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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-3 : Strength of Materials [All Topics]

Highway Engg. - 1 + Surveying and Geology-1 + Geo-technical & Foundation Engg. - 2 + Environmental Engg. - 2 [Part Syllabus]

Name :

Roll No

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Kolkata ☐	Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	40
Q.2	44
Q.3	58
Q.4	
Section-B	
Q.5	28
Q.6	40
Q.7	
Q.8	
Total Marks Obtained	210

Signature of Evaluator: Ujjawal

Cross Checked by: ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

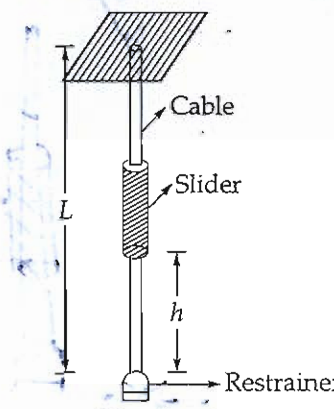
Remarks

- Practice more questions to avoid silly mistakes.
- Presentation can be improved.
- Revise theory portion continuously.

Section A : Strength of Materials

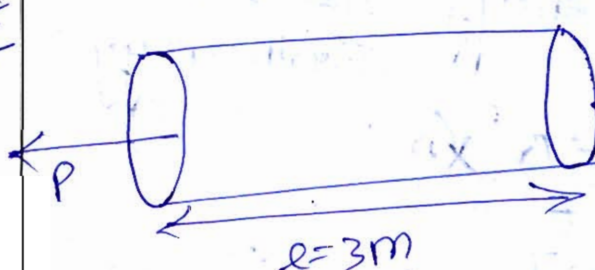
Q.1 (a) (i) A prismatic bar of circular cross-section is loaded by an axial tensile force of 85 kN. The bar has length of 3.0 m and diameter of 30 mm. It is made of aluminium with modulus of elasticity E = 70 GPa and Poission's ratio of $\frac{1}{3}$. Calculate the decrease in diameter 'Δd' and the change in volume of the bar. Assume the stresses in the bar are below proportionality limit.

(ii) A cable with a restrainer at the bottom hangs vertically from its upper end as shown in figure. The cable has cross sectional area of 50 mm^2 and modulus of elasticity E = 130 GPa. A slider of mass m = 40 kg drops from a height of h = 1.2 m on to the restrainer. If the allowable stress in the cable under an impact load is 700 MPa, then, what is the minimum permissible length L of the cable? (Take g = 9.81 m/s^2)



[6 + 6 = 12 marks]

Ans:



Axial tensile force = 85 kN

d = 30 mm

E_{al} = 70 GPa

$\mu = \frac{1}{3}$

Now axial stress $\sigma_x = \frac{P}{A} = \frac{85\times10^3}{\frac{\pi}{4}(30)^2} = 120.25\ \frac{N}{mm^2}$

Now

$\frac{\Delta V}{V} = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$

Now longitudinal strain $= \frac{\sigma_x}{E} = \frac{120.25}{70\times10^3} = 1.7178\times10^{-3}$

(6)

$\frac{-\text{lateral strain}}{\text{longitudinal strain}}$ = Poisson's ratio

$\Rightarrow$ lateral strain $= -1.7178\times10^{-3}\times\frac{1}{3} = -5.726\times10^{-4}$

Now $\Delta d = -5.726\times10^{-4}\times30 = \boxed{-0.01718\ mm}$

decrease in diameter.

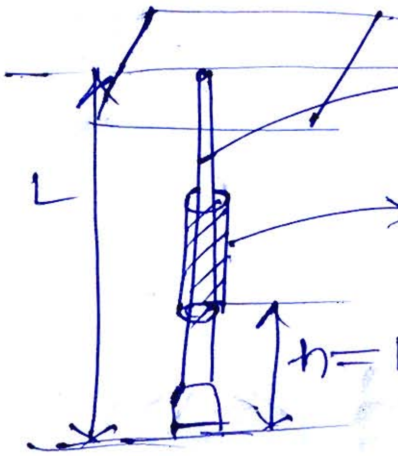
$\because \sigma_y = \sigma_z = 0$

$\Rightarrow \frac{\Delta V}{V} = \frac{120.25\,(1-2\times\frac{1}{3})}{70\times10^3} = 5.7262\times10^{-4}$

∴ change in vol^m $\Rightarrow \delta V = 5.7262 \times 10^{-4} \times \frac{\pi}{4}(30)^2 \times 3000$

$= \boxed{1214.281 \, mm^3}$

(iii)



Now Decrease in potential energy = Stored at Strain energy

$\Rightarrow mg(h + \delta_{max}) = \frac{\sigma^2}{2E} \times (AL)$

$\Rightarrow 40 \times 9.81 \left(1200 + \frac{\sigma}{E} \times L\right) = \frac{\sigma^2}{2E}(AL)$

$\left[\because \delta = \text{deformation under impact load} = \frac{\sigma}{E} \times L\right]$

$\Rightarrow 40 \times 9.81 \left(1200 + \frac{700}{130 \times 10^3} \times L\right) = \frac{700^2 \times 50 \times L}{2 \times 130 \times 10^3}$

$\Rightarrow 470880 + 2.113 L = 94.23 L$

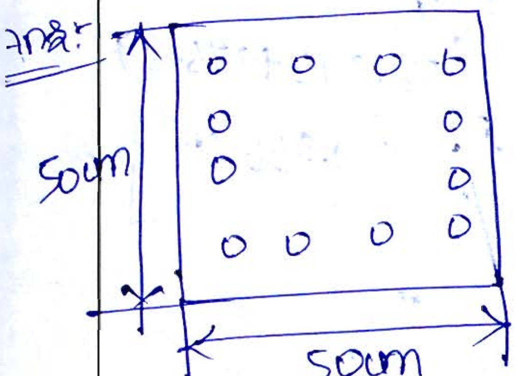
$\Rightarrow \boxed{L = 5111.76 \, mm}$ or $\boxed{5.11 \, m}$

6

Q.1 (b) A reinforced concrete pedestal of height h and square cross-section having side of 50 cm is constructed with 12 steel reinforcing bars each has a diameter, $d = 25$ mm. The pedestal supports a compressive load P applied through a rigid bearing plate. Assuming linear elastic behaviour, calculate the maximum permissible value of the load if the allowable stress in the concrete and steel are 8 MPa and 70 MPa respectively. Neglect the self weight of the pedestal itself. (Assume $E_s = 200$ GPa and $E_c = 25$ GPa).

[12 marks]

Ans:



$E_s = 200$ GPa
$E_c = 25$ GPa

$\phi_{st} = 25$ mm, $n = 12$.

$\sigma_{permissible\ concrete} = 8$ MPa

$\sigma_{permissible\ steel} = 70$ MPa

Now we know that $P_{Total} = P_{steel} + P_{concrete}$

Now $\delta_{steel} = \delta_{concrete}$; $\therefore \delta = \frac{Pl}{AE}$

$\Rightarrow \frac{P_{st} \times l}{5890 \times 200 \times 10^3} = \frac{P_{con} \times l}{244109.514 \times 25 \times 10^3}$

$A_{st} = 12 \times \frac{\pi}{4} \times 25^2$
$= 5890.486\ mm^2$

$A_c = 500^2 - A_{st}$
$= 500^2 - 5890.486$
$= 244109.514\ mm^2$

$\Rightarrow P_{st} = 0.193 P_{conc}$

$\Rightarrow \frac{\sigma_{st}}{E_{st}} = \frac{\sigma_{conc}}{E_{con}}$

Now $\frac{\sigma_{st}}{200 \times 10^3} = \frac{\sigma_{conc}}{25 \times 10^3} \Rightarrow \sigma_{st} = 8\sigma_{conc}$.

Taking $\sigma_{st} = 70$ MPa $\Rightarrow \sigma_{conc} = \frac{70}{8} = 8.75$ MPa > $\sigma_{permissible}$ not O.K

Taking $\sigma_{con} = 8$ MPa $\Rightarrow \sigma_{st} = 8 \times 8 = 64$ MPa < 70 MPa

So O.K.

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∴ Load carried by concrete = $\sigma_{conc} \cdot A_{conc} \times 10^{-3}$ kN

$= 8 \times 244109.514 \times 10^{-3} = 1952.876$ kN

Load carried by steel $= 64 \times 5890.486 \times 10^{-3}$ kN

$= 376.99$ kN.

∴ Total load carrying capacity $= 1952.876 + 376.99$

$= 2329.866$ kN

8

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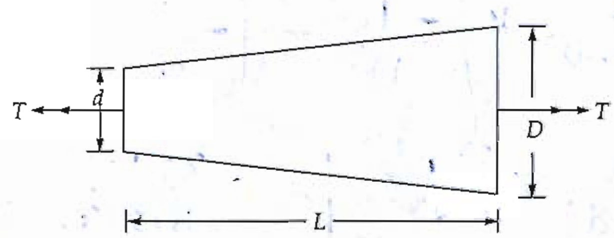
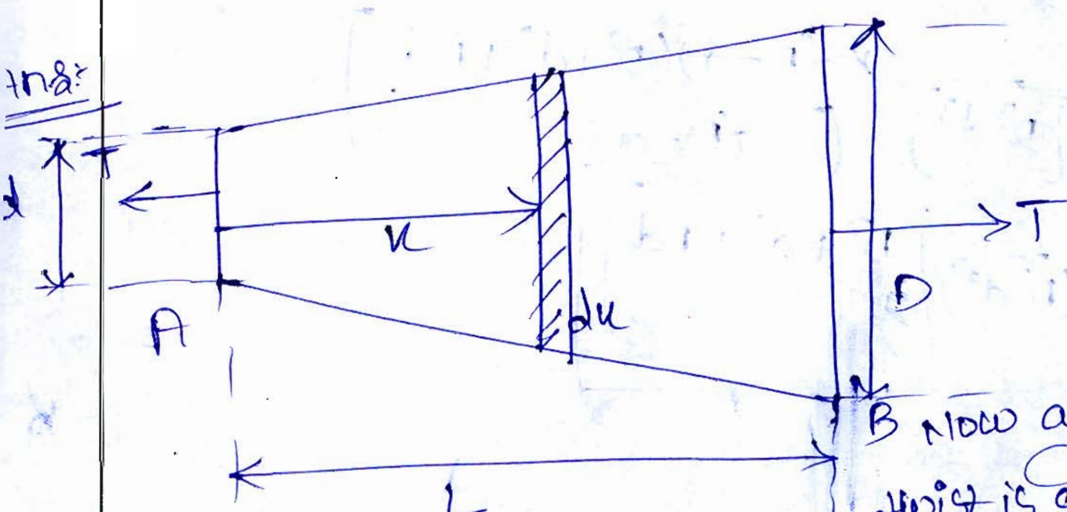
Q.1 (c) A tapered bar AB of solid circular cross section is twisted by torques 'T' applied at the ends. The diameter of the bar varies uniformly from 'd' at left end to 'D' at the right end. Derive an expression for the angle of twist 'ϕ' of the bar. The length of the bar is 'L' and shear modulus of the material used in the bar is G..



[12 marks]

Ans:



Now angle of twist is given by $\boxed{\theta = \frac{TL}{GI_p}}$

∴ Considering an element at x distance from end A of thickness dx.

∴ $d_x = d + \left[\frac{D-d}{L}\right] \cdot x$ ⇒ $\boxed{d_x = d + Ax}$, $A = \left[\frac{D-d}{L}\right]$

Now $d\theta_x = \frac{T\,dx}{G I_x}$, $I_x = \frac{\pi}{32}(d_x)^4$

⇒ $d\theta_x = \frac{T\,dx}{G \times \frac{\pi}{32}(d+Ax)^4}$ → angle of twist for small thickness dx

Integrating it over length

⇒ $\int d\theta_x = \int_0^L \frac{32T\,dx}{G\pi(d+Ax)^4} = \frac{32T}{G\pi}\int_0^L \frac{dx}{(d+Ax)^4}$

$$\theta = \frac{32T}{G\pi}\left[\frac{1}{A}\cdot\frac{(d+Ax)^{-4+1}}{-4+1}\right]_0^L = \frac{32T}{3G\pi}\times\frac{1}{A}\left[\frac{-1}{(d+Ax)^3}\right]_0^L$$

$$= \frac{32T}{3G\pi}\times\frac{L}{(D-d)}\times\left[-\frac{1}{D^3}-\left(-\frac{1}{d^3}\right)\right]$$

$$= \frac{32TL}{3G\pi(D-d)}\times\left[\frac{1}{d^3}-\frac{1}{D^3}\right] = \frac{32TL}{3G\pi(D-d)}\left[\frac{D^3-d^3}{D^3d^3}\right]$$

$$= \frac{32TL}{3G\pi[D^3d^3]}\times\left[\frac{(D-d)(D^2+d^2+Dd)}{(D-d)}\right]$$

$$= \frac{32TL}{3G\pi(D^3d^3)}\left[D^2+d^2+Dd\right]$$

(12)

Q.1 (d) Briefly explain the need of theories of elastic failure. Describe the maximum strain theory and maximum shear strain energy theory along with formulae (derivation not required) and limitations associated with them.

[4 + 4 + 4 = 12 marks]

Ans: The materials used often fail abruptly without indicating any clear failure point, so to deal with such situations & providing a safe design, we need a particular design stress which can be used to design & predict the failure mode of various materials. Hence the theories of elastic failure helps us to predict the failure stress of various materials, & various situations in which they are applicable.

(i) maxm strain theory : von mises theory

$$\frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\nu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]=\frac{\sigma_y^2}{FOS}\times\frac{1}{2E}.$$

failure stress = Elliptical.

(ii) maxm shear strain energy theory:

$$\frac{1}{2\times 6G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]=\frac{1}{12G}\frac{\sigma_y^2}{FOS}$$

8

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Q.1 (e) An aluminium bar has length $L = 120$ cm and diameter $d = 2.5$ cm. The initial straight line part of the stress-strain curve has a slope (modulus of elasticity) of 6.89×10^{10} N/m^2. The bar is loaded by tensile force, $P = 250$ kN beyond elastic limit. It reaches to a strain value of 0.045 and then unloaded.

(a) What is the permanent set of the bar?

(b) What is the modulus of resilience after the load application?

[12 marks]

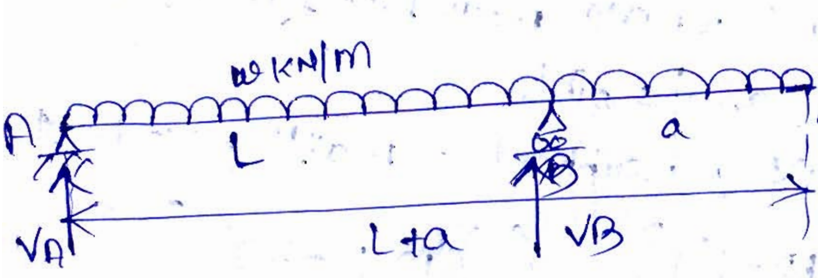
Ans:

Q.2 (a) (i) A 12 m long beam of uniform section carries a uniformly distributed load of 18 kN/m over the whole length. If the beam is freely supported at the left end, find the position of second support so that the maximum bending moment for the beam shall be as small as possible. Also, find the maximum bending moment for this case.

(ii) State Castigilano's first theorem and Maxwell's reciprocal theorem.

[16 + 4 = 20 marks]

Ans:



$w = 18\,kN/m$

$L + a = 12\,m$

$(a = 12 - L)$

Now $V_A + V_B = w(L+a)$, $\Sigma M_A = 0$

$\Rightarrow V_B \times L = \frac{w}{2}(L+a)^2$

$$V_B = \frac{w}{2L}(L+a)^2$$

Putting in values

$V_B = \frac{18}{2L}(12)^2 = \frac{1296}{L}$

& $V_A = 18 \times 12 - \frac{1296}{L} = \frac{216L - 1296}{L}$

Now B.M at support B = $\boxed{\dfrac{-wa^2}{2}}$

Now point of 0 shear force b/w supports A & B ⇒ $V_A - wK = 0$

$\Rightarrow \dfrac{216L - 1296}{L} = 18K \Rightarrow \boxed{K = \dfrac{12L - 72}{L}}$

∵ point where shear force is 0, is where bending moment is maxm

∴ Maxm of +ve B.M = $V_A \cdot K - \dfrac{wK^2}{2}$

$\Rightarrow \dfrac{216L - 1296}{L} \cdot \dfrac{12L - 72}{L} - \dfrac{18}{2}\left[\dfrac{12L - 72}{L}\right]^2$

$\Rightarrow \dfrac{18}{L^2}\left[12L - 72\right]^2 - \dfrac{9}{L^2}\left[12L - 72\right]^2$

$\Rightarrow \boxed{\dfrac{9}{L^2}\left[12L - 72\right]^2}$

Now for point where maxm B.M for this beam is as small as possible

⇒ −ve B.M @ support = maxm +ve B.M

$\Rightarrow \dfrac{18}{2}\left[12 - L\right]^2 = \dfrac{9}{L^2}\left[12L - 72\right]^2$

$\Rightarrow L^2\left[12 - L\right]^2 = \left[12L - 72\right]^2$

$\Rightarrow L^2\left[144 + L^2 - 24L\right] = \left[144L^2 + 5184 - 1728L\right]$

$\Rightarrow 144L^2 + L^4 - 24L^3 = 144L^2 + 5184 - 1728L$

$\Rightarrow L^4 - 24L^3 + 1728L - 5184 = 0$

$\Rightarrow L = 8.485\ m,\ 3.515\ m$

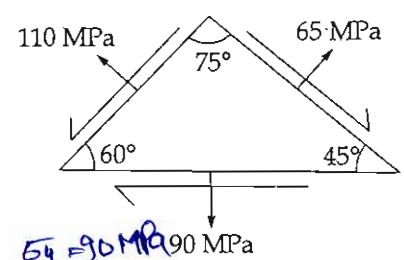
$\therefore\ a = 12 - 8.485 = \boxed{3.515 m}$

$L = 8.485 m$ & $\boxed{a = 3.515 m}$

BM

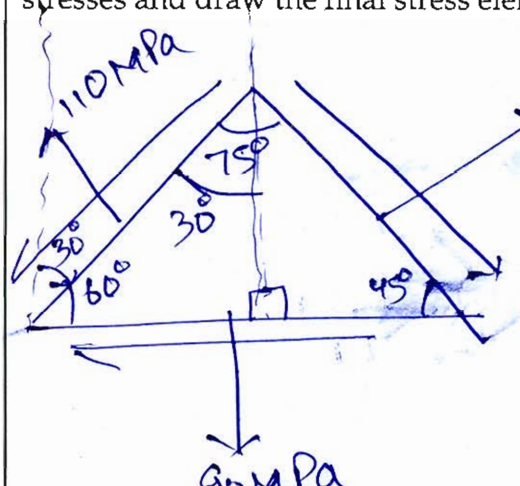
12

Q.2 (b) In a strained body, the normal stresses on three planes inclined as shown in figure are 65 MPa (Tensile), 90 MPa (Tensile) and 110 MPa (Tensile). Determine the shear stresses acting on these planes. Also find the principle stresses and draw the final stress element.

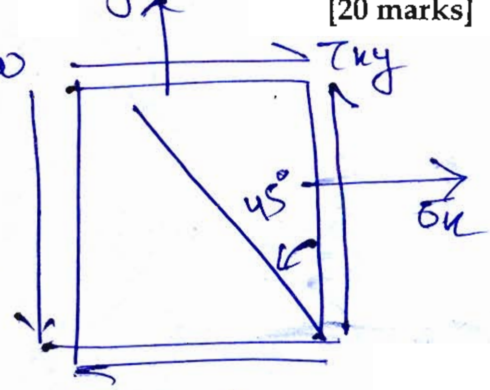


[20 marks]

Ans:



Now



σ_x ?
τ_{xy} ?
$\sigma_y = 90$ MPa

@ $\theta = 45^\circ$, $\sigma_x' = 65$ MPa

& @ $\theta = -30^\circ = 150^\circ$, $\sigma_x' = 110$ MPa.

$\sigma_x' = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \cos\theta \sin\theta$

Putting $\theta = 45^\circ$

$\Rightarrow 65 = \sigma_x \cos^2 45^\circ + 90 \sin^2 45^\circ + 2\tau_{xy} \sin 45^\circ \cos 45^\circ$

$\Rightarrow 65 = \frac{\sigma_x}{2} + 45 + \tau_{xy} \Rightarrow \frac{\sigma_x}{2} + \tau_{xy} = 20$ — (I)

Putting $\theta = 150^\circ$

$\Rightarrow 110 = \sigma_x \cos^2 150^\circ + 90 \sin^2 150^\circ + 2\tau_{xy} \sin 150^\circ \cos 150^\circ$

$\Rightarrow 110 = 0.75\sigma_x + 22.5 + (-0.866\tau_{xy})$

$\Rightarrow 87.5 = 0.75\sigma_x - 0.866\tau_{xy}$ — (II)

Solving (I)st & (II)nd we get

$\boxed{\sigma_x = 88.605 \text{ MPa}}$, $\boxed{\tau_{xy} = -24.302 \text{ MPa}}$

$\tau_{x'y'} = (\sigma_y - \sigma_x)\cos\theta \sin\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$

$\therefore \tau_{x'y'}$ @ $\theta = 45^\circ$

$\tau_{x'y'} = (90 - 88.605)\cos 45^\circ \sin 45^\circ - 24.302(\cos^2 45^\circ - \sin^2 45^\circ)$

$\Rightarrow \boxed{\tau_{x'y'} = 0.6975 \text{ MPa}}$

$\tau_{x'y'}$ @ $\theta = 150^\circ$

$\tau_{x'y'} = (90 - 88.605)\cos 150^\circ \sin 150^\circ - 24.302(\cos^2 150^\circ - \sin^2 150^\circ)$

$\boxed{\tau_{x'y'} = -12.755 \text{ MPa}}$

$\sigma_{p1}/\sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}$

$= \frac{90 + 88.605}{2} \pm \frac{1}{2}\sqrt{(88.605 - 90)^2 + 4 \times (-24.302)^2}$

$\sigma_{p1} | \sigma_{p2} = 89.3025 \pm 24.312$

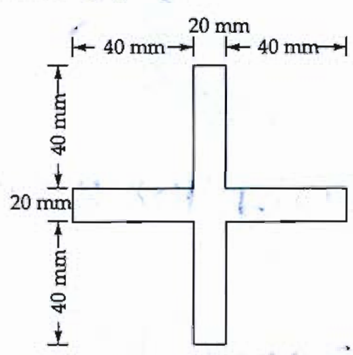
$\Rightarrow \sigma_{p1} = 113.6145$ MPa , $\sigma_{p2} = 64.9905$ MPa

$\tan 2\theta_{pmax} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times (-24.302)}{88.605 - 90}$

$\Rightarrow \theta_{pmax} = \boxed{44.178^\circ}$

20

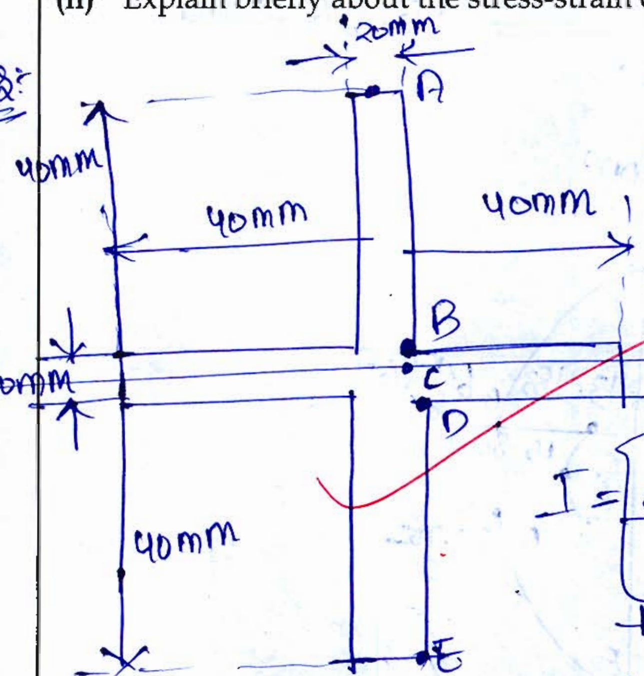
Q.2 (c) (i) Plot the shear stress distribution over the beam cross-section as shown in figure due to a shear force of 100 kN.



(ii) Explain briefly about the stress-strain diagram for mild steel.

[12 + 8 = 20 marks]

Ans:



S.F = 100 kN

12

Now $\tau_A = \tau_E = 0$

$\because A\bar{y} = 0$

$$\tau = \frac{V(A\bar{y})}{It}$$

$$I = \left[\frac{20 \times 40^3}{12} + 20 \times 40 \times (30)^2\right] \times 2 + \frac{100 \times 20^3}{12} = 1720000 \text{ mm}^4$$

Now τ_B = (just above)

$$= \frac{100\times10^3 \times \left[40\times20\times\left[\frac{40}{2}+10\right]\right]}{172\times10^4 \times 20}$$

$= 69.767$ MPa

τ_B (just below)

$$= \frac{100\times10^3 \times \left[40\times20\times\left[\frac{40}{2}+10\right]\right]}{172\times10^4 \times 100}$$

$= 13.953$ MPa.

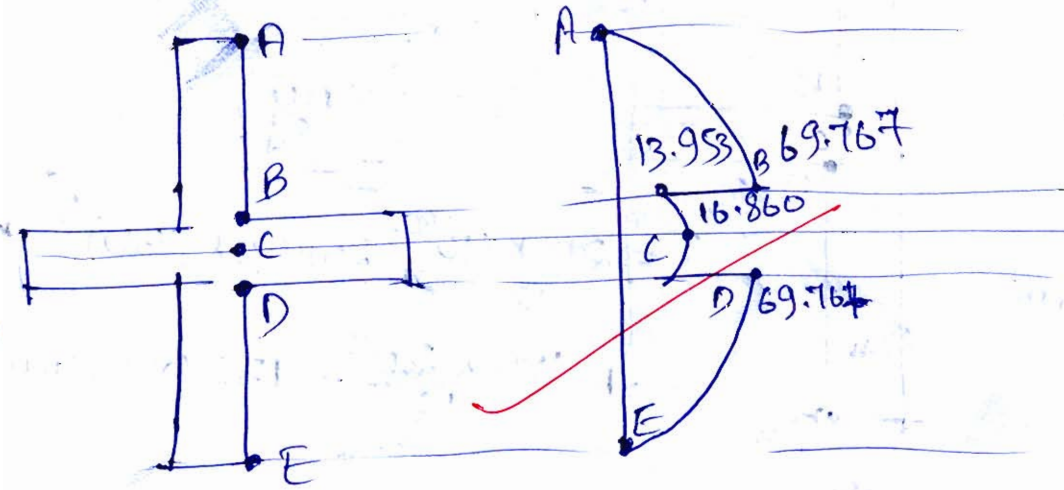
Now similarly for τ_D →
- just below = 69.767 MPa
- just above = 13.953 MPa

Now for C:-

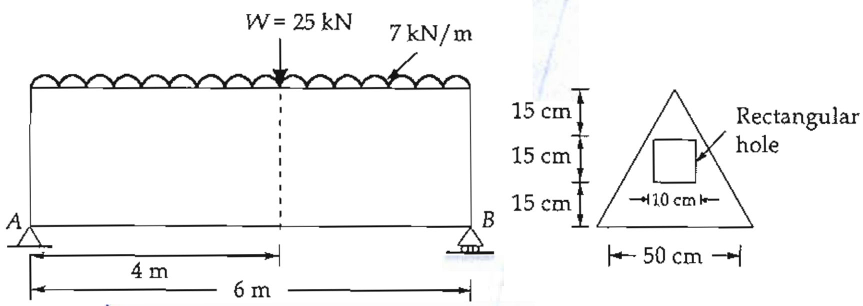
$A\bar{y} = 40\times20\times\left[\frac{40}{2}+10\right] + 10\times100\times\frac{10}{2} = 29000\ mm^3$

$$\therefore \tau_C = \frac{100\times10^3\times[29000]}{172\times10^4\times100} = 16.860\ MPa.$$

∴ Shear Stress Distribution:

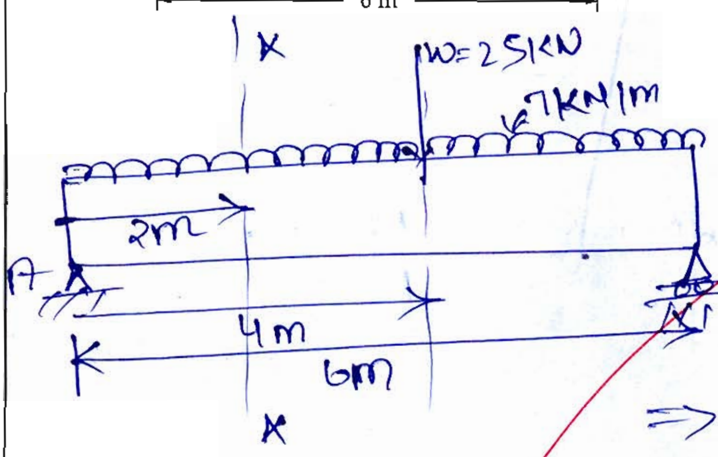


Q.3 (a) A simply supported beam and it's cross-section are shown below. The beam carries a point load of 25 kN as shown in figure. The self weight of the beam is 7 kN/m. Determine the maximum value of bending compressive stress at a section 2 m from end *A*.



[20 marks]

Ans:



$V_A + V_B = 7 \times 6 + 25$

$= 67\ kN$

$\& \ \Sigma M_B = 0$

$\Rightarrow V_A \times 6 = 25 \times 2 + 7 \times 6 \times \frac{6}{2}$

$V_A = 29.33\ kN$

$\Rightarrow V_B = 37.67\ kN$

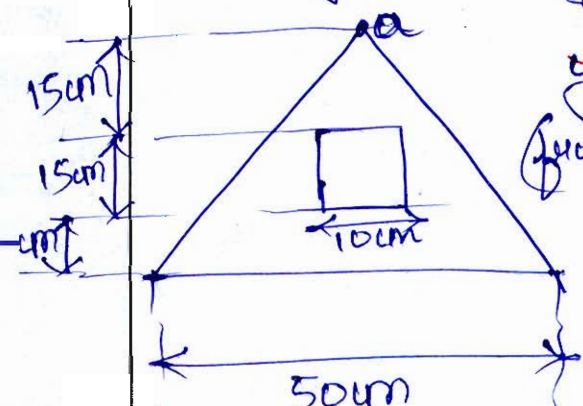
Now B.M at section x – x

$\Rightarrow V_A \times 2 - 7 \times 2 \times \frac{2}{2} = 29.33 \times 2 - 7 \times 2 \times \frac{2}{2}$

$= \boxed{44.66 \text{ kN-m}}$

Now for the section Bending moment stress

$\frac{\sigma}{y} = \frac{M}{I} \Rightarrow \boxed{\sigma = \frac{M}{I} \times y}$



$y_{c.m} = \dfrac{\frac{1}{2} \times 50 \times 45 \times \left[\frac{45}{3}\right] - 15 \times 10 \times \left[\frac{15}{2} + 15\right]}{\frac{1}{2} \times 50 \times 45 - 15 \times 10}$ (from bottom)

Depth of neutral axis $= 13.846$ cm from bottom

18

Now $I_{NA} = \frac{bh^3}{36} + \frac{1}{2} bh (y_1 - y_{NA})^2 - \left[\frac{Bd^3}{12} + Bd (y_2 - y_{NA})^2\right]$

$= \frac{50 \times 45^3}{36} + \frac{1}{2} \times 50 \times 45 (15 - 13.846)^2 - \left[\frac{10 \times 15^3}{12} + 10 \times 15 \times (22.5 - 13.846)^2\right]$

$= 128060.6805 - 14046.2574 = 114014.4231 \text{ cm}^4$

$\therefore \sigma_{bending}$ = stress at top fibre A

$\Rightarrow \frac{44.66 \times 10^6}{114014.4231 \times 10^4 \text{ mm}^4/\text{cm}^4} \times [45 - 13.846] = \boxed{122.0317 \ \frac{N}{mm^2}}$

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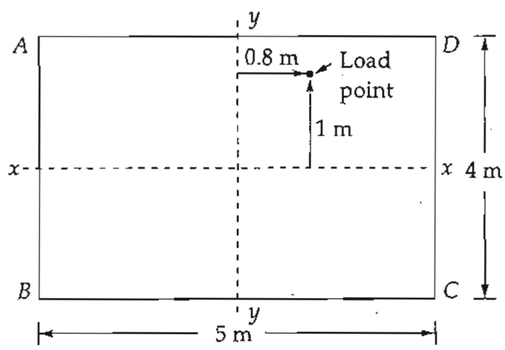
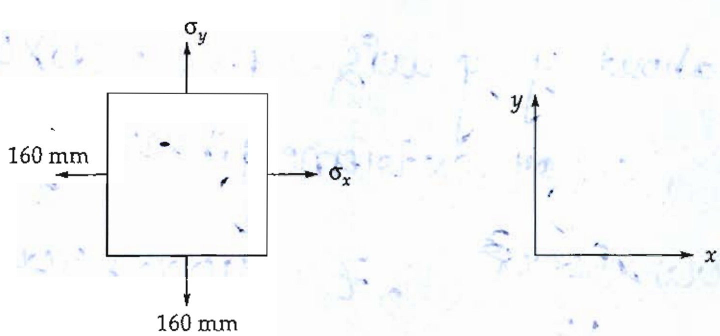
Q.3 (b) (i) A masonry pier as shown in the figure supports a load of 60 kN.



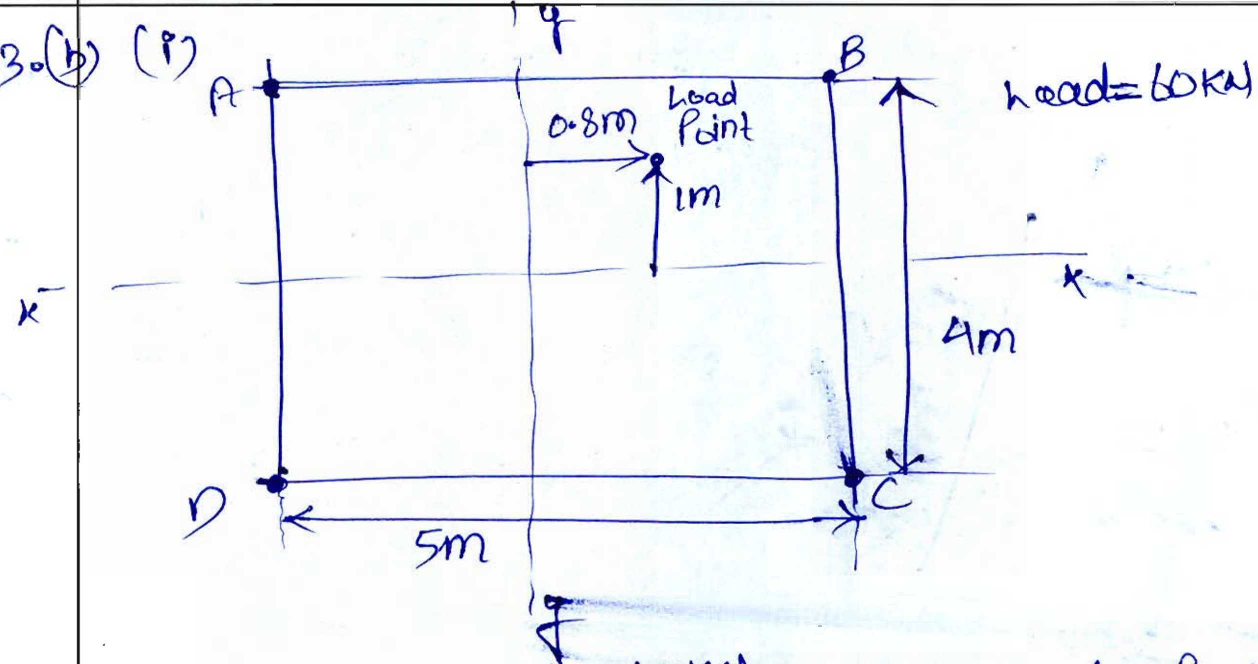
1. Find the stress developed at each corner.
2. Find the value of additional load required to be placed at centre of pier so that there is no tension anywhere in the column section.

(ii) A thin rectangular steel plate 100 mm by 100 mm undergoes elongation of 0.05 mm and 0.03 mm in x and y directions due to stresses σ_x and σ_y respectively as shown in figure. Determine the magnitude of σ_x and σ_y if modulus of elasticity of steel is 200 GPa and Poisson's ratio is 0.3.



[15 + 5 = 20 marks]

Q3.(b) (i)



Now Direct load = 60kN

(i) Compressive stress due to direct point load = $\frac{P}{A}$

$$= \frac{60 \times 10^3}{5000 \times 4000} = 3 \times 10^{-3} \frac{N}{mm^2}$$

(ii) Moment about x-x axis = $P.e_y = 60 \times 1 = 60$ kN-m

$$I_{xx} = \frac{(5000) \times (4000)^3}{12}, \quad Z = \frac{5000 \times (4000)^2}{6}$$

$\therefore \sigma = \frac{M}{I} \times y$, for extreme fibres $= \frac{M}{Z}$

$$\Rightarrow \sigma_{xx} = \frac{6 \times 60 \times 10^6}{5000 \times 4000^2} = 4.5 \times 10^{-3} \frac{N}{mm^2}$$

(iii) Moment about y-y axis $\Rightarrow P.e_x = 60 \times 0.8 = 48$ kN-m

$\therefore \sigma = \frac{M}{I} \times y$, $\therefore$ for extreme fibres $= \frac{M}{Z}$

$$I_{yy} = \frac{(4000) \times (5000)^3}{12}, \quad Z = \frac{4000 \times 5000^2}{6}$$

$$\therefore \sigma_{yy} = \frac{6 \times 48 \times 10^6}{4000 \times 5000^2} = 2.88 \times 10^{-3} \frac{N}{mm^2}$$

Stress at various corner points:
(tensile = -ve, compressive = +ve)

$\Rightarrow A = 3\times10^{-3} + 4.5\times10^{-3} - 2.88\times10^{-3} = 4.62\times10^{-3}\ \frac{N}{mm^2}$

$B = 3\times10^{-3} + 4.5\times10^{-3} + 2.88\times10^{-3} = 10.38\times10^{-3}\ \frac{N}{mm^2}$

$C = 3\times10^{-3} - 4.5\times10^{-3} + 2.88\times10^{-3} = 1.38\times10^{-3}\ \frac{N}{mm^2}$

$D = 3\times10^{-3} - 4.5\times10^{-3} - 2.88\times10^{-3} = -4.38\times10^{-3}\ \frac{N}{mm^2}$

15

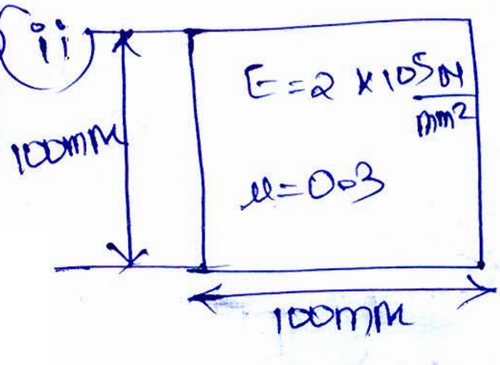
(ii) Additional load required to be placed at centre of pier so that there is no tension anywhere

$\Rightarrow -4.38\times10^{-3} = \frac{P}{A}$ (∴ tension at D where max tension develops in entire pier = 0)

$\Rightarrow$ ~~P = 4.38 × 10^-3 × 5000 × 4000~~

$P = 4.38\times10^{-3}\times5000\times4000\ \frac{kN}{10^3 N} = \boxed{87.6\ kN}$

Q3(b)(ii)



~~εx =~~ $\Delta L_x = 0.05\,mm$

$\varepsilon_x = \frac{0.05}{100} = 5\times10^{-4}$

$\Delta L_y = 0.03\,mm$

$\varepsilon_y = \frac{0.03}{100} = 3\times10^{-4}$

Now $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} \Rightarrow 5\times10^{-4} = \frac{\sigma_x}{2\times10^5} - \frac{0.3\sigma_y}{2\times10^5}$

$\Rightarrow \sigma_x - 0.3\sigma_y = 100$ — (1)

Similarly $\varepsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} \Rightarrow 3\times10^{-4} = \frac{\sigma_y}{2\times10^5} - \frac{0.3\sigma_x}{2\times10^5}$

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$\Rightarrow 3\times10^{-4} \times 2\times10^{5} = \sigma_y - 0.3\sigma_x$

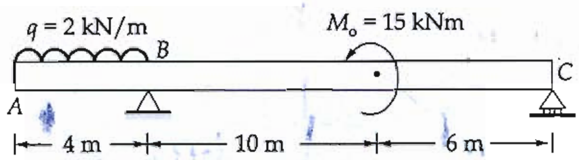
$\Rightarrow \boxed{60 = \sigma_y - 0.3\sigma_x}$ — (ii)

(3)

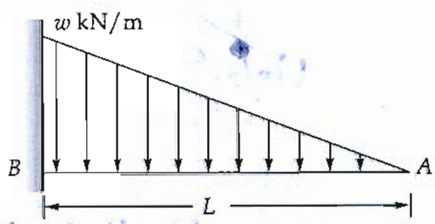
Solving (i)st & (ii)nd we get:

$\boxed{\sigma_x = 129.67\ MPa}$, $\boxed{\sigma_y = 98.901\ MPa}$

Q.3 (c) (i) A beam *ABC* with an overhang at the left hand end is shown in figure below. The beam is subjected to a uniform load of intensity $q = 2$ kN/m on the overhang *AB* and a counterclockwise couple $M_o = 15$ kN-m as shown. Draw the shear-force and bending moment diagram for this beam.

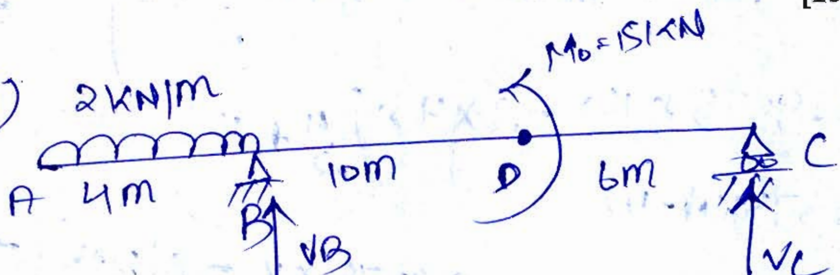


(ii) Find slope and deflection at free end using double integration method for the beam shown in figure.



[10 + 10 = 20 marks]

Ans: (c)(i)



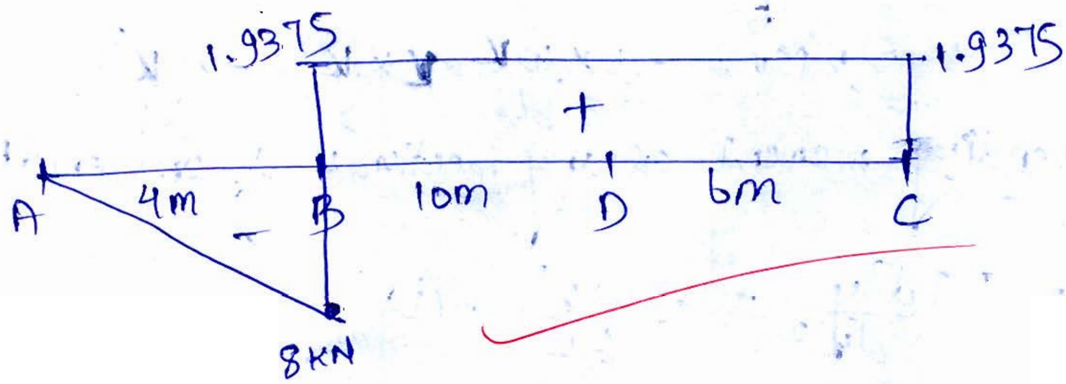
Now $V_B + V_C = 2 \times 4 \Rightarrow V_B + V_C = 8\text{ kN}$

$\Sigma M_C = 0 \Rightarrow V_B \times 16 = 2 \times 4 \times \left[\frac{4}{2} + 16\right] + 15$

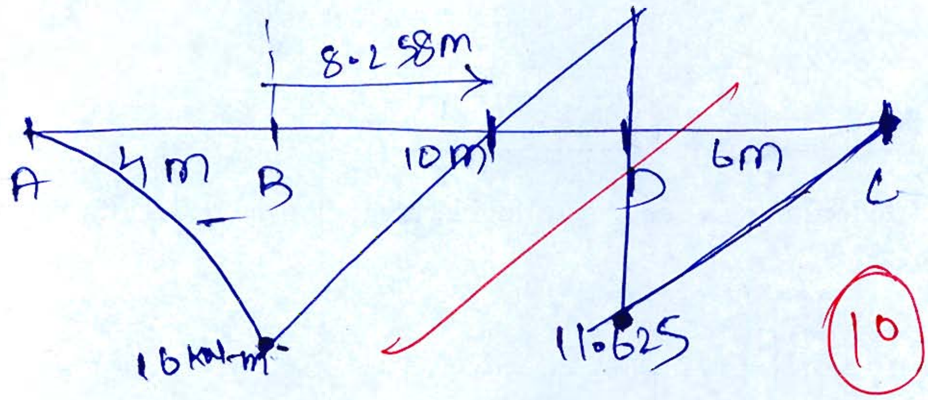
$\Rightarrow \boxed{V_B = 9.9375\text{ kN}}$ & $\boxed{V_C = -1.9375\text{ kN}}$

S.F.D: For part $AB \Rightarrow -2 \times x$

@ $B \Rightarrow -2 \times 4 = -8\text{ kN}$



B.M.D:- For Part AB ⇒ $B.M = \frac{-wx^2}{2}$, @ $B = \frac{-2\times4^2}{2} = -16 kNm$



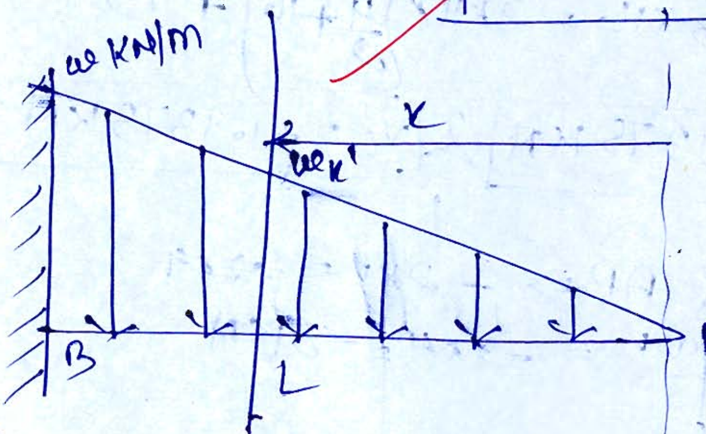
For part BD : $B.M = V_B \cdot x - 2\times4\times\left[\frac{4}{2}+x\right]$

@ D $= 9.9375\times10 - 2\times4\times\left[\frac{4}{2}+10\right]$
(x=10)
$= 3.375 kN\text{-}m$

Point of contraflexure $= 9.9375\times x - 8[2+x] = 0$

$\Rightarrow \boxed{x = 8.258m}$

(ii)



$\Rightarrow \frac{wx'}{x} = \frac{w}{L}$

$\Rightarrow wx' = \frac{wx}{L}$

$B.M_x = -\frac{1}{2}\times wx'\times x\times\frac{x}{3}$

$\Rightarrow B.M_x = -\frac{1}{2}\times\frac{wx}{L}\times x\times\frac{x}{3} = \frac{-wx^3}{6L}$

Bending moment at any section x from end A.

$\Rightarrow EI\frac{d^2y}{dx^2} = \frac{-wx^3}{6L}$ — (1)

Integrating w.r.t x

$$EI\frac{dy}{dx} = \frac{-wx^4}{24L} + C_1 \quad \text{—(ii)}$$

Integrating again w.r.t x

$$EIy = \frac{-wx^5}{120L} + C_1x + C_2 \quad \text{—(iii)}$$

Now we know that:- (i) @ $x = L$, $\frac{dy}{dx} = 0$

$$\Rightarrow 0 = \frac{-wL^3}{24} + C_1 \Rightarrow \boxed{C_1 = \frac{wL^3}{24}}$$

(ii) @ $x = L$, $y = 0$

$$\Rightarrow 0 = \frac{-wL^4}{120} + \frac{wL^3}{24} \times L + C_2$$

$$\Rightarrow C_2 = \frac{wL^4}{120} - \frac{wL^4}{24} \Rightarrow C_2 = \boxed{\frac{-wL^4}{30}}$$

Putting value of C_1 & C_2 in eqn (ii) & (iii)

$$EI\theta = \frac{-wx^4}{24L} + \frac{wL^3}{24}, \ @ x=0, \ \boxed{\theta = \frac{wL^3}{24EI}}$$

$$EIy = \frac{-wx^5}{120L} + \frac{wL^3}{24}x - \frac{wL^4}{30}$$

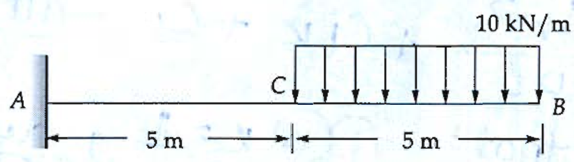
$$\Rightarrow @ x=0, \ \boxed{y = \frac{-wL^4}{30EI}}$$

→ Slope & Deflection @ free end.

10

Q.4 (a) (i) Explain the application of the moment area method to determine the slope and deflection.

(ii) Find the angle of rotation θ_B and deflection δ_B by using moment area method at the free end B of cantilever beam ACB supporting a uniform load of intensity 10 kN/m acting over the beam as shown. (The beam has length 10 m and constant flexural rigidity EI)



[8 + 12 = 20 marks]

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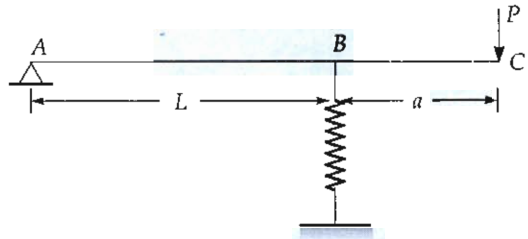
Q.4 (b) **(i)** A steel shaft is to be manufactured as a circular tube. The shaft is required to transmit a torque of 1250 N-m without exceeding an allowable shear stress of 40 MPa nor an allowable rate of twist of 0.75/m. The shear modulus of elasticity of the steel is 78 GPa. Determine the required diameter of the shaft if the thickness of the shaft is $1/10^{th}$ of the outer diameter.

(ii) A prismatic shaft consists of a solid aluminum rod of diameter 35 mm, which is inside a steel tube of external diameter 52 mm. Both the shafts are firmly jointed and subjected to a torque of 1025 N-m. Find the maximum stresses developed in aluminium and steel shafts.

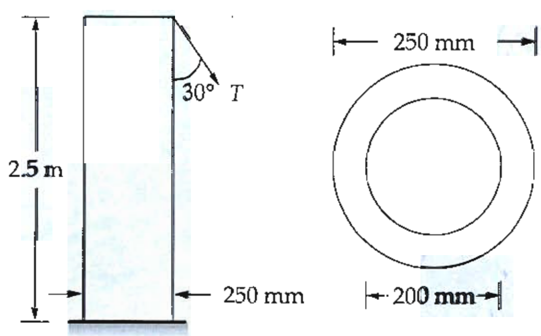
[Take, $G_{Al} = 70 \times 10^9$ N/m² and $G_{st} = 80 \times 10^9$ N/m²]

[12 + 8 = 20 marks]

Q.4 (c) (i) An overhanging beam ABC rests on a simple support at A and a spring support at B (see figure). A concentrated load P acts at the end of the overhang. Span AB has length L, the overhang has length 'a' and the spring has stiffness k. Determine the downward displacement at the end of overhang i.e. at C using Castingliano's theorem.



(ii) A vertical pole of aluminium is fixed at the base and pulled at the top by a cable having a tensile force T as shown. The cable is attached to the outer surface and makes an angle of 30° at the point of attachment. The dimensions of the pole are shown in figure. Determine the allowable tensile force T in the cable if the allowable compressive stress in the aluminium pole is 100 MPa.



[12 + 8 = 20 marks]

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Section B : Highway Engineering - 1 + Surveying and Geology-1 + Geo-technical and Foundation Engineering - 2 + Environmental Engineering - 2

Q.5 (a) What are the various factors that control the highway alignment? List out the special care which should be taken while aligning roads in hilly areas?

[12 marks]

Ans: There are many special factors which needed to be controlled for highway alignment:-

(i) super-elevation provided is tobe controlled as the highways are designed for more design speed as compared to urban roads & hence adequate super-elevation needs to be provided.

(ii) Longitudinal gradient:- It should be ensured that not excessive gradient is provided which may hinder the movement of vehicles at high speed & becomes a cause for accidents.

(iii) Sight distance:- It should be ensured that adequate sight distance is available for the driver as ~~Highway~~ it's absence may result in accidents.

(iv) Extra-widening:- It should be ensured that sufficient extra widening is provided to the highway so that vehicles can safely negotiate the curves.

2

only geometric part

Q.5 (b) (i) Classify different types of survey based on their purposes.

(ii) A 30 m chain was found to be 5 cm too long after chaining 1650 m. It was 8 cm too long at the end of day's work after chaining a total distance of 3125 m. If the chain was correct before commencement of the work, determine the true distance.

[8 + 4 = 12 marks]

Ans:

(ii) Distance 1 = 1650 m & ΔL = 5 cm too long.

chain was correct before commencement of the work

∴ ΔL = 0 cm @ Distance = 0

∴ $\Delta L_{avg} = \frac{5+0}{2} = 2.5$ cm & $L_1' = 30.025$ m

Now True length × True Distance = False length × False Distance

$\Rightarrow 30 \times D_1 = 30.025 \times 1650$

$\Rightarrow D_1 = 1651.375$ m

For Distance 2: chain was 8 cm too long at end

$\Rightarrow \Delta L_{avg} = \frac{5+8}{2} = 6.5$ cm. $L_2' = 30.065$

$D_2' = 3125 - 1650 = 1475$ m

$\therefore 30 \times D_2 = 30.065 \times 1475$

$\Rightarrow D_2 = 1478.196$ m

∴ Total distance chained = 1651.375 + 1478.196

= 3129.571 m

4

(i) Types of survey based on their purpose:

(a) Archeological survey ⇒ It is done to search for archeological sites & ancient ruins so that we can rediscover our lost history.

(ii) Geological Survey :- It is done to know about the soil crusts, different types & layers of soils present, & unearth different rare minerals and elements found in our earth.

(iii) Topographical survey :- It is done to map out the various topographical features present e.g. like lakes, rivers, mountains etc. So we appropriately draw the map.

4

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Q.5 (c) Briefly explain the two commonly used geophysical methods of soil exploration.

[12 marks]

Q.5 (d) The speed of overtaking and overtaken vehicles are 80 kmph and 50 kmph, respectively on a two way traffic road. The average acceleration during overtaking may be assumed as 0.99 m/sec^2.

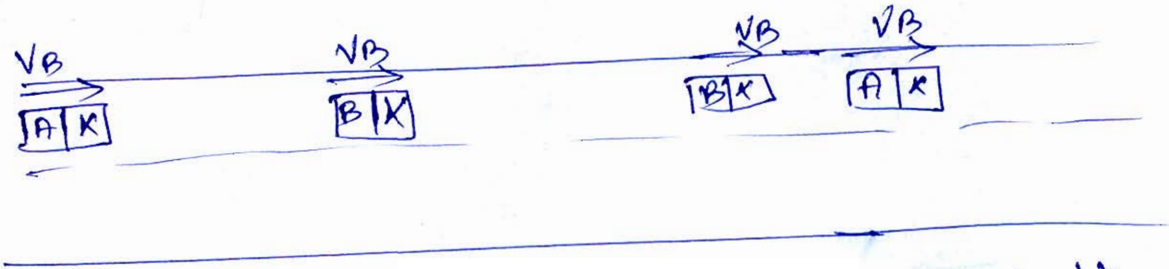
(i) Calculate safe overtaking sight distance.

(ii) What are the minimum and desirable length of overtaking zones?

Assume any other data as per IRC-37.

[12 marks]

Ans:



$a = 0.99\ m/s^2$, $V_B = 50\ kmph$, $V_d = 80\ kmph$.

(i) Total safe overtaking sight distance (OSD) $= d_1 + d_2 + d_3$

$d_1 = 0.278 \times V_B \times t_r$ Let reaction time be 2 sec

$= 0.278 \times 50 \times 2 = 27.8\ m$.

$d_2 = 0.278\, V_B T + \frac{1}{2} a T^2$

Now $T = \sqrt{\frac{4s}{a}}$, s = space headway b/w vehicles A & B before & after overtaking operation.

$s = 0.2 V_B + 6$

$\Rightarrow s = 0.2 \times 50 + 6 = 16\ m$

$\therefore T = \sqrt{\frac{4 \times 16}{0.99}} = 8.04\ sec$

$\therefore d_2 = 0.278 \times 50 \times 8.04 + \frac{1}{2} \times 0.99 \times 8.04^2$

$= 143.753\ m$

$d_3 = 0.278 V_d T = 0.278 \times 80 \times 8.04 = 178.809\ m$

$\therefore$ Total length of overtaking sight distance $= 27.8 + 143.753 + 178.809 = 350.362\ m$

(ii) minimum length of overtaking zone = 3×OSD

= 3×350.362 = **1051.086m**

Desirable length of overtaking zone = 5×OSD

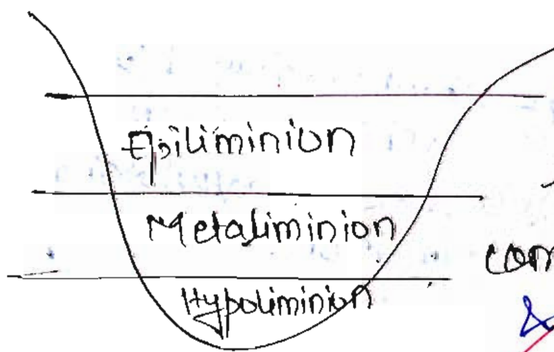
= 5×350.362 = **1751.81m**

12

Q.5 (e) Describe stratification of lakes and biological zones in lakes.

[12 marks]

Ans: Stratification of lakes:-



(i) The 1st layer of the lake is known as epiliminion here sufficient sunlight comes, so fishes, algae, & oxygen is present abundantly plants and various other organisms are present in this zone. Here sufficient amount of food is also present due to presence of different organisms.

(ii) 2nd layer is Metaliminion :- It is a type of transition layer present where adequate amount of oxygen & very rare sunlight just comes, here, organisms are present rarely, but some organisms which are capable of sustaining themselves without oxygen can live here.

(iii) 3rd Layer is Hypoliminion :- It is the bottommost layer where there is no sunlight, very least amount of oxygen, hence, humans need oxygen tanks to explore this region, very rarely any organisms are found here & this layer have very poor visibility.

6

Q.6 (a) A national highway passing through a plain terrain has a horizontal curve of radius equal to the ruling minimum radius.

(i) Design all the geometric features of this horizontal curve, assuming suitable data.

(ii) What is the safest intermediate sight distance provided for the given national highway?

[16 + 4 = 20 marks]

Ans: For a national highway passing through a plain terrain

Design speed = 100 km/h.

Lateral friction co-efficient $f = 0.15$

& Superelevation $e = 0.07$.

(i) Ruling minimum radius $R = \frac{V^2}{127(e+f)}$

$$R_{min} = \frac{100^2}{127(0.07+0.15)} = 357.909\ m$$

$R_{min} \simeq 358\ m$

(ii) Super-elevation to be provided:

(a) For mixed traffic $e = \frac{V^2}{225R} = \frac{100^2}{225 \times 358} = 0.124 > 0.07$

hence not O.K.

Let's provide $e = 0.07$ & check for f:

$$\Rightarrow f = \frac{V^2}{127R} - e = \frac{100^2}{127 \times 358} - 0.07 = 0.15 \leq 0.15$$

hence O.K

(iii) Sight distance required:

$$= \text{~~0.278~~} V t_r + \frac{V^2}{2gf} \quad \text{taking V in kmph}$$

$$SSD = 0.278 V t_r + \frac{V^2}{254 f}, \quad f = \text{longitudinal frictional coefficient} = 0.35.$$

Let Reaction time $T_r = 2.5$ sec.

$$\Rightarrow SSD = 0.278 \times 100 \times 2.5 + \frac{100^2}{254 \times 0.35}$$

$$= 181.986 \text{ m} \approx \boxed{182 \text{ m}}$$

(iv) Extra widening to be provided:

$$= \frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}}, \quad \text{Let no. of lanes } n = 2$$

& length of axle = 6m.

$$\therefore W_E = \frac{2 \times 6^2}{2 \times 358} + \frac{100}{9.5\sqrt{358}} = 0.656 \text{ m.}$$

(v) Length of transition curve:

(a) Based on rate of change of superelevation.

$$L = \frac{80}{75+V}, \quad L = \frac{80}{75+100} = 0.457 \not< 0.5$$

$$\Rightarrow C = 0.5$$

$$\therefore L_T = \frac{0.0215 V^3}{CR} = \frac{0.0215 \times 100^3}{0.5 \times 358} = 120.11 \text{ m.}$$

(b) Based rate of introduction of superelevation:
(Assuming rotation about inner edge)

$$L_T = eN[W + W_E], \quad \text{Let } N = 150 \text{ for plain area.}$$

$L_T = 0.07 \times 150\ [7 + 0.656] = 80.388\ m.$

(c) Based on IRC formula:

$$L_T = \frac{2.7V^2}{R} = \frac{2.7 \times 100^2}{358} = 75.419\ m.$$

$\therefore L_T = \max\ [120.11,\ 80.388,\ 75.42]$

$= 120.11\ m$

(vi) Set back distance to be provided:

$\because [L_c < SSD]$

$$\frac{\alpha}{2} = \frac{180L}{2\pi(R-d)} \qquad d = \frac{1}{4}[nd + W_E] = \frac{1}{4} \times 7.656 = 1.914\ m.$$

$$\Rightarrow \frac{\alpha}{2} = \frac{180 \times 120.11}{2\pi(358 - 1.914)} = 9.663^\circ$$

16

$$\therefore m = \left[\frac{S - L_c}{2}\right] \sin\frac{\alpha}{2} + R - (R-d)\cos\frac{\alpha}{2}$$

$$= \left[\frac{182 - 120.11}{2}\right] \sin 9.663^\circ + 358 - (358 - 1.914)\cos 9.663^\circ$$

$m = 12.16\ m$

(2) Safest intermediate sight distance provided for the given highway $= 2 \times SSD = 2 \times 182 = 364\ m$

4

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Q.6 (b) (i) The following latitudes and departures were obtained for a closed traverse *ABCDEFA* survey:

Line	Latitude (m)	Departure (m)
AB	0.00	183.79
BC	128.72	98.05
CD	177.76	–140.85
DE	–76.66	–154.44
EF	–177.09	0.00
FA	–52.43	13.08

Adjust the traverse by Bowditch's method and compute corrected latitudes and departures of all the traverse lines. Also calculate the bearing of *CD*.

(ii) What do you understand by the following forms of curves and where are they generally used?

1. Compound curve 2. Reverse curve

[14 + 6 = 20 marks]

Ans:

Line	Latitude (m)	Departure (m)	Length (m)
AB	0	183.79	183.79
BC	128.72	98.05	161.81
CD	177.76	-140.85	226.798
DE	-76.66	154.44	172.42
EF	-177.09	0	177.09
FA	-52.43	13.08	54.037
Σ =	$e_L = 0.3$	$e_D = -0.37$	$975.945 = \Sigma l$

Now total closing error $= \sqrt{e_D^2 + e_L^2} = \sqrt{0.3^2 + 0.37^2}$

$= 0.476$

angle of closing error $\Rightarrow \theta = \tan^{-1}\left(\frac{e_D}{e_L}\right) = \tan^{-1}\left(\frac{-0.37}{0.3}\right)$

$= -50.964°$

Line	Length	Correction		Corrected Measurement	
		CL	CD	Latitude	Departure
AB	183.79	-0.056	0.069	-0.056	183.859
BC	161.81	-0.049	0.061	128.671	98.111
CD	226.798	-0.069	0.086	177.691	-140.764
DE	172.42	-0.053	0.065	-76.713	-154.375
EF	177.09	-0.0544	0.067	-177.144	0.067
FA	54.037	-0.0166	0.0204	52.020	13.1004
				0	0

Sample calculations :-

By Bowditch's Rule :-

$C_L = \left[\frac{-l_1}{\Sigma l} \times e_L\right]$

For AB :-

$C_{L1} = -\frac{183.79}{975.945} \times 0.3$

$C_{L1} = -0.056$

corrected Latitude

$= L_1 + C_{L1}$

$= 0 - 0.056$

$= -0.056$

$C_D = \left[\frac{-l_1}{\Sigma l} \times e_D\right]$

For AB :-

$C_{D1} = -\frac{183.79}{975.945} \times -0.37$

$= 0.069$

corrected Departure

$= D_1 + C_{D1}$

$= 183.79 + 0.069$

$= 183.859$

(10/13)

CD bearing?

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Q.6 (c) (i) A conventional activated sludge process of municipal waste having discharge of 1000 m^3/day disposes its digested sludge on relatively impervious farm land. Raw sludge suspended solids concentration is 225 mg/lt [70% volatile], BOD = 190 mg/*lt* (Excess activated sludge returned to primary). Primary settling suspended solids removal is 50% and BOD removal is 30%. Excess activated sludge is 0.4 gm volatile solids produced per gm of BOD applied.

Compute: (1) Total volatile solids to be anaerobically digested in kg/day.

(2) If anaerobic digestor produces 50% volatile solids and digested sludge solids concentration is 6%, then compute area required in hectares for disposal of sludge on the farm land. Specific gravity of sludge is 1 and rate of application on farm land is 2 m^3/ha/day.

(ii) A control sample BOD water containing seeded diluted water has a drop of 1.5 mg/lt in its dissolved oxygen over 5 days of incubation. If BOD sample is of 300 ml with 20 ml waste water in it and remaining seeded water in contribution has a drop of 6.8 mg/lt in its DO, then calculate the BOD of the test sample.

[14 + 6 = 20 marks]

Ans: (ii) Now BOD drop of seeded diluted water = $1.5 \frac{mg}{l}$, $V = 300$ ml

∴ BOD requirement of 280 ml seeded water $= 1.5 \times \frac{280}{300} = 1.4 \frac{mg}{l}$

∴ BOD requirement of w/w = Total Drop of D.O − D.O requirement of seeded water

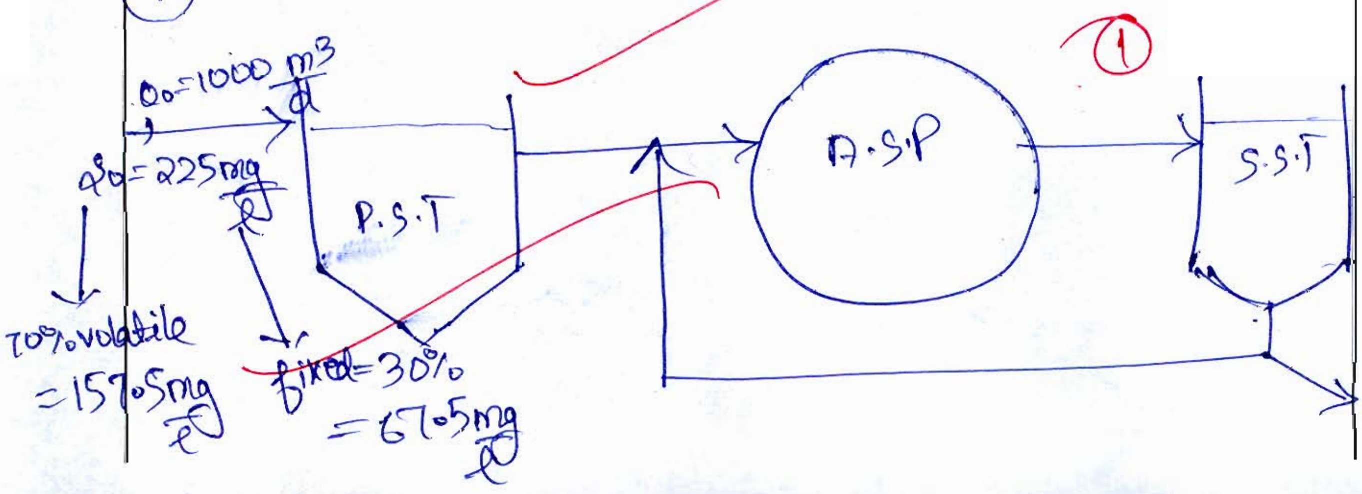
$= 6.8 \frac{mg}{l} - 1.4 \frac{mg}{l} = \boxed{5.4 \frac{mg}{l}}$

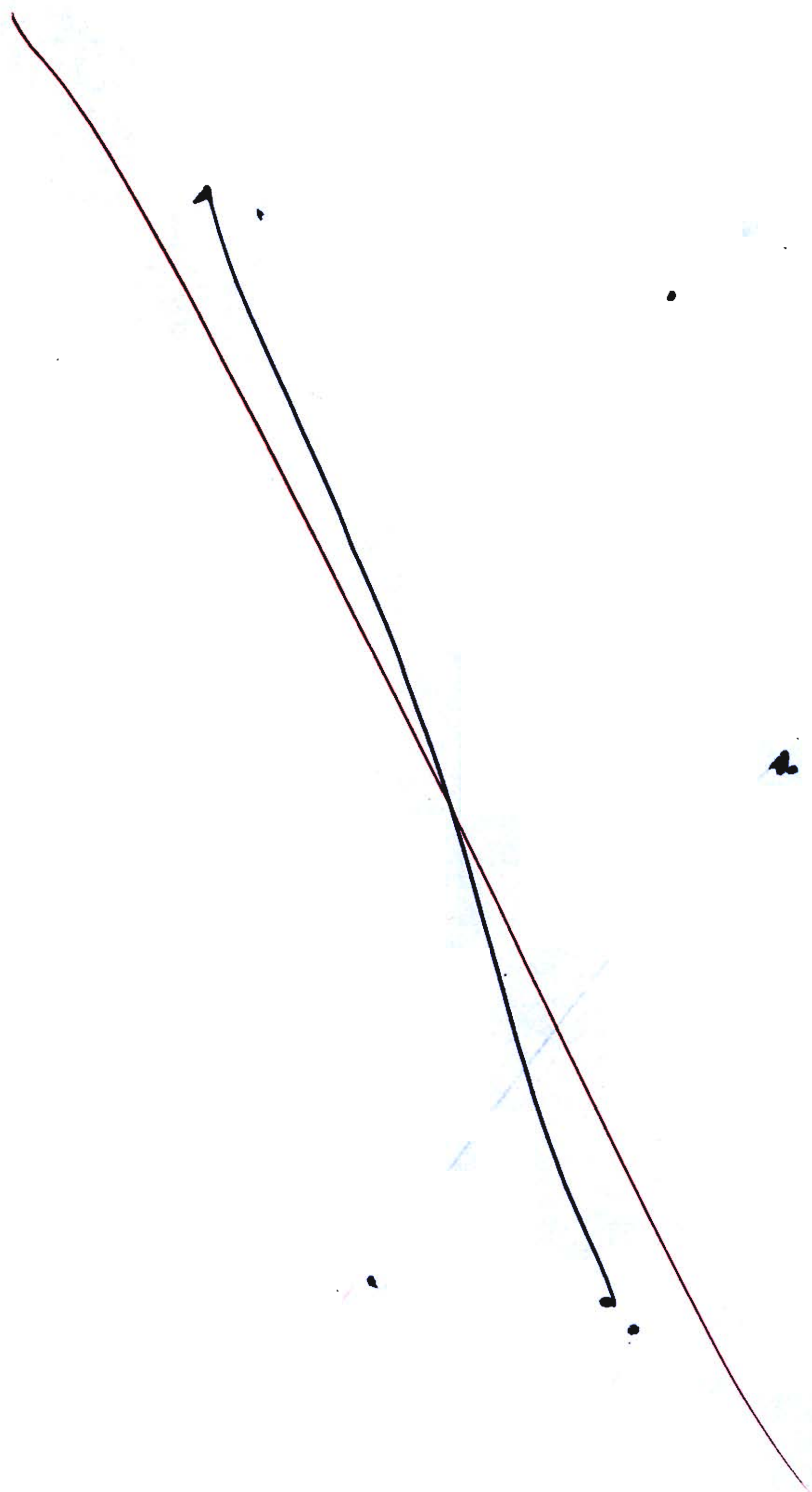
∴ BOD_5 of w/w $= 5.4 \frac{mg}{l} \times \frac{300}{20} = \boxed{81 \frac{mg}{l}}$

6

(i)

1





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Q.7 (a) Two sets of tacheometric readings were taken from an instrument station A (RL of A = 100 m) to a staff station B as shown below.

Instruments	P	Q
Multiplying constant	100	95
Additive constant	0.30	0.45
Height of instrument	1.40 m	1.45 m
Staff held	Vertical	Normal

Instruments	Instruments station	Staff station	Vertical angle	Stadia readings
P	A	B	5°44′	1.090, 1.440, 1.795
Q	A	B	5°44′	?

Determine:

(i) The distance between instrument station and staff station.

(ii) The R.L. of staff station B.

(iii) Stadia readings with instrument Q.

[20 marks]

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Q.7 (b) (i) Explain the factors influencing the geometric design of hills roads.

(ii) Why should the psychological widening be added to the mechanical widening of roads?

[12 + 8 = 20 marks]

7 (c) (i) Design a group of friction piles which is required to carry a load of 3500 kN including the weight of the pile cap at a site where the soil is uniform clay to a depth of 20 m, underlain by rock. Average unconfined compressive strength of the clay is 65 kN/m^2. The clay may be assumed to be of normal sensitivity and normally loaded with liquid limit of 55%. A factor of safety of 3 is required against shear failure.

(ii) A square mass concrete footing supporting a load of 3500 kN extends from ground level to 4 m deep into a clay stratum. What will be the size of the footing allowing for a factor of safety of 3.0? Unit weight of concrete is 24 kN/m^3. Shear strength of the soil is 0.15 N/mm^2. Adhesion of clay with footing is 30 kN/m^2. The adhesion may be supported to act over a depth of 2 m from the bottom of the foundation. For $\phi = 0°$, $N_C = 5.7$, $N_q = 1$ and $N_\gamma = 0$. Take unit weight of soil as 21 kN/m^3.

[10 + 10 = 20 marks]

Q.8 (a) **(i)** What is gradient in vertical alignment? Also, define the types of gradient and their values for different types of terrain as per Indian practices.

(ii) There is a horizontal curve of radius 500 m and length 250 m on a highway. Determine the setback distance required from the centre line of the curve so as to provide a stopping sight distance of 100 m. (The distance between the centre lines of the road and the inner lane is 1.9 m.)

[14 + 6 = 20 marks]

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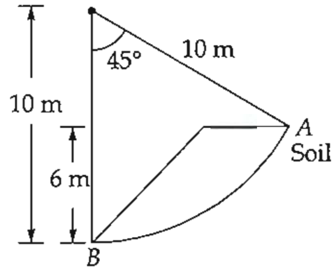
8 (b) (i) Find the factor of safety (1) with respect to shear strength and, (2) with respect to height along the indicated sliding surface *AB* in the figure below.



$c = 55$ kN/m^2
$\phi = 15°$
$\gamma = 17$ kN/m^3

Normal effective pressure on sliding surface *AB* is 225 kN/m^2.

Downward tangential disturbing force along *AB* is 850 kN/m.

(ii) Explain sludge treatment process.

[12 + 8 = 20 marks]

Q.8 (c) **(i)** Find the maximum permissible error in laying off the direction of an offset so that maximum displacement may not exceed 0.025 cm on paper given that length of the offset is 25 m, the scale is 1 cm to 75 m and the maximum error in the length of the offset is 0.50 m.

(ii) Write short notes on the following methods of plane table surveying:

1. Radiation
2. Traversing
3. Intersection
4. Resection

(iii) Explain the two basic principles of surveying.

[4 + 12 + 4 = 20 marks]

■■■■

Space for Rough Work

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