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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-3 : Power Systems [All topics] + Systems and Signal Processing-1 + Microprocessor-1 Electrical Circuits-2 + Control Systems-2 [Part Syllabus]

Name :

Roll No :

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Delhi ☑	Bhopal ☐	Jaipur ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	36
Q.2	50
Q.3	
Q.4	30
Section-B	
Q.5	36
Q.6	
Q.7	
Q.8	34
Total Marks Obtained	186

Signature of Evaluator: Sourabh Kumar

Cross Checked by: ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Do not write in this margin

Section A : Power Systems

Q.1 (a) A 66 kV concentric cable with intersheath has a core diameter of 1.6 cm. 3 mm thick dielectric materials constitute the three zones of insulation. Determine the maximum stress in each of the three layers if 20 kV is maintained across each of the inner two layers.

[12 marks]

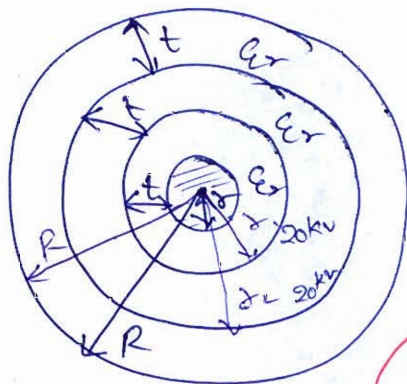
$V = 66\,kV$

$d = 1.6\,cm$ (dia of core)

$\Rightarrow r = 0.8\,cm = 0.8\times10^{-2}\,m$

$t = 3\,mm = 3\times10^{-3}\,m$ (thickness of dielectric)

Supply 66 kV.



$R = r + 3t$

$= 0.8\times10^{-3} + 3\times3\times10^{-3}$

$R = 9.8\times10^{-3}\,m$

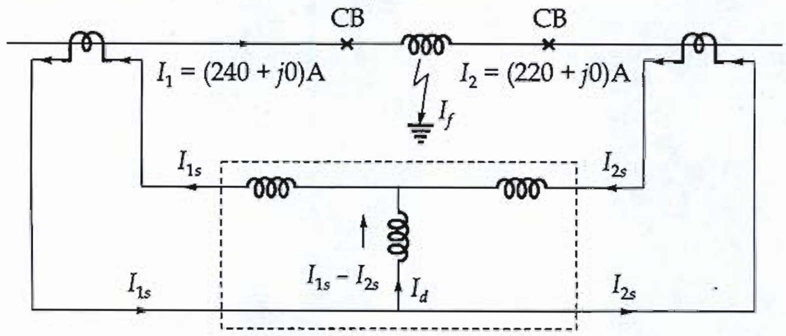
$\Rightarrow$ $R = 9.8\,mm$

$V_1 = x E_{max} \ln\left(\frac{r_1}{x}\right)$

2

Incomplete solution

Q.1 (b) Figure below shows percentage differential relay is applied for the protection of a generator winding. The relay has 10% slope of its operating characteristic on $\frac{(I_{1s}+I_{2s})}{2}$ versus $(I_{1s}-I_{2s})$ diagram. A high resistance ground fault occurred near the grounded neutral end of the generator winding while generator is carrying load. As a consequence, the currents flowing at each end of the winding are shown in the figure below. Assuming CT ratio of 400/5 ampere, will the relay operate to trip the breaker?



[12 marks]

$I_1 = 240$A

$I_2 = 220$ A.

CT. Ratio = 400/5 A

$K = 10\% = 0.1$

$I_{2s} = \frac{220}{\text{CT Ratio}} = \frac{220}{400/5} = 25$ A

$I_{1s} = \frac{240}{\text{CT Ratio}} = \frac{240}{400/5} = 30$ A.

Now $(I_{1s} - I_{2s}) = (30-25) = 5$ A. —— (1)

& $\left(\frac{I_{1s}+I_{2s}}{2}\right) = \left(\frac{30+25}{2}\right) = 27.5$ A.

Now restraining current

$i_R = K\left[\frac{I_{1s}+I_{2s}}{2} + I_0\right]$ $I_0 = 0$A default

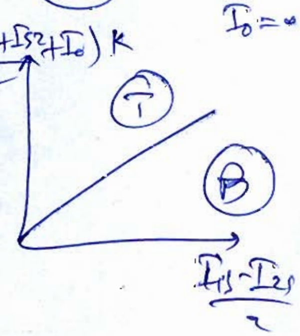
(9)

$\Rightarrow i_R = 0.1[27.5+0]$

$\Rightarrow i_R = 2.75$ A. —— (2)

Since $i_R < (I_{1s} - I_{2s})$ [∵ 2.75 < 5A] from (1) & (2)

∴ Relay Blocks



Q.1 (c) A 50 Hz, 4 pole, turbo-generator rated 100 MVA, 11 kV has an inertia constant of 8 MJ/MVA. ⇒ $H = 8\ MJ/MVA$

(i) Determine the stored energy in the rotor at synchronous speed.

(ii) If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, determine acceleration in elec-degree/sec^2, neglecting mechanical and electrical losses.

(iii) If the acceleration calculated in part (ii) is maintained for 10 cycle, determine the change in torque angle and rotor speed in revolutions per minute at the end of the period.

[2 + 5 + 5 marks]

No of Poles, $P = 4$

(i) K.E. Stored $= GH$

$= 100 \times 8$

$\boxed{K.E. = 800\ J}$

(ii) $P_m = 80\ MW$ $\quad P_e = 50\ MW$

$P_a = P_m - P_e = 30\ MW.$

Also Swing eq^n: $\dfrac{GH}{180 f} \dfrac{d^2\delta}{dt^2} = P_a$

$\dfrac{100 \times 8}{180 \times 50} \dfrac{d^2\delta}{dt^2} = 30$

$\boxed{\text{acceleration, } \dfrac{d^2\delta}{dt^2} = 337.5\ \text{elec. degree/sec}^2}$

(iii) $t = 10$ cycles $= \dfrac{10}{50} = 0.2$ sec.

Now $\dfrac{d^2\delta}{dt^2} = 337.5$ elec. degree/sec^2

Integrating $\dfrac{d\delta}{dt} = 337.5t + K_1$

At $t = 0^+$, $\dfrac{d\delta}{dt} = \omega_s - \omega_s = 0 \Rightarrow K_1 = 0$

$\therefore \dfrac{d\delta}{dt} = 337.5t$ —— (1) elec degree/sec

Integrating again

$$\delta(t) = \frac{337.5}{2}t^2 + \delta_0$$

$$\Rightarrow \Delta\delta = \delta(t) - \delta_0 = \frac{337.5}{2}t^2$$

$$\Rightarrow \Delta\delta = 168.75\,t^2 \quad \text{electrical degre}$$

After $t = 10$ cycles $= 0.2$ sec.

$$\Delta\delta = 168.75\,(0.2)^2$$

$$\Rightarrow \boxed{\Delta\delta = 6.75^\circ}$$

from eqn (1)

$$\omega = \frac{d\delta}{dt} = 337.5t \quad \text{elec degree/sec.}$$

$$\Rightarrow \omega_m = \frac{\omega}{P/2} = \frac{337.5t}{P/2} = \frac{337.5t}{4/2} = \frac{337.5\times0.2}{2}$$

$\omega_m =$ ~~168.75~~ 33.75 mech degree/sec.

$\omega_m = 0.589$ mech rad/sec

$$\Rightarrow N = \frac{60\omega_m}{2\pi} = \text{~~}\frac{60\times168.75}{2\pi}\text{~~}$$

$$N = \text{~~1611.44 rpm~~}\ \frac{60\times0.589}{2\pi} = 5.625 \text{ rpm}$$

Synchronous speed $N_s = \frac{120f}{P} = \frac{120\times50}{4}$

$$N_s = 1500 \text{ rpm.}$$

$\therefore$ Rotor speed, $N_r = N_s + N$

$= 1500 +$ ~~1611.44~~ 5.625

$$\boxed{N_r = 1505.625 \text{ rpm}}$$

11

Good Approach

Q.1 (d) The per phase impedance of 3-ϕ short transmission line is $(0.3 + j0.4)\Omega$. The sending-end line to line voltage is 3300 V and the load at the receiving end is 300 kW per phase at 0.8 pf lagging. Calculate receiving end voltage and line current.

[12 marks]

We know $V_S = V_R + I_R\cos\phi_R + I_R\sin\phi_R$ —— (1)

Also, $I_R = \frac{P_R}{\sqrt{3}V_{RL}\cos\phi_R} = \frac{300\times10^3}{\sqrt{3}\times V_{RL}\times 0.8}$

$\Rightarrow I_R = \frac{300\times10^3}{\sqrt{3}\times(\sqrt{3}V_R)\times0.8} = \frac{10^5}{0.8V_R}$

$\Rightarrow I_R = \frac{125\times10^3}{V_R}$ —— (2)

3

Putting (2) in (1)

$V_S = V_R + \frac{125\times10^3}{V_R}\times0.8 + \frac{125\times10^3}{V_R}\times0.6$

∵ $\cos\phi_R = 0.8 \Rightarrow \sin\phi_R = 0.6$
& $V_S = \frac{3300}{\sqrt{3}}$ V

∴ $\frac{3300}{\sqrt{3}} = V_R + \frac{125\times10^3}{V_R}[1]$

$\Rightarrow V_R^2 - \frac{3300}{\sqrt{3}}V_R + 125\times10^3 = 0$

$\Rightarrow V_R = 1837.218V, 68.04V$ ✓ ✗

(Can't be too low at 0.8 pf.)

∴ $V_R = 1837.218$ V per phase
(line-line) $V_{RL} = 3182.155$ Volts

Putting V_R in (2), $I_R = 125\times10^3/1837.218$

$\Rightarrow I_R = 68.04$ A per phase.

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Q.1 (e) A three phase generator delivers 1.0 p.u. power to an infinite bus through a transmission network when a fault occurs. The maximum power which can be transferred in pre-fault, during fault and post fault conditions are 1.75 p.u., 0.4 p.u and 1.25 p.u. respectively. Find the critical angle.

[12 marks]

Given :- $P_m = 1$ p.u.

$P_{max_1} = 1.75$ p.u.

$P_{max_2} = 0.4$ p.u.

$P_{max_3} = 1.25$ p.u.

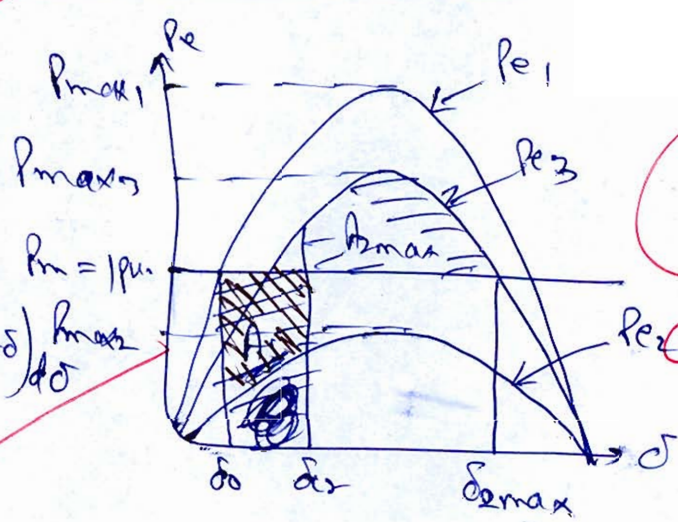
$\Rightarrow \delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max_1}}\right] = \sin^{-1}\left[\frac{1}{1.75}\right] \Rightarrow \delta_0 = 34.85^\circ$

$\Rightarrow$ $\boxed{\delta_0 = 34.85^\circ \text{ or } 0.608 \text{ radian}}$

& $\delta_{2max} = 180^\circ - \sin^{-1}\left[\frac{P_m}{P_{max_3}}\right] = 180^\circ - \sin^{-1}\left[\frac{1}{1.25}\right]$

$\Rightarrow$ $\boxed{\delta_{2max} = 126.87^\circ \text{ or } 2.214 \text{ radian}}$

Now



$A_{2max} = A_1$

$\int_{\delta_{cr}}^{\delta_{2max}} (P_{max_3}\sin\delta - P_m)\, d\delta = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{max_2}\sin\delta)\, d\delta$

Integrating above

$\Rightarrow$
$$\delta_{cr} = \cos^{-1}\left[\frac{P_m(\delta_{2max} - \delta_0) - P_{max_2}\cos\delta_0 + P_{max_3}\cos\delta_{2max}}{P_{max_3} - P_{max_2}}\right]$$

$$\Rightarrow \delta_{cr} = \cos^{-1}\left[\frac{1(2.214 - 0.608) - 0.4\cos 34.85^\circ + 1.25\cos 126.87^\circ}{1.25 - 0.4}\right]$$

$\Rightarrow \delta_{cr} = \cos^{-1}(0.62087) \Rightarrow \boxed{\delta_{cr} = 51.62^\circ}$

11

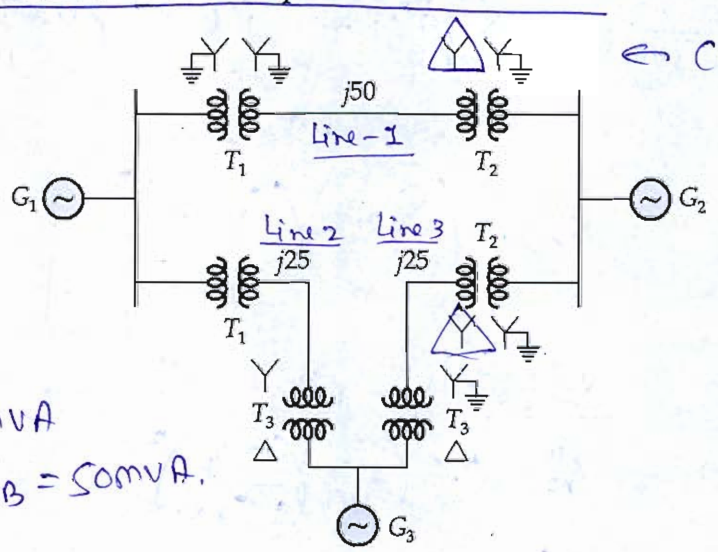
Good Approach

Q.2 (a) A 3-bus system is given in figure below. The ratings of the various components are listed below :

Generator 1 = 50 MVA;	13.8 kV;	$X'' = 0.15$ pu
Generator 2 = 40 MVA;	13.2 kV;	$X'' = 0.20$ pu
Generator 3 = 30 MVA;	11 kV;	$X'' = 0.25$ pu
Transformer 1 = 45 MVA,	11 kV Δ/110 kV Y,	$X = 0.1$ pu
Transformer 2 = 25 MVA,	12.5 kV Δ/115 kV Y,	$X = 0.15$ pu
Transformer 3 = 40 MVA,	12.5 kV Δ/115 kV Y,	$X = 0.1$ pu

The line impedances are shown in figure below. Determine the reactance diagram based on 50 MVA and 13.8 kV as base quantities in Generator 1.

← Correction. $T_2: \Delta/Y$



System MVA Base, $S_B = 50$ MVA.

On Generator 1, G1 side, Base voltage, $V_{B_1} = 13.8$ kV **[20 marks]**

on line 1, $V_{BL_1} = (13.8\text{ kV})\left(\frac{110}{11}\right) = 138$ kV

line 2, $V_{BL_2} = (13.8\text{ kV})\left(\frac{110}{11}\right) = 138$ kV

~~line 3, $V_{BL_3} = (V_{BL_2})$~~

On G2 side, $V_{B2} = (V_{BL_1}) \times \left(\frac{12.5}{115}\right) = (138\text{ kV})\left(\frac{12.5}{115}\right) = 15$ kV

On G3 side, $V_{B3} = (V_{BL2}) \times \left(\frac{12.5}{115}\right)$ ~~$= (138\text{ kV})\left(\frac{115}{12.5}\right) = 1269.6$ kV~~
$= (138\text{ kV})\left(\frac{12.5}{115}\right) = 15$ kV

One line 3, $V_{BL_3} = (V_{B3}) \times \left(\frac{115}{12.5}\right) = 15 \times \frac{115}{12.5} = 138$ kV.

Base Impedance

Line 1 :- $Z_{B_1} = \frac{V_{BL_1}^2}{S_B} = \frac{(138)^2}{50} = 380.88\ \Omega$

Line 2 :- $Z_{B_2} = \frac{V_{BL_2}^2}{S_B} = \frac{(138)^2}{50} = 380.88\ \Omega$

Line 3 :- $Z_{B_3} = V_{BL_3}^2 / S_B = \frac{(138)^2}{50} = 380.88\ \Omega$.

Per unit line impedances

$jX_{L_1} = \frac{j50}{Z_{B_1}} = \frac{j50}{380.88}$ [using $Z_{pu} = \frac{Z_{actual}}{Z_{Base}}$]

$\Rightarrow$ $X_{L_1} = j0.131$ p.u.

& $jX_{L_2} = \frac{j25}{380.88} \Rightarrow$ $jX_{L_2} = j0.066$ p.u.

& $jX_{L_3} = \frac{j25}{380.88} \Rightarrow$ $jX_{L_3} = j0.066$ p.u.

per unit reactance on new base,

$$X_{new} = X_{old} \times \left(\frac{S_{new}}{S_{old}}\right)\left(\frac{V_{old}}{V_{new}}\right)^2$$

$\therefore$ For $G_1 \Rightarrow X''_{G_1 new} = 0.15$ p.u. (Same on system Base)

$G_2 \Rightarrow X''_{new} = 0.2\left(\frac{50}{40}\right)\left(\frac{13.2kV}{15kV}\right)^2$

$G_2 \Rightarrow X''_{G_2 new} = 0.194$ p.u.

For G_3: $X''_{new} = 0.25\left(\frac{50}{30}\right)\left(\frac{11kV}{15kV}\right)^2$

$\Rightarrow$ G_3: $X''_{G_3 new} = 0.224$ p.u.

T_1: $X_{t_1 new} = 0.1\left(\frac{50}{45}\right)\left(\frac{11kV}{13.8kV}\right)^2$

$\Rightarrow$ $X_{t_1 new} = 0.0706$ p.u.

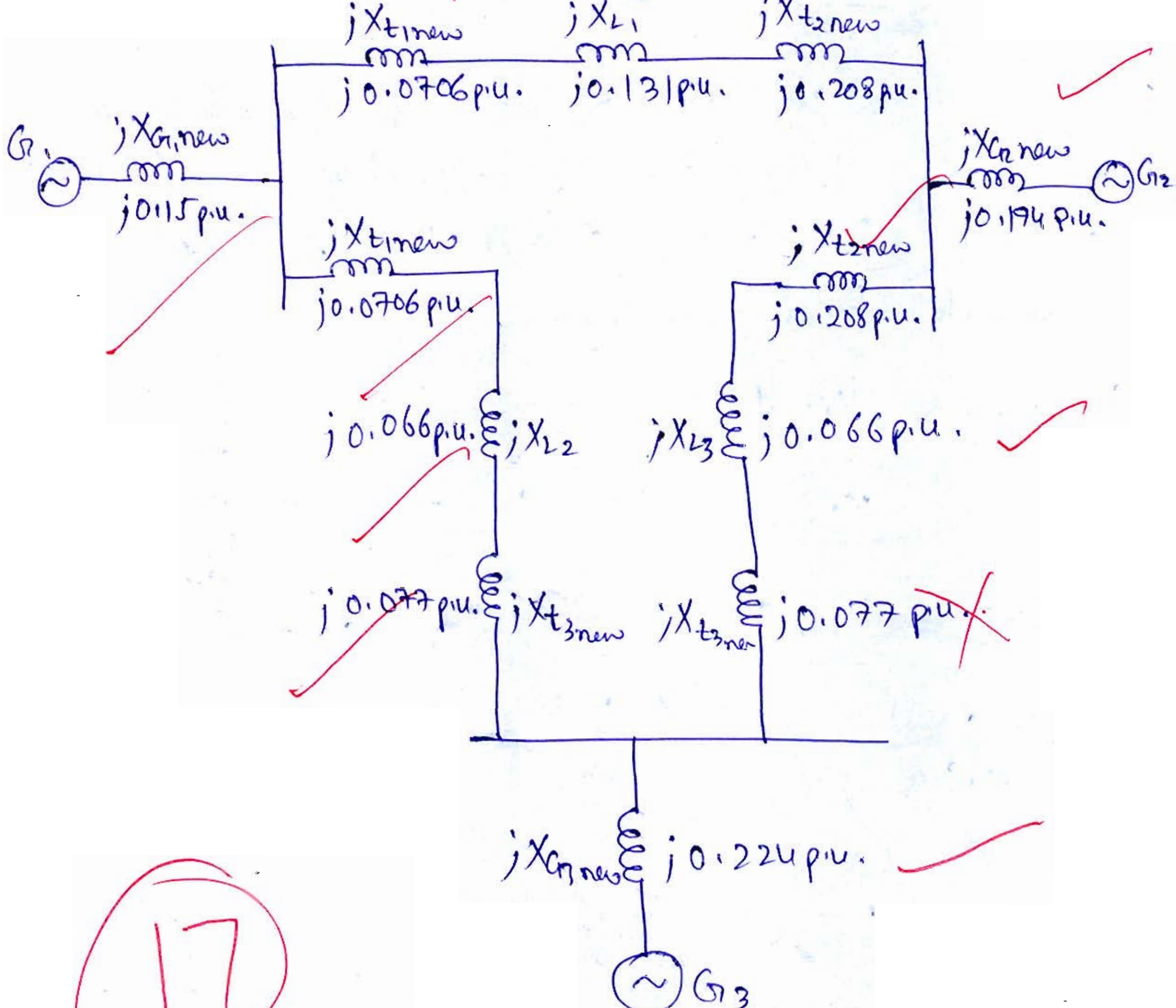
T_2: $X_{t_2 new} = 0.15\left(\frac{50}{25}\right)\left(\frac{115kV}{138kV}\right)^2$

$\Rightarrow$ $X_{t_2 new} = 0.208$ p.u.

T_3: $X_{t_3 new} = 0.1\left(\frac{50}{40}\right)\left(\frac{115kV}{138kV}\right)^2$

$\Rightarrow$ $X_{t_3 new} = 0.077$ p.u.

Thus reactance diagram is –



17

Q.2 (b) Explain briefly what is swing equation and use dynamics of angular motion with time to formulate the equation for a synchronous generator of inertia constant H in seconds run by a mechanical turbine with input power P_m in p.u. to deliver electrical power P_e in p.u. to the electrical network at f Hz in terms of power angle δ in radians measured from rotating reference of generator axis.

[20 marks]

Swing Equation: It is the equation of representing the dynamics of the rotor of a synchronous machine connected to a large/infinite Bus system when subjected to a disturbance.

We have kinetic energy

$$K.E. = \frac{1}{2} J\omega_m^2 \quad \text{---} (1)$$

J → moment of inertia of the machine (kg-m²)

ω_m ⇒ angular velocity in mech rad/sec

Also

$$K.E. = H.G \quad \text{---} (2)$$

H : inertia constant (in sec)

G : MVA Rating of machine

from (1) & (2)

$$\frac{1}{2} J\omega_m^2 = H.G$$

$$\Rightarrow \frac{1}{2}(J\omega_m)\omega_m = H.G.$$

$$\Rightarrow \frac{1}{2} M\omega_m = H.G.$$

(M : angular moment of inertia)

$$\Rightarrow M = \frac{2H.G}{\omega_m} \quad \text{---} (3)$$

Also we know that

$$M\frac{d^2\theta_m}{dt^2} = P_m - P_e \quad \text{---} (4)$$

from (3), in per unit $\frac{M}{G} = \frac{2H}{\omega_m}$

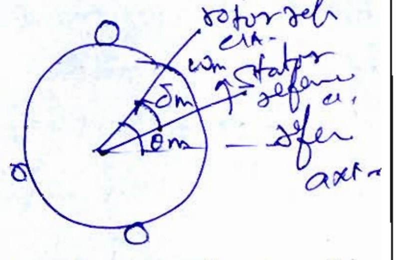
$$\Rightarrow M_{pu} = \frac{2H}{\omega_m} \quad \text{---} (5)$$

Substituting (5) in (4)

$$\frac{2H}{\omega_m}\frac{d^2\theta_m}{dt^2} = P_m - P_e \quad \text{—— (6)}$$

P_m : per unit input power
P_e : per unit o/P power

Now $\theta_m = \omega_m t + \delta_m$



$$\Rightarrow \frac{d\theta_m}{dt} = \omega_m + \frac{d\delta_m}{dt}$$

Again $\Rightarrow$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \quad \text{—— (7)}$$

δ_m : mech angle of rotor.

Putting (7) in (6)

$$\frac{2H}{\omega_m}\frac{d^2\delta_m}{dt^2} = P_m - P_e$$

Dividing above by $(P/2)$: P: No of poles.
& multiplying

$$\Rightarrow \frac{(P/2)\,2H}{(P/2)\,\omega_m}\frac{d^2\delta_m}{dt^2} = P_m - P_e$$

$$\Rightarrow \frac{2H}{\omega_s}\frac{d^2}{dt^2}\left(\frac{P}{2}\delta_m\right) = P_m - P_e$$

$$\Rightarrow \frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

∵ δ_m → mechanical degree
δ → electrical degree
$\theta_e = \frac{P}{2}\theta_m$
& $\omega_s = \frac{P}{2}\omega_m$

$$\Rightarrow \boxed{\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e}$$

Basic Swing Equation

where f is the system frequency in Hz.
H : is in sec.
P_m & P_e is in p.u.

18

Q.2 (c) A 3-ϕ, 400 km, 50 Hz long transmission line with series impedance of $(0.15 + j0.78)\ \Omega/\text{km}$ and shunt admittance of $j5.0 \times 10^{-6}$ ℧/km. Determine A, B, C, D parameter of line assuming :

(i) The line could be represented by nominal-T.

(ii) The line could be represented by nominal-π.

(iii) The exact representation.

[20 marks]

Here, $l = 400\text{km}$

$z = (0.15 + j0.78)\ \Omega/\text{km}$

$\Rightarrow Z = zl = (0.15 + j0.78) \times 400$

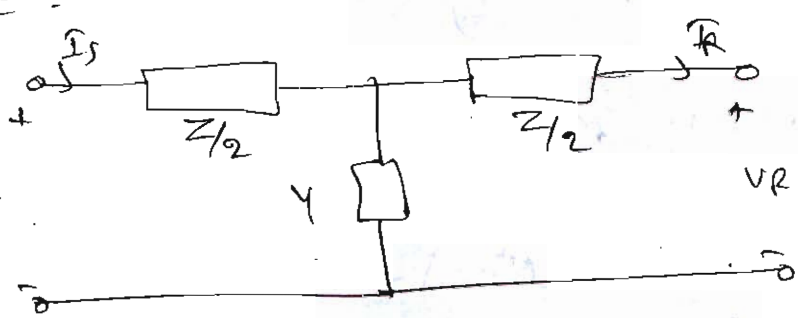
$\Rightarrow \boxed{Z = (60 + j312)\ \Omega = 317.717\angle 79.11^\circ\ \Omega}$

& $y = j5.0 \times 10^{-6}$ ℧/km

$\Rightarrow Y = yl = j5 \times 10^{-6} \times 400$

$\Rightarrow \boxed{Y = j2 \times 10^{-3}\ ℧}$

(i) T- n/w :-



$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & \left(1 + \frac{YZ}{4}\right)Z \\ Y & \left(1 + \frac{YZ}{2}\right) \end{bmatrix}$$

where $A = D = 1 + \frac{YZ}{2} = 1 + \frac{(j2 \times 10^{-3})(60 + j312)}{2}$

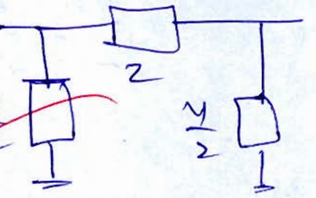
$\Rightarrow \boxed{A = D = 0.6906\angle 4.98^\circ}$

& $\boxed{C = Y = j2 \times 10^{-3}\ ℧}$

& $B = \left(1 + \frac{YZ}{4}\right)Z = \left[1 + \frac{(j2 \times 10^{-3})(60 + j312)}{4}\right](60 + j312)$

$\Rightarrow \boxed{B = 268.322\angle 81.15^\circ\ \Omega}$

(ii) Nominal π :-



$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1+\frac{YZ}{2}\right) & Z \\ Y\left(1+\frac{YZ}{4}\right) & \left(1+\frac{YZ}{2}\right) \end{bmatrix}$$

$$\Rightarrow A = D = \left(1+\frac{YZ}{2}\right) = 1 + \frac{(j2\times10^{-3})(60+j312)}{2}$$

$$\Rightarrow \boxed{A = D = 0.6906 \angle 4.98^\circ}$$

$$\boxed{B = Z = 60+j312 = 317.717 \angle 79.11^\circ \ \Omega}$$

$$\& \quad C = Y\left[1+\frac{YZ}{4}\right] = (j2\times10^{-3})\left[1+\frac{(j2\times10^{-3})(60+j312)}{4}\right]$$

$$\Rightarrow \boxed{C = 1.689 \angle 92.04 \times 10^{-3} \ \mho}$$

(iii) Exact representation. (a long line of uniformly distributed.

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh\gamma l & Z_c \sinh\gamma l \\ \frac{1}{Z_c}\sinh\gamma l & \cosh\gamma l \end{bmatrix}$$

Now Characteristic impedance, $Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{0.15+j0.78}{j5\times10^{-6}}}$

$$\Rightarrow \boxed{Z_c = 398.57 \angle -5.45^\circ \ \Omega}$$

& $\gamma l = \alpha l + j\beta l$: propagation constant,

$$\gamma l = \alpha l + j\beta l = \sqrt{ZY} = \sqrt{(0.15+j0.78)(j5\times10^{-6})}$$

$$\Rightarrow \gamma l = \alpha l + j\beta l = 0.06302 \angle 84.56$$

$$\text{or} \quad \boxed{\gamma l = \alpha l + j\beta l = (0.00597 + j0.0627)}$$

$$\Rightarrow \boxed{\begin{aligned} \alpha l &= 0.00597 \text{ neper} \\ \& \quad \beta l &= 0.0627 \text{ radian} = 3.59^\circ \end{aligned}}$$

Now $\cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{e^{\alpha l}.e^{j\beta l} + e^{-\alpha l}.e^{-j\beta l}}{2}$

$\Rightarrow = \frac{e^{0.00597}.e^{j3.59^\circ} + e^{-0.00597}.e^{-j3.59^\circ}}{2}$

$\Rightarrow \boxed{\cosh \gamma l = 0.9981 \angle 0.02^\circ}$

& $\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{e^{\alpha l}.e^{j\beta l} - e^{-\alpha l}.e^{-j\beta l}}{2}$

$\Rightarrow = \frac{e^{0.00597}.e^{j3.59^\circ} - e^{-0.00597}.e^{-j3.59^\circ}}{2}$

$\Rightarrow \boxed{\sinh \gamma l = 0.0629 \angle 84.56^\circ}$

Thus, $\boxed{A = D = \cosh \gamma l = 0.9981 \angle 0.02^\circ}$

& $B = Z_c \sinh \gamma l = (398.57 \angle -5.45^\circ)(0.0629 \angle 84.56^\circ)$

$\Rightarrow \boxed{B = 25.07 \angle 79.11^\circ \ \Omega}$

& $C = \frac{1}{Z_c} \sinh \gamma l = \frac{1}{(398.57 \angle -5.45^\circ)} (0.0629 \angle 84.56^\circ)$

$\Rightarrow \boxed{C = 1.578 \times 10^{-4} \angle 90.01^\circ \ \mho}$

15

Q.3 (a) A 50 Hz generator is delivering 50% of the power that it is capable of delivering through a transmission line to an infinite bus. A fault occurs that increases the reactance between the generator and infinite bus to 400% of the value before the fault. When the fault is isolated, the maximum power that can be delivered is 80% of the original maximum value. Determine critical clearing angle for the condition described.

[20 marks]

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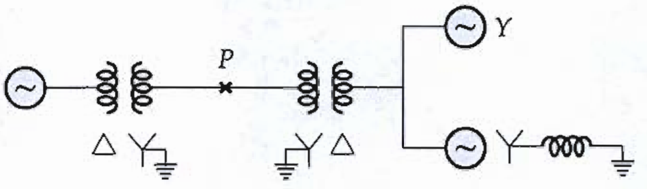
Q.3 (b) A 30 MVA, 13.8 kV, 3-phase alternator has a subtransient reactance of 15% and negative and zero sequence reactance of 15% and 5% respectively. The alternator supplies two motors over a transmission line having tensiometers of both-ends as shown on one line diagram. The motors having rated input of 20 MVA and 10 MVA both with 12.5 kV with 20% subtransient reactance and negative and zero sequence reactances are 20% and 5% respectively. Current limiting reactor of 2 Ω each are in the alternator and larger motor. The 3-phase transformers are both rated 35 MVA, 13.2 Δ - 115 Y kV with leakage reactance of 10%. Series reactance of the line is 80 Ω. The zero sequence reactance of the line is 200 Ω. Determine the fault current when (i) L-G, (ii) L-L, (iii) LLG and fault takes place at point P.



(Assume, $V_f = 120$ kV)

[20 marks]

Q.3 (c) (i) Give the methods of improving string efficiency for an insulator.

(ii) A transmission line has a span of 375 m between level supports. The conductor has an effective diameters of 1.96 c.m. and weight 0.865 kg/m. Its ultimate strength is 9060 kg. If the conductor has ice coating of radial thickness 1.27 c.m. and subjected to a wind pressure of 3.9 gm/cm^2 of projected area. Calculate sag for a safety factor of 2. (Weight of 1 c.c. of ice is 0.91 gm).

[8 + 12 marks]

Q.4 (a) A star connected 3-phase, 10 MVA, 6.6 kV alternator has a per phase reactance of 20%. It is protected by Merz-price circulating current principle not less than 170 A. Calculate of the value of earthing resistance to be provided in order to ensure that only 20% of the alternator winding remains unprotected.

[20 marks]

$V_L = 6.6 \text{ kV}$

$V_{ph} = 3810.51$ Volts.

$$\% W_{unprotected} = \frac{I_N R_N}{V_{ph}} \times 100$$

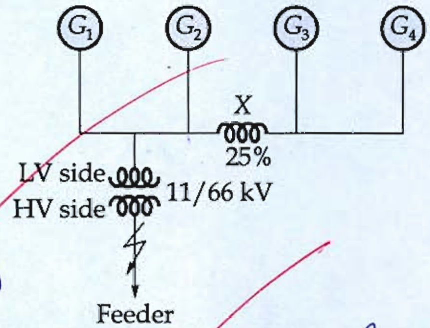
$$\Rightarrow 20\% = \frac{170 \times R_N}{3810.51} \times 100$$

$$\Rightarrow R_N = 0.04485 \ \Omega$$

(2)

$r = 4.48\ \Omega$

Q.4 (b) A generating station has four identical generators, G_1, G_2, G_3 and G_4 each of 20 MVA, 11 kV having 20% reactance. They are connected to a busbar which has a busbar reactor of 25% reactance on 20 MVA base, inserted between G_2 and G_3 as shown below. A 66 kV feeder is taken off from the bus bars through a 15 MVA, 11/66 kV transformer having 7.5% reactance. A symmetrical 3-phase fault occurs at the high voltage terminals of the transformers. Calculate the current fed into the fault.



[20 marks]

Let Base Values

$S_B = 20\text{MVA}$

$V_{B_1} = 11\text{kV}$ (LV side)

$V_{B_2} = 66\text{kV}$ (HV side)

For Generators: (On 20MVA, 11kV Base)

$G_1: X'' = 0.2$ p.u. $G_3: X'' = 0.2$ pu.

$G_2: X'' = 0.2$ pu. $G_4: X'' = 0.2$ pu.

Bus Bar :- $X = 0.25$ p.u. (On 20MVA, 11kV)

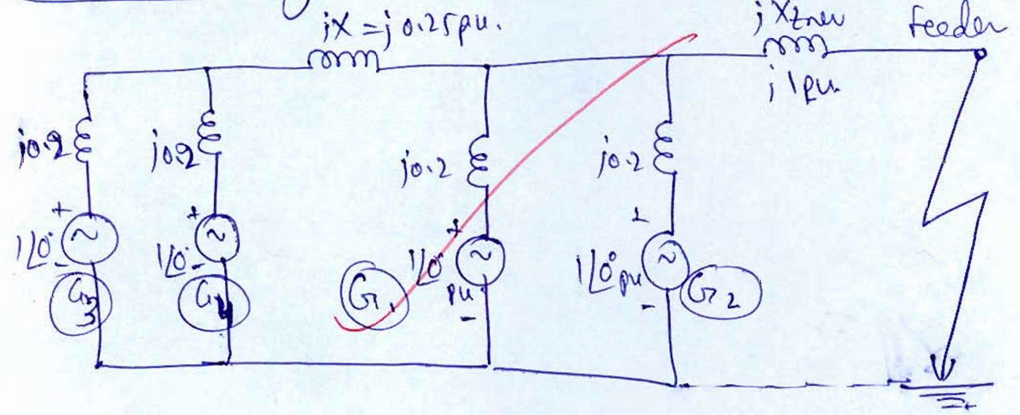
Transformer : $X_t = 7.5\%$ on 15MVA, 11kV

At new Base, $X_{t\,new} = 0.75 \times \left(\frac{S_{new}}{S_{old}}\right)\left(\frac{V_{old}}{V_{new}}\right)^2$

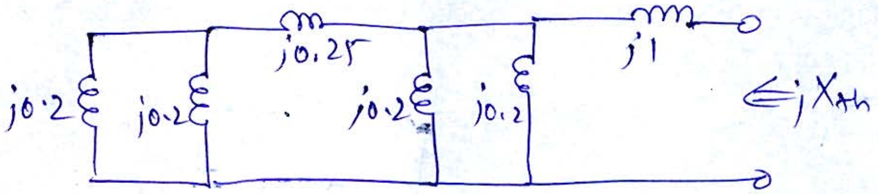
$X_{t\,new} = 0.75 \times \left(\frac{20}{15}\right)\left(\frac{11}{11}\right)^2$

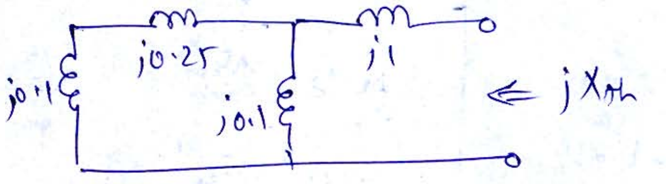
$\Rightarrow X_{t\,new} = 1$ p.u.

Reactance Diagram :- (Assume $1\angle 0°$ pu. internal voltage on no load)



Thevenin Reactance (voltage short circuit.)





$$\Rightarrow X_{th} = 1 + \frac{0.35 \times 0.1}{0.35 + 0.1}$$

$$\Rightarrow jX_{th} = j1.078 \text{ p.u.}$$

Therefore, p.u. fault current, $I_{f(pu)} = \frac{1}{jX_{th}}$

$$\Rightarrow I_{f(pu)} = \frac{1}{j1.078}$$

$$\Rightarrow \boxed{I_{f(pu)} = -j0.928 \text{ p.u.}}$$

Base current on HV side of transformer

$$I_B = \frac{S_B}{\sqrt{3} V_{B2}} = \frac{20 \times 10^3}{\sqrt{3} \times 66}$$

$$\Rightarrow I_B = 174.955 \text{ A}$$

$\therefore$ Actual fault current, $|I_f| = |I_{fpu}| I_B$

$$= 0.928 \times 174.955$$

$$\Rightarrow \boxed{|I_f| = 162.358 \text{ A}}$$

10

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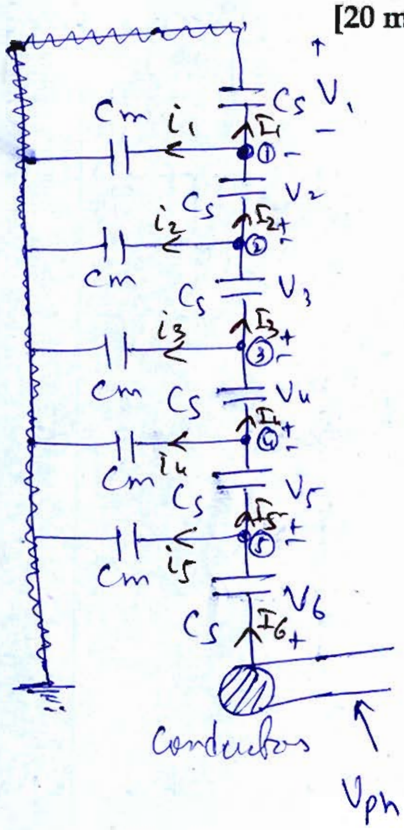
Q.4 (c) A string of six insulation unit has mutual capacitance 10 times the capacitance to ground. Determine the voltage across each unit as a fraction of the operating voltage. Also, determine string efficiency.

[20 marks]

Given: $n = 6$

& $10 C_m = C_s$

$\Rightarrow C_m = 0.1 C_s$



Here, $V_{ph} = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$ ——— (1)

By KCL at node (1)

$I_2 = i_1 + I_1$

$\Rightarrow V_2(j\omega C_s) = V_1(j\omega C_m) + V_1(j\omega C_s)$

$\Rightarrow V_2 C_s = V_1 C_m + V_1 C_s$

$\Rightarrow V_2 = \left[\frac{0.1 C_s + C_s}{C_s}\right] V_1$

$\Rightarrow V_2 = 1.1 V_1$ ——— (2)

KCL at node (2)

$I_3 = i_2 + I_2$

$V_3(j\omega C_s) = (V_1 + V_2)(j\omega C_m) + V_2(j\omega C_s)$

$\Rightarrow V_3 C_s = (V_1 + V_2)(0.1 C_s) + V_2 C_s$

$\Rightarrow V_3 = 0.1 V_1 + 1.1 V_2$

$\Rightarrow V_3 = 0.1 V_1 + 1.1(1.1) V_1$ (from (1))

$V_3 = 1.31 V_1$ ——— (3)

KCL at (3)

$I_4 = i_3 + I_3$

$\Rightarrow V_4(j\omega C_s) = (V_1 + V_2 + V_3)(j\omega C_m) + V_3(j\omega C_s)$

$\Rightarrow V_4 = \frac{(V_1 + 1.1 V_1 + 1.31 V_1)\,0.1 C_s + 1.31 V_1 C_s}{C_s}$ (using (2) & (3))

$\Rightarrow$ ~~$V_4 = 4.72 V_1$~~ ——— (4)

$\Rightarrow V_4 = 1.651 V_1$ ——— (4)

KCL at (4) $I_5 = i_4 + I_4$

$\Rightarrow V_5(j\omega C_s) = (V_1+V_2+V_3+V_4)(j\omega C_m) + V_4(j\omega C_s)$

$\Rightarrow V_5 C_s = (V_1 + 1.1V_1 + 1.31V_1 + 1.651V_1)\, j\omega\, 0.1C_s + 1.651V_1 C_s$

$\Rightarrow \boxed{V_5 = 2.1571V_1}$ — (5)

KCL at (5) $I_6 = i_5 + I_5$

$\Rightarrow V_6(j\omega C_s) = (V_1+V_2+V_3+V_4+V_5)(j\omega C_m) + V_5(j\omega C_s)$

$\Rightarrow V_6 C_s = (V_1 + 1.1V_1 + 1.31V_1 + 1.651V_1 + 2.1571V_1)\, 0.1C_s + 2.1571C_s V_1$

$\Rightarrow \boxed{V_6 = 2.879\,V_1}$ — (6)

Now, Phase Voltage will be

$V_{ph} = V_1+V_2+V_3+V_4+V_5+V_6$

$V_{ph} = V_1 + 1.1V_1 + 1.31V_1 + 1.651V_1 + 2.1571V_1 + 2.879V_1$

$\Rightarrow$ ~~$V_{ph} = 21.8871\,V_1$~~ $\boxed{V_{ph} = 10.0971\,V_1}$

∴ Voltage across each unit in terms of V_{ph} (operating vtg).

$\Rightarrow V_1 = \frac{1}{10.0971} V_{ph} \Rightarrow \boxed{V_1 = 0.099\,V_{ph}}$ — (α)

from (2), $V_2 = 1.1\,V_1 = 1.1 \times 0.099\,V_{ph}$ (∵ from α)

$\Rightarrow \boxed{V_2 = 0.1089\,V_{ph}}$

from (3), $V_3 = 1.31V_1 = 1.31 \times 0.099\,V_{ph}$ (∵ from α)

$\boxed{V_3 = 0.1297\,V_{ph}}$

from (4), $V_4 = 1.651V_1 = 1.651 \times 0.099\,V_{ph}$ (∵ using α)

$\boxed{V_4 = 0.1634\,V_{ph}}$

from (5), $V_5 = 2.1571V_1 = 2.1571 \times 0.099\,V_{ph}$ (∵ using α)

$\boxed{V_5 = 0.2136\,V_{ph}}$

from (6), $V_6 = 2.879V_1 = 2.879 \times 0.099\,V_{ph}$ (∵ using α)

$\boxed{V_6 = 0.285\,V_{ph}}$

Now, String efficiency

$$\%\eta = \frac{V_{ph}}{n \times V_6} \times 100$$

$$\%\eta = \frac{V_{ph}}{6[0.285 V_{ph}]} \times 100$$

$$\Rightarrow \boxed{\%\eta = 58.48\%}$$

18

Good Approach

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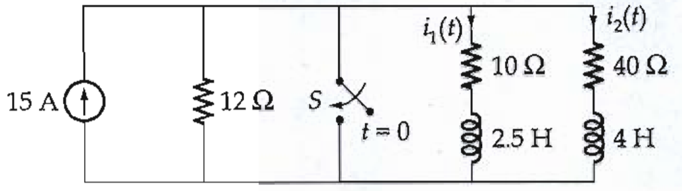
Section B : Systems and Signal Processing-1 + Microprocessor-1 + Electrical Circuits-2 + Control Systems-2

Q.5 (a) The switch 'S' in the circuit shown below is opened for a long time and closed at $t = 0$. Find the time domain expressions for currents $i_1(t)$ and $i_2(t)$ for $t > 0$.



[12 marks]

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Q.5 (b) The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(s+2)} \quad \text{——— (1)}$$

The system is to have 25% maximum overshoot and peak time 1.0 second. Determine the value of K and tachometer feedback constant K_r.

[12 marks]

$1 + q(s) = 1 + G(s)H(s) = 0$

$\Rightarrow 1 + \frac{K}{s(s+2)} = 0$

$\Rightarrow s^2 + 2s + K = 0$ ——— (2)

Also $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ ——— (3)

(2) & (3), $\omega_n = \sqrt{K}$ ——— (α)

$2\xi\omega_n = 2$

$\xi = \frac{1}{\omega_n} = \frac{1}{\sqrt{K}}$ ——— (β)

Try to avoid

Given $\%M_p = 25\% = 0.25$

$e^{-\xi\pi/\sqrt{1-\xi^2}}$

Given: $\% M_p = 25\% = 0.25$

$\Rightarrow e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.25$

$\Rightarrow \frac{-\xi\pi}{\sqrt{1-\xi^2}} = \ln(0.25)$

$\Rightarrow \boxed{\xi = 0.4}$ ——— (2)

& $t_p = \frac{\pi}{\omega_d} = 1 \text{ sec.}$

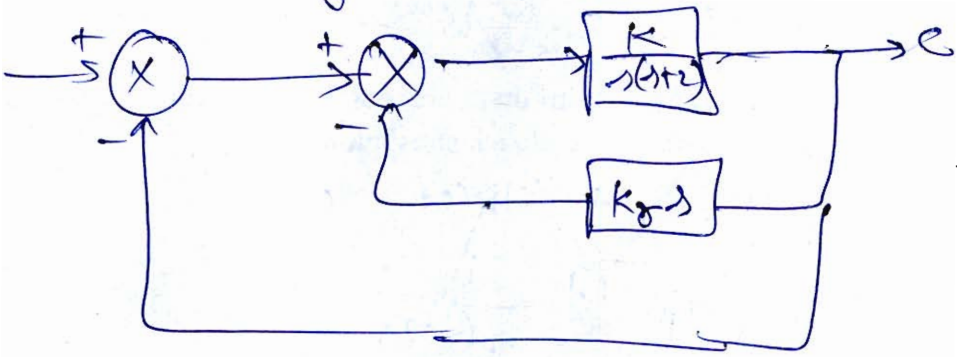
$\Rightarrow \omega_d = \pi$

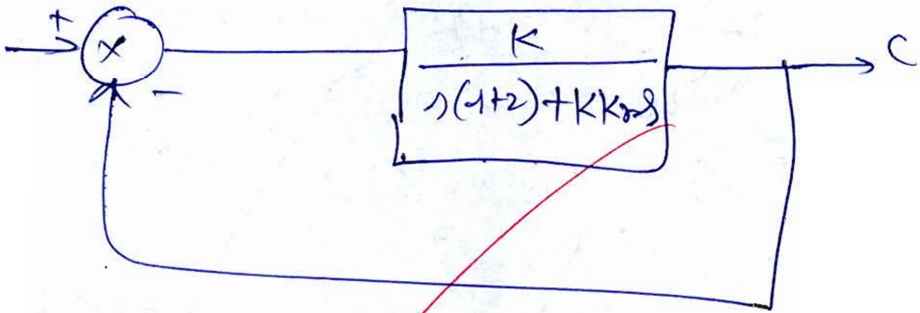
$\Rightarrow \omega_n\sqrt{1-\xi^2} = \pi$

$\Rightarrow \omega_n = \pi/\sqrt{1-\xi^2} = \pi/\sqrt{1-0.4^2}$

$\Rightarrow \boxed{\omega_n = 3.428 \text{ rad/sec}}$

with tachometer feedback.





$1 + G(s)H(s) = 0$

$\Rightarrow 1 + \frac{K}{s(s+2)+KK_r s} = 0$

$\Rightarrow s(s+2) + KK_r s + K = 0$

$\Rightarrow s^2 + (2+KK_r)s + K = 0$ —— (3)

Also $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ —— (4)

for (3) & (4) $\Rightarrow$

$\omega_n^2 = K$

$\Rightarrow K = (3.428)^2 \Rightarrow$ $K = 11.75$

& substituting all

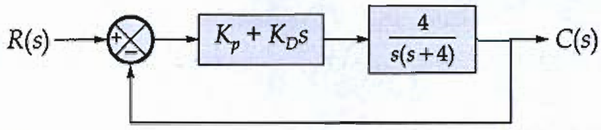
$2\xi\omega_n = 2 + KK_r$

$2 \times 0.4 \times 3.428 = 2 + 11.75 \times K_r$

$\Rightarrow$ $K_r = 0.0632$

11

Q.5 (c) A control system with PD controller is shown below :



Determine the value of K_P and K_D such that the damping ratio of the system will be 0.75 and the steady state error for unit ramp input will be 0.20.

[12 marks]

$G(s) = \frac{4(K_p + K_D s)}{s(s+4)}$ $\quad H(s) = 1 \quad , R(s) = \frac{1}{s^2}$

$e_{ss} = \lim_{s\to 0} s E(s) = \lim_{s\to 0} s \frac{R(s)}{[1 + G(s)H(s)]}$

$0.2 = \lim_{s\to 0} s \times \frac{1/s^2}{\left[1 + \frac{4(K_p + K_D s)}{s(s+4)}\right]}$

$0.2 = \lim_{s\to 0} \frac{1}{\left[s + \frac{4(K_p + K_D s)}{(s+4)}\right]}$

$\Rightarrow 0.2 = \frac{1}{0 + \frac{4(K_p + 0)}{4}} = \frac{1}{K_p}$

$\Rightarrow \boxed{K_p = 5}$

Also, $1 + G(s)H(s) = 0$

$1 + \frac{4(K_p + K_D s)}{s(s+4)} = 0$

$\Rightarrow s(s+4) + 4K_p + 4K_D s = 0$

$\Rightarrow s^2 + (4 + 4K_D)s + 4K_p = 0$

$\Rightarrow s^2 + (4 + 4K_D)s + 4 \times 5 = 0$

$s^2 + (4 + 4K_D)s + 20 = 0$

& $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

Given $\xi = 0.25$

& $\omega_n^2 = 20 \Rightarrow \omega_n = \sqrt{20} = 4.472$ r/s.

$\Rightarrow 2\xi\omega_n = 4 + 4K_D$

$\Rightarrow 2 \times 0.25 \times 4.472 = 4 + 4K_D$

$\Rightarrow K_D = 0.677$

11

Good Approach

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Q.5 (d) The Fourier transform $X(\omega)$ of a continuous time periodic signal $x(t)$ is given by

$$X(\omega) = j\delta\left(\omega - \frac{\pi}{3}\right) + 2\delta\left(\omega - \frac{\pi}{7}\right) \quad \text{——— (1)}$$

Determine :

(i) The fundamental frequency of the signal $x(t)$.

(ii) The complex Fourier series coefficients of the signal $x(t)$.

(iii) The time domain expression of $x(t)$.

[12 marks]

(i) From (1), $\omega_{01} = \frac{\pi}{3}$

$\omega_{02} = \pi/7$

$\therefore \omega_0 = HCF(\omega_{01}, \omega_{02})$

$\omega_0 = \frac{\pi}{21}$ rad/sec.

(ii) & (iii) :

$1 \xrightarrow{F.T} 2\pi\delta(-\omega) = 2\pi\delta(\omega)$

$e^{j\omega_0 t} \xrightarrow{F.T} 2\pi\,\delta(\omega - \omega_0)$

or $\frac{A}{2\pi} e^{j\omega_0 t} \xrightarrow{F.T.} A\delta(\omega - \omega_0)$

Using above.

$$x(t) = \frac{1}{2\pi}\left[j\, e^{j\frac{\pi}{3}t}\right] + \frac{1}{2\pi}\left[2\, e^{j\frac{\pi}{7}t}\right]$$

$$\Rightarrow x(t) = \frac{j}{2\pi} e^{j\pi/3\, t} + \frac{1}{\pi} e^{j\pi/7\, t}$$

$$\Rightarrow x(t) = \frac{j}{2\pi} e^{j\left(\frac{\pi}{21}\right)\times 7t} + \frac{1}{\pi} e^{j\left(\frac{\pi}{21}\right)3t}$$

$$\boxed{x(t) = \frac{j}{2\pi} e^{j7\omega_0 t} + \frac{1}{\pi} e^{j3\omega_0 t}}$$

$$\frac{j}{2\pi} e^{\frac{j\pi}{3}t} + \frac{1}{\pi} e^{\frac{j\pi}{7}t}$$

Also, $x(t) = C_7 e^{j7\omega_0 t} + C_3 e^{j3\omega_0 t}$

Complex Exponential F.S. Coefficient

$C_7 = \frac{j}{2\pi}$ & $C_3 = \frac{1}{\pi}$

& $x(t) = \frac{j}{2\pi} e^{j7\omega_0 t} + \frac{1}{2\pi} e^{j3\omega_0 t}$

when $\omega_0 = \frac{\pi}{21}$ rad/sec.

10

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Q.5 (e) Explain the following instructions:

(i) XCHG **(ii)** IN **(iii)** OUT **(iv)** DAA

[12 marks]

(i) XCHG :- exchange the value of b/w two register pair

(ii) IN :- increment the value of default register

(iii) OUT :- Serial output of the accumulator bit.

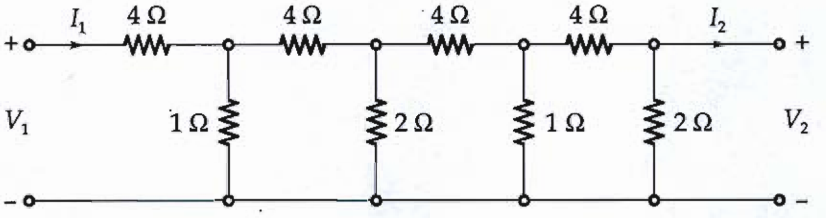
(iv) DAA :- Double addition of content with accumulator

Elaborate it more

4

Q.6 (a) **(i)** Two 2-port network are connected in cascade. Prove that the overall transmission parameter matrix equals to the multiplication of individual transmission parameter matrices.

(ii) Determine the transmission parameters of the 2-port network shown in the figure below:



[20 marks]

Q.6 (b) **(i)** Explain the similarities and differences between :

1. JUMP and CALL instructions.
2. STA and STAX instructions.

[10 marks]

Q.6 (b) (ii) Write an assembly language program for an 8085 microprocessor, to find 2's complement of a 16-bit number. Write comments for selected instructions.

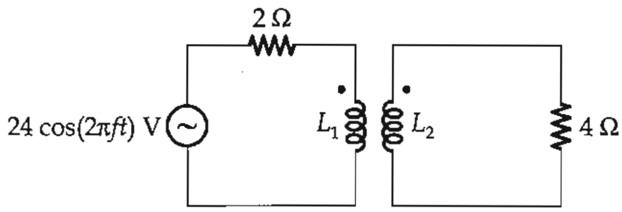
[10 marks]

Q.6 (c) Check whether given signal $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)} u(2t-n)$ is periodic. If yes, compute its average power.

[20 marks]

Q.7 (a) The coupled circuit shown below has a coefficient of coupling $K = 1$. Determine the energy stored in the mutually coupled inductor at $t = 5$ msec.

$L_1 = 3.185$ mH; $L_2 = 12.74$ mH; $f = 50$ Hz



[20 marks]

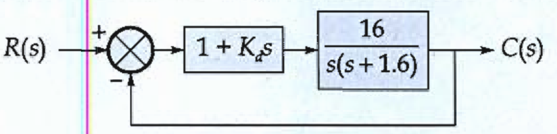
Q.7 (b) Obtain eigen values, eigen vectors and the state model in canonical form for a system described by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = [1 \;\; 0 \;\; 0]\, x(t)$$

[20 marks]

Q.7 (c) A control system employing proportional and derivative control as shown below, has damping ratio equal to 0.8. Find the time instant at which the step response of system attains the peak value. Also find the percent maximum overshoot of system.



[20 marks]

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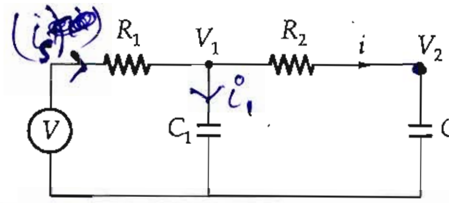
Q.8 (a) (i) Determine the state model for the network shown below considering $V_1 = x_1$; $V_2 = x_2$ and $y = i$.



Given → State variables ★
$x_1 = V_1$
& $x_2 = V_2$
⇒ Input $U = V$

[10 marks]

$V_2 = \frac{1}{C_2}\int i\,dt$ —— ① (Ohm's law)

$i = \frac{V_1 - V_2}{R_2}$ —— ② (Ohm's law)

Do not use like this symbol

By KCL at V_1

~~$V_1 = \frac{1}{C_1}\int i_1 dt = \frac{1}{C_1}\int (i + i_1)$~~ $i_s = i_1 + i$

$\Rightarrow \frac{V - V_1}{R_1} = i_1 + \frac{V_1 - V_2}{R_2}$

$\Rightarrow i_1 = \frac{V - V_1}{R_1} - \frac{V_1 - V_2}{R_2}$

$i_1 = \frac{V}{R_1} - V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{V_2}{R_2}$ —— ③

Now $V_1 = \frac{1}{C_1}\int i_1 dt = \frac{1}{C_1}\int\left[\frac{V}{R_1} - V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{V_2}{R_2}\right]dt$ —— ④

⇒ ~~$V_1 = \ldots$~~

Differentiating eqn ① w.r.t. t

$\frac{dV_2}{dt} = \dot{x}_2 = \frac{1}{C_2} i$ (using eq ②)

$\Rightarrow \dot{x}_2 = \frac{1}{C_2}\left[\frac{V_1 - V_2}{R_2}\right] = \frac{1}{R_2C_2}V_1 - \frac{1}{R_2C_2}V_2$

$\Rightarrow \dot{x}_2 = \frac{1}{R_2C_2}x_1 - \frac{1}{R_2C_2}x_2$ —— (α)

Differentiating eqn. ④ w.r.t. 't'

$\frac{dV_1}{dt} = \dot{x}_1 = \frac{1}{R_1C_1}V - \frac{1}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_1 + \frac{1}{R_2C_1}V_2$

$$\Rightarrow \dot{x}_1 = \frac{1}{R_1C_1}U - \frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)x_1 + \frac{1}{R_2C_2}x_2 \quad \text{———} (\beta)$$

& output $y = i = \dfrac{V_1 - V_2}{R_2}$

$$\Rightarrow y = \frac{1}{R_2}V_1 - \frac{1}{R_2}V_2$$

$$\Rightarrow y = \frac{1}{R_2}x_1 - \frac{1}{R_2}x_2 \quad \text{———} (\gamma)$$

State Eq.

$$[\dot{x}] = \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + [B]U$$

Output

$$y = [C]\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + [D]U$$

8

6.5

∴ From eq: $(\alpha), (\beta) \& (\gamma)$

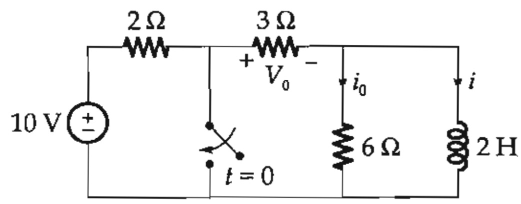
State Eq.

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\left[-\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right] & \frac{1}{R_2C_2} \\ \frac{1}{R_2C_2} & -\frac{1}{R_2C_2}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{R_1C_1} \\ 0\end{bmatrix}U$$

Output Eq.

$$y = \begin{bmatrix}\frac{1}{R_2} & -\frac{1}{R_2}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + 0.U$$

Q.8 (a) (ii) In the circuit shown below:



Find i_0, V_0 and i for all time, assuming that the switch was open for a long time.

[10 marks]

Q.8 (b) Consider a discrete time system with input $x(n)$ and output $y(n)$ related by

$$y(n) = \sum_{k=n-n_0}^{n+n_0} x(k) \quad \text{——— (1)}$$

where n_0 is a finite positive integer

(i) Is this system linear?

(ii) Is this system time-invariant?

(iii) If $x(n)$ is known to be bounded by a finite integer B_x [i.e. $|x(n)| < B_x$ for all n], it can be shown that $y(n)$ is bounded by a finite number C. We conclude that the given system is stable. Express C in terms of B_x and n_0.

[20 marks]

(i) Linearity:

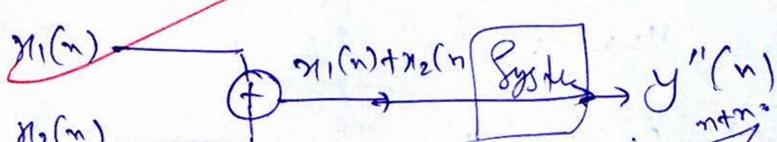
Additivity:-

Path 1: $x_1(n) \xrightarrow{Sys} y_1(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k)$

$x_2(n) \xrightarrow{System} y_2(n) = \sum_{k=n-n_0}^{n+n_0} x_2(k)$

$y_1(n) + y_2(n) = y'(n) = \sum_{k=n-n_0}^{n+n_0} (x_1(k) + x_2(k))$

Path 2



$= \sum_{k=n-n_0}^{n+n_0} (x_1(k) + x_2(k))$

$\because y'(n) = y''(n)$

$\therefore$ Additive.

Homogeneity:-

Path 1: $x(n) \xrightarrow[\text{Gain}]{A} Ax(n) \xrightarrow{System} y'(n) = \sum_{k=n-n_0}^{n+n_0} Ax(k)$

$\Rightarrow y'(n) = A \sum_{k=n-n_0}^{n+n_0} x(k)$

Path 2: $x(n) \xrightarrow{System} \sum_{k=n-n_0}^{n+n_0} x(k) \xrightarrow[A]{Gain} y''(n) = A \sum_{k=n-n_0}^{n+n_0} x(k)$

$\because y'(n) = y''(n)$

$\therefore$ Homogeneous

$\because$ Both additive & homogeneous

$\therefore$ System linear.

(ii) Path 1

$$x(n) \xrightarrow[m]{\text{delay}} x(n-m) \xrightarrow{\text{System}} \sum_{k=n-n_0}^{n+n_0} x(k-m) = y_1(n)$$

Path 2

$$x(n) \xrightarrow{\text{System}} \sum_{k=n-n_0}^{n+n_0} x(k) \xrightarrow[m]{\text{Delay}} y_2(n) = \sum_{k=n-m-n_0}^{n-m+n_0} x(k)$$

$\therefore \; y_1(n) \neq y_2(n)$

So System is Time variant

$\therefore$ System is not time-invariant.

(iii) $$y(n) = \sum_{k=n-n_0}^{n+n_0} x(k) \quad \text{—— (A)}$$

where $|x(n)| < B_x$

i.e. $-B_x < x(n) < B_x$ for all n —— (1)

Also, $|y(n)| < C$

$\Rightarrow -C < y(n) < C$ —— (2)

from (1)

$$\sum_{k=n-n_0}^{n+n_0} -B_x < \sum_{k=n-n_0}^{n+n_0} x(k) < \sum_{k=n-n_0}^{n+n_0} B_x$$

$$\Rightarrow -B_x \sum_{k=n-n_0}^{n+n_0} 1 < y(n) < B_x \sum_{k=n-n_0}^{n+n_0} 1$$

$$\Rightarrow -B_x [1+1+ \ldots + 1 \ldots] < y(n) < B_x [1+ \ldots +1] \text{ till } (n+n_0)$$

$$\Rightarrow -B_x [1(n+n_0) - 1(n-n_0)] < y(n) < B_x [1(n+n_0) - 1(n-n_0)]$$

$$\Rightarrow -B_x [2n_0] < y(n) < B_x [2n_0]$$

$$\Rightarrow -2B_x n_0 < y(n) < 2B_x n_0 \quad \text{--- (3)}$$

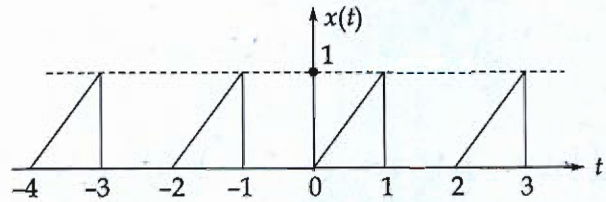
Comparing eqn. (2) & (3)

$$C = 2B_x n_0$$

$$C \leq (2n_0 + 1) \cdot B_x$$

(10)

Q.8 (c) Find the trigonometric Fourier series for the waveform shown in figure and sketch the line spectrum.



[20 marks]

$$x(t) = \begin{cases} t & ; \quad 0 \le t \le 1 \\ 0 & ; \quad 1 < t \le 2 \end{cases}$$ with period $T_0 = 2$ units.

Fourier series, $x(t) = a_0 + \sum_{n=1}^{\infty}[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$ —— ①

The fundamental angular frequency, $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi$

∴ eqn ①, $x(t) = a_0 + \sum_{n=1}^{\infty}[a_n \cos n\pi t + b_n \sin n\pi t]$ —— (α)

Now, $a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt = \frac{1}{2}\left[\int_0^2 x(t)dt\right] = \frac{1}{2}\left[\int_0^1 t\,dt + \int_1^2 0\,dt\right]$

$\Rightarrow a_0 = \frac{1}{2}\left[\frac{t^2}{2}\Big|_0^1 + 0\right] = \frac{1}{2}\left(\frac{1}{2}(1^2 - 0^2)\right)$

$\Rightarrow \boxed{a_0 = 1/4}$

& $a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\,dt = \frac{2}{2}\int_{T_0} x(t)\cos n\pi t\,dt = \int_0^2 x(t)\cos n\pi t\,dt$

$\Rightarrow a_n = \int_0^1 t\cos n\pi t\,dt \int_1^2 0 \cos n\pi t\,dt = \left[\frac{t\sin n\pi t}{n\pi}\right]_0^1 + \left[\frac{\cos n\pi t}{(n\pi)^2}\right]_0^1 + 0$

$\Rightarrow a_n = \frac{\sin n\pi}{n\pi} + \frac{1}{(n\pi)^2}[\cos n\pi - \cos 0]$

$\Rightarrow a_n = \frac{0}{n\pi} + \frac{1}{(n\pi)^2}[(-1)^n - 1]$

$\Rightarrow a_n = \frac{(-1)^n - 1}{(n\pi)^2}$

$$\Rightarrow a_n = \begin{cases} 0 & ; \ n \text{ is even} \\ -2/(n\pi)^2 & ; \ n \text{ is odd.} \end{cases}$$

& $b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega_0 t\,dt = \frac{2}{2}\int_0^2 x(t)\sin n\pi t\,dt$

$\Rightarrow b_n = \int_0^1 t\sin n\pi t\,dt + \int_1^2 0\cdot\sin n\pi t\,dt = \int_0^1 t\sin n\pi t\,dt$

$\Rightarrow b_n = \frac{t(-\cos n\pi t)}{n\pi}\Big|_0^1 + \left[\frac{\sin n\pi t}{(n\pi)^2}\right]_0^1$

$\Rightarrow b_n = -\frac{1}{n\pi}\left[\cos n\pi - 0\right] + \frac{1}{(n\pi)^2}\left[\sin n\pi - \sin 0\right]$

$\Rightarrow b_n = -\frac{(-1)^n}{n\pi} + \frac{1}{(n\pi)^2}\left[0-0\right]$

$\Rightarrow b_n = \frac{-(-1)^n}{n\pi} = \frac{(-1)(-1)^n}{n\pi}$

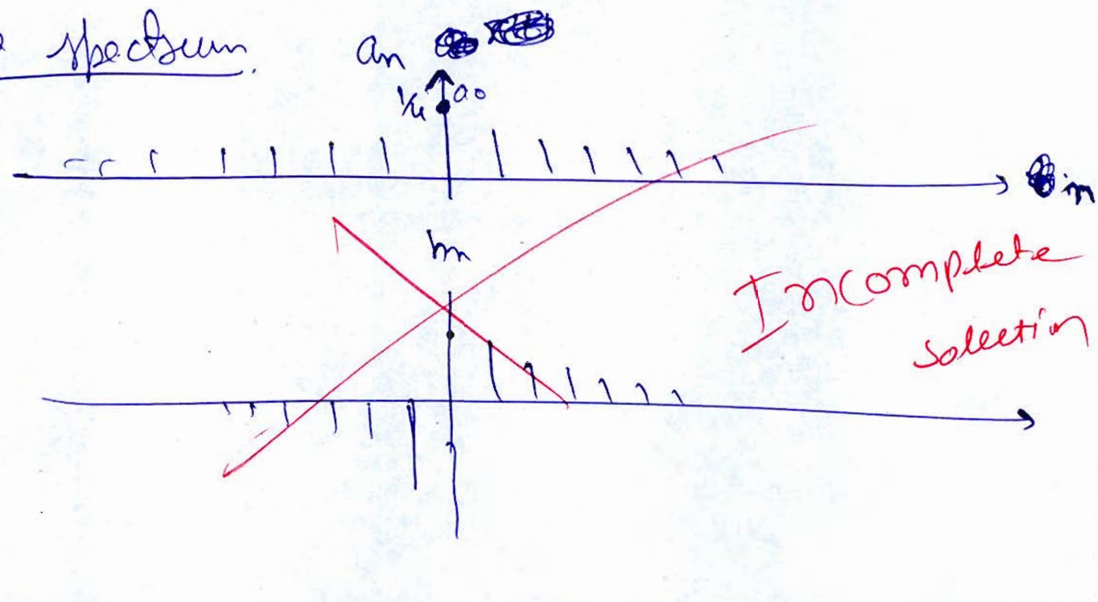
or $\boxed{b_n = \frac{(-1)^{n+1}}{(n\pi)}}$

16

Substituting a_0, a_n & b_n in eq^n (α)

$$\boxed{x(t) = \frac{1}{4} + \sum_{n=1,3,5,7,\ldots}^{\infty}\left[\frac{-2}{(n\pi)^2}\cos n\pi t\right] + \sum_{n=1,2,3,4,\ldots}^{\infty}\frac{(-1)^{n+1}}{(n\pi)}\sin n\pi t}$$

Line spectrum.



Incomplete solution

OOOO

Space for Rough Work

Space for Rough Work