow \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad \ldots (5)$$

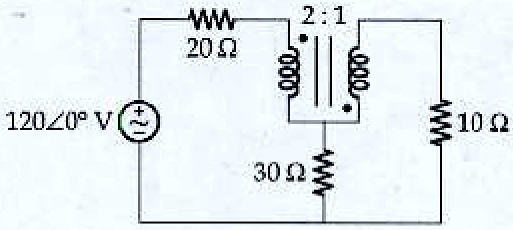
multiplying Equation (4) and (5)

$$\omega_1 \omega_2 = -\left(\frac{R}{2L}\right)^2 + \left(\frac{R}{2L}\right)^2 + \frac{1}{LC}$$

$$\Rightarrow \omega_1 \omega_2 = \frac{1}{LC} \Rightarrow \omega_1 \omega_2 = \omega_0^2 \Rightarrow \boxed{\omega_0 = \sqrt{\omega_1 \omega_2}}$$

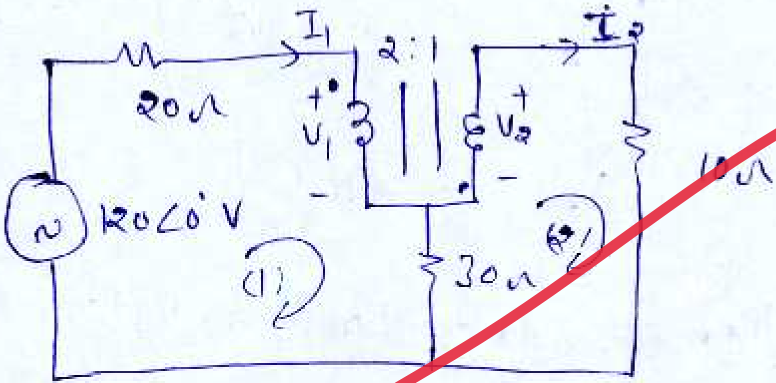
then ω_0 is geometric mean of ω_1 and ω_2.

Q.3 (c) Calculate the power supplied to the 10 Ω resistor in the ideal transformer circuit given in the figure below.



[20 marks]

Sol: Circuit can be redrawn as



$$\frac{V_1}{V_2} = -2 \quad \ldots(1) \qquad \text{and} \qquad \frac{I_1}{I_2} = \frac{-1}{2} \quad \ldots(2)$$

KVL in Loop (1)

$$20 I_1 + V_1 + 30(I_1 - I_2) = 120$$

$$\Rightarrow \quad 50 I_1 - 30 I_2 + V_1 = 120 \quad \ldots(3)$$

Sustitute value from Eq. (1) and (2)

$$50\left(-\frac{I_2}{2}\right) - 30 I_2 - 2V_2 = 120$$

$$\Rightarrow \quad \cancel{-75} - 55 I_2 - 2V_2 = 120 \quad \ldots(4)$$

KVL in Loop (2)

$$10 I_2 + 30(I_2 - I_1) = V_2$$

$$\Rightarrow \quad 40 I_2 - 30 I_1 = V_2 \quad \ldots(5)$$

Sustitute value in Eq (5) from Eq (2)

$$40 I_2 - 30\left(-\frac{I_2}{2}\right) = V_2$$

$$\Rightarrow \quad 55 I_2 = V_2 \quad \dots (6)$$

by solving Eq (4) and (6)

then,

$$\boxed{I_2 = -\frac{8}{11} \text{ A}}$$

$$\boxed{V_2 = -40 \text{ V}}$$

power supplied to the 10Ω resistor is given by

$$P_{supplied} = I^2 R$$

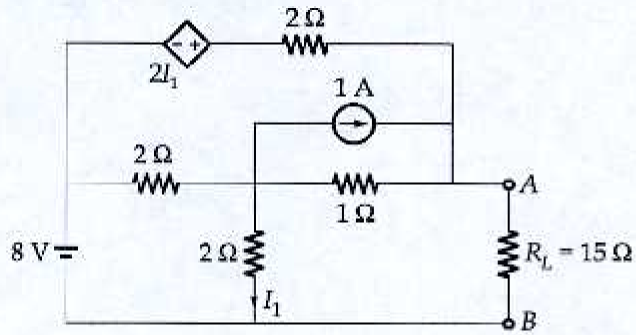
$$= \left(-\frac{8}{11}\right)^2 \times 10$$

$$\boxed{P_{10\Omega} = 5.289 \text{ watt}}$$

18

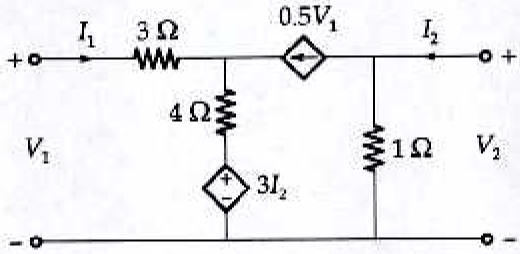
Good Approach

Q.4 (a) Determine the current through the load resistance $R_L = 15\ \Omega$ across the terminal *A*-*B* of the circuit shown in figure below, using Thevenin's theorem. Also find the maximum power that can be transferred to the load resistance R_L.



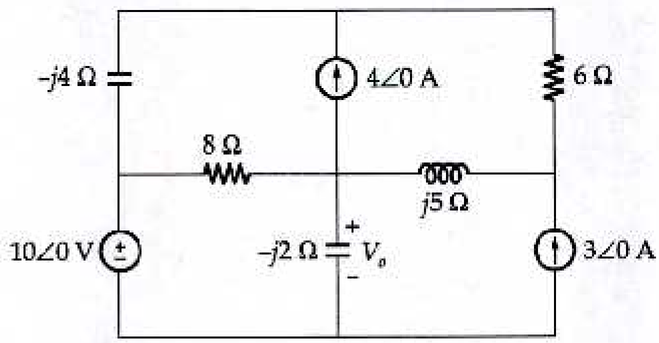
[20 marks]

Q.4 (b) Find the h-parameters for the two-port network shown



[20 marks]

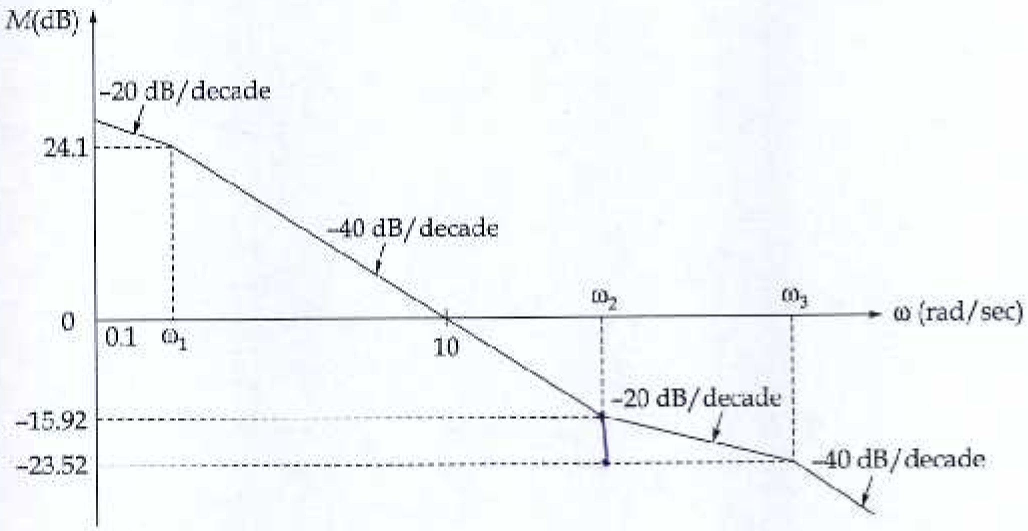
Q.4 (c) Solve for V_o in the circuit of figure using mesh analysis.



[20 marks]

Section B : Control System

Q.5 (a) Obtain the open loop transfer function for a unity negative feedback system whose bode magnitude plot is shown below:



[12 marks]

Sol: finding ω_1

$$-40 = \frac{24.1 - 0}{\log\left(\frac{\omega_1}{10}\right)}$$

$$\Rightarrow \boxed{\omega_1 = 2.4974 \simeq 2.5 \text{ rad/sec}}$$

finding ω_2,

$$-40 = \frac{-15.92 - 0}{\log\left(\frac{\omega_2}{10}\right)}$$

$$\Rightarrow \boxed{\omega_2 = 25 \text{ rad/sec}}$$

finding ω_3,

$$-20 = \frac{-23.52 + 15.92}{\log_{10}\left(\frac{\omega_3}{25}\right)}$$

$\Rightarrow \boxed{\omega_3 = 60 \text{ rad/sec}}$

Open Loop TF is given as

$$G(s)H(s) = \frac{K\left(1+\frac{s}{\omega_2}\right)}{s\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_3}\right)}$$

- initial slope is -20 dB/decade $\rightarrow$ pole at origin
- slope change -20 dB/decade $\rightarrow -40$ dB/decade $\rightarrow$ So pole
- slope change -40 dB/decade $\rightarrow -20$ dB/decade zero ~~pole~~ at ω_2

$$G(s)H(s) = \frac{K\left(1+\frac{s}{25}\right)}{s\left(1+\frac{s}{2.5}\right)\left(1+\frac{s}{60}\right)}$$

finding K

$$24.1 = -20\log_{10}(\omega_1) + 20\log_{10}K$$

$$24.1 = -3.979 + 20\log_{10}K$$

$\Rightarrow \cancel{K = 25.3517}$ ~~39.81~~ $\boxed{K = 40.08}$

then

$$G(s)H(s) = \frac{40.08\left(1+\frac{s}{25}\right)}{s\left(1+\frac{s}{2.5}\right)\left(1+\frac{s}{60}\right)}$$

Do not write in this margin

Q.5 (b) A servo mechanism is represented by the equation :

$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144E$$

where $E = C - 0.5y$ is the actuating signal. Find the value of damping ratio, damped and undamped frequency of oscillation. Draw the block diagram of the system described by the above equation.

[12 marks]

given differetial Eq.

Sol. $$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144E$$

given $E = C - 0.5y$

Sustitute in Eq.

$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144(C - 0.5y)$$

$$\Rightarrow \frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} + 72y = 144C$$

taking Laplace Transform

$$\frac{Y(s)}{C(s)} = \frac{144}{s^2 + 4.8s + 72}$$

Comparing with $\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ then

$\omega_n^2 = 72$ and $2\zeta\omega_n = 4.8$

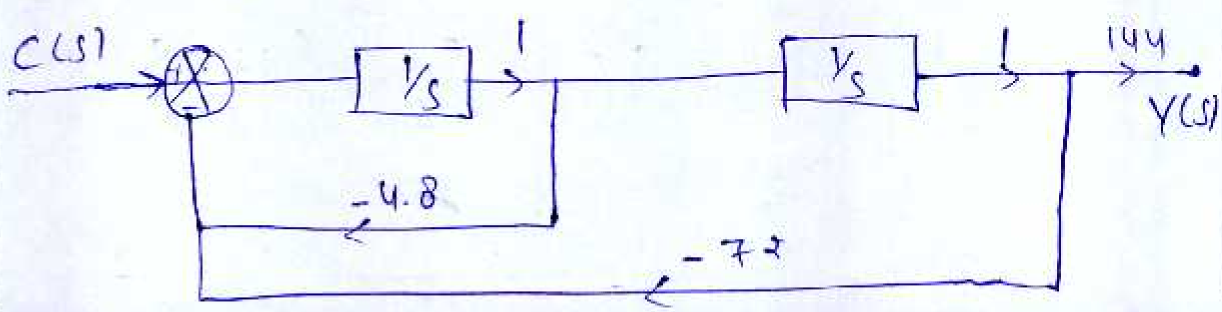
undamped frequency → Damping ratio

$$\Rightarrow \boxed{\omega_n = 8.485 \text{ rad/sec}} \qquad \boxed{\zeta = 0.2828}$$

Damped Natural frequency $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\boxed{\omega_d = 8.1385 \text{ rad/sec}}$$

Block Diagram:



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Q.5 (c) Closed loop system with unity feedback has the forward loop transfer function as :

$$G(s) = \frac{28.8}{s(1+0.2s)}$$

Modify the design using cascaded compensation to satisfy the optimum performance criterion, so that the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot. Take gain of proportional controller equal to 5.

[12 marks]

Q.5 (d) A unity negative feedback system has open loop transfer fucntion, $G(s) = \frac{K}{s(1+sT)}$, where K and T are positive constants. Determine the factor by which the amplifier gain K be reduced so that peak overshoot of the unit step response is reduced from 80% to 50%?

[12 marks]

given open loop Transfer function

$$G(s)H(s) = \frac{K}{s(1+sT)}$$

Characteristics Equation is given as

$$1 + G(s)H(s) = 0$$

$$\Rightarrow \quad 1 + \frac{K}{s(1+sT)} = 0$$

$$\Rightarrow \quad s^2T + s + K = 0$$

$$\Rightarrow \quad s^2 + \frac{1}{T}s + \frac{K}{T} = 0$$

by comparing standard 2nd order Eq. $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$2\zeta\omega_n = \frac{1}{T} \quad \text{and} \quad \omega_n^2 = \frac{K}{T}$$

$$\Rightarrow \omega_n = \sqrt{\frac{K}{T}}$$

$$\Rightarrow \quad 2\zeta\sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$\Rightarrow \quad \zeta = \frac{1}{2}\sqrt{\frac{T}{K}} \quad \dots (1)$$

for, $M_{p_1} = 80\%$ corresponding ζ_1

then $\quad e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.8$

$$\Rightarrow \quad \zeta_1 = \frac{-\ln(M_p)}{\sqrt{\pi^2 + (\ln(M_p))^2}}$$

$$\zeta_1 = -\frac{0.223}{\sqrt{\pi^2 + (0.221)^2}}$$

$$\Rightarrow \quad \zeta_1 = 0.07085 \dots$$

Similaryly for $M_{P_2} = 50\%$

Corresponding

$$\zeta_2 = \frac{-\ln(0.5)}{\sqrt{\pi^2 + (\ln(0.5))^2}}$$

$$\Rightarrow \zeta_2 = \frac{0.693}{\sqrt{\pi^2 + (0.693)^2}}$$

$$\Rightarrow \boxed{\zeta_2 = 0.2154}$$

~~for~~ from Equation (1)

$$\frac{\zeta_1}{\zeta_2} = \sqrt{\frac{K_2}{K_1}}$$

$$\Rightarrow \frac{K_2}{K_1} = \left(\frac{\zeta_1}{\zeta_2}\right)^2$$

$$\Rightarrow K_2 = 0.1082 K_1$$

gain K is reduced to 0.1082 times of initial value.

11

Q.5 (e) The open loop transfer function of a unity negative feedback system is given as, $G(s) = \dfrac{K}{2s(1+0.1s)(1+s)}$. Determine the value of 'K' for which the gain margin of the system is 14 dB.

[12 marks]

given open loop TF:

$$G(s)H(s) = \frac{K}{2s(1+0.1s)(1+s)}$$

for gain margin, determine the phase cross over frequency such that

$$\angle G(j\omega)H(j\omega) = -180$$

$$\Rightarrow -90 - \tan^{-1}(\omega) - \tan^{-1}(0.1\omega) = -180^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{\omega + 0.1\omega}{1 - 0.1\omega^2}\right) = 90^\circ$$

Now $1 - 0.1\omega^2 = 0$

$$\Rightarrow \boxed{\omega_{pc} = 3.1622 \text{ rad/sec}}$$

magnitude at $\omega = \omega_{pc}$

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = \frac{K}{2\omega\sqrt{1+\omega^2}\sqrt{1+(0.1\omega)^2}}$$

$$= \frac{K}{2(3.1622)\sqrt{1+(3.1622)^2}\sqrt{1+(0.1\times3.1622)^2}}$$

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = \frac{K}{22} = M$$

given $GM = 14\ dB$

$$20\log_{10}\frac{1}{M} = 14$$

$$\Rightarrow 20\log_{10}\frac{22}{K} = 14$$

$$\Rightarrow \frac{22}{K} = 5.0118$$

$$\Rightarrow K = 4.389$$

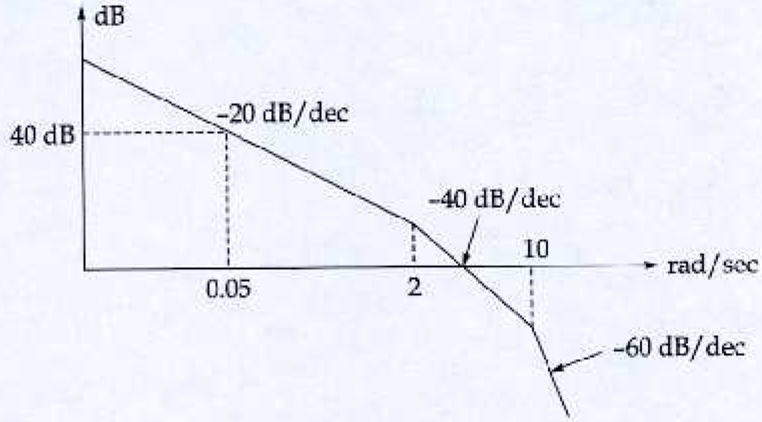
$$\Rightarrow \boxed{K \simeq 4.39}$$

Value of K for Gain Margin 14 dB is $\boxed{K = 4.39}$

11

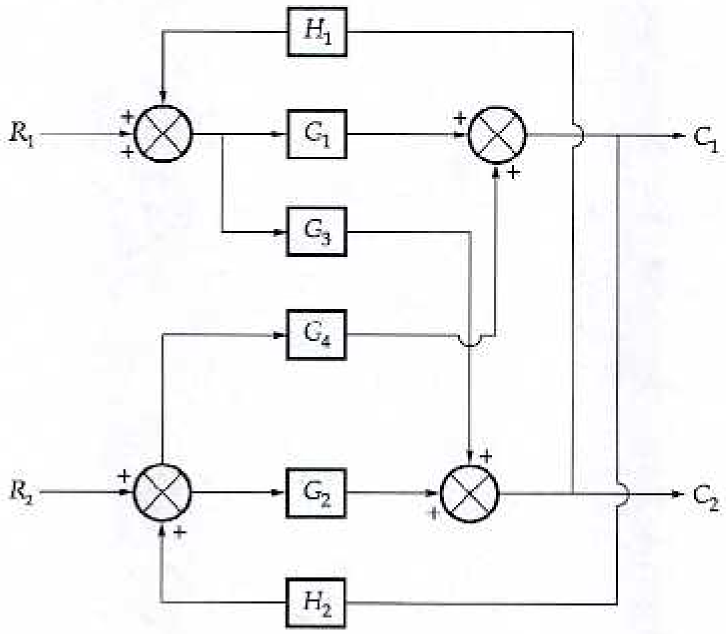
Good Approach

Q.6 (a) The open loop transfer function of a unity feedback system is given by $G(s)H(s) = e^{-Ts}G_1(s)$, where $G_1(s)$ is minimum phase system. The approximate bode magnitude plot of the open loop transfer function is shown in the figure below. If the phase margin of the system is -18.19°, determine the transportation lag T.

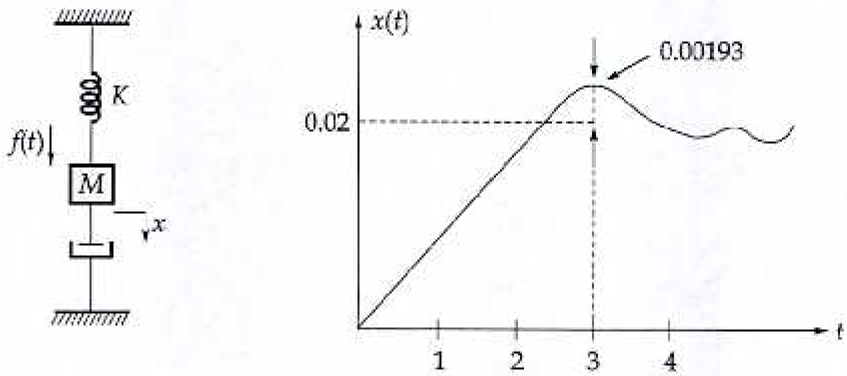


[20 marks]

Q.6 (b) (i) Evaluate $\frac{C_2}{R_1}$ for the system whose block diagram representation is shown in figure below. (Use block diagram reduction technique to solve).



(ii) Figure below shows a mechanical system and the response when 10 N of force is applied to the system. Determine the values of M, F, K. The dimension 'x' is in meter.



[10 + 10 marks]

Q.6 (c) Derive the expression for the transfer function of an ac servomotor and obtain the same in respect of a servomotor having following data :

(i) Starting torque = 0.166 N-m

(ii) Moment of inertia, $J = 1 \times 10^{-5}$ kgm^2

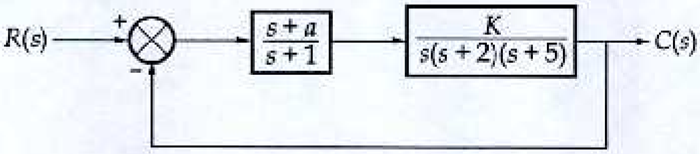
(iii) Supply voltage = 115 Volts

(iv) No load speed = 2904 rpm

(Assume friction to be zero)

[15 + 5 = 20 marks]

Q.7 (a) (i) A position control system is shown in figure below :



K and a are the parameters of the system. Determine the range of K and a for which system is stable.

(ii) Sketch the root-locus of $G(s) = \dfrac{K(s+1)}{s^2(s+2)}$.

[10 + 10 marks]

Sol: (i)

Characteristics Equation from the given position Control System is

$$1 + G(S)H(S) = 0$$

$$\Rightarrow \quad 1 + \frac{K(S+a)}{S(S+2)(S+1)(S+5)} = 0$$

$$\Rightarrow \quad S(S+2)(S+1)(S+5) + KS + aK = 0$$

$$\Rightarrow \quad (S^2+S)(S+2)(S+5) + KS + aK = 0$$

$$\Rightarrow \quad (S^3+3S^2+2S)(S+5) + KS + aK = 0$$

$$\Rightarrow \quad S^4 + 8S^3 + 17S^2 + (10+K)S + aK = 0$$

Necessary Condition for Stability

$$10 + K > 0$$

$$\Rightarrow \quad K > -10 \quad \ldots(1)$$

and $\quad aK > 0 \quad \ldots(2)$

forming Routh Array

S^4	1	17	aK
S^3	8	$(10+K)$	0
S^2	$\frac{136-(10+K)}{8}$	aK	0

Do not write in this margin

$$s^1 \quad \frac{\frac{(126-K)(10+K)}{8} - 8aK}{\left(\frac{126-K}{8}\right)} \qquad 0$$

$$s^0 \qquad aK$$

then for system to be stable, No sign change in first column od Routh Array

$$\frac{126-K}{8} > 0 \Rightarrow K < 126 \quad \ldots (2)$$

and $(126-K)(10+K) - 64aK > 0$

$$\Rightarrow \quad 1260 + 116K - K^2 - 64aK > 0$$

$\Rightarrow$

(ii) Given $G(s) = \frac{K(s+1)}{s^2(s+2)}$

Open Loop poles $s = 0, 0, -2$ $P = 3$

Open Loop Zeros $s = -1$ $Z = 1$

No. of branches terminating at infinity

$= P - Z$

$= 3 - 1 = 2$

Angle of Asymptotes is

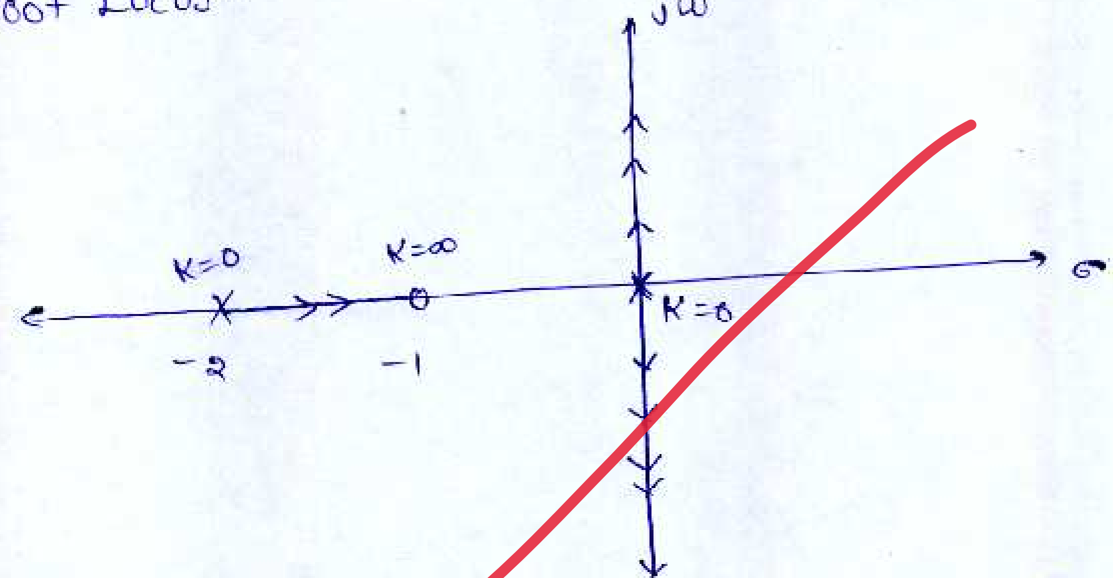
$\phi = \frac{(2q+1)180}{2}$

$\phi = 90°, 270°$

Centroid : $\frac{\Sigma P - \Sigma Z}{(P-Z)} = \frac{-2-(-1)}{2}$

$= -\frac{1}{2}$

Root Locus :



Breakaway points is given by solution of

$\frac{dK}{ds} = 0$

$$1 + \frac{K(s+1)}{s^2(s+2)} = 0$$

$$\Rightarrow \quad s^3 + 2s^2 + K(s+1) = 0$$

$$\Rightarrow \quad K = -\frac{(s^3+2s^2)}{(s+1)}$$

$$\Rightarrow \quad \frac{dK}{ds} = \frac{(s+1)(3s^2+4s) - (s^3+2s^2)}{(s+1)^2} = 0$$

$$3s^3 + 4 7 s^2 + 4s - s^3 - 2s^2 = 0$$

$$\Rightarrow \quad 2s^3 + 5s^2 + 4s = 0$$

$$\Rightarrow \quad s(2s^2 + 5s + 4) = 0$$

$$\Rightarrow \quad s = 0 \quad \text{and} \quad s = -1.25 + j0.66$$

$$-1.25 - j0.66$$

1

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Q.7 (b) Sketch the polar plot of the transfer function given below. Determine whether the plot crosses the real axis. If so, determine the frequency at which the plot cross the real axis and the corresponding magnitude $|G(j\omega)|$.

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

[20 marks]

Sol: given Open Loop Transfer function

$$G(S) = \frac{1}{S^2(1+S)(1+2S)}$$

$$G(j\omega) = \frac{1}{-\omega^2(j\omega+1)(1+2j\omega)} \quad \dots(1)$$

$$\angle G(j\omega) = 180 - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \quad \dots(2)$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{\omega^2+1}\sqrt{1+4\omega^2}} \quad \dots(3)$$

~~$\omega = 0$~~

~~$\omega = \infty$~~

	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
$\omega = 0$	∞	180
$\omega = \infty$	0	0

from Eq (1)

$$G(j\omega) = \frac{1}{-\omega^2(1+j\omega)(1+2j\omega)} \frac{(1-j\omega)(1-2j\omega)}{}$$

$$G(j\omega) = \frac{1-3j\omega-2\omega^2}{-\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

Separating Real and Imaginary parts.

$$G(j\omega) = \frac{1-2\omega^2}{-\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} - j\frac{3\omega}{-\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

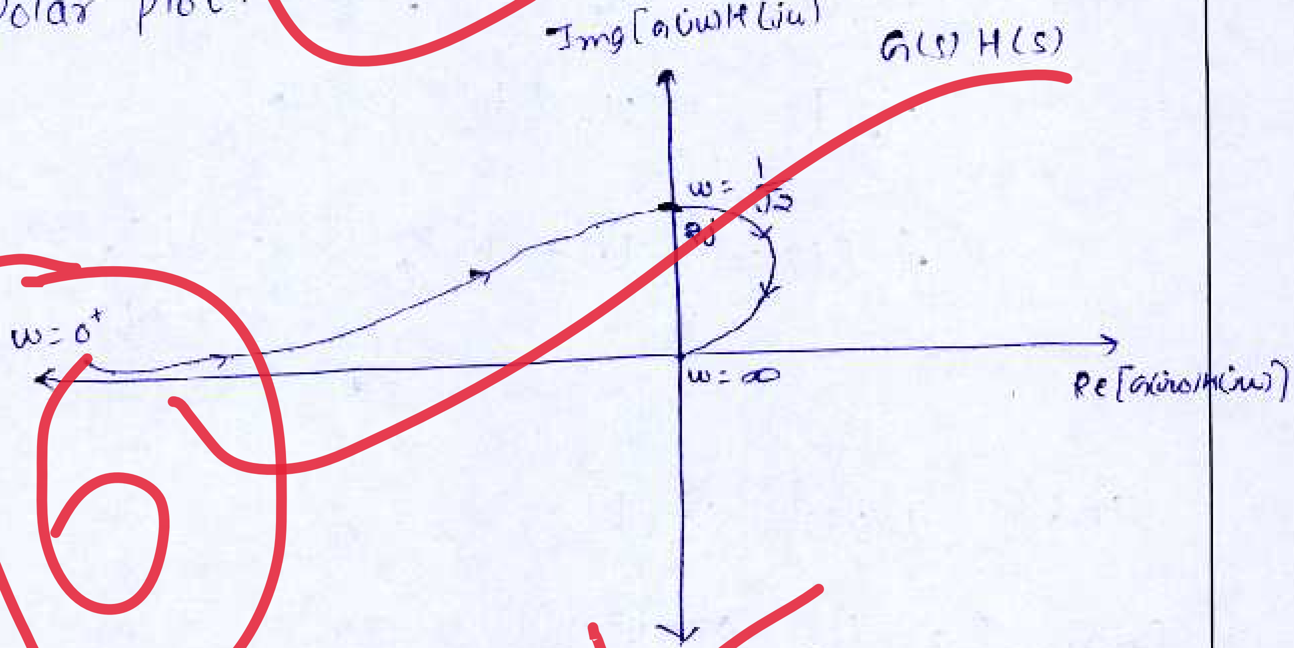
$$G(j\omega) = \frac{1-2\omega^2}{-\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} + \frac{3j}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \quad \ldots(4)$$

→ plot doesn't crosses the Real Axis. hence frequency at which plot crosses the real axis cann't determine.

→ plot crosses the Imaniary axis at from Eq. (4) Real [G(jω)] = 0

$$\Rightarrow \boxed{\omega = \frac{1}{\sqrt{2}}}$$ ← plot crosses Imaginary axis ← magnitude at this fre $|G(j\omega)| = 2$

Polar plot:



$$\boxed{|G(j\omega)|_{at\ \omega=\frac{1}{\sqrt{2}}} = \cancel{2\sqrt{2}}}$$

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Q.7 (c) Construct the state model for a system characterised by the differential equation :

$$\frac{d^3y}{dt^3}+\frac{6d^2y}{dt^2}+\frac{11dy}{dt}+6y=u$$

Give the block diagram representation of the state model.

[15 + 5 = 20 marks]

Soution: given differential Equation

$$\frac{d^3y}{dt^3}+6\frac{d^2y}{dt^2}+11\frac{dy}{dt}+6y=U \quad \dots(1)$$

Consider the state ~~val~~ variable as follows:

$$y(t)=x_1(t)$$

$$\frac{dy(t)}{dt}=\dot{x}_1(t)=x_2(t)$$

$$\frac{d^2y(t)}{dt^2}=\dot{x}_2(t)=x_3(t)$$

and

$$\dot{x}_1(t)=x_2(t) \quad \dots(1)$$

$$\dot{x}_2(t)=x_3(t) \quad \dots(2)$$

$$\dot{x}_3(t)=U-6x_3(t)-11x_2(t)-6x_1(t) \quad \dots(3)$$

and

$$y(t)=x_1(t) \quad \dots(4)$$

from Eq. (1) (2) (3) (4)

State model of system is given as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}U$$

and $y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$

Comparing with Standard State Space representati

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

System matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Input matrix

Output maxic

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

BLOCK Diagram Representation (BDR)

taking Laplace Transform of Eq. (1)

$$Y(s)\left[s^3 + 6s^2 + 11s + 6\right] = U(s)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{1}{s^3+6s^2+11s+6} = \frac{1/s^3}{1 + \frac{6}{s} + \frac{11}{s^2} + \frac{6}{s^3}}$$

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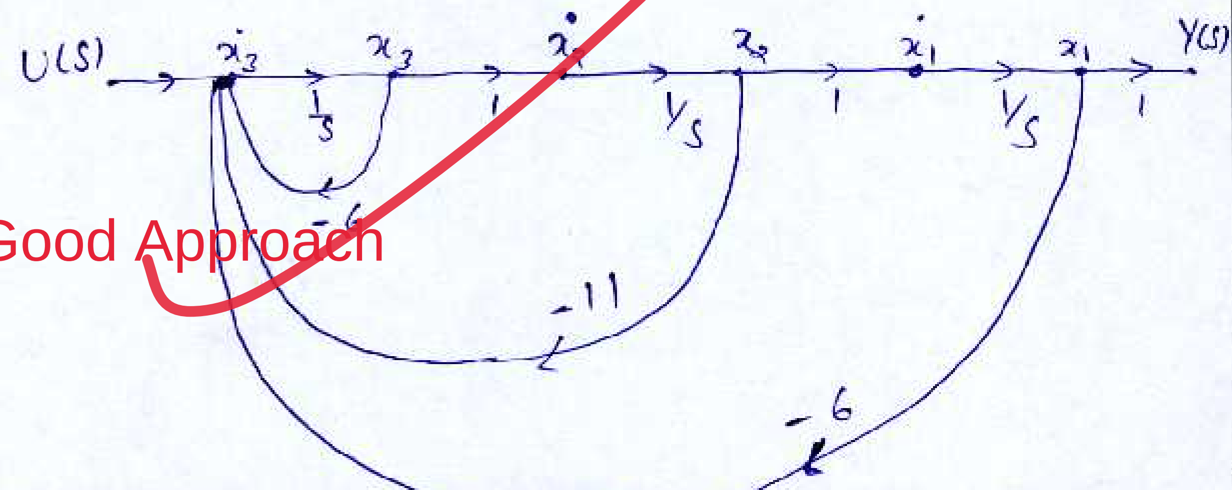
Transfer function $= \frac{Y(s)}{U(s)} = \frac{1/s^3}{1 - \left(-\frac{6}{s} - \frac{11}{s^2} - \frac{6}{s^3}\right)}$

Then BDR:



Good Approach

Q.8 (a) The open-loop transfer function of a unity feedback control system is given below:

$$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}$$

Plot the root locus and determine the value of K at the breakaway point.

[20 marks]

Q.8 (b) The open loop transfer function of a feedback control system is

$$G(s)H(s) = \frac{K(1+2s)}{s(1+s)(1+s+s^2)}$$

Find the restriction on K for stability. Find the value of K for the system to have a gain margin of 3 dB. With this value of K, find the gain cross over frequency and phase margin. Use Nyquist Approach.

[20 marks]

Q.8 (c) The state space model of a second order system given below is designed using feedback control system.

$$\dot{x} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

(i) What are the conditions for the desired response? Also check whether desired response is possible or not.

(ii) Design an observer system such that the above system has settling time of 0.5 sec and damping frequency of 6 rad/sec.

[8 + 12 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work