India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

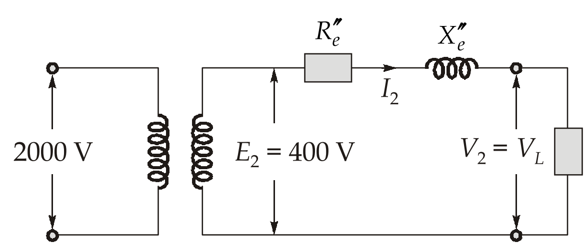
Electrical Engineering Test No : 4

Section A : Electrical Machines

Q.1 (a) Solution:

Assume, $$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Referring the primary quantities to the secondary side



$$R_e'' = R_2 + R_1\left[\frac{N_2}{N_1}\right]^2 = R_2 + \left[\frac{V_2}{V_1}\right]^2 R_1$$

$$= 0.2 + \left[\frac{400}{2000}\right]^2 \times 5.5 = 0.42\,\Omega$$

$$X_e'' = X_2 + \left[\frac{N_2}{N_1}\right]^2 X_1 = 0.45 + \left[\frac{400}{2000}\right]^2 \times 12 = 0.93\,\Omega$$

$$I_2 = \frac{10\times 1000}{400} = 25\,\text{A}$$ ...(Approximate value)

$$Z_e'' = \sqrt{(R_e'')^2 + (X_e'')^2} = 1.02\Omega$$

Voltage across the load at full-load and 0.8 p.f. lagging

$$V_L = 400 - I_2(0.8 - j0.6)\,(R_e'' + jX_e'')$$
$$= 400 - 25(0.8 - j0.6)\,(0.42 + j0.93)$$
$$|V_L| = 377.85 \text{ V} \qquad \text{...(Approximate value)}$$
$$\%\text{ voltage regulation} = \frac{400 - 377.85}{377.85} \times 100 = 5.86\%$$

Q.1 (b) Solution:

Assuming infinite permeability of iron path,

Reluctance of air gap

$$R = \frac{l}{\mu_o \mu_r A} = \frac{2(D-x)}{\mu_o A}$$
$$L(x) = \frac{N^2}{R} = \frac{\mu_o A N^2}{2(D-x)}$$
$$\lambda = N\phi = NAB_m \sin \omega t$$

(i)
$$e = \frac{d\lambda}{dt} = NAB_m \omega \cos \omega t$$

(ii)
$$W_f(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L} = \frac{1}{2} \times \frac{N^2 A^2 B_m^2 \sin^2 \omega t}{(\mu_o N^2 A)/2(D-x)}$$
$$W_f(\lambda, x) = \frac{AB_m^2}{\mu_o}(D-x)\cdot \sin^2 \omega t$$
$$F_f = -\frac{\partial W_f}{\partial x} = \frac{AB_m^2 \sin^2 \omega t}{\mu_o}$$
$$F_f = \frac{1}{2\mu_o} AB_m^2 (1 - \cos 2\omega t)$$

(iii)
$$F_f = M\frac{d^2 x_1}{dt^2} + B\frac{dx_1}{dt} + K(x_0 + x_1)$$
$$x_o = \frac{AB_m^2}{2K}$$

Equating constant terms of F_f;

$$\frac{1}{2\mu_0} AB_m^2 = K X_0$$

$$X_0 = \frac{AB_m^2}{2K\mu_0}$$

x can be obtained by equating (i) and (ii),

$$-\frac{1}{2\mu_o} AB_m^2 \cos 2\omega t = M\frac{d^2x_1}{dt^2} + B\frac{dx_1}{dt} + Kx_1$$

$$H(j\omega) = \frac{1}{M(j2\omega)^2 + B(j2\omega) + K}$$

$$= \frac{1}{\sqrt{(K-4M\omega^2)^2 + 4B^2\omega^2}} \angle -\tan^{-1}\left(\frac{K-4M\omega^2}{2B\omega}\right)$$

$$\therefore \quad x_1(t) = \frac{-\mu_o^{-1} AB_m^2}{2\sqrt{(K-4M\omega^2)^2 + 4B^2\omega^2}} \cos(2\omega t - \psi)$$

$$\left(\because \psi = \tan^{-1}\left(\frac{K-4M\omega^2}{2B\omega}\right)\right)$$

Net moment is $\quad x(t) = x_0 + x_1(t)$

Q.1 (c) Solution:

Synchronous speed, $N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{6} = 1000$ rpm

Rotor speed, $N_r = 975$ rpm

Slip, $S = \dfrac{N_s - N_r}{N_s} = \dfrac{1000-975}{1000} = 0.025$

Power input to stator = 40 kW

Stator losses = 1 kW

Output from stator = Input to rotor

Therefore, Rotor input = Stator input – Stator losses

= 40 – 1 = 39 kW

Rotor copper losses = Slip × Rotor input

= 0.025 × 39 = 0.975 kW

Rotor output i.e. power obtaining from the rotor shaft,

P = Rotor input – Rotor copper loss – frictional windage losses (assuming iron loss in the rotor core as negligible)

or $P = 39 - 0.975 - 2 = 36.025$ kW

$$\text{Output horsepower} = \frac{36025}{735.5} = 48.98$$

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{36025 \times 100}{40000} = 90\%$$

Q.1 (d) Solution:

At no load,

$$E = V - I_a R_a$$

$$= 240 - 2 \times 0.4 = 239.2 \text{ V}$$

$$E = K \phi N$$

$$= K_1 I_f N$$

$$I_f = \frac{V}{R_f} = \frac{240}{160} \text{A} = 1.5 \text{ A}$$

Substituting,

$$239.2 = K_1 \frac{240}{160} \times 800$$

$$\therefore \quad K_1 = \frac{239.2 \times 160}{240 \times 800} = 0.199$$

At a load of 30 A

$$30 = I_a' + I_f'$$

$$E_a' = 240 - (30 - I_f') \times 0.4$$

$$= 240 - 12 + 0.4 I_f$$

$$= 228 + 0.4 I_f = 228 + 0.4 I_f$$

$$E_a' = K_1 I_f' N'$$

$$228 + 0.4\, I_f' = 0.199 \times I_f' \times 950$$

$$I_f' = 1.21 \text{ A}$$

$$I_a' = 28.79 \text{ A}$$

$$\text{Overall field resistance} = \frac{240}{1.21}$$

$$R_f' = 198.35\ \Omega$$

$$R_{ext} = R_f' - 160$$

$$R_{ext} = 38.35\ \Omega$$

Q.1 (e) Solution:

Transformer-*A*:

Rated output = 40 kVA,

Core loss = 500 W = 0.5 kW

Full-load copper-loss = 500 W = 0.5 kW

$$\text{Copper-loss at half-load} = \frac{500}{4} = 125\ \text{W} = 0.125\ \text{kW}$$

Assuming, power factor for lighting load = 1.0

All-day efficiency of transformer-*A*,

$$= \frac{\text{Output energy in kWh in 24 h}}{\text{Input engery in kWh in 24 h}}$$

$$= \frac{(40\times1\times4+20\times1\times8)\times100}{40\times1\times4+20\times1\times8+0.500\times24+0.500\times4+0.125\times8}$$

$$= 95.5\%$$

Similarly the all-day efficiency of transformer-*B* is calculated as

All-day efficiency of transformer-*B*,

$$= \frac{(40\times1\times4+20\times1\times8)\times100}{40\times1\times4\times20\times1\times8+0.250\times24+0.750\times4+0.1875\times8}$$

$$= 96.8\%$$

Q.2 (a) Solution:

(i) The high voltage side of this transformer has a base line voltage of 13800 V and a base apparant power of 50 kVA,

Since the primary is Δ-connected

$$V_{line} = V_{phase}$$

Using, $$S_{3\text{-}\phi} = \sqrt{3}V_L \cdot I_L$$

$$I_L = \frac{50\times10^3}{\sqrt{3}\times13800} = 2.091\ \text{A}$$

$$I_{phase} = \frac{I_L}{\sqrt{3}} = \frac{2.091}{\sqrt{3}} = 1.207 \text{ A}$$

$$Z_{base} = \frac{(V_{phase})_{rated}}{(I_{phase})_{rated}} = \frac{13800}{1.207}$$

$$Z_{base} = 11426.4 \ \Omega$$

Given, $R_{p.u.} = 1\% = 0.01$ p.u.

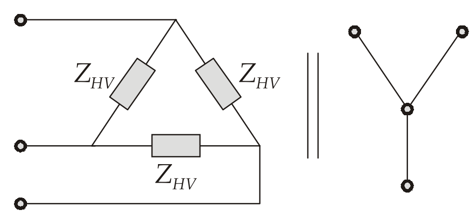
$$R_{HV} = R_{pu} \times Z_{base} = 0.01 \times 11426.4$$

$$R_{HV} = 114.26 \ \Omega$$

Given, $X_{p.u.} = 7\% = 0.07$ p.u.

$$X_{HV} = X_{p.u.} \times Z_{base} = 0.07 \times 11426.4$$

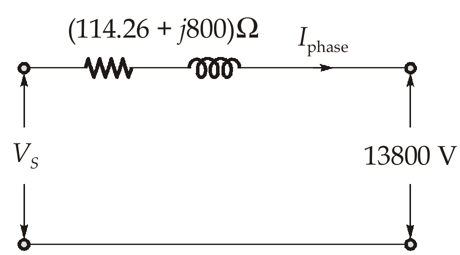
$$X_{HV} \approx 800 \ \Omega$$



$$Z_{HV} = R_{HV} + jX_{HV}$$

$$= (114.26 + j800) \ \Omega \text{ in each phase of } \Delta$$

(ii) The referred circuit of transformer to HV side per phase,



Given, full load current is flowing (0.8 p.f. lagging)

$$I_{phase} = 1.207\angle{-36.87°} \text{ A}$$

From the circuit,

$$V_S = 13800\angle 0° + (1.207\angle{-36.86})\,(114.26 + j800)$$

$$= 14505.983\angle 2.725 \text{ V}$$

Percentage voltage regulation,

$$\% VR = \frac{V_S - 13800}{13800} \times 100 = \frac{14505.983 - 13800}{13800} \times 100 = 5.11\%$$

(iii) In the per unit system, the output voltage is $1\angle 0°$ and the current is $1\angle -36.87°$ A, therefore, the input voltage is

$$V_S = 1\angle 0° + (1\angle -36.86)(0.01 + j0.07)$$

$$V_S = 1.0511\angle 2.726$$

$$\%V_R = \frac{V_S - V_R}{V_R} \times 100 = \frac{1.0511 - 1}{1} \times 100$$

$$= 5.11\%$$

Q.2 (b) (i) Solution:

On per phase base is

The terminal voltage, $$V_t = \frac{13800}{\sqrt{3}} = 7967.43 \text{ Volt}$$

The load current, $$I_a = \frac{70 \times 10^6}{\sqrt{3} \times 13800} = 2928.59 \text{ Amp}$$

So, $$\tan\psi = \frac{V_t \sin\phi + I_a X_q}{V_t \cos\phi + I_a R_a}$$

$$\tan\psi = \frac{7967.43 \times 0.6 + 2928.59 \times 1.21}{7967.43 \times 0.8 + 0} = 1.3059$$

$$\psi = \tan^{-1}(1.3059) = 52.56°$$

So, for lagging load

$$\psi = \phi + \delta$$

$$\delta = \psi - \phi = 52.56 - \cos^{-1} 0.8 = 15.69°$$

The value of $$I_d = I_a \sin\psi$$

$$I_d = 2928.59 \times \sin(52.56°)$$

$$I_d = 2325.27 \text{ Amp}$$

So, excitation voltage $$E_g = V_t \cos\delta + I_d X_d$$

$$E_g = 7967.43 \times \cos(15.69°) + 2325.27 \times 1.83$$

$$E_g = 11925.80 \text{ V}$$

(i) So, % regulation $$VR = \frac{E_g - V_t}{V_t} \times 100$$

$$= \frac{11925.80 - 7967.43}{7967.43} \times 100$$

$$VR = 49.68\%$$

(ii) Power developed,

$$P_d = P_{output}$$

$$P_d = 70 \times 0.8 = 56 \text{ MW}$$

Q.2 (b) (ii) Solution:

In the first case,

$$E = V - I_a(R_a + R_{sc})$$

$$= 200 - 20(0.5 + 0.2) = 186 \text{ V}$$

$$E = K\,\phi\,N$$

or

$$186 = K\,\phi \times 1000$$

$$K\,\phi = 0.186$$

When a resistance R of value 0.2 Ω is connected in parallel with the series field, 20 A current will be equally divided between the series field winding and the parallel resistance called the diverter. In this case flux will be produced due to 10 A current flowing through the series field,

Induced emf,

$$E_1 = K\,\phi_1\,N_1$$

$$= 0.7 \times K\,\phi \times N_1 \quad (\because K\,\phi_1 = 0.7\,K\,\phi)$$

$$E_1 = 0.7 \times 0.186 \times N_1$$

But

$$E_1 = V - (I_a R_a + I_{se}\,R_{se})$$

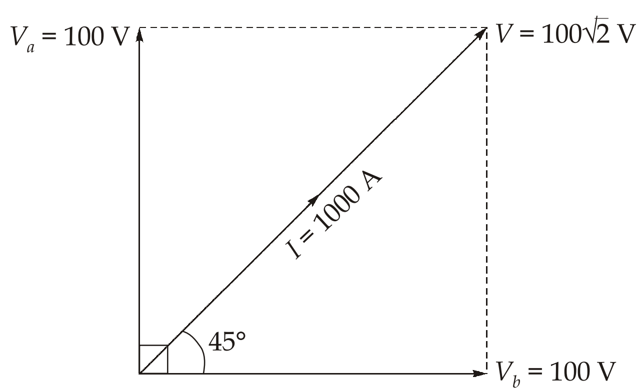
$$= 200 - (20 \times 0.5 + 10 \times 0.2) = 188 \text{ V}$$

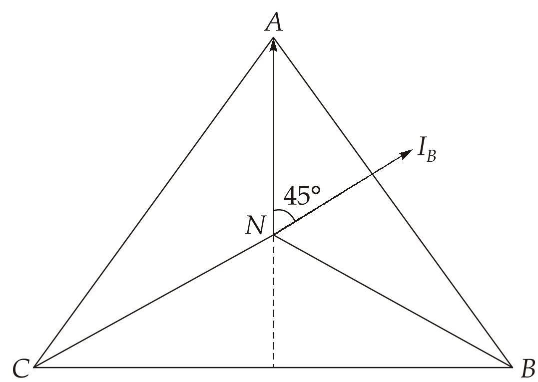
Thus,

$$188 = 0.7 \times 0.186 \times N_1$$

$$N_1 = \frac{188}{0.7 \times 0.186} = 1444 \text{ rpm}$$

Q.2 (c) Solution:





(i) 1. $$|V_a| = |V_b| = \frac{V_L}{\sqrt{2}}$$

In balanced 2-ϕ system,

$$\text{Phase system} = \frac{\text{Line voltage}}{\sqrt{2}}$$

Since in Scott connection, teaser voltage leads by 90°,

$$V_a = \frac{100\sqrt{2}}{\sqrt{2}}\angle +90°$$

$$V_a = 100\angle 90°$$

Main transform secondary voltage,

$$V_b = 100\angle 0°$$

Teaser transformer turn ratio

$$= \frac{\sqrt{3}}{2}\frac{N_1}{N_2} : 1 \quad \text{...(i)}$$

Now, $$\frac{N_1}{N_2} = \frac{V_1}{V_2}$$

where, V_1 = 11000 volts and V_2 = 100 volts

So, $$\frac{N_1}{N_2} = \frac{11000}{100} = 110$$

From equation (i),

Teaser transformer turns ratio

$$= \frac{\sqrt{3}}{2}\times 110 = 95.261 : 1$$

2. Line current in A phase

$$I_A = \frac{I_{\text{teaser secondary}}}{\text{Teaser turns ratio}} = \frac{1000}{95.262}\angle 90°$$

$$\bar{I}_A = 10.497\angle 90°$$

and

$$\text{current } I_{BC} = \frac{\text{Main secondary current}}{\text{Main turns ratio}} = \frac{1000}{(N_1 / N_2)}\angle 0°$$

$$\bar{I}_{BC} = \frac{1000}{110} = 9.09\angle 0° \text{ A}$$

Current in Line-B

$$I_B = \bar{I}_{BC} - \frac{\bar{I}_A}{2} = 9.09\angle 0° - \frac{10.497\angle 90°}{2}$$

$$= 10.496\angle -30°$$

Q.3 (a) Solution:

$$\text{Output power} = \sqrt{3}V_L I_L = 1200 \times 1000$$

$$I_L = \frac{1200 \times 1000}{\sqrt{3} \times 3300} = 210 \text{ A}$$

For stat-connection,

$$\text{Per phase, } I_a = I_L = 210 \text{ A}$$

$$\text{V per phase} = \frac{3300}{\sqrt{3}} = 1905 \text{ V}$$

Synchronous impedance,

$$Z_s = \frac{1100}{\sqrt{3} \times 200} 3.175\,\Omega$$

$$X_S = \sqrt{Z_s^2 - R_a^2}$$

$$= \sqrt{(3.175)^2 - (0.25)^2} = 3.165\,\Omega$$

(i) For lagging in pf load,

$$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2}$$

$$= \sqrt{(1905 \times 0.8 + 210 \times 0.25)^2 + (1905 \times 0.6 + 210 \times 3.165)^2}$$

$$= 2398 \text{ V}$$

$$\text{Regulation} = \frac{E-V}{V} \times 100$$

$$= \frac{2398-1905}{1905} \times 100 = 25.9\%$$

(ii) For leading power factor load,

$$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi - I_a X_s)^2}$$

$$= \sqrt{(1905 \times 0.8 + 210 \times 0.25)^2 + (1905 \times 0.6 - 210 \times 3.165)^2}$$

or $E = 1647$ V

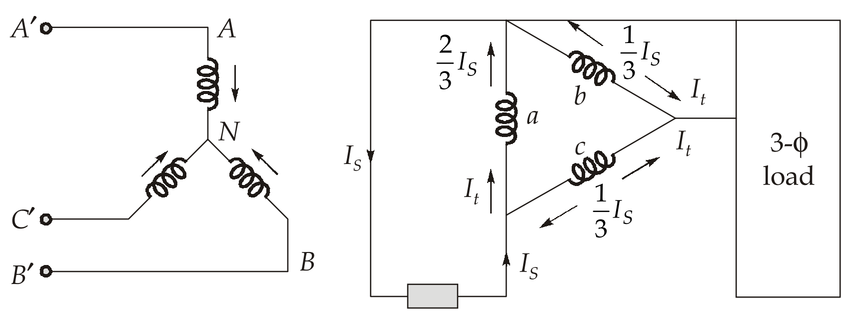
$$\text{Regulation} = \frac{E-V}{V} \times 100 = \frac{1647-1905}{1905} \times 100 = -13.54\%$$

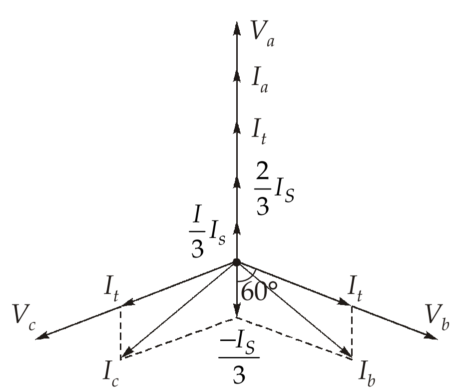
It can be noticed that for leading power factor load, the regulation is negative.

This is because a synchronous generator operating on leading power factor is under-excited. The low value of excitation EMF is due to the underexcitation.

Q.3 (b) Solution:

The figure shown illustrates the circuit and phasor diagrams, where subscripts '*s*' and '*t*' denotes the single phase and three phase loads respectively,





Let the single phase load be connected to 'a' phase secondary.

Let the current due to single phase load be I_S.

$$I_a = I_s \times \frac{Z_b + Z_c}{Z_a + Z_b + Z_c} = I_s \times \frac{Z+Z}{Z+Z+Z} = \frac{2I_s}{3}$$

$$I_b = I_c = \frac{I_S}{3}$$

Current due to signal phase load,

$$I_s = \frac{60000}{400} = 150 \text{ A}$$

Current I_S divides in the three phase delta winding. The current $\frac{2}{3}I_S$ and I_t in secondary phase must flow in the same direction because the single phase load is connected across this phase,

Current in secondary phase winding due to 3-phase load,

$$I_t = \frac{300}{3(0.4)} = 250 \text{ A}$$

$$I_a = I_t + \frac{2}{3}I_s = 250 + \frac{2}{3}(150) = 350 \text{ A}$$

Current in phase winding b and c are equal to phasor difference of I_t and $\frac{1}{3}I_s$

$$\bar{I}_b = \bar{I}_c = \bar{I}_t - \frac{1}{3}\bar{I}_s$$

$$\bar{I}_b = \bar{I}_c = \sqrt{I_t^2 + \left(\frac{I_s}{3}\right)^2 + 2I_x\left(\frac{I_s}{3}\right)\cos 60°}$$

$$= \sqrt{(250)^2 + (50)^2 + 2 \times 250 \times 50 \times \frac{1}{2}} = 278.4 \text{ A}$$

$$\text{Turn ratio for each phase} = \frac{11000}{\sqrt{3} \times 400} = 15.88$$

The current in primary side are

$$I_{AA'} = I_{AN} = \frac{I_a}{15.88} = \frac{350}{15.88} = 22.04 \text{ A}$$

$$I_{BB'} = I_{BN} = \frac{I_b}{15.88} = \frac{278.04}{15.88} = 17.5\text{ A}$$

$$I_{CC'} = I_{CN} = \frac{I_c}{15.88} = \frac{278.04}{15.88} = 17.5\text{ A}$$

Q.3 (c) Solution:

The terminal voltage (per phase)

$$V_t = \frac{600}{\sqrt{3}}\text{ V}$$

$$V_t = 346.41\text{ Volt}$$

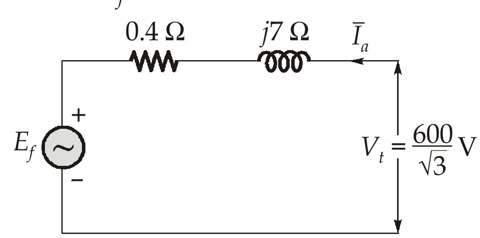
The synchronous impedance, $Z_s = 0.4 + j7 = 7.011\angle 86.73°\ \Omega$

The armature current, $I_a = 15\angle 0°\text{ A}$

Excitation emf, $E_f = V_t - \bar{I}_a Z_s$

$$E_f = 346.41\angle 0° - 15\angle 0°(0.4 + j7)$$

$$E_f = 356.23\angle -17.142°\text{ Volt}$$



Now load torque is increased until the motor draw a current of 50 A, by keep field current constant.

So, $E_g = 356.23\text{ Volt}$

$$I_a = 50\text{ Amp}$$

Let angle is ϕ (power factor angle).

So, $E_f\angle -\delta = V_t - \bar{I}_a Z_s$

$$356.23\angle -\delta = 346.41\angle 0° - (50\angle\phi)(7.011\angle 86.73°)$$

$$356.23\angle -\delta = 346.41\angle 0° + 350.57\angle\phi - 93.27°$$

So, $356.23 = \sqrt{346.41^2 + 350.57^2 + 2\times 346.41 \times 350.57\times\cos(\phi - 93.27°)}$

$$356.23^2 = 120000 + 122900 + 242881.907\cos(\phi - 93.27°)$$

$$\cos(\phi - 93.27) = -0.47759$$

$$\phi - 93.27 = -118.528°$$

$$\phi = -25.26° \text{ (lagging)}$$

So, New power factor $= \cos(\phi) = \cos(25.26°)$

Power factor $= 0.904$ lagging

and developed power, $P_{m(\text{out})} = P_{\text{in}} - \text{losses}$

$$P_{\text{in}} = \sqrt{3} V_t I_a \cos\phi$$

$$P_{\text{in}} = \sqrt{3} \times 600 \times 50 \times \cos(25.26)$$

$$P_{\text{in}} = 47 \text{ kW}$$

and $P_{\text{loss}} = 3I_a^2(R_a) = 3 \times 50^2 \times 0.4 = 3 \text{ kW}$

So, $P_{m(\text{out})} = 47 - 3 = 44 \text{ kW}$

$$T_{m(\text{developed})} = \frac{P_{m(\text{out})}}{\omega_s} = \frac{44 \times 10^3}{\frac{2}{6} \times 2\pi \times 50}$$

$$T_{m(\text{developed})} = 420.17 \text{ N.m}$$

Q.4 (a) (i) Solution:

∵ At maximum efficiency,

$$P_{\text{cu}} = P_i$$

and P_{cu} at 75% laod = $(0.75)^2 P_{\text{cu}\,fl}$

$$\therefore \quad 0.97 = \frac{500 \times 0.75 \times 1.0}{500 \times 0.75 \times 1.0 + 2 \times 0.75^2 \times P_{cu\,fl}}$$

$$P_{\text{cu}\,fl} = 10.309 \text{ kW}$$

$$P_{\text{cu}\,fl\,(\text{p.u.})} = \frac{10.309}{500} = 0.02062 \text{ p.u.}$$

$$R_{\text{p.u.}} = 0.02062 \text{ p.u.}$$

$$Z_{\text{p.u.}} = 0.10 \text{ p.u.}$$

$$X_{\text{p.u.}} = \sqrt{0.10^2 - 0.02062^2}$$

$$= 0.09785 \text{ p.u.}$$

$$V' = 1.0 + 1.0\angle -36.87°(0.02062 + j0.09785)$$

$$V' = 1.07723\angle 3.5° \text{ p.u.}$$

$$\% \text{ V.R.} = \frac{1.07723 - 1.0}{1.0} \times 100 = 7.72\%$$

Q.4 (a) (ii) Solution:

$$E = V - I_a R_a$$
$$= 250 - 50 \times 0.3 = 235 \text{ V}$$
$$E = K\phi N$$

or
$$K\phi = \frac{235}{1000} = 0.235$$

To calculate E at 800 rpm,

$$E = K\phi \times 800 = 0.235 \times 800 = 188 \text{ V}$$

Let an extra resistance R be put in series with the armature circuit, then

$$E = V - I_a(R_a + R)$$
$$188 = 250 - 50\,(0.3 + R)$$
$$R = 0.94\ \Omega$$

If the load torque is halved, I_a is halved since there is a linear relationship between torque and I_a, now

$$I_a = 25 \text{ A}$$
$$E_1 = V - I_a(R_a + R)$$
$$= 250 - 25(0.3 + 0.94)$$
$$= 219 \text{ V}$$
$$\frac{E}{E_1} = \frac{N}{N_1}$$
$$N_1 = N\frac{E_1}{E} = 800 \times \frac{219}{188} = 932 \text{ rpm}$$

Q.4 (b) Solution:

$$p = 6, \quad f = 50$$

Total number of slots $= 12 \times 6 = 72$

Total number of slots per phase $= \frac{72}{3} = 24$

Total number of conductors per phase $= 24 \times 4 = 96$

Total number of turns per phase $= \frac{96}{2} = 48$

Number of slots per pole per phase $= \frac{72}{6 \times 3} = 4 = \text{m}$

$$\text{Slot angle} = \frac{\text{Total mech. angle}}{\text{Total number of slots}}$$

$$= \frac{360}{72} = 5^\circ \text{ mechanical}$$

$$\text{Since } 1^\circ \text{ mechanical} = \frac{P^\circ}{2} \text{ electrical}$$

$$\text{Slot angle, } \alpha = 5^\circ \text{ mech.} = 5 \times \frac{P^\circ}{2} \text{ electrical}$$

$$= 5 \times \frac{6}{2} = 15^\circ \text{ electrical}$$

Distribution factor,
$$K_d = \frac{\sin \frac{m\alpha}{2}}{m \sin \alpha / 2}$$

Substituting the values of m and α,
$$K_d = \frac{\sin \frac{4 \times 15}{2}}{4 \sin \frac{15}{2}} = \frac{\sin 30^\circ}{4 \sin 7.5^\circ} = \frac{0.5}{4 \times 0.13} = 0.96$$

Coil pitch is $\frac{5}{6}$ of full-pitch

A full-pitch coil has 180° electrical between the coil sides A $\frac{5}{6}$ pitched coil will have an electrical angle of $\frac{5}{6} \times 180^\circ$, i.e., 150° between the coil sides,

Thus, short pitch angle, $\beta = 180^\circ - 150^\circ = 30^\circ$

Pitch factor,
$$K_p = \cos \frac{\beta}{2}$$

Substituting the value β,
$$K_p = \cos \frac{\beta}{2} = \cos 15^\circ = 0.96$$

Induced emf, $E = 4.44\, \phi f\, T\, K_p K_d$ volts

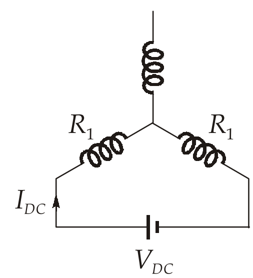
Substituting the values we get,

$$E = 4.44 \times 1.5 \times 50 \times 48 \times 0.96 \times 0.96$$

$$= 14730 \text{ volts} = 14.73 \text{ kV}$$

Q.4 (c) Solution:

From the dc test values,



$$R_1 = \frac{V_{DC}}{2I_{DC}} = \frac{13.6}{2 \times 28} = 0.2428 \text{ A}$$

From no load test,

$$I_{L,\,\text{av}} = \frac{8.12 + 8.20 + 8.18}{3} = 8.166 \text{ A}$$

$$V_{\phi,\,nl} = \frac{208}{\sqrt{3}} = 120.088 \text{ V}$$

$$|Z_{nl}| = \frac{120.088}{8.17} = 14.688\ \Omega = X_1 + X_m$$

Also, $$P_{sc,\,nl} = 3I_1^2 R_1 = 3 \times (8.166)^2\,(0.2428)$$
$$= 48.572 \text{ W}$$
$$P_{\text{rot}} = P_{in,\,nl} - P_{sc,\,nl}$$
$$= 420 - 48.572 = 371.428 \text{ W}$$

From blocked rotor test,

$$I_{L\,,\,\text{av}} = \frac{28.1 + 28.0 + 27.6}{3} = 27.9 \text{ A}$$

Blocked rotor impedance,

$$|Z_{BR}| = \frac{V_\phi}{I_a} = \frac{V_T}{\sqrt{3}I_a} = \frac{25}{\sqrt{3}(27.9)} = 0.517\ \Omega$$

Impedance angle, $$\phi = \cos^{-1}\frac{P_{\text{in}}}{\sqrt{3}V_T I_L} = \cos^{-1}\frac{920}{\sqrt{3}(25)(27.9)}$$

$$\cos^{-1}(0.7615) = 40.40°$$

Therefore, $R_{BR} = 0.517 \cos 40.4° = 0.394\ \Omega$

$= R_1 + R_2$

As, $R_1 = 0.243\ \Omega,$

$R_2 = 0.394 - 0.243 = 0.151\ \Omega$

Reactance value (at 15 Hz) $= 0.517 \times \sin 40.4°$

$= 0.335\ \Omega$

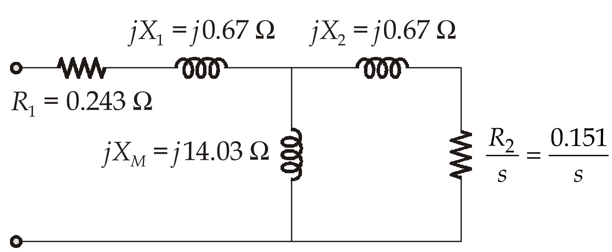
The equivalent reactance at 60 Hz

$$X_{BR} = \frac{f_{\text{rated}}}{f_{\text{test}}} \times X'_{BR} = \left(\frac{60}{15}\right) \times 0.335 = 1.34\ \Omega$$

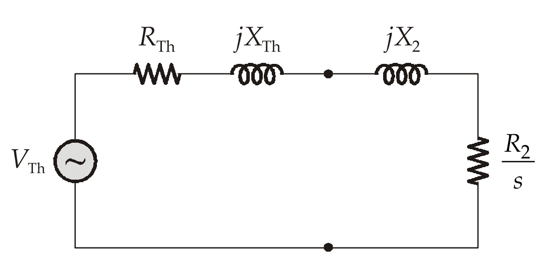
Given, $X_1 = X_2 = \dfrac{X_{LR}}{2} = 0.67\ \Omega$

$$X_m = |Z_{nl}| - X_1 = 14.7 - 0.67 = 14.03\ \Omega$$

Final per phase equivalent circuit will be



$$V_{\text{Th}} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$



$$\approx V_\phi \frac{X_M}{X_1 + X_M} = \frac{208}{\sqrt{3}} \times \frac{j14.03}{j(14.03 + 0.67)} = 114.61\ \text{V}$$

$$Z_{\text{Th}} = jX_m \parallel (R_1 + jX_1)$$

$$= \frac{(j14.03)(0.243 + j0.67)}{(0.243 + j14.03 + j0.67)} = 0.221 + j0.643\ \Omega$$

$$S_{max} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}} = \frac{0.151}{\sqrt{(0.243)^2 + (0.643 + 0.67)^2}}$$

$$= 0.1131 \text{ or } 11.31\%$$

The maximum torque of the motor,

$$T_{max} = \frac{3}{\omega_s} \times \frac{V_{Th}^2 \times \frac{R_2}{S_{max}}}{\left[\left(R_{Th} + \frac{R_2}{S_{max}}\right)^2 + (X_{Th} + X_2)^2\right]}$$

$$\omega_{syn} = \frac{120 f \times 2\pi}{P \times 60} = 188.5 \text{ rad/sec}$$

$$= \frac{3 \times (114.61)^2 \times \frac{0.151}{0.1131}}{(188.5) \times \left[\left(0.221 + \frac{0.151}{0.1131}\right)^2 + (0.643 + 0.67)^2\right]}$$

$$= 67.33 \text{ N-m}$$

Section B : Power Systems-1 + Systems and Signal Processing-2 + Microprocessors-2

Q.5 (a) Solution:

(i)

$$V = 60 \text{ kV (rms)}, \quad g_{max} = 4 \text{ kV/mm (rms)}$$

$$V_1 = \frac{V}{e} = \frac{60}{2.718} = 22.1 \text{ kV}$$

$$r = \frac{V}{e\, g_{max}} = \frac{60}{2.718 \times 4} = 5.5 \text{ mm}$$

Diameter of core $= 2r = 2 \times 5.5 = 11$ mm

Radius of intersheath, $r_1 = \frac{V}{g_{max}} = \frac{60}{4} = 15$ mm

Diameter of intersheath, $d_1 = 2r_1 = 2 \times 15 = 30$ mm

$$V_2 = V - V_1 = 60 - 22.1 = 37.9 \text{ kV}$$

$$R = 1.881 \frac{V}{g_{max}} = 1.881 \times \frac{60}{4} = 28.2 \text{ mm}$$

Minimum overall diameter of the cable,

$$D = 2R = 2 \times 28.2 = 56.4 \text{ mm}$$

(ii) Cable without intersheath

For economic cable size, $\frac{R}{r} = e = 2.718$; $\ln\frac{R}{r} = 1$

$$V = g_{max} r \ln\frac{R}{r} = g_{max} r$$

$$r = \frac{V}{g_{max}} = \frac{60}{4} = 15 \text{ mm}$$

$$\text{Diameter of conductor} = 2r = 2 \times 15 = 30 \text{ mm}$$

$$R = e\,r = 2.718 \times 15 = 40.77 \text{ mm}$$

$$D = 2R = 2 \times 40.77 = 81.54 \text{ mm}$$

Q.5 (b) Solution:

Analytical method:

(i) The maximum power received by the load is

$$P_{rm} = \frac{V_s V_r}{B} - \frac{A V_r^2}{B}\cos(\beta - \alpha)$$

For a short line

$A = 1$, $\alpha = 0$, $B = Z$

$\beta = \tan^{-1}\frac{X}{R}$, $\cos\beta = \frac{R}{Z}$, $\sin\beta = \frac{X}{Z}$

In this problem,

$$R = 10\ \Omega/\text{phase}$$

$$X = 30\ \Omega/\text{phase},$$

$$V_s = V_r = 132 \text{ kV}$$

$$B = Z = R + jX = 10 + j30 = 31.62\angle 71.56°$$

$$B = B\angle\beta,\ B = 31.62\ \Omega,$$

$$\beta = 71.56°$$

$$P_m = \frac{132 \times 132}{31.62} - \frac{1 \times (132)^2}{31.62}\cos\left(71.56^\circ - 0^\circ\right)$$

$$= 551(1 - \cos 71.56°) = 376.7 \text{ MW}$$

(ii) For the given values of V_s and V_r, the power transfer is maximum when $\delta = \beta$, where δ is the angle between V_s and V_r

$$\therefore \quad \delta = \beta = 71.56°$$

(iii) The equation of the receiving-end circuit is

$$(P_r - P_{ro})^2 + (Q_r - Q_{ro})^2 = \rho_r^2$$

$$P_{r0} = -\frac{V_r^2}{Z^2}R = -\frac{(132)^2 \times 10}{(10)^2 + (30)^2} = -174.24\text{MW}$$

$$Q_{r0} = -\frac{V_r^2}{Z^2}X = -\frac{(132)^2 \times 30}{(10)^2 + (30)^2} = -522.72\text{MVAr}$$

$$\rho_r = \frac{V_s V_r}{Z} = \frac{132 \times 132}{31.62} = 551\text{MVA}$$

Putting P_r = 100 MW in the equation of the receiving-end circle we get,

$$(100 + 174.24)^2 + (Q_r + 522.72)^2 = (551)^2$$

$$(Q_r + 522.72)^2 = (551)^2 - (274.24)^2$$

$$Q_r + 522.72 = (303601 - 75207.6)^{1/2}$$

$$Q_r = 477.90 - 522.72 = -44.82 \text{ MVAr}$$

Rating of phase modifier at full load

$$Q_{pm} = P_r \tan\varphi_r - Q_r = 100 \times 0.4843 - (-44.82)$$

$$= 93.25 \text{ MVAr (leading)}$$

Q.5 (c) Solution

$$V(z) = X(z) + \frac{1}{2}z^{-1}V(z)$$

$$V(z) = \frac{X(z)}{1 - \frac{1}{2}z^{-1}}$$

$$Y(z) = 2[3X(z) + V(z)] + 2z^{-1}V(z) = 6X(z) + 2(1 + z^{-1})V(z)$$

$$= \left(6 + \frac{2(1+z^{-1})}{1-0.5z^{-1}}\right)X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{8 - z^{-1}}{1 - 0.5z^{-1}}$$

By taking inverse z-transform, we get,

$$h(n) = 8(0.5)^n u(n) - (0.5)^{n-1} u(n-1)$$

Q.5 (d) Solution:

(i) Given : $$y(n)-\frac{1}{2}y(n-1) = x(n)$$

$$\Downarrow Z.T.$$

$$Y(z)-\frac{1}{2}[z^{-1}Y(z)+y(-1)] = X(z)$$

$$Y(z)\left[1-\frac{1}{2}z^{-1}\right] = X(z)+\frac{1}{2}y(-1)$$

$$Y(z) = \frac{X(z)}{\left[1-\frac{1}{2}z^{-1}\right]}+\frac{\frac{1}{2}}{1-\frac{1}{2}z^{-1}} \quad \text{...(i)}$$

Given : $$x(n) = \left(\frac{1}{3}\right)^n u(n)$$

$$\Downarrow Z.T.$$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} \quad \text{...(ii)}$$

On putting eqn. (ii) in eqn. (i)

$$Y(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}+\frac{1}{2}\times\frac{1}{1-\frac{1}{2}z^{-1}}$$

$$Y(z) = \frac{A}{\left(1-\frac{1}{2}z^{-1}\right)}+\frac{B}{\left(1-\frac{1}{3}z^{-1}\right)}+\frac{1}{2}\cdot\frac{1}{1-\frac{1}{2}z^{-1}}$$

where, $$A = \left.\frac{1}{1-\frac{1}{3}z^{-1}}\right|_{z^{-1}=2} = \frac{1}{1-\frac{1}{3}\times 2} = 3$$

and $$B = \left.\frac{1}{1-\frac{1}{2}z^{-1}}\right|_{z^{-1}=3} = \frac{1}{1-\frac{3}{2}} = -2$$

Therefore, $$X(z) = \frac{3.5}{1-\frac{1}{2}z^{-1}}-\frac{2}{1-\frac{1}{3}z^{-1}}$$

$$\Big\downarrow\!\!\uparrow \text{I.Z.T.}$$

$$x(n) = 3.5\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^n u(n);\ \text{ROC } |z| > \frac{1}{2}$$

(ii) Given :

$$y(n) + \frac{1}{2}y(n-1) + \frac{1}{4}y(n-2) = 0$$

$$\Big\downarrow\!\!\uparrow \text{Z.T.}$$

$$Y(z) + \frac{1}{2}[z^{-1}Y(z) + y(-1)] + \frac{1}{4}[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)] = 0$$

$$Y(z)\left[1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right] = -\frac{1}{2}y(-1) - \frac{1}{4}z^{-1}y(-1) - \frac{1}{4}y(-2)$$

$$= -\frac{1}{2} - \frac{1}{4}z^{-1} - \frac{1}{4}$$

$$Y(z) = \frac{-\left[\frac{3}{4} + \frac{1}{4}z^{-1}\right]}{1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}$$

Using long division method

$$\begin{array}{l}
1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} \;\Big)\; \frac{3}{4} + \frac{1}{4}z^{-1} \;\Big(\; +\frac{3}{4} - \frac{1}{8}z^{-1} - \frac{1}{8}z^{-2} + \frac{3}{32}z^{-3}\ldots\ldots \\
\qquad \frac{3}{4} + \frac{3}{8}z^{-1} + \frac{3}{16}z^{-2} \\
\hline
\qquad -\frac{1}{8}z^{-1} - \frac{3}{16}z^{-2} \\
\qquad -\frac{1}{8}z^{-1} - \frac{1}{16}z^{-2} - \frac{1}{32}z^{-3} \\
\hline
\qquad\qquad -\frac{1}{8}z^{-2} + \frac{1}{32}z^{-3} \\
\qquad\qquad -\frac{1}{8}z^{-2} - \frac{1}{16}z^{-3} - \frac{1}{32}z^{-4} \\
\hline
\qquad\qquad\qquad \frac{3}{32}z^{-3} + \frac{1}{32}z^{-4} \\
\qquad\qquad\qquad \frac{3}{32}z^{-3} + \frac{3}{64}z^{-4} + \frac{3}{128}z^{-5} \\
\hline
\qquad\qquad\qquad\qquad -\frac{1}{64}z^{-4} - \frac{3}{128}z^{-5}
\end{array}$$

Therefore,

$$Y(z) = -\frac{3}{4}+\frac{1}{8}z^{-1}+\frac{1}{8}z^{-2}-\frac{3}{32}z^{-3}.....$$

$$\Downarrow\Uparrow I.Z.T.$$

$$y(n) = -\frac{3}{4}\delta(n)+\frac{1}{8}\delta(n-1)+\frac{1}{8}\delta(n-2)-\frac{3}{32}\delta(n-3)-\frac{1}{64}\delta(n-4).....$$

Q.5 (e) Solution:

Memory mapping of an I/O device	I/O mapping of an I/O device
1. 16-bit address are provided for input/output devices.	1. 8-bit address are provided for input/output devices.
2. The devices are accessed by memory read or memory write cycles.	2. The devices are accessed by I/O read or I/O write cycle. During these cycles, the 8-bit address is available on both low order address lines and high order address lines.
3. The input/output ports or peripherals can be treated like memory locations and so all instruction related to memory can be used for data transfer between the I/O device and the processor.	3. Only IN and OUT instructions can be used for data transfer between the I/O device and the processor.
4. In memory mapped ports, the data can be moved from any register to the ports and vice versa.	4. In I/O mapped ports, the data transfer can take place only between the accumulator and the ports.
5. When memory mapping is used for I/O devices the full memory address space cannot be used for addressing memory. Hence memory mapping is useful only for small systems where memory requirement is less.	5. When I/O mapping is used for I/O devices, the full memory address space can be used for addressing the memory. Hence it is suitable for systems which requires a large memory capacity.
6. In memory mapped I/O devices, a large number of I/O ports can be interfaced.	6. In I/O mapping, only 256 port ($2^8 = 256$) can be interfaced.
7. For accessing memory mapped devices the processor executes the memory read or write cycle. During this cycle $IO/\overline{M}$ is asserted low ($IO/\overline{M} = 0$)	7. For accessing the I/O mapped devices the processor executes the I/O read or write cycle during this cycle, $IO/\overline{M}$ is asserted high ($IO/\overline{M} = 1$)

Q.6 (a) (i) Solution:

$$\text{Feeder relay current, } I_{RF} = \frac{5000}{400/5} = 62.5 \text{ A}$$

$$\text{Feeder relay pick-up current} = 5 \times \frac{125}{100} = 6.25 \text{ A}$$

$$\text{Feeder relay PSM} = \frac{62.5}{6.25} = 10$$

From given data, the operating time corresponding to PSM of 10 is 3 seconds

Actual operating time for feeder relay,

$$= 3 \times \text{TMS} = 3 \times 0.3 = 0.9 \text{ sec}$$

$$\text{Transformer overload current} = 1.3 \times \frac{20 \times 10^6}{\sqrt{3} \times 11000} = 1365 \text{ A}$$

$$\text{Transformer relay current, } I_{RT} = \frac{1365}{1000/5} = 6.825 \text{ A}$$

Transformer relay pick up current = $PS \times 5$

Where PS means plug setting, since the transformer relay must not operate to overload current,

$$\text{plug setting (PS)} > \frac{6.825}{5}$$

$$\Rightarrow \quad PS = 1.365 \text{ (or) } 136.5\%$$

The plug setting are restricted to standard values in steps of 25%, so the nearest values but higher than 136.5% is 150%.

So transformer relay pick up current

$$= \frac{150}{100} \times 5 = 7.5 \text{ A}$$

Transformer relay current corresponding to fault current of 5000 A

$$= \frac{5000}{1000/5} = 25 \text{ A}$$

$$\text{Transformer relay PSM} = \frac{25}{7.5} = 3.33$$

Given, time corresponding to PSM of 3.33 is 5.6 seconds

Time setting for transformer relay = Actual operating time of feeder relay + time grading margin

$$= 0.9 + 0.5 = 1.4 \text{ seconds}$$

$$\text{Time setting multiplier} = \frac{1.4}{5.6} = 0.25$$

Q.6 (a) (ii) Solution:

$$\text{Industrial load, } P_1 = 4000 \text{ kW}$$

$$\text{pf of industrial load, } \cos\phi_1 = 0.8 \text{ lag}$$

$$\text{Phase angle, } \phi_1 = \cos^{-1}(0.8) = 36.87° \text{ lag}$$

$$\text{Motor load, } P_2 = \frac{\text{Additional load}}{\text{Motor efficiency}} = \frac{1103.25}{0.8} = 1379.06 \text{ kW}$$

Total load on the system including additional load supplied by the synchronous motor

$$P = P_1 + P_2$$

$$= 4000 + 1379.06$$

$$= 5379.06 \text{ kW}$$

$$\text{Improved power factor, } \cos\phi_2 = 0.95 \text{ lagging}$$

$$\text{Phase angle } \phi_2 = \cos^{-1} 0.95$$

$$= 18.195° \text{ lagging}$$

Reactive kVAR supplied by the synchronous motor

= reactive kVAR drawn by industrial load of 4000 kW – Reactive kVAR drawn by the combined load of 5379.06 kW

$$= 4000 \tan 36.87° - 5379.06 \tan 18.195°$$

$$= 1232 \text{ kVAR}$$

$$\text{kVA rating of the motor} = \sqrt{P_2^2 + (\text{kVAR supplied by motor})}$$

$$= \sqrt{(1379.06)^2 + (1232)^2} = 1849 \text{ kVA}$$

Power factor of synchronous motor

$$= \frac{1379.06}{1849} = 0.7458 \text{ (leading)}$$

Q.6 (b) Solution:

(i) To check whether the given system is linear on non-linear. We assume,

$$y_1(n) = \begin{cases} x_1(n)\,; & n \geq 1 \\ 0\,; & n = 0 \\ x_1(n+1)\,; & n \leq -1 \end{cases} \quad \text{...(i)}$$

and

$$y_2(n) = \begin{cases} x_2(n)\,; & n \geq 1 \\ 0\,; & n = 0 \\ x_2(n+1)\,; & n \leq -1 \end{cases} \quad \text{...(ii)}$$

Let, $x_3'(n) = ax_1(n) + bx_2(n)$ ...(iii)

$y_3'(n) = T\{x_3'(n)\}$ ...(iv)

Here, 'T' represents the transformation from $x(n)$ to $y(n)$.

Now, from equation (iv) and (iii), we get

$$y_3'(n) = T\{ax_1(n) + bx_2(n)\}$$

$$y_3'(n) = ay_1(n) + by_2(n) \quad \text{...(v)}$$

Since, $$y_3'(n) = \begin{cases} x_3'(n)\,; & n \geq 1 \\ 0\,; & n = 0 \\ x_3'(n+1)\,; & n \leq -1 \end{cases}$$

Now, $x_3'(n+1) = ax_1(n+1) + bx_2(n+1)$

and let, $y_3(n) = ay_1(n) + by_2(n)$

Since, $y_3(n) = y_3'(n)$

So, the system is linear.

(ii) Since, $y(n) = x(n+1)$ for $n \leq -1$

So, we can say $y(n)$ depends on the future value of input sequence, so we may say that given system is not causal.

(iii) As given, $$y(n) = \begin{cases} x(n)\,; & n \geq 1 \\ 0\,; & n = 0 \\ x(n+1)\,; & n \leq -1 \end{cases}$$

So, $$y(n-k) = \begin{cases} x(n-k)\,; & n-k \geq 1 \\ 0\,; & n-k = 0 \\ x(n-k+1)\,; & n-k \leq -1 \end{cases}$$

Now, $y(n, k) = T[x(n, k)]$

$y_1(n, k) = T[x_1(n, k)]$

where, 'T' is the transformation from $x(n)$ to $y(n)$.

Let, $x_1(n, k) = x(n-k)$ i.e., shifted by 'k' unit

$y_1(n, k) = T[x(n-k)]$

$$= \begin{cases} x(n-k)\,; & n \geq 1 \\ 0\,; & n = 0 \\ x(n-k+1)\,; & n \leq -1 \end{cases}$$

Because, $y_1(n, k) \neq y(n - k)$

So, the system is time-variant.

(iv) For this system when the input is bounded, the output is also bounded. So, the system is stable.

Q.6 (c) Solution:

(i)

$$X(z) = \log(1 + az^{-1})\ ;\ |z| > |a|$$

$$\frac{dX(z)}{dz} = \frac{-az^{-2}}{1+az^{-1}}$$

$$-z\frac{dX(z)}{dz} = az^{-1}(1 - az^{-1} + a^2z^{-2} - a^3z^{-3} + a^4z^{-4} - a^5z^{-5} + ...)$$

$$= az^{-1} - a^2z^{-2} + a^3z^{-3} - a^4z^{-4} + a^5z^{-5} - a^6z^{-6} + ...$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} a^n z^{-n} = \sum_{n=-\infty}^{\infty} \left[(-1)^{n+1} a^n u(n-1)\right] z^{-n}$$

From the properties of z-transform,

$$x(n) \overset{z}{\longleftrightarrow} X(z)$$

$$nx(n) \overset{z}{\longleftrightarrow} -z\frac{dX(z)}{dz}$$

So,
$$nx(n) = (-1)^{n+1} a^n u(n-1)$$

$$x(n) = \frac{1}{n}(-1)^{n+1} a^n u(n-1)$$

(ii)

$$X(z) = \frac{z^3 - 10z^2 - 4z + 4}{2z^2 - 2z - 4}$$

$$2z^2 - 2z - 4\overline{\big)\, z^3 - 10z^2 - 4z + 4\,}\big(\, 0.5z - 4.5$$

$$\underline{z^3 - z^2 - 2z}$$

$$-9z^2 - 2z + 4$$

$$\underline{-9z^2 + 9z + 18}$$

$$-11z - 14$$

So, $X(z)$ can be written as,

$$X(z) = 0.5z - 4.5 - \frac{11z+14}{2z^2 - 2z - 4} = 0.5z - 4.5 - \frac{5.5z+7}{z^2 - z - 2}$$

$$= 0.5z - 4.5 - \frac{5.5z + 7}{(z+1)(z-2)}$$

Using the partial fraction expansion, we get,

$$X(z) = 0.5z - 4.5 + \frac{0.5}{z+1} - \frac{6}{z-2}$$

$$= 0.5z - 4.5 + \frac{0.5z^{-1}}{1-(-1)z^{-1}} - \frac{6z^{-1}}{1-2z^{-1}}$$

Given that the ROC of $X(z)$ is $|z| > 2$.

So, the inverse z-transform of $X(z)$ yields,

$$x(n) = 0.5\delta(n+1) - 4.5\delta(n) + 0.5(-1)^{(n-1)}u(n-1) - 6(2)^{(n-1)}u(n-1)$$

Q.7 (a) Solution:

Phase voltage at the receiving end,

$$V_r = 60 \text{ kV} = 60 \times 10^3 \text{ V}$$

$$\text{Power per phase} = \frac{1}{3} \times 36 \text{ MW} = 12 \times 10^6 \text{ W}$$

Therefore,

The receiving-end current, $I_r = \dfrac{12 \times 10^6}{60 \times 10^3 \times 0.8} = 250 \text{ A}$

Taking V_r as the reference phasor,

$$V_r = V_r + j0$$

$$\cos\phi_r = 0.8,$$

$$\phi_r = 36.8699°$$

Resistance per phase, $R = 2.5\ \Omega$

Inductive reactance per phase,

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.416\ \Omega$$

Series impedance per phase, $Z = R + jX = 2.5 + j31.4 = 31.499\angle 85.448°\ \Omega$

Shunt admittance per phase, $Y = 2\pi fC = 2 \times \pi \times 50 \times 0.25 \times 10^{-6} = 7.854 \times 10^{-5}$ S

$$Y = 0 + j7.854 \times 10^{-5} = 7.854 \times 10^{-5}\angle 90° \text{ S}$$

Nominal T-model:

$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z\left(1+\frac{YZ}{2}\right)$$

$$C = Y$$

$$A = D = 0.9988\angle 5.6 \times 10^{-3°}$$

$$B = 31.48\angle 85.45°\ \Omega$$

$$C = j7.854 \times 10^{-5}\ \mho$$

$$V_s = A\,V_R + B\,I_R$$

$$V_s = (0.9988\angle 5.6 \times 10^{-3})\,(60 \times 10^3\angle 0°) + (31.48\angle 85.45)\,(250\angle -36.87°)$$

$$= 65401.85\angle 5.18°\ \text{Volt/phase}$$

$$I_s = CV_R + DI_R$$

$$= (j7.854 \times 10^{-5} \times 60 \times 10^3) + (0.9988\angle 5.18 \times 10^{-3})\,(250\angle -36.87°)$$

$$I_s = 246.90\angle -35.99°\ \text{A}$$

Power factor, $\phi_s = \cos(5.18 + 35.99) = 0.7527$ lagging

Active power, $P_s = 3 \times 65401.85 \times 246.90 \times 0.7527 = 36.47$ MW

Reactive power, $Q_s = 3\times 35401.85 \times 246.90 \times \sin(5.18 + 35.99)$

$$= 31.89\ \text{MVAR}$$

$$\eta = \frac{P_R}{P_s} = \frac{36}{36.47}\times 100 = 98.71\%$$

$$\text{V.R.} = \frac{V_s - V_R}{V_R}\times 100 = \frac{65401.85 - 6000}{6000}\times 100 = 9\%$$

Nominal Π model:

$$A = D = \left(1+\frac{YZ}{Z}\right)$$

$$B = Z$$

$$C = Y\left(1+\frac{YZ}{Y}\right)$$

$$A = D = 0.9986\angle 5.6 \times 10^{-3}$$

$$B = 31.499\angle 85.45°\ \Omega$$

$$C = 7.85 \times 10^{-5}\angle 90°\ \mho$$

$$V_S = AV_R + BI_R$$

$$= (0.9988\angle 5.6 \times 10^{-3})\,(60 \times 10^3\angle 0°) + (31.499\angle 85.45)\,(250\angle -36.87°)$$

$$V_S = 65403.55\angle 5.185° \text{ volt/phase}$$

$$I_s = CV_R + DI_R$$

$$= (j7.854 \times 10^{-5} \times 60 \times 10^3) + (0.9988\angle 5.18 \times 10^{-3})\,(250\angle -36.87°)$$

$$I_s = 246.90\angle -35.98° \text{ A}$$

Power factor, $\phi_s = \cos(5.185 + 35.98) = 0.7528$ lagging

Active power, $P_s = 3 \times 65403.55 \times 246.895 \times 0.7528 = 36.46$ MW

Reactive power, $Q_s = 3 \times 65403.55 \times 246.895 \times \sin(5.185 + 35.98)$

$= 31.88$ MVAR

$$\eta = \frac{P_R}{P_s} = \frac{36}{36.46} \times 100 = 98.73\%$$

$$\text{V.R.} = \frac{V_s - V_R}{V_R} \times 100 = \frac{65403.55 - 6000}{6000} \times 100 = 9\%$$

Q.7 (b) Solution:

Given that, $f_c = 1$ kHz and $f_s = 5$ kHz

So, $$\omega_c = 2\pi f_c T = \frac{2\pi f_c}{f_s} = \frac{2\pi \times 1000}{5000} = \frac{2\pi}{5}$$

Therefore, $$H_d(e^{j\omega}) = \begin{cases} 1\,; & |\omega| \le \frac{2\pi}{5} \\ 0\,; & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-2\pi/5}^{2\pi/5} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-2\pi/5}^{2\pi/5} = \frac{\sin\left(\frac{2\pi}{5}n\right)}{\pi n}$$

The rectangular window, for the length of impulse response of 7, can be given by,

$$w_R(n) = \begin{cases} 1\,; & -3 \le n \le 3 \\ 0\,; & \text{otherwise} \end{cases}$$

The filter coefficients can be given by,

$$h(n) = h_d(n)\, w_R(n)$$

$$h(0) = h_d(0) w_R(0) = \lim_{n \to 0} \frac{\sin \frac{2\pi}{5} n}{\pi n} = \frac{2}{5} = 0.40$$

$$h(1) = h(-1) = \frac{\sin\frac{2\pi}{5}}{\pi} = 0.3027$$

$$h(2) = h(-2) = \frac{\sin\frac{4\pi}{5}}{2\pi} = 0.0935$$

$$h(3) = h(-3) = \frac{\sin\frac{6\pi}{5}}{3\pi} = -0.0624$$

$$h(n) = \{-0.0624, 0.0935, 0.3027, \underset{\uparrow}{0.40}, 0.3027, 0.0935, -0.0624\}$$

The transfer function of the filter is,

$$H(z) = -0.0624z^3 + 0.0935z^2 + 0.3027z + 0.40 + 0.3027z^{-1} + 0.0935z^{-2} - 0.0624z^{-3}$$

But this filter is non-causal, which is practically non-realizable. The transfer function of the practically realizable filter can be given by,

$$H'(z) = z^{-3}H(z)$$

$$= -0.0624 + 0.0935z^{-1} + 0.3027z^{-2} + 0.40z^{-3} + 0.3027z^{-4} + 0.0935z^{-5} - 0.0624z^{-6}$$

Q.7 (c) (i) Solution:

'ROTATE' command group has four instructions; two are for rotating left and two are for rotating right.

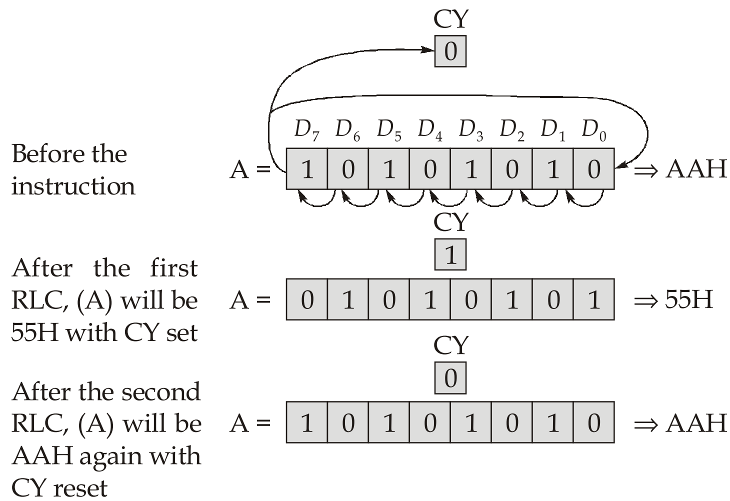
- RLC : Rotate Accumulator Left
- RAL : Rotate Accumulator Left Through Carry
- RRC : Rotate Accumulator Right
- RAR : Rotate Accumulator Right Through Carry

The difference between these instructions are illustrated in the following examples.

1. **RLC : Rotate Accumulator Left**
   - Each bit is shifted to the adjacent left position. Bit D_7 becomes D_0.
   - CY flag is modified according to bit D_7.

   Example: Assume the accumulator contents are AAH and CY = 0.

   Figure below shows the contents of the accumulator and the CY flag after the execution of the RLC instruction twice. The first RLC instruction shifts each bit to the left by one position, places bit D_7 in bit D_0 and sets the CY flag because D_7 = 1. The accumulator byte AAH becomes 55H after the first rotation. In the second rotation, the byte is again AAH, and the CY flag is reset because bit D_7 of 55H is 0.



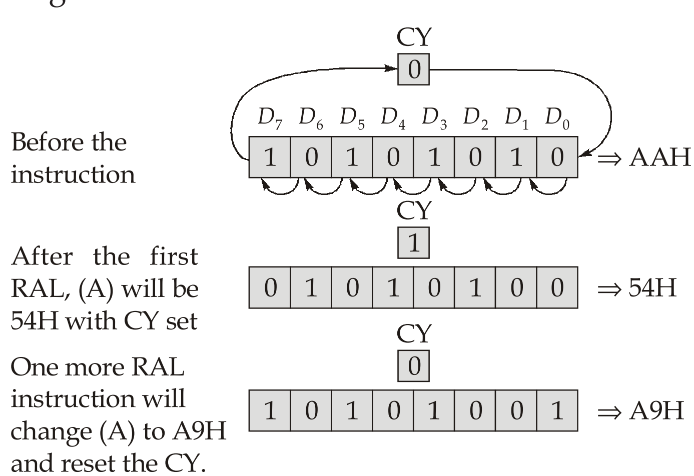
2. RAL : Rotate Accumulator Left Through Carry

- Each bit is shifted to the adjacent left position. Bit D_7 becomes the carry bit and the carry bit is shifted into D_0 .
- The Carry flag is modified according to bit D_7.

Example: Assume the accumulator contents are AAH and CY = 0.

Figure below show the contents of the accumulator and the CY flag after the execution of the RAL instruction twice. The first RAL instruction shifts each bit to the left by one position, places bit D_7 in the CY flag, and the C bit in bit D_0. This is a 9-bit rotation; CY is assumed to be the ninth bit of the accumulator. The accumulator type AAH becomes 54H after the first rotation. In the second rotation, the byte becomes A9H, and the CY flag is reset.

By these two examples, it can be seen that the primary difference between these two instruction is that the instruction RLC rotates through eight bits, and the instruction RAL rotates through nine bits.



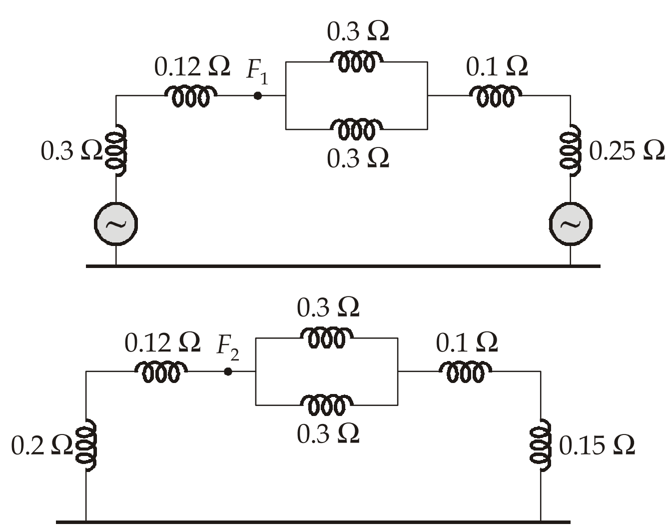
3. **RRC : Rotate Accumulator Right**
   - Each bit is shifted to the adjacent right position. Bit D_0 becomes D_7.
   - The Carry flag is modified according to bit D_0.
4. **RAR : Rotate Accumulator Right Through Carry**
   - Each bit is shifted right to the adjacent position. Bit D_0 becomes the carry bit and the carry bit is shifted into D_7.

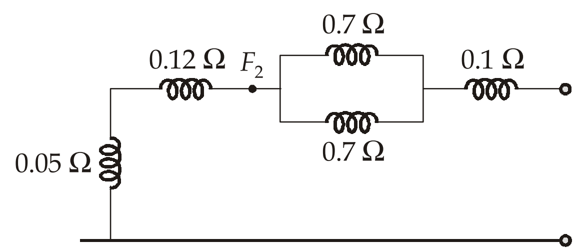
Q.7 (c) (ii) Solution:

Program given below transfers sixteen bytes of data stored in memory to another location starting from XX70H

```
START :   LXI H, XX50 H   ; Set up HL as a pointer for source memory.
          LXI D, XX70 H   ; Set up DE as a pointer for destination memory.
          MVI B, 10 H     ; Set up B to count 16 bytes
NEXT :    MOV A, M        ; Get data byte from source memory.
          STAX D          ; Store data byte at destination.
          INX H           ; Point HL to next source location.
          INX D           ; Point DE to next destination
          DCR B           ; One transfer complete
          JNZ NEXT        ; If counter is not 0, go back to transfer next byte
          HLT             ; End
```

Q.8 (a) Solution:





From the positive-sequence network, the equivalent impedance upto the point of the fault is given by

$$Z_1 = j\left[(0.3+0.12) \,||\, \left(0.25+0.1+\frac{0.3}{2}\right)\right]$$

$$= j[(0.42) \,||\, (0.5)] = j\frac{0.42\times 0.5}{0.42+0.5} = j0.22826 \text{ p.u.}$$

From the negative-sequence network, the equivalent impedance upto F is given by

$$Z_2 = j\left[(0.2+0.12) \,||\, \left(0.15+0.1+\frac{0.3}{2}\right)\right]$$

$$= j(0.32) \,||\, (0.4) = j\frac{0.32\times 0.4}{0.32+0.4} = j0.1778 \text{ p.u.}$$

From the zero-sequence network, the equivalent impedance upto F is given by

$$Z_0 = j(0.05 + 0.12) = j0.17 \text{ p.u.}$$

(i) LLG fault at F

If phase a is assumed to be the reference phasor and phases b and c are shorted at the fault, then from equation

$$I_{a1} = \frac{V_f}{Z_{a1}+\frac{Z_{a0}Z_{a2}}{Z_{a0}+Z_{a2}}} = \frac{1\angle 0^\circ}{j\left(0.22826+\frac{0.17\times 0.1778}{0.17+0.1778}\right)}$$

$$= -j3.1729 \text{ p.u.} = 3.1729\angle -90^\circ \text{ p.u.}$$

If we put $Z_f = 0$ and $Z_g = 0$ then by current division rule

$$I_{a0} = -I_{a1}\frac{Z_{a2}}{Z_{a0}+Z_{a2}} = j\left(3.1729\times\frac{0.1778}{0.17+0.1778}\right)$$

$$= j1.622 \text{ p.u.} = 1.622\angle 90^\circ \text{ p.u.}$$

$$I_{a2} = -I_{a1}\frac{Z_{a0}}{Z_{a0}+Z_{a2}} = j\left[3.1729\times\frac{0.17}{0.17+0.1778}\right]$$

$$= j1.551 \text{ p.u.} = 1.551\angle 90^\circ \text{ p.u.}$$

Check

$$I_a = I_{a0} + I_{a1} + I_{a2} = j1.622 - j3.1729 + j1.551 \approx 0$$

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$
$$= j1.622 + (1\angle 240°)(3.1729\angle -90°) + (1\angle 120°)(1.551\angle 90°)$$
$$= j1.622 + 3.1729\angle 150° + 1.551\angle 210°$$
$$= j1.622 - 2.7478 + j1.5864 - 1.3432 - j0.7755$$
$$= -4.091 + j2.4729$$

$$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$$
$$= j1.622 + (1\angle 120°)(3.1729\angle -90°) + (1\angle 240°)(1.551\angle 90°)$$
$$= j1.622 + 2.7478 + j1.5864 + 1.3432 - j0.7755$$
$$= 4.091 + j2.4729$$
$$|I_b| = |I_c| = \sqrt{(4091)^2 + (24729)^2} = 4.78 \text{ p.u.}$$

(ii) LL fault at *F*

If the line-to-line fault is between phases *b* and *c*, then

$$I_{a1} = \frac{V_f}{Z_{a1} + Z_{a2}} = \frac{1\angle 0^{\circ}}{j(0.22826 + 0.1778)}$$
$$= -j2.4627 \text{ p.u.} = 2.4627\angle -90° \text{ p.u.}$$

The phase *a* negative-sequence current is given by

$$I_{a2} = -I_{a1} = j2.4627 \text{ p.u.} = 2.4627\angle 90° \text{ p.u.}$$

The phase *a* fault current

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 - j2.4627 + j2.4627 = 0$$

The phase *b* fault current

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$
$$= 0 + (1\angle 240°)(2.4627\angle -90°) + (1\angle 120°)(2.4627\angle 90°)$$
$$= 2.4627\angle 150° + 2.4627\angle 210°$$
$$= -2.133 + j1.231 - 2.133 - j1.231 = -4.266 \text{ p.u.}$$

The phase *c* fualt current

$$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$$
$$= 0 + (1\angle 120°)(2.4627\angle -90°) + (1\angle 240°)(2.4627\angle 90°)$$

$$= 2.4627\angle 30° + 2.4627\angle 330°$$

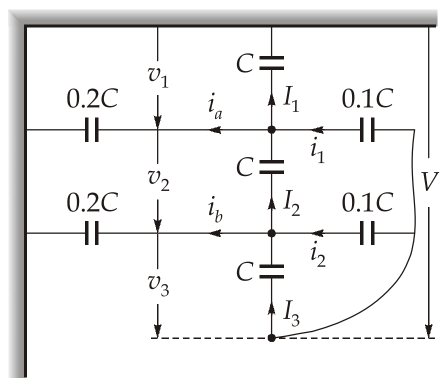
$$= 2.133 + j1.231 + 2.133 - j1.231 = 4.266 \text{ p.u.}$$

Thus, it is calcualted that

$$I_b = -I_c = -4.266 \text{ p.u.}$$

Q.8 (b) Solution:

Let C represent the self capacitance of each unit. The capacitances of the link pins to earth and to the line will, therefore, be $0.2C$ and $0.1C$ respectively. The capacitances, voltages and currents are shown in below,



We have,

$$I_1 = v_1(j\omega C), \qquad I_a = v_1(j\omega)(0.2C)$$

$$i_1 = (V - v_1)\, j\omega(0.1\, C)$$

$$I_2 = I_1 + I_a - i_1$$

$$= v_1\, j\omega C + 0.2v_1\; j\omega C - 0.1(V - v_1)\, j\omega C$$

$$= j\omega C(1.3v_1 - 0.1V)$$

$$v_2 = \frac{I_2}{j\omega C} = 1.3v_1 - 0.1V$$

$$I_b = (v_1 + v_2) \times 0.2\, j\omega C$$

$$= (v_1 + 1.3v_1 - 0.1\; V) \times 0.2j\omega C = j\omega C(0.46v_1 - 0.02\text{ V})$$

$$i_2 = v_3 \times 0.1\, j\omega C$$

$$\therefore \qquad I_3 = I_2 + I_b - i_2$$

$$= j\omega C(1.3v_1 - 0.1V) + j\omega C(0.46v_1 - 0.02V) - 0.1j\omega C\, v_3$$

$$= j\omega C(1.76v_1 - 0.12\; V - 0.1v_3)$$

$$v_3 = \frac{I_3}{j\omega C} = 1.76v_1 - 0.12V - 0.1v_3$$

$$v_3 = \frac{1.76v_1}{1.1} - \frac{0.12V}{1.1} = 1.6v_1 - 0.109\text{ V}$$

Also, $v_1 + v_2 + v_3 = V$

$$v_1 + 1.3v_1 - 0.1V + 1.6v_1 - 0.109V = V$$

$$3.9v_1 = 1.209\text{ V}$$

$$v_1 = \frac{1.209}{3.9}V = 0.31\text{ V}$$

$$v_2 = 1.3v_1 - 0.1V = 1.3 \times 0.31V - 0.1V = 0.303\text{ V}$$

$$v_3 = 1.6v_1 - 0.109V = 1.6 \times 0.31V - 0.109V = 0.387\text{ V}$$

$$\text{String efficiency} = \frac{V}{3v_3} = \frac{V}{3 \times 0.387V} = 0.8613\text{ p.u.}$$

With the grading ring

The capacitance of the lower link pin now becomes 0.35C instead of 0.1C,

Therefore, $i_2 = v_3 \times 0.35\, j\omega C$

and $I_3 = I_2 + I_b - i_2 = j\omega C\,(1.76\,v_1 - 0.12V - 0.35v_3)$

$$v_3 = \frac{I_3}{j\omega C} = 1.76v_1 - 0.12V - 0.35v_3$$

$$v_3 = \frac{1.76}{1.35}v_1 - \frac{0.12}{1.35}V = 1.3037v_1 - 0.0389\text{ V}$$

Again, $v_1 + v_2 + v_3 = \text{V}$

$$v_1 + 1.3v_1 - 0.1V + 1.3037v_1 - 0.0889\text{V} = \text{V}$$

$$v_1(1 + 1.3 + 1.3037) = (1 + 0.1 + 0.0889)\text{V}$$

$$v_1 = \frac{1.1889}{3.6037} = 0.3299\text{ V}$$

$$v_2 = 1.3v_1 - 0.1V = 0.3288\text{V}$$

$$v_3 = V - (v_1 + v_2) = V - (0.3299 + 0.32887)\text{V}$$

$$= 0.34123\text{V}$$

It is found that v_1 , v_2 and v_3 are practically equal to each other, that is, the voltage distribution becomes practically uniform,

$$\text{String efficiency} = \frac{V}{3v_3} \times 100 = 97.68\%$$

Q.8 (c) (i) Solution:

1. **PROM (Programmable Read-Only Memory) :** This memory has nichrome or polysilicon wires arranged in a matrix; these wires can be functionally viewed as diodes or fuses. This memory can be programmed by the user with a special PROM programmer that selectively burns the fuses according to the bit pattern to be stored. The process is known as "burning the PROM" and the information stored is permanent.
2. **EPROM (Erasable Programmable Read-Only Memory) :** This memory stores a bit by charging the floating gate of an FET. Information is stored by using an EPROM programmer, which applies high voltages to charge the gate. All the information can be erased by exposing the chip to ultraviolet light through its quartz window, and the chip can be reprogrammed. Because the chip can be reused many times, this memory is ideally suited for product development, experimental projects, and college laboratories. The disadvantages of EPROM are (1) is must be taken out of the circuit to erase it, (2) the entire chip must be erased, and (3) the erasing process takes 15 to 20 minutes.
3. **EE-PROM (Electrically Erasable PROM) :** This memory is functionally similar to EPROM, except that information can be altered by using electrical signals at the register level rather than erasing all the information. This has an advantage in field and remote control applications. In microprocessor system, software update is a common occurrence. If EE-PROMs are used in the systems, they can be updated from a central computer by using a remote link via telephone lines. Similarly, in a process control where timing information needs to be changed, it can be changed by sending electrical signals from a central place. This memory also includes a Chip Erase mode, whereby the entire chip can be erased in 10 ms vs 15 to 20 min. to erase an EPROM. However, this memory is expensive compared to EPROM or flash memory.
4. **MASKED ROM :** In this ROM, a bit pattern is permanently recorded by the masking and metalization process. Memory manufacturers are generally equipped to do this process. It is an expensive and specialized process, but economical for large production quantities.
5. **CONTROL BUS :** The control bus comprises of various single lines that carry synchronization signals. The MPU uses such lines to perform the third function : providing timing signals.

The MPU generates specific control signals for every operation (such as Memory Read or I/O Write) it performs. These signals are used to identify a device type with which the MPU intends to communicate.

To communicate with a memory - for example, to read an instruction from a memory location - the MPU place the 16-bit address on the address bus. The address on the bus is decoded by an external logic circuit and the memory location is identified. The MPU sends a pulse called Memory Read as the control signal. The pulse activates the memory chip, and the contents of the memory location (8-bit data) are placed on the data bus and brought inside the microprocessor.

Q.8 (c) (ii) Solution:

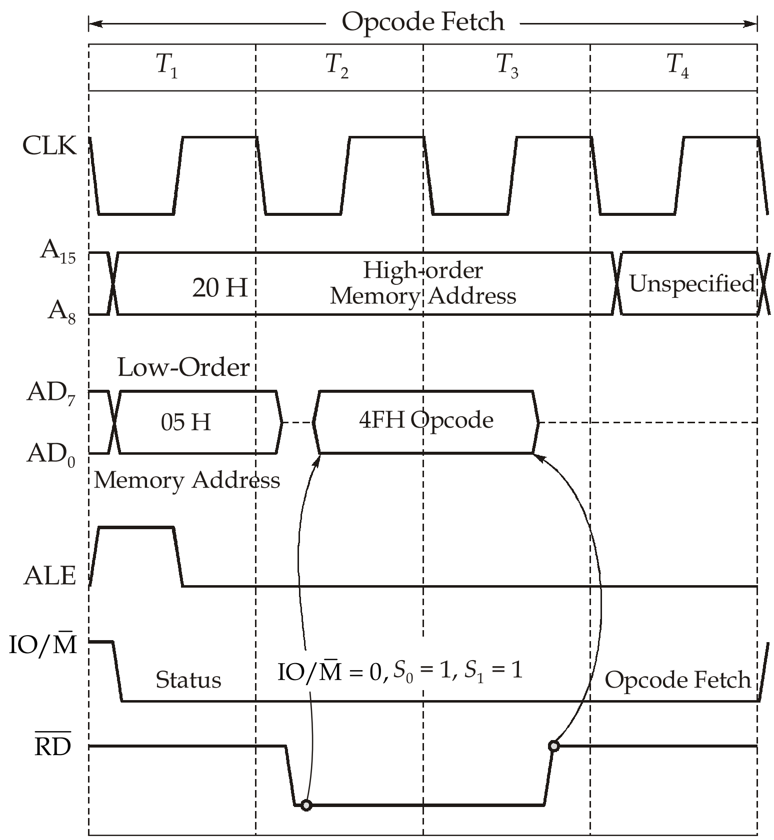
Data Flow from Memory of the MPU:

Below figure shows the timing of how a data byte is transferred from memory to the MPU; it shows for different groups of signals in relation to the system clock. The address bus and data bus are shown as two parallel lines. This is commonly used practice to represent logic levels of groups of lines; some lines are high and others are low. The crossover between the lines indicates that a new byte (information) is placed on the bus, and a dashed straight line indicates the high impedance state. To fetch the byte, the MPU performs the following steps:

- The microprocessor places the 16-bit memory address from the program counter (PC) on the address bus.

  Below figure shows that at T_1 the high-order memory address 20H is placed on the address lines A_{15} - A_8, the low-order memory address 05H is placed on the bus AD_7 - AD_0, and the ALE signal goes high. Similarly, the status signal $IO/\overline{M}$ goes low, indicating that this is memory-related operation.
- The control unit sends the control signal $\overline{RD}$ to enable the memory chip. The control signal $\overline{RD}$ is sent out during the clock period T_2, thus enabling the memory chip. The $\overline{RD}$ signal is active during two clock periods.
- The byte from the memory location is placed on the data bus.

  When the memory is enabled, the instruction byte (4FH) is placed on the bus AD_7 - AD_0 and transferred to the microprocessor. The $\overline{RD}$ signal causes 4FH to be placed on bus AD_7 - AD_0 (shown by the arrow), and when $\overline{RD}$ goes high, it cause the bus to go into high impedance.



- The byte is placed in the instruction decoder of the microprocessor and the task is carried out according to the instruction. The machine code or the byte (4FH) is decoded by the instruction decoder, and the contents of the accumulator are copied into register C. This task is performed during the period T_4.

❍❍❍❍