SAKET.

MADE EASY

India's Best Institute for IES, GATE & PSUs

ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-3 : Strength of Materials [All Topics]

Highway Engg. - 1 + Surveying and Geology-1 + Geo-technical & Foundation Engg. - 2 + Environmental Engg. - 2 [Part Syllabus]

Name :

Roll No :

Test Centres

Delhi ☑ Bhopal ☐ Jaipur ☐

Pune ☐ Kolkata ☐ Hyderabad ☐

Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	52
Q.2	56
Q.3	44
Q.4	
Section-B	
Q.5	24
Q.6	
Q.7	36
Q.8	
Total Marks Obtained	214

Signature of Evaluator ....

Cross Checked by ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

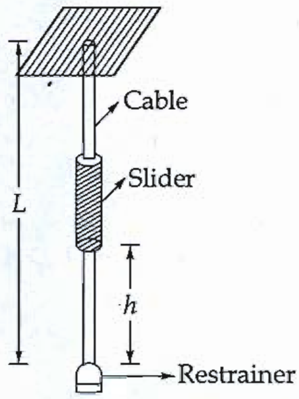
- Few Concepts are really good.
- Avoid Calculation error.
- Read all the Concepts again to avoid Conceptual error.
- Read theory part also.

Do not write in this margin

Section A : Strength of Materials

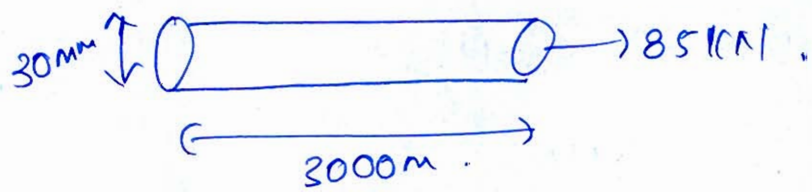
Q.1 (a) (i) A prismatic bar of circular cross-section is loaded by an axial tensile force of 85 kN. The bar has length of 3.0 m and diameter of 30 mm. It is made of aluminium with modulus of elasticity E = 70 GPa and Poission's ratio of $\frac{1}{3}$. Calculate the decrease in diameter 'Δd' and the change in volume of the bar. Assume the stresses in the bar are below proportionality limit.

(ii) A cable with a restrainer at the bottom hangs vertically from its upper end as shown in figure. The cable has cross sectional area of 50 mm^2 and modulus of elasticity E = 130 GPa. A slider of mass m = 40 kg drops from a height of h = 1.2 m on to the restrainer. If the allowable stress in the cable under an impact load is 700 MPa, then, what is the minimum permissible length L of the cable? (Take g = 9.81 m/s^2)



[6 + 6 = 12 marks]

(i)



$$\epsilon_x = \frac{\sigma_x}{E} = \frac{-85\times10^3}{\frac{\pi}{4}(30)^2 \times 70\times10^3}.$$

$$= -1.717\times10^{-3} \text{ (length } \downarrow\text{ve)}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} = -\frac{1}{3}(-1.717\times10^{-3})$$

$$\frac{\Delta d}{d} = 5.723\times10^{-4} \Rightarrow \Delta d = +0.01717\text{mm}.$$

(increase)

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z.$$

$$= \epsilon_x + 2\epsilon_y$$

$$= \epsilon_x + 2(-\mu\epsilon_x) \Rightarrow \epsilon_x(1-2\mu)$$

$$\epsilon_v = -1.717\times10^{-3}\left(1-\frac{2}{3}\right)$$

$$= -5.7233\times10^{-4}.$$

$$\frac{\Delta V}{\frac{\pi}{4}(30)^2 \times 3000} = 5.723 \times 10^{-4}$$

$$\Delta V = 1213.67 \, mm^3.$$

6

(ii) PE = strain Energy

$$mg(h + \delta_{max}) = \frac{\sigma^2}{2E} \times volume.$$

$$40 \times 9.81 \left(1.2 \times 10^3 + \frac{\sigma}{E} \times \ell \times 10^3\right) = \frac{700^2}{2 \times 130 \times 10^3} \times 50 \times (\ell \times 1000)$$

converting
h = 1200mm $\cancel{470.88 + 2.113\ell = 94.23\ell}$

$$\cancel{470.88 = 92.117\ell}$$

$$\cancel{\ell = 5.11m}$$

6

$$470.88 + 2.1129\ell = 94.23\ell$$

$$470.88 = 92.117\ell$$

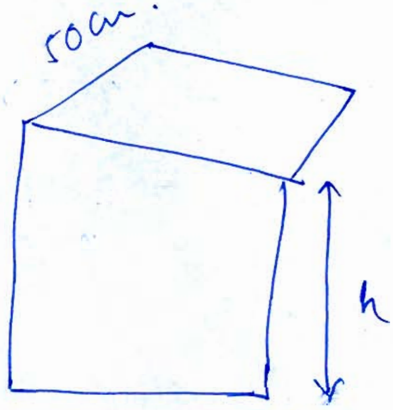
$$\boxed{\ell = 5.11m}$$

Q.1 (b) A reinforced concrete pedestal of height h and square cross-section having side of 50 cm is constructed with 12 steel reinforcing bars each has a diameter, d = 25 mm. The pedestal supports a compressive load P applied through a rigid bearing plate. Assuming linear elastic behaviour, calculate the maximum permissible value of the load if the allowable stress in the concrete and steel are 8 MPa and 70 MPa respectively. Neglect the self weight of the pedestal itself. (Assume E_s = 200 GPa and E_c = 25 GPa).

[12 marks]

linear elastic means $\delta_c = \delta_{steel}$.

$P_c + P_{steel} = P$.



$(\sigma_c) A_c + (\sigma_{st})(A_{st}) = P$.

$$\frac{P_c l_c}{A_c E_c} = \frac{P_s l_s}{A_s E_s}.$$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}.$$

$\sigma_s = \frac{200}{25}\sigma_c$ $\quad \underline{\sigma_s = 8\sigma_c}$.

if $\sigma_c = 8$

$\sigma_s = 8 \times 8 = 64 < 70$ Mpa (Hence ok)

if $\sigma_s = 70$

$\sigma_c = 70/8 = 8.7578$ Mpa (Not safe)

$\therefore \sigma_c = 8$ Mpa $(\sigma_{steel}) = 64$ Mpa.

$P_{steel} = \sigma_{st} A_{st} = 64 \times 12 \times \frac{\pi}{4}(25)^2 = 64 \times (5890.486)$

$= 376991.11$

$= 376.99$ kN

$P_{concrete} = \sigma_c A_c$

$= 8 \times (500 \times 500 - A_{st}) =$

$= 8(500 \times 500 - 5890.486)$

$= 1952876.11$

$= 1952.87\,kN.$

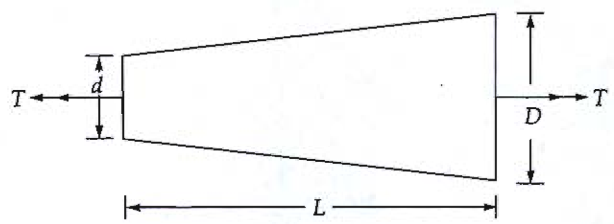
12

$P_{total} = P_s + P_{concrete}$

$\Rightarrow 1952.87 + 377$

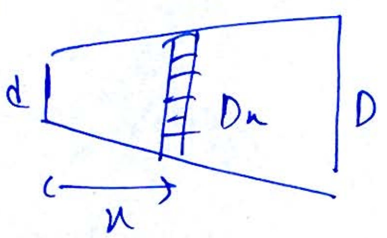
$= 2329.87\,kN.$

Q.1 (c) A tapered bar AB of solid circular cross section is twisted by torques 'T' applied at the ends. The diameter of the bar varies uniformly from 'd' at left end to 'D' at the right end. Derive an expression for the angle of twist 'ϕ' of the bar. The length of the bar is 'L' and shear modulus of the material used in the bar is G.



[12 marks]

$$\phi = \int_0^L \frac{T\,dx}{GJ} \qquad J = \frac{\pi}{32} D_x^4$$



$$\frac{D-d}{L} = \frac{D_x - d}{x}$$

$D_x = d + Kx$ assume $= \frac{D-d}{L} = K$.

$$\phi = \int_0^L \frac{T\,dx}{G\frac{\pi}{32}(d+Kx)^4}$$

$$\Rightarrow \frac{32T}{\pi G}\int_0^L \frac{dx}{(d+Kx)^4} = \frac{32T}{\pi G}\left[\frac{1}{(d+Kx)^3}\right]_0^L\left(\frac{-1}{3K}\right)$$

12

$$= \frac{32T}{\pi G}\left(\frac{-1}{3\left(\frac{D-d}{L}\right)}\right)\left[\frac{1}{(d+KL)^3} - \frac{1}{d^3}\right]$$

$$= \frac{32TL}{3\pi G(D-d)}\left[\frac{1}{d^3} - \frac{1}{D^3}\right]$$

$$\boxed{\phi = \frac{32TL}{3\pi G(D-d)}\left[\frac{1}{d^3} - \frac{1}{D^3}\right].}$$ ↳ Angle of twist

Do not write in this margin

Q.1 (d) Briefly explain the need of theories of elastic failure. Describe the maximum strain theory and maximum shear strain energy theory along with formulae (derivation not required) and limitations associated with them.

[4 + 4 + 4 = 12 marks]

max strain theory

→ by st venant.

→ the maximum strain produced by complex stresses should be less than strain produced by uniaxial load under yielding condition

$$\epsilon_{p1} = \frac{\sigma_p}{E_1} - \frac{(\sigma_{p2} + \sigma_{p3})\mu}{E}$$

uniaxial cond^n $\epsilon_p = \frac{\sigma_y}{E\,FOS}$

$$\therefore \epsilon_{p1} \le \epsilon_p$$

$$\frac{\sigma_{p1} - \mu(\sigma_{p2} + \sigma_{p3})}{E} = \frac{\sigma_y}{E(FOS)}$$

Limitation

-> overestimate strength of ductile material

-> Not suitable for pure shear condition

-> Not suitable for hydrostatic loading condition

Shear strain Energy / Distortion Energy

-> by ~~Haigh theory~~ von misses.

-> the max shear strain energy produced by group of loading should be less than shear strain energy produced by uniaxial loading under yielding condition.

by Complex loading

$$U = \frac{1}{12G}\left((\sigma_{p1}-\sigma_{p2})^2 + (\sigma_{p2}-\sigma_{p3})^2 + (\sigma_{p3}-\sigma_{p1})^2\right)$$

$$U = \frac{2}{12G}\left(\frac{\sigma_y}{FOS}\right)^2 \rightarrow \text{uniaxial}$$

8

$$\Rightarrow \frac{1}{12G}\left((\sigma_{p1}-\sigma_{p2})^2 + (\sigma_{p2}-\sigma_{p3})^2 + (\sigma_{p3}-\sigma_{p1})^2\right) \le \frac{2}{12G}\left(\frac{\sigma_y}{FOS}\right)^2$$

-> Best for ductile materials & pure shear condn

Limitation

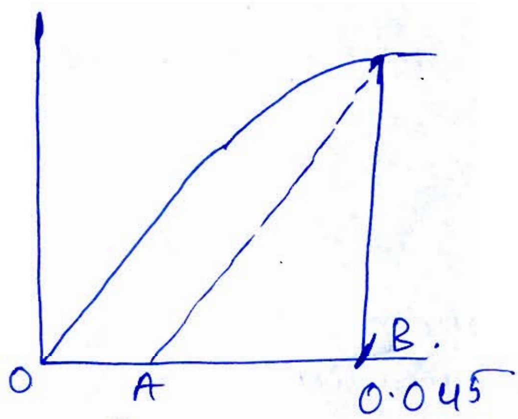
-> Not suitable for hydrostatic loading case

Q.1 (e) An aluminium bar has length $L = 120$ cm and diameter $d = 2.5$ cm. The initial straight line part of the stress-strain curve has a slope (modulus of elasticity) of 6.89×10^{10} N/m^2. The bar is loaded by tensile force, $P = 250$ kN beyond elastic limit. It reaches to a strain value of 0.045 and then unloaded.

(a) What is the permanent set of the bar?

(b) What is the modulus of resilience after the load application?

[12 marks]



$$\sigma = \frac{250 \times 10^3}{\frac{\pi}{4}(25)^2} = \underline{509.29}$$

$$\epsilon_{AB} = \frac{\sigma}{E} = \frac{509.29}{6.89 \times 10^4 \text{ N/mm}^2}$$

$$= 7.39 \times 10^{-3}.$$

B

(a) permanent set strain $= OA = OB - AB$

$$= 0.045 - 7.39 \times 10^{-3}$$

$$= 0.0376.$$

permanent set $= 0.0376 \times 1200$

$$= 45.13 \text{mm}.$$

(b) $$MOR = \frac{\frac{1}{2} \times P \times \delta_{elastic}}{volume} = \frac{\sigma^2}{2E} \times \epsilon_{Elastic\ part}$$

$$= \frac{509.29^2}{2 \times 6.89 \times 10^4} \times \epsilon_{AB}.$$

$$MOR = \frac{509.29^2}{2 \times 6.89 \times 10^4} \times 7.39 \times 10^{-3}$$

$$= 0.0139$$

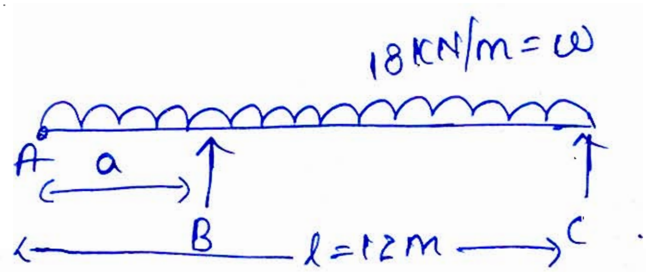
$$\simeq 0.014 \ N/mm^2$$

Q.2 (a) (i) A 12 m long beam of uniform section carries a uniformly distributed load of 18 kN/m over the whole length. If the beam is freely supported at the left end, find the position of second support so that the maximum bending moment for the beam shall be as small as possible. Also, find the maximum bending moment for this case.

(ii) State Castigilano's first theorem and Maxwell's reciprocal theorem.

[16 + 4 = 20 marks]

(i)



In this there is chance of hogging & sagging moment

$\Rightarrow$ (max hogg) = max sagg then BM of beam will be min.

$$\Sigma M_C = 0$$

$$-\omega(l)\left(\frac{l}{2}\right) + R_B(l-a) = 0.$$

$$R_B = \frac{\omega l^2}{2(l-a)}$$

$$R_B + R_C = wl .$$

$$R_C = wl - \frac{wl^2}{2(l-a)}$$

$$= \frac{2wl^2 - wla(2) - wl^2}{2(l-a)}$$

$$= \frac{wl^2 - 2wal}{2(l-a)} = \frac{wl(l-2a)}{2(l-a)}$$

SF from c must be zero at x. (from c)

$$\Rightarrow R_C - wx = 0$$

$$\frac{wl(l-2a)}{2(l-a)} = wx$$

$$x = \frac{l(l-2a)}{2(l-a)}$$

$\therefore$ max sagg moment occur at $x = \frac{l(l-2a)}{2(l-a)}$ from c

$$BM_x = R_C(x) - \frac{wx^2}{2}$$

$$= \frac{wl(l-2a)}{2(l-a)} \times \frac{l(l-2a)}{2(l-a)} - \frac{w}{2}\left[\frac{l(l-2a)}{2(l-a)}\right]^2$$

$$= \frac{w}{2}\left(\frac{(l-2a)l}{l-a}\right)^2\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{w}{8}\left(\frac{l(l-2a)}{l-a}\right)^2 \rightarrow ①$$

max hogg moment occur at B

$$BM_{hogg} = \frac{wa^2}{2} \rightarrow ② .$$

$\therefore$ for min BM ① = ②

$$\frac{w}{8}\left(\frac{l(l-2a)}{l-a}\right)^2 = \frac{w}{2}a^2$$

$$\frac{l(l-2a)}{l-a} = 2a \Rightarrow l^2 - 2al = 2al - 2a^2$$

$$l^2 - 4al + 2a^2 = 0 .$$

$$a = \frac{+4l \pm \sqrt{16l^2 - 4(l^2)(2)}}{2(2)} = \frac{4l + 2.82l}{4} .$$

$a = 1.707l, 0.292l .$
$\underbrace{}_{\text{not possible}}$

$a = 0.292l$

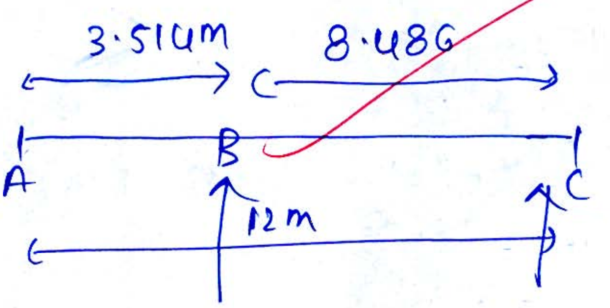
$a = 0.292 \times 12 = 3.514 m$

16

$$(BM)_{max} = (BM)_{sag} = (BM)_{hogg} = \frac{wa^2}{2} = \frac{18(3.514)^2}{2}$$

$= 111.179 kN-m .$

$= 111.1 kN-m .$

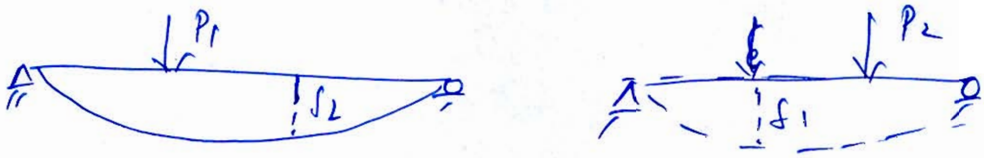


B, C are supports

(ii) Castiglonos first theorem

$\Rightarrow$ It states that the partial derivative of strain energy with respect to the deflection caused at the point gives the force which is the main cause of deflection

$\frac{dU}{d\delta} = P$ $\Rightarrow$ it applies for both linearly elastic & non elastic body maxwell reciprocal theorem



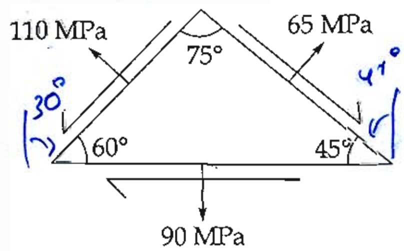
It states that deflection @ 2 if the load applied at ① will be same as deflection at ① if same load applied at ②.

2

$$P_1\delta_1 = P_2\delta_2 \qquad \text{if } P_1 = P_2$$

$$\boxed{\delta_1 = \delta_2}$$

Q.2 (b) In a strained body, the normal stresses on three planes inclined as shown in figure are 65 MPa (Tensile), 90 MPa (Tensile) and 110 MPa (Tensile). Determine the shear stresses acting on these planes. Also find the principle stresses and draw the final stress element.



[20 marks]

$\sigma_x = ?. \quad \sigma_y = +90\,\text{Mpa}.$

$$(\sigma_n)_{\theta=45^\circ\,ACW} = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta.$$

$$65 = \frac{\sigma_x+90}{2} + \tau_{xy}(1) \longrightarrow ①.$$

$$(\sigma_n)_{at\ \theta=30^\circ\,cw} = 110\,\text{Mpa}$$

$$110 = \frac{\sigma_x+90}{2} + \left(\frac{\sigma_x-90}{2}\right)\cos(-60) + \tau_{xy}\sin(-60)$$

$$110 = \left(\frac{\sigma_x+90}{2}\right) + \left(\frac{\sigma_x-90}{2}\right)\left(\frac{1}{2}\right) + \frac{\tau_{xy}\sqrt{3}(-1)}{2}$$

$$110 = \frac{\sigma_x}{2} + \frac{\sigma_x}{4} + 45 - 22.5 - \frac{\sqrt{3}}{2}\tau_{xy}.$$

$$\cancel{110} \quad 87.5 = \frac{3\sigma_x}{4} - \frac{\sqrt{3}}{2}\tau_{xy} \longrightarrow (2).$$

by (1) & (2).

$$\sigma_x = 88.6 \text{ Mpa}$$

$$\tau_{xy} = -24.302 \text{ Mpa}.$$

∴ shear stress on right side plane.

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (\theta = 45° \text{ ACW}.$$

$$\tau_{xy} = -\left(\frac{88.6 - 90}{2}\right)\sin(90)$$

$$= +0.7 \text{ Mpa}.$$

shear stress on left side plane $\theta = -30°$ (since cw)

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(-60) + \tau_{xy}\cos(-60)$$

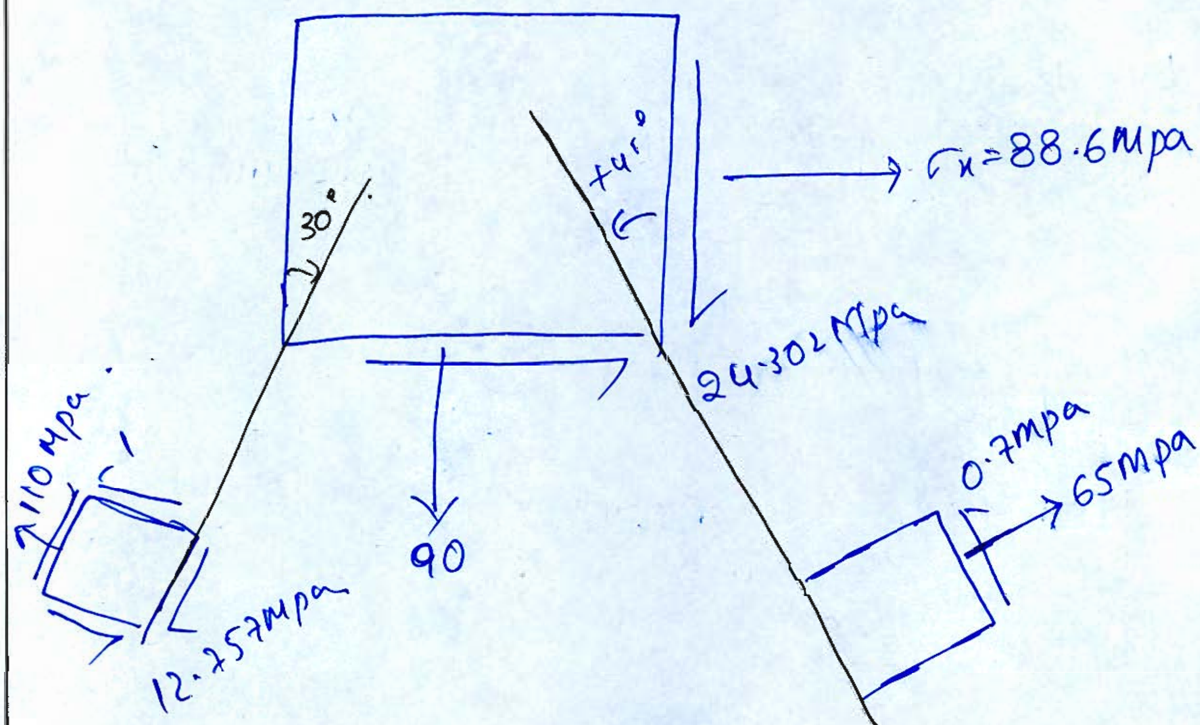
$$= \left(\frac{88.6 - 90}{2}\right)\sin 60 - 24.302\cos 60$$

$$= -0.606 - 12.151$$

$$= -12.757 \text{ Mpa}.$$

$$= -12.757 \text{ Mpa}.$$

Final Diagram



Principal stress

$$\sigma_{P1}/\sigma_{P2} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \frac{88.6+90}{2} \pm \sqrt{\left(\frac{88.6-90}{2}\right)^2+(24.302)^2}$$

$$= 89.3 \pm 24.312$$

$\sigma_{P1} = 113.612\,Mpa$

$\sigma_{P2} = 64.988\,Mpa$.

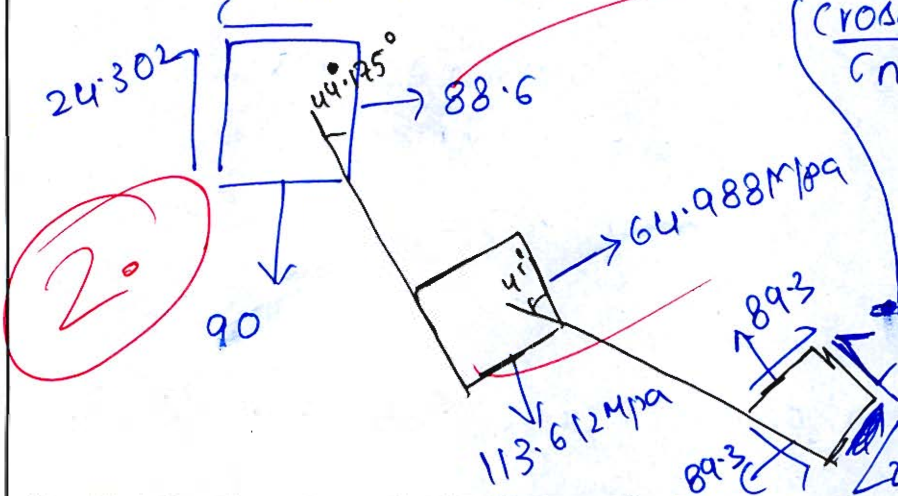
$(\tau_{max})_{in\ plane} = \frac{\sigma_{P1}-\sigma_{P2}}{2} = \frac{113.612-64.988}{2} = 24.307\ Mpa$

$(\tau_{max})_{absolute} = \frac{\sigma_{P1}}{2} = \frac{113.612}{2} = 56.806$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-24.302)}{88.6-90} = 34.717$$

$$\theta_{p_1} = 44.175^\circ \quad \theta_{p_2} = 134.175^\circ$$

$$\theta_{s_1} = \theta_{p_1} + 45^\circ = 89.175^\circ.$$



Crosscheck

σ_n @ 44.175

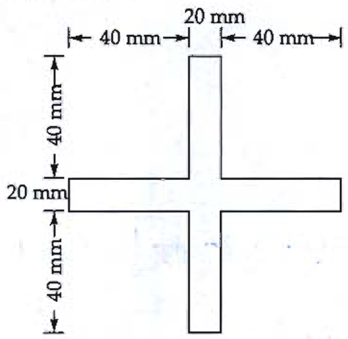
$= \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos(88.35) + \tau_{xy}\sin(88.35)$

$= 64.988$ MPa

$\tau_{xy} = +24.302$ MPa

+ve.

Q.2 (c) **(i)** Plot the shear stress distribution over the beam cross-section as shown in figure due to a shear force of 100 kN.



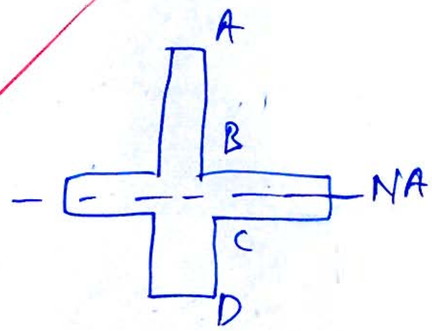
(ii) Explain briefly about the stress-strain diagram for mild steel.

[12 + 8 = 20 marks]

(i) $I = I_{vertical} + I_{horizontal}$

$$= \frac{20\times100^3}{12} + \frac{80\times20^3}{12} =$$

$$= 1.72\times10^6 \text{ mm}^4.$$



$$\tau = \frac{VA\bar{y}}{Ib}.$$

$$\tau_{A,D} = 0.$$

$$\tau_B = \frac{100 \times 20 \times 40 \times (10+20) \times 10^3}{20 \times 1.72 \times 10^6} \quad \text{(in vertical part)}$$

$$= 69.767 \text{ Mpa}$$

$$\tau_B \text{ (in horizontal part)} = \frac{100 \times 10^3 \times 20 \times 40 \times (30)}{100 \times 1.72 \times 10^6}$$

$$= 13.953 \text{ Mpa}$$

Since it is symmetric

C point also have same shear stress like B.

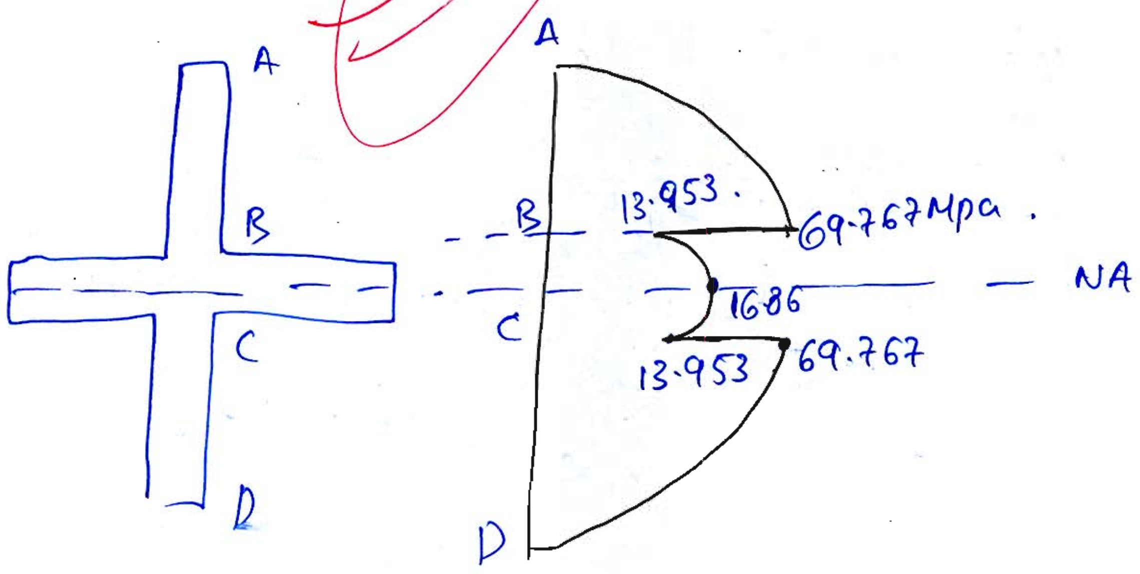
2

shear stress at NA.

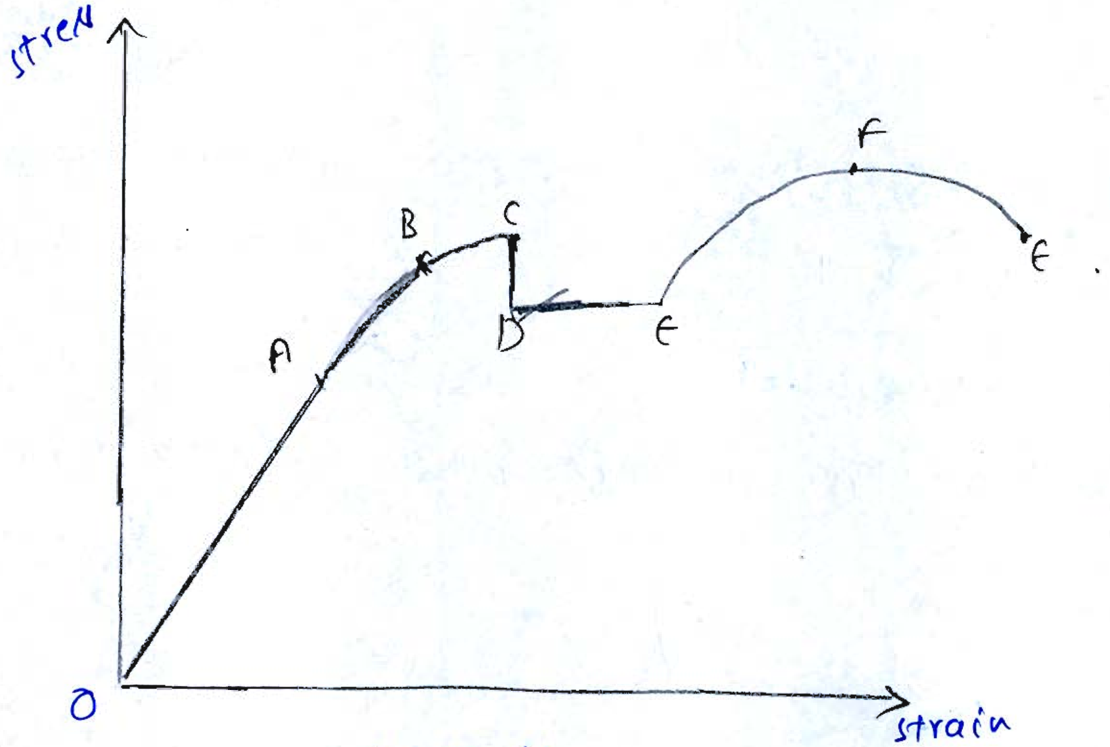
$$\tau = \frac{VA\bar{y}}{Ib}$$

$$= \frac{100 \times 10^3 (20 \times 40 \times 30 + 100 \times 10 \times 5)}{1.72 \times 10^6 \times 100}$$

$$= 16.86 \text{ Mpa}$$



(iii)



A is proportionality limit
→ upto A hooks law is valid
→ $\sigma \propto \epsilon$.
→ body is in elastic

B is elastic limit
→ strain rate is faster than stress
→ b/w AB hooks ~~law is~~ not valid.
→ when load is removed, body comes to original position

C upper yield point
→ beyond B when load applied body undergo permanent deformation
→ strain at c 0.02% (found by offset mtd)

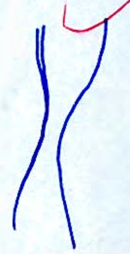
D lower yield point
A sudden drop in stress due to cotrell break

DE is yielding zone increase in strain for constant stress

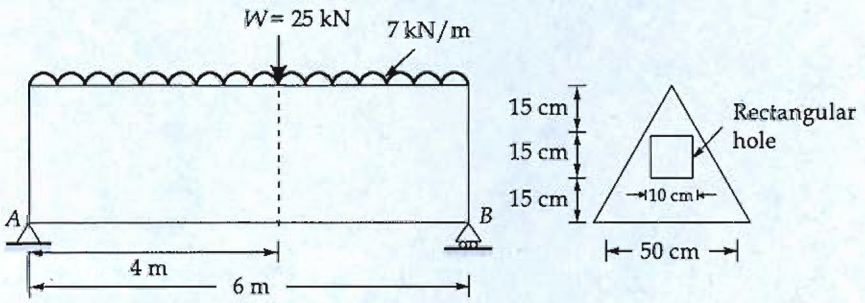
F is ultimate strength - max tensile strength of the body

G rupture point - beyond F area decreases and at G cup cone failure takes place (u/s failure)

6



Q.3 (a) A simply supported beam and it's cross-section are shown below. The beam carries a point load of 25 kN as shown in figure. The self weight of the beam is 7 kN/m. Determine the maximum value of bending compressive stress at a section 2 m from end A.



[20 marks]

$\cancel{V_A = V_B = \frac{wl}{2} + \frac{W}{2}}$

$V_A(6) - 25(2) - 7(6)\left(\frac{6}{2}\right) = 0$

$V_A = \frac{88}{3} = 29.33\,kN$

$V_B = 25 + 7(6) - V_A = 67 - 29.33$

$= 37.67\,kN$

centroid of A^{le} about base.

$$y_1 = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{\frac{1}{2}(50)(45)(15) - (10)(15)\left(15 + \frac{15}{2}\right)}{\frac{1}{2}(50)(45) - 10 \times 15}$$

$$= \frac{16875 - 3375}{975} = 13.846\,cm \text{ from base.}$$

$$I_{net} = \frac{bh^3}{36} + \frac{1}{2}(b)(h)\left(13.846 - \frac{h}{3}\right)^2$$
I_{NA}

$$- \left[\frac{10 \times 15^3}{12} + 10 \times 15\,(13.846 - 22.5)^2\right)$$

$$= \frac{50 \times 45^3}{36} + \frac{1}{2}(50)(45)(13.846 - 15)^2 - \left[14046.2874\right)$$

$$= \cancel{128060.6801}\ 38118568 - 14046.2874$$

$$= \cancel{114014.4cm}^4 \quad = 114014.4 \times 10^4\,mm^4$$

$$= 11.4 \times 10^8\,mm^4$$

$$= 367139.42 \times 10^4\,mm^4$$

$$= 36.714 \times 10^8\,mm^4$$

It was correct

BM at $x = 2$ from A

$$BM = V_A(2) - 7(2)\left(\frac{2}{2}\right)$$

$$= 29.33(2) - 7(2)$$

$$= 44.66\,kN\text{-}m.$$

since beam has downward load it sas max compressive force is at top

$\sigma = \frac{M}{I}(y)$

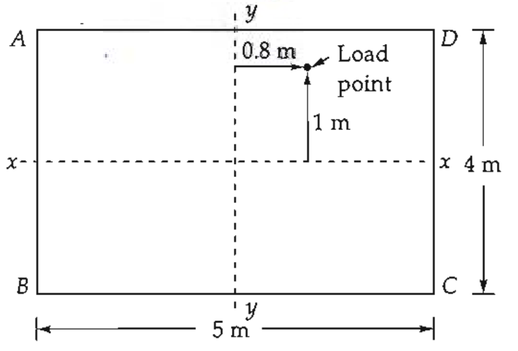
$\sigma = \frac{44.66 \times 10^6}{36.714 \times 10^8} (45 - 13.846)$

$\sigma = 0.3789 \text{Mpa}$

max comp stress $\sigma = 0.38 \text{Mpa}$.

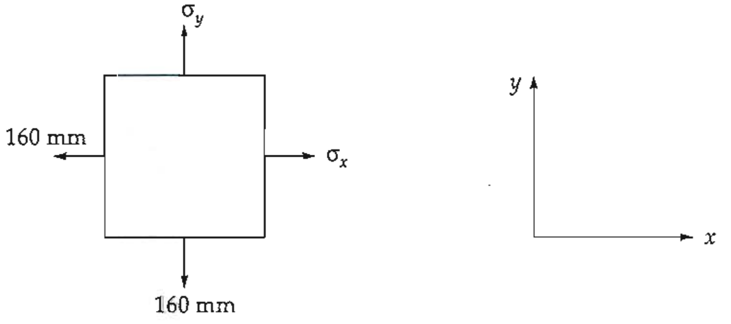
4

Q.3 (b) (i) A masonry pier as shown in the figure supports a load of 60 kN.



1. Find the stress developed at each corner.
2. Find the value of additional load required to be placed at centre of pier so that there is no tension anywhere in the column section.

(ii) A thin rectangular steel plate 100 mm by 100 mm undergoes elongation of 0.05 mm and 0.03 mm in x and y directions due to stresses σ_x and σ_y respectively as shown in figure. Determine the magnitude of σ_x and σ_y if modulus of elasticity of steel is 200 GPa and Poisson's ratio is 0.3.



[15 + 5 = 20 marks]

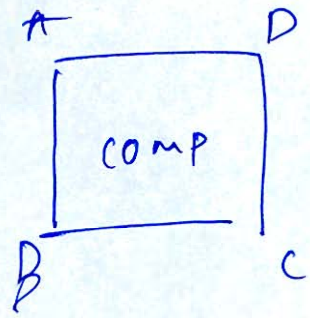
Do not write in this margin

(i) ①

① $\sigma_{direct} = P/A$

$= \dfrac{60 \times \cancel{10^3}}{5 \times 4}$

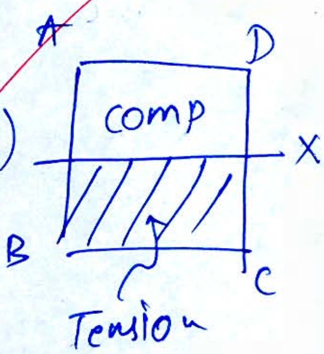
$= 3 kN/m^2$



② (σ bending) about $X\text{-}X$ $= \dfrac{M}{I}(y)$

$= \dfrac{60 \times 1 kN\text{-}m}{\dfrac{5 \times 4^3}{12}} (2)$

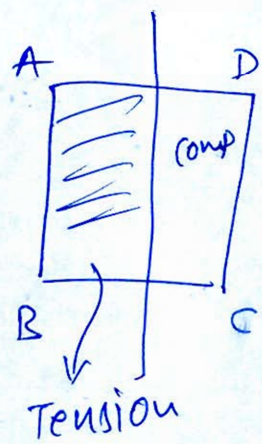
$= 4.5 \cancel{Mpa} kN/m^2$



③ σ bending about $yy = \dfrac{M_y (z)}{I_y}$

$= \dfrac{60 \times 0.8 \times 5/2}{4 \times 5^3/12}$

$= 2.88 \cancel{Mpa} kN/m^2$



Stress at corner.

$A = \sigma_1 + \sigma_2 - \sigma_3$

$= 3 + 4.5 - 2.88$

$= 4.6 \cancel{Mpa}$ kN/m^2 (comp)

Assuming compression = +ve

tension = -ve

$B = \sigma_1 - \sigma_2 - \sigma_3 = 3 - 4.5 - 2.88$

$= -4.38 \cancel{Mpa}$ kN/m^2. (-ve indicate tensile)

Ⓒ $= \sigma_1 - \sigma_2 + \sigma_3$

$= 3 - 4.5 + 2.88 = 1.38$ ~~Mpa~~ kN/m^2 (comp)

Ⓓ $= \sigma_1 + \sigma_2 + \sigma_3$.

$= 3 + 4.8 + 2.88 = 10.38$ ~~Mpa~~ kN/m^2 (comp)

· ② since tension created only at B

$= -4.38$ ~~Mpa~~ kN/m^2

∴ Additional load to compensate this

15

$4.38 = \frac{P}{5 \times 4}$.

$P = 87.6\ kN$

∴ Additional load at center $= 87.6\ kN$.

(ii) $\epsilon_x = \frac{0.05}{100} = 5 \times 10^{-4}$.

$\epsilon_y = \frac{0.03}{100} = 3 \times 10^{-4}$.

$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E}$.

$(5 \times 10^{-4})(200 \times 10^3) = \sigma_x - 0.3(\sigma_y)$ ——①.

$100 = \sigma_x - 0.3(\sigma_y)$ ——①

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu \sigma_x}{E}.$$

$$(3\times10^{-4})(200\times10^3) = \sigma_y - 0.3(\sigma_x)$$

$$\sigma_y - 0.3(\sigma_x) = 60 \longrightarrow ②.$$

① $\times 0.3$.

$$30 = 0.3\sigma_x - 0.3(0.3)\sigma_y . \rightarrow ③$$

③ + ②

$$90 = \sigma_y(1 - 0.3\times0.3)$$

$$\sigma_y = 98.9\,\text{Mpa}$$

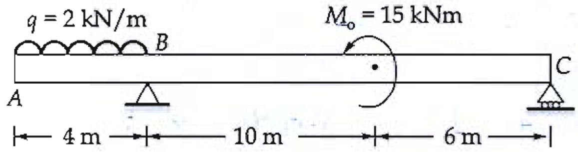
$$\sigma_x = 129.67\,\text{Mpa}.$$

Final stresses

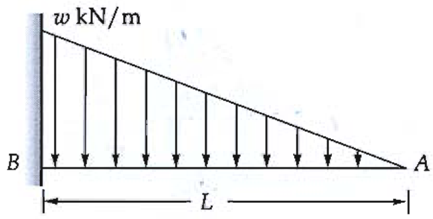
$\sigma_x = 129.67$ Mpa
$\sigma_y = 98.9$ Mpa } both are tensile.

5

Q.3 (c) (i) A beam ABC with an overhang at the left hand end is shown in figure below. The beam is subjected to a uniform load of intensity $q = 2$ kN/m on the overhang AB and a counterclockwise couple $M_o = 15$ kN-m as shown. Draw the shear-force and bending moment diagram for this beam.

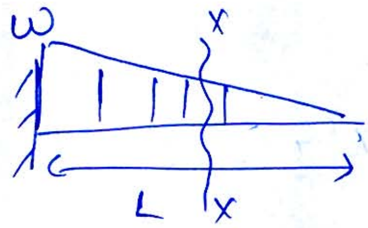


(ii) Find slope and deflection at free end using double integration method for the beam shown in figure.



[10 + 10 = 20 marks]

(ii)



$\omega \rightarrow L$

$\omega' \rightarrow x$.

$\therefore$ ω' at $x-x = \dfrac{\omega x}{L}$

total load $= \dfrac{1}{2}(\omega')(x) = \dfrac{1}{2}\left(\dfrac{\omega x}{L}\right)(x) = \dfrac{\omega x^2}{2L}$.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$M_x = \frac{1}{2}(\omega')(x)\left(\frac{x}{3}\right) = \cancel{\frac{1}{2}}\left(\frac{\omega x^2}{2L}\right)\left(\frac{x}{3}\right) = \frac{\omega x^3}{6L} .$$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{\omega x^3}{6L} .$$

integrate $EI \dfrac{dy}{dx} = \left(\dfrac{\omega x^4}{24L} + C\right) \longrightarrow (1)$.

at $x = L$ $\dfrac{dy}{dx} = \theta = 0$ (since fixed end

$$0 = \frac{wL^4}{24L} + C$$

$$\Rightarrow C = \frac{-wL^3}{24}.$$

$\therefore$ slope at free end at $(x=0)$

$$\frac{dy}{dx}(EI) = \frac{w(0)^3}{24L} - \frac{wL^3}{24}$$

$$\boxed{\frac{dy}{dx} = \theta = \frac{-wl^3}{24EI}}$$ (–) indicate cw slope.

for deflection

integrate ①

$$EI(y) = \frac{wx^5}{120L} + C_1(x) + C_2.$$

at $x = L \Rightarrow y = 0$

$$\Rightarrow 0 = \frac{wl^4}{120} - \frac{wl^3}{24}(l) + C_2$$

$$C_2 = \frac{-wl^4}{120} + \frac{wl^4}{24} = \frac{wl^4}{30}$$

$\therefore$ At $x=0$ $y = y_{max}$

$$EI(y) = 0 + C_1(0) + \frac{wl^4}{30}.$$

$$\boxed{y = \frac{wl^4}{30EI}}$$ downward.

$\therefore$ slope $= \frac{wl^3}{24EI}$

deflection $= \frac{wl^4}{30EI}$

10

(ii) $\Sigma M_B = 0$.

$-R_C(16) - 15 - 2(4)(2) = 0$

$R_C(16) = -31$ $\quad R_C = -1.9375\,kN$.

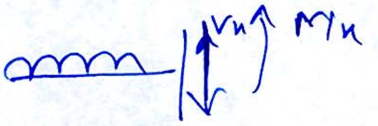
Assume moment act at D.

$R_A + R_C = 8\,kN$

$R_A = 9.9375\,kN$

Section SF | BM

AB



$2(x) + V_x = 0$.

at A $x=0$ $V=0$

at B $x=4$ $V=-8\,kN$.

$M_x + 2(x)\left(\frac{x}{2}\right) = 0$

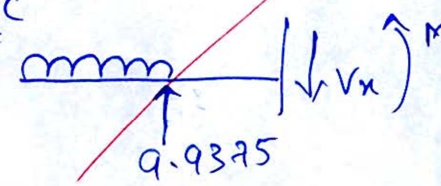
at $x=0$ $M_x = 0$

at $x=4$ $M_x = -2(4)(2)$

$= -16\,kN\text{-}m$.

10

BC



$8 - 9.9375 + V_x = 0$

$V_x = 1.9375\,kN$

$V = 1.9375$ constant

$-2(4)(2+x) + 9.9375(x) - M_x = 0$

$M_x = 9.9375(x) - 8(2+x)$

at $x=0$

$M_B = -16$

at $x = 10$

$M_D = +3.375\,kN$.

CD

$V_x = 1.9375\,kN$ constant.

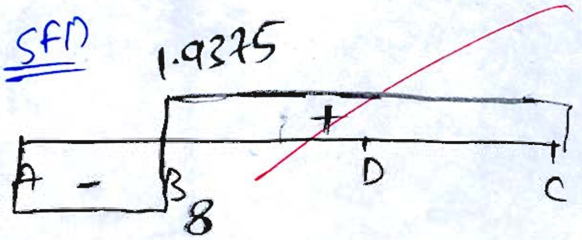
$M_x + R_C x = 0$.

$M_x = -1.9375(x)$

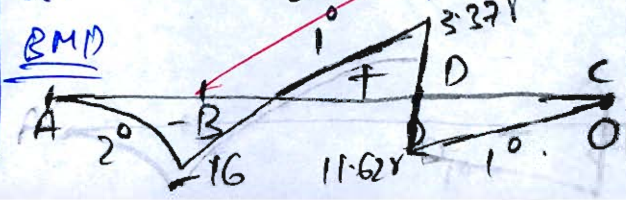
at $x=0$ (C) $M_C = 0$

at $x=6$ $M_D = -11.625\,kN\text{-}m$

SFD

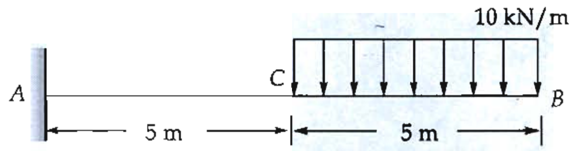


BMD



Q.4 (a) **(i)** Explain the application of the moment area method to determine the slope and deflection.

(ii) Find the angle of rotation θ_B and deflection δ_B by using moment area method at the free end B of cantilever beam ACB supporting a uniform load of intensity 10 kN/m acting over the beam as shown. (The beam has length 10 m and constant flexural rigidity EI)



[8 + 12 = 20 marks]

Do not write in this margin

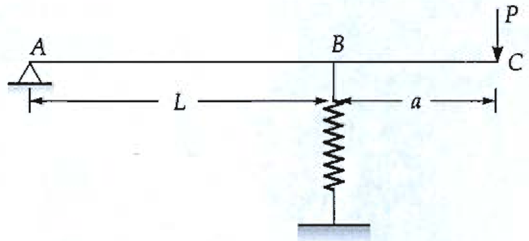
Do not write in this margin

Q.4 (b) (i) A steel shaft is to be manufactured as a circular tube. The shaft is required to transmit a torque of 1250 N-m without exceeding an allowable shear stress of 40 MPa nor an allowable rate of twist of 0.75/m. The shear modulus of elasticity of the steel is 78 GPa. Determine the required diameter of the shaft if the thickness of the shaft is $1/10^{th}$ of the outer diameter.

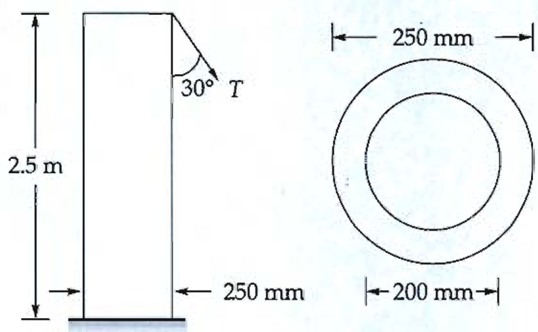
(ii) A prismatic shaft consists of a solid aluminum rod of diameter 35 mm, which is inside a steel tube of external diameter 52 mm. Both the shafts are firmly jointed and subjected to a torque of 1025 N-m. Find the maximum stresses developed in aluminium and steel shafts.
[Take, $G_{Al} = 70 \times 10^9$ N/m^2 and $G_{st} = 80 \times 10^9$ N/m^2]

[12 + 8 = 20 marks]

Q.4 (c) **(i)** An overhanging beam ABC rests on a simple support at A and a spring support at B (see figure). A concentrated load P acts at the end of the overhang. Span AB has length L, the overhang has length 'a' and the spring has stiffness k. Determine the downward displacement at the end of overhang i.e. at C using Castingliano's theorem.



(ii) A vertical pole of aluminium is fixed at the base and pulled at the top by a cable having a tensile force T as shown. The cable is attached to the outer surface and makes an angle of 30° at the point of attachment. The dimensions of the pole are shown in figure. Determine the allowable tensile force T in the cable if the allowable compressive stress in the aluminium pole is 100 MPa.



[12 + 8 = 20 marks]

Do not write in this margin

Section B : Highway Engineering - 1 + Surveying and Geology-1 + Geo-technical and Foundation Engineering - 2 + Environmental Engineering - 2

Q.5 (a) What are the various factors that control the highway alignment? List out the special care which should be taken while aligning roads in hilly areas? **[12 marks]**

Spl features care which are taken for Aligning of hilly road.

① Shear strength of soil

② drainage conditions.

③ bends along the road.

④ Gradient & curvature

⑤ wild life existence.

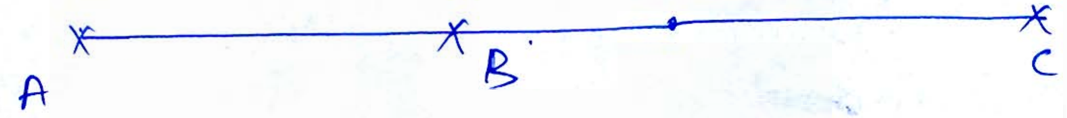
⑥ land slide conditions

Q.5 (b) (i) Classify different types of survey based on their purposes.

(ii) A 30 m chain was found to be 5 cm too long after chaining 1650 m. It was 8 cm too long at the end of day's work after chaining a total distance of 3125 m. If the chain was correct before commencement of the work, determine the true distance.

[8 + 4 = 12 marks]

(ii)



$l'_{AB} = 1650m$.

$L'_{AB} = \frac{30+30.05}{2}$

$= 30.025m$.

$l'_{BC} = 1475$

$L'_{BC} = \frac{30.05+30.08}{2}$

$= 30.065m$.

$l_{AB} = L'_{AB} \frac{l'_{AB}}{L_{AB}}$

$= 1650 \times \frac{30.025}{30}$

$= 1651.375m$.

$l_{BC} = \frac{l_{BC}'}{L_{BC}} L_{BC}'$

$= \frac{1475 \times 30.065}{30}$

$= 1478.195m$

4

$l_{true} = l_{AB} + l_{BC} = 3129.57m$.

(i) classification of survey

1) land survey → survey which is done on the land.

a) topographical survey

survey which is used to identify natural features like hills, river, valley

b) control survey

→ survey which is used to fix the control points, it is useful in seperation of boundaries b/w two places.

(c) city survey

It is used for the urban planning and development of city

(ii) Hydro~~graphic~~ survey

→ It is used in water

→ Instrument used is fathometer

→ it is done to find the depth of various water bodies

(iii) Astronomical survey

→ It is done on the heavenly bodies like mars, moon

4

other survey

⇒ profile and crossection survey ⇝ used for road and drainage purpose

⇒ Archeological survey ⇝ for relics of antiquity

→ Geological survey → for strata of earth

Q.5 (c) Briefly explain the two commonly used geophysical methods of soil exploration. **[12 marks]**

1) seismic refraction mtd

In this method seismic rays are passed through the surface of the earth based on the ~~reactio~~ time travel & their response to the rays we will find the depth of soil layers.

2) pressure meter test

→ stress strain curves are plotted with this test

→ load is applied and the strain is found w.r.t load

4

3) Electric analogy

→ In this method it is used to find the water table below the ground by passing the cathode rays

→ it is costly method.

Do not write in this margin

Q.5 (d) The speed of overtaking and overtaken vehicles are 80 kmph and 50 kmph, respectively on a two way traffic road. The average acceleration during overtaking may be assumed as 0.99 m/sec^2.

(i) Calculate safe overtaking sight distance.

(ii) What are the minimum and desirable length of overtaking zones?

Assume any other data as per IRC-37.

[12 marks]

(i) $V_B = 50$ kmph $V_A = V_C = 80$ kmph.

$OSD = D_1 + D_2 + D_3$.

$D_1 = 0.278 V_B (t_r) = 0.278 (50)(2)$

$= 27.8$ m.

$D_2 = S_1 + S_2 + 0.278 V_B (T)$

$T = \sqrt{\frac{2(S_1+S_2)}{a}}$. $S_1 = 0.2 V_B + l$

assume length of vehicle = 6m.

$S_1 = S_2 = 0.2(50) + 6$

$= 16$ m

$T = \sqrt{\frac{4(16)}{0.99}} = 8.04$ sec

$D_2 = 16 + 16 + 0.278(50)(8.04) = 143.76$ m

$D_3 = 0.278 V_C (T) = 0.278(80)(8.04) = 178.809$ m.

$\therefore OSD = D_1 + D_2 + D_3$

$= 27.8 + 143.76 + 178.81$ m

$= 350.37$ m $\simeq 350.4$ m → safe OSD

Do not write in this margin

(ii) min OSD length $= 3(OSD) = 3(350.4)$

$= 1051.2m$

desirable $OSD = 5(OSD) = 5(350.4) = 1752m$.

12

Q.5 (e) Describe stratification of lakes and biological zones in lakes.

[12 marks]

Q.6 (a) A national highway passing through a plain terrain has a horizontal curve of radius equal to the ruling minimum radius.

(i) Design all the geometric features of this horizontal curve, assuming suitable data.

(ii) What is the safest intermediate sight distance provided for the given national highway?

[16 + 4 = 20 marks]

Q.6 (b) (i) The following latitudes and departures were obtained for a closed traverse *ABCDEFA* survey:

Line	Latitude (m)	Departure (m)
AB	0.00	183.79
BC	128.72	98.05
CD	177.76	–140.85
DE	–76.66	–154.44
EF	–177.09	0.00
FA	–52.43	13.08

Adjust the traverse by Bowditch's method and compute corrected latitudes and departures of all the traverse lines. Also calculate the bearing of *CD*.

(ii) What do you understand by the following forms of curves and where are they generally used?

1. Compound curve 2. Reverse curve

[14 + 6 = 20 marks]

Q.6 (c) (i) A conventional activated sludge process of municipal waste having discharge of $1000\ m^3/day$ disposes its digested sludge on relatively impervious farm land. Raw sludge suspended solids concentration is 225 mg/lt [70% volatile], BOD = 190 mg/*lt* (Excess activated sludge returned to primary). Primary settling suspended solids removal is 50% and BOD removal is 30%. Excess activated sludge is 0.4 gm volatile solids produced per gm of BOD applied.

Compute: (1) Total volatile solids to be anaerobically digested in kg/day.

(2) If anaerobic digestor produces 50% volatile solids and digested sludge solids concentration is 6%, then compute area required in hectares for disposal of sludge on the farm land. Specific gravity of sludge is 1 and rate of application on farm land is $2\ m^3/ha/day$.

(ii) A control sample BOD water containing seeded diluted water has a drop of 1.5 mg/lt in its dissolved oxygen over 5 days of incubation. If BOD sample is of 300 ml with 20 ml waste water in it and remaining seeded water in contribution has a drop of 6.8 mg/lt in its DO, then calculate the BOD of the test sample.

[14 + 6 = 20 marks]

Q.7 (a) Two sets of tacheometric readings were taken from an instrument station A (RL of A = 100 m) to a staff station B as shown below.

Instruments	P	Q
Multiplying constant	100	95
Additive constant	0.30	0.45
Height of instrument	1.40 m	1.45 m
Staff held	Vertical	Normal

Instruments	Instruments station	Staff station	Vertical angle	Stadia readings
P	A	B	5°44′	1.090, 1.440, 1.795
Q	A	B	5°44′	?

Determine:

(i) The distance between instrument station and staff station.

(ii) The R.L. of staff station B.

(iii) Stadia readings with instrument Q.

[20 marks]

for instrument P

$L = KS\cos\theta + C$

$L = 100(1.795 - 1.090)\cos 5^\circ 44' + 0.3$

$= 70.447m$

$D_{AB} = L\cos\theta = 70.447\cos 5^\circ 44'$

$= 70.095m$.

$V_{AB} = L\sin\theta = 70.447\sin 5^\circ 44'$

$= 7.037m$.

(i) Distance b/w A & B = 70.095m.

(ii) $(RL)_B = (RL)_A + 1.4 + 7.037 - 1.440$.

$= 100 + 1.4 + 7.037 - 1.440$

$(RL)_B = 106.997m$

(iii) staff held normal

Assume staff reading are S_1 S_2 S_3

$S_2 - S_1 = S_3 - S_2 = h$.

$L = KS + C$.

$L = 95(2h) + 0.45$

$L = 190h + 0.45 \rightarrow (1)$.

$D_{AB} = L\cos\theta + S_2\sin\theta$.

$70.095 = L(\cos 5^\circ 44') + S_2\sin(5^\circ 44') \rightarrow (2)$.

$V_{AB} = L\sin\theta$

$(RL)_B = (RL)_A + 0.45 + L\sin\theta - S_2\cos\theta$.

$106.997 = 100 + 0.45 + L(\sin 5^\circ 44') - S_2(\cos 5^\circ 44')$

$6.547 = L(\sin 5^\circ 44') - S_2(\cos 5^\circ 44') \rightarrow (3)$

solving (2), (3).

$L = 70.398\,m$.

$S_2 = 0.489\,m$.

Calculation error

12

$L = 70.398 = 190(h) + 0.45$

$h = 0.368\,m$.

$\therefore S_2 = 0.489\,m$

$S_1 = S_2 - h = 0.489 - 0.368 = 0.121\,m$ } staff reading

$S_3 = S_2 + h = 0.489 - 0.368 = 0.857\,m$.

Q.7 (b) (i) Explain the factors influencing the geometric design of hills roads.

(ii) Why should the psychological widening be added to the mechanical widening of roads?

[12 + 8 = 20 marks]

(i) Geometric design factors

1) Gradient → the Gradient should not be too high.

2) Curvature – the curve along with gradient makes vehicle to drive slowly

3) Shear strength of soil → soil strength is imp parameter because in rainy season soil looses its strength.

4) weep holes

these are used for the drainage of the rain water without build up of pressure

5) Retaining walls/ parpet walls

these walls are constructed to make the soil stable and prevent land sliding

6) Traffic markings and signals

⇒ Traffic signs should be present at each and every turning so that driver can be caution

⇒ street lights are to be provided at neccessary location

⇒ sign boards of steeper Gradient sites & wild animal zone should be present

6

(ii)

⇒ mechanical widening is just based on the theoritical approach

⇒ psychological widening is added

1) because driver tries to move outer lane during the curve journey to have the greater sight distance

2) driver feel that he may get collided with the inner vehicles or ~~po~~ kerb so that he move outer side of road during curve

→ Also extra width offers greater skid resistance so that centrifugal force can easily balanced .

→ offtracking issue since backward wheels donot follow the forward wheels .

2

formulae??

Q.7 (c) (i) Design a group of friction piles which is required to carry a load of 3500 kN including the weight of the pile cap at a site where the soil is uniform clay to a depth of 20 m, underlain by rock. Average unconfined compressive strength of the clay is 65 kN/m^2. The clay may be assumed to be of normal sensitivity and normally loaded with liquid limit of 55%. A factor of safety of 3 is required against shear failure.

(ii) A square mass concrete footing supporting a load of 3500 kN extends from ground level to 4 m deep into a clay stratum. What will be the size of the footing allowing for a factor of safety of 3.0? Unit weight of concrete is 24 kN/m^3. Shear strength of the soil is 0.15 N/mm^2. Adhesion of clay with footing is 30 kN/m^2. The adhesion may be supported to act over a depth of 2 m from the bottom of the foundation. For $\phi = 0°$, $N_C = 5.7$, $N_q = 1$ and $N_\gamma = 0$. Take unit weight of soil as 21 kN/m^3.

[10 + 10 = 20 marks]

(i) $Q_{ulti} = Q_{safe} \times FOS$

$= 3500 \times 3$

$= 10500 kN$.

Since given friction pile. $\therefore \frac{Q_g}{nQ_i} > 100\%$.

$Q_{ultimate} = n \times \pi \times d \times l \times c \times \alpha$.

Assume dia $= 0.5m$

$L = 12m$ since we can go upto 20m

$\alpha = 0.8$

$10500 = n(\pi)(0.5)(12)\left(\frac{65}{2}\right)(0.8)$

$n = 21.42$

provides 25 piles of 5×5.

As per IS code min spacing $= 3d$ for frcition

$= 3 \times 0.5 = 1.5m$.

$(Q_{ultimate})_{group} = 2L(L_0+B_0)C$ $\quad s = 1.5m$

$= 2(12)(2)(4s+d)\left(\frac{65}{2}\right)$

$= 2(12)(2)(6.5)\left(\frac{65}{2}\right)$

$= \cancel{68404} \; 20280 \text{ kN}$

$Q_{individual} = 25 \times \pi \times d \times l \times C \times \alpha$

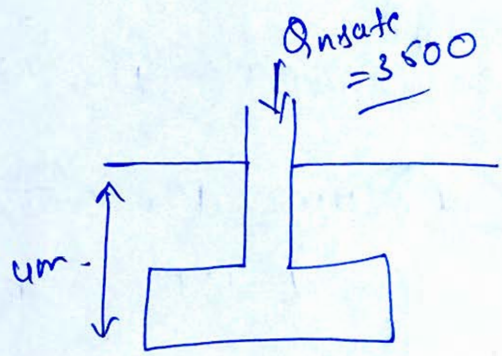
$= 25 \times \pi \times 0.5 \times 12 \times \frac{65}{2} \times 0.8$

$= 12252.21 \text{ kN}$.

$\frac{Q_g}{Q_i} > 100\%$. Hence ok.

(ii)

FOS = 3



$S = C = .15 \text{ N/mm}^2$

$= 150 \text{ kN/m}^2$.

$q_{u} = 1.3CN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma$

$q_{nu} = 1.3CN_c + \gamma D_f(N_q - 1)$ $\quad N_q = 1$

$q_{nu} = 1.3CN_c$.

$q_{ns} = \frac{1.3CN_c}{FOS}$.

Assuming load acting is net external load

$$\frac{1.3 C N_c}{FOS} = \frac{3500}{B \times B}.$$

$$\frac{1.3 \times 150 \times 5.7}{3} = \frac{3500}{B \times B}$$

$B = 3.07m$.

$\therefore$ provide square footing of size $3.1m \times 3.1m$.

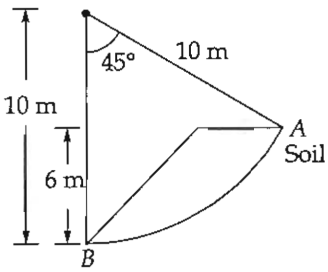
10

Q.8 (a) **(i)** What is gradient in vertical alignment? Also, define the types of gradient and their values for different types of terrain as per Indian practices.

(ii) There is a horizontal curve of radius 500 m and length 250 m on a highway. Determine the setback distance required from the centre line of the curve so as to provide a stopping sight distance of 100 m. (The distance between the centre lines of the road and the inner lane is 1.9 m.)

[14 + 6 = 20 marks]

Q.8 (b) **(i)** Find the factor of safety (1) with respect to shear strength and, (2) with respect to height along the indicated sliding surface AB in the figure below.



$c = 55\ \text{kN/m}^2$
$\phi = 15°$
$\gamma = 17\ \text{kN/m}^3$

Normal effective pressure on sliding surface AB is 225 kN/m^2.

Downward tangential disturbing force along AB is 850 kN/m.

(ii) Explain sludge treatment process.

[12 + 8 = 20 marks]

Do not write in this margin

Q.8 (c) **(i)** Find the maximum permissible error in laying off the direction of an offset so that maximum displacement may not exceed 0.025 cm on paper given that length of the offset is 25 m, the scale is 1 cm to 75 m and the maximum error in the length of the offset is 0.50 m.

(ii) Write short notes on the following methods of plane table surveying:

1. Radiation
2. Traversing
3. Intersection
4. Resection

(iii) Explain the two basic principles of surveying.

[4 + 12 + 4 = 20 marks]

■■■■

Space for Rough Work

Space for Rough Work