2020

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WORKBOOK

Detailed Explanations of Try Yourself *Questions*

Electronics Engineering

Signals & Systems

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1 Introduction

Detailed Explanation of Try Yourself Questions

T1. Sol.

$$y_2(t) = [x_1(t)\cos\omega_c t + \cos\omega_c t] + [x_1(t)\cos\omega_c t + k\cos\omega_c t]$$
$$= 2x_1(t)\cos\omega_c t + (1+k)\cos\omega_c t$$

For system to be linear, $1 + k = 0 \Rightarrow k = -1$

T2. Sol.

$$\text{Energy of } x_1[n] = \sum_{h=-\infty}^{\infty} |x_1[n]|^2$$

$$= \sum_{h=-\infty}^{\infty} \alpha^2 \left(\frac{1}{4}\right)^n = \alpha^2 \cdot \frac{1}{1-\frac{1}{4}} = \alpha^2 \cdot \frac{4}{3}$$

Energy of $x_2[n] = 1.5 + 1.5 = 3$

$\Rightarrow \quad \alpha^2 \frac{4}{3} = 3$

$\Rightarrow \quad \alpha^2 = \frac{9}{4}$

$\Rightarrow \quad \alpha = 1.5$

T3. Sol.

(*i*) $\quad y(t) = \text{Even}[x(t)] = \frac{x(t) + x(-t)}{2}$

At $t = -1$, $\quad y(t) = \frac{x(1) + x(-1)}{2}$

$\Rightarrow$ Present depends on future input

$\Rightarrow$ System is non-causal

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(*ii*) $$y(t) = \int_{-\infty}^{t+1} x(t)\,dk$$

At $t = -1$, $y(-1) = \int_{-\infty}^{0} x(k)dk$, that is present output $y(-1)$ depends on future input $x(0)$

$\Rightarrow$ System is non-causal.

T4. Sol.

(*i*) $$y(t) = \frac{x(t)}{\sin t}$$

At $t = \pi$, $y(t) = \frac{x(t)}{0}$ = infinte

$\Rightarrow$ System is unstable.

(*ii*) $$y(t) = \frac{x(t)}{t-1}$$

At $t = 1$, $y(t)$ = infinite for any bounded $x(t)$

$\Rightarrow$ System is unstable.

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2 LTI Systems

Detailed Explanation of Try Yourself Questions

T1. Sol.

Given that,

$$h_1(t) = 2\delta(t+2) - 3\delta(t+1)$$
$$h_2(t) = \delta(t-2)$$

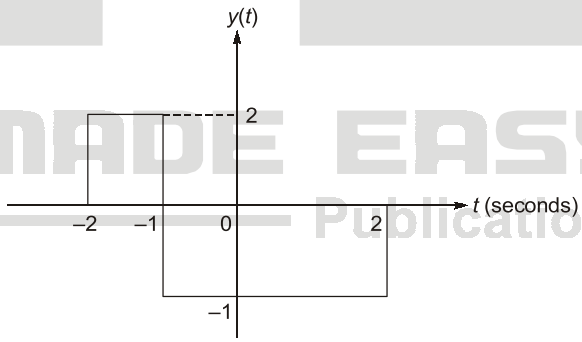
Overall impulse response is,

$$h(t) = h_1(t) + h_2(t)$$
$$= 2\delta(t+2) - 3\delta(t+1) + \delta(t-2)$$

If input, $x(t) = u(t)$, then the output will be,

$$y(t) = x(t) * h(t)$$
$$= u(t) * [2\delta(t+2) - 3\delta(t+1) + \delta(t-2)]$$
$$= 2u(t+2) - 3u(t+1) + u(t-2)$$

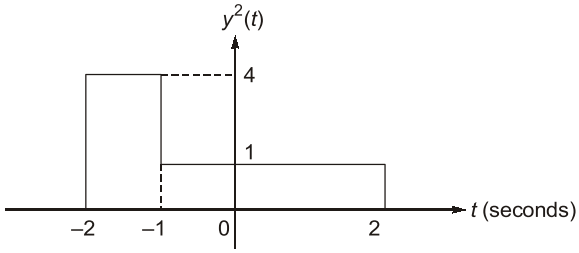
By plotting $y(t)$, we get,



The energy of $y(t)$ can be given as,

$$E_y = \int_{-\infty}^{\infty} y^2(t)\,dt$$

By plotting $y^2(t)$, we get,

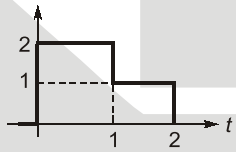


E_y = Area under the plot of $y^2(t)$
= (4 × 1) + (3 × 1) = 7

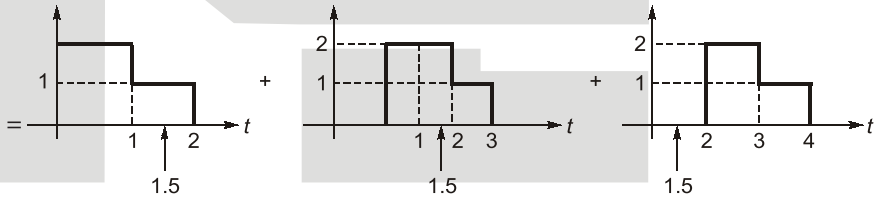
T2. Sol.

$x(t) = \delta(t) + \delta(t-1) + \delta(t-2)$
$h(t) = 2\,u(t) - u(t-1) - u(t-2)$

$h(t) =$



$y(t) = \delta(t) \otimes h(t)\big|_{t=1.5} + \delta(t-1) \otimes h(t)\big|_{t=1.5} + \delta(t-2) \otimes h(t)\big|_{t=1.5}$



$y(t)\big|_{t=1.5} = 1 + 2 + 0 = 3$

T3. Sol.

$$y[n] = \sum_{k=n-3}^{n+2} x[k]$$

$= x[n-3] + x[n-2] + x[n-1] + x[n] + x[n+1] + x[n+2]$
$= x[n] \otimes \delta[n-3] + x[n]\, x\, \delta[n-2] + x[n] \otimes \delta[n-1]$
$\quad + x[n]\, x\, \delta[n] + x[n]\, x\, \delta(n+1] + x[n] \otimes \delta[n+2]$
$= x[n] \otimes [\delta[n-3] + \delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1] + \delta[n+2]]$
$= x[n] \otimes h[n]$

$\therefore$ $h[n\}$ contains 6 unit impulse.

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3 Fourier Series

Detailed Explanation of Try Yourself Questions

T1. Sol.

$$x(t) \xrightarrow{F.S.} a_k$$

$$x(t-t_0) \xrightarrow{F.S.} e^{-j\frac{2\pi}{T}kt_0}\, a_k$$

$$x(t+t_0) \xleftrightarrow{F.S.} a_k e^{j\frac{2\pi}{T}kt_0}$$

$$x(t-t_0)+x(t+t_0) \xleftrightarrow{F.S.} a_k\left[e^{j\frac{2\pi}{T}kt_0}+e^{-j\frac{2\pi}{T}kt_0}\right] \xleftrightarrow{F.S.} 2a_k\left(\cos\frac{2\pi}{T}kt_0\right)$$

T2. Sol.

$$x(t)=\sum_{k=-\infty}^{\infty} a_k\, e^{j\omega_0 kt}$$

where, $\quad a_k=\left(-\frac{1}{3}\right)^k \;;\; k\geq 0$

$$=0 \qquad ;\; k<0 \quad \text{and} \quad \omega_0=2\pi$$

$$\therefore \qquad x(t)=\sum_{k=0}^{\infty}\left[-\frac{1}{3}e^{j2\pi t}\right]^k=\frac{1}{1+\frac{1}{3}e^{j2\pi t}}$$

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T3. Sol.

$$\frac{dy(t)}{dt} + 4y(t) = x(t)$$

$$h(t) = e^{-4t}u(t)$$

$$H(j\omega) = \int_0^\infty e^{-4\tau}\, e^{-j\omega\tau}\, d\tau = \frac{1}{4 + j\omega}$$

$$x(t) = \cos 2\pi t + \sin 8\pi t, \text{ and } \omega_0 = 2\pi$$

$$= \frac{1}{2}\left[e^{j2\pi t} + e^{-j2\pi t}\right] + \frac{1}{2j}\left[e^{j8\pi t} - e^{-j8\pi t}\right]$$

$$= \sum_{n=1,-1,4,-4} a_n\, e^{jn2\pi t}$$

where, $a_1 = a_{-1} = \frac{1}{2}$

and $a_4 = \frac{1}{2j}, a_{-4} = -\frac{1}{2j}$

Now, $y(t) = \sum_{n=1,-1,4,-4} C_n\, e^{j2\pi nt}$

where, $C_n = a_n H(jn2\pi)$

$$C_1 = a_1 \cdot \frac{1}{4 + j2\pi} = \frac{1}{2}\frac{1}{4 + j2\pi}$$

$$C_{-1} = a_{-1}\frac{1}{4 - j2\pi} = \frac{1}{2}\frac{1}{4 - j2\pi}$$

$$C_4 = a_4\frac{1}{4 + j8\pi} = \frac{1}{2j}\frac{1}{4 + j8\pi}$$

$$C_{-4} = a_{-4}\frac{1}{4 - j8\pi} = -\frac{1}{2j}\frac{1}{4 - j8\pi}$$

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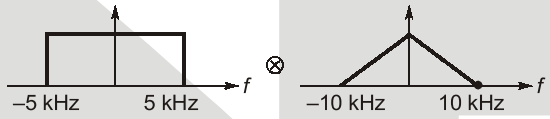
4 Fourier Transform

Detailed Explanation of Try Yourself Questions

T1. Sol.

The output of the multiplier is given by $Sa(10^4\pi t)\cdot Sa^2(10^4\pi t)$.

The spectrum of the above signal is given by



Now $Y(f)$ is given by

The convolution of both the signals will give the bandwidth 15 kHz.

Now $H(\omega)$ has bandwidth 20kHz which is in cascade with multiplier which has bandwidth of 15 kHz.

$\therefore$ $y(t)$ has bandwidth of 15 kHz.

T2. Sol.

$$Y(f) = H(f)\cdot X(f)$$

Now, $$y(0) = \int_{-\infty}^{\infty} Y(f)\,df = \text{Area of } Y(f) = 2\times 5\times 10 = 100$$

T3. Sol.

$$m(t) = \underbrace{Sa^2(3\pi\times 10^6 t)}_{m_1(t)}\cdot\underbrace{Sa^4(3\pi\times 10^6 t)}_{m_2(t)}$$

ω_{max} of $m_1(t) = 6\pi\times 10^6$ rad/sec

ω_{max} of $m_2(t) = 12\pi\times 10^6$ rad/sec

ω_{max} of $m_1(t)\cdot m_2(t)$ is $18\pi\times 10^6$ rad/sec

$\therefore$ Nyquist rate $= 2\,\omega_{max} = 36\pi\times 10^6$ rad/sec

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T4. Sol.

$$h(t) = \begin{cases} 3 & \text{for } 0 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = 5$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{0}^{3} 3 \times 5\, dt = 45$$

T5. (b)

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5 Laplace Transform

Detailed Explanation of Try Yourself Questions

T1. Sol.

$$x_1(t) = e^{-2t}\,u(t)$$
$$x_2(t) = e^{-3t}\,u(t)$$
$$X_1(s) = \frac{1}{s+2}$$
$$X_2(s) = \frac{1}{s+3}$$
$$x_1(t-2) \leftrightarrow X_1(s)\cdot e^{-2s} = \frac{e^{-2s}}{s+2}$$
$$x_2(-t+3) \leftrightarrow X_2(s)\cdot e^{-3s} = \frac{e^{-3s}}{-s+3}$$

Now, $$x_1(t-2) * x_2(-t+3) = \frac{e^{-5s}}{(3-s)(s+2)}$$

T2. Sol.

$$F(s) = \frac{s^2+2s+5}{(s+3)(s+5)} = \frac{s^2+2s+5}{s^2+8s+15}$$
$$= 1-\frac{6s+10}{(s+3)(s+5)}$$
$$= 1-\left[\frac{A}{s+3}+\frac{B}{s+5}\right] = 1-\left[\frac{-4}{s+3}+\frac{10}{s+5}\right]$$
$$= 1+\frac{4}{s+3}-\frac{10}{s+5}$$

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(*i*) for $\sigma > -3$

$$f(t) = \delta(t) + 4e^{-3t}u(t) - 10e^{-5t}u(t)$$

(*ii*) for $\sigma < -5$

$$f(t) = \delta(t) - 4e^{-3t}u(-t) + 10\,e^{-5t}u(-t)$$

(*iii*) for $-5 < \sigma < -3$

$$f(t) = \delta(t) - 4e^{-3t}u(-t) - 10e^{-5t}u(t)$$

T3. Sol.

$$Y(s) = \frac{1}{1-e^{-sT}}Y_1(s)$$

where, $T = 10$ and $Y_1(s)$ is given by

$$Y_1(s) = [u(t) - u(t-2)] = \frac{1}{s}[1-e^{-2s}]$$

$\therefore$

$$Y(s) = \frac{1}{1-e^{-10s}}\cdot\frac{1}{s}[1-e^{-2s}]$$

T4. Sol.

$$y(t) = H(s)\,e^{st}$$

Given,

$$y(t) = \frac{1}{6}e^{2t} \Rightarrow H(2) = \frac{1}{6}$$

Given, differential equation,

$$\frac{d}{dt}h(t) + 2h(t) = e^{-4t}u(t) + k\,u(t)$$

Taking, L.T on both sides,

$$s\,H(s) + 2\,H(s) = \frac{1}{s+4} + \frac{k}{s}$$

$$H(s)\,[s+2] = \frac{1}{s+4} + \frac{k}{s}$$

$$H(s) = \frac{1}{(s+2)(s+4)} + \frac{k}{s(s+2)}$$

At $s = 2$,

$$H(2) = \frac{1}{6}$$

$$\frac{1}{6} = \frac{1}{4\times 6} + \frac{k}{2(4)}$$

$\Rightarrow$

$$4 = 1 + 3k \Rightarrow k = 1$$

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6 Z-Transform

Detailed Explanation of Try Yourself Questions

T1. Sol.

Let, $$X(z) = z^{-2}\log(1-2z^{-1})$$

and $$ZT[n\cdot x(n)] = -z\frac{d}{dz}X(z) = -z\frac{1}{1-2z^{-1}}\cdot 2z^{-2}\cdot z^{-2}$$

$$ZT[n\cdot x(n)] = z^{-2}\cdot\frac{-2z^{-1}}{1-2z^{-1}}$$

Inverse of the above function is i.e.,

$$n\,x(n) = -2\cdot 2^{(n-3)}\,u(n-3)$$

$$x(n) = \frac{-0.25}{n}2^{n}\ ;\ n\geq 3$$

$$= 0 \qquad ;\ n<3$$

T2. Sol.

$$X(z) = \frac{1+z^{-1}}{1+\frac{1}{3}z^{-1}}$$

(*i*) For $|z| < \frac{1}{3}$,

$$\begin{array}{r|l} & \quad x(0)\ \ x(-1)\ \ x(-2) \\ \frac{1}{3}z^{-1}+1 & z^{-1}+1\ (\ 3 - 6z + 18z^{2} \\ & \underline{z^{-1}+3} \\ & -2 \\ & \underline{-2-6z} \\ & \quad 6z \\ & \quad \underline{6z+18z^{2}} \\ & \qquad -18z^{2}\ldots \end{array}$$

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$\therefore \quad x(0) = 3,\ x(-1) = -6,\ 2x(-2) = 36$

(*ii*) For $|z| > \frac{1}{3}$

$$1+\frac{1}{3}z^{-1}\ \Big)\ 1+z^{-1}\ \Big(\ 1+\frac{2}{3}z^{-1}-\frac{1}{3}z^{-2}\ldots$$

$$\frac{1+\frac{1}{3}z^{-1}}{\frac{2}{3}z^{-1}}$$

$$\frac{\frac{2}{3}z^{-1}+\frac{2}{6}z^{-2}}{-\frac{1}{3}z^{-2}}$$

$$\frac{-\frac{1}{3}z^{-2}-\frac{1}{9}z^{-3}}{\frac{1}{9}z^{-3}}$$

$x(0) = 1$

$x(1) = \frac{2}{3}$

$x(2) = -\frac{1}{3}$

T3. Sol.

$$y(n)+\frac{1}{4}y(n-1)-\frac{1}{8}y(n-2) = 2x(n)+\frac{5}{4}x(n-1)$$

Taking Z.T on both sides,

$$Y(z)\left(1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}\right) = X(z)\left(2+\frac{5}{4}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2+\frac{5}{4}z^{-1}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

Peforming long division,

$$1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}\ \Big)\ 2+\frac{5}{4}z^{-1}\ \Big(\ \underset{\downarrow}{h(0)}\ \ \underset{\downarrow}{h(1)}\quad 2+\frac{3}{4}z^{-1}\ldots$$

$$(-)\ \frac{2+\frac{1}{2}z^{-1}-\frac{1}{4}z^{-2}}{\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}}$$

$$(-)\ \frac{\frac{3}{4}z^{-1}+\frac{3}{16}z^{-2}-\frac{3}{32}z^{-3}}{\frac{1}{6}z^{-2}\ldots.}$$

$$h[n] = \left(\underset{\uparrow}{2}, \frac{3}{4}\ldots\right)$$

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7 DTFT and DFT

Detailed Explanation of Try Yourself Questions

T1. Sol.

$$y\{n\} = n\,x[n]$$
$$= [0, 1, 4, 9, 16]$$

$$\text{Energy of } y\{n\} = \sum_{n=-\infty}^{\infty} [y(n)]^2$$
$$= 0 + 1 + 16 + 81 + 256 = 354$$

T2. Sol.

$$x[n] = 1 \;;\; |n| \le 10$$
$$= 0 \;;\; |n| > 10$$

For
$$x\{n\} = 1 \;;\; |n| \le N$$
$$= 0 \;;\; |n| > N$$

$$X(e^{j\omega}) = \frac{\sin\left[\left(N+\frac{1}{2}\right)\omega\right]}{\sin\left(\frac{\omega}{2}\right)}$$

$\therefore$ $N = 10 \Rightarrow k = N + \frac{1}{2} = \frac{21}{2} = 10.5$

T3. Sol.

$$a^n u[n] * a^n u[n] * a^n u[n] \longleftrightarrow \frac{1}{(1 - ae^{-j\omega})^3}$$

$$a^n u[n] * a^n u[n] = \sum_{k=-\infty}^{\infty} a^k u[k]\, a^{n-k}\, u[n-k]$$

$$= \sum_{k=0}^{n} a^n = (n+1)\, a^n\, u[n]$$

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$$a^n u[n] * a^n u[n] * a^n u[n] = \sum_{k=-\infty}^{\infty} (k+1)^k\, u[k]\, a^{n-k}\, u[n-k]$$

$$= \sum_{k=0}^{n} (k+1)\, a^n = a^n \sum_{k=0}^{n} (k+1)$$

$$= \frac{(n+1)(n+2)}{2} a^n\, u[n]$$

T4. Sol.

$$H(e^{j\omega}) = e^{-j2\omega}\,[3 + 4\cos\omega + 2\cos 2\omega]$$

$$\text{Group delay} = -\frac{d\phi}{d\omega} = -\frac{d}{d\omega}(-2\omega) = 2$$

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