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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electrical Circuits [All Topics]

Control Systems [All Topics]

Name :

Roll No

Test Centres			**Student's Signature**
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Kolkata ☐	Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are *compulsory* and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum *Answer Booklet. Candidate should write* the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages *after completion of the examination.*

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	41
Q.2	56
Q.3	
Q.4	45
Section-B	
Q.5	53
Q.6	
Q.7	51
Q.8	
Total Marks Obtained	246

Signature of Evaluator   Cross Checked by

Nikhil

→ Approach and accuracy is very good.
→ Do not forget to mention units
→ Do not write outside borders marked.

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

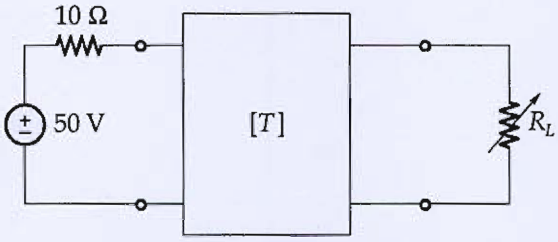
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Electrical Circuits

Q.1 (a) The *ABCD* parameter of the two-port network in figure are $\begin{bmatrix} 4 & 20\,\Omega \\ 0.1S & 2 \end{bmatrix}$.



The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.

[12 marks]

ABCD parameter of the network = $\begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix}$

$$V_1 = 4V_2 + 20I_2$$

$$I_1 = 0.1V_2 + 2I_2$$

from figure

$$V_1 = 50 - 10I_1$$

for finding equivalent resistance

50V voltage source deactivated

$$V_1 = -10I_1$$

$$-10I_1 = 4V_2 + 20I_2$$

$$-10I_1 = -V_2 + 20I_2$$

$$+ \quad + \quad +$$

$$0 = 5V_2 + 40I_2$$

$$+\cancel{40}\,8\,I_2 = \cancel{5}V_2$$

$$V_2 = +8I_2$$

$$V_2/I_2 = 8\,\Omega$$

$$\underline{R_{th} = 8\,\Omega}$$

for finding V_{th} $\boxed{I_2 = 0}$

$V_1 = 50 - 10I_1$

$V_1 = 4V_2 - 20(I_2)$

$I_2 = 0 \quad V_1 = 4V_2$ —— (I)

$I_1 = 0.1V_2 - \cancel{2I_2}^{\,0}$

$I_1 = 0.1V_2$ —— (II)

by solving equation (I) & (II)

$I_1 = 0.1\left(\frac{V_1}{4}\right) = \frac{0.1}{4}(50 - 10I_1)$

$4I_1 = 5 - I_1$

$5I_1 = 5$

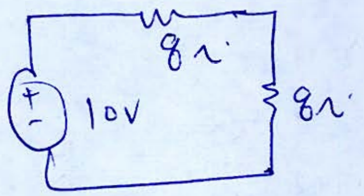
$\boxed{I_1 = 1}$

from equation (II)

$V_2 = 10 \text{ volt} = V_{th}$

$R_L = R_{th} = 8\Omega$

(by max MPT)



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"Good"

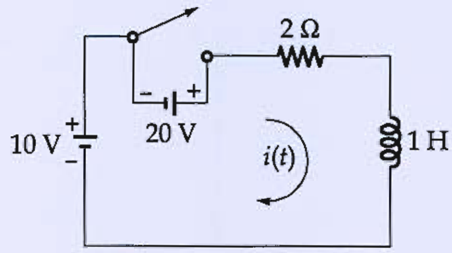
$P_{max} = \left(\frac{V_{th}^2}{4R_{th}}\right)$

$= \left(\frac{\cancel{100}^{\,25}}{\cancel{4} \times 8}\right)$

$= 25/8 \text{ watt} = 3.125 \text{ watt}$

Do not write in this margin

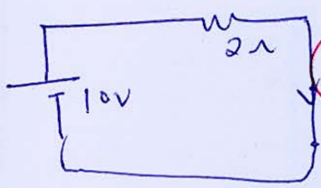
Q.1 (b) Determine the current $i(t)$ in the circuit shown in figure at an instant t, after opening the switch at $t = 0$, if a current of 1 A had been passed through the circuit at the instant of opening.



[12 marks]

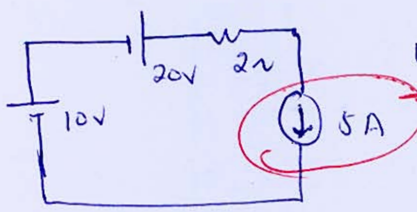
before $t=0$

at $t=0^-$



$i_L(0^-) = 5A$

after $t=0^+$

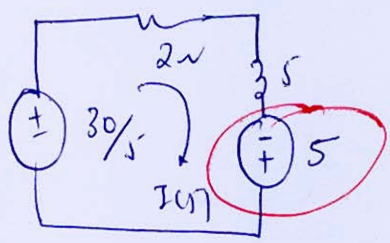


$i_L(0^+) = 5A$

2

current in inductor does not change instantly

convert ckt time domain to s domain



$$I(s) = \left(\frac{30/s + 5}{s+2}\right)$$

$$I(s) = \frac{5(s+6)}{s(s+2)}$$

$$I(s) = \frac{A}{s} + \frac{B}{(s+2)}$$

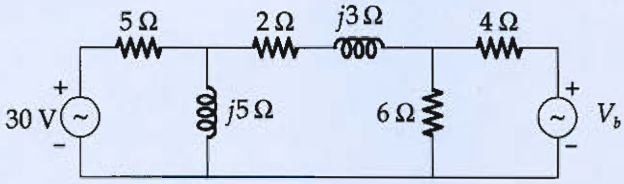
$$A = 15, \quad B = -10$$

$$I(s) = 15/s - 10/(s+2)$$

apply laplace inverse transform

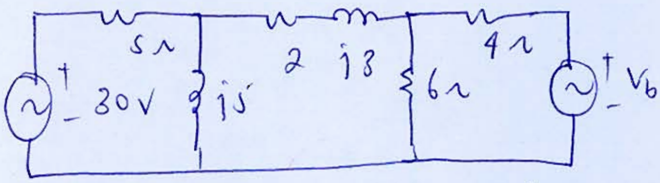
$$\boxed{I(t) = (15 - 10e^{-2t})\,u(t)}$$

Q.1 (c) For the circuit shown below:



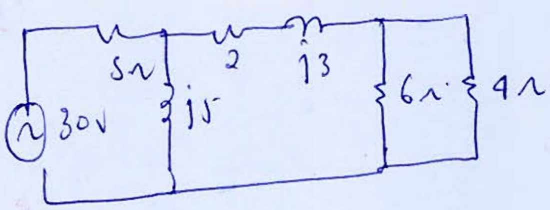
Determine the voltage V_b which results in a zero current through the $(2 + j3)\Omega$ impedance branch. Using superposition theorem.

[12 marks]



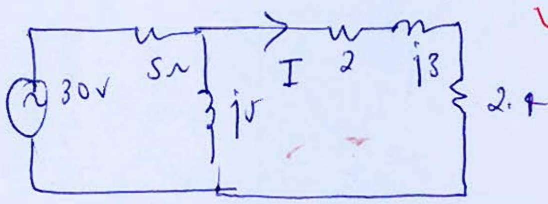
by using superposition theorem

(1) consider 30V voltages source and deactivate V_b voltage source and replace by internal resistance.



6 & 4 ohm are in parallel

$$= \frac{6\times4}{10} = 2.4$$



$$Z_{eq} = 5 + \frac{j5 \times (4.4 + j3)}{(j5 + 4.4 + j3)} = 6.8338\angle 22.369^\circ$$

$$I_S = \left(\frac{30}{Z_{eq}}\right) = 4.389\angle -22.369$$

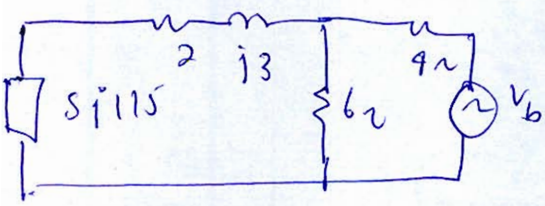
$$I = \left(\frac{I_S \times j5}{j5 + 4.4 + j3}\right)$$

$= 4.389 \angle -22.369 \times \frac{(5j)}{(4.4 + j8)}$

$\boxed{I_1 = 2.404 \angle 6.44}$

(11) by considering V_b voltage source, deactivate 30 voltage source by replacing its internal resistance.

not write here



$Z_{eq} = 6 \parallel (4.5 + 5.5i) + 4$

$= 7.444 \angle 10.912°$

$I_{S_2} = \left(\frac{V_b}{Z_{eq}}\right) = 0.134 V_b \angle -10.912°$

$I_1' = \left(\frac{I_{S_2} \times 6}{6 + 4.5 + 5.5j}\right)$

$I_1' = 0.068 V_b \angle -38.558$

to make zero current through $(2+j3)$

then $I_1 = I_1'$ — $I_1 = -I'$

$7.444 \angle 10.912 = 0.068 V_b \angle -38.558$

$V_b = 71.144 + j83.208$

$\boxed{V_b = 109.474 \angle 49.47°}$

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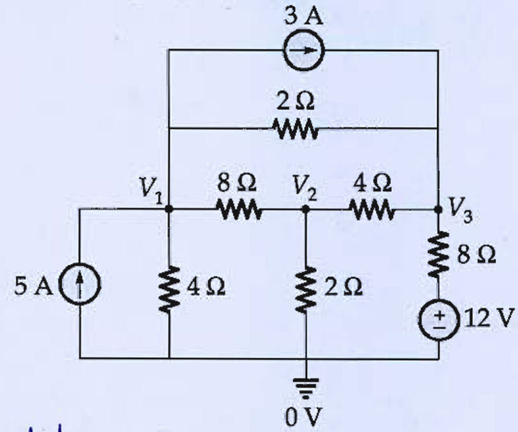
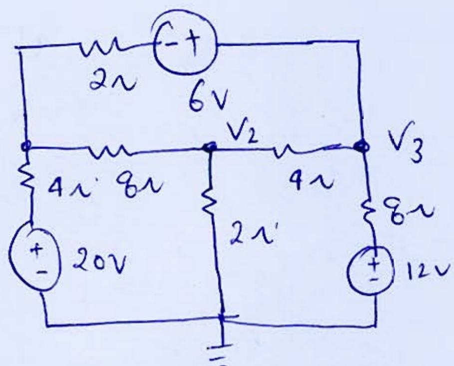
Q.1 (d) Use nodal analysis to find V_1, V_2 and V_3 in the circuit of figure.



[12 marks]

simply fig ckt.



by using nodal analysis

apply nodal at 1 node

$$\frac{V_1-20}{4} + \frac{V_1-V_2}{8} + \frac{V_1+6-V_3}{2} = 0$$

$$2V_1 - 40 + V_1 - V_2 + 4V_1 + 24 - 4V_3 = 0$$

$$7V_1 - V_2 - 4V_3 = 16 \quad \text{—— (I)}$$

apply nodal at 2 node.

$$\frac{V_2}{2} + \frac{V_2-V_3}{4} + \frac{V_2-V_1}{8} = 0$$

$$4V_2 + 2V_2 - 2V_3 + V_2 - V_1 = 0$$

$$-V_1 + 7V_2 - 2V_3 = 0 \quad \text{—— (II)}$$

apply nodal at node 3.

$$\frac{V_3-12}{8}+\frac{V_3-V_2}{4}+\frac{V_3-6-V_1}{2}=0$$

$$V_3-12+2V_3-2V_2+4V_3-24-4V_1=0$$

$$-4V_1-2V_2+7V_3=36 \quad \text{(iii)}$$

by solving equation (i) (ii) & (iii)

$$\begin{bmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 16 \\ 0 \\ 36 \end{bmatrix}$$

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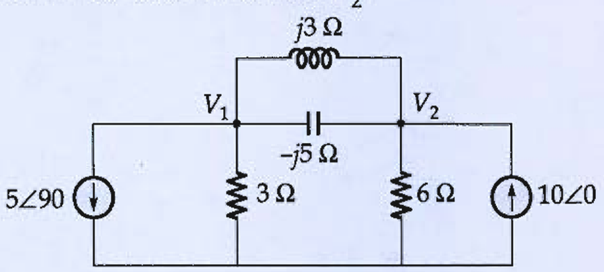
$V_1 = 10$ Volt

$V_2 = 4.933$ Volt

$V_3 = 12.267$ Volt.

Q.1 (e) Use nodal analysis on the circuit to find V_2.



[12 marks]

apply nodal at V_1

$$5\angle 90 + \frac{V_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j3} = 0$$

$$V_1\left(\frac{1}{3} - \frac{1}{j5} + \frac{1}{j3}\right) + V_2\left(\frac{1}{j5} - \frac{1}{j3}\right) = -5\angle 90$$

$$V_1(0.333 - 0.133i) + V_2(0.133i) = -5j \quad (1)$$

apply nodal at V_2

$$\frac{V_2}{6} + \frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j3} = 10\angle 0$$

$$V_2\left(\frac{1}{6} - \frac{1}{j5} + \frac{1}{j3}\right) + V_1\left(\frac{1}{j5} - \frac{1}{j3}\right) = 10 \quad (11)$$

11

$$V_2(0.166 - 0.133j) + V_1(.133j) = 10 \quad (11)$$

$$\begin{bmatrix} 0.333 - 0.133i & 0.133i \\ 0.133i & .166 - 0.133i \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -5\angle 90 \\ 10 \end{bmatrix}$$

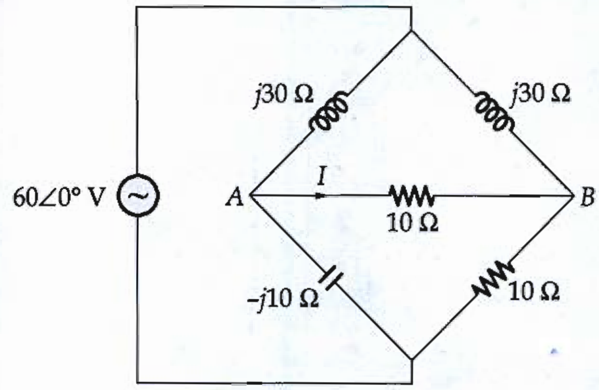
$$\boxed{V_2 = \frac{\Delta_2}{\Delta} = \frac{2.665 - 1.33i}{0.0556 - 0.0665} = 34.358\angle 23.57}$$

volt

$$\Delta_2 = \begin{bmatrix} 0.333 - 0.133i & -5j \\ 0.133i & 10 \end{bmatrix} = 2.665 - j.33i$$

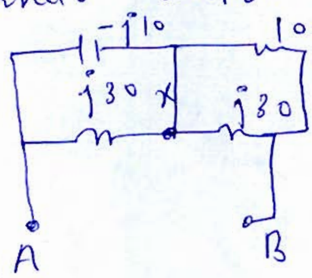
$$\Delta = \begin{bmatrix} 0.333 - 0.133i & 0.133i \\ 0.133i & 0.167 - 0.133i \end{bmatrix} = 0.05561\text{-}0.066$$

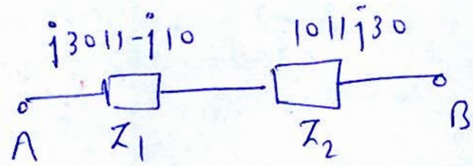
Q.2 (a) Determine the current I through the terminal AB of the network shown below:



[20 marks]

find equivalent resistance across AB
deactivate 60 volt voltage source and replace by its internal resistance.



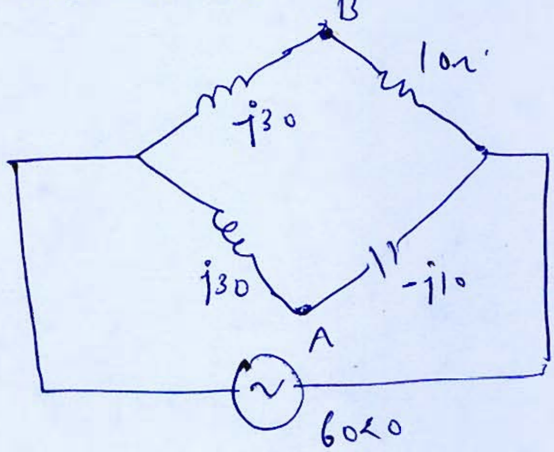


$Z_1 = \dfrac{j30 \times -j10}{j30 - j10}$ $\quad Z_2 = \dfrac{10 \times j30}{10 + j30}$

$= \dfrac{\cancel{j}30 \times -j1\cancel{0}}{\cancel{j}2\cancel{0}}$ $\quad = \left(\dfrac{j30}{j3+1}\right)$

$= -j15$ $\quad = 9 + 3i$

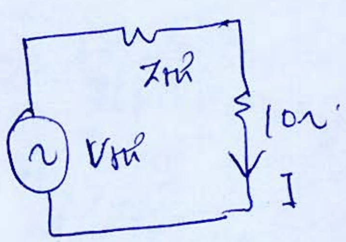
$Z_{eq_{AB}} = -j15 + 9 + 3i = \underline{9 - 12i}$ Ω find Vth across AB



$$V_A = \frac{60 \times -j10}{j30 - j10} = \frac{60 \times -\cancel{j}1\cancel{0}}{\cancel{j}2\cancel{0}} = -30 \text{ Volt}$$

$$V_B = \frac{60 \times 1\cancel{0}}{1\cancel{0} + j3\cancel{0}} = \left(\frac{60}{j3+1}\right) = 6 - 18i$$

$$V_{AB} = V_{th} = V_A - V_B$$
$$= -30 - (6 - 18i)$$
$$= -36 + 18i = 40.25 \angle 153.434^\circ \text{ Volt}$$



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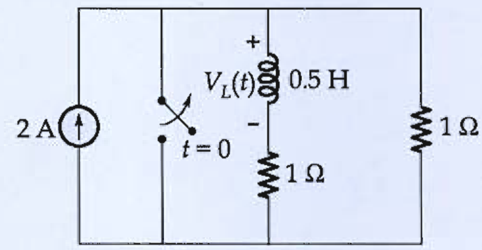
$$I = \left(\frac{V_{th}}{Z_{th} + 10}\right)$$

$$= \frac{40.25 \angle 153.434^\circ}{(9 - 12i + 10)}$$

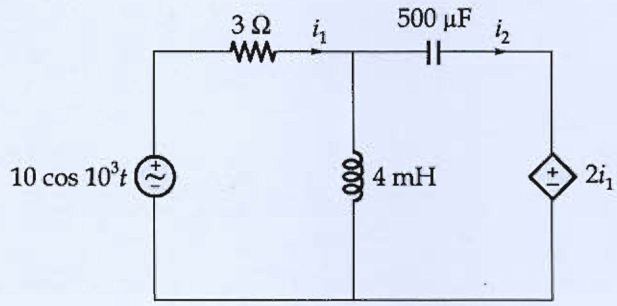
$$\boxed{I = 1.79 \angle -174.28^\circ \text{ A}}$$

Q.2 (b) **(i)** For the network shown in figure below, the switch is closed for a long time and at $t = 0$, the switch is opened.



Determine the voltage across inductor for $t > 0$.

(ii) Obtain expressions for the time domain currents i_1 and i_2 in the circuit given as figure.

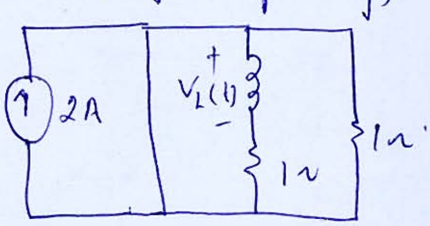


[10 + 10 marks]

2b(i)

"do not write here"

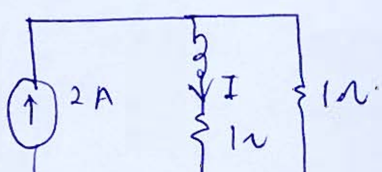
at $t=0^-$ (before opening)



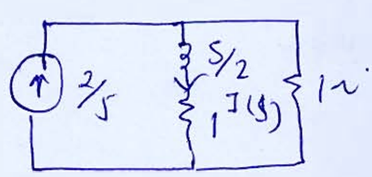
$\boxed{i_L(0^-) = 0A}$

switch closed.

after $t=0^+$
switch open



convert ckt into s domain



$$I(s) = \frac{2/s \times 1}{1 + \left(1 + s/2\right)}$$

$$I(S) = \frac{2}{S\left(\frac{4+S}{2}\right)} = \frac{4}{S(S+4)}$$

$$V_2(S) = I(S) \times S/2$$

$$= \frac{\cancel{4}^{2}}{\cancel{S}(S+4)} \times \frac{\cancel{S}}{\cancel{2}}$$

$$V_2(S) = \left(\frac{2}{S+4}\right)$$

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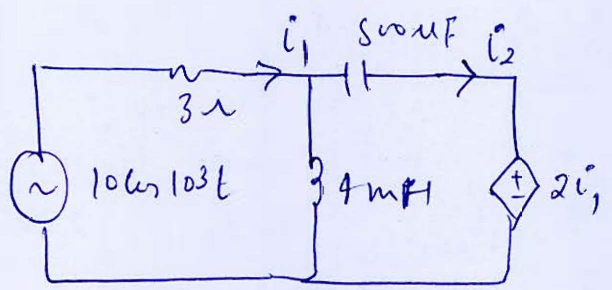
apply laplace inverse transform

$$\boxed{V_2(t) = 2e^{-4t}u(t)}$$

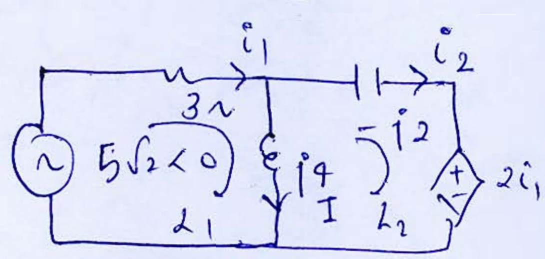
2b(ii)



$(\omega = 1000 \text{ rad/sec})$

$$jX_L = j(\omega L) = 1000 \times 4 \times 10^{-3} j = 4j$$

$$X_c = \left(\frac{1}{\omega C}\right) = \left(\frac{1 \times 2}{\cancel{2} \times 1\cancel{000} \times \cancel{500} \times \cancel{10^{-6}}}\right) = 2$$



$i_1 = I + i_2$

$I = i_1 - i_2$

apply KCL at loop 1.

$$5\sqrt{2} - 3i_1 - 4j(I_1 - I_2) = 0$$

$$i_1(+3+4j) - 4j I_2 = 5\sqrt{2} \quad \text{—— (1)}$$

apply KCL at loop 2.

$$-(-j2)i_2 - 2i_1 - j4(I_2 - i_1) = 0$$

$$j2i_2 - j4i_2 - 2i_1 + j4I_1 = 0$$

$$\cancel{-}j2i_2 = \cancel{-}(-2+j4)i_1$$

$$i_2 = i_1\left(\frac{-1+j2}{j}\right) = i_1(2+i) \quad \text{(11)}$$

put value of i_2 to (1) eqn.

$$i_1(3+9i) - 4i(2+i)i_1 = 5\sqrt{2}$$

$$i_1 = \left(\frac{5\sqrt{2}}{7-4i}\right)$$

$$i_{1\,rms} = 0.877\angle 29.744$$

$$\boxed{i_1 = 1.24\cos(1000t + 29.744)\,A}$$

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$$i_2 = (2+i)i_1$$

$$= 1.96\angle 56.301^\circ$$

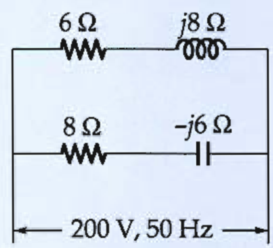
$$\boxed{i_2 = 2.77\cos(1000t + 56.301^\circ)\,A}$$

Q.2 (c) For the circuit shown below, calculate,

(i) Total admittance, total conductance and total susceptance.

(ii) Total current and total power factor (pf).

(iii) The value of pure capacitance to be connected in parallel with the above combination to make the total power factor (pf) unity.



[20 marks]

(i)

admittance $Y = \left(\frac{1}{6+j8} + \frac{1}{8-j6}\right)$

$Y = \left(\frac{6-j8}{6^2+8^2} + \frac{8+j6}{8^2+6^2}\right)$

$Y = \frac{1}{100}(6-j8+8+j6)$

$Y = \left(\frac{14-j2}{100}\right) = 0.1414 \angle -8.13°$

$\boxed{\text{total admittance} = 0.1414 \mho}$

$\boxed{\text{total conductance} = \left(\frac{14}{100}\right)}$

$= .14 \mho$

$\boxed{\text{total susceptance} = 2/100 = 0.025}$

(ii)

total current $= (YV)$

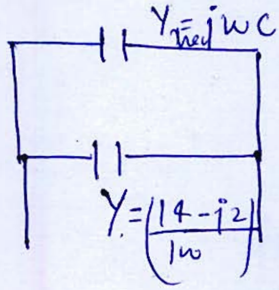
$= 0.1414 \times 200$

$\boxed{I = 28.28 \angle -8.13°}$

power factor = $\cos(8.13)$

$$\boxed{pf = 0.989 \text{ lagg.}}$$

(11)



$Y_{net} = Y + Y_{new}$

$Y_{net} = \frac{14}{100} - \frac{j2}{100} + j\omega C$

to make powerfactor unity

Img(Y_{net}) must be zero.

$Y(net)\ (img) = 0$

$\left(-\frac{2}{100} + \omega C\right) = 0$

$\omega C = \frac{2}{100}$

$C_{new} = \frac{2}{100\omega}$

$C_{new} = \frac{2}{100 \times 2\pi \times 50}$

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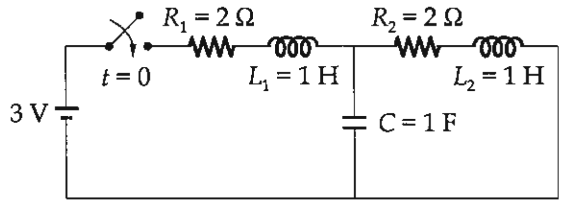
$C_{new} = \frac{2 \times \cancel{10^6}^{\,2}}{\cancel{100 \times 100}\,\pi \times 10^6}$

$C_{new} = \frac{200}{\pi}\ \mu F$

$$\boxed{C_{new} = 63.662\ \mu F.}$$

Q.3 (a) In the network shown in figure the switch is closed at time $t = 0$. Assuming all the initial currents and voltages as zero, find the current through the inductor L_2 by the use of Norton's theorem.



[20 marks]

Q.3 (b) Show that the resonant frequency ω_0 of a series R-L-C circuit is geometric mean of ω_1 and ω_2, i.e., the upper and lower half power frequencies respectively.

[20 marks]

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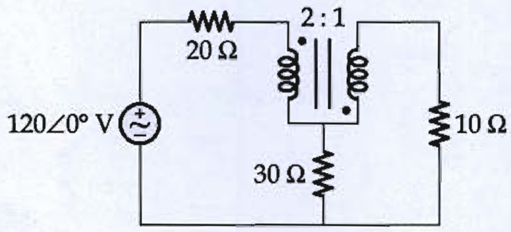
Q.3 (c) Calculate the power supplied to the 10 Ω resistor in the ideal transformer circuit given in the figure below.



[20 marks]

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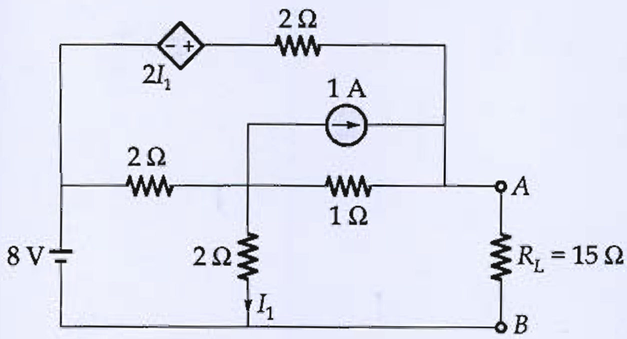
Q.4 (a) Determine the current through the load resistance $R_L = 15\ \Omega$ across the terminal A-B of the circuit shown in figure below, using Thevenin's theorem. Also find the maximum power that can be transferred to the load resistance R_L.



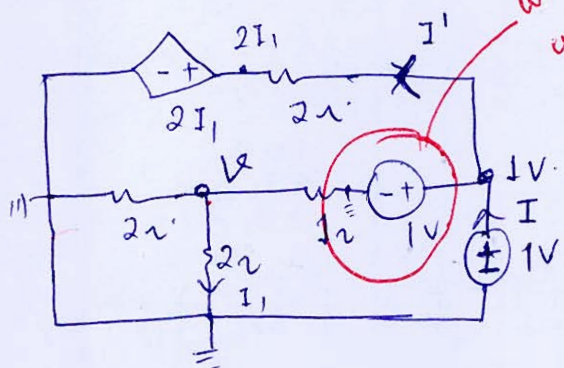
[20 marks]

apply ~~Thevenin~~ Thevenin theorem

find R_{th}.

deactivate the all the sources and replace by internal resistances.

will be SC while calculating R_{th}.



$$\frac{1-2I_1}{2} = I'$$

$$\frac{V}{2} + \frac{V}{2} + \frac{V}{1} = 0$$

$$V = 0$$

$$I_1 = V/2 = 0.$$

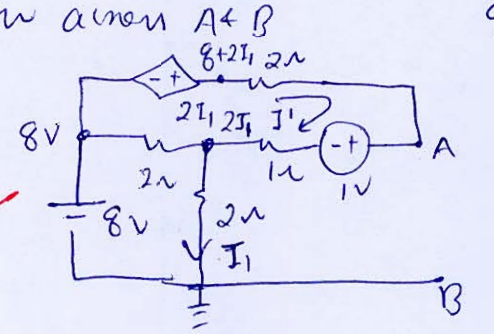
$$I = I' = \frac{1-2\times 0}{2}$$

$$I = 1/2$$

$$R_{th} = \left(\frac{1}{I}\right) = \left(\frac{1}{1/2}\right)$$

$$\boxed{R_{th} = 2\Omega}$$

find V_{th}. open across A & B



$$8 + \cancel{2I_1} - 2I' - 1 - I' = \cancel{2I_1}$$

$$7 = 3I'$$

$$I' = 7/3$$

$$V_A - 1 - 7/3 - 2I_1 = V_B$$

$$I' = \left(\frac{2I_1 - 8}{2}\right) + I_1 = \frac{4I_1 - 8}{2} = 7/3$$

$$12I_1 - 24 = 14$$

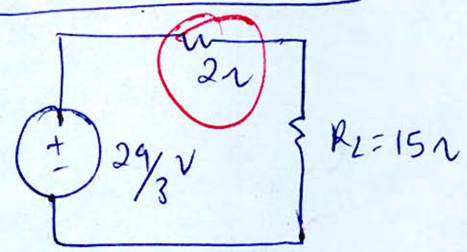
$$6I_1 - 12 = 7$$

$$I_1 = \left(\frac{19}{6}\right)$$

$$V_{AB} = 1 + 7/3 + \cancel{2}\left(\frac{19}{\cancel{6}_3}\right)$$

$$= 1 + 26/3 = 29/3$$

$$\boxed{V_{AB} = 29/3 \text{ Volt.}}$$



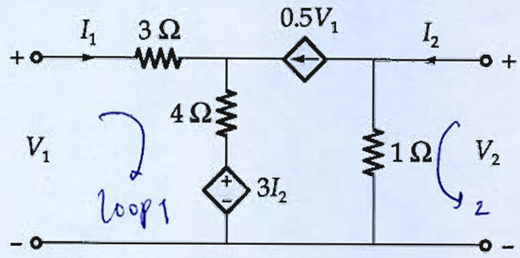
$$I = \left(\frac{29/3}{2+15}\right) = \left(\frac{29}{3\times17}\right) = \left(\frac{29}{51}\right)$$

6

$$P_{max\ transfer} = \left(\frac{29}{51}\right)^2 \times 15$$

$$\boxed{P_{max\ transfer} = 4.85 \text{ watt}}$$

Q.4 (b) Find the h-parameters for the two-port network shown



[20 marks]

h parameter

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

apply KCL loop 1.

$$V_1 - 3I_1 - 4(I_1 + 0.5V_1) - 3I_2 = 0$$

$$V_1 - 3I_1 - 4I_1 - 2V_1 - 3I_2 = 0$$

$$+V_1 + 3I_2 + 7I_1 = 0 \quad \text{--- (1)}$$

apply KCL loop 2.

$$V_2 - 1(I_2 - 0.5V_1) = 0$$

$$V_2 - I_2 + 0.5V_1 = 0 \quad \text{--- (11)}$$

from equation (1)

$$V_1 = -7I_1 - 3I_2$$

put value of V_1 in eqn (2)

$$V_2 - I_2 + 0.5(-7I_1 - 3I_2) = 0$$

$$V_2 - I_2 - 3.5I_1 - 1.5I_2 = 0$$

$2.5 I_2 = V_2 - 3.5 I_1$

$I_2 = \frac{-3.5}{2.5} I_1 + \frac{V_2}{2.5}$

put value of I_2 in equation (11)

$V_2 + \frac{3.5}{2.5} I_1 - \frac{V_2}{2.5} + \frac{V_1}{2} = 0$

$\frac{V_1}{2} = -0.6 V_2 - 1.4 I_1$

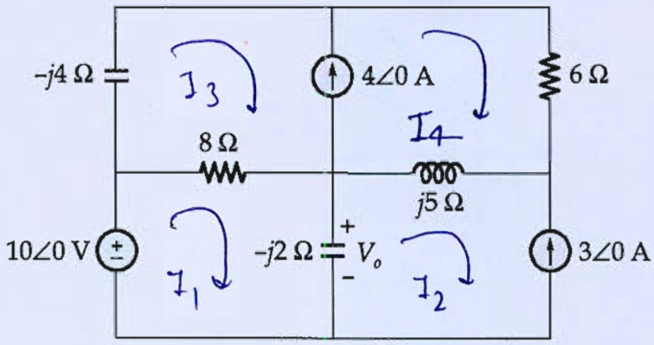
$V_1 = -2.8 I_1 - 1.2 V_2$

$I_2 = \underline{-1.4} I_1 + \underline{0.4} V_2$

h parameter $= \begin{bmatrix} -2.8 & -1.2 \\ -1.4 & 0.4 \end{bmatrix}$.

19

Q.4 (c) Solve for V_o in the circuit of figure using mesh analysis.



[20 marks]

apply mesh at loop 1.

$$10 - 8(I_1 - I_3) + j2(I_1 - I_2) = 0$$

$$10 - 8I_1 + 8I_3 + j2I_1 - j2I_2 = 0 \text{ —— (I)}$$

$$I_2 = -3\,A \text{ —— (II)}$$

$$I_4 - I_3 = 4\,A \text{ —— (III)}$$

by applying current in same branch are same.

by apply mesh in (1) (3) & (4) at same time

$$10 - (-j4)I_3 - 6I_4 - j5(I_4 - I_2) + j2(I_1 - I_2) = 0$$

$$(10 + j6) = (8 - j2)I_1 - 8I_3 \text{ —— (IV)}$$

$$10 + j4I_3 - 6(4 + I_3) - j5(4 + I_3 + 3) + j2(I_1 + 3) = 0$$

$$10 + j4I_3 - 24 - 6I_3 - 35j - j5I_3 + j2I_1 + 6j = 0$$

$$14 + 29i = j2I_1 + I_3(-6 - j) \text{ —— (V)}$$

$$\begin{bmatrix} 8 - j2 & -8 \\ j2 & -6 - j \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ 14 + 29i \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 8-j2 & -8 \\ j2 & -6-j \end{bmatrix}$$

$$= -50+20i$$

$$\Delta_1 = \begin{bmatrix} 10+j6 & -8 \\ 14+29i & -6-j \end{bmatrix}$$

$$= 58+186i$$

$$I_1 = \frac{\Delta_1}{\Delta} = \left(\frac{58+186i}{-50+20i}\right)$$

$$\underline{I_1 = 3.618\angle -85.517^\circ}$$

$$V_o = (I_1 - I_2) \times -j2$$

$$= (3.618\angle -85.517^\circ + 3) \times -j2$$

$$V_o = -7.214 - 6.565i$$

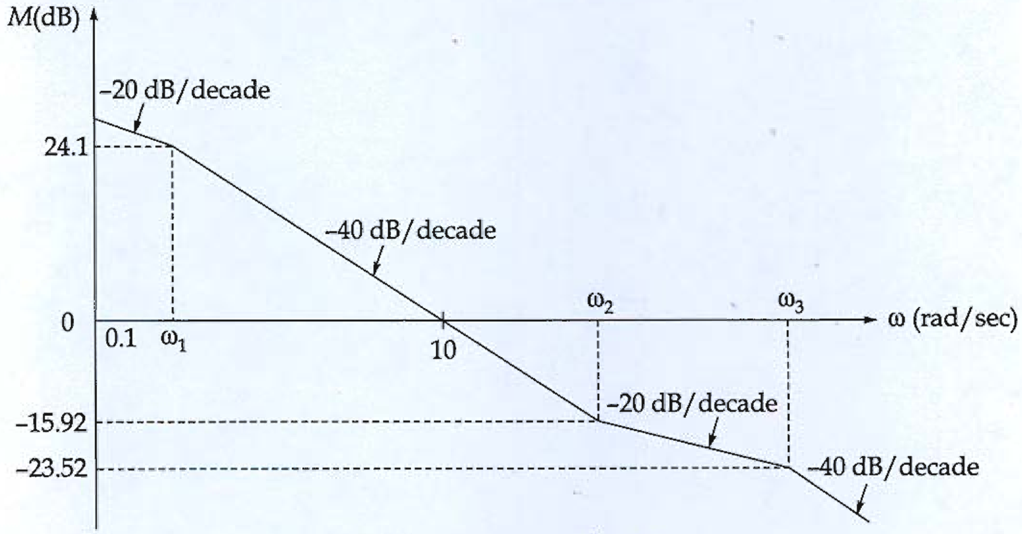
$$\boxed{V_o = 9.754\angle -137.69^\circ} \ \underline{\text{Volt}}$$

20

" Good Approach "

Section B : Control System

Q.5 (a) Obtain the open loop transfer function for a unity negative feedback system whose bode magnitude plot is shown below:



[12 marks]

(i) bode plot start -20 dB/decade slope means it has one pole at origin

(ii) at $\omega = \omega_1$ slope changes -20 dB/decade to -40 dB/decade means it has one pole at $\omega = \omega_1$

(iii) at $\omega = \omega_2$ slope changes -40 dB/decade to -20 dB/decade means it has one zero at $\omega = \omega_2$

(iv) at $\omega = \omega_3$ slope changes -20 dB/dec to -40 dB/dec means it has one pole at $\omega = \omega_3$

$$\text{OLTF} = \frac{K\left(\frac{s}{\omega_2}+1\right)}{s\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_3}+1\right)}$$

$$-40 = \left(\frac{-15.92 - 0}{\log \omega_2 - \log 10}\right)$$

$$\log(\omega_2/10) = \left(\frac{15.92}{40}\right)$$

$$\omega_2 = 10\,10^{\left(\frac{15.92}{40}\right)} = 25 \text{ rad/sec.}$$

$$-20 = \frac{-23.52 + 15.92}{\log(\omega_3/\omega_2)}$$

$$\omega_3 = \omega_2\, 10^{(0.38)}$$

$$\omega_3 = 25 \times 10^{0.38} = 60 \text{ rad/sec.}$$

$$-40 = \frac{0 - 24.1}{\log(10/\omega_1)}$$

$$\left(\frac{10}{\omega_1}\right) = 10^{\frac{24.1}{40}} = 4$$

$$\omega_1 = 10/4 = 2.5 \text{ rad/sec.}$$

OLTF $\underline{0.1 < \omega < \omega_1}$

$$OLTF = \frac{K \quad 1}{s\,(s/\omega_1)\;1} = \frac{K\omega_1}{s^2}$$

$$24.1 = -40\log\omega_1 + 20\log K\omega_1$$

$$K\omega_1 = 100$$

$$K = \frac{10\cancel{0} \times 4}{1\cancel{0}} = 40$$

12

$$OLTF = \frac{40\left(s/25 + 1\right)}{s\left(s/2.5 + 1\right)\left(s/60 + 1\right)}$$

$$\boxed{OLTF = \frac{240(s+25)}{s(s+2.5)(s+60)}}$$

Q.5 (b) A servo mechanism is represented by the equation :

$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144E$$

where $E = C - 0.5y$ is the actuating signal. Find the value of damping ratio, damped and undamped frequency of oscillation. Draw the block diagram of the system described by the above equation.

[12 marks]

$$\frac{d^2y}{dt^2} + 4.8\left(\frac{dy}{dt}\right) = 144\,(C - 0.5y)$$

$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} + 72y = 144C$$

taking laplace inverse transform

$$s^2Y(s) + 4.8s\,Y(s) + 72Y(s) = 144\,C(s)$$

$$\frac{Y(s)}{C(s)} = \left(\frac{144}{s^2 + 4.8s + 72}\right)$$

$CE = s^2 + 4.8s + 72.$

by comparing standard $CE = s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\omega_n = \sqrt{72}$ rad/sec. $= 8.485$ rad/sec

$2\xi\omega_n = 4.8$

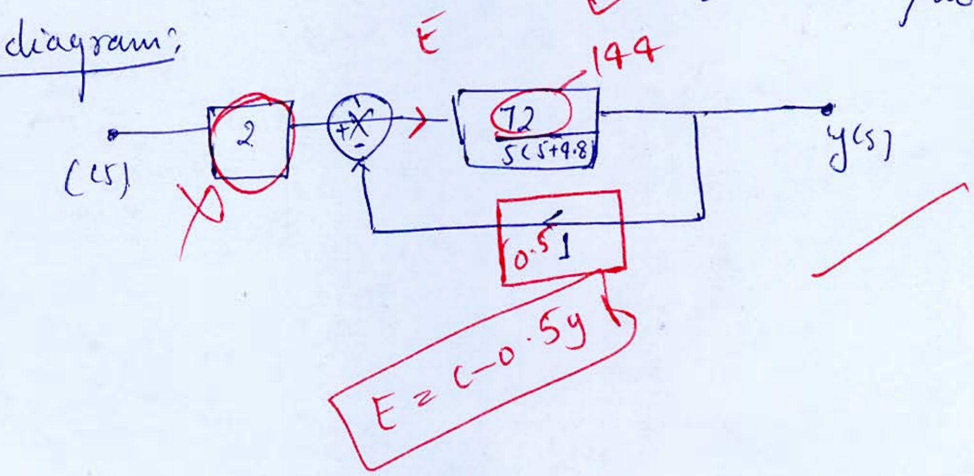
$\xi = 2.4/\sqrt{72} = 0.2828$

7

damped freq $= \omega_n\sqrt{1-\xi^2} = \sqrt{72}\sqrt{1-0.2828^2}$

$= 8.138$ rad/sec

blockdiagram:



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Q.5 (c) Closed loop system with unity feedback has the forward loop transfer function as :

$$G(s) = \frac{28.8}{s(1+0.2s)}$$

Modify the design using cascaded compensation to satisfy the optimum performance criterion, so that the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot. Take gain of proportional controller equal to 5.

[12 marks]

modify by using PD controller

gain = $(K_p + K_D s)$

given $(K_p = 5)$

gain = $(5 + K_D s)$

Now $G_{overall}(s) = \frac{28.8(5+K_D s)}{s(1+0.2s)}$

CE (characteristic equation)

$s + 0.2s^2 + 28.8 K_D s + 28.8\times5 = 0$

$s^2 + 5s + 28.8\times5 K_D s + 28.8\times25 = 0$

by comparing standard CE,

$\omega_n^2 = 28.8\times5\times5$

$\boxed{\omega_n = 12\sqrt{5}\ \text{rad/sec} = 26.8328\ \text{rad/sec}}$

$2\xi\omega_n = 28.80 K_D\times5 + 5$

no overshoot means

$\boxed{\xi = 1}$

$2\xi\omega_n = 5 + 28.80\times5 K_D$

$2\times12\sqrt{5}\times1 - 5 = 28.80\times5 K_D$

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$$K_D = \left[\frac{2\times 1\times 12\sqrt{5}-5}{28.8\times 5}\right]$$

$$= 0.3379$$

PD controller $= (5+0.338\,s)$

$$\boxed{\text{overall } G(s) = \frac{144(5+0.338\,s)}{s(s+5)}}$$

12

Q.5 (d) A unity negative feedback system has open loop transfer fucntion, $G(s) = \dfrac{K}{s(1+sT)}$, where K and T are positive constants. Determine the factor by which the amplifier gain K be reduced so that peak overshoot of the unit step response is reduced from 80% to 50%?

[12 marks]

$$G(s) = \frac{K}{s(1+sT)}$$

$$CE = s^2T + s + K$$

$$CE = s^2 + \frac{1}{T}s + \frac{K}{T}$$

$$\omega_n = \sqrt{K/T}, \quad 2\xi\sqrt{K/T} = 1/T$$

$$\xi = \left(\frac{1}{2\sqrt{KT}}\right), \quad \xi \propto \frac{1}{\sqrt{K}}$$

Let ξ_1 when overshoot is 80%.

$$\text{overshoot} = \underline{e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

$$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.8$$

$$\xi_1 = \sqrt{\frac{\ln(0.8)^2}{\pi^2 + \ln(0.8)^2}}$$

$$\xi_1 = 0.07$$

when overshoot is 50%.

$$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.5$$

$$\xi_2 = \sqrt{\frac{\ln(0.5)^2}{\pi^2 + \ln(0.5)^2}}$$

$$\xi_2 = 0.2154$$

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$$\xi \propto \frac{1}{\sqrt{K}}.$$

$$\xi^2 \propto \frac{1}{K}$$

$$K \propto \frac{1}{\xi^2}$$

<u>Now.</u> $$\frac{K_1}{K_2} = \left(\frac{\xi_2^{\,2}}{\xi_1^{\,2}}\right)$$

$$\frac{K_1 \text{ at } (80\%)}{K_2 \text{ at } (50\%) \text{ overshoot}} = \left(\frac{0.2154^2}{0.07^2}\right)$$

$$\frac{K_1}{K_2} = 9.4735$$

$$K_2 = \cdot 1055\, K_1$$

<u>gain K reduced by 89.44%</u>

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Q.5 (e) The open loop transfer function of a unity negative feedback system is given as, $G(s) = \dfrac{K}{2s(1+0.1s)(1+s)}$. Determine the value of 'K' for which the gain margin of the system is 14 dB.

[12 marks]

$$G(s) = \left(\frac{10K}{2s(s+1)(s+10)}\right)$$

$$\angle G(j\omega) = -90° - \tan^{-1}\left[\frac{\omega}{1}\right] - \tan^{-1}\left[\frac{\omega}{10}\right]$$

for finding phase cross over frequency

$$\angle G(j\omega) = -180°$$

$$\angle G(j\omega) = -180°$$

$$\cancel{-}90 \cancel{-} \tan^{-1}[\omega] \cancel{-} \tan^{-1}\left[\frac{\omega}{10}\right] = \cancel{-}180$$

$$\tan^{-1}\left(\frac{\omega + \omega/10}{1 - \omega^2/10}\right) = 90$$

$$\left(\frac{\frac{11\omega}{10}}{1-\omega^2/10}\right) = \frac{1}{0}$$

$$1 - \omega^2/10 = 0$$

$$\omega = \sqrt{10} \text{ rad/sec}$$

$$\text{gain at } \omega_{pc} = \left(\frac{10K}{2\sqrt{10}\sqrt{1+10}\sqrt{100+10}}\right)$$

$$= \frac{10K}{2\sqrt{10} \times \sqrt{11}\sqrt{110}}$$

$$= \frac{\cancel{10}K}{11 \times 2 \times \cancel{10}} = K/22$$

Gain Margin = 14 dB

$$20\log(GM) = 14$$

$$GM = 10^{0.7}$$

$$\text{gain} = \left(\frac{1}{10^{0.7}}\right) = \left(\frac{1}{5}\right)$$

$$\text{gain at } \omega_{pc} = \left(\frac{K}{22}\right) = \frac{1}{5}$$

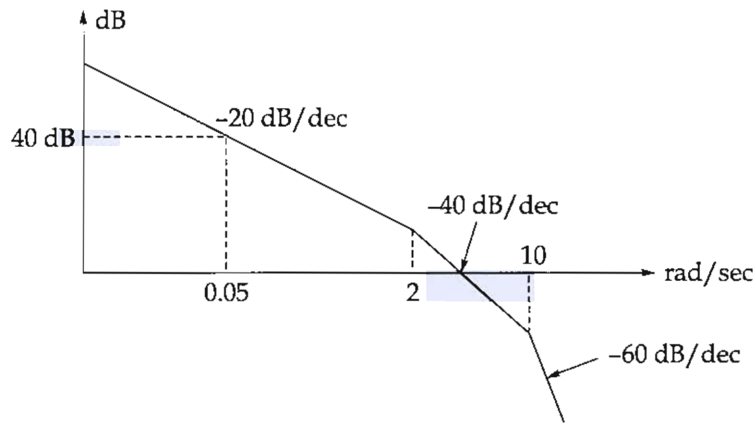
$$K = \frac{22}{5}$$

$$\boxed{K = 4.4}$$

$\boxed{\text{at } K = 4.4 \text{ GM is } 14\,dB}$

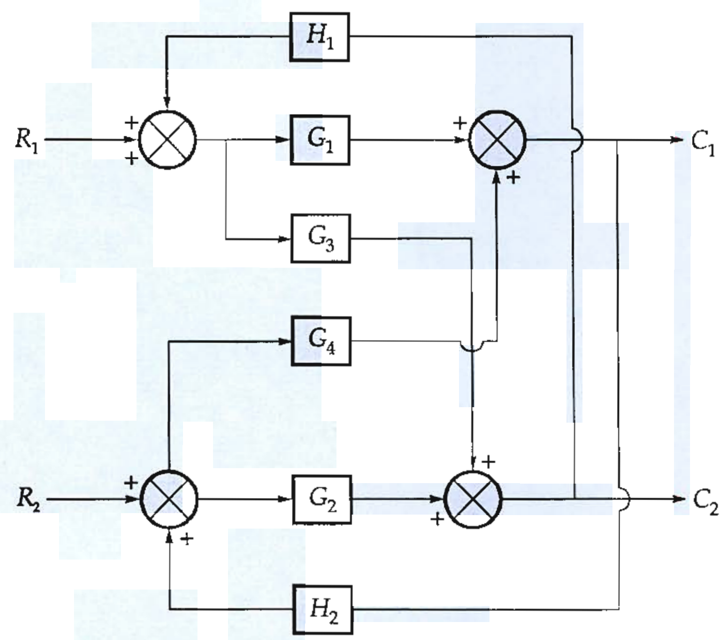
11

Q.6 (a) The open loop transfer function of a unity feedback system is given by $G(s)H(s) = e^{-Ts}G_1(s)$, where $G_1(s)$ is minimum phase system. The approximate bode magnitude plot of the open loop transfer function is shown in the figure below. If the phase margin of the system is –18.19°, determine the transportation lag T.

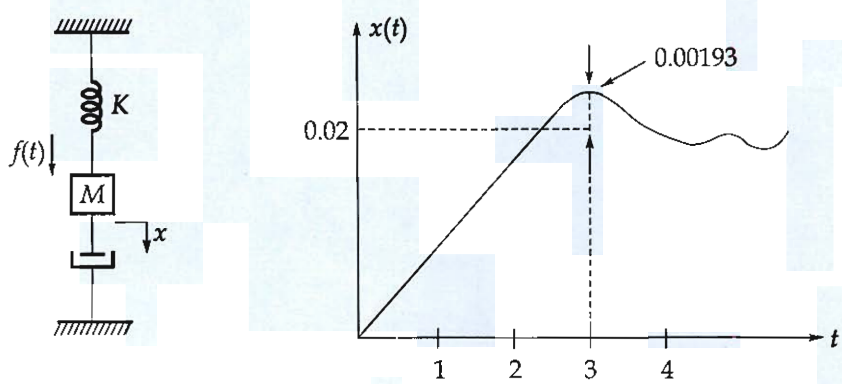


[20 marks]

Q.6 (b) (i) Evaluate $\frac{C_2}{R_1}$ for the system whose block diagram representation is shown in figure below. (Use block diagram reduction technique to solve).



(ii) Figure below shows a mechanical system and the response when 10 N of force is applied to the system. Determine the values of M, F, K. The dimension 'x' is in meter.



[10 + 10 marks]

Q.6 (c) Derive the expression for the transfer function of an ac servomotor and obtain the same in respect of a servomotor having following data :

(i) Starting torque = 0.166 N-m

(ii) Moment of inertia, $J = 1 \times 10^{-5}$ kgm^2

(iii) Supply voltage = 115 Volts

(iv) No load speed = 2904 rpm

(Assume friction to be zero)

[15 + 5 = 20 marks]

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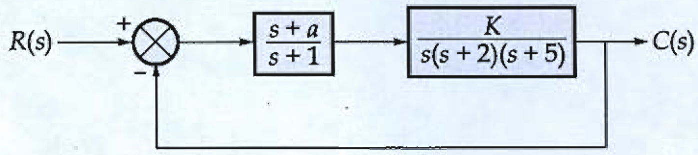
Q.7 (a) **(i)** A position control system is shown in figure below :

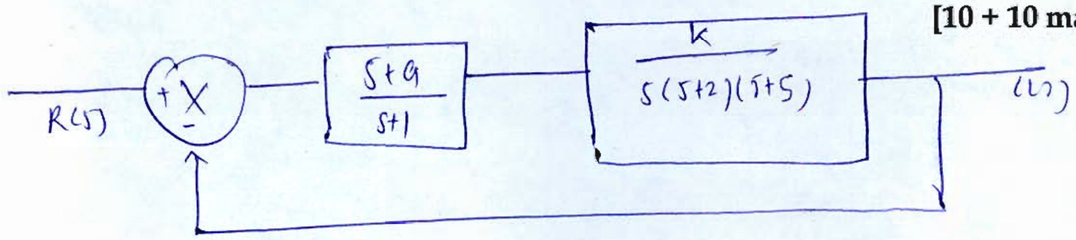


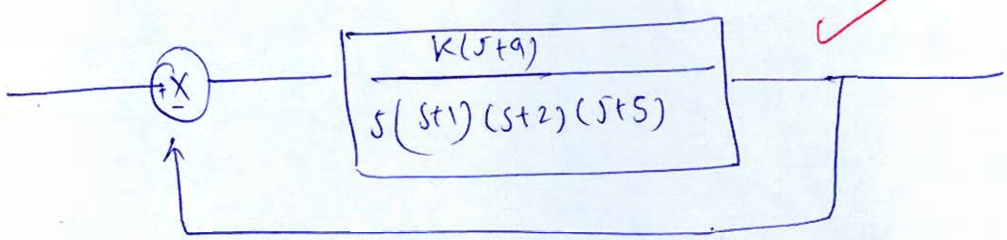
K and *a* are the parameters of the system. Determine the range of *K* and *a* for which system is stable.

(ii) Sketch the root-locus of $G(s) = \dfrac{K(s+1)}{s^2(s+2)}$.

[10 + 10 marks]

7a(i)





$$CE = s(s^2+3s+2)(s+5)+ks+ka$$

$$CE = s(s^3+3s^2+2s+5s^2+15s+10)+ks+ka=0$$

$$CE = s^4+8s^3+17s^2+(10+k)s+ka=0$$

apply hurtwitz criteria to find stability.

s^4	1	17	ka
s^3	8	$10+k$	
s^2	$\frac{136-10-k}{8}$	ka	
s^1	$\frac{\left(\frac{126-k}{8}\right)(10+k)-8ka}{\left(\frac{126-k}{8}\right)}$		
s^0	ka		

for stable system
first coloumn of all element should have same sign

$\frac{126-k}{8} > 0$ | $ka > 0$

$k < 126$ | $a > 0$

$k > 0$

$\left(\frac{126-k}{8}\right)(10+k) - 8ka > 0$

$(126-k)(10+k) - 64ka > 0$

$64ka < (126-k)(10+k)$

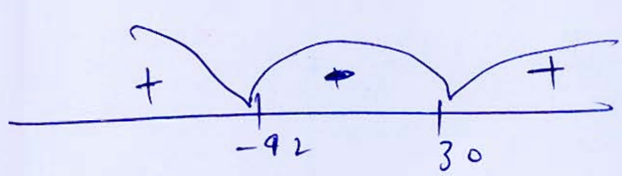
Let $a = 2$.

$128k < 1260 - 10k + 126k - k^2$

$k^2 + 116k + 128k - 1260 < 0$

$k^2 + 12k - 1260 < 0$

$(k-30)(k+42) < 0$



$k \in (-42, 30)$

10

$0 < k < 30$

$\vee$ $k < 126$

when $a = 2$, k lies between $(0, 30)$

$\boxed{0 < k < 30}$ — $a = 2$.

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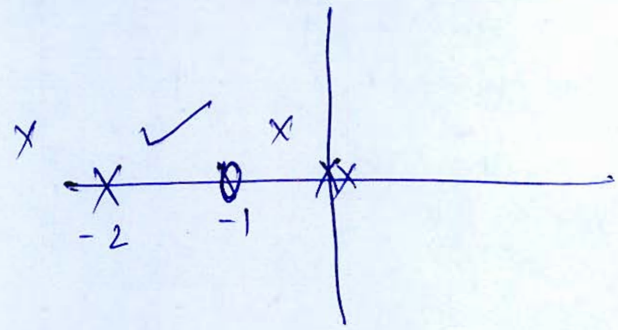
7a (ii) $G(s) = \dfrac{K(s+1)}{s^2(s+2)}$

(i) no of poles = 3

no of zero = 1.



(ii) centroid $= \dfrac{-0-2-(-1)}{2}$

$= \dfrac{-2+1}{2} = -0.5$

(iii) asymptotes $= \dfrac{(2n+1)\pi}{P-Z}$

$= \dfrac{(2n+1)\pi}{2}$

$= 90^\circ, 270^\circ$

(iv)

$K = -\dfrac{(s^3+2s^2)}{s+1}$

$dK/ds = -\left[\dfrac{(s+1)(3s^2+4s) - (s^3+2s^2)(1)}{(s+1)^2}\right]$

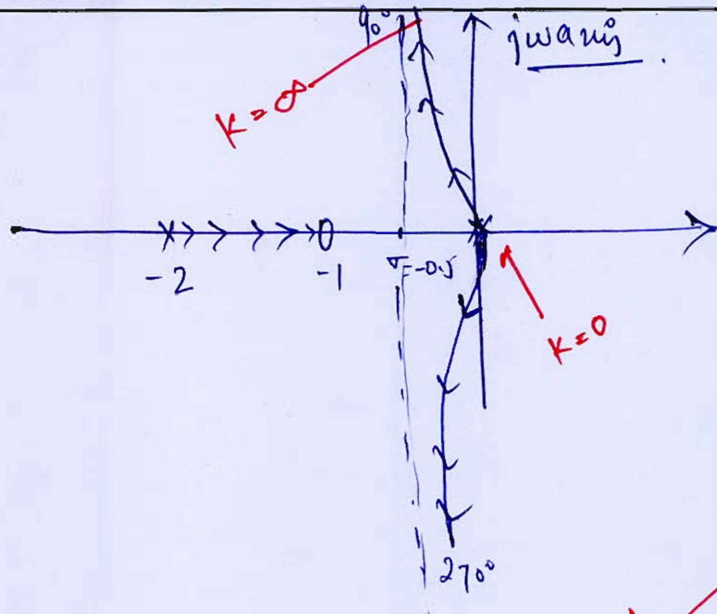
$0 = 3s^3 + 3s^2 + 4s^2 + 4s - s^3 - 2s^2$

$2s^3 + 5s^2 + 4s = 0$

$s(2s^2+5s+4) = 0$

$s = 0, \quad -1.25 \pm 0.66143.$

Only one break point lies at s=0 on real axis.



jω axis

$s^3 + 2s^2 + Ks + k = 0$

1	K	
2	K	
$\frac{2K-K}{2} = K/2$		
K		

when K>0

always stable.

08

Q.7 (b) Sketch the polar plot of the transfer function given below. Determine whether the plot crosses the real axis. If so, determine the frequency at which the plot cross the real axis and the corresponding magnitude $|G(j\omega)|$.

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

[20 marks]

Polar plot.

$s = j\omega$

$$G(j\omega) = \frac{1}{(j\omega)^2(j\omega+1)(2j\omega+1)}$$

$$= \frac{1(-j\omega+1)(-2j\omega+1)}{-\omega^2(\omega^2+1)(4\omega^2+1)}$$

$$G(j\omega) = \frac{(-2\omega^2+1)}{-\omega^2(\omega^2+1)(4\omega^2+1)} + \frac{j3}{\omega(\omega^2+1)(4\omega^2+1)}$$

at $\omega = 0$

$G(j\omega) = -\infty + j\infty$

at $\omega \to \infty$

$G(j\omega) = 0 + j0$

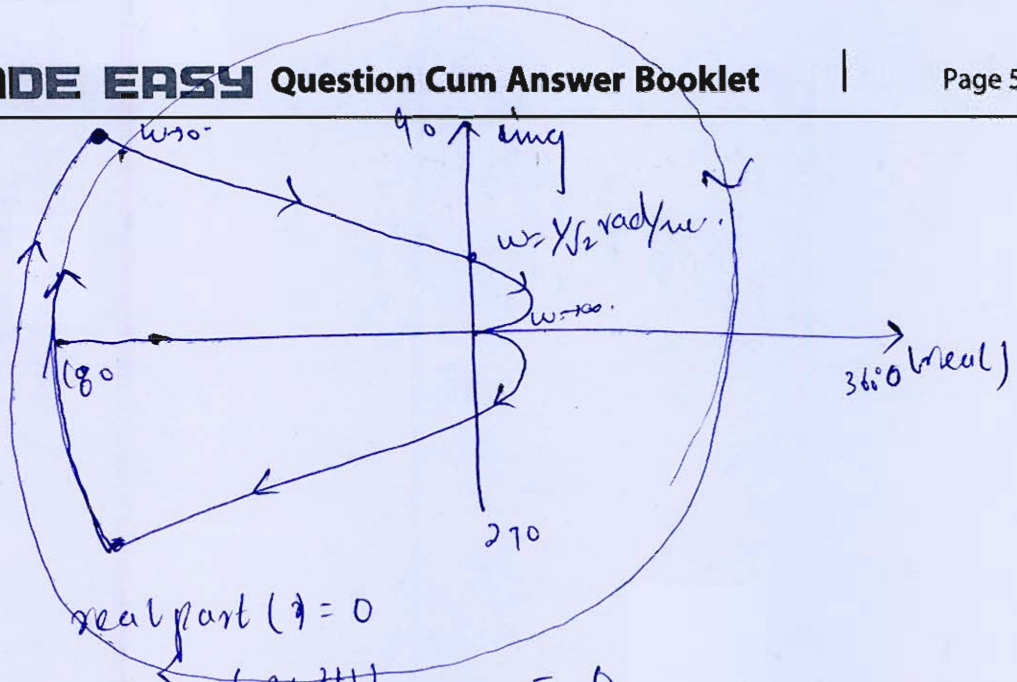
$\angle G(j\omega) = -180° - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$

at $\omega = 0$

$\angle G(j\omega) = -180°$

at $\omega = \infty$

$\angle G(j\omega) = -360°$



real part $(\cdot) = 0$

$$\frac{(-2\omega^2+1)}{-\omega^2(\omega^2+1)(4\omega^2+1)} = 0$$

$$\underline{\omega = 1/\sqrt{2} \text{ rad/sec}}$$

at $\omega = 1/\sqrt{2}$ rad/sec polar plot cut at imaginary axis

when $R \to \infty$

$$G(Re^{j\theta}) = \frac{1}{R^2 e^{j2\theta}(Re^{j\theta})(2Re^{j\theta})}$$

$$= \frac{1}{2R^4} e^{-j4\theta}$$

2 semicircle from.

18

Polar plot does not intersect at real axis hence w. is undefined.

Q.7 (c) Construct the state model for a system characterised by the differential equation :

$$\frac{d^3y}{dt^3}+\frac{6d^2y}{dt^2}+\frac{11dy}{dt}+6y=u$$

Give the block diagram representation of the state model.

[15 + 5 = 20 marks]

$$\frac{d^3y}{dt^3}+\frac{6d^2y}{dt^2}+\frac{11dy}{dt}+6y=u.$$

Let $y = x_1$ —— (I)

$$\frac{dy}{dt} = \dot{x}_1 = x_2 \quad \text{—— (II)}$$

$$\frac{d^2y}{dt} = \dot{x}_2 = x_3 \quad \text{—— (III)}$$

$$\frac{d^3y}{dt^3} = \dot{x}_3$$

$$\dot{x}_3 + 6x_3 + 11x_2 + 6x_1 = u$$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 + u \quad \text{—— (IV)}$$

state model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0]$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0], \quad D = [0]$$

$$\frac{d^3y}{dt^3} + \frac{6d^2y}{dt^2} + \frac{11dy}{dt} + 6y = u$$

by applying inverse laplace transform

$$Y(s)\left(s^3+6s^2+11s+6\right) = U(s)$$

$$\frac{Y(s)}{U(s)} = T/F = \left(\frac{1}{s^3+6s^2+11s+6}\right)$$

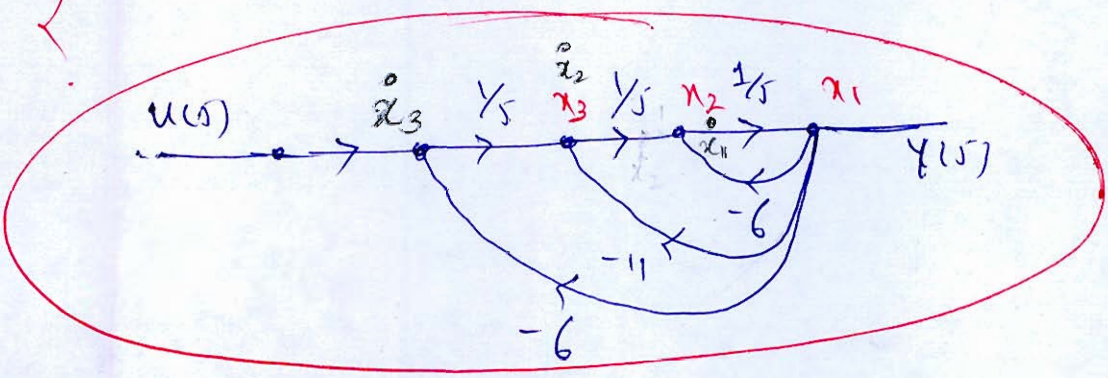
$$T/F = \left(\frac{1/s^3}{1+6/s^1+11/s^2+6/s^3}\right)$$

order of transfer $f^n = 3$

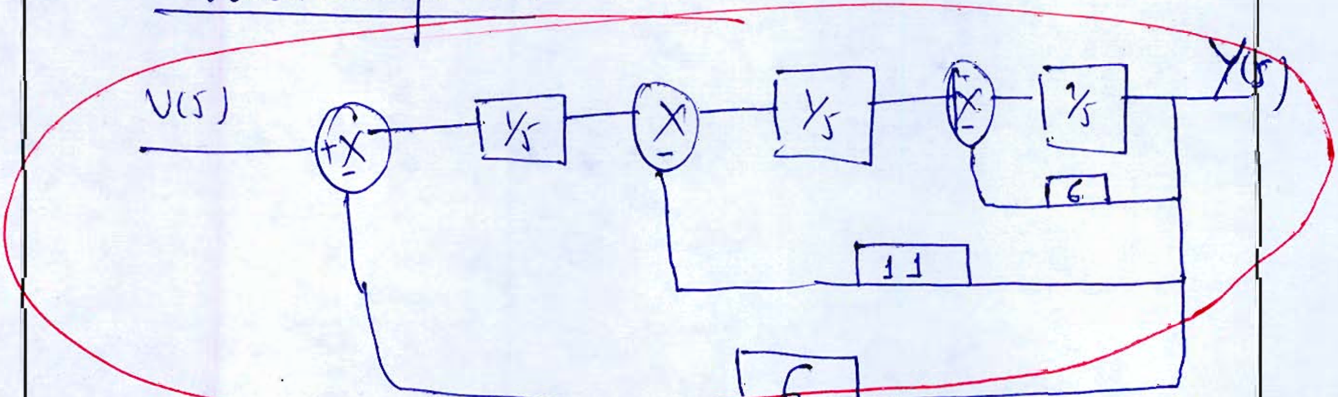
node $= (3+2) = 5$

15

?



block diagram:



Q.8 (a) The open-loop transfer function of a unity feedback control system is given below:

$$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}$$

Plot the root locus and determine the value of K at the breakaway point.

[20 marks]

Q.8 (b) The open loop transfer function of a feedback control system is

$$G(s)H(s) = \frac{K(1+2s)}{s(1+s)(1+s+s^2)}$$

Find the restriction on K for stability. Find the value of K for the system to have a gain margin of 3 dB. With this value of K, find the gain cross over frequency and phase margin. Use Nyquist Approach.

[20 marks]

Do not write in this margin

Q.8 (c) The state space model of a second order system given below is designed using feedback control system.

$$\dot{x} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

(i) What are the conditions for the desired response? Also check whether desired response is possible or not.

(ii) Design an observer system such that the above system has settling time of 0.5 sec and damping frequency of 6 rad/sec.

[8 + 12 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work

$$\frac{144}{s(s+5)}$$

$$\frac{144(s+K_D s)}{s^2+5s}$$

$$s^2+5s+144 \times s+144 K_D$$

$$\omega_n = 12\sqrt{s}$$

$$\frac{2 \cdot 12\sqrt{5}-5}{144}$$