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Saket Centre

ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-1 : Network Theory + Control Systems [All Topics]

Name :

Roll No

Test Centres				Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	Pune ☐	
Kolkata ☐	Hyderabad ☐			

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	13
Q.2	46
Q.3	—
Q.4	—
Section-B	
Q.5	30
Q.6	—
Q.7	41
Q.8	37
Total Marks Obtained	167

Signature of Evaluator: Ch. R... | Cross Checked by: ....

• Good presentation, keep it up.

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

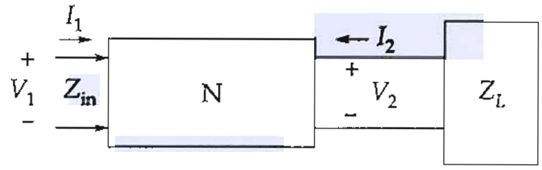
1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Network Theory

Q.1 (a) The Z parameter of a two port device N are $Z_{11} = Ks$, $Z_{12} = Z_{21} = 10\,Ks$ and $Z_{22} = 100\,Ks$. A 1 Ω resistor is connected as load across the output port.

(i) Find the input impedance $Z_{in} = \frac{V_1}{I_1}$ and construct its equivalent circuit.

(ii) Give the values of the element for $K = 1$ and $K = 10^6$.



[12 marks]

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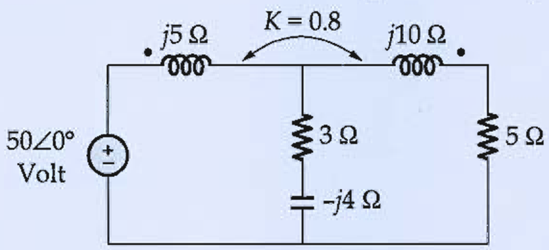
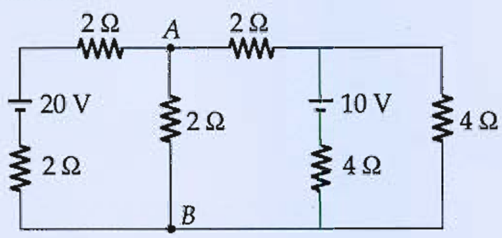
Q.1 (b) **(i)** Find voltage across the 5 Ω resistor using Mesh analysis.

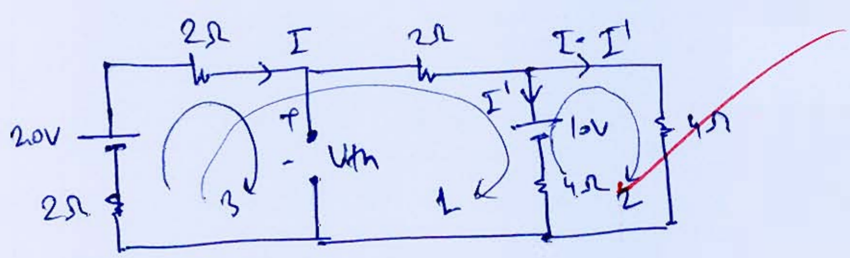


(ii) Determine the current through the branch AB of the network shown below using Thevenin's equivalent.



[6 + 6 marks]

(ii) Calculation for V_{th} : Remove load resistor 2Ω across A-B terminal.



By KVL @ loop 1

$20 = 6I + 10 + 4I' \Rightarrow 6I + 4I' = 10$ —— (1)

KVL @ loop 2

$4(I - I') - 4I' - 10 = 0 \Rightarrow 4I - 8I' = 10$

$\Rightarrow 2I - 4I' = 5$ —— (2)

Adding ① & ② $8I = 15$

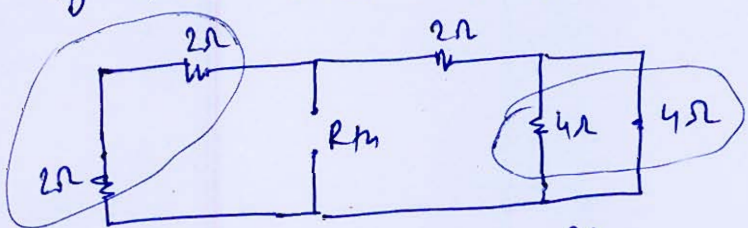
$I = 1.875\ A$

By KVL at loop 3

$20 = 4I + V_{th} \Rightarrow 20 - 7.5 = V_{th}$

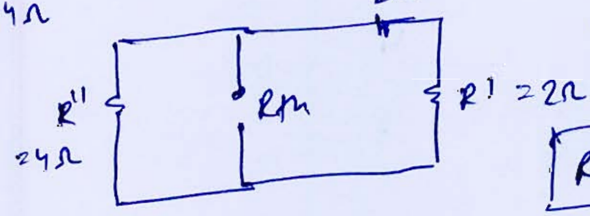
$\Rightarrow V_{th} = 12.5$ volt

Calculation for R_{th} : Replace all independent sources by their internal resistance & remove load resistor



$R' = 4||4 = 2\Omega$

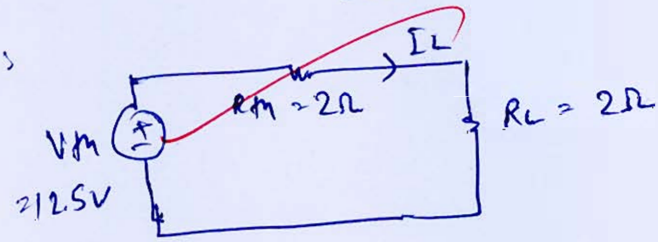
$R'' = 2+2 = 4\Omega$



$\Rightarrow R_{th} = 4||4$

$R_{th} = 2\Omega$

Thevenin circuit :



By KVL

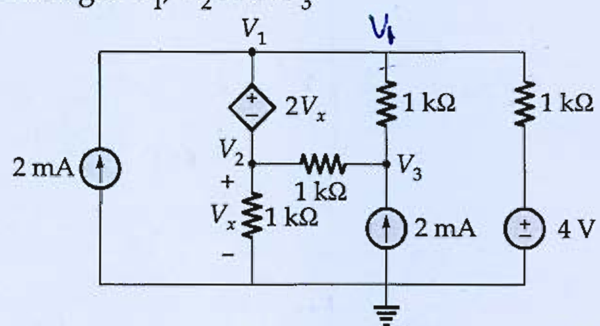
$I_L = \frac{12.5}{2+2}$

$I_L = 3.125\ A$ {current through load resistor}

6

Do not write in this margin

Q.1 (c) Determine node voltages V_1, V_2 and V_3.



[12 marks]

Applying KCL @ node (V_1, V_2) (supernode)

$$\frac{V_1 - V_3}{1} + \frac{V_1 - 4}{1} + \frac{V_2}{1} + \frac{V_2 - V_3}{2} = 2$$

$$V_1[1+1] + V_2\left[1+\frac{1}{2}\right] - V_3\left[1+\frac{1}{2}\right] = 6$$

$$2V_1 + 1.5V_2 - 1.5V_3 = 6 \quad \text{——— (1)}$$

KCL @ node V_3

$$\frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{1} = 2$$

$$-V_1 - V_2 + 2V_3 = 2 \quad \text{——— (2)}$$

Also, $V_1 - V_2 = 2V_x$ (By KVL)

& $V_x = V_2$ [By Ohm's law)

$\Rightarrow V_1 - V_2 = 2V_2$

$\Rightarrow V_1 - 3V_2 + 0V_3 = 0$ —— (3)

By using crammer's rule in equation (1), (2), (3)

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} 6 & 1.5 & -1.5 \\ 2 & -1 & 2 \\ 0 & -3 & 0 \end{bmatrix}}{\begin{bmatrix} 2 & 1.5 & -1.5 \\ -1 & -1 & 2 \\ 1 & -3 & 0 \end{bmatrix}}$$

$V_1 = 5$ volt

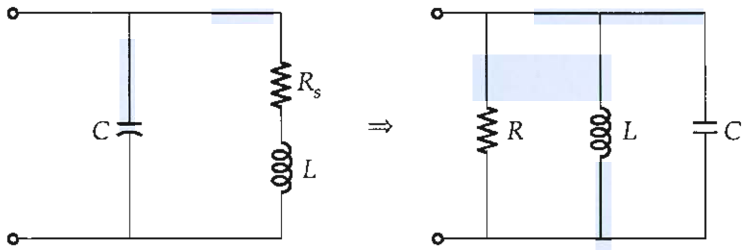
$$V_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} 2 & 6 & -1.5 \\ -1 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix}}{\begin{bmatrix} 2 & 1.5 & -1.5 \\ -1 & -1 & 2 \\ 1 & -3 & 0 \end{bmatrix}}$$

$V_2 = \frac{5}{3} = 1.67$ volt

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{bmatrix} 2 & 1.5 & 6 \\ -1 & -1 & 2 \\ 1 & -3 & 0 \end{bmatrix}}{\begin{bmatrix} 2 & 1.5 & -1.5 \\ -1 & -1 & 2 \\ 1 & -3 & 0 \end{bmatrix}}$$

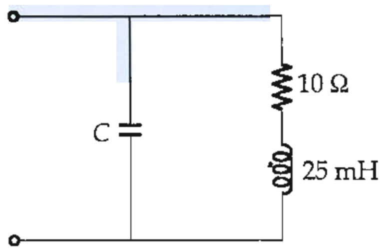
$V_3 = \frac{13}{3}$ (or) 4.33 volt

Q.1 (d) Show that a high Q-coil resonant circuit can be approximated as shown in figure.



High Q-coil resonant circuit   Approximated equivalent circuit

For a practical tank circuit shown in figure below, the resonance occurs at 1 MHz. Assume a high Q-coil, find out the quality factor of high Q-coil at resonant frequency and the value of capacitance C.



[12 marks]

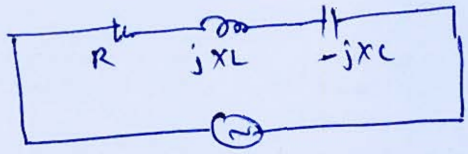
Q.1 (e) A series R-L-C circuit having $R = 25\ \Omega$, $L = 2$ H and $C = 30\ \mu F$ is connected across an a.c. variable frequency source. At what frequencies will the phase angle of circuit be

(i) 45° lagging and

(ii) 45° leading, the applied voltage.

[6 + 6 marks]

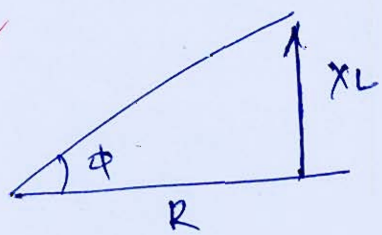
Consider a series RLC circuit



(i) Given, $\phi = 45°$ ~~lagging~~ (it means circuit is inductive)

current lags voltage by angle ϕ

By phasor diagram



~~$\tan\phi = \frac{X_L}{R}$~~

$\tan 45° = \frac{\omega L}{R}$

$1 = \frac{\omega \times 2}{25}$

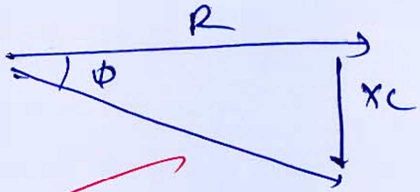
$\omega = 12.5$ rad/sec

$f = 1.989$ Hz

(ii) Given 45° leading

hence the circuit behaves as capacitive circuit. current leads applied voltage by angle ϕ.

By phasor diagram



$$\tan\phi = \frac{X_C}{R}$$

$$\tan 45 = \frac{1}{\omega C R}$$

$$\omega = \frac{1}{RC} = \frac{1}{25 \times 30 \times 10^{-6}}$$

$$\omega = 1333.33 \text{ rad/sec}$$ (or)

$$f = 212.206 \text{ Hz}$$

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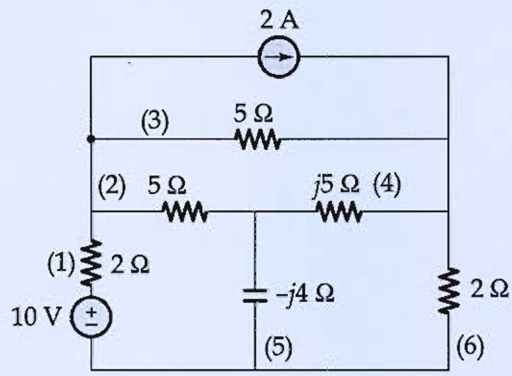
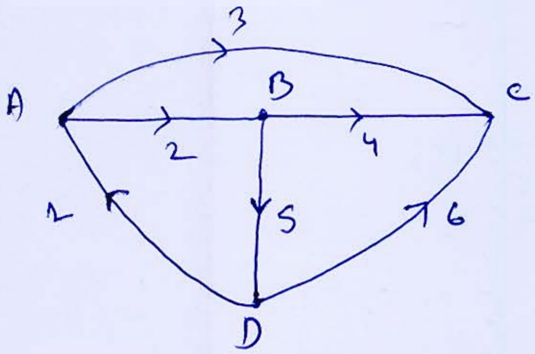
Q.2 (a) For the network shown below, draw its graph and obtain tie set matrix $[B_f]$, taking branches 2, 4, 5 as tree branches. Also, determine the loop impedance matrix and find the loop equations.

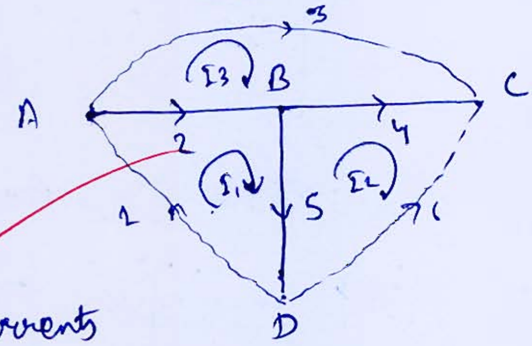


[20 marks]

Graph : voltage source – short circuited
current source – open circuited
& replace elements by their respective branches.



Tree



Let I_1, I_2 & I_3 be loop currents

Tie set matrix $[B_f]$

$$[B_f] = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

$$[Z_B] = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \qquad [B_f]^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

(loop impedence matrix)

We know, $\boxed{Z_L = [B_f][Z_B][B_f]^T}$

$$[Z_L] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & -1 & 0 & 0 \end{bmatrix}_{3\times6} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}_{6\times6} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_{6\times3}$$

$$= \begin{bmatrix} 2 & 5 & 0 & 0 & -j4 & 0 \\ 0 & 0 & 0 & j5 & j4 & -2 \\ 0 & -5 & 5 & -j5 & 0 & 0 \end{bmatrix}_{3\times6} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{6\times3}$$

$$[Z_L] = \begin{bmatrix} (7-j4) & -j4 & -5 \\ j4 & (-2+j) & -j5-2 \\ -5 & -j5 & 10+j5 \end{bmatrix}_{3\times 3}$$

By equilibrium equation :

$$[Z_L][I_L] = [B][V_s] - [B][Z_B][I_s] \quad —— (1)$$

~~$I_s = [0]$~~ $[I_s] = [0]$; $[V_s] = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Substituting all values in equation (1) we get

$$\begin{bmatrix} (7-j4) & -j4 & -5 \\ j4 & (-2+j) & -(2+5j) \\ -5 & -j5 & 5+10j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & -1 & 0 & 0 \end{bmatrix}_{3\times 6}\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}$$

$$\begin{bmatrix} (7-j4)I_1 & -j4 I_2 & -5I_3 \\ (j4)I_1 & (-2+j)I_2 & -(2+5j)I_3 \\ -5I_1 & (-j5)I_2 & +(5+10j)I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

Hence we get :

$$\left.\begin{aligned} (7-j4)I_1 - (j4)I_2 - 5I_3 &= 10 \\ (j4)I_1 + (-2+j)I_2 - (2+5j)I_3 &= 0 \\ -5I_1 + (-j5)I_2 + (5+10j)I_3 &= 0 \end{aligned}\right\} \text{loop equations}$$

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Q.2 (b) (i) A heater takes 10 A at 50 V. Calculate the impedance of a choke of 5 Ω resistance to be placed in series with it in order that it may work at 200 V, 50 Hz supply. Find the power factor of the circuit.

(ii) State the maximum power transfer theorem. For the given circuit, what resistance should be connected across *x*-*y*, such that maximum power is developed across this load resistance? What is the amount of this maximum power?

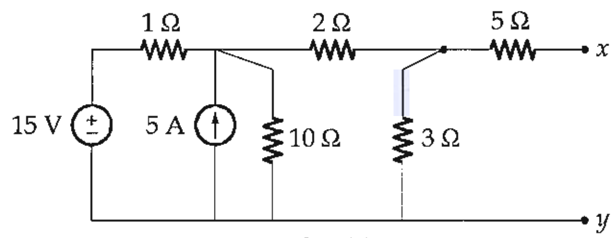

fig. (a)

[10 + 10 marks]

(ii) Maximum power transfer theorem : According to this theorem maximum power is transferred from source to load when Thevenin resistance of the circuit is equal to load resistance i.e $R_{th} = R_L$

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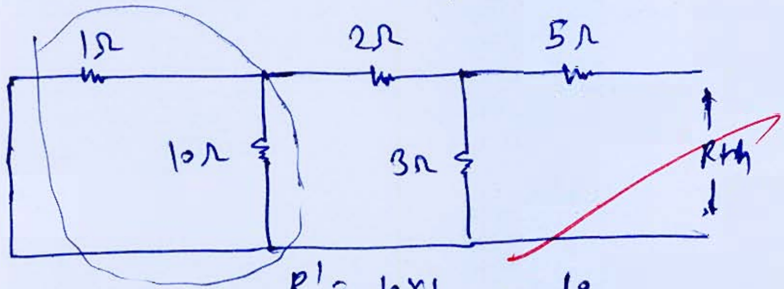
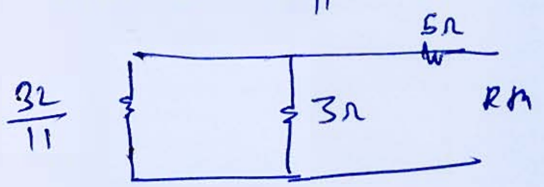
Calculation for R_{th}: Replace all independent sources by their internal impedance.



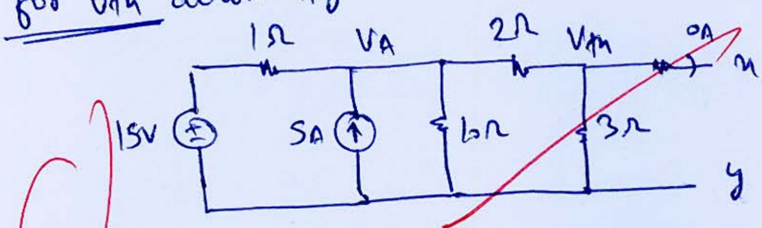
$R' = \frac{10 \times 1}{10+1} = \frac{10}{11}$



$\Rightarrow R_{th} = \frac{32}{11} \parallel 3\Omega + 5\Omega$

$\Rightarrow \boxed{R_{th} = 6.4769\ \Omega}$

for V_{th} across x, y



By KCL at V_A: $\frac{V_A - 15}{1} + \frac{V_A}{10} + \frac{V_A - V_{th}}{2} = 5$

$V_A \left[1 + \frac{1}{10} + \frac{1}{2}\right] - \frac{V_{th}}{2} = 20$

$1.6 V_A - 0.5 V_{th} = 20$ — (1)

By KCL at V_{th}: $\frac{V_{th} - V_A}{2} + \frac{V_{th}}{3} = 0$

$0.8333 V_{th} - 0.5 V_A = 0$ — (2)

solving (1) & (2) by cramer's rule we get $\boxed{V_{th} = 9.231 \text{ volt}}$

hence maximum power transferred

$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(9.231)^2}{4(6.4769)} = \boxed{3.289 \text{ Watt}}$

Do not write in this margin

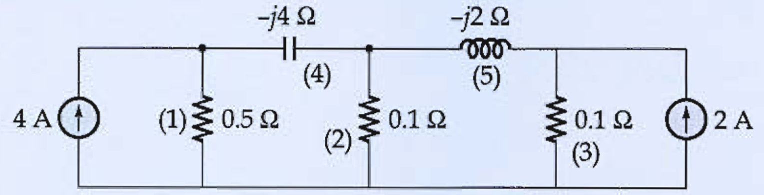
Q.2 (c) **(i)** A load Z draws 12 kVA at a power factor of 0.856 lagging from a 120 V rms sinusoidal source.

Calculate:

1. the average and reactive power delivered to the load.
2. the peak current and,
3. the load impedance.

(ii) For the circuit diagram shown below, draw its graph and

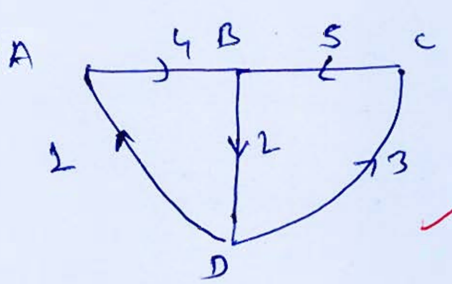
1. Obtain incidence matrix and cut-set matrix.
2. How many trees are possible for this circuit?



[10 + 10 marks]

(ii) Graph — current source → open circuit

1.) replace elements by respective branches

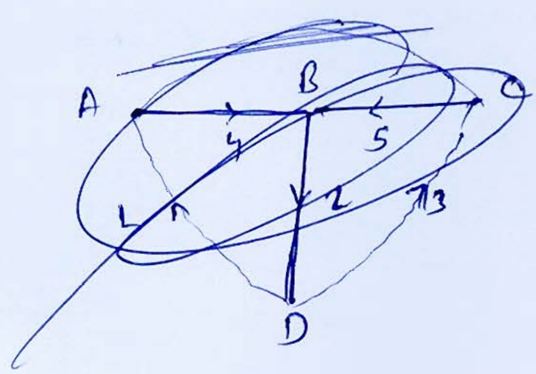


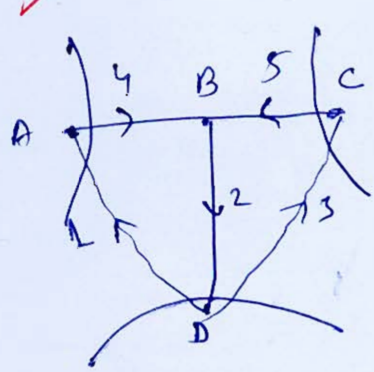
Incidence matrix

	1	2	3	4	5
A	−1	0	0	1	0
B	0	1	0	−1	−1
C	0	0	−1	0	1
D	1	−1	1	0	0

— (1)

Now Tree





Cutset matrix

$$[C] = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 2 & -1 & 1 & -1 & 0 & 0 \\ 4 & -1 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & -1 & 0 & 1 \end{array}$$

2) No. of possible trees = $\left|[A_r][A_r]^T\right|$, reduced incident matrix

{from ①} (removing the last row)

$$[A_r][A_r]^T = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}_{3\times5} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_{5\times3}$$

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$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} = [A_r][A_r]^T$$

$$\left|(A_r)(A_r)^T\right| = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 8$$

Hence possible no. of trees = 8

(i) Given Power factor, $\cos\phi = 0.856$ lagging

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$$\phi = \cos^{-1}(0.856) \Rightarrow \boxed{\phi = 31.1296}$$

Power = 12×10^3 VA

Voltage = 12 volt

$$\text{Current } (I_{rms}) = \frac{12\times10^3}{12}$$

$$\boxed{I_{rms} = 1000\text{ A}}$$

1) Average power = $V_{rms} I_{rms} \cos\phi$

$= 120 \times 1000 \times 0.856$

$= 102.720$ KW

Reactive power : $V_{rms} I_{rms} \sin\phi$

$= 120 \times 1000 \times \sin(31.1296)$

$= 62.037$ KVAR

2) Peak current : $I_{peak} = I_{rms} \times \sqrt{2}$

$I_{peak} = 1000 \times 1.414$

$I_{peak} = 1414.21$ A

3) Load impedence : Since power factor is lagging in nature, so load impedence will be inductive in nature

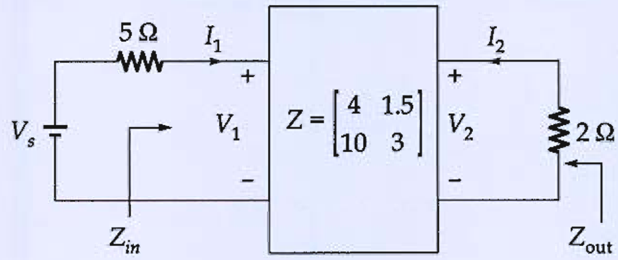
$Z_L = \frac{V}{I} =$

$Z_L = \frac{120 \angle 31.1296}{1000} = 0.12 \angle 31.1296$

$Z_L = 0.1027 + j0.06203$

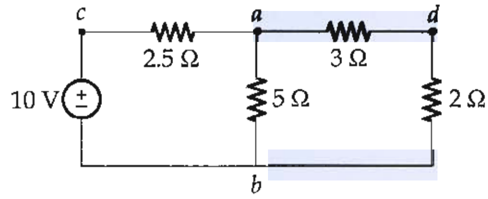
inductive

Q.3 (a) For the network shown below, find internal current gain G_I, voltage gain G_V, power gain G_P, input impedance Z_{in} and output impedance Z_{out}.

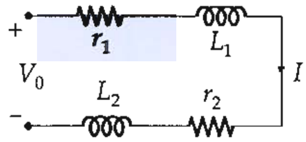


[20 marks]

Q.3 (b) **(i)** Consider the circuit shown below. If the resistance of 5 Ω branch increases to 6 Ω, determine the compensation source and verify the results.



(ii) In the circuit given below, $r_1 = 8.2\ \Omega$, $r_2 = 2.7\ \Omega$, $L_1 = 0.01$ H, $L_2 = 0.03$ H, $f = 25$ Hz and the circulating current $I = 10$ A.

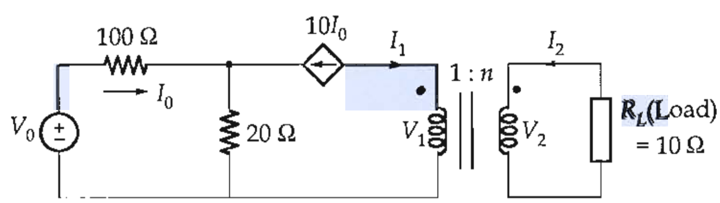


Find:

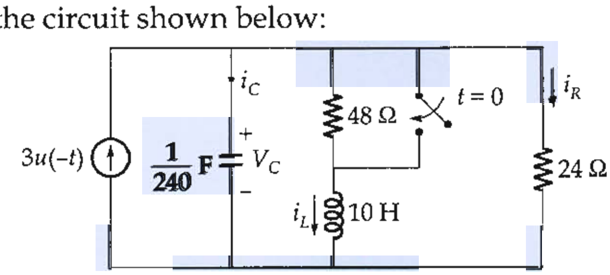
1. Voltage drop across each element.
2. Total resistive and inductive voltage drop.
3. Supply voltage.
4. Impedance angle of each of the R-L branch and power factor of the circuit.

[10 + 3 + 3 + 1 + 3 marks]

Q.3 (c) (i) What is the voltage and power gain of the circuit shown in figure? Assume $n = \frac{1}{10}$.



(ii) Consider the circuit shown below:



After being open for a long time, the switch is closed at $t = 0$. Find

1. $i_L(0^-)$ 2. $V_C(0^-)$ 3. $i_R(0^+)$

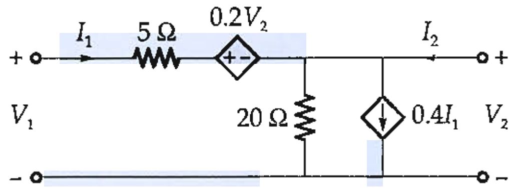
4. $i_C(0^+)$ 5. $V_C(0.2)$ using Laplace transform approach.

[10 + 10 marks]

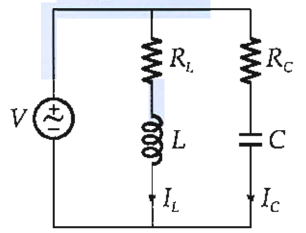
Q.4 (a) Synthesize Cauer-I form and Cauer-II form of the network with driving point immitance function $Y(s) = \dfrac{(s^2+1)(s^2+5)}{s(s^2+3)}$

[20 marks]

Q.4 (b) (i) Find the Y-parameters for the 2-port network shown below.

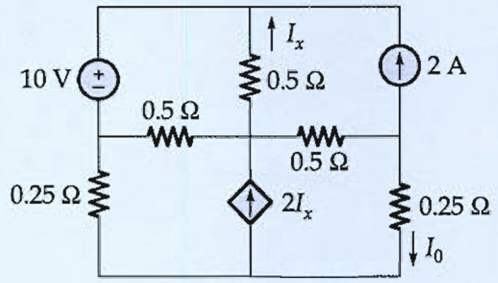


(ii) For the circuit shown, draw the phasor diagram. Derive the condition for the two branch currents, I_L and I_C to be in quadrature.

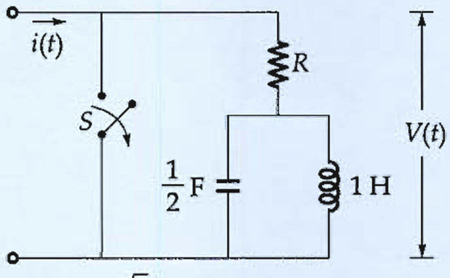


[10 + 10 marks]

Q.4 (c) **(i)** Determine I_0 in figure using nodal analysis:



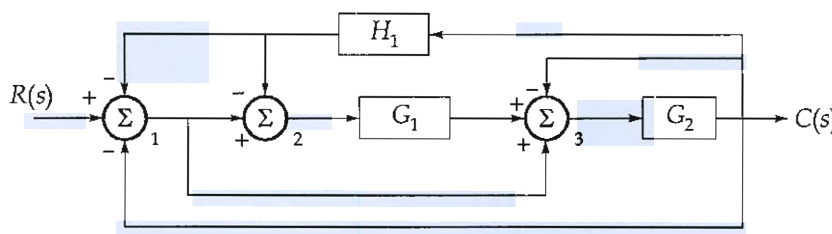
(ii) The circuit shown has zero initial energy. At $t = 0$, the switch 'S' is opened. Find the value of resistor R for the given excitation such that the response is $V(t) = 0.5\sin\sqrt{2}t\, u(t)$.



The excitation is $i(t) = te^{-\sqrt{2}t}\, u(t)$. **[10 + 10 marks]**

Section B : Control Systems

Q.5 (a) Using block diagram reduction technique, find the transfer function $\frac{C(s)}{R(s)}$ for the system shown below:



[12 marks]

Q.5 (b) The closed loop transfer function of a control system is given as $\frac{10}{s^3 + 0.1s^2 + 10}$.

Determine the steady state error of the system when input is $5 + 10t + 4t^2$.

[12 marks]

Characteristic equation = $s^3 + 0.1s^2 + 10$

$= 1 + \frac{10}{s^3 + 0.1s^2} \Rightarrow 1 + GH$

hence, open loop transfer function:

$GH(s) = \frac{10}{s^2(s + 0.1)}$

Input $r(t) = 5 + 10t + 4t^2$

By Laplace transform

$R(s) = \frac{5}{s} + \frac{10}{s^2} + \frac{4 \times 2}{s^3}$

$R(s) = \frac{5}{s} + \frac{10}{s^2} + \frac{8}{s^3} = \frac{5s^2 + 10s + 8}{s^3}$

steady state error. (ess)

$$e_{ss} = \lim_{s\to 0} \frac{s \times R(s)}{1 + GH(s)}$$

(Putting all values)

$$e_{ss} = \lim_{s\to 0} \frac{s \times \frac{(5s^2+10s+8)}{s^3}}{1 + \frac{10}{s^2(s+0.1)}}$$

$$\Rightarrow \lim_{s\to 0} \frac{(5s^2+10s+8)(s+0.1)}{s^2(s+0.1)+10}$$

(Putting limits)

$$e_{ss} = \frac{8 \times 0.1}{10}$$

~~ess = 0.08~~

$$\boxed{e_{ss} = 0.08}$$

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Q.5 (c) The open loop transfer function of a unity feedback system is given by $G(s) = \dfrac{K}{s(1+sT)}$, where T and K are constants having positive values. By what factor the amplifier gain be reduced so that

(i) the peak overshoot of unit step response of the system is reduced from 75% to 20%.

(ii) the damping ratio increases from 0.2 to 0.6.

[12 marks]

(i) Given $M_{p1} = 0.75$

$$M_{p1} = e^{\frac{-\pi \xi_1}{\sqrt{1-\xi_1^2}}} = 0.75$$

$$\Rightarrow \boxed{\xi_1 = 0.0912}$$

$M_{p2} = 0.20$, $M_{p2} = e^{\frac{-\pi \xi_2}{\sqrt{1-\xi_2^2}}}$

$$\Rightarrow \boxed{\xi_2 = 0.456}$$

Closed loop transfer function $= \dfrac{K}{Ts^2 + s + K}$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + \frac{1}{T}s + \frac{K}{T}}$$

On comparing with we get relation is

① — $\xi_1 = \sqrt{\dfrac{T}{K_1}}$, $\xi_2 = \sqrt{\dfrac{T}{K_2}}$ —②

divide ① & ②

$$\boxed{\frac{\xi_1}{\xi_2} = \sqrt{\frac{T}{K_1} \times \frac{K_2}{T}}} \quad \text{—③}$$

$$\frac{\xi_1}{\xi_2} = \sqrt{\frac{K_2}{K_1}} \Rightarrow K_2 = \left(\frac{\xi_1}{\xi_2}\right)^2 K_1$$

$$K_2 = \frac{1}{25} K_1$$

$$\boxed{K_2 = 0.04 K_1}$$

(it should be reduced by 25 times)

(ii) for $e_{ss1} = 0.2$

$e_{ss2} = 0.6$

{from equation ③}

$$\frac{e_{ss1}}{e_{ss2}} = \sqrt{\frac{K_2 \times T}{K_1 T_b}}$$

$$\frac{K_2}{K_1} = \left(\frac{e_{ss1}}{e_{ss2}}\right)^2$$

(Putting values of e_{ss1} & e_{ss2})

$$\frac{K_2}{K_1} = \left(\frac{0.2}{0.6}\right)^2$$

$$\frac{K_2}{K_1} = \frac{1}{9} \Rightarrow \boxed{K_2 = \frac{1}{9} K_1}$$

(or) $\boxed{K_2 = 0.111 K_1}$

Hence, gain should be reduced by 9 times.

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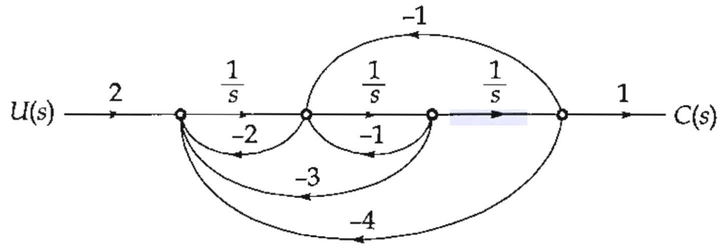
Q.5 (d) A control system is represented using the signal flow graph shown below:



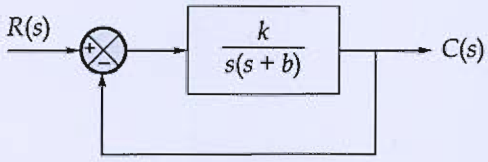
(i) Construct a state model for the above system.

(ii) Using the state model obtained in part (i), find the transfer function $\frac{C(s)}{U(s)}$.

[4 + 8 marks]

Q.5 (e) (i) Consider the feedback system shown in figure. Find the values of k and b to satisfy the following frequency-domain specifications: $M_r = 1.6$, $\omega_r = 15$ rad/s.

(ii) For the values of k and b determined in part (a), calculate the settling time and bandwidth of the system.



[12 marks]

(i) Given, $M_r = 1.6$ (resonant peak)

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.6$$

$$\Rightarrow \quad 4\xi^4 - 4\xi^2 + \left(\frac{1}{1.6}\right)^2 = 0$$

$$= \xi = 0.444, \; 0.3312$$

(resonant frequency) $\omega_r = \omega_n\sqrt{1-2\xi^2} = 15$

for $\xi = 0.3312$ $\quad \omega_n = \frac{15}{\sqrt{1-2(0.3312)^2}}$ = ~~15.897 rad/sec~~ = 16.974 rad/sec

for $\xi = 0.444$ $\quad \omega_n = \frac{15}{\sqrt{1-(0.444)^2}}$ (invalid)

Hence, $\xi = 0.331$, $\omega_n = 16.974$ rad/sec

Closed loop transfer function $\frac{C(s)}{R(s)} = 1 + \frac{K}{s(s+b)}$

Characteristic equation = $s^2 + bs + K = 0$

Comparing from standard 2nd order equation

$2\xi\omega_n = b$ $\qquad \omega_n^2 = K$

$2\times(0.331)(16.974) = b$ $\qquad$ $K = 288.1167$

$b = 11.2368$

(ii) Settling time $(t_s) = \frac{4}{\xi\omega_n}$ (2% tolerence)

$$= \frac{4}{(0.331)(16.974)}$$

$$\boxed{t_s = 0.71194}$$ sec

$$t_s = \frac{3}{\xi\omega_n}$$ (for 5% tolerence)

$$= \frac{3}{(0.331)(16.974)} \Rightarrow \boxed{t_s = 0.534 \text{ sec}}$$

Bandwidth $BW = \omega_n\sqrt{1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}}$

$$= 16.974\sqrt{1-2\times(0.331)^2+\sqrt{4(0.331)^4-4(0.331)^2+2}}$$

$$\boxed{\text{Bandwidth} = 35.147 \text{ Hz}}$$

Calculation error

Q.6 (a) The open loop transfer function of a control system with unity feedback is given by

$$G(s) = \frac{(2K+5)}{s(s-(2+K))}$$

Calculate K for which

(i) The system is stable.

(ii) Both the poles of characteristic equation lies in the left of $s + 1 = 0$ line.

(iii) One pole of the characteristic equation is present in left of $s + 1 = 0$ line.

(iv) Poles are at $-0.125 + 0.7i$ and at $-0.125 - 0.7i$. **[20 marks]**

Q.6 (b) Write a short note on the following compensators:

(i) Lag compensator

(ii) Lead compensator

(iii) Lead-lag compensator

[6 + 6 + 8 marks]

Q.6 (c) A linear time-invariant system is characterized by the homogeneous state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i) Compute the solution of the homogeneous equation assuming the initial state vector

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(ii) Consider now that the system has a forcing function and is represented by the following non homogeneous state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

where u is a unit-step function. Compute the solution of this equation assuming initial conditions of part (a).

[10 + 10 marks]

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Q.7 (a) The block diagram of a unity feedback system is shown in figure (i) and its step response is shown in figure (ii). With the help of the given figures, calculate:

(i) closed loop transfer function.

(ii) the minimum value of 'K' for which the step response of the system would exhibit an overshoot as shown in figure (ii).

(iii) If 'K' is taken twice of the minimum value, then calculate the time period 'T' indicated in figure (ii)

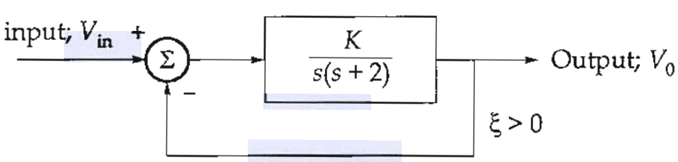

Fig. (i)

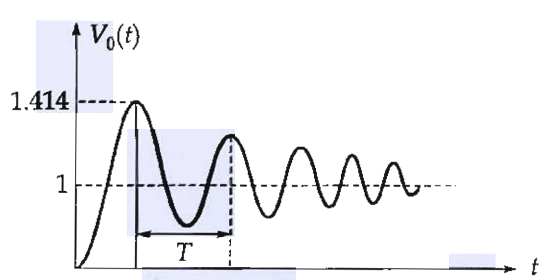

Fig. (ii)

[20 marks]

Peak overshoot, $M_p = \frac{1.414 - 1}{1} = 0.414$

$M_p = e^{\frac{-n\pi\xi}{\sqrt{1-\xi^2}}} = 0.414$

damping ratio $\Rightarrow$ $\boxed{\xi = 0.27}$

(i) Closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + 2s + K} \quad \text{— (1)}$$

(ii) On comparing with 2nd order standard equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

(2) — $\omega_n^2 = K$ $\qquad$ $2\zeta\omega_n = 2$ — (3)

$\omega_n = \sqrt{K}$ $\qquad$ $\sqrt{K} = \frac{2}{2 \times 0.27}$

$$K = 13.717$$

Hence

$$\frac{C(s)}{R(s)} = \frac{13.717}{s^2 + 2s + 13.717}$$

(iii) K is taken twice the minimum

$$K' = 2K = 27.434$$

from (2) & (3)

$$\omega_n = 5.238 \text{ rad/sec}$$

$$2\zeta\omega_n = 2$$

$$\zeta = \frac{1}{5.230} \Rightarrow \zeta = 0.2$$

~~Now, first overshoot~~

~~$$M_{p1} = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$~~

~~$$\Rightarrow M_{p1} = e^{-\frac{\pi 0.2}{\sqrt{1-(0.2)^2}}}$$~~

~~$M_{p1} = 0.527$~~ $\qquad$ ~~$M_{p1} = 0.527$~~

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Peak time 1, $t_{p1} = \frac{n\pi}{\omega_n\sqrt{1-\zeta^2}}$

$$t_{p1} = \frac{(1)\pi}{5.238\sqrt{1-(0.2)^2}} = 0.61214 \text{ sec}$$

Peak time(2) during 2 overshoot $(n=3)$

$$t_{p2} = \frac{3\pi}{\omega_n\sqrt{1-(0.2)^2}} = 1.8364 \text{ sec}$$

hence time ~~period~~ period 'T' = $t_{p2} - t_{p1}$

$$T = 1.8364 - 0.61214$$

$$\boxed{T = 1.22426 \text{ sec}}$$

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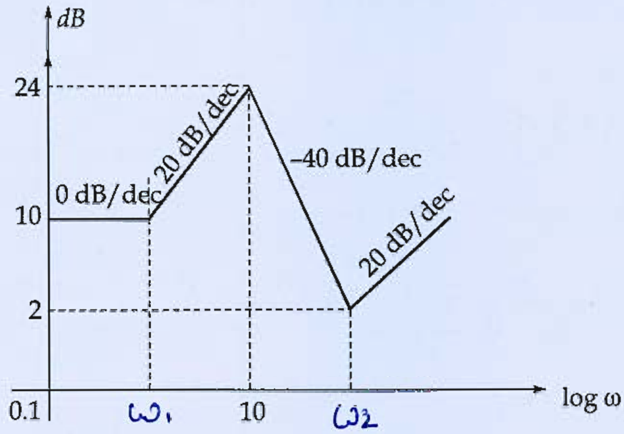
Q.7 (b) **(i)** Find the transfer function for the bode plot shown below:



(ii) Sketch the polar plot for the following transfer function:

$$G(s) = \frac{1+5s}{s^2(1+s)(1+2s)}$$

Also, calculate: Phase crossover frequency and corresponding gain margin.

[10 + 10 marks]

(i) ⇒ No pole (or) zero at origin, hence 0dB/dec

let corner frequency be ω_1 & ω_2.

$$\frac{24-10}{\log\left(\frac{10}{\omega_1}\right)} = 20$$

$$5.0119 = \frac{10}{\omega_1} \Rightarrow \boxed{\omega_1 = 1.995 \text{ rad/sec}}$$

$$\boxed{\omega_1 \simeq 2 \text{ rad/sec}}$$

for ω_2

Coordinates $(24, 10)$ $(\omega_2, 2)$

$$\frac{2-24}{\log\left(\frac{\omega_2}{10}\right)} = -40$$

$$3.548 = \frac{\omega_2}{10}$$

$$\boxed{\omega_2 = 35.48 \text{ rad/sec}}$$

⇒ zero at ω_1 so 20 dB/dec slope

$$GH(s) = K\left(\frac{s}{2}+1\right)$$

⇒ 3 pole at ω=10 rad/sec

$$G H(s) = \frac{K\left(\frac{s}{2}+1\right)}{\left(\frac{s}{10}+1\right)^3}$$

⇒ 3 zeros at ω = 35.48 rad/sec

$$G(s)H(s) = \frac{K\left(\frac{s}{2}+1\right)\left(\frac{s}{35.48}+1\right)^3}{\left(\frac{s}{10}+1\right)^3}$$

Calculation for K

$$20\log K = 10$$

$$\boxed{K = 3.162}$$

~~G(s)H(s) = 3.162 [(s+2)(s+3~~

10

$$G(s)H(s) = \frac{3.162\left(\frac{s}{2}+1\right)\left(\frac{s}{35.48}+1\right)^3}{\left(\frac{s}{10}+1\right)^3}$$

$$= \frac{3.162\,(s+2)(s+35.48)^3 \times 10^3}{(s+10)^3\,(35.48)^3 \times 2}$$

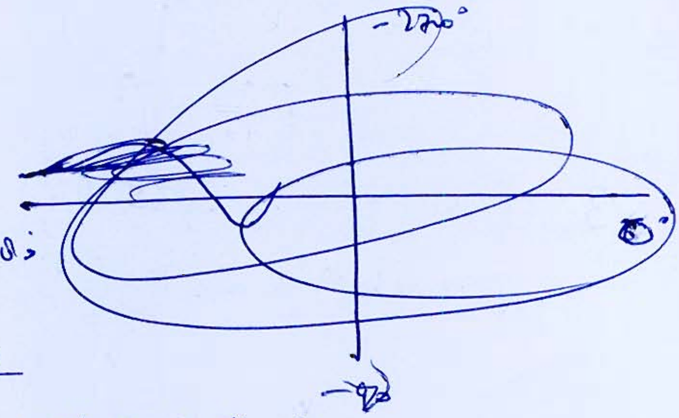
$$\boxed{G(s)H(s) = \frac{0.0354\,(s+2)\,(s+35.48)^3}{(s+10)^3}}$$

(ii) $$G(j\omega)H(j\omega) = \frac{1+5j\omega}{(j\omega)^2(1+j\omega)(1+2j\omega)}$$

$$|M| = \text{Magnitude} = \frac{\sqrt{1+25\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \quad \text{—(1)}$$

$\phi = \tan^{-1}(5\omega) - 180 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$

	$\omega = 0$	$\omega = \infty$
M	∞	0
ϕ	-180	-270



Intersection with -real axis

$-180 = \tan^{-1}[5\omega] - 180 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$

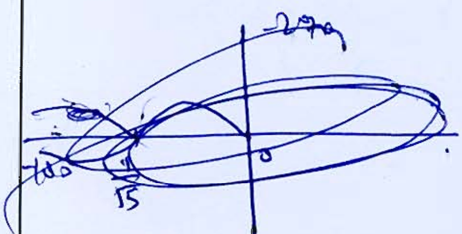
$0 = \tan^{-1}[5\omega] - \tan^{-1}\left[\frac{3\omega}{1-2\omega^2}\right]$

$5\omega = \frac{3\omega}{1-2\omega^2} \Rightarrow 5\omega - 10\omega^3 = 3\omega$

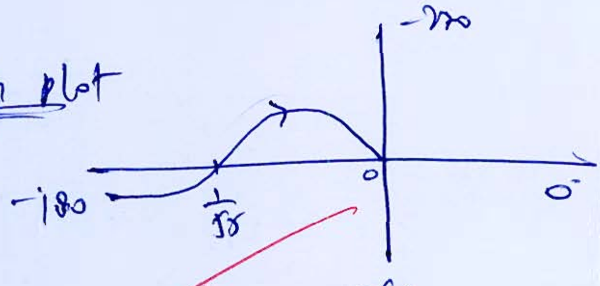
$2\omega - 10\omega^3 = 0$

$2\omega(1-5\omega^2) = 0$

$\boxed{\omega = \frac{1}{\sqrt{5}} \text{ rad/sec} = \omega_{pc}}$



Polar plot



Phase crossover frequency
$(\omega_{pc} = \frac{1}{\sqrt{5}}$ rad/sec$)$

$\text{Gain} = \frac{\sqrt{1+25\times\frac{1}{5}}}{\frac{1}{5}\sqrt{1+\frac{1}{5}}\sqrt{1+\frac{4}{5}}}$ { from ① }

$\text{Gain} = \frac{5\times\sqrt{6}\times\sqrt{5}\times\sqrt{5}}{\sqrt{6}\ \sqrt{9}} = \frac{25}{3}$

Gain margin $= \frac{3}{25} = \frac{1}{\text{Gain}} \Rightarrow \boxed{= 0.12}$

$\boxed{\text{Gain margin (dB)} = -18.416}$

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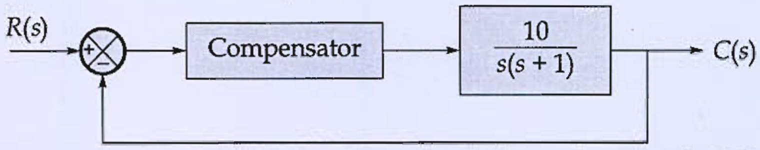
7 (c) Consider the feedback control system shown below:



The compensator block of the system is to be designed, such that the overall system will have a velocity error coefficient of 10 and a minimum phase margin of 43°. Compare the phase margin of the uncompensated system and compensated system.

[20 marks]

Phase Margin of uncompensated system

$$G(s)H(s) = \frac{10}{s(s+1)}$$

for ω_{gc}

$$|G(j\omega)H(j\omega)| = 1$$

$$\left|\frac{10}{(j\omega)(j\omega+1)}\right|_{\omega_{gc}} = 1$$

$$\frac{10}{\omega\sqrt{\omega^2+1}} = 1 \Rightarrow \text{squaring both sides}$$

$$100 = \omega^2(\omega^2+1)$$

$$\omega^4 + \omega^2 - 100 = 0$$

$$\omega^2 = 9.5125, \; -10.512 \times$$

$\omega_{gc} = 3.084$ rad/sec (Gain crossover frequency)

$$\angle G(j\omega)H(j\omega) = -90 - \tan^{-1}[\omega]\Big|_{\omega=\omega_{gc}}$$

$$\phi = -90 - \tan^{-1}[3.084]$$

$$\phi = -162.0345$$

Phase margin (PM) $= 180 + \phi$

$$\boxed{PM = 17.966°}$$ uncompensated system

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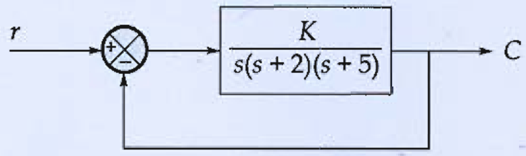
Q.8 (a) Consider the following control system.



(i) Sketch the root locus diagram for $0 < K < \infty$

(ii) Without the help of root locus diagram, determine the value of K that gives the system characteristic equation with a damping ratio of 0.5.

[10 + 10 marks]

Poles (P) are at 0, -2, -5

zeros (z) are at 0

P-Z = 3

(a) Angle of asymptotes $= \frac{(2k+1)\pi}{P-Z}$

$\theta_A = 60, 180^\circ, 300^\circ$

(b) centroid $= \frac{\sum Re(poles) - \sum Re(zeros)}{P-Z}$

$= \frac{-2-5-0}{3} = -2.33$

(c) Breakaway point Characteristic equation

$= 1 + GH(s) = 1 + \frac{K}{s(s+2)(s+5)} = 0$

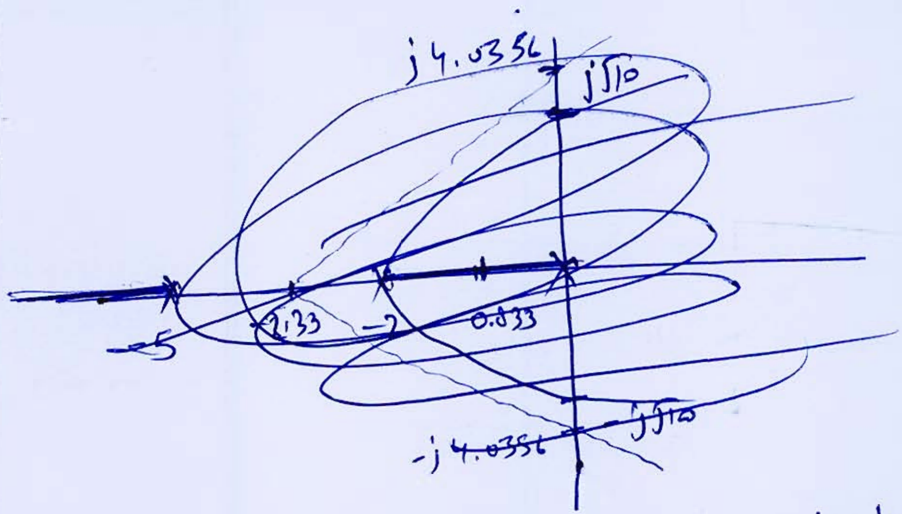
$s^3 + 7s^2 + 10s + K = 0$ ——— (1)

$K = -(s^3 + 7s^2 + 10s + K)$

(for breakaway point) $\frac{dK}{ds} = 0$

$\Rightarrow 3s^2 + 14s + 10 = 0$

$s = -0.88, -3.786$ (invalid) doesn't lie on locus



(d) Intersection of Root locus with jω axis by Routh array

from (1) $s^3 + 7s^2 + 10s + K = 0$

s^3	1	10
s^2	7	K
s	$\frac{70-K}{7}$	
s^0	K	

(stability)

$70 - K > 0$

$K < 70$

for marginally stable

$K = 70$

Auxillary equation

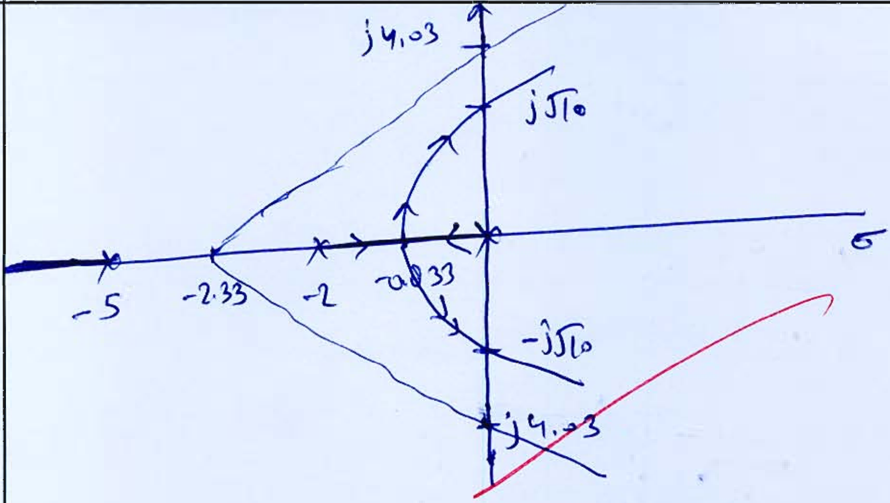
$7s^2 + K = 0 \Rightarrow 7s^2 + 70 = 0$

$s^2 = -10 \Rightarrow s = \pm j\sqrt{10}$

(e) Intersection of asymptodes

$\tan 60 = \frac{y}{2.33}$

$y = 4.0356$



(ii) By dominant pole concept

$(s+p)(s^2+2\zeta\omega_n s+\omega_n^2)$

$= s^3+2\zeta\omega_n s^2+s\omega_n^2+p s^2+2\zeta\omega_n p s+p\omega_n^2$

$s^3+s^2(2\zeta\omega_n+p)+s(\omega_n^2+2\zeta\omega_n p)+p\omega_n^2$

Comparing from characteristic equation

$s^3+7s^2+10s+K$

$2\zeta\omega_n+p=7 \qquad \omega_n^2+2\zeta\omega_n p=10 \qquad p\omega_n^2=K$

$(\zeta=0.5)$ $\quad \omega_n+p=7 \qquad \omega_n^2+\omega_n p=10 \qquad p\omega_n^2=K$

$\omega_n=7-p \qquad (7-p)^2+(7-p)p=10$

$49+p^2-14p+7p-p^2=10$

$+7p=39 \Rightarrow$ $p=5.5714$

$\omega_n=7-p=7-5.5714$

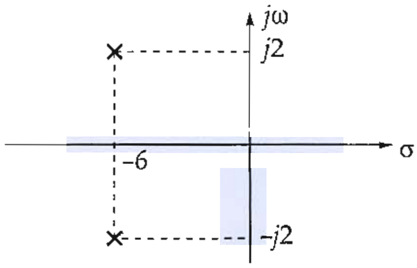
$\omega_n=1.4286$

$K=(5.5714)(1.4286)^2$

$K=11.3706$

19

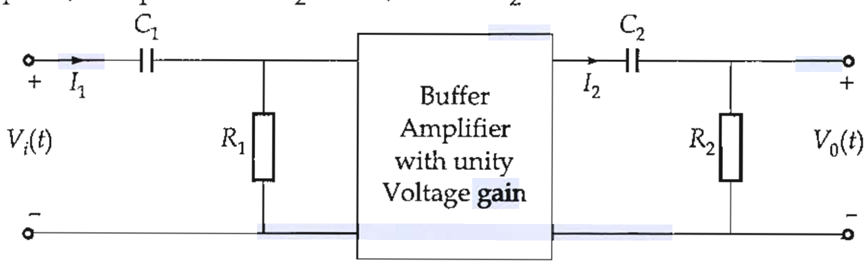
3 (b) (i) The closed-loop poles of a system is shown in figure below:



Find the

1. Transfer function of the system
2. Settling time for 2% tolerance band.
3. Percentage peak overshoot.
4. Rise time
5. Delay time

(ii) Determine the transfer function relating $V_0(s)$ and $V_i(s)$ for network shown in figure below. Calculate output voltage, $t \geq 0$ for a unit step voltage input at $t = 0$ when $C_1 = 1\ \mu F$, $R_1 = 1\ M\Omega$, $C_2 = 0.5\ \mu F$ and $R_2 = 1\ M\Omega$.



[10 + 10 marks]

$s = -6 + j2$, $s = -6 - j2$

$(s+6-j2)(s+6+j2)$

$(s+6)^2 + 4 \Rightarrow s^2 + 36 + 12s + 4$

$q(s) = s^2 + 12s + 40$

$\omega_n^2 = 40 \Rightarrow \omega_n = 6.324$ rad/sec

$2\xi\omega_n = 12$

$\xi = \dfrac{12}{2 \times 6.234}$

$\boxed{\xi = 0.9624}$

10

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(ii) settling time = $\frac{4}{\zeta_1 \omega_n} = \frac{4}{\zeta_1 \omega_n}$

$t_s = 0.667$ sec

(iii) Peak overshoot $\left(M_p = e^{\frac{-n\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} \right)$

$M_p = 1.4625 \times 10^{-5}$

(iv) $$t_r = \frac{\pi - \cos^{-1}(\zeta_1)}{\omega_n \sqrt{1-\zeta_1^2}}$$ (Rise time)

$t_r = 1.6686$ sec

(i) Transfer function

$$\frac{C(s)}{R(s)} = \frac{(6.234)^2}{s^2 + 11.99s + 38.862}$$

$$\frac{C(s)}{R(s)} = \frac{38.8627}{s^2 + 11.99s + 38.862}$$

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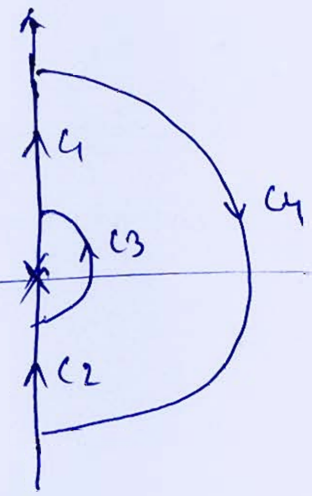
(c) Sketch the Nyquist plot and using the plot, assess the stability of the closed loop system whose open-loop transfer function is

$$G(s)H(s) = \frac{K(s+4)}{s^2(s+2)}$$

[20 marks]

$$G(j\omega)H(j\omega) = \frac{K(4+j\omega)}{(j\omega)^2(2+j\omega)}$$



Nyquist contour

Assuming direction of contour is clockwise. direction

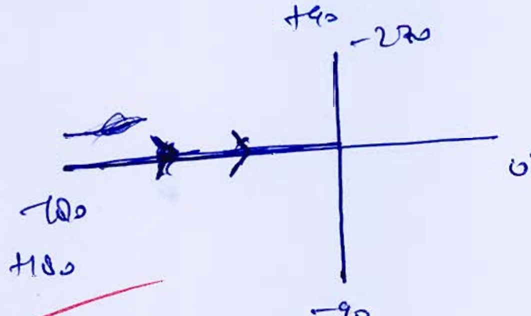
To map contour C_1

$s = j\omega$ ω tends from $0 \to \infty$

$$|M| = \frac{K\sqrt{4+\omega^2}}{\omega^2\sqrt{2+\omega^2}}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{4}\right) + 180 - \tan^{-1}\left(\frac{\omega}{2}\right)$$

	$\omega=0$	$\omega=\infty$
M	∞	0
ϕ	+180	+180



To map contour C_2

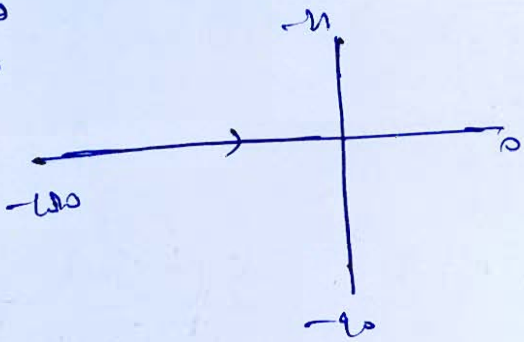
$s = -j\omega$ ω tends to ∞ to 0

~~$|M| = K\sqrt{4+}$~~ $G(-j\omega) = \frac{K(4-j\omega)}{(-j\omega)^2(-j\omega+2)}$

$$\phi = 180 - \tan^{-1}\left(\frac{\omega}{4}\right) - 180 - 180 + \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$= -\tan^{-1}\left(\frac{\omega}{4}\right) - 180 + \tan^{-1}\left(\frac{\omega}{2}\right)$$

ϕ	$\omega = 0$	$\omega = \infty$
	-180	-180



To map contour C_3

$\lim_{r \to 0} r e^{j\theta}$ $\quad \theta \to -\frac{\pi}{2}$ to $\pi/2$

$|M| = \infty$

for contour C_4

$\lim_{R \to \infty} R e^{j\theta}$ $\quad \theta \to \frac{\pi}{2}$ to $-\pi/2$

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Space for Rough Work

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Space for Rough Work

$$\frac{0.0083854}{1.0083854} = 0.0083157 04$$

$q = 0.0911$

$q_1^2 = 0.8903$

0.1297

$$\frac{1}{4\varepsilon_1^2(1-\varepsilon_1^2)} = (1.1)^2$$

$4\varepsilon_1^2 - 1$

$$-4\varepsilon_1^4 + 4\varepsilon_1^2 = \left(\frac{1}{1.1}\right)^2$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\frac{\omega_0}{B\omega}$$

$$B\omega = \frac{1}{2} = \frac{R}{L}$$

$$\left(\frac{\omega_0 \times L}{R}\right) = \tan$$