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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-3 : Strength of Materials [All Topics]

Highway Engg. - 1 + Surveying and Geology-1 + Geo-technical & Foundation Engg. - 2 + Environmental Engg. - 2 [Part Syllabus]

Name :

Roll No :

Test Centres			Student's Signature
Delhi ☐	Bhopal ☐	Jaipur ☐	
Pune ☐	Kolkata ☐	Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	50
Q.2	59
Q.3	38
Q.4	
Section-B	
Q.5	32
Q.6	58
Q.7	
Q.8	
Total Marks Obtained	237

Signature of Evaluator: Ujjawal

Cross Checked by

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Remarks

- Keep revising & practicing questions.
- Keep doing good.

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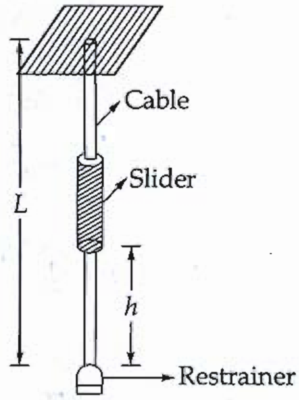
Section A : Strength of Materials

Q.1 (a) (i) A prismatic bar of circular cross-section is loaded by an axial tensile force of 85 kN. The bar has length of 3.0 m and diameter of 30 mm. It is made of aluminium with modulus of elasticity E = 70 GPa and Poission's ratio of $\frac{1}{3}$. Calculate the decrease in diameter 'Δd' and the change in volume of the bar. Assume the stresses in the bar are below proportionality limit.

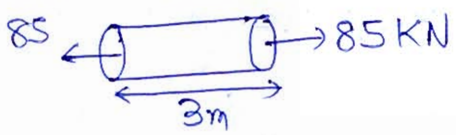
(ii) A cable with a restrainer at the bottom hangs vertically from its upper end as shown in figure. The cable has cross sectional area of 50 mm^2 and modulus of elasticity E = 130 GPa. A slider of mass m = 40 kg drops from a height of h = 1.2 m on to the restrainer. If the allowable stress in the cable under an impact load is 700 MPa, then, what is the minimum permissible length L of the cable? (Take g = 9.81 m/s^2)



[6 + 6 = 12 marks]

(i)

$\phi = 30mm.$



$E = 70 GPa.$

$\mu = \frac{1}{3}.$

$V = \pi r^2 h.$

$$\frac{dV}{V} = \frac{\pi(2r)dr\,h}{\pi r^2 h} + \frac{\pi r^2 dh}{\pi r^2 h}$$

$$\boxed{\frac{\Delta V}{V} = 2\frac{\Delta r}{r} + \frac{dh}{h}}$$

$$\epsilon_h = \frac{\sigma_h}{E} - \mu\frac{\sigma_r}{E}^{\to 0}$$

$$\epsilon_h = \frac{\frac{85\times10^3}{\frac{\pi}{4}\times30^2}}{70\times10^3} = 1.718\times10^{-3}.$$

$$\epsilon_r = \frac{\sigma_r}{E}^{\to 0} - \frac{\mu\sigma_h}{E} = -\left(\frac{1\times}{3\times70\times10^3}\times\frac{85\times10^3}{\frac{\pi}{4}\times30^2}\right).$$

$$= -5.729\times10^{-4}$$

$$\Delta d = -5.729\times10^{-4}\times30 = \boxed{0.0171\ mm}$$

$$\frac{\Delta V}{V} = 2(-5.729\times10^{-4}) + (1.718\times10^{-3}) = 5.722\times10^{-4}$$

$$\Delta V = 5.722\times10^{-4}\times\pi\times(15)^2\times3000 = \boxed{1213.39\ mm^3}$$

6

(ii) $A = 50mm^2$

$E = 130 GPa$.

$m = 40 Kg$

$h = 1.2m$.

$\sigma_{allowable} = \boxed{700 MPa}$

$\boxed{L = ?}$

$130 \times 10^3 \frac{N}{mm^2}$

$130 \times 10^3 \times 10^{-6}$

$\delta = \boxed{\frac{PL}{AE}}$

from Conservation of Energy:—

$$mg(h+\delta) = \frac{\sigma^2}{2E} \times A \times l.$$

$$\Rightarrow 40 \times 9.81(1.2+\delta) = \frac{(700)^2}{2 \times 130 \times 10^9} \times 50 \times 10^{-6} \times l.$$

$$\Rightarrow 40 \times 9.81\left(1.2 + \left(\frac{40 \times 9.81 \times l}{50 \times 130 \times 10^3}\right)\right) = 9.423 \times 10^{-11} \times l.$$

$$\Rightarrow 470.88 + 0.0236\,l = 9.423 \times 10^{-11} \times l \times 10^{12}$$

$$= 470.88 + 0.0236\,l = 94.23\,l.$$

$$470.88 = 94.2064\,l$$

$$\boxed{l = 4.998\,m}$$

6

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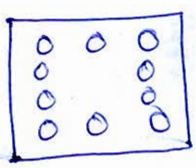
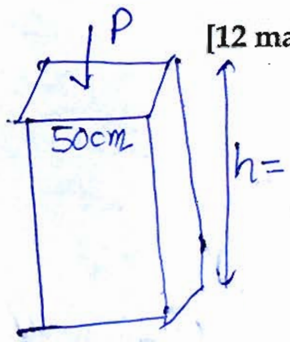
Q.1 (b) A reinforced concrete pedestal of height h and square cross-section having side of 50 cm is constructed with 12 steel reinforcing bars each has a diameter, d = 25 mm. The pedestal supports a compressive load P applied through a rigid bearing plate. Assuming linear elastic behaviour, calculate the maximum permissible value of the load if the allowable stress in the concrete and steel are 8 MPa and 70 MPa respectively. Neglect the self weight of the pedestal itself. (Assume E_s = 200 GPa and E_c = 25 GPa).

[12 marks]



12 bars
$\phi = 25\,mm$



$\sigma_{concrete} = 8\,MPa$.

$\sigma_{steel} = 70\,MPa$.

$E_S = 200 \times 10^3\,MPa$

$E_C = 25 \times 10^3\,MPa$.

from strain compatibility :-

$$\frac{\sigma_{concrete}}{E_C} = \frac{\sigma_{steel}}{E_S}$$

$$\Rightarrow [\sigma_c] \times \frac{\cancel{200}^{\,8}}{\cancel{25}} = \sigma_S$$

$$\boxed{\sigma_S = 8\sigma_C}$$

Assume $\sigma_S = 70\,MPa \Rightarrow \sigma_C = \frac{70}{8} = 8.75 > Per$ Not safe

Assume $\sigma_C = 8\,MPa$, $\sigma_S = 8 \times 8 = 64 < 75$ ✓

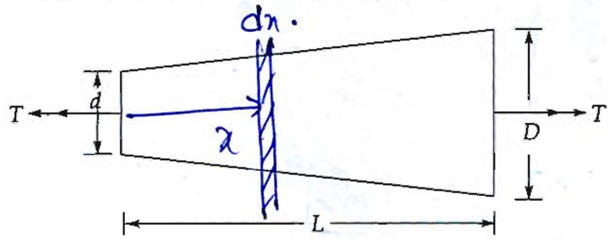
$$P = 12 \times \frac{\pi}{4} \times (25)^2 \times \sigma_S + \left[500^2 - \left[12 \times \frac{\pi}{4} \times 25^2\right]\right]\sigma_C$$

$$P = 12 \times \frac{\pi}{4} \times 25^2 \times 64 + 1952900.$$

$$\boxed{P = 2329.7\,KN.}$$

6

Q.1 (c) A tapered bar AB of solid circular cross section is twisted by torques 'T' applied at the ends. The diameter of the bar varies uniformly from 'd' at left end to 'D' at the right end. Derive an expression for the angle of twist 'ϕ' of the bar. The length of the bar is 'L' and shear modulus of the material used in the bar is G.



[12 marks]

$$\frac{\tau}{r} = \frac{T}{I_p} = \frac{G\theta}{l}$$

$$I_{px} = \frac{\pi}{32} \times (D_x)^4$$

$$\Rightarrow D_x = d + \left[\frac{D-d}{L}\right]x$$

$$= \boxed{d + Kx}$$

$$d\theta = \frac{T\,dl}{G\,I_{px}}$$

$$\int d\theta = \int \frac{T \times dx \times 32}{G \times \pi \times (d+Kx)^4}$$

$$\int d\theta = \frac{32T}{G\pi}\int_0^L \frac{dx}{(d+Kx)^4}$$

$$d + Kx = t$$

$$K\,dx = dt$$

$$\frac{32T}{G\pi}\int_0 \frac{dt}{K(t)^4}$$

$$\frac{32T}{G\pi K}\left[\frac{t^{-4+1}}{-4+1}\right]$$

12

$$\int -\frac{32T}{3G\pi K}\left[\frac{1}{t^3}\right]$$

$$-\frac{32T}{3G\pi K}\left[\frac{1}{(d+Kx)^3}\right]_0^l$$

$$\frac{32T}{3G\pi K}\left[\frac{1}{(d+Kx)^3}\right]_l^0$$

$$\frac{32T \times L}{3G\pi(D-d)}\left[\frac{1}{d^3} - \frac{1}{[d+(D-d)]^3}\right]$$

$$\frac{32TL}{3G\pi(D-d)}\left[\frac{1}{d^3}-\frac{1}{D^3}\right]$$

$$\frac{32TL}{3G\pi(D-d)}\left[\frac{D^3-d^3}{D^3 \cdot d^3}\right]$$

$$\frac{32TL}{3G\pi}\left[\frac{D^2+d^2+Dd}{D^3 \cdot d^2}\right] \qquad a^3-b^3=(a-b)(a^2+ab+b^2)$$

Proved

Q.1 (d) Briefly explain the need of theories of elastic failure. Describe the maximum strain theory and maximum shear strain energy theory along with formulae (derivation not required) and limitations associated with them.

[4 + 4 + 4 = 12 marks]

- Since while performing different uniaxial tests on the specimen in Universal testing machine we are only getting failure condition for uniaxial loading but in this Universe there will be general Loading so to derieve the failure condition from uniaxial loading to general loading, we need theories of failure for different types of materials.
- Each failure theory is suitable for a particular type of material.

Maximum strain Energy :-

For no failure, the energy stored in the system due to general loading should be less than the energy stored due to uniaxial Loading

$$\frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]\le\frac{\sigma_y^2}{2E}$$

→ It is not suitable for Brittle material.

→ It is also not suitable for tensile material which fails in shear.

$\sigma_1=\tau,\quad \sigma_2=-\tau.$

$2\tau^2\le\sigma_y^2$

$\tau\le\frac{1}{\sqrt{2}}\sigma_y\quad \tau\le 0.707\sigma_y$ But actual $\tau\le 0.577\sigma_y$

Max shear strain Energy :-

- It is also called Von-Mises theory
- for no failure the shear strain energy due to uniaxial loading should be less than Shear strain Energy due to general Loading

$$\frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]\le\frac{\sigma_y^2}{6G}$$

→ It is no suitable for Brittle material

→ It is suitable for Ductile material

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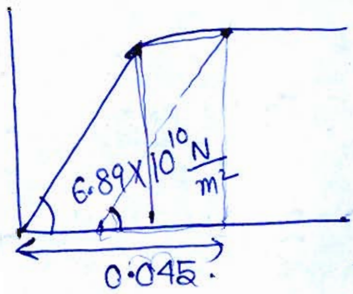
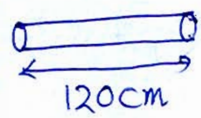
Q.1 (e) An aluminium bar has length $L = 120$ cm and diameter $d = 2.5$ cm. The initial straight line part of the stress-strain curve has a slope (modulus of elasticity) of $6.89 \times 10^{10}\ \text{N/m}^2$. The bar is loaded by tensile force, $P = 250$ kN beyond elastic limit. It reaches to a strain value of 0.045 and then unloaded.

(a) What is the permanent set of the bar?

(b) What is the modulus of resilience after the load application?

[12 marks]





$\phi = 2.5cm$

$P = 250kN$ (Beyond elastic limit)

$\epsilon = 0.045$

$\Delta l_{final} = 0.045 \times 1200 = \boxed{54mm}$

$$\text{Stress} = \frac{250 \times 10^3}{\frac{\pi}{4} \times (25)^2} = 509.55\ MPa$$

$\sigma = E \times \text{Strain}$

$$\text{Strain} = \frac{509.55}{\frac{6.89 \times 10^{10}}{10^6}} = 7.395 \times 10^{-3}$$

$$\Delta L_{elastic} = 7.395 \times 10^{-3} \times 1200 = \boxed{8.874mm}$$

$$\boxed{\text{Permanent set} = 45.125mm}$$

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$$\text{Modulus of resilence} = \left[\frac{\sigma^2}{2E}\right] \text{volume}$$

$$= \frac{(509.55)^2}{2 \times 6.89 \times 10^4} \times \frac{\pi}{4} \times (25)^2 \times 1200$$

$$\frac{N^2}{m^4} \frac{m^2}{\times N} \times m^3$$

$$= \boxed{1.109\ KN\text{-}m}$$

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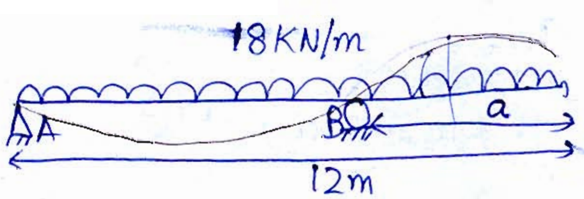
2 (a) (i) A 12 m long beam of uniform section carries a uniformly distributed load of 18 kN/m over the whole length. If the beam is freely supported at the left end, find the position of second support so that the maximum bending moment for the beam shall be as small as possible. Also, find the maximum bending moment for this case.

(ii) State Castigilano's first theorem and Maxwell's reciprocal theorem.

[16 + 4 = 20 marks]

(i)



$R_A + R_B = 18 \times 12$.

$\Sigma M_A = 0$;

$18 \times \frac{12^2}{2} - R_B \times (12-a) = 0 \Rightarrow \boxed{R_B = \frac{9 \times 144}{(12-a)}}$

Moment (-ve) Maximum will be at B :-

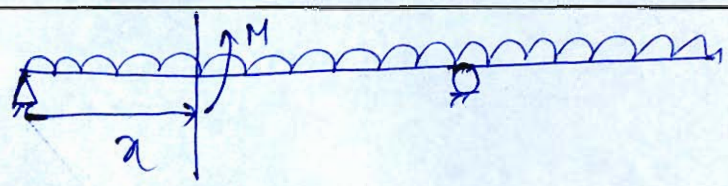
$M_B = -\frac{18 \times a^2}{2} = \boxed{-9a^2}$

$R_A = 216 - \frac{1296}{12-a}$

$\frac{2592 - 216a - 1296}{(12-a)}$

$= \left[\frac{1296 - 216a}{12-a}\right]$.

$\frac{216(6-a)}{12-a}$



$$M - R_A x + \frac{18x^2}{2} = 0$$

$$M_+ = R_A x - 9x^2$$

$$\frac{216(6-a)}{(12-a)} x - 9x^2$$

from $M_+ = M_{ax}$ $\frac{\partial M_+}{\partial x} = 0$.

$$\frac{216(6-a)}{(12-a)} - 18x = 0$$

$$\boxed{x = \frac{12(6-a)}{(12-a)}}$$

$$M_{max\oplus} = \frac{216(6-a)}{(12-a)} \times \frac{12(6-a)}{(12-a)} - 9\left(\underbrace{\frac{12(6-a)}{(12-a)}}_{z}\right)^2$$

$$\Rightarrow 18\left[z^2\right] - 9z^2$$

$$= \boxed{9z^2}$$

from M_{max} to be small $M_{\oplus} = M_{\ominus}$

$$\cancel{9}a^2 = \cancel{9}z^2$$

$$a^2 = \left(\frac{12(6-a)}{(12-a)}\right)^2$$

$$a(12-a) = 12(6-a)$$

$$12a - a^2 = 72 - 12a$$

$$a^2 - 24a + 72 = 0$$

$a = 20.48$ X (Not possible)

$a_2 = 3.514$

$$\boxed{a = 3.514\ m.}$$

$$M_{max} \Rightarrow x = \frac{12(6-3.519)}{(12-3.519)} = \boxed{3.514\,m}$$

$$M = \frac{216(6-3.514)}{(12-3.514)} \times 3.514 - 9(3.514)^2$$

$$222.358 - 111.224$$

$$= \boxed{111.134\ KN\text{-}m}$$

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(ii) Castangiliano's Theorom :-

According to this theorom, the derivative of Energy stored in a body with respect to the deflection at a point gives the corresponding Load at that point

$$\boxed{\frac{\partial U}{\partial \Delta} = P}$$

According to 2nd theorom, the derivative of energy stored with respect to the load at that point gives deflection.

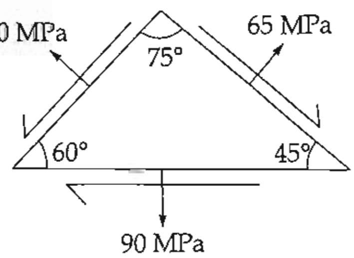
$$\boxed{\frac{\partial U}{\partial P} = \Delta}$$

Maxwell Resiprocal theorom :- It is an application of Betti's theorom.

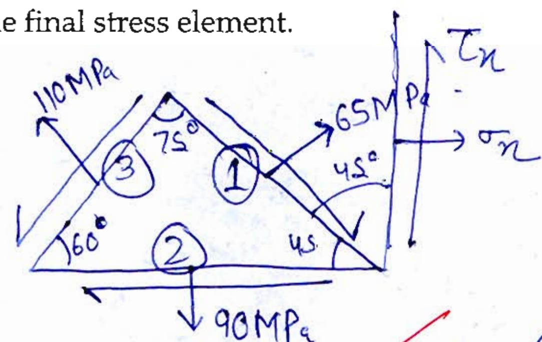
4

$$\boxed{P_1\Delta_1 = P_2\Delta_2}$$

Q.2 (b) In a strained body, the normal stresses on three planes inclined as shown in figure are 65 MPa (Tensile), 90 MPa (Tensile) and 110 MPa (Tensile). Determine the shear stresses acting on these planes. Also find the principle stresses and draw the final stress element.



[20 marks]



Let the stress on verticle plane be σ_x & Shear stress be τ_x

<u>for plane 1:-</u>

$$\sigma'_x = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos(90^\circ) + \tau_{xy}\sin 90^\circ$$

$$= \left(\frac{\sigma_x+90}{2}\right) + \left(\frac{\sigma_x-90}{2}\right)\times 0 + \boxed{\tau_x}$$

$$65 = \left(\frac{\sigma_x+90}{2}\right) + \tau \quad \text{--- (1)}$$

for plane 3°; $\theta = [-30°]$

$$\sigma'_x = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos(2\times 30°) - \tau \sin(60°)$$

$$110 = \left(\frac{\sigma_x+90}{2}\right) + \left(\frac{\sigma_x-90}{2}\right)\times\frac{1}{2} - \frac{\tau\sqrt{3}}{2} \quad \text{--(II)}$$

from (I) + (II)

$$110 = \left(\frac{\sigma_x+90}{2}\right) + \frac{(\sigma_x-90)}{2}\times\frac{1}{2} - \cancel{\frac{\tau\sqrt{3}}{2}}$$

$$65 = \left(\frac{\sigma_x+90}{2}\right) \quad + \tau \;\Big]\times\frac{\sqrt{3}}{2}.$$

$$\frac{65\sqrt{3}}{2} = \left(\frac{\sigma_x+90}{2}\right)\frac{\sqrt{3}}{2} + \cancel{\frac{\tau\sqrt{3}}{2}}$$

$$110+\frac{65\sqrt{3}}{2} = \left(\frac{\sigma_x+90}{2}\right)\left[1+\frac{\sqrt{3}}{2}\right] + \frac{(\sigma_x-90)}{4}$$

$$166.291 = 0.933\sigma_x + 83.971 + 0.25\sigma_x - 22.5$$

$$104.82 = 1.183\sigma_x$$

$$\boxed{\sigma_x = 88.605\text{ MPa}}$$

$$65 = \left[\frac{88.605+90}{2}\right] + \tau \Rightarrow \boxed{\tau = -24.302\text{ MPa}}.$$

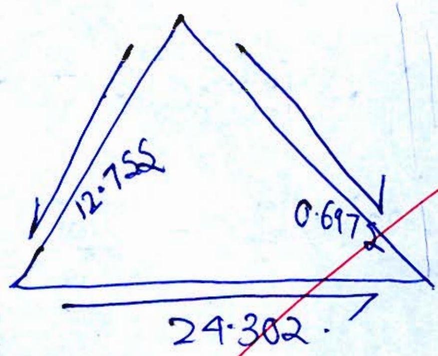
$$\tau_{(I)} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \underbrace{\cancel{\tau_{xy}\cos 2\theta}}_{\to 0}.$$

$$= -\left(\frac{88.605-90}{2}\right) = \boxed{-0.6975\text{ MPa}}$$

$$\tau_3 = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin(-60) + (-24.302)\cos(60°)$$

$$= -0.604 - 12.151$$

$$= \boxed{-12.755\text{ MPa}}$$



$$\sigma_1/\sigma_2 = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \left(\frac{88.605+90}{2}\right) \pm \sqrt{\left(\frac{88.605-90}{2}\right)^2 + (24.302)^2}$$

$$= 89.3025 \pm 24.312$$

$$\Rightarrow \boxed{\sigma_1 = 113.614 \text{ MPa}}$$

$$\boxed{\sigma_2 = 64.99 \text{ MPa}}$$

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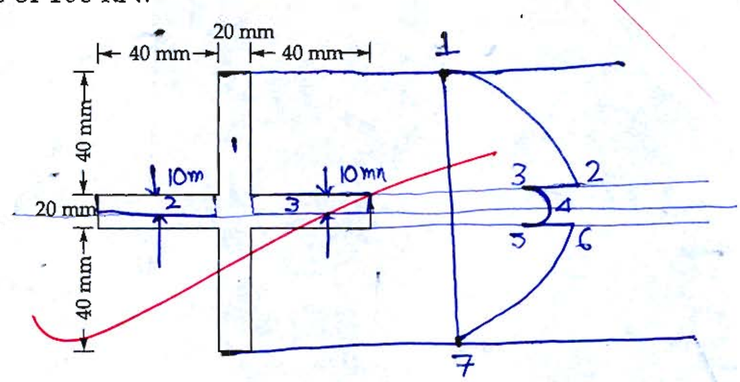
Q.2 (c) (i) Plot the shear stress distribution over the beam cross-section as shown in figure due to a shear force of 100 kN.



(ii) Explain briefly about the stress-strain diagram for mild steel.

[12 + 8 = 20 marks]

$\boxed{\tau_1 = 0 = \tau_7 .}$

$\tau = \dfrac{VA\bar{Y}}{Ib}.$

$I_{section} = 2 \times \left[\dfrac{20 \times 50^3}{3} + \dfrac{40 \times 10^3}{3} + \dfrac{40 \times 10^3}{3} \right]$

$= \underline{1.72 \times 10^6 \ mm^4}.$

Do not write in this margin

$$\tau_2 = \frac{VA\bar{Y}}{Ib} = \frac{100\times10^3 \times 40\times20\times30}{1.72\times10^6 \times 20} = 69.767\,MPa$$

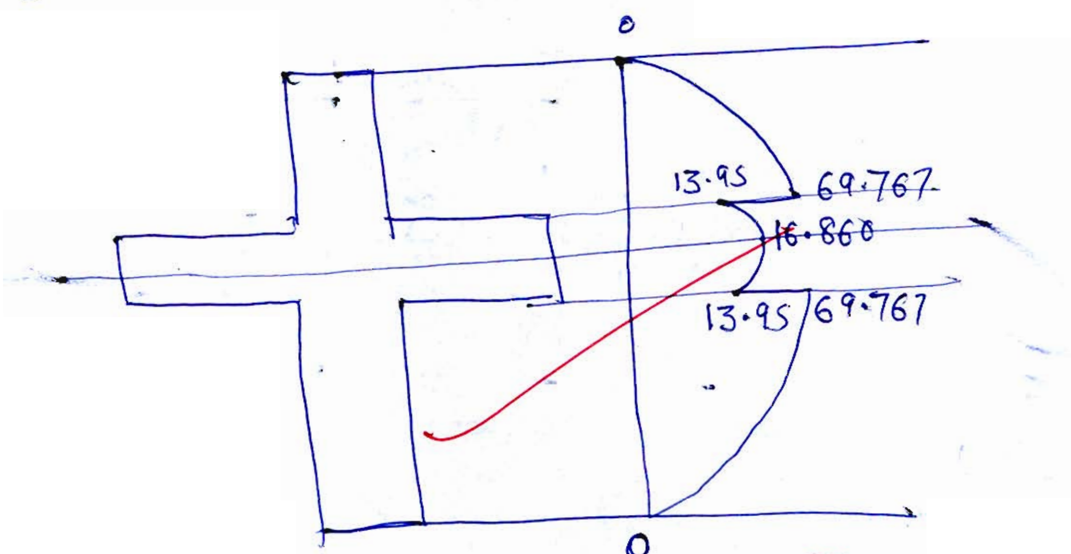
$$\tau_3 = \frac{69.767\times20}{100} = 13.95\,MPa.$$

$$\tau_4 = \frac{VA\bar{Y}}{Ib} = \frac{100\times10^3 \times A\bar{Y}}{1.72\times10^6 \times 100} = 16.860\,MPa$$

$$A\bar{Y} = A_1Y_1 + A_2Y_2$$
$$= (20\times40)\times30 + (100\times10)\times5$$
$$= 29000$$

$$\tau_5 = \tau_3 = 13.95\,MPa$$

$$\tau_6 = \tau_2 = 69.767\,MPa.$$



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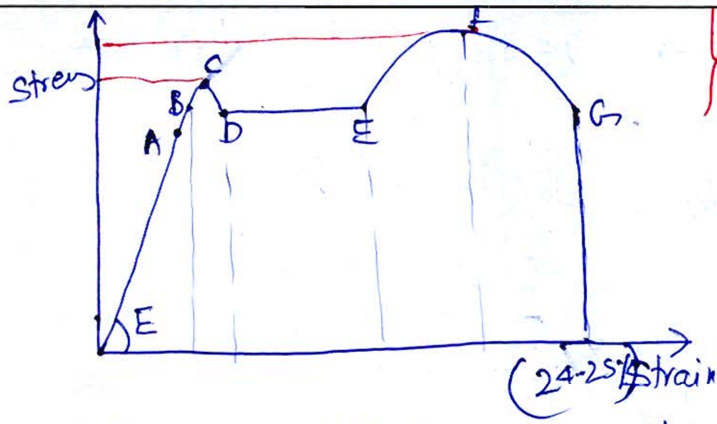

Stress values ?

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A = Proportional limit → Upto this point stress is directly proportional to strain.

B = Elastic limit :- Upto this point if load is applied and removed the material will come back to its normal position.

C & D :- Upper Yield point & Lower Yield point
→ It is the point after which there is plastic deformation in the material.
→ for stability Lower yield point is considered.

DE :- Yield pleatue :- In this region, without changing of stress, strain continuously increases.

EF :- Strain Hardening :- Due to re-orientation of atoms at molecular level, the material again show resistance and Hence stress increases.

F → Ultimate point :- It is the maximum stress that a material can reach.

FG :- Necking Zone → Area of specimen starts to decrease.

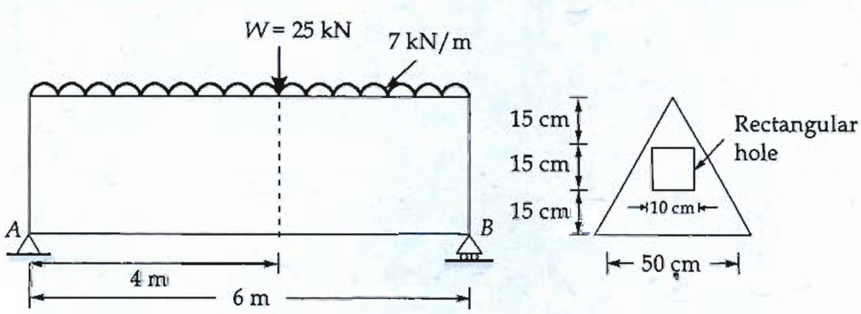
G = Fracture point → It is point where the material fractures.

This test is performed in UTM

Specimen $\phi = 0.5''$

Gauge length = 2''

Q.3 (a) A simply supported beam and it's cross-section are shown below. The beam carries a point load of 25 kN as shown in figure. The self weight of the beam is 7 kN/m. Determine the maximum value of bending compressive stress at a section 2 m from end *A*.



[20 marks]

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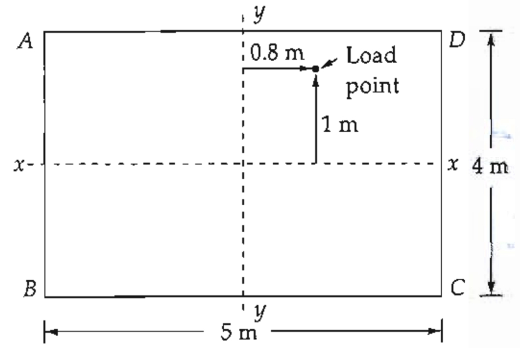
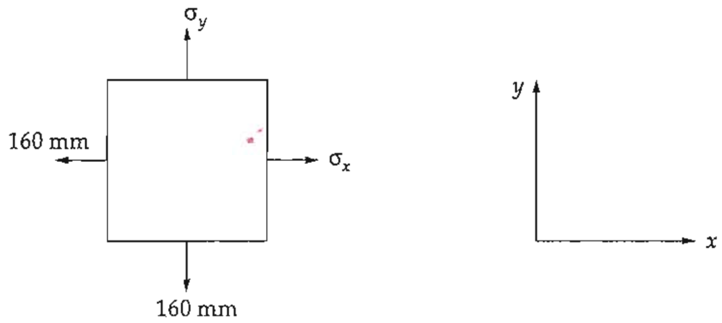
Q.3 (b) (i) A masonry pier as shown in the figure supports a load of 60 kN.



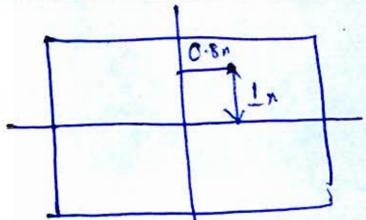
1. Find the stress developed at each corner.
2. Find the value of additional load required to be placed at centre of pier so that there is no tension anywhere in the column section.

(ii) A thin rectangular steel plate 100 mm by 100 mm undergoes elongation of 0.05 mm and 0.03 mm in x and y directions due to stresses σ_x and σ_y respectively as shown in figure. Determine the magnitude of σ_x and σ_y if modulus of elasticity of steel is 200 GPa and Poisson's ratio is 0.3.



[15 + 5 = 20 marks]

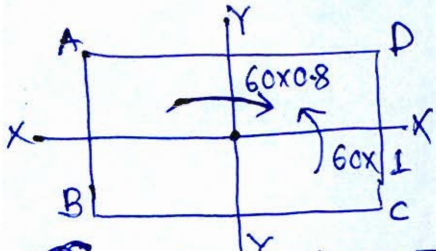
6 (i)



$P = 60\,KN$.

Transfer Load to centre:-

Compr = +ve
Tens = −ve



$$\sigma_A = \frac{60}{20} - \frac{(60\times 0.8)\times 2.5\times 12}{4\times 5^3} + \frac{(60\times 1)\times 2\times 12}{5\times 4^3}$$

$$= 3 - 2.88 + 4.5 = \boxed{4.62\,KN/m^2}$$

$$\sigma_B = 3 - 2.88 - 4.5 = \boxed{-4.38\,KN/m^2}$$

$$\sigma_C = 3 + 2.88 - 4.5 = \boxed{1.38\,KN/m^2}$$

$$\sigma_D = 3 + 2.88 + 4.5 = \boxed{10.38\,KN/m^2}$$

Since tension developed at B only

So, $\frac{P}{A} = 4.38$

$\Rightarrow \frac{P}{20} = 4.38 \Rightarrow \boxed{P = 87.6\,KN}$

13.

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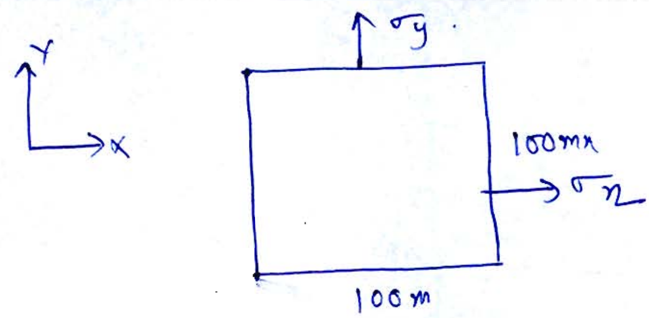
(ii)



$\epsilon_x = 0.05mm$
$\epsilon_y = 0.03\ mm$

$E_s = 200GPa, \quad \mu = 0.3$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\frac{0.05}{100} = \frac{\sigma_x}{200\times10^3} - \left.\frac{0.3\sigma_y}{200\times10^3}\right] \times 0.3$$

$$\frac{0.03}{100} = \frac{\sigma_y}{200\times10^3} - \frac{0.3\sigma_x}{200\times10^3}$$

$$\frac{0.05\times0.3}{100} = \cancel{\frac{0.3\sigma_x}{E}} - \frac{(0.3)^2\sigma_y}{E}$$

$$\frac{0.03}{100} = -\cancel{\frac{0.3\sigma_x}{E}} + \frac{\sigma_y}{E}$$

$$\frac{0.045}{100} = \frac{\sigma_y}{E}\left(1 - 0.3^2\right)$$

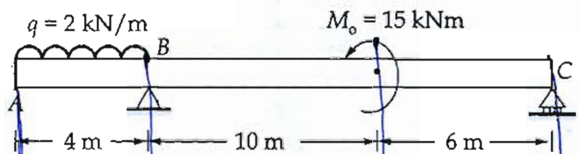
$$\boxed{\sigma_y = 98.901\ MPa}$$

$$\frac{0.03}{100} = \frac{98.901}{200\times10^3} - \frac{0.3\sigma_x}{200\times10^3}$$

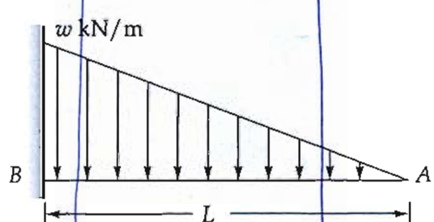
$$\boxed{\sigma_x = 129.6703\ MPa}$$

5

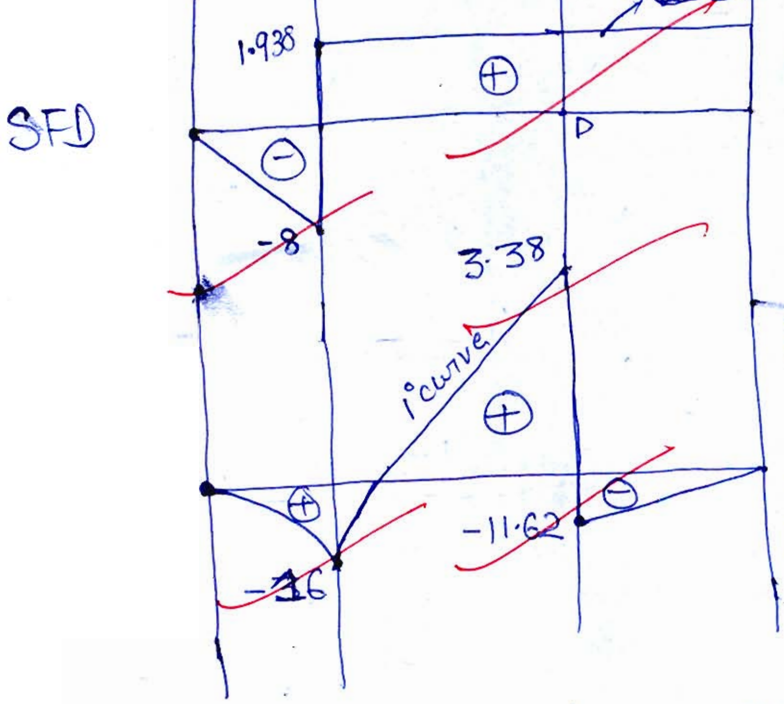
Q.3 (c) (i) A beam *ABC* with an overhang at the left hand end is shown in figure below. The beam is subjected to a uniform load of intensity q = 2 kN/m on the overhang *AB* and a counterclockwise couple M_o = 15 kN-m as shown. Draw the shear-force and bending moment diagram for this beam.



(ii) Find slope and deflection at free end using double integration method for the beam shown in figure.



[10 + 10 = 20 marks]



Constant as there is no force

$SF_B - SF_A =$ Area of Lo

$x - 0 = -2 \times 4$

$\boxed{x = -8}$

$BM_B - BM_A =$ Ar. of SFD

$x - 0 = \frac{1}{2} \times 4 \times -8$

$BM_D - BM_B = 19.38$

$BM_D = 19.38 + (-16)$

$= 3.38$

8

$R_B + R_C = 8$ — (1)

$\Sigma M_B = 0;$ ↺+

$\frac{2 \times 4^2}{2} + 15 = R_C \times 16$

$\frac{31}{16} = R_C = 1.9375$

$R_B = 8 - \frac{31}{16}$

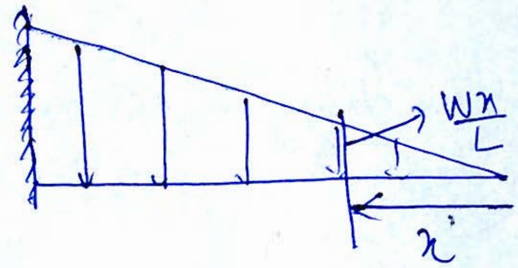
$= \boxed{6.062 \text{ kN}}$

Explain at CP8.

X

BM

(ii)



$$\frac{\omega}{L} = \frac{\omega'}{x}$$

$$EI\frac{d^2y}{dx^2} = M.$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{2} \times x \times \frac{\omega x}{L} \times \frac{x}{3}.$$

$$EI\frac{d^2y}{dx^2} = -\frac{Wx^3}{6L}$$

$$EI\frac{dy}{dx} = \frac{-Wx^4}{24L} + C_1$$

10

$$EIy = \frac{-Wx^5}{120L} + C_1x + C_2$$

@ $x = L$, $\frac{dy}{dx} = 0$, $x = L$, $y = 0$

$$\boxed{C_1 = \frac{WL^3}{24}}, \quad \frac{WL^4}{120} - \left(\frac{WL^4}{24}\right) = C_2$$

$$= \frac{(1-5)WL^4}{120}$$

$$C_2 = -\frac{4WL^4}{120}$$

$$= \left[\frac{-WL^4}{30}\right]$$

Slope @ $x = 0$;

$$\boxed{\frac{dy}{dx} = \frac{WL^3}{24EI}}$$

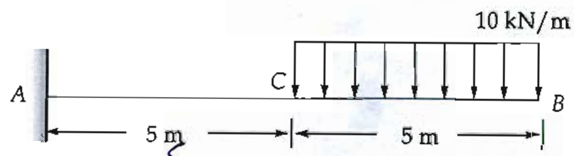
Δ @ $x = 0$;

$$\boxed{y = \frac{-WL^4}{30EI}}$$

Do not write in this margin

Q.4 (a) (i) Explain the application of the moment area method to determine the slope and deflection.

(ii) Find the angle of rotation θ_B and deflection δ_B by using moment area method at the free end B of cantilever beam ACB supporting a uniform load of intensity 10 kN/m acting over the beam as shown. (The beam has length 10 m and constant flexural rigidity EI)



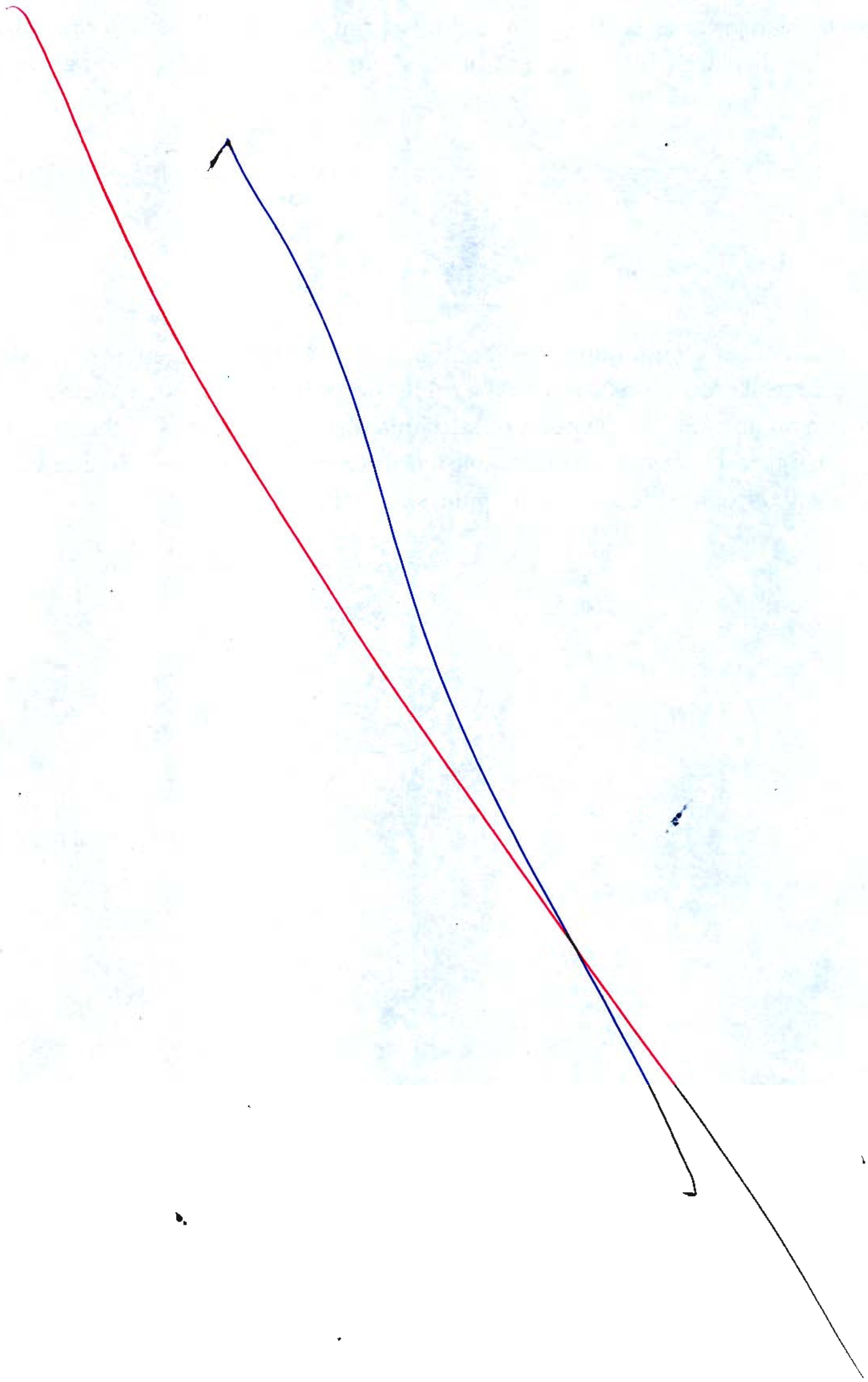
[8 + 12 = 20 marks]

Q.4 (b) (i) A steel shaft is to be manufactured as a circular tube. The shaft is required to transmit a torque of 1250 N-m without exceeding an allowable shear stress of 40 MPa nor an allowable rate of twist of 0.75/m. The shear modulus of elasticity of the steel is 78 GPa. Determine the required diameter of the shaft if the thickness of the shaft is $1/10^{th}$ of the outer diameter.

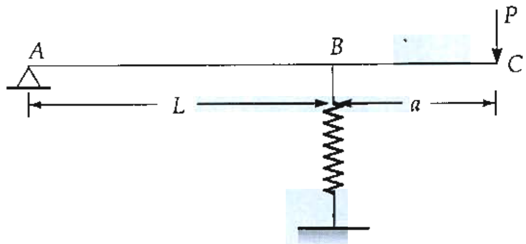
(ii) A prismatic shaft consists of a solid aluminum rod of diameter 35 mm, which is inside a steel tube of external diameter 52 mm. Both the shafts are firmly jointed and subjected to a torque of 1025 N-m. Find the maximum stresses developed in aluminium and steel shafts.

[Take, $G_{Al} = 70 \times 10^9$ N/m^2 and $G_{st} = 80 \times 10^9$ N/m^2]

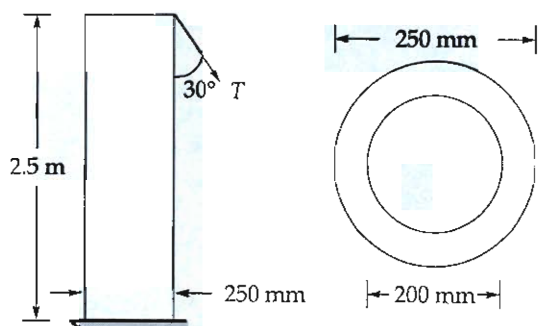
[12 + 8 = 20 marks]



Q.4 (c) (i) An overhanging beam ABC rests on a simple support at A and a spring support at B (see figure). A concentrated load *P* acts at the end of the overhang. Span AB has length *L*, the overhang has length '*a*' and the spring has stiffness k. Determine the downward displacement at the end of overhang i.e. at C using Castingliano's theorem.



(ii) A vertical pole of aluminium is fixed at the base and pulled at the top by a cable having a tensile force T as shown. The cable is attached to the outer surface and makes an angle of 30° at the point of attachment. The dimensions of the pole are shown in figure. Determine the allowable tensile force T in the cable if the allowable compressive stress in the aluminium pole is 100 MPa.



[12 + 8 = 20 marks]

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Section B : Highway Engineering - 1 + Surveying and Geology-1 + Geo-technical and Foundation Engineering - 2 + Environmental Engineering - 2

Q.5 (a) What are the various factors that control the highway alignment? List out the special care which should be taken while aligning roads in hilly areas?

[12 marks]

Q.5 (b) (i) Classify different types of survey based on their purposes.

(ii) A 30 m chain was found to be 5 cm too long after chaining 1650 m. It was 8 cm too long at the end of day's work after chaining a total distance of 3125 m. If the chain was correct before commencement of the work, determine the true distance.

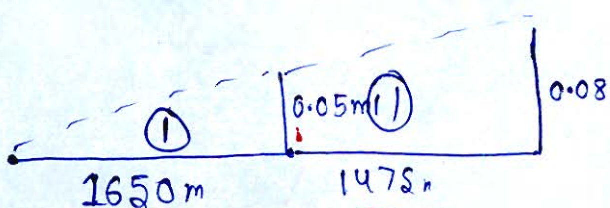
[8 + 4 = 12 marks]

5.(b) (ii)

$l' = 30.05\,m$.

$l = 30\,m$.

$L = 1650\,m$.



4

for s/c ①:-

$$\text{Avg. change} = \frac{0 + 0.05}{2} = 0.025$$

$$L_1' = \left(\frac{1650}{30}\right) \times 30.025 = \boxed{1651.375\,m}$$

for s/c -2:-

$$\text{Avg. change} = \frac{0.05 + 0.08}{2} = 0.065\,m$$

$$L_2' = \left(\frac{1475}{30}\right) \times 30.065 = \boxed{1478.195\,m.}$$

$$L_{final} = 1651.375 + 1478.195$$

$$= \boxed{3129.570\,m}$$

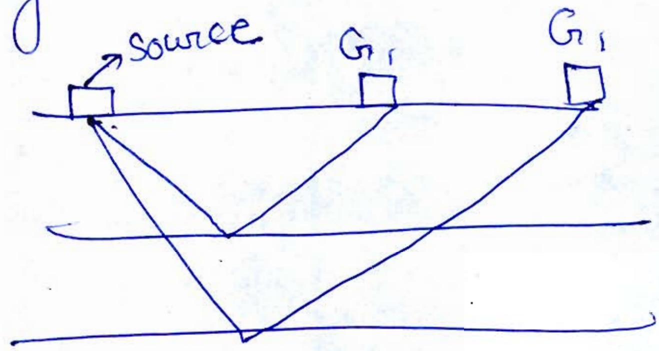
Q.5 (c) Briefly explain the two commonly used geophysical methods of soil exploration.

[12 marks]

Two Methods :-

(i) ~~Electrical resistivity Methods~~ Geo-Phone Method :-

- In this method a source of wave is being applied at a point on the surface of soil and 2 geo-phones are kept at a particular distance from point of radiation.
- The waves after travelling through soil & reflecting from a particular depth is detected by the Geo-phones.
- This gives the clear idea of strata of soil as time is being measured and velocity is also measured.



4

(ii) Electrical Resistivity :-

- In this method electrodes are being drawn in soil
- The voltage is being applied on the assembly
- The Resistance is being measured which is caliberated with the soil property.

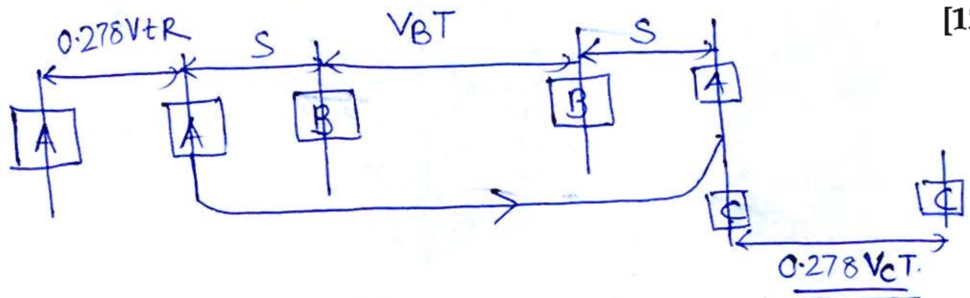
Q.5 (d) The speed of overtaking and overtaken vehicles are 80 kmph and 50 kmph, respectively on a two way traffic road. The average acceleration during overtaking may be assumed as 0.99 m/sec^2.

(i) Calculate safe overtaking sight distance.

(ii) What are the minimum and desirable length of overtaking zones?

Assume any other data as per IRC-37.

[12 marks]



$S = 0.2V_B + l$ [Assume $l = 6m$] → IRC-37

$= 0.2 \times 50 + 6$

$= 16m.$

$2S + \cancel{V_BT} = \cancel{V_BT} + \frac{1}{2}aT^2$

$T = \sqrt{\frac{4S}{a}} = \boxed{8.04 \text{ sec}}$

$OSD = 0.278 \times (50) \times 2\text{sec} + 2 \times (16)$ [$t_R = 2\text{sec}$ for OSD]

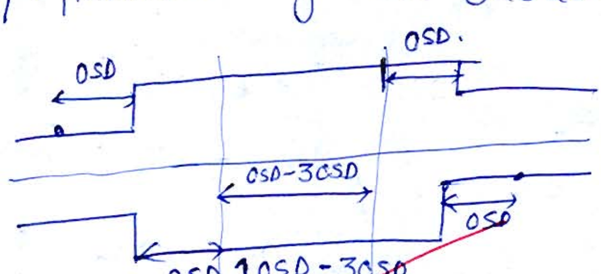
$+ [50 \times 0.278 \times 8.04]$

$+ [0.278 \times 80 \times 8.04]$

$= \boxed{350.3656 m.}$

Desirable/Minimum Length $=$ 3OSD - 5OSD

$= 3 \times 350.365 - 5 \times 350.365$

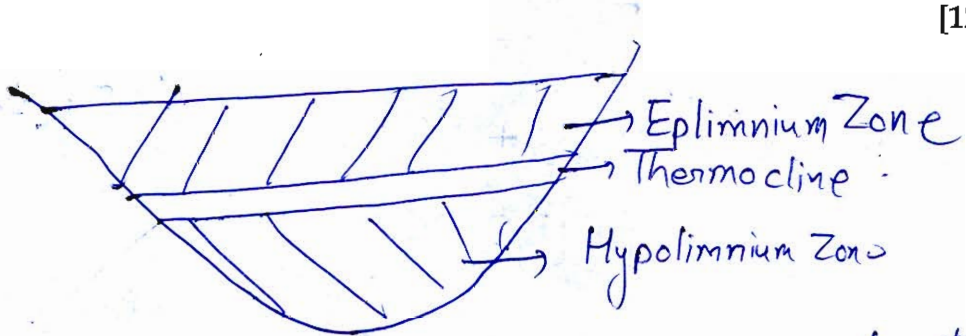


12

Min $= 1051.0968 m$, Max $= 1751.828 m.$

Q.5 (e) Describe stratification of lakes and biological zones in lakes.

[12 marks]

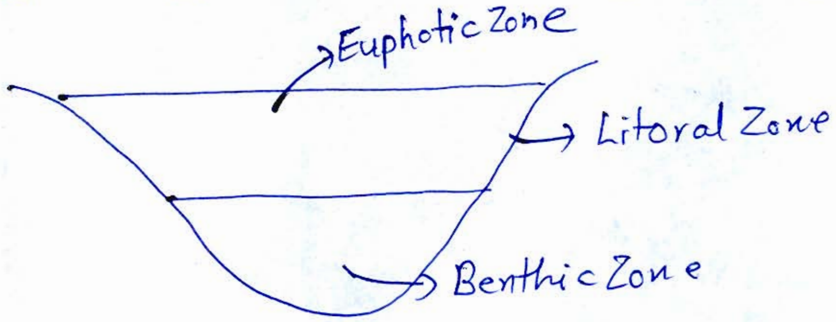


The stratification of lake takes place due to temprature change of water and further Density.

Eplimnium Zone :- It is the top zone in which sunlight can easily penetrate. The water of this Zone is well mixed and this Zone has good growth of aerobic M/O

Thermocline :- It is a zone of steep temprature change. The temprature change more rapidly in this zone.

Hypolimnium Zone :- It is zone where sunlight can't reach properly. There is no sufficient oxygen also in this zone. The temprature of this zone is less in summer.



Euphotic Zone :- This zone has sufficient supply of oxygen and sunlight. The sunlight can easily penetrate through this zone. The algae types of plant grow in mid of lake in this zone.

12

Littoral zone :- The side/Bank part of Euphotic zone is called Littoral zone.

Benthic Zone :- The bottom of Lake where there is deficiency of sunlight & oxygen is called Benthic Zone.

→ Dead cell-mass are present in this zone.

→ Anaerobic decomposition takes place in this zone

Q.6 (a) A national highway passing through a plain terrain has a horizontal curve of radius equal to the ruling minimum radius.

(i) Design all the geometric features of this horizontal curve, assuming suitable data.

(ii) What is the safest intermediate sight distance provided for the given national highway?

[16 + 4 = 20 marks]

$V_{Ruling} = 100 kmph \rightarrow [IRC]$.

$R = \frac{V_R^2}{127[e_{max}+f]} = \frac{(100)^2}{127[0.07+0.15]} = \boxed{357.9098 m}$

$e = \frac{V^2}{225 \times R} = \frac{(100)^2}{225 \times 357.9098} = 0.124 m > 0.07 m$

$\boxed{e_{provided} = 0.07}$

$e + f = \frac{V^2}{127R} \Rightarrow 0.07 + f = \frac{(100)^2}{127 \times 357.9098} =$

$\boxed{f = 0.15}$ OK No speed restriction.

$\boxed{\text{Extra widening}} = \frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}}$

n = no. of lanes = assume $\boxed{2 \text{ lane}}$, $\boxed{l = 6m}$ IRC-37

$$\frac{(6)^2}{\cancel{2}\times 357.909} + \frac{100}{9.5\sqrt{357.909}} = \boxed{0.6569\ m}$$

(*) Length of Transition curve :-

(i) $L = \frac{V^3}{CR}$

$C = \frac{80}{75+V} = \frac{80}{175} = 0.45$

But $C \equiv (0.5 - 0.8)$

$C = 0.5$ assume.

$$= \frac{(100)^3}{0.5 \times 357.909}$$

$$= 119.77 m$$

(ii) (1) in (N) Assume $N = \underline{150}$.

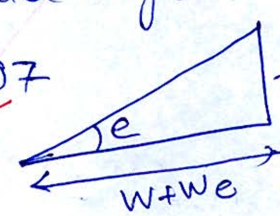
$L = Nn$.

Assume rotation about outer edge

$= Nn$

$= 150 \times (7 + 0.6569) \times 0.07$

$= 80.397 m$



$e = \frac{x}{w}$

(iii) $L = \frac{2.7V^2}{R} = \frac{2.7 \times 100^2}{357.909} = 75.4381 m$

Provide $\boxed{L_T = 119.77 m.}$

(*) $\text{Shift} = \frac{L_T^2}{24R} = \frac{119.77^2}{24 \times 357.909} = \boxed{1.669 m}$

(b) SSD:-

$SSD = 0.278 V t_R + \frac{V^2}{254[f \pm S]}$ $[S=0]$

$f = 0.35$ (IRC)

$t_R = 2.5 sec$

$= 0.278 \times 100 \times 2.5 + \frac{100^2}{254 \times 0.35}$

$= \boxed{181.985 m}$

$ISD = 2(SSD)$

$= \boxed{363.971\text{ m}}$

20

Q.6 (b) (i) The following latitudes and departures were obtained for a closed traverse *ABCDEFA* survey:

Line	Latitude (m)	Departure (m)
AB	0.00	183.79
BC	128.72	98.05
CD	177.76	–140.85
DE	–76.66	–154.44
EF	–177.09	0.00
FA	–52.43	13.08

Adjust the traverse by Bowditch's method and compute corrected latitudes and departures of all the traverse lines. Also calculate the bearing of *CD*.

(ii) What do you understand by the following forms of curves and where are they generally used?

1. Compound curve 2. Reverse curve

[14 + 6 = 20 marks]

(i)

Line	l_i	Latitude	Depar.	C_L	C_D	L	D
AB ①	183.79	0	183.79	-0.056	0.069	-0.056	183.859
BC ②	161.81	128.72	98.05	-0.049	0.0613	128.671	98.1113
CD ③	226.79	177.76	-140.85	-0.069	0.0859	+177.691	-140.764
DE ④	172.42	-76.66	-154.44	-0.0529	0.0653	-76.712	-154.374
EF ⑤	177.09	-177.09	0.00	-0.054	0.0671	-177.144	0.0671
FA ⑥	54.03	-52.43	13.08	-0.016	0.020	-52.446	13.1
						Σ 0.00	Σ 0

Σ latitude = +0.3

Correction = (−0.3)

$C_{Latitud} = \frac{l_i}{\Sigma l_i} \times (-0.3)$

Σ departure = −0.37

C = +0.37

$C_{Dep.} = +0.37 \times \frac{l_i}{\Sigma l_i}$

$l \sin\theta = 183.79$

$l \cos\theta = 0$

for BC:

$L\cos\theta = 128.72$

$L\sin\theta = 98.05$

$L = 161.810$

Bearing of CD?

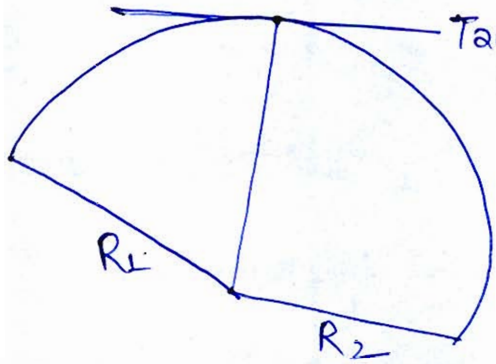
$\Sigma l_i = \boxed{975.936 m}$

$C_L = -3.07 \times 10^{-4} \times l_i$, $C_D = 3.79 \times 10^{-4} \times l_i$

12

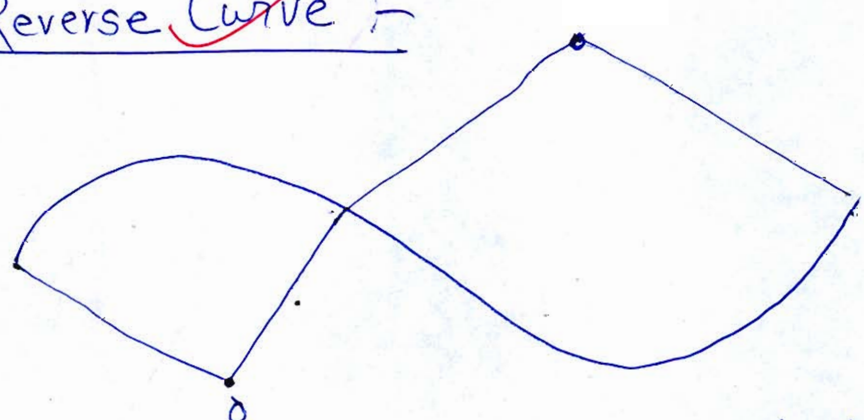
(ii)

(i) Compound curve:- This is a type of curve in which there are two different curves of different radius forming a smooth curve.



These curves are mainly used when it is not possible to draw curve with same radius throughout or where merging traffic from two different curves are coming.

(ii) Reverse Curve :-



When a curve in the form of above mentioned curve is set it is known as Reverse curve. In this curve there are 2 different centres for each side of turn. But both centres lie on same line.

Reverse curve is provided mainly in forest areas where appropriate set back distance is to be provided.

6

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Q.6 (c) (i) A conventional activated sludge process of municipal waste having discharge of $1000\ m^3/day$ disposes its digested sludge on relatively impervious farm land. Raw sludge suspended solids concentration is 225 mg/lt [70% volatile], BOD = 190 mg/lt (Excess activated sludge returned to primary). Primary settling suspended solids removal is 50% and BOD removal is 30%. Excess activated sludge is 0.4 gm volatile solids produced per gm of BOD applied.

Compute: (1) Total volatile solids to be anaerobically digested in kg/day.

(2) If anaerobic digestor produces 50% volatile solids and digested sludge solids concentration is 6%, then compute area required in hectares for disposal of sludge on the farm land. Specific gravity of sludge is 1 and rate of application on farm land is $2\ m^3/ha/day$.

(ii) A control sample BOD water containing seeded diluted water has a drop of 1.5 mg/lt in its dissolved oxygen over 5 days of incubation. If BOD sample is of 300 ml with 20 ml waste water in it and remaining seeded water in contribution has a drop of 6.8 mg/lt in its DO, then calculate the BOD of the test sample.

[14 + 6 = 20 marks]

$Q = 1000\ m^3/day$.

Raw slude → SS → 225 mg/Lt (70% Volatie)

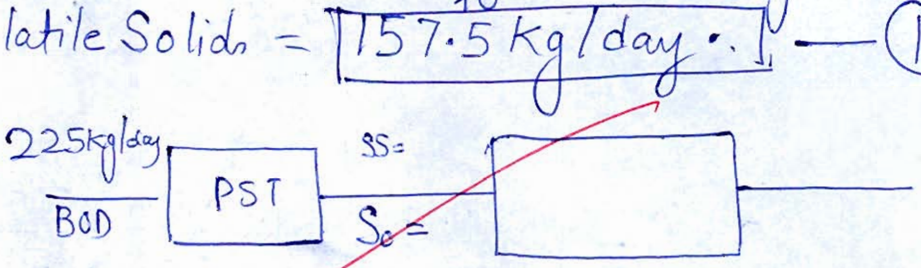
$S_0 = 190\ mg/Lt$

$\eta_{PST(SS)} = 50\%$, $\eta[BOD] = 30\%$.

Conversion faction = (0.4) gm/gm of volatile solid.

$$\text{Total SS} = 225\ mg \frac{\times \cancel{1000} \times 10^3}{10^6}\ kg/day = 225\ kg/day$$

$$\text{Volatile Solids} = \boxed{157.5\ kg/day.} \text{ — (I)}$$



$$[BOD]_{PST} = \frac{1000 \times 10^3 \times 190}{10^6} \frac{mg}{Lt} = 190\ kg/day$$

$$[S_0]_{ASP} = 190 \times 0.7 = 133\ kg/day$$

$$\text{Volatile Biomass} = 133 \times 0.4 = \boxed{53.2\ kg/day} \text{ — (II)}$$

Total Volatile solid going into Anaerobi (I) + (II)

$157.5 + 53.2$

$= \boxed{210.7\,Kg/day}$

After Digestor :-

Solids Coming out $= 0.5 \times 210.7$

$= 105.35\,Kg/day$

6kg solid $\longrightarrow$ 100kg.

1kg " $\rightarrow \frac{100}{6}$ kg Sludge

$105.35 \rightarrow \frac{100}{6} \times 105.35$

$= 1755.833\,kg/day$

Volume of sludge $= 1.7558\,m^3/day$.

14

Area Buried $= \frac{1.7558}{2}$

$= \boxed{0.8779 \text{ hactare}}$

(ii)

$[BOD] = \left[(6.8) - [1.5]\times\frac{280}{300}\right]\left(\frac{300}{20}\right)$

$[DO_i - DO_f]$ sample

(Seeded Sample)

Dilution factor.

$\boxed{[BOD] = 81\,mg/L}$

6

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Q.7 (a) Two sets of tacheometric readings were taken from an instrument station A (RL of A = 100 m) to a staff station B as shown below.

Instruments	P	Q
Multiplying constant	100	95
Additive constant	0.30	0.45
Height of instrument	1.40 m	1.45 m
Staff held	Vertical	Normal

Instruments	Instruments station	Staff station	Vertical angle	Stadia readings
P	A	B	5°44′	1.090, 1.440, 1.795
Q	A	B	5°44′	?

Determine:

(i) The distance between instrument station and staff station.

(ii) The R.L. of staff station B.

(iii) Stadia readings with instrument Q.

[20 marks]

Q.7 (b) (i) Explain the factors influencing the geometric design of hills roads.

(ii) Why should the psychological widening be added to the mechanical widening of roads?

[12 + 8 = 20 marks]

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Q.7 (c) (i) Design a group of friction piles which is required to carry a load of 3500 kN including the weight of the pile cap at a site where the soil is uniform clay to a depth of 20 m, underlain by rock. Average unconfined compressive strength of the clay is 65 kN/m^2. The clay may be assumed to be of normal sensitivity and normally loaded with liquid limit of 55%. A factor of safety of 3 is required against shear failure.

(ii) A square mass concrete footing supporting a load of 3500 kN extends from ground level to 4 m deep into a clay stratum. What will be the size of the footing allowing for a factor of safety of 3.0? Unit weight of concrete is 24 kN/m^3. Shear strength of the soil is 0.15 N/mm^2. Adhesion of clay with footing is 30 kN/m^2. The adhesion may be supported to act over a depth of 2 m from the bottom of the foundation. For $\phi = 0°$, $N_C = 5.7$, $N_q = 1$ and $N_\gamma = 0$. Take unit weight of soil as 21 kN/m^3.

[10 + 10 = 20 marks]

Q.8 (a) **(i)** What is gradient in vertical alignment? Also, define the types of gradient and their values for different types of terrain as per Indian practices.

(ii) There is a horizontal curve of radius 500 m and length 250 m on a highway. Determine the setback distance required from the centre line of the curve so as to provide a stopping sight distance of 100 m. (The distance between the centre lines of the road and the inner lane is 1.9 m.)

[14 + 6 = 20 marks]

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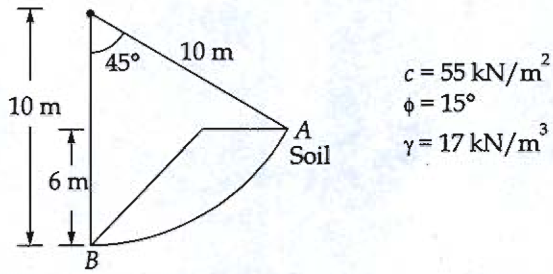
Do not write in this margin

.8 (b) (i) Find the factor of safety (1) with respect to shear strength and, (2) with respect to height along the indicated sliding surface AB in the figure below.



Normal effective pressure on sliding surface AB is 225 kN/m^2.

Downward tangential disturbing force along AB is 850 kN/m.

(ii) Explain sludge treatment process.

[12 + 8 = 20 marks]

Q.8 (c) **(i)** Find the maximum permissible error in laying off the direction of an offset so that maximum displacement may not exceed 0.025 cm on paper given that length of the offset is 25 m, the scale is 1 cm to 75 m and the maximum error in the length of the offset is 0.50 m.

(ii) Write short notes on the following methods of plane table surveying:

1. Radiation
2. Traversing
3. Intersection
4. Resection

(iii) Explain the two basic principles of surveying.

[4 + 12 + 4 = 20 marks]

Space for Rough Work

Space for Rough Work