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Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 4

Section A : Electronic Devices & Circuits + Advanced Communication Topics

Q.1 (a) Solution:

Given that,

Energy band gap of silicon, $E_g = 1.17\text{ eV} - 4.73\times10^{-4}\dfrac{T^2}{T+636}$

The intrinsic carrier concentration is given by,

$$n_i^2 \propto T^3 e^{-E_g/KT}$$

$$n_i \propto T^{3/2} e^{-\frac{E_g}{2KT}}$$

Let at temperature, $T_1 = 300$ K,

$$n_i(T_1) \propto (T_1)^{3/2} e^{-\frac{E_g}{2KT_1}} \quad \text{...(i)}$$

Let at temperature, $T_2 = 77$ K

$$n_i(T_2) \propto (T_2)^{3/2} e^{-\frac{E_g}{2KT_2}} \quad \text{...(ii)}$$

Dividing equation (ii) with equation (i),

$$\frac{n_i(T_2)}{n_i(T_1)} = \left(\frac{T_2}{T_1}\right)^{3/2} e^{-\left[\frac{E_g(T_2)}{2KT_2} - \frac{E_g(T_1)}{2KT_1}\right]}$$

$$\therefore \quad n_i(77\text{ K}) = n_i(300\text{ K})\left(\frac{77}{300}\right)^{3/2} e^{-\left[\frac{E_g(T_2)}{2KT_2}-\frac{E_g(T_1)}{2KT_1}\right]}$$

where, $E_g(300\text{ K}) = E_g(T_1) = 1.12$ eV

We have, $KT_1 = 0.026$ eV

$$KT_2 = \frac{T_2}{11600} = \frac{77}{11600} = 0.0066\text{ eV}$$

$$E_g(T_2) = E_g(77\text{K}) = 1.17\text{ eV} - 4.73\times10^{-4}\frac{(77)^2}{77+636}$$

$$E_g(77\text{ K}) = 1.16\text{ eV}$$

$$n_i(77\text{ K}) = 1.05\times10^{10}\times\left(\frac{77}{300}\right)^{3/2} e^{-\left[\frac{1.16}{0.0132}-\frac{1.12}{0.052}\right]}$$

$$n_i(77\text{ K}) = 2.11\times10^{-20}\text{ cm}^{-3}$$

Q.1 (b) Solution:

Given, $N_A = 2\times10^{19}$ cm^{-3}

$N_D = 8\times10^{15}$ cm^{-3}

Since $N_A > N_D$, it is p^+n junction

The junction capacitance per unit area,

$$C_j = \frac{\epsilon_s}{W}$$

Where W is width of the one-sided abrupt junction, $W = \sqrt{\frac{2\epsilon_s(V_{bi}+|V_R|)}{qN_D}}$ for p^+n junction

$$V_{bi} = V_T\ln\left[\frac{N_AN_D}{n_i^2}\right] = 0.0259\ln\left[\frac{2\times10^{19}\times8\times10^{15}}{(9.65\times10^9)^2}\right]$$

$$\therefore \quad V_{bi} = 0.906\text{ V}$$

Depletion width at $V_R = 0$ V,

$$W = \sqrt{\frac{2\epsilon_s V_{bi}}{qN_D}}$$

$$= \sqrt{\frac{2\times11.9\times8.85\times10^{-14}\times0.906}{1.6\times10^{-19}\times8\times10^{15}}}$$

$$W = 3.86\times10^{-5}\text{ cm}$$

$$C_j(V_R = 0) = \frac{\epsilon_s}{W} = \frac{11.9 \times \epsilon_0}{3.86 \times 10^{-5}}\ \text{F/cm}^2$$

$$\therefore \quad C_j(V_R = 0) = 2.728 \times 10^{-8}\ \text{F/cm}^2$$

Depletion width at $V_R = -4$ V,

$$W = \sqrt{\frac{2 \epsilon_s (V_{bi} + 4)}{qN_D}}$$

$$= \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14}(0.906 + 4)}{1.6 \times 10^{-19} \times 8 \times 10^{15}}}$$

$$W = 8.99 \times 10^{-5}\ \text{cm}$$

Junction capacitance at $V_R = -4$ V is,

$$C_j(V_R = -4\ \text{V}) = \frac{\epsilon_s}{W}$$

$$= \frac{\epsilon_s}{W} = \frac{11.9 \times 8.85 \times 10^{-14}}{8.99 \times 10^{-5}}$$

$$\therefore \quad C_j(V_R = -4\ \text{V}) = 1.172 \times 10^{-8}\ \text{F/cm}^2$$

$$\therefore \quad \frac{C_j(V_R = 0\ \text{V})}{C_j(V_R = -4\ \text{V})} = \frac{2.728 \times 10^{-8}}{1.172 \times 10^{-8}} = 2.33$$

Q.1 (c) Solution:

(i) The relative refractive index difference is given as:

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

Hence,
$$n_1^2 - n_2^2 = 2\Delta n_1^2$$

$$n_2^2 = n_1^2 - 2\Delta n_1^2$$

$$\therefore \quad n_2^2 = (2.250) - 2 \times 0.03 \times (2.250) = 2.115$$

Now, we know

$$R_c \simeq \frac{3n_1^2\lambda}{4\pi(n_1^2 - n_2^2)^{3/2}}$$

$$R_c \simeq \frac{3 \times (1.5)^2 \times 0.82 \times 10^{-6}}{4\pi\left[[1.5]^2 - [2.115]\right]^{3/2}} = 8.88\ \mu\text{m}$$

(ii)

$$n_2^2 = n_1^2 - 2\Delta n_1^2$$

$$n_2^2 = (1.5)^2 - 2 \times 3 \times 10^{-3} \times (1.5)^2 = 2.2365 \Rightarrow n_2 = 1.4955$$

The cutoff wavelength for the single mode fiber is given as

$$\lambda_c = \frac{2\pi a n_1 (2\Delta)^{1/2}}{2.405}$$

$$\lambda_c = \frac{2\pi(4\times10^{-6})\times1.5\times(2\times3\times10^{-3})^{1/2}}{2.405}$$

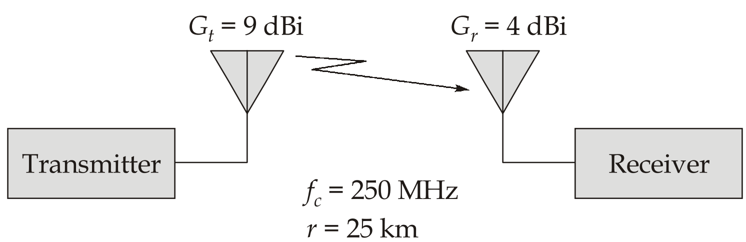
$$\lambda_c = 1.214\ \mu\text{m}$$

$$\therefore \quad R_{CS} \cong \frac{20\lambda}{(n_1-n_2)^{3/2}}\left[2.748 - 0.996\frac{\lambda}{\lambda_c}\right]^{-3}$$

$$R_{CS} \cong \frac{20\times1.55\times10^{-6}}{(0.0045)^{3/2}}\left[2.748 - \frac{0.996\times1.55\times10^{-6}}{1.214\times10^{-6}}\right]^{-3}$$

$$\cong 31.914\times10^{-3} \cong 31.914\ \text{mm}$$

Q.1 (d) Solution:



Given:

$P_t = 10\ \text{W} = 10^4\ \text{mW}$ — P_t : Transmitter output power

$G_t = 9$ dBi — G_t : Transmitter antenna gain

$G_r = 4$ dBi — G_r : Receiver antenna gain

Carrier frequency, $f_c = 250$ MHz

Distance between transmitter and receiver,

$$r = 25\ \text{km}$$

To convert $P_t(W)$ in P_t (dBm)

We know that, $P_t\,(\text{dBm}) = 10 \log_{10} P_t\,(\text{mW})$

$$P_t\,(\text{dBm}) = 10 \log_{10}[P_t\,(\text{mW})]$$

Therefore, $P_t\,(\text{dBm}) = 10 \log_{10} 10^4 = +40$ dBm

To determine free space path loss, L_{pf} (dB)

$$L_{pf}\,(\text{dB}) = 32.44 + 20\log_{10} r(\text{km}) + 20\log_{10} f_c(\text{MHz})$$
$$= 32.44 + 20\log_{10}25 + 20\log_{10}250$$
$$= 108.35 \text{ dB}$$

To find the transmitter antenna RF cable loss, L_t (dB)

$$\text{Cable length} = 20 \text{ m} \quad \text{(given)}$$
$$\text{Cable attenuation} = 3 \text{ dB/100 m} \quad \text{(given)}$$

Therefore,

T_X antenna RF cable loss, $$L_t = 20 \times \frac{3}{100}\text{dB} = \frac{3}{5}\text{dB} = 0.6 \text{ dB}$$

To calculate the power delivered to the receiver, P_r (dBm)

R_X antenna, RF cable loss, $L_r = 0.2$ dB (given)

$$P_r\,(\text{dBm}) = P_t\,(\text{dBm}) - L_t\,(\text{dB}) + G_t\,(\text{dB}) - L_{pf}\,(\text{dB}) + G_r\,(\text{dB}) - L_r\,(\text{dB})$$
$$P_r\,(\text{dBm}) = 40 - 0.6 + 9 - 108.35 + 4 - 0.2$$
$$= -56.15 \text{ dBm}$$

Hence, power delivered to the receiver is –56.15 dBm.

Q.1 (e) Solution:

(i) Given, $L = 6$ mm; $W = 2$ mm; $D = 1$ mm

$$I = 2.83 \text{ mA}$$
$$V = 10 \text{ V}$$

1. We know that, for photoconductor the change in current due to illumination

$$\Delta I = q(\mu_n + \mu_p)\Delta n\, EA$$

where, Δn is electron-hole pairs generated and E is the electric field across the device given by $E = V/L$.

$$\therefore \quad \Delta n = \frac{\Delta I}{q(\mu_n + \mu_p)EA}$$

$$\Delta n = \frac{2.83 \times 10^{-3}}{1.6 \times 10^{-19}(3600 + 1700)\left(\dfrac{10}{0.6}\right)(2 \times 1 \times 10^{-2})}$$

$$= 10^{13} \text{ cm}^{-3}$$

Equilibrium density of electron-hole pairs generated under radiation,

$$\therefore \quad \Delta n = 10^{13} \text{ cm}^{-3}$$

2. The current across the device varies as

$$\Delta I(t) = \Delta I \exp(-t/\tau)$$

$$|d\Delta I(t)/dt| = (1/\tau)\Delta I(t)$$

$$\tau = \frac{\Delta I}{|d\Delta I(t)/dt|} = \frac{2.83\times 10^{-3}}{23.6} = 120\ \mu\text{sec}$$

3. Excess concentration of electron and holes varies with time as

$$\Delta n(t) = \Delta n \exp(-t/\tau)$$

At $t = 1$ msec,

$$= 10^{13}\exp\left(-\frac{10^{-3}}{1.2\times 10^{-4}}\right)$$

$$\Delta n(t)\big|_{\text{at } t=1\text{ msec}} = 2.4\times 10^{9}\ \text{cm}^{-3}$$

(ii) Given,

$$N_A = 10^{17}\ \text{cm}^{-3}$$

$$\frac{Q_f}{q} = 10^{11}\ \text{cm}^{-2}$$

$$\phi_{ms} = -0.98\ \text{V}$$

Threshold voltage,

$$V_{th} = \phi_{ms} - \frac{Q_f}{C_{ox}} + 2\psi_B + \frac{2\sqrt{\epsilon_s qN_A\psi_B}}{C_{ox}}$$

$$\psi_B = V_T\ln\left(\frac{N_A}{n_i}\right) = 0.026\ln\left(\frac{10^{17}}{9.65\times 10^{9}}\right) = 0.42\ \text{V}$$

$$V_{\text{th}} = -0.98 - \frac{1.6\times 10^{-19}\times 10^{11}}{C_{ox}} + 0.84 + \frac{2\sqrt{11.9\times 8.85\times 10^{-14}\times 10^{17}\times 0.42\times 1.6\times 10^{-19}}}{C_{ox}}$$

$$V_{\text{th}} = -0.14 + \frac{15.2\times 10^{-8}}{C_{ox}} \quad \text{...(i)}$$

But oxide capacitance,

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9\times 8.85\times 10^{-14}}{t_{ox}}$$

$$C_{ox} = \frac{3.45\times 10^{-13}}{t_{ox}} \quad \text{...(ii)}$$

Given, $V_{th} > 20$ V

Using equation (i) and (ii),

$$\frac{t_{ox} \times 15.2 \times 10^{-8}}{3.45 \times 10^{-13}} > 20 + 0.14$$

$$\frac{t_{ox} \times 15.2 \times 10^{-8}}{3.45 \times 10^{-13}} > 20.14$$

$\therefore \quad t_{ox} > 4.57 \times 10^{-5}$ cm

(or) $\quad t_{ox(\text{min})} = 0.457\ \mu\text{m}$

Q.2 (a) Solution:

Given, Substrate doping concentration,

$$N_a = 10^{16}/\text{cm}^3,$$

oxide thickness, $t_{ox} = 50$ nm

(i) At flatband,

The hole concentration at $x = 0$ is equal to the doping concentration in the semiconductor.

i.e., $\quad P(x = 0) = N_a = 10^{16}/\text{cm}^3$

(ii) At threshold,

by the definition of threshold, the electron concentration at $x = 0$ is equal to the doping level of the substrate.

$$n(x = 0) = N_a = 10^{16}/\text{cm}^3$$

Since in the MOS structure under bias

$$np = n_i^2$$

$$P(x = 0) = \frac{n_i^2}{n(x=0)} = \frac{(10^{10})^2}{10^{16}} = 10^4/\text{cm}^3$$

(iii) From the Boltzman relation to relate carrier concentration across the depletion region of a MOS structure under bias.

$$\phi(x = 0) - \phi(x = x_d) = \frac{kT}{q}\ln\left(\frac{P(x_d)}{P(x=0)}\right)$$

for, $$P(x = 0) = P(x_d)\exp\left(\frac{-q}{kT}[\phi(x=0) - \phi(x = x_d)]\right)$$

$$= 10^{16}\exp\left(\frac{-0.5}{0.026}\right)$$

$\therefore \quad P(x = 0) = 4.5 \times 10^7\ \text{cm}^{-3}$

(iv) Given, capacitance per unit area of the MOS structure, $C = 50\ \text{nF/cm}^2$

the capacitance of oxide,

$$C_{ox} = \frac{\in_{oxide}}{t_{ox}} = \frac{3.45\times10^{-13}\ \text{F/cm}}{50\times10^{-7}\ \text{cm}}$$

$\therefore$ $$C_{ox} = 6.9 \times 10^{-8}\ \text{F/cm}^2$$

(or) $$C_{ox} = 69\ \text{nF/cm}^2$$

Since the oxide capacitance, C_{ox} is high than the given capacitance of MOS structure. Hence, the MOS structure is in depletion mode.

Therefore, the extent of the depletion region,

$$x_d = \frac{\in_{si}}{C_s}$$

where C_s is the capacitance of semiconductors.

We have, $$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_s}$$

$\therefore$ $$C_s = \frac{1}{\frac{1}{C} - \frac{1}{C_{ox}}} = \frac{1}{\frac{1}{50} - \frac{1}{69}}$$

$\therefore$ $$C_s = 182\ \text{nF/cm}^2$$

$\therefore$ $$x_d = \frac{\in_{si}}{C_s} = \frac{1.05\times10^{-12}}{182\times10^{-9}} = 5.8\times10^{-6}\ \text{cm},$$

$$x_d = 58\ \text{nm}$$

$\therefore$ The built-in potential across this depletion region (x_d) is

$$V_{bi} = \frac{qN_a x_d^2}{2\in_{si}} = \frac{1.6\times10^{-19}\times10^{16}\times(5.8\times10^{-6})^2}{2\times1.05\times10^{-12}}$$

$$V_{bi} = 2.6 \times 10^{-2}\ \text{V}$$

$\therefore$ The hole concentration at oxide-semiconductor interface ($x = 0$) is

$$P(x=0) = P(x_d)e^{\frac{-qV_{bi}}{KT}}$$

Since $$P(x_d) = N_a = 10^{16}\ \text{cm}^{-3}$$

$$P(x=0) = 10^{16}\exp\left(\frac{-qV_{bi}}{kT}\right) = 10^{16}\exp\left(\frac{-2.6\times10^{-2}}{0.026}\right)$$

$$P(x=0) = 3.67 \times 10^{15}/\text{cm}^3$$

Q.2 (b) Solution:

(i) The major sources of noise of photodetector are

- Dark current noise
- Shot noise
- Thermal noise

1. Dark Current Noise: The dark current noise is developed on account of dark current, meaning, the current is flowing in the circuit when the photodiode is not illuminated under bias condition. The value of the current is equal to the reverse saturation current of the photodiode. The magnitude of this current depends on the following:

(a) Operating temperature

(b) Bias voltage

(c) Type of detector

The main features of the dark current noise are as follows:

- The dark current develops a noise floor for the detectable signal power level. It is to be reduced by proper device design and fabrication.
- The typical values of dark current are the following:
  – At SiPIN photodiodes, it is 100 pA.
  – At GaAs PIN photodiodes and avalanche photodiodes, it is in the order of 2-5 nA.
  – At Ge avalanche photodiodes, it is 100 nA.
- The dark current noise is sometimes termed surface leakage current noise.

2. Shot Noise: This is the noise developed from the statistical nature of generation and collection of photoelectrons at the time of falling of optical signal to the photodiode. Its main features are as follows:

- A Poisson process is followed in case of shot noise.
- The shot noise current is expressed as

$$(i_{shn})^2 = 2eI_pB$$

where i_{shn} = short noise current

I_p = average photocurrent

B = single sided receiver bandwidth

- If dark noise current is high enough in comparison to signal current, signal current will be covered by the noise. As a result, the system cannot be used.

- If the value of dark noise current is low, then the effect of dark noise current is ignored since the average photocurrent is the sum of average dark current and average signal current.

3. **Thermal Noise:** Thermal noise generally occurs due to photodiode load resistance. The main features are:
   - Electrons inside a resistor always move in a resistor. This is due to the thermal energy acquired by them. Obviously, without applied voltage, the above scenario is observed.
   - The motion of the electrons is random in nature.
   - The movement of the charge generally occurs towards the electrode at any moment.
   - In this way, a randomly changing current remains in the resistor.
   - The current, so developed, is expressed as:

$$(i_{\text{thermal}})^2 = 4kTB/r$$

where i_{thermal} = thermal current

k = Boltzmann's constant

T = absolute temperature in kelvin

r = photodiode load resistance

(ii) The responsivity of a photodetector is given by

$$R = \frac{q\eta}{hf} = \frac{0.9\times1.6\times10^{-19}\times0.8\times10^{-6}}{6.63\times10^{-34}\times3\times10^{8}}$$

$$R = 0.576 \text{ A/W}$$

The photocurrent is

$$I_P = P_0R = 0.5\times10^{-6}\times0.576$$

$$I_p = 0.288\times10^{-6} \text{ A}$$

Multiplication Factor, $$M = \frac{I}{I_p} = \frac{13\times10^{-6}}{0.288\times10^{-6}} \approx 45.2$$

Q.2 (c) Solution:

(i) From the given C-V plot it is clear that, for positive values of gate voltage V_G, the device is in accumulation mode. Hence, the semiconductor is n-type. The applied positive voltage attracts more electrons towards the gate. In this mode of operation, the interface has more accumulated numbers of electrons near the oxide interface than the bulk.

(ii) From the given plot,

$$AC_{ox} = 200 \text{ pF}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{200\times10^{-12}}{2\times10^{-3}} \text{ F/cm}^2 = 100\times10^{-9} \text{ F/cm}^2$$

$$t_{ox} = \frac{\varepsilon_{ox}}{C_{ox}} = \frac{3.45\times10^{-13}}{100\times10^{-9}} \text{ cm} = 3.45\times10^{-6} \text{ cm}$$

$$t_{ox} = 345 \text{ Å}$$

(iii)

$$V_{FB} = \phi_{ms} - \frac{Q_s}{C_{ox}} = -0.8 \text{ V}$$

$$\frac{Q_s}{C_{ox}} = \phi_{ms} - V_{FB} = -0.5 \text{ V} + 0.8 \text{ V} = 0.3 \text{ V}$$

$$Q_s = 0.3 \times 100 \times 10^{-9} \text{ C/cm}^2 = 3 \times 10^{-8} \text{ C/cm}^2$$

The density of trapped oxide charges can be given by,

$$N_s = \frac{Q_s}{q} = \frac{3\times10^{-8}}{1.6\times10^{-19}} \text{ cm}^{-2} = 1.875\times10^{11} \text{ cm}^{-2}$$

(iv) Given that, $V_{FB} = -0.8 \text{ V}$

The MOS total capacitance at $V_G = V_{fb}$ is given by

$$C_{FB} = \left(\frac{C_{ox} C_{S(FB)}}{C_{ox} + C_{S(FB)}}\right) A \qquad ...(i)$$

where, $C_{S(FB)}$ = capacitance of the semiconductor at flat band condition

$$= \frac{\varepsilon_{si}}{L_D}$$

Where L_D = Debye length $= \sqrt{\frac{\varepsilon_{si}}{qN_D} V_t}$; $V_t = \frac{kT}{q} = 0.026 \text{ V}$

$$= \sqrt{\frac{1.06\times10^{-12}\times0.026}{1.6\times10^{-19}\times2\times10^{16}}} \text{ cm} = 2.935\times10^{-6} \text{ cm}$$

$$\therefore \quad C_{S(FB)} = \frac{1.06\times10^{-12}}{2.935\times10^{-6}} \text{ F/cm}^2 = 3.6\times10^{-7} \text{ F/cm}^2$$

From equation (i),

$$C_{FB} = \frac{(100\times10^{-9})(3.6\times10^{-7})}{(1+3.6)\times10^{-7}} \times 2\times10^{-3} \text{ F} = 156.52 \text{ pF}$$

Q.3 (a) Solution:

Given,
$$N_a = N_d = 10^{17}\text{ cm}^{-3}$$
$$A = 10^{-3}\text{ cm}^2$$
$$\mu_n = 3000\text{ cm}^2/\text{V-sec}$$
$$\mu_p = 200\text{ cm}^2/\text{V-sec}$$
$$\tau_{p0} = \tau_{n0} = \tau_0 = 10^{-8}\text{ sec}$$

The reverse saturation current density of pn junction,
$$J_s = qn_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$$

where,
$$D_n = \mu_n V_t = 3000 \times 0.0259 = 77.7\text{ cm}^2/\text{V-s}$$
$$D_p = \mu_p V_t = 200 \times 0.0259 = 5.18\text{ cm}^2/\text{V-s}$$
$$J_s = (1.6\times10^{-19})(1.8\times10^{6})^2\left[\frac{1}{10^{17}}\sqrt{\frac{77.7}{10^{-8}}}+\frac{1}{10^{17}}\sqrt{\frac{5.18}{10^{-8}}}\right]$$

$\therefore$
$$J_s = 5.75 \times 10^{-19}\text{ A/cm}^2$$

The reverse saturation current,
$$I_s = J_s \cdot A = 5.75 \times 10^{-19} \times 10^{-3} = 5.75 \times 10^{-22}\text{ A}$$

Total reverse bias current,
$$I_{TS} = I_s + I_{gen}$$

I_{gen} is the current due to electron-hole generation at junction.

$\therefore$
$$I_{gen} = \frac{qn_iWA}{\tau_{n0}+\tau_{p0}} = \frac{qn_iWA}{2\tau_0}$$

Where, width of depletion region in reverse bias,
$$W = \left[\frac{2\in_G(V_{bi}+V_R)}{q}\left(\frac{N_d+N_a}{N_aN_d}\right)\right]^{\frac{1}{2}}$$
$$V_{bi} = V_t\ln\left(\frac{N_aN_d}{n_i^2}\right)$$
$$= 0.0259\ln\left[\frac{10^{17}\times10^{17}}{(1.8\times10^{6})^2}\right]$$
$$V_{bi} = 1.28\text{ V}$$

$$\therefore \quad W = \left[\frac{2\times(13.1\times 8.85\times 10^{-14})(1.28+5)}{1.6\times 10^{-19}}\times\left(\frac{10^{17}+10^{17}}{10^{17}\times 10^{17}}\right)\right]^{\frac{1}{2}}$$

$$\therefore \quad W = 0.427\times 10^{-4}\text{ cm}$$

$$\therefore \quad I_{gen} = \frac{(1.6\times 10^{-19})(1.8\times 10^{6})(0.427\times 10^{-4})(10^{-3})}{2(10^{-8})}$$

$$\therefore \quad I_{gen} = 6.15\times 10^{-13}\text{ A}$$

$\therefore$ Total diode reverse bias current,

$$I_{Ts} = I_s + I_{gen}$$

$$= 5.75\times 10^{-22} + 6.15\times 10^{-13}$$

$$\therefore \quad I_{Ts} \simeq 6.15\times 10^{-13}\text{ A}$$

At forward bias voltage, $V_a = 0.5$ V,

The diode current, $I_D = I_s\exp(V_a/V_t) = 5.75\times 10^{-22}\exp\left(\dfrac{0.5}{0.0259}\right)$

$$I_D = 1.39\times 10^{-13}\text{ A}$$

During forward bias, $I_{gen} = \dfrac{qn_iWA}{2\tau_0}\exp\left(\dfrac{V_a}{2V_t}\right)$

At $V_a = 0.5$ V, $W = \left[\dfrac{2(13.1)(8.85\times 10^{-14})(1.28-0.5)}{1.6\times 10^{-19}}\left(\dfrac{2\times 10^{17}}{10^{34}}\right)\right]^{\frac{1}{2}}$

$$\therefore \quad W = 0.15\times 10^{-4}\text{ cm}$$

$$\therefore \quad I_{gen} = \frac{(1.6\times 10^{-19})(1.8\times 10^{6})(0.15\times 10^{-4})(10^{-3})}{2\times 10^{-8}}\exp\left(\frac{0.5}{2(0.0259)}\right)$$

$$I_{gen} = 3.36\times 10^{-9}\text{ A}$$

Total forward current,

$$I_{Fs} = 1.39\times 10^{-13} + 3.36\times 10^{-9}\text{ A}$$

$$\therefore \quad I_{FS} \approx 3.36\times 10^{-9}\text{ A}$$

Q.3 (b) Solution:

(i) During mid day, $f_{c1} = 4.5$ MHz

During sun set time, $f_{c2} = 1.5$ MHz

The critical frequency is given by the expression

$$f_c = 9\sqrt{N_{max}}$$

where, N is the electron density

During mid-day time,

$$4.5 \times 10^6 = 9\sqrt{N_{max_1}} \quad ...(i)$$

Similarly, during sunset time,

$$1.5 \times 10^6 = 9\sqrt{N_{max_2}} \quad ...(ii)$$

From (i) and (ii), we get

$$(4.5 \times 10^6)^2 - (1.5 \times 10^6)^2 = 81(N_{max_1} - N_{max_2})$$

$$(N_{max1} - N_{max2}) = \frac{18 \times 10^{12}}{81}$$

$$N_{max_1} - N_{max_2} = 0.222 \times 10^{12}/m^3$$

Hence, the change in the electron density of the E-layer is $0.222 \times 10^{12}/m^3$.

(ii) Given two prime numbers, $p = 13$, $q = 17$

RSA algorithm:

1. $p = 13$, $q = 17$
2. $n = p \times q = 13 \times 17 = 221$

   $z = (p - 1) \times (q - 1) = 12 \times 16 = 192$
3. Given public key (e) as 35, let the private key be d, then

$$(d \times e) \bmod z = 1$$

$$(d \times 35) \bmod 192 = 1$$

$$d = 11$$

Hence, the private key of A is 11.

Q.3 (c) Solution:

(i) IPv4;

1. In IPv4, there are only 2^{32} possible ways how to represent the address (about 4 billion possible addresses).
2. The IPv4 address is written by dotted decimal notation, e.g. 121.2.8.12.
3. The basic length of the IPv4 header comprises a minimum of 20 bytes (without optional fields). The maximum total length of the IPv4 header is 60 bytes (with optional fields), and it uses 13 fields to identify various control settings.

4. IPv4 header has a checksum, which must be computed by each router.
5. IPv4 contains an 8 bit field service type. The service type field is composed of a *TOS* (type of service) field and a procedure field.
6. The IPv4 node has only stateful auto configuration.
7. Security in IPv4 networks is limited to tunneling between two networks.
8. Source and destination addresses are 32 bits (4 bytes) in length.
9. IP sec support is optional.
10. No identification of packet flow for QoS handling by routers is present within the IPv4 header.
11. Address Resolution Protocol (ARP) uses broadcast ARP request frames to resolve an IPv4 address to a link layer address.
12. Must be configured either manually or through DHCP.
13. ICMP Router Discovery is used to determine the IPv4 address of the best default gateway and is optional.
14. Header includes options. There is no separate extension header.

IPv6;

1. In IPv6, there are 2^{128} possible ways (about $3.4 * 10^{38}$ possible addresses)
2. IPv6 is written in hexadecimal and consists of 8 groups of 16 bits each e.g. FABC : AC77 : 7834 : 2222 : FACB : AB98 : 5432 : 4567.
3. The IPv6 header is a static header of 40 bytes in length, and has only 8 fields. Option information is carried by the extension header, which is placed after the IPv6 header.
4. IPv6 does not use header checksum.
5. The IPv6 header contains an 8 bit field called the traffic class field. This field allows the traffic source to identify the desired delivery priority of its packets.
6. The IPv6 has both a stateful and a stateless address autoconfiguration mechanism.
7. IPv6 has been designed to satisfy the growing and expanded need for network security.
8. Source and destination address are 128 bits (16 bytes) in length.
9. IPsec support is required.
10. Packet flow identification for QoS handling by routers is included in the IPv6 header using the flow label field.

11. ARP request frames are replaced with multicast neighbour solicitation messages.
12. Does not require manual configuration or DHCP.
13. ICMP Router Discover is replaced with ICMP Router solicitation and Router advertisement messages and is required.
14. All optional data is moved to IPv6 extension headers.

(ii) 1. To determine the minimum cluster size, K for a signal to co-channel interference,

$$\frac{C}{I} = 18 \text{ dB}$$

$$\therefore \quad 18 \text{ dB} = 10\log_{10}\frac{C}{I}$$

$$\text{or} \quad \left(\frac{C}{I}\right) = 10^{1.8} = 63.1$$

To determine the frequency reuse ratio, q we know that

$$q = \left(6 \times \frac{C}{I}\right)^{\frac{1}{n}}, \text{ where } n = \text{path-loss exponent} = 4$$

$$q = (6 \times 63.1)^{\frac{1}{4}} = 4.41$$

To determine the cluster size, K

We know that, $q = \sqrt{3K}$

or, $K = \dfrac{q^2}{3}$

or, $K = \dfrac{(4.41)^2}{3} = 6.48$

Hence, the nearest valid cluster size, $K = 7$.

2. We know that, $q = \sqrt{3K}$

For $K = 7$, $q = \sqrt{21} = 4.58$

To determine $\dfrac{C}{I}$ for $K = 7$

We know that for six equidistant co-channel cells in the first tier,

$$\frac{C}{I} = \frac{1}{6} \cdot (q)^4$$

$$\frac{C}{I} = \frac{1}{6} \cdot (4.58)^4$$

Hence, $\frac{C}{I}\text{(ratio)} = 73.5$

To convert $\frac{C}{I}\text{(ratio)}$ in $\frac{C}{I}\text{(dB)}$

We know that, $\frac{C}{I}\text{(dB)} = 10\log_{10}\left(\frac{C}{I}\right)$

$$= 10 \log_{10} 73.5$$

$$= 18.66 \text{ dB}$$

Acceptable $\frac{C}{I}$ = 20 dB (given)

The required $\frac{C}{I}$ of 18.73 dB is less than the acceptable $\frac{C}{I}$ of 20 dB for the given situation. Hence, $K = 7$ cannot meet the desired $\frac{C}{I}$ requirement.

To convert given $\frac{C}{I}$ in dB to $\frac{C}{I}$ in ratio

We know that,

$$\left(\frac{C}{I}\right)_{\text{dB}} = 10\log_{10}\left(\frac{C}{I}\right)$$

$$20 = 10\log_{10}\frac{C}{I}$$

$$\frac{C}{I} = 10^2 = 100$$

To determine new frequency re-use ratio, q.

The required frequency reuse ratio q, corresponding to $\frac{C}{I}$ = 20 dB or 100 can be determined from the relationship.

$$q = \left(6 \times \frac{C}{I}\right)^{\frac{1}{4}}$$

$$q = (6 \times 100)^{\frac{1}{4}} = 4.95$$

To determine new cluster size K for $\frac{C}{I}$ = 20 dB

We know that, $K = \frac{(q)^2}{3}$

or, $$K = \frac{(4.95)^2}{3} = 8.165 \cong 9$$

Hence, the nearest valid cluster size, $K = 9$ for enhanced C/I of 20 dB.

Q.4 (a) Solution:

(i) Given: $T_0 = 290$ K

For the main receiver, $F = 10^{1.2} = 15.85$

For the cable, $L = 10^{0.5} = 3.16$

For the LNA, $G_1 = 10^5 = 10,0000$

Hence,

Overall system noise temperature is given by

$$T_s = T_{\text{ant}} + T_{e1} + \frac{(L-1)T_0}{G_1} + \frac{L(F-1)T_0}{G_1}$$

$$T_s = 35 + 150 + \frac{(3.16-1)\times 290}{10^5} + \frac{3.16\times(15.85-1)\times 290}{10^5}$$

$$T_s = 185.14 \text{ K}$$

Consider figure (b),

In this case, the cable precedes the LNA, and therefore, the equivalent noise temperature referred to the cable input is

$$T_s = T_{ant} + (L-1)T_0 + T_{e1}\times L + \frac{L(F-1)T_0}{G_1}$$

$$T_s = 35 + (3.16-1)\times 290 + 3.16\times 150 + \frac{3.16\times(15.85-1)\times 290}{10^5}$$

$$T_s = 1136 \text{ K}$$

(ii) 1. Given: $\alpha = 0.2$

$\text{BER} = 10^{-5}$

$\text{losses} = 200$ dB

$\frac{G}{T} = 32$ dB K^{-1}

$B = 36$ MHz

We have, $$B = \frac{R_b(1+\alpha)}{2}$$

$$\frac{2B}{(1+\alpha)} = R_b$$

$$\therefore \qquad R_b = \frac{2 \times 36 \times 10^6}{1.2} = 60 \text{ Mbps}$$

In decibels, $[R_b] = 10 \log_{10} 60 \times 10^{+6}$

$$= 77.78 \text{ dB bps}$$

For BER = 10^{-5}, we get $\frac{E_b}{N_0} = 9.6$ dB [from graph]

We know,
$$\left(\frac{C}{N_0}\right) = \left(\frac{E_b}{N_0}\right) + [R_b]$$
$$= 9.6 + 77.78$$
$$= 87.38 \text{ dB Hz}$$

2.
$$(\text{EIRP})_{\text{dB}} = \left(\frac{C}{N_0}\right)_{\text{dB}} - \left[\frac{G}{T}\right]_{\text{dB}} + (losses)_{\text{dB}} - [K]_{\text{dB}}$$
$$(\text{EIRP}) = 87.38 - 32 + 200 - 228.6$$
$$= 26.78 \text{ dBW}$$

Q.4 (b) Solution:

Given,

Doping concentrations in emitter, base and collector as

$$N_E = 10^{18} \text{ cm}^{-3},$$
$$N_B = 10^{16} \text{ cm}^{-3},$$
$$N_C = 10^{15} \text{ cm}^{-3}$$

Metallurgical base width, $= 1.2\ \mu\text{m} = x_B + x_n$ ...(i)

Minority concentration in Base,

$$P_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$P_B(0) = P_{B0} \exp\left(\frac{V_{EB}}{V_t}\right)$$
$$= 2.25 \times 10^{+4} \exp\left(\frac{0.625}{0.0259}\right)$$
$$= 6.8 \times 10^{14} \text{ cm}^{-3}$$

(i) The hole diffusion current density,

$$J_h = qD_B \frac{dP_B}{dx}$$

$$= qD_B\left(\frac{P_B(0)}{x_B}\right) = \frac{(1.6\times10^{-19})(10)(6.8\times10^{14})}{x_B}$$

$$J_h = \frac{1.09\times10^{-3}}{x_B}$$

$$\text{Width, } x_n = \left\{\frac{2\in_s(V_{bi}+V_R)}{q}\left(\frac{N_C}{N_B}\right)\frac{1}{(N_C+N_B)}\right\}^{\frac{1}{2}} \quad \text{...(ii)}$$

$$V_{bi} = (0.0259)\ln\left[\frac{10^{16}\times10^{15}}{(1.5\times10^{10})^2}\right] = 0.635\text{ V}$$

$$x_n = \left\{\frac{2\times11.7\times8.85\times10^{-14}(V_{bi}+V_R)}{1.6\times10^{-19}}\times\frac{10^{15}}{10^{16}}\left(\frac{1}{10^{15}+10^{16}}\right)\right\}^{\frac{1}{2}}$$

$$x_n = \left\{(1.177\times10^{-10})(V_{bi}+V_R)\right\}^{\frac{1}{2}}$$

For $V_R = V_{BC} = 10$ V,

$$x_n = \left\{\left(1.177\times10^{-10}\right)(0.635+10)\right\}^{\frac{1}{2}}$$

$$\therefore \quad x_n = 0.354\times10^{-4}\text{ cm} \quad \text{...(iii)}$$

From equation (i) and (iii),

$$x_B = 1.2\times10^{-4} - x_n$$

$$\therefore \quad x_B = 1.2\times10^{-4} - 0.354\times10^{-4}$$

$$\therefore \quad x_B = 0.846\times10^{-4}\text{ cm}$$

From equation (ii), $J_h = \dfrac{1.09\times10^{-3}}{0.846\times10^{-4}} = 12.9\text{ A/cm}^2$

(ii) We know that,

$$J_h = g'(V_{EC}+V_A) \quad \text{...(iv)}$$

where,

$$\text{conductance } g' = \frac{\Delta J_p}{\Delta V_{BC}}$$

given, $V_{BC} = 5$ V

From part (i),

$$x_n = \left\{(1.177\times10^{-10})(0.635+5)\right\}^{\frac{1}{2}}$$

$\therefore \quad x_n = 0.258 \times 10^{-4}$ cm

$\therefore \quad x_B = 1.2 \times 10^{-4} - 0.258 \times 10^{-4} = 0.942 \times 10^{-4}$ cm

From equation (ii), J_h' for $V_{BC} = 5$ V is,

$$J_h' = \frac{1.09 \times 10^{-3}}{0.942 \times 10^{-4}} = 11.6 \text{ A/cm}^2$$

$$\therefore \quad g' = \frac{\Delta J_p}{\Delta V_{BC}} = \frac{12.9 - 11.6}{10 - 5} = 0.26 \text{ A/cm}^2\text{/V}$$

For pnp bipolar junction transistor,

$$V_{EC} = V_{BC} + V_{EB}$$
$$= 5 + 0.625$$
$$\therefore \quad V_{EC} = 5.625 \text{ V}$$

From equation (iv),

$$11.6 = 0.26\,(5.625 + V_A)$$
$$\therefore \quad V_A = 38.99 \text{ V}$$

Q.4 (c) Solution:

(i) Given: Service area of cellular system,

$$A_{sys} = 4200 \text{ km}^2$$

Coverage area of a cell,

$$A_{cell} = 12 \text{ km}^2$$

Total number of available channels, $N = 1001$

1. The coverage area of a cluster,

$$A_{cluster} = K \times A_{cell}$$

Therefore, $A_{cluster} = 7 \times 12 = 84 \text{ km}^2$

The number of times that cluster has to be replicated to cover the entire service area of cellular system

$$= \frac{A_{sys}}{A_{cluster}}$$

Number of clusters, $M = \dfrac{4200}{84} = 50$ clusters

Since total number of available channels are allocated to one cluster, therefore, the number of channels per cell,

$$J = \frac{N}{K}$$

or, Cell capacity, $J = \frac{1001}{7}$

Hence, Cell capacity, $J = 143 \frac{\text{Channels}}{\text{Cell}}$

The system capacity, $C = N \times M$

or System capacity, $C = 1001 \times 50$

Hence,

the system capacity, C = 50,050 channels

2. To calculate new system capacity for reduced K

New cluster size, $K = 4$ (given)

The coverage area of a cluster,

$$A_{cluster} = K \times A_{cell}$$

Therefore, $A_{cluster} = 4 \times 12 = 48 \text{ km}^2$

The number of times that the cluster has to be replicated to cover the entire service area of a cellular system

$$= \frac{A_{sys}}{A_{cluster}}$$

Number of clusters, $M = \frac{4200}{48}$

Hence,

Number of clusters, $M = 87.5$

$$M \simeq 87 \text{ (approx)}$$

The system capacity, $C = N \times M$

or, System capacity, $C = 1001 \times 87$

Hence,

the system capacity, C = 87,087 channels

Comments on the results:

From (i) and (ii) above, it is seen that decrease in cluster size from 7 to 4 results in the increase in system capacity from 50,050 channels to 87,087 channels.

Therefore, decreasing the cluster size does increase the system capacity. However, the average signal to co-channel interference also increases which has to be kept at an acceptable level in order to achieve desirable signal quantity.

(ii)

Characteristic Description	UDP	TCP
General Description	Simple, high speed, low functionality wrapper that interfaces applications to the network layer and does little else.	Full featured protocol that allows applications to send data reliably without worrying about network layer issues.
Protocol connection setup	Connectionless; data is sent without setup.	Connection-oriented, connection must be established prior to transmission.
Data interface to applications	Message based; data is sent in discrete packages by the application.	Stream based; data is sent by the application with no particular structure.
Reliability and Acknowledgements	Unreliable, best efforts delivery without acknowledgements.	Reliable devliery of messages; all data is acknowledged.
Retransmissions	Not performed. Application must detect lost data and retransmit if needed.	Delivery of all data is managed, and lost data is retransmitted automatically.
Features provided to manage data flow	None	Flow control using sliding windows; window size adjustment heuristics; congestion avoidance algorithms.
Overhead	Very low	Low, but higher than UDP.
Transmission speed	Very high	High, but not as high as UDP.
Data quantity suitability	Small to moderate amounts of data (upto a few hundred bytes)	Small to very large amounts of data (upto gigabytes)
Types of Applications that use the protocol	Applications where data delivery speed matters more than completeness, where small amounts of data are sent or where multicast/broadcast are used.	Most protocols and applications sending data that most be received reliably, including most file and message transfer protocols.
Well known applications and protocols	Multimedia applications, DNA, BOOTP, DHCP, TFTP, SNMP, RIP, NFS (early versions)	FTP, Telnet, SMTP, DNS, HTTP, POP, NNTP, IMAP, BGP, IRC, NFS (later versions)

Section B : Analog & Digital Communication Systems-1 + Signals and Systems-2 + Microprocessors and Microcontroller-2

Q.5 (a) Solution:

(i) **1.** The first stage is an attenuator section that has a loss factor of 1.5 dB

$$L = 10^{0.15} = 1.412$$

Gain factor of this stage has to be interpreted as

$$G_1 = \frac{1}{1.412} = 0.708$$

The second stage has $G_2 = 10$, $F_2 = 10^{0.2} = 1.58$

The third stage has $G_3 = 100$, $F_3 = 10^{0.2} = 1.58$

The noise figure of the cascaded system is

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$= 1.412 + (1.58 - 1) \times 1.412 + \frac{(1.58 - 1) \times 1.412}{10}$$

$$= 2.31 = 3.64 \text{ dB}$$

2. If P_{in} = –90 dBm then we get

P_{out} = –90 dBm – 1.5 dB + 10 dB + 20 dB

= –61.5 dBm

The noise power output is

$$P_n = G_{cas} KT_{e\,cas} B = K(F_{cas} - 1)\, T_0\, B\, G_{cas}$$

$$= (1.38 \times 10^{-23})(2.31 - 1) \times 290 \times 10^8 \times 28.5$$

$$[\because G_{cas} = 10 \log(0.708 \times 10 \times 100) = 28.5]$$

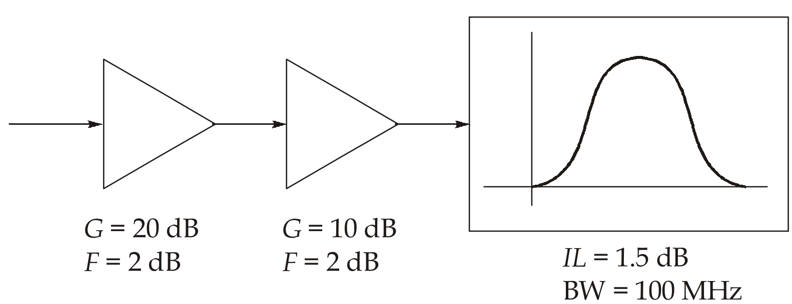
$$= 1.494 \times 10^{-11} \text{ W}$$

$$= -78.26 \text{ dBm}$$

Thus $S_0/N_0 = -61.5 + 78.26$

$$= 16.76 \text{ dB}$$

3. The best noise figure would be achieved with the arrangement as shown in figure.



Then the noise figure is

$$F_{cas} = F_3 + \frac{F_2 - 1}{G_3} + \frac{F_1 - 1}{G_2 G_3}$$

$$= 1.58 + \frac{1.58 - 1}{100} + \frac{1.412 - 1}{1000} = 1.586$$

$$10 \log_{10} 1.586 = 2 \text{ dB}$$

In practice, however, the essential filter may serve to prevent overload of the amplifier and may not be allowed to be moved.

(ii) The phase modulated signal $x_c(t)$ is given by

$$x_c(t) = A_c \cos(\omega_c t + K_p x(t))$$

Instantaneous frequency,

$$f_i = \frac{1}{2\pi}\frac{d}{dt}\left[\omega_c t + K_p x(t)\right]$$

$$f_i = f_c + \frac{1}{2\pi} K_p \frac{d(x(t))}{dt}$$

$$f_i - f_c = \text{Maximum frequency deviation}$$

$$= \frac{1}{2\pi} \cdot K_p \left|\frac{dx(t)}{dt}\right|_{\max}$$

$$\left|\frac{dx(t)}{dt}\right|_{\max} = \frac{18}{2 \times 10^{-3}} = 9 \times 10^3 \text{ V/sec}$$

∴ Maximum frequency deviation

$$\Delta f = \frac{1}{2\pi} \times K_p \times 9 \times 10^3$$

$$= 75 \times 10^3 \text{ (Given)}$$

We get, $$K_p = \frac{50\pi}{3} \text{ rad/volt}$$

From $t = 0$ to $t = 10$ ms

$$\frac{dx(t)}{dt} = 1.8 \text{ V/ms} = 1800 \text{ V/sec}$$

During this period, frequency deviation

$$\Delta f = \frac{1}{2\pi} K_p \frac{dx(t)}{dt} = \frac{1}{2\pi} \times \frac{50\pi}{3} \times 1800 = 15 \text{ kHz}$$

From 0 ms to 10 ms, frequency of the modulated signal is

$$1000 \text{ kHz} + 15 \text{ kHz} = 1015 \text{ kHz}$$

From 10 ms to 12 ms, frequency of modulated signal is

$$1000 \text{ kHz} - 75 \text{ kHz} = 925 \text{ kHz}$$

∴ Frequency of modulated signal varies between 925 kHz and 1015 kHz.

Q.5 (b) Solution:

(i) 1. Frequency response $H(\omega)$ of the system is

$$H(\omega) = FT[h(t)] = \frac{1}{j\omega + \beta}$$

So,
$$|H(\omega)|^2 = \frac{1}{\omega^2 + \beta^2}$$

$$S_{XX}(\omega) = FT[R_{XX}(\tau)] = A\frac{2\alpha}{\omega^2 + \alpha^2}$$

The power spectral density of $y(t)$ is

$$S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$$

$$= \frac{1}{\omega^2 + \beta^2} \times A\frac{2\alpha}{\omega^2 + \alpha^2}$$

$$= \frac{2A\alpha}{(\omega^2 + \beta^2)(\omega^2 + \alpha^2)}$$

$$S_{YY}(\omega) = \frac{2A\alpha}{(\omega^2 + \beta^2)(\omega^2 + \alpha^2)}$$

Apply partial fraction

$$\frac{2A\alpha}{(\omega^2 + \alpha^2)(\omega^2 + \beta^2)} = \frac{C}{\omega^2 + \alpha^2} + \frac{D}{\omega^2 + \beta^2}$$

$$2A\alpha = C(\omega^2 + \beta^2) + D(\omega^2 + \alpha^2)$$

$$C\beta^2 + D\alpha^2 = 2A\alpha \qquad \text{...(1)}$$

$$C + D = 0 \quad \Rightarrow \quad C = -D$$

Using equation (1), we can write

$$C(\beta^2 - \alpha^2) = 2A\alpha$$

$$C = \frac{2A\alpha}{\beta^2 - \alpha^2} = \frac{-2A\alpha}{\alpha^2 - \beta^2}$$

$$D = \frac{-2A\alpha}{\beta^2 - \alpha^2} = \frac{2A\alpha}{\alpha^2 - \beta^2}$$

$$S_{YY}(\omega) = \frac{2A\alpha}{\alpha^2 - \beta^2}\left[\frac{1}{\omega^2 + \beta^2}\right] - \frac{2A\alpha}{\alpha^2 - \beta^2}\left[\frac{1}{\omega^2 + \alpha^2}\right]$$

$$= \frac{A\alpha}{(\alpha^2 - \beta^2)\beta}\left(\frac{2\beta}{\omega^2 + \beta^2}\right) - \frac{A}{\alpha^2 - \beta^2}\frac{2\alpha}{\omega^2 + \alpha^2}$$

The inverse fourier transform of the power spectral density gives the auto correlation function. Hence, taking Inverse Fourier Transform, we get

$$R_{YY}(\tau) = \frac{A\alpha}{(\alpha^2-\beta^2)\beta}e^{-\beta|\tau|} - \frac{A}{\alpha^2-\beta^2}e^{-\alpha|\tau|}$$

2. (a) The auto correlation of $Z(t)$ is given by

$$R_{ZZ}(t_1\, t_2) = E[Z(t_1)\, Z(t_2)]$$

$$= E\{[X(t_1)+Y(t_1)][X(t_2)+Y(t_2)]\}$$

$$= E[X(t_1)\, X(t_2)] + E[Y(t_1)Y(t_2)] + E[Y(t_1)\, X(t_2)] + E[X(t_1)\, Y(t_2)]$$

$$= R_{XX}(t_1, t_2) + R_{XY}(t_1, t_2) + R_{YX}(t_1, t_2) + R_{YY}(t_1, t_2)$$

If $X(t)$ and $Y(t)$ are jointly WSS, then we have

$$R_{ZZ}(\tau) = R_{XX}(\tau) + R_{XY}(\tau) + R_{YX}(\tau) + R_{YY}(\tau) \quad \text{...(1)}$$

where $\tau = t_2 - t_1$

Taking the Fourier transform of both sides of equation (1), we obtain

$$S_{ZZ}(\omega) = S_{XX}(\omega) + S_{XY}(\omega) + S_{YX}(\omega) + S_{YY}(\omega)$$

(b) If $X(t)$ and $Y(t)$ are orthogonal

$$R_{XY}(\tau) = R_{YX}(\tau) = 0$$

From equation (1)

$$R_{ZZ}(\tau) = R_{XX}(\tau) + R_{YY}(\tau)$$

$$R_{ZZ}(0) = R_{XX}(0) + R_{YY}(0)$$

$$E[Z^2(t)] = E[X^2(t)] + E[Y^2(t)]$$

Which indicates that the mean square of $Z(t)$ is equal to the sum of the mean squares of $X(t)$ and $Y(t)$.

(ii) For a random variable X,

$$\text{Variance, } \sigma_X^2 = E[X^2] - \{E[X]\}^2$$

$$\text{Standard deviation, } \sigma_X = \sqrt{\text{variance}(\sigma_x^2)}$$

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\cdot dx$$

$$= \int_0^1 x(1-x^2)dx$$

$$= \int_0^1 (x - x^3)dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{4}$$

Similarly, $$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_0^1 x^2(1-x^2)dx$$

$$= \int_0^1 (x^2 - x^4)dx = \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$= \left[\frac{1}{3} - \frac{1}{5}\right] = \frac{2}{15}$$

Hence, $$\sigma_X^2 = E[X^2] - \{E[X]\}^2 = \frac{2}{15} - \frac{1}{16} = \frac{32-15}{15\times 16} = \frac{17}{15\times 16}$$

$$\sigma_X = \sqrt{\frac{17}{15}} \times \frac{1}{4} = 0.266$$

Q.5 (c) Solution:

(i) Using tabular method given as follows, the linear convolution of the two sequences can be determined as shown below:

$x_2(n)$ \ $x_1(n)$	1	a	b	2
c	c	ac	bc	$2c$
2	2	$2a$	$2b$	4
d	d	ad	bd	$2d$
4	4	$4a$	$4b$	8

Hence, $$x_3(n) = x_1(n) * x_2(n)$$

$$= \{\underset{\uparrow}{c}, 2+ac, d+2a+bc, 4+ad+2b+2c, 4a+bd+4, 2d+4b, 8\}$$

We have,

$$x_3(n) = \{\underset{\uparrow}{1}, 3, 7, 13, 14, 14, 8\}$$

On comparison, we get

$$c = 1;\ 2 + ac = 3 \qquad ; \qquad d + 2a + bc = 7$$

$$2 + a = 3 \qquad d + 2(1) + b(1) = 7$$

$$a = 1 \qquad d + 2 + b = 7$$

$$d + b = 5 \qquad \text{...(i)}$$

$$4 + ad + 2b + 2c = 13$$
$$4 + (1)d + 2b + 2(1) = 13$$
$$d + 2b = 7 \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$d = 3 \text{ and } b = 2$$

Now, we have, $x_1(n) = \{1, a, b, 2\}$ and $x_2(n) = \{c, 2, d, 4\}$

$= \{1, 1, 2, 2\}$ $\quad = \{1, 2, 3, 4\}$

(ii) Similarly, circular convolution of the two sequences is

$$x_3(m) = \sum_{0}^{N-1} x_1(n)x_2(m-n, (\text{mod}\, N)), \; m = 0, 1, ...N-1$$

For $m = 0$

$$x_3(0) = \sum_{n=0}^{3} x_1(n)x_2(-n, (\text{mod}\, 4))$$

$x_2(-n, (\text{mod } 4))$ is the sequence $x_2(n)$ folded. The folded sequence is obtained by plotting $x_2(n)$ in clockwise direction.

$$x_2(0, (\text{mod } 4)) = x_2(0) = c$$
$$x_2(-1, (\text{mod } 4)) = x_2(3) = 4$$
$$x_2(-2, (\text{mod } 4)) = x_2(2) = d$$
$$x_2(-3, (\text{mod } 4)) = x_2(1) = 2$$

$x_3(0)$ is obtained by computing the product sequence, i.e. multiplying the sequences $x_1(n)$ and $x_2(-n, \text{mod } 4)$, point by point and taking the sum, we get $x_3(0) = c + 4a + bd + 4$

For $m = 1$

$$x_3(1) = \sum_{n=0}^{3} x_1(n) \cdot x_2(1-n, (\text{mod}\, 4))$$

$x_2(1 - n, (\text{mod } 4))$ is the sequence $x_2(-n, (\text{mod } 4))$ rotated counter clockwise by one unit in time. From the product sequence, the sum is $x_3(1) = 2 + ac + 2d + 4b$

For $m = 2$

$$x_3(2) = \sum_{n=0}^{3} x_1(n) \cdot x_2(2-n, (\text{mod}\, 4))$$

$x_2(2 - n, (\text{mod } 4))$ is the sequence $x_2(-n, (\text{mod } 4))$ rotated counterclockwise by two units in time. From the product sequence, the sum is $x_3(2) = d + 2a + bc + 8$

For $m = 3$

$$x_3(3) = \sum_{n=0}^{3} x_1(n) \cdot x_2(3-n, (\text{mod}\,4))$$

$x_2(3 - n, \text{mod } 4)$ is the sequence $x_2(-n, (\text{mod } 4))$ rotated counter clockwise by three units in time. From the product sequence, the sum is $x_3(3) = 4 + ad + 2b + 2c$

Hence, the circular convolution of the two sequence $x_1(n)$ and $x_2(n)$ is

$x_3(n) = \{c + 4a + bd + 4, 2 + ac + 2d + 4b, d + 2a + bc + 8, 4 + ad + 2b + 2c\}$

Alternate Solution:

In part (i), the linear convolution of $x_1(n)$ and $x_2(n)$ is obtained as

$y(n) = \{c, 2 + ac, d + 2a + bc, 4 + ad + 2b + 2c, 4a + bd + 4, 2d + 4b, 8\}$

The circular convolution of $x_1(n)$ and $x_2(n)$ can be calculated as

$x_3(n) = \{y(0) + y(4), y(1) + y(5), y(2) + y(6), y(3)\}$

$= \{c + 4a + bd + 4, 2 + ac + 2d + 4b, d + 2a + bc + 8, 4 + ad + 2b + 2c\}$

(iii) We have calculated $a = 1, b = 2, c = 1$ and $d = 3$

Thus,

$x_3(n) = \{1 + 4(1) + (2)(3) + 4, 2 + (1)(1) + 2(3) + 4(2), 3 + 2(1) + 2(1) + 8, 4 + (1)(3) + 2(2) + 2(1)\}$

$x_3(n) = \{15, 17, 15, 13\}$

Q.5 (d) Solution:

The Program Status Word (PSW) contains several status bits that reflect the current state of the CPU. The PSW is one of the special function registers and contains the carry bit, the auxiliary carry bit (used for BCD operations), two register select bits, the overflow flag bit, a parity bit, and two user definable status flag bits. Following are the brief descriptions of these bits:

- CY (PSW.7) is set, if the operation results in a carry out of (during addition) or a borrow into (during subtraction) the high-order bit of the result; otherwise CY is cleared.
- AC (PSW.6) is set, if the operation results in a carry out of low-order 4 bits of the result (during addition) or a borrow from the high-order bits into the low-order 4 bits (during subtraction); otherwise AC is cleared.
- RS1, RS0 (PSW.4, PSW.3) represent the current register bank in the internal Data RAM selected out of the four register banks

RS1	RS0	
0	0	Register bank 0
0	1	Register bank 1
1	0	Register bank 2
1	1	Register bank 3

- OV (PSW.2) is set, whenever the result of a signed number operation is too large causing the high-order bit to overflow into the sign bit; otherwise OV is cleared. OV has a significant role in two's complement arithmetic, since it becomes set when the signed result can't be represented in 8-bits.
- P (PSW.0) is set, if the modulo 2 sum of the eight bits in the accumulator is 1 (odd Parity); otherwise P is cleared (even parity). When a value is written to the PSW register, the P bit remains unchanged as it always reflects the parity of the accumulator. F0(PSW.5) represents Flag 0 available to user for general purpose and UD(PSW.1) represents a user definable flag.

7	6	5	4	3	2	1	0
CY	AC	F0	RS1	RS0	OV	UD	P

Q.5 (e) Solution:

(i) When PLL is not initially locked to the signal, the frequency of the VCO will be free running f_0.

The phase angle difference between the signal and the VCO output voltage will be

$$\phi = (\omega_s t + \theta_s) - (\omega_0 t + \theta_0)$$
$$= (\omega_s - \omega_0)t + \Delta\theta$$

Thus the phase angle difference does not remain constant but will change with time at a rate given by

$$\frac{d\phi}{dt} = \omega_s - \omega_0$$

The phase detector output voltage is given by $V_e = K_\phi(\phi\text{-}\pi/2)$. It will therefore, not have a dc component but will produce an ac voltage with a triangular waveform of peak amplitude $K_\phi(\pi/2)$ and a fundamental frequency $(f_s - f_0) = \Delta f$.

The low pass filter (LPF) used in PLL is a simple RC network having transfer function,

$$T(f) = \frac{1}{(1 + j\, f/f_1)}$$

where $f_1 = \dfrac{1}{2\pi RC}$ is the 3-dB point of LPF. In the slope portion of LPF, where $\left(\dfrac{f}{f_1}\right)^2 >> 1,$

then, $$T(f) \cong \frac{f_1}{jf}$$

The fundamental frequency term supplied to the LPF by the phase detector will be the difference frequency,

$$\Delta f = f_s - f_0$$

If $\Delta f > 3f_1$, then LPF transfer function will be approximately,

$$T(f) \cong \frac{f_1}{\Delta f} = \frac{f_1}{f_s - f_0}$$

The voltage V_c to drive the VCO is

$$V_c = V_e \times T(f) \times A$$

$$V_{c\,max} = V_{e\,max} \times T(f) \times A$$

$$V_{c\,max} = \pm K_\phi \left(\frac{\pi}{2}\right) A \left(\frac{f_1}{\Delta f}\right) \qquad \left[\because V_{e\,max} = \pm K_\phi \left(\frac{\pi}{2}\right)\right]$$

Then corresponding value of maximum phase shift is

$$(f - f_0)_{max} = K_V V_{c\,max} = \pm K_V K_\phi \left(\frac{\pi}{2}\right) A \left(\frac{f_1}{\Delta f}\right)$$

For the acquisition of signal frequency, we should put $f = f_s$, so that maximum signal frequency range that can be acquired by PLL is

$$(f_s - f_0)_{max} = K_V K_\phi \left(\frac{\pi}{2}\right) A \left(\frac{f_1}{\Delta f_c}\right)$$

Now, $$\Delta f_c = (f_s - f_0)_{max}$$

$$(\Delta f_c)^2 = K_V K_\phi \frac{\pi}{2} A f_1$$

But $$\pm K_V K \frac{\pi}{2} A = \Delta f_L, \text{ where } \Delta f_L \text{ is the lock-in frequency range.}$$

$$\Delta f_c = \pm\sqrt{f_1 \Delta f_L}$$

Therefore total capture range is,

$$2\Delta f_c = 2\sqrt{f_1 \Delta f_L}$$

(ii) Given: $\Delta f = 8$ kHz

From the given message signal, we obtain input frequency, $f_1 = 2 \times 10^3$ Hz which should be the cut-off frequency of LPF

$$\therefore \quad \text{Capture range (total)} \cong 2\sqrt{f_1 \Delta f}$$

$$= 2 \times \sqrt{2 \times 10^3 \times 8 \times 10^3}$$

$$= 2 \times 4 \times 10^3 = 8 \text{ kHz}$$

Q.6 (a) Solution:

As X and Y are two independent random variables, the joint probability density function of X and Y can be given as

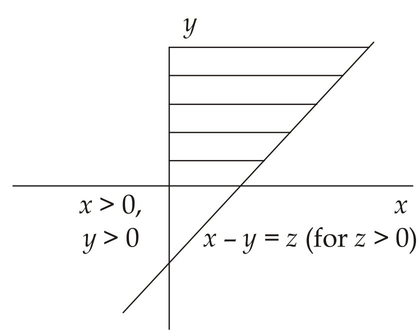
$$F_{XY}(x, y) = f_X(x) f_Y(y)$$

(i) $Z = X - Y$

The cumulative distribution function of Z can be given by

$$F_Z(z) = P(Z \leq z) = P(X - Y \leq z)$$

$$F_Z(z) = \int_0^\infty \left[\int_0^{z+y} f_{XY}(x, y) dx \right] dy$$



The probability density function of Z can be given by

$$f_Z(z) = \frac{dF_Z(z)}{dz} = \int_0^\infty \left[\frac{d}{dz} \int_0^{z+y} f_{XY}(x, y) dx \right] dy$$

$$f_Z(z) = \int_0^\infty f_{XY}(z + y, y) \cdot dy = \int_0^\infty f_X(z + y) \cdot f_Y(y) \cdot dy$$

$$f_Z(z) = ab \int_0^\infty e^{-a(z+y)} \cdot e^{-by} dy$$

$$f_Z(z) = abe^{-az} \left[\frac{e^{-(a+b)y}}{(a+b)} \right]_0^\infty ; z \geq 0$$

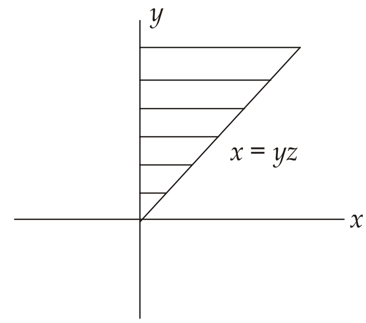
$$f_Z(z) = \frac{ab}{a+b}e^{-az}u(z)$$

(ii) $Z = \frac{X}{Y};$

The cumulative distribution function of Z can be given by

$$f_z(z) = P(Z \le z) = P\left(\frac{X}{Y} \le z\right)$$

$$F_Z(z) = \int_{y=0}^{\infty}\left[\int_{x=0}^{yz} f_{XY}(x,y)dx\right]dy$$



The probability density function of Z can be given by,

$$f_Z(z) = \frac{dF_Z(z)}{dz} = \int_{y=0}^{\infty}\left[\frac{d}{dz}\int_{x=0}^{yz} f_{XY}(x,y)dx\right]dy$$

$$f_Z(z) = \int_{y=0}^{\infty}\frac{d}{dz}(yz)f_{XY}(yz,y)\cdot dy = \int_{y=0}^{\infty} yf_X(yz)\cdot f_y(y)dy$$

$$f_Z(z) = ab\int_{y=0}^{\infty} ye^{-ayz}e^{-by}dy;\ z > 0$$

$$f_Z(z) = ab\int_{y=0}^{\infty} ye^{-(az+b)y}dy;\ z > 0$$

$$f_Z(z) = ab\left[\frac{-ye^{-(az+b)y}}{az+b} - \frac{e^{-(az+b)y}}{(az+b)^2}\right]_{y=0}^{\infty},\ z > 0$$

$$f_Z(z) = \frac{ab}{(az+b)^2};\ z > 0$$

$$f_Z(z) = \frac{ab}{(az+b)}u(z)$$

(iii) $Z = \min(X, Y)$:

The cumulative distribution function of Z can be given by

$$F_Z(z) = P(Z \le z) = P[\min(X, Y) \le z]$$

$$F_Z(z) = P(X \le z, Y > X) + P(Y \le z, X > Y)$$

$$F_Z(z) = \int_{x=0}^{z}\int_{y=x}^{\infty} f_{XY}(x,y)dydx + \int_{y=0}^{z}\int_{x=y}^{\infty} f_{XY}(x,y)dxdy$$

$$F_Z(z) = ab\int_{x=0}^{z} e^{-ax}\left(\int_{y=x}^{\infty} e^{-by}dy\right)dx + ab\int_{y=0}^{z} e^{-by}\left(\int_{x=y}^{\infty} e^{-ax}dx\right)dy$$

$$F_Z(z) = ab\int_{x=0}^{z} e^{-ax}\left[\frac{e^{-by}}{-b}\right]_{y=x}^{\infty} dx + ab\int_{y=0}^{z} e^{-by}\left[\frac{e^{-ax}}{-a}\right]_{x=y}^{\infty} dy;\ z > 0$$

$$F_Z(z) = a\int_{x=0}^{z} e^{-(a+b)x}dx + b\int_{y=0}^{z} e^{-(a+b)y}dy;\ z > 0$$

$$F_Z(z) = \frac{a}{a+b}\left[-e^{-(a+b)x}\right]_{x=0}^{z} + \frac{b}{a+b}\left[-e^{-(a+b)y}\right]_{y=0}^{z};\ z > 0$$

$$F_Z(z) = \frac{-a}{a+b}e^{-(a+b)z} + \frac{a}{a+b} - \frac{b}{a+b}e^{-(a+b)z} + \frac{b}{a+b};\ z > 0$$

$$F_Z(z) = 1 - e^{-(a+b)z};\ z > 0$$

The probability density function of Z can be given by

$$f_Z(z) = \frac{dF_Z(z)}{dz} = \frac{d}{dz}\left\{1 - e^{-(a+b)z}\right\}$$

$$f_Z(z) = (a+b)e^{-(a+b)z};\ z > 0$$

$$f_Z(z) = (a+b)e^{-(a+b)z};\ u(z)$$

(iv) $Z = \max(X, Y)$

The cumulative distribution function of Z can be given by

$$F_Z(z) = P(Z \le Z) = P[\max(X, Y) \le Z]$$

$$F_Z(z) = P(X \le Z, Y < X) + P(Y \le Z, X < Y)$$

$$F_Z(z) = \int_{x=0}^{z}\int_{y=0}^{x} f_{XY}(x,y)dy\,dx + \int_{y=0}^{z}\int_{x=0}^{y} f_{XY}(x,y)dx\,dy$$

$$F_Z(z) = ab\int_{x=0}^{z} e^{-ax}\left[\frac{e^{-by}}{-b}\right]_{y=0}^{x} dx + ab\int_{y=0}^{z} e^{-by}\left[\frac{e^{-ax}}{-a}\right]_{x=0}^{y} dy$$

$$F_Z(z) = a\int_{x=0}^{z}\left[e^{-ax} - e^{-(a+b)x}\right]dx + b\int_{y=0}^{z}\left[e^{-by} - e^{-(a+b)y}\right]dy$$

$$F_Z(z) = a\left[\frac{-e^{-ax}}{a} + \frac{e^{-(a+b)x}}{a+b}\right]_{x=0}^{z} + b\left[\frac{-e^{-by}}{b} + \frac{e^{-(a+b)y}}{a+b}\right]_{y=0}^{z}$$

$$F_Z(z) = \left[-e^{-az} + \frac{a}{a+b}e^{-(a+b)z} + 1 - \frac{a}{a+b}\right] + \left[-e^{-bz} + \frac{b}{a+b}e^{-(a+b)z} + 1 - \frac{b}{a+b}\right]$$

$$F_Z(z) = 1 - e^{-az} - e^{-bz} + e^{-(a+b)z};\ z > 0$$

The probability density function of Z can be given by

$$f_Z(z) = \frac{dF_Z(z)}{dz}$$

$$f_Z(z) = ae^{-az} + be^{-bz} - (a + b)e^{-(a + b)z}; \; z > 0$$

$$f_Z(z) = [ae^{-az} + be^{-bz} - (a + b)e^{-(a + b)z}]u(z)$$

Q.6 (b) Solution:

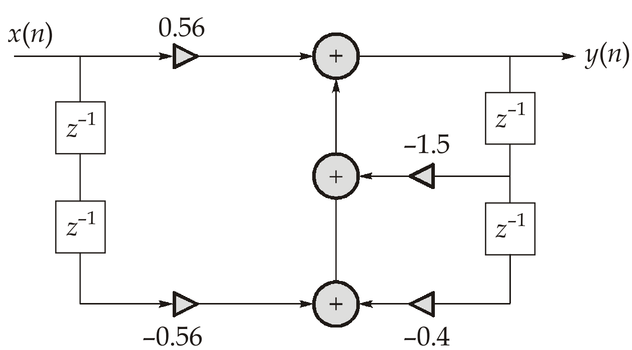
(i) To perform the filter realization using the direct form I and direct form II, we rewrite the given second-order transfer function as

$$H(z) = \frac{0.56(1 - z^{-2})}{1 + 1.5z^{-1} + 0.4z^{-2}} = \frac{0.56 - 0.56z^{-2}}{1 + 1.5z^{-1} + 0.4z^{-2}} = \frac{Y(z)}{X(z)}$$

Using inverse z - transform, we can write difference equation for the direct-form I realization as

$$y(n) = 0.56x(n) - 0.56x(n - 2) - 1.5y(n - 1) - 0.4y(n - 2)$$

Direct Form-I Realization:



To obtain Direct Form-II realization, we can write

$$H(z) = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)}$$

where
$$\frac{W(z)}{X(z)} = \frac{1}{1 + 0.5z^{-1} + 0.4z^{-2}}$$

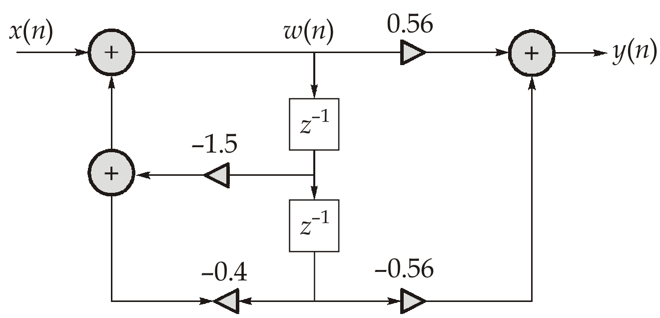
$$\frac{Y(z)}{W(z)} = 0.56 - 0.56z^{-2}$$

Hence, we obtain the difference equations for Direct Form - II realization as

$$w(n) = x(n) - 0.5\,w(n - 1) - 0.4w(n - 2)$$

$$y(n) = 0.56w(n) - 0.56\,w(n - 2)$$

The Direct Form-II realization is thus obtained as below:



(ii) To realize the filter using cascade of first-order sections, we write,

$$H(z) = \frac{0.56 - 0.56z^{-1}}{1 + 0.35z^{-1}} \times \frac{1 + z^{-1}}{1 + 1.15z^{-1}}$$

Assume, $$H_1(z) = \frac{0.56 - 0.56z^{-1}}{1 + 0.35z^{-1}};\ H_2(z) = \frac{1 + z^{-1}}{1 + 1.15z^{-1}}$$

or,

We can take $$H_1(z) = \frac{0.56 - 0.56z^{-1}}{1 + 1.15z^{-1}} \text{ and } H_2(z) = \frac{1 + z^{-1}}{1 + 0.35z^{-1}}$$

Now,

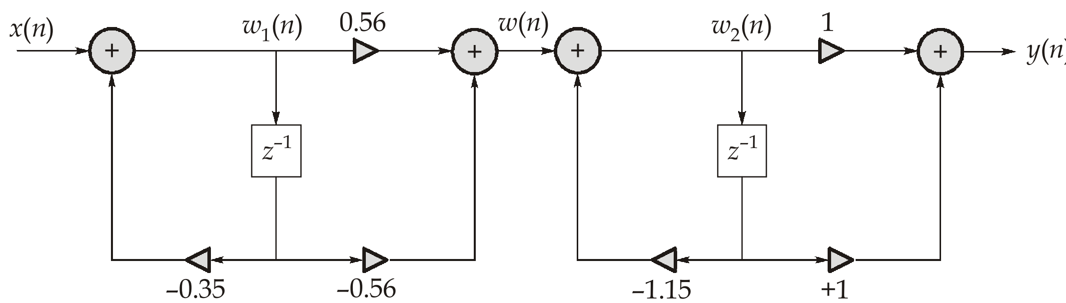
Considering $$H_1(z) = \frac{0.56 - 0.56z^{-1}}{1 + 0.35z^{-1}} = \frac{W(z)}{X(z)} = \frac{W_1(z)}{X(z)} \times \frac{W(z)}{W_1(z)}$$

$$H_2(z) = \frac{1 + z^{-1}}{1 + 1.15z^{-1}} = \frac{Y(z)}{W(z)} = \frac{W_2(z)}{W(z)} \times \frac{Y(z)}{W_2(z)}$$

$H_1(z)$ and $H_2(z)$ can be realized in direct-II form using the following difference equations

$$w_1(n) = x(n) - 0.35\, w_1(n-1)$$
$$w(n) = 0.56 w_1(n) - 0.56\, w_1(n-1)$$
$$w_2(n) = w(n) - 1.15\, w_2(n-1)$$
$$y(n) = w_2(n) + w_2(n-1)$$

The filter can thus be realized as below:



Cascade realization

(iii) To realize the filter using parallel addition of first-order sections, we write

$$\frac{H(z)}{z} = \frac{0.56(z^2-1)}{z(z+0.35)(z+1.15)} = \frac{A}{z} + \frac{B}{z+0.35} + \frac{C}{z+1.15}$$

Now,
$$A = \left.\frac{0.56(z^2-1)}{(z+0.35)(z+1.15)}\right|_{\text{at } z=0} = \frac{-0.56}{0.35\times 1.15} = -1.40$$

$$B = \left.\frac{0.56(z^2-1)}{z(z+1.15)}\right|_{\text{at } z=-0.35} = \frac{0.56\left((-0.35)^2-1\right)}{-0.35(-0.35+1.15)} = 1.755$$

$$C = \left.\frac{0.56(z^2-1)}{z(z+0.35)}\right|_{\text{at } z=-1.15} = \frac{0.56\left((-1.15)^2-1\right)}{-1.15(-1.15+0.35)} = 0.20$$

Therefore,
$$H(z) = -1.40 + \frac{1.755z}{z+0.35} + \frac{0.20z}{z+1.15}$$
$$= -1.40 + 1.755H_1(z) + 0.20H_2(z)$$

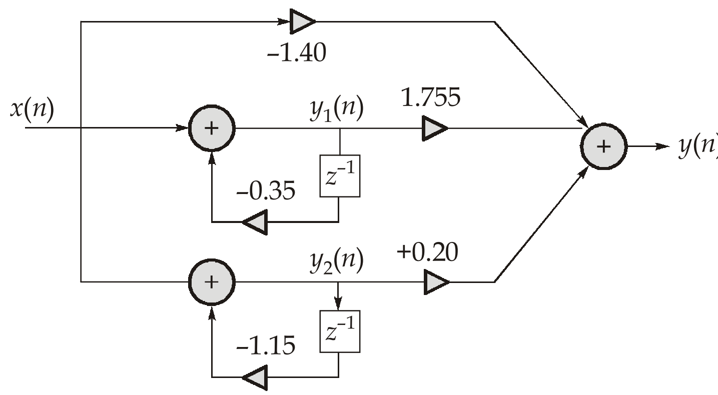
where,
$$H_1(z) = \frac{1}{1+0.35z^{-1}} = \frac{Y_1(z)}{X(z)}$$

and
$$H_2(z) = \frac{1}{1+1.15z^{-1}} = \frac{Y_2(z)}{X(z)}$$

From above, the difference equations are obtained as below:

$$y_1(n) = x(n) - 0.35y_1(n-1)$$
$$y_2(n) = x(n) - 1.15y_2(n-1)$$
$$y(n) = -1.40 + 1.755y_1(n) + 0.20y_2(n)$$

The filter can thus be realized as below:



Parallel form via the first-order sections

Q.6 (c) Solution:

(i) Processor to memory communication:

The following sequence of events takes place when information is transferred from memory to the processor:

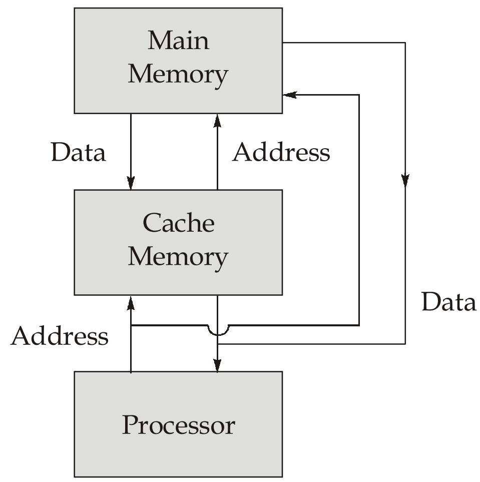
1. The processor places the address in MAR through the address bus.
2. The processor issues a READ command through the control bus.
3. The memory places retrieved data on the data bus, which is then transferred to processor.

Similarly, the following sequence of events takes place when information is written into the memory:

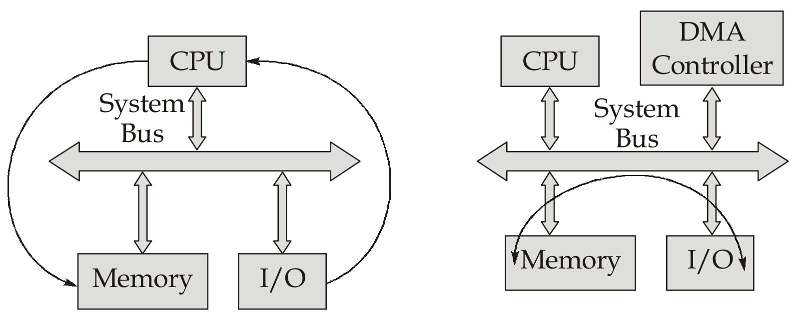
1. The processor places the address in MAR through the address bus.
2. The processor transmits the data to be written in memory using the data bus.
3. The processor issues a WRITE command to memory by control bus.
4. The data is written in memory at address specified in MAR.

The main concern in processor-memory communication is the speed mismatch between the memory and processor.

Memory access time is generally slower than the CPU's access time. Hence to eliminate speed mismatch fast memory as an intermediate buffer between processor and memory called cache memory is used.



Processor to I/O communication:



Without DMA Transfer With DMA Transfer

I/O units can be connected to the computer system through the system bus. Here, the CPU facilitates the data transfer between I/O and Memory. There is no direct communication between I/O and Memory. When a large amount of data are to be transferred, a DMA controller can be used. Each I/O device in a computer system is first met with controller, called DMA (Direct Memory Access) Controller which controls the operation of that device.

The controller is connected to the buses to perform a sequence of data transfer on behalf of the CPU. It is capable of taking over control of the system bus from the CPU, which is required to transfer data to and from memory over the system bus. A DMA controller can use the system bus only when the CPU doesn't require it or it should suspend the operations currently being processed by CPU.

DMA allows I/O unit exchange data directly with memory without going through CPU except at beginning (to issue the command) and at end (to clean up after the command is processed). While the I/O is being performed by the DMA, the CPU can start execution of some other part of the same program or can start executing some other program.

(ii)

Microcontroller	Microprocessor
1. A microcontroller is a dedicated chip which is also called single chip computer.	1. A microprocessor is a general purpose device which is called a CPU.
2. A microcontroller includes RAM, ROM series and parallel interface, timers, interrupts circuitry (in addition to CPU) in a single chip.	2. A microprocessor don't contain on chip I/O ports, timers, memories etc.
3. Microcontroller are used in small, minimum component designs embedded systems performing control oriented applications.	3. Microprocessor are most commonly used as the CPU in micro computer systems eg. Personal computer
4. Microcontroller instructions are both bit addressable as well as byte addressable.	4. Microprocessor instructions are mainly nibble or byte addressable.
5. Microcontroller based system design is simple and cost-effective.	5. Microprocessor based system design is complex and expensive.
6. The instruction set of a microcontroller is very simple with less number of instructions.	6. The instruction set of microprocessor is complex with large number of instructions.

Q.7 (a) Solution:

A simple schematic for interfacing the 8255 with an 8085 processor is shown in figure (a).

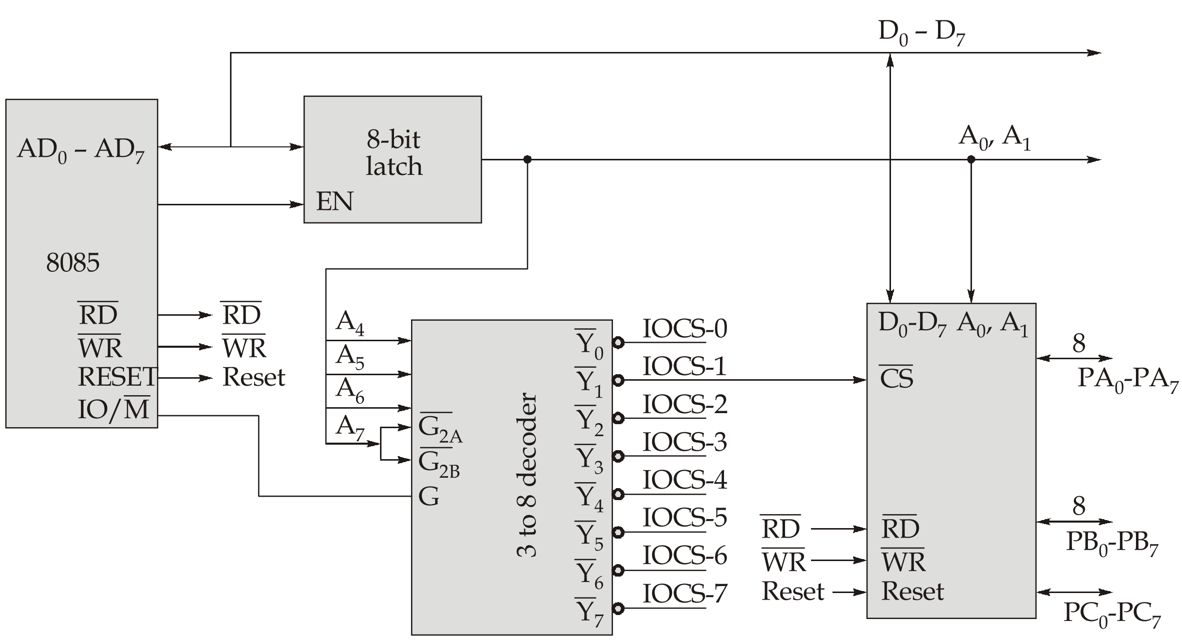


Fig. (a): Interfacing 8255 with 8085 processor

The 8255 can be either memory-mapped or IO-mapped in the system. In the schematic shown in fig. (a), the 8255 is IO-mapped in the system. The chip select signals for IO-mapped devices are generated by using a 3-to-8 decoder.

The address lines A_4, A_5 and A_6 are decoded to generate eight chip select signals IOCS-0 to IOCS-7) and in this, the chip select IOCS-1 is used to select 8255. The address line A_7 and the control signal $IO/\bar{M}$ are used as enable for the decoder.

The address line A_0 of 8085 is connected to A_0 of 8255 and A_1 of 8085 is connected to A_1 of 8255 to provide the internal addresses. The IO addresses allotted to the internal device of 8255 are listed in table given below. The Data lines D_0-D_7 of 8255 are connected to D_0-D_7 of the processor to achieve parallel data transfer.

In the schematic shown in fig. (a), the interrupt scheme is not included and so the data transfer can be performed only by checking the status of 8255 and not by interrupt method. For interrupt driven data transfer scheme, the interrupt controller 8259 has to be interfaced to the system and the interrupts of port-A (PC_3) and Port-B (PC_0) should be connected to two IR inputs of 8259.

Ports/Control Register	Address lines								Hexa address
	Decoder input and enable				Input to address Pin of 8255				
	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	
Port A	0	0	0	1	X	X	0	0	10
Port B	0	0	0	1	X	X	0	1	11
Port C	0	0	0	1	X	X	1	0	12
Control Register	0	0	0	1	X	X	1	1	13

where X denote "Don't care".

Q.7 (b) Solution:

(i) For the given system,

$$y(t) = 6.5\left[m(t)+\cos(\omega_c t)\right]+12\left[m(t)+\cos\omega_c t\right]^2$$

$$= 6.5\, m(t) + 6.5\cos(\omega_c t) + 12\, m^2(t) + 24 m(t)\cos\omega_c t + 12\cos^2\omega_c t$$

$$= 6.5\, m(t) + 6.5\cos(\omega_c t) + 12\, m^2(t) + 24 m(t)\cos\omega_c t + 6 + 6\cos 2\omega_c t$$

Passing this signal through band pass filter with cut-off frequency centered around ω_c, We get the AM signal as

$$Y_{AM}(t) = 6.5\cos(\omega_c t) + 24\, m(t)\cos\omega_c t$$

$$= 6.5\left[1 + \frac{24}{6.5}m(t)\right]\cos\omega_c t$$

Since, maximum value of $m(t)$ is

$$\max[\,|m(t)|\,] = A_m$$

therefore, the modulation index is given by

$$m_a = \max\{3.7\,|m(t)|\,\}$$

$$m_a = 3.7\,A_m$$

As the modulation index for the AM is 0.85, thus we get,

$$3.7\,A_m = 0.85$$

$$A_m = \frac{0.85}{3.7}$$

$$A_m = 0.23$$

(ii) The SSB AM signal can be described by

$$S(t) = \frac{A_c m(t)}{2}\cos(\omega_c t) - \frac{A_c \hat{m}(t)}{2}\sin(\omega_c t)$$

After demodulation, we get

$$Y(t) = \frac{A_c m(t)}{4} + \frac{n_c(t)}{2}$$

where $n_c(t)$ is the in-phase component of noise

The noise at the input of the demodulator is given by

$$n(t) = n_c(t)\cos(\omega_c t) - n_q(t)\sin\omega_c t$$

Output signal power, $S_0 = \dfrac{A_c^2 P_m}{16}$

where P_m is the power of the message signal $m(t)$.

Input signal power i.e., the average power of the modulated signal,

$$S_i = \frac{A_c^2 P_m}{4}$$

With the two-sided noise power spectral density of $\eta/2$, the average noise power in the message bandwidth 2 W is

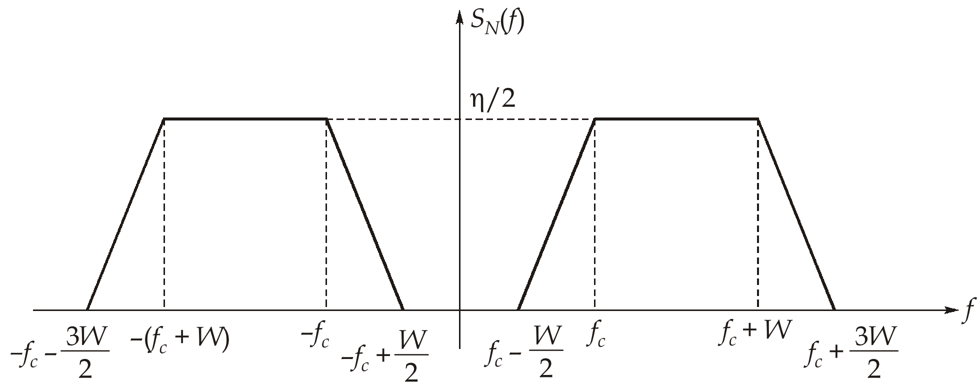
$$N_i = 2W \times \left(\frac{\eta}{2}\right) = \eta W$$

Hence, SNR at the input of demodulator,

$$(\text{SNR})_i = \frac{S_i}{N_i} = \frac{A_c^2 P_m}{4\eta W}$$

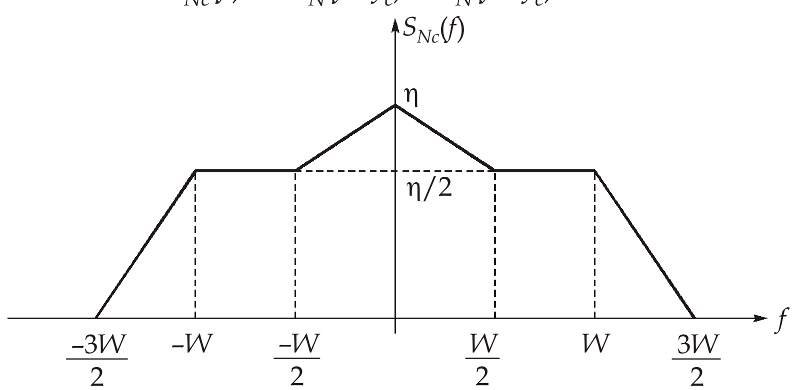
The power-spectral density of the noise, $n(t)$ at the output of receiver is

$$S_N(f) = |H_{eq}(f)|^2 (\eta/2)$$



The power spectral density of the in-phase component of the noise, $n_c(t)$ can be obtained as

$$S_{Nc}(f) = S_N(f - f_c) + S_N(f + f_c)$$



Since, $\frac{n_c(t)}{2}$ component of noise appears at the output, hence output noise power,

$$N_0 = \frac{1}{4}\int_{-W}^{W} S_{Nc}(f)df$$

$$= \frac{1}{4} \times 2 \times \left[\frac{2}{2} \times \frac{\eta}{2} \times \frac{W}{2} + \frac{\eta}{2} \times W\right]$$

$$= \frac{1}{2}\left[\frac{W\eta}{4} + \frac{\eta W}{2}\right]$$

$$= \frac{1}{2}\frac{3W\eta}{4} = \frac{5W\eta}{8} = \frac{3W\eta}{8}$$

Hence, SNR at the output of demodulator

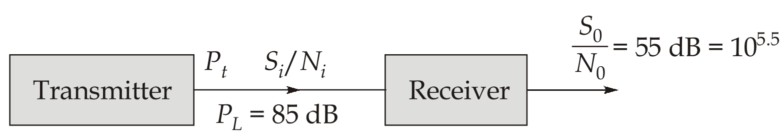
$$(SNR)_0 = \frac{S_0}{N_0} = \frac{\left(\frac{A_c^2 P_m}{16}\right)}{\frac{3W\eta}{8}} = \frac{A_c^2 P_m}{3W\eta} \times \frac{1}{2} = \frac{A_c^2 P_m}{6W\eta}$$

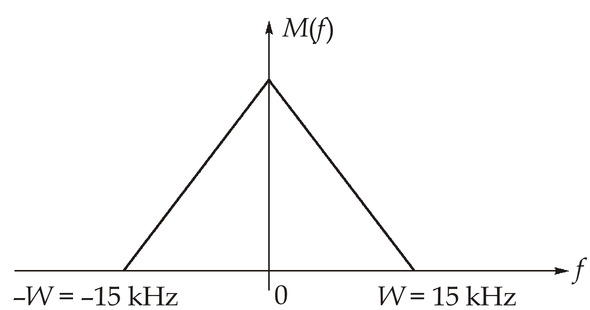
FOM of the system is

$$FOM = \frac{(SNR)_0}{(SNR)_i} = \frac{A_c^2 P_m}{6W\eta} \times \frac{4\eta W}{A_c^2 P_m} = \frac{4}{6} = 0.67$$

Q.7 (c) Solution:

(i) Given:





Message signal, BW = 15 kHz

Message power, P_m = 18 W

White noise PSD: $\frac{N_0}{2} = 10^{-13}$ Watt/Hz

N_i = White noise power affecting message signal

Average noise power affecting message signal,

$$N_i = \frac{N_0}{2} \times 2\text{ W} = N_0\text{ W}$$

$$N_i = (2 \times 10^{-13}\text{ W/Hz}) \times (15 \times 10^3\text{ Hz})$$

$$= 3 \times 10^{-9}\text{ Watts}$$

1. DSB AM,

$$\text{FOM} = 1 \quad \Rightarrow \quad \frac{S_0/N_0}{S_i/N_i} = 1 \text{ (figure of merit)}$$

$$\frac{S_0}{N_0} = \frac{S_i}{N_i} \Rightarrow \frac{S_i}{N_i} = 10^{5.5}$$

$$S_i = 10^{5.5} \times 3 \times 10^{-9} = 9.486 \times 10^{-4} \text{ W}$$

We have,

$$[S_i]_{dB} = [P_t]_{dB} - (P_L)_{dB}$$

$$10 \log_{10} S_i = 10 \log_{10} P_t - 10 \log_{10} P_L$$

$$10 \log_{10} S_i = 10 \log \frac{P_t}{P_L}$$

$$S_i = \frac{P_t}{P_L}$$

Given,

$$P_L = 85 \text{ dB} = 10^{8.5}$$

$$P_t = S_i P_L = 9.486 \times 10^{-4} \times 10^{8.5} \qquad N_i = (N_0/4) \times 2 \text{ W}$$

$$= 3 \times 10^5 \text{ Watt}$$

Transmitted power, $P_t = 300$ kW

For DSB AM, BW = 2 W = 2 × 15 = 30 kHz

2. SSB AM,

$$\text{FOM} = 1 \quad \Rightarrow \quad \frac{S_0/N_0}{S_i/N_i} = 1 \text{ (figure of merit)}$$

$$\frac{S_0}{N_0} = \frac{S_i}{N_i} = 10^{5.5}$$

$$S_i = 10^{5.5} N_i$$

$$N_i = N_0 W$$

$$S_i = 10^{5.5} \times 2 \times 10^{-13} \times 15 \times 10^3$$

$$= 9.486 \times 10^{-4} \text{ W}$$

We have,

$$[S_i]_{dB} = [P_t]_{dB} - (P_L)_{dB}$$

$$10 \log_{10} S_i = 10 \log_{10} P_t - 10 \log_{10} P_L$$

$$10 \log_{10} S_i = 10 \log \frac{P_t}{P_L}$$

$$S_i = \frac{P_t}{P_L}$$

Given, $P_L = 85 \text{ dB} \Rightarrow 10^{8.5}$

$$P_t = S_i P_L = 9.486 \times 10^{-4} \times 10^{8.5}$$
$$= 3 \times 10^5 \text{ Watt}$$

Transmitted power, $P_t = 300$ kW

For SSB AM, $BW = W = 15$ kHz

3. Conventional AM,

Given, $\mu = 0.65$

$$\text{FOM} = \frac{K_a^2 P_m}{1 + K_a^2 P_m}$$

$$\mu = K_a A_m = 0.65$$

$$K_a \times 8 = 0.65 \quad \Rightarrow \quad K_a = 0.08125$$

$$P_m = 18 \text{ W (Given)}$$

$$\text{FOM} = \frac{(0.08125)^2 \times 18}{1 + (0.08125)^2 \times 18} = 0.1062$$

$$\frac{S_0 / N_0}{S_i / N_i} = 0.1062$$

$$\frac{S_i}{N_i} = \frac{10^{5.5}}{0.1062} = 2.97 \times 10^6$$

$$S_i = 2.97 \times 10^6 \times 3 \times 10^{-9} = 8.932 \times 10^{-3}$$

$$P_t = S_i P_L = 2.824 \times 10^6 \text{ W}$$
$$= 2824.85 \text{ kW}$$

For AM, BW = 2 W = 2 × 15 = 30 kHz

(ii) $Q_A = 120$, $f_{IF} = 455$ kHz

For tuned frequency at 1500 kHz,

$$f_{si} = f_s + 2f_{IF} = 1500 + 2 \times 455$$
$$= 2410 \text{ kHz}$$

We have, $$P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{2410}{1500} - \frac{1500}{2410} = 0.984$$

$$\alpha = \sqrt{1 + P^2 Q^2} = \sqrt{1 + (0.984)^2 \times (120)^2} = 118$$

1. To achieve the same image rejection at 30 MHz, RF amplifier is used, let us say with $Q = Q_{RF}$. Hence, the total $Q(Q_T)$ is given by

$$Q_T = Q_R Q_{RF} \qquad \text{...(1)}$$

At $f_s = 30$ MHz,

$$f_{si} = f_s + 2f_{If} = (30 + 2 \times 0.455) \text{ MHz}$$
$$= 30.91 \text{ MHz}$$

$$P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{30.91}{30} - \frac{30}{30.91} = 0.0597$$

To achieve $\alpha = 118$,

$$118 = \sqrt{1 + Q_T^2 \times (0.0597)^2}$$

$$Q_T^2 = 3906467$$

$$Q_T = 1976.47$$

From equation (i)

$$Q_A Q_{RF} = 1976.47$$

$$Q_{RF} = \frac{1976.47}{120} = 16.47$$

2. To achieve the same image rejection as good as 1500 kHz at 30 MHz in the absence of RF amplifier, let us assume new intermediate frequency to be f'_{IF}. The corresponding image frequency,

$$f_{si} = f_s + 2f'_{IF} = (30 + 2f'_{IF}) \text{ MHz} \quad ...(ii)$$

To get the same $\alpha = 118$, preselector, P should be same as for

$$f_s = 1500 \text{ kHz}$$

Hence, $$\frac{f_{si}}{30} - \frac{30}{f_{si}} = 0.984$$

$$\frac{f_{si}^2 - 900}{30 f_{si}} = 0.984$$

$$f_{si}^2 - 29.52 f_{si} - 900 = 0$$

$$f_{si} = 48.194 \text{ MHz}$$

Using equation (ii),

$$30 + 2f'_{IF} = 48.194$$

$$2f'_{IF} = 48.194 - 30 = 18.194$$

$$f'_{IF} = 9.097 \text{ MHz}$$

Q.8 (a) Solution:

(i) We convert $X(z)$ into a ratio of polynomials in z^{-1}, by factoring z^3 from the numerator and $2z^2$ from the denominator, yielding

$$X(z) = \frac{1}{2}z\left(\frac{1-10z^{-1}-4z^{-2}+4z^{-3}}{1-z^{-1}-2z^{-2}}\right)$$

Using long division to reduce the order of the numerator polynomial,

$$\begin{array}{r|l} & -2z^{-1}+3 \\ -2z^{-2}-z^{-1}+1 & 4z^{-3}-4z^{-2}-10z^{-1}+1 \\ & 4z^{-3}+2z^{-2}-2z^{-1} \\ \hline & -6z^{-2}-8z^{-1}+1 \\ & -6z^{-2}-3z^{-1}+3 \\ \hline & -5z^{-1}-2 \end{array}$$

Thus, we may write

$$\frac{1-10z^{-1}-4z^{-2}+4z^{-3}}{1-z^{-1}-2z^{-2}} = -2z^{-1}+3+\frac{-5z^{-1}-2}{1-z^{-1}-2z^{-2}}$$

$$= -2z^{-1}+3+\frac{-5z^{-1}-2}{(1+z^{-1})(1-2z^{-1})}$$

Next, using partial-fraction expansion, we have

$$\frac{-5z^{-1}-2}{(1+z^{-1})(1-2z^{-1})} = \frac{1}{(1+z^{-1})}-\frac{3}{1-2z^{-1}}$$

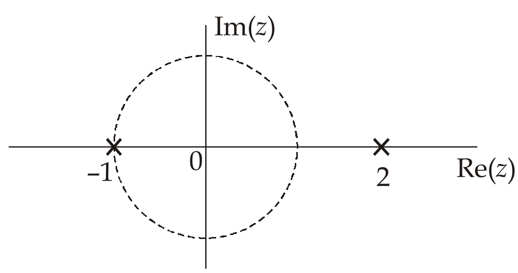
and thus, we get

$$X(z) = \frac{1}{2}z\left[-2z^{-1}+3+\frac{1}{1+z^{-1}}-\frac{3}{1-2z^{-1}}\right]$$

with ROC $|z| < 1$

Let us assume $\quad W(z) = -2z^{-1}+3+\frac{1}{1+z^{-1}}-\frac{3}{1-2z^{-1}}$

$$X(z) = \frac{z^3-10z^2-4z+4}{2z^2-2z-4}$$ has poles at $z = 2$ and at $z = -1$



For ROC: $|z| < 1$, the inverse z-transform of $W(z)$ is

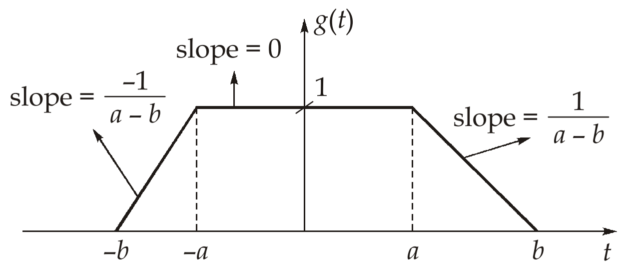
$$w(n) = -2\delta(n-1) + 3\delta(n) - (-1)^n u(-n-1) + 3(2)^n u(-n-1) \quad ...(i)$$

Finally, we apply time shifting property in equation (i) to obtain

$$x(n) = \frac{1}{2} w[n+1]$$

Thus,
$$x(n) = -\delta(n) + \frac{3}{2}\delta(n+1) - \frac{1}{2}(-1)^{n+1}u(-n-2) + 3(2)^{n+1}u(-n-2)$$

(ii) 1. We have,



For $-b < t < -a$, $\quad g(t) = \frac{t+b}{b-a}$ ...(i)

For $-a < t < a$, $\quad g(t) = 1$ ...(ii)

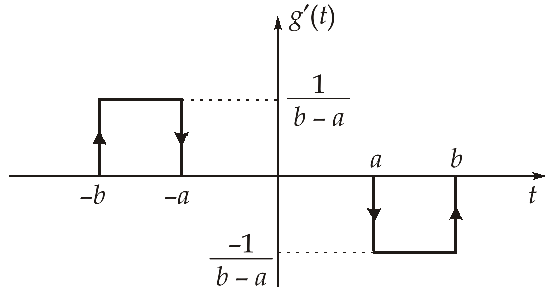
For $a < t < b$, $\quad g(t) = \frac{t-b}{a-b}$ ...(iii)

On combining all three equations, we can write,

$$g(t) = \begin{cases} \frac{t+b}{b-a} & ; \ -b \le t \le -a \\ 1 & ; \ -a \le t \le a \\ \frac{t-b}{a-b} & ; \ a \le t \le b \end{cases}$$

Differentiating $g(t)$ with respect to t,

$$g'(t) = \begin{cases} \frac{1}{b-a} & ; \ -b \le t \le -a \\ 0 & ; \ -a \le t \le a \\ \frac{1}{a-b} & ; \ a \le t \le b \end{cases}$$



Differentiating again w.r.t t, (for getting impulse form)

$$g''(t) = \frac{1}{b-a}[\delta(t+b) - \delta(t+a) - \delta(t-a) + \delta(t-b)]$$

Fourier transform of $g''(t)$,

$$G''(j\omega) = \frac{1}{(b-a)}\left[e^{j\omega b} - e^{j\omega a} - e^{-j\omega a} + e^{-j\omega b}\right]$$

Using the differentiation property of Fourier Transform,

$$G'(j\omega) = \frac{G''(j\omega)}{j\omega} = \frac{1}{j\omega(b-a)}\left[e^{j\omega b} - e^{j\omega a} - e^{-j\omega a} + e^{-j\omega b}\right]$$

Similarly,

$$G(j\omega) = \frac{1}{(j\omega)^2(b-a)}[e^{j\omega b} - e^{j\omega a} - e^{-j\omega a} + e^{-j\omega b}]$$

$$= \frac{-2}{\omega^2(b-a)}\left[\left(\frac{e^{j\omega b} + e^{-j\omega b}}{2}\right) - \left(\frac{e^{j\omega a} + e^{-j\omega a}}{2}\right)\right]$$

$$= \frac{-2}{\omega^2(b-a)}[\cos\omega b - \cos\omega a]$$

$$\because \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$G(j\omega) = \frac{4}{\omega^2(b-a)}\frac{\sin\omega(a+b)}{2}\frac{\sin(b-a)\omega}{2}$$

This procedure of differentiation is valid till $g(t) \to 0$ as $|t| \to \infty$

2. As we have, $$G(j\omega) = \frac{4}{\omega^2(b-a)}\frac{\sin\omega(a+b)}{2}\frac{\sin(b-a)\omega}{2}$$

For $a = \frac{b}{2} = 1 \quad \Rightarrow \quad a = 1$ and $b = 2$

$$G(2j) = \frac{4}{(2)^2(2-1)}\frac{\sin 2(2+1)}{2}\frac{\sin(2-1)2}{2} = 0.12$$

Q.8 (b) Solution:

Envelope Detector:

An envelope detector is simple and yet highly effective method that is well-suited for the demodulation of an AM signal for which the percentage modulation is less then 100%. Ideally, an envelope detector produces an output signal that follows the envelope of the input signal waveform exactly.

Figure below shows the circuit diagram of an envelope detector that consists of a diode and a resistor-capacitor filter. The operation of this envelope detector is as follows: On the positive half cycle of the input signal (i.e., modulated signal), the diode is forward biased and the capacitor charges up rapidly to the peak value of the input. When the signal falls below this value, the diode becomes reverse-biased and the capacitor discharges slowly through the resistance R. The discharging process continues until the next positive cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again the process is repeated.

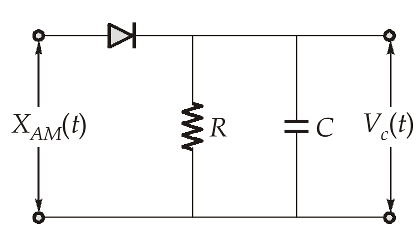
The discharging time constant RC must be large enough to ensure that the capacitor discharge slowly through the resistor R between positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave. Therefore,

$$\frac{1}{f_c} << RC << \frac{1}{W}$$

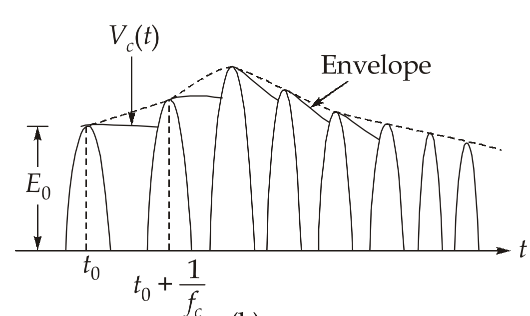
where W = message bandwidth

Figure (a) below shows the envelope of $X_{AM}(t)$ and the output of the detector (the voltage across the capacitor of envelope detector) is shown figure (b). Assume that the capacitor discharge from the peak value $E_0 = A(1 + \mu \cos \omega_m t)$ at $t_0 = 0$. Then the voltage $V_c(t)$ across the capacitor of figure (a) is given by

$$V_c(t) = E_0 e^{-t/RC}$$



(a)



(b)

The interval between two successive carrier peaks is $\frac{1}{f_c} = \frac{2\pi}{\omega_c}$ and $RC >> \frac{1}{f_c}$. This means that the time constant RC is much larger than the interval between two successive carrier peaks. Therefore, $V_c(t)$ can be approximated by

$$V_c(t) \cong E_0(1 - t/RC)$$

Thus, if the $V_c(t)$ is to follow the envelope of $X_{AM}(t)$, it is required that any time t_0

$$(1+\mu\cos\omega_m t_0)\left(1-\frac{1}{RCf_c}\right) \le 1+\mu\cos\omega_m\left(t_0+\frac{1}{f_c}\right)$$

Now, if $\omega_m << \omega_c$, then,

$$1+\mu\cos\omega_m\left(t_0+\frac{1}{f_c}\right) = 1+\mu\cos\left(\omega_m t_0+\frac{\omega_m}{f_c}\right)$$

$$= 1+\mu\cos\omega_m t_0\cos\frac{\omega_m}{f_c} - \mu\sin\omega_m t_0\sin\frac{\omega_m}{f_c}$$

$$\cong 1+\mu\cos\omega_m t_0 - \mu\frac{\omega_m}{f_c}\sin\omega_m t_0$$

Hence, $$(1+\mu\cos\omega_m t_0)\left(\frac{1}{RCf_c}\right) \ge \frac{\mu\omega_m\sin\omega_m t_0}{f_c}$$

We have, $$\frac{1}{RC}+\frac{\mu}{RC}\cos\omega_m t_0 \ge \mu\omega_m\sin\omega_m t_0$$

or $$\mu\left(\omega_m\sin\omega_m t_0 - \frac{1}{RC}\cos\omega_m t_0\right) \le \frac{1}{RC}$$

or $$\mu\sqrt{\omega_m+\left(\frac{1}{RC}\right)^2}\sin\left(\omega_m t_0 - \tan^{-1}\left(\frac{1}{\omega_m RC}\right)\right) \le \frac{1}{RC}$$

Since this inequality must hold for every t_0,

we must have

or $$\mu\sqrt{\omega_m^2+\left(\frac{1}{RC}\right)^2} \le \frac{1}{RC}$$

or $$\mu^2\left[\omega_m^2+\left(\frac{1}{RC}\right)^2\right] \le \left(\frac{1}{RC}\right)^2$$

From which we obtain,

$$RC \le \frac{1}{\omega_m}\frac{\sqrt{1-\mu^2}}{\mu}$$

Synchronous Detector:

The synchronous detector consists of a multiplier and a Low Pass Filter (LPF). The multiplier multiplies the AM signal with a locally generated carrier having the same frequency and phase as that of the carrier signal.

The output of multiplier is passed through LPF to recover the message signal.

The general expression for AM wave is given as,

$$S_{AM}(t) = A_c(1 + K_a m(t)) \cos 2\pi f_c t$$

If the locally generated carrier is $\cos 2\pi f_c t$, the output of multiplier is given as

$$(Mul)_{o/p} = A_c(1 + K_a m(t)) \cos^2 2\pi f_c t$$

$$= \frac{A_c}{2} + \frac{A_c}{2}\cos 4\pi f_c t + \frac{A_c}{2}K_a m(t) + \frac{A_c}{2}K_a m(t)\cos 4\pi f_c t$$

The low pass filter only allows the third term to pass through it.

$$m'(t) = \frac{A_c}{2}K_a m(t)$$

Hence, the message signal is recovered. However, if the phase of the locally generated carrier differs by ϕ.

$$(Mul)_{o/p} = A_c(1 + K_a m(t)) \cos 2\pi f_c t \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c}{2}(1 + K_a m(t))\left[\cos(4\pi f_c t + \phi) + \cos\phi\right]$$

$$= \frac{A_c}{2}\cos(4\pi f_c t + \phi) + \frac{A_c K_a}{2} m(t)\cos\phi + \frac{A_c}{2}\cos\phi$$

$$+ \frac{A_c}{2}K_a m(t)\cos(4\pi f_c t + \phi)$$

The output of low pass filter is now given as

$$m'(t) = \frac{A_c}{2}K_a m(t)\cos\phi$$

If the phase difference (ϕ) is 90°, then $m(t) = 0$ and this effect is known as Quadrature Null Effect.

Q.8 (c) Solution:

The 8255 A is a widely used, programmable, parallel I/O device. It can be programmed to transfer data under various conditions, from simple I/O to interrupt I/O. It is flexible, versatile and economical, but somewhat complex. It is an important general-purpose I/O device that can be used with almost any microprocessor.

The 8255A has 24 I/O pins that can be grouped primarily in two 8-bit parallel ports: A and B, with the remaining eight bits as port C.

The eight bits of port C can be used as individual bits or be grouped in two 4-bits ports: C_{UPPER} (C_U) and C_{LOWER} (C_L), as shown in fig. (a).

The functions of these ports are defined by writing a control word in the control register. The table (b) shows all the function of the 8255 A, classified according to two modes, whereby ports A and/or B use bits from port C as handshake signals. In the handshake mode, the Bit set/Reset (BSR) mode and the I/O mode. The BSR mode is used to set or reset the bits in port C.

The I/O mode is further divided into three modes: Mode 0, Mode 1 and Mode 2. In Mode 0, all ports function as simple I/O ports. Mode 1 is a handshake mode, whereby ports A and/or B use bits from port C as handshake signals. In the handshake mode, two type of I/O data transfer can be implemented: status check and interrupt. In Mode 2, port A can be set up for bidirectional data transfer using handshake signals from port C, and port B can be setup either in Mode 0 or Mode 1.

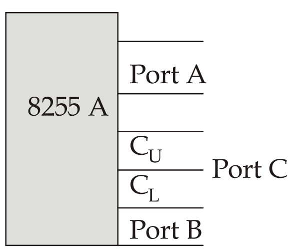


Fig. (a)

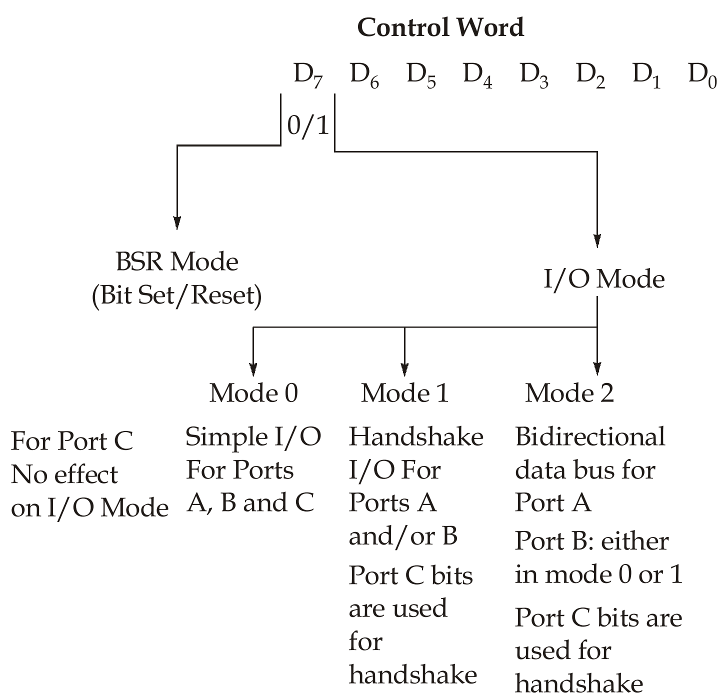


Table (b)

The Block Diagram of the 8255A:

The block diagram in fig (c) shows two 8-bit ports (A and B), two 4-bits ports (C_U and C_L), the data bus buffer, and control logic. This block diagram includes all the elements of a programmable device; port C performs functions similar to that of the status register in addition to providing handshake signals.

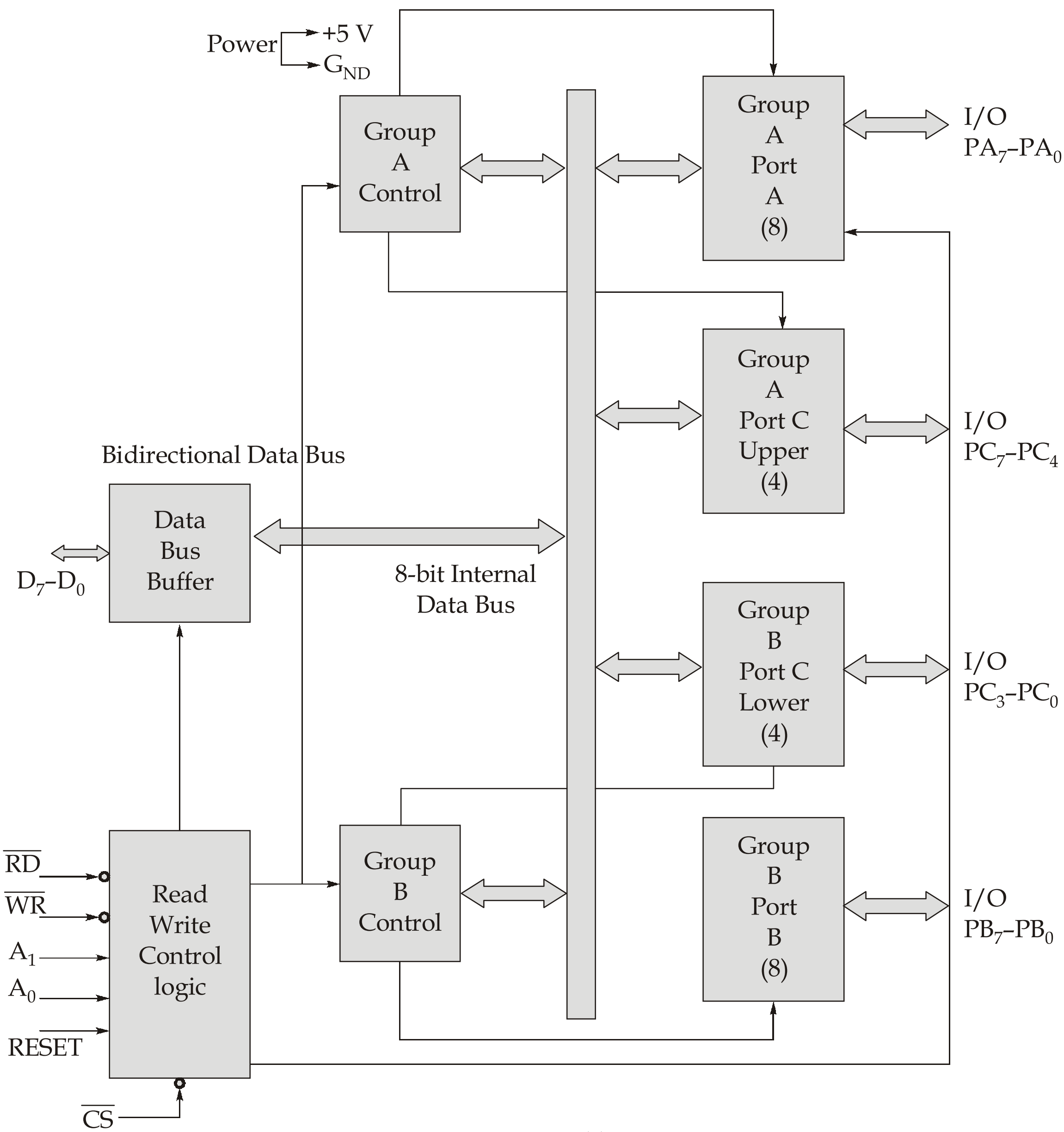

Power
+5 V
GND
Group A Control
Group A Port A (8)
I/O
PA7–PA0
Group A Port C Upper (4)
I/O
PC7–PC4
Bidirectional Data Bus
Data Bus Buffer
D7–D0
8-bit Internal Data Bus
Group B Port C Lower (4)
I/O
PC3–PC0
RD
WR
A1
A0
RESET
Read Write Control logic
Group B Control
Group B Port B (8)
I/O
PB7–PB0
CS

Fig. (c)

8255 A IC has 40 pins with details as below:

D_7-D_0	Data Bus (Bidirectional)
RESET	Reset Input
$\overline{CS}$	Chip select
$\overline{RD}$	Read Input
$\overline{WR}$	Write Input
A_0 A_1	Port address
PA_7–PA_0	Port A (8-Bit)
PB_7–PB_0	Port B (8-Bit)
PC_7–PC_0	Port C (8-Bit)
V_{CC}	+5 V
GND	0 Volt

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