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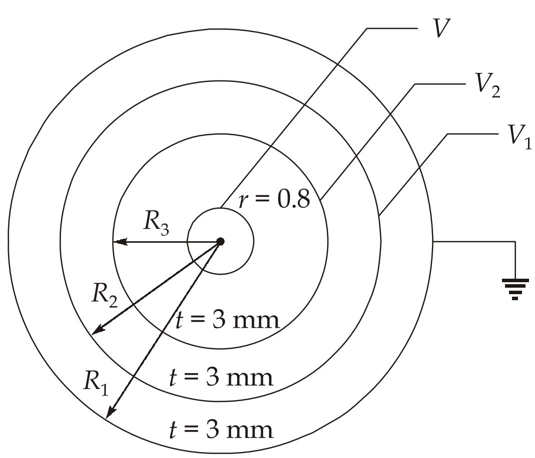
Detailed Solutions

# ESE-2024 Mains Test Series

# Electrical Engineering Test No : 3

## Section A : Power Systems

**Q.1 (a) Solution:**




$$\text{Overall radius of the cable} = 0.8 + 0.3 \times 3 = 1.7 \text{ cm}$$

$$R_1 = 0.8 + 0.9 = 1.7 \text{ cm}$$

$$R_2 = 0.8 + 0.6 = 1.4 \text{ cm}$$

$$R_1 = 0.8 + 0.3 = 1.1 \text{ cm}$$

$$r = 0.8 \text{ cm}$$

Let, $E_{max}$ be maximum stress on the surface of conductor

$E_{2,\,max}$ be the maximum stress on first intersheath

$E_{1,\,max}$ be the maximum stress on second intersheath

$$V_2 - V_1 = E_{2max} R_3 \ln \frac{R_2}{R_3}$$

$$= E_{2\max} \times 1.1 \ln \frac{1.4}{1.1} = 0.265\, E_{2\,\max} = 20$$

$$V - V_1 = E_{\max} \times r \ln \frac{R_3}{1.1}$$

$$= E_{\max} \times 0.8 \times \ln \frac{1.1}{0.8} = 0.255\, E_{\max}$$

$$V_1 = E_{1\max} R_2 \ln \frac{R_1}{R_2} = E_{1\max} 1.4 \times \ln\left(\frac{1.7}{1.4}\right)$$

$$V_1 = 0.272\, E_{1,\,\max}$$

$$V - V_2 = 0.255\, E_{\max} = 20$$

$$E_{\max} = \frac{20}{0.255} = 78.43 \text{ kV/cm}$$

$$V_2 - V_1 = 20 = 0.265\, E_{2\,\max}$$

$$E_{2,\,\max} = \frac{20}{0.265} = 78.43 \text{ kV/cm}$$

$$V_1 = 66 - 40 = 26 = 0.272\, E_{1,\,\max}$$

$$E_{1,\,\max} = \frac{26}{0.272} = 95.39 \text{ kV/cm}$$

**Q.1 (b) Solution:**

Given,

$$I_1 = (240 + j0) \text{ Amp}$$

$$I_2 = (220 + j0) \text{ Amp}$$

$$\text{CT ratio} = \frac{400}{5} = 80$$

Therefore, CT secondary current will be

$$I_{1s} = \frac{(240 + j0) \times 5}{400} = (3 + j0) \text{Amp}$$

$$I_{2s} = \frac{(220 + j0) \times 5}{400} = (2.75 + j0) \text{Amp}$$

Differential operating current, i.e., current in the operating coil,

$$I_d = I_{1s} - I_{2s}$$

$$= 3 - 2.75 = 0.25 \text{ Amp}$$

Restraining current, i.e, current in the restraining coil,

$$I_r = \frac{I_{1s} + I_{2s}}{2} = \frac{3 + 2.75}{2} = 2.875 \text{ Amp}$$

Slope of the characteristics, $K = 10\% = 0.1$

The differential operating current required for the operation of the relay corresponding to current of 2.75 Amp in the restraining coil,

$$I_{\text{trigger}} = K \cdot I_r = 0.1 \times 2.875 = 0.2875 \text{ Amp}$$

Since the actual current in the operating coil is 0.25 Amp,

$$I_d = (0.25 \text{ A}) < I_{\text{trigger}} (0.2875 \text{ A})$$

So, the relay will not operate to trip the circuit breaker.

**Q.1 (c) Solution:**

**(i)** Stored energy $= GH$

$$= 100 \times 8 = 800 \text{ MJ}$$

**(ii)** Using swing equation,

$$\frac{2HG}{W_s}\frac{d^2\delta}{dt^2} = (P_m - P_e)$$

$$\frac{2 \times 8 \times 100}{360 \times 50}\frac{d^2\delta}{dt^2} = 30$$

$$\frac{d^2\delta}{dt^2} = 337.5 \text{ elec-deg/sec}^2 \qquad ...(i)$$

$$\text{Acceleration, } \alpha = 337.5 \text{ elec-deg/sec}^2$$

**(iii)** Time, $t = 10$ cycle

$$= \frac{10}{50} = 0.2 \text{ sec}$$

Change in $\delta \Rightarrow$

$$\Delta\delta = \frac{1}{2}\alpha \cdot t^2 = \frac{1}{2}(337.5) \times (0.2)^2$$

$$= 6.75 \text{ ele-deg}$$

As,

$$\alpha = 337.5 \text{ elec. deg/sec}^2$$

$$= 337.5 \times \left(\frac{2}{P}\right) \text{mech-deg/sec}^2$$

$$= 337.5 \times \frac{2}{4} \times \frac{60}{360} \text{ rpm/sec}$$

$$\alpha = 28.125 \text{ rpm/sec}$$

Rotor speed at the end of 10 cycle

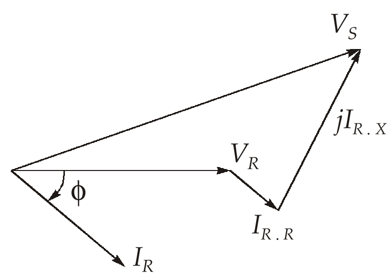
$$N_r = \frac{120 \times 50}{4} + 28.125 \times 0.2 = 1505.625 \text{ rpm}$$

**Q.1 (d) Solution:**

The analysis is done on a per phase basis,

So sending end voltage

$$V_{S\ (\text{phase})} = \frac{3300}{\sqrt{3}} = 1905.25 \text{ V}$$




$$\bar{V}_S = \bar{V}_R + \bar{I} \cdot (R + jX_S)$$

Sending end voltage can be written as

$$V_S = \sqrt{(V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX)^2} \quad \text{...(i)}$$

The current, $$I = \frac{P_R}{V_R \cos\phi_R} = \frac{300 \times 10^3}{(0.8)V_R} = \frac{375000}{V_R}$$

So equation (i) becomes,

$$(V_S)^2 = (1905.25)^2$$

$$= \left(0.8V_R + \frac{0.3 \times 375000}{V_R}\right)^2 + \left(0.6V_R + \frac{0.4 \times 375000}{V_R}\right)^2$$

Solving this,

$$(V_R)_{\text{phase}} = 1805.32 \text{ Volt}$$

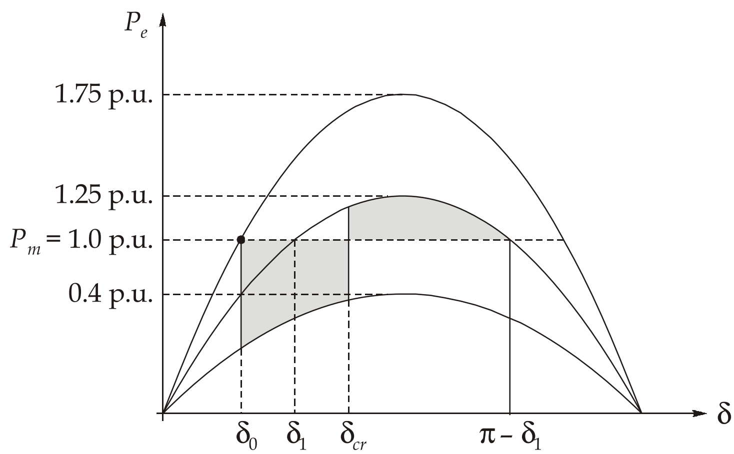
The receiving end voltage,

$$V_{R\ (\text{L-L})} = \sqrt{3} \times 1805.32 \approx 3127 \text{ V (L-L)}$$

The line current,

$$I = \frac{375000}{V_R} = \frac{375000}{1805.32} = 207.72 \text{ Amp}$$

**Q.1 (e) Solution:**




Given,

$$P_m = 1 \text{ p.u.}$$
$$P_{\text{max (pre-fault)}} = 1.75 \text{ p.u.}$$
$$P_{\text{max (during fault)}} = 0.4 \text{ p.u.}$$
$$P_{\text{max (post fault)}} = 1.25 \text{ p.u.}$$

From the power angle curve,

$$\delta_0 = \sin^{-1}\left(\frac{1}{1.75}\right) = 34.85° = 0.608 \text{ rad}$$

$$\delta_1 = \sin^{-1}\left(\frac{1}{1.25}\right) = 53.13° = 0.927 \text{ rad}$$

$$\pi - \delta_1 = 126.87° = 2.214 \text{ rad}$$

Using equal area criteria,

$$\int_{\delta_0}^{\delta_{cr}} (P_m - 0.4\sin\delta)d\delta = \int_{\delta_{cr}}^{\pi-\delta_1} (1.25\sin\delta - 1)d\delta$$

$$\int_{\delta_0}^{\pi-\delta_1} P_m \cdot d\delta = 0.4\int_{\delta_0}^{\delta_{cr}} \sin\delta \cdot d\delta + 1.25\int_{\delta_{cr}}^{\pi-\delta_1} \sin\delta \cdot d\delta$$

$$1[\pi - \delta_1 - \delta_0] = 0.4[\cos\delta_0 - \cos\delta_{cr}] + 1.25[-\cos(\pi - \delta_1) + \cos\delta_{cr}]$$

$$1.606 = 0.4[\cos 34.85° - \cos \delta_{cr}] + 1.25[-\cos (126.87) + \cos \delta_{cr}]$$

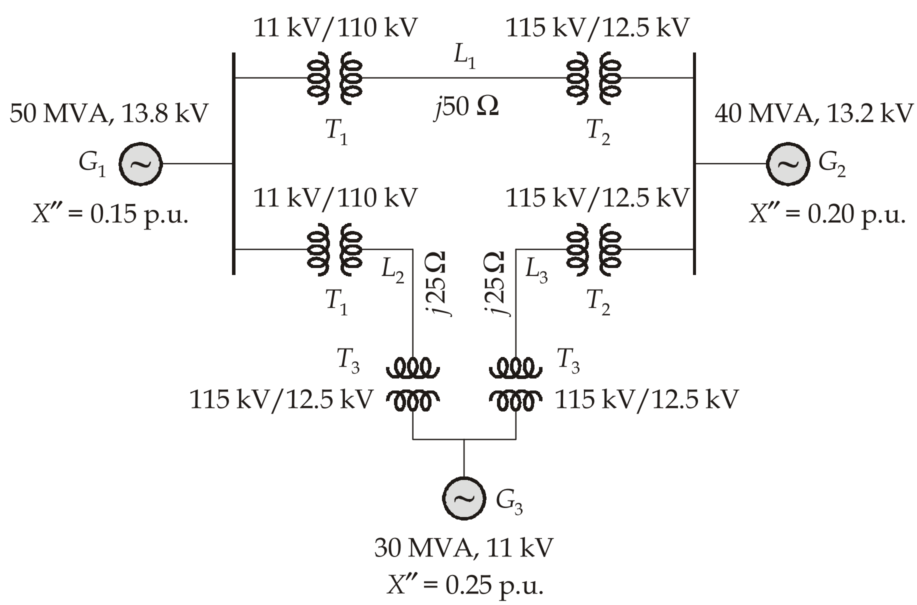
$$1.606 = 0.4[0.820 - \cos \delta_{cr}] + 1.25[0.6 + \cos \delta_{cr}]$$

$$\delta_{cr} = 0.9 \text{ rad}$$

$$= 51.566°$$

**Q.2 (a) Solution:**

The given power system can be redrawn as below :




Given : Base : $\text{MVA}_b = 50 \text{ MVA}, V_b = 13.8 \text{ kV}$

Let $\text{MVA}_{new} = 50 \text{ MVA and } V_{new} = V_b = 13.8 \text{ kV}$

For generator 1, p.u. reactance in the given base quantities = 0.15 pu

$$X_{g1} = 0.15 \text{ pu} \quad \text{...(a)}$$

To calculate the reactances in the given base quantities, apply the relation

$$(Z_{pu})_{new} = (Z_{pu})_{old} \times \frac{\text{MVA}_{new}}{\text{MVA}_{old}} \times \left(\frac{\text{KV}_{old}}{\text{KV}_{new}}\right)^2 \quad \text{...(1)}$$

For transformer 1 :

$$(Z_{pu})_{new} = (Z_{pu})_{old} \times \left(\frac{\text{MVA}_{new}}{\text{MVA}_{old}}\right) \times \left(\frac{\text{KV}_{old}}{\text{KV}_{new}}\right)^2$$

$$= 0.1 \times \left(\frac{50}{45}\right) \times \left(\frac{11}{13.8}\right)^2$$

$$X_{T1} = 0.0706 \text{ pu} \quad \text{...(b)}$$

Base voltage at $L_1$ side $(V_{bL1})$

$$\frac{V_b}{V_{b_{L1}}} = \frac{11}{110}$$

$$V_{bL1} = \frac{110}{11} \times 13.8 \text{ kV}$$

$$V_{bL1} = 138 \text{ kV}$$

Base impedance of line 1 ($L_1$)

$$Z_{BL1} = \frac{(\text{kV})^2}{\text{MVA}} = \frac{(138)^2}{50} = 380.88\ \Omega$$

$$X_{L1} = \frac{50}{380.88} = 0.1313 \text{ pu} \quad \text{...(c)}$$

For transformer $T_2$ :

$$V_{\text{base}} = 138 \text{ kV}$$

$$\text{MVA}_b = 50 \text{ MVA} = \text{MVA}_{\text{new}}$$

$$(X_{T2})_{\text{new}} = (X_{T2})_{\text{old}} \times \left(\frac{\text{MVA}_{\text{new}}}{\text{MVA}_{\text{old}}}\right)\left(\frac{\text{kV}_{\text{old}}}{\text{kV}_{\text{new}}}\right)^2$$

$$= 0.15\left(\frac{50}{25}\right)\left(\frac{115}{138}\right)^2 = 0.2083 \text{ pu} \quad \text{...(d)}$$

Base voltage on generator $G_2$ side :

$$\frac{V_{bG_2}}{V_{bL_1}} = \frac{12.5}{115}$$

$$V_{bG2} = \frac{12.5}{115} \times 138 \text{ kV}$$

$$V_{bG2} = 15 \text{ kV}$$

New pu impedance of Generator $G_2$ on $V_{\text{Base}} = 15$ kV and $\text{MVA}_B = 50$ MVA

$$(X_{G2})_{\text{new}} = (X_{G2})_{\text{old}} \times \left(\frac{\text{MVA}_{\text{new}}}{\text{MVA}_{\text{old}}}\right)\left(\frac{\text{kVA}_{\text{old}}}{\text{kV}_{\text{new}}}\right)^2$$

$$= 0.2 \times \left(\frac{50}{40}\right)\left(\frac{13.2}{15}\right)^2$$

$$(X_{G2})_{\text{new}} = 0.1936 \text{ pu} \quad \text{...(e)}$$

Now, base voltage at the side of line $L_2$

$$(V_B)_{L2} = (V_B)_{L1} = 138 \text{ kV}$$

Now, new base reactance of line $L_2$

$$X_{L2} = \frac{25}{(138)^2} \times 50$$

$$= 0.0656 \text{ pu}$$

$$\text{Base voltage of line } L_3 = \text{Base voltage of line } L_1$$

$$(V_b)_{L3} = 138 \text{ kV}$$

$$\text{Base impedance of line } L_3 = \frac{(\text{kV})^2}{\text{MVA}} = \frac{(138)^2}{50} = 380.88\ \Omega$$

pu impedance of line $L_3$

$$X_{L3} = \frac{25}{380.88}$$

$$X_{L3} = 0.0656 \text{ pu} \quad \text{...(f)}$$

New pu reactance of transformer $T_3$

$$(X_{T3})_{\text{new}} = (X_{T3})_{\text{old}} \times \left(\frac{\text{kV}_{\text{old}}}{\text{kV}_{\text{new}}}\right)^2 \left(\frac{\text{MVA}_{\text{new}}}{\text{MVA}_{\text{old}}}\right)$$

$$= 0.1 \times \left(\frac{115}{138}\right)^2 \left(\frac{50}{40}\right)$$

$$X_{T3} = 0.0868 \text{ pu} \quad \text{...(g)}$$

Base voltage at generator $G_3$ side

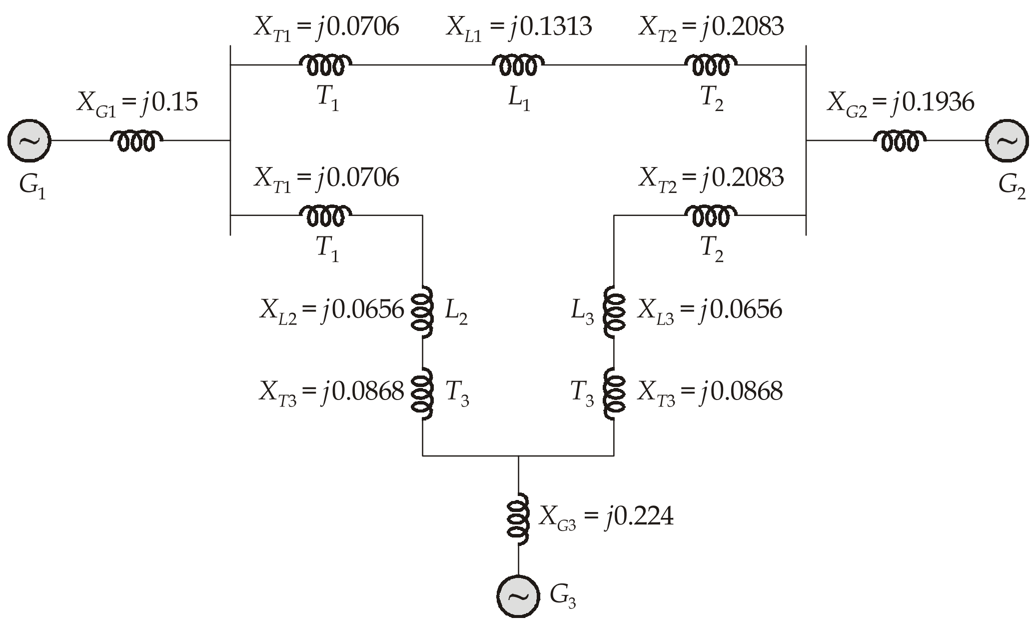
$$\frac{(V_{G3})_B}{138 \text{ kV}} = \frac{12.5}{115}$$

$$(V_{G3})_B = 15 \text{ kV}$$

$$(X_{G3})_{\text{new}} = (X_{G3})_{\text{old}} \left(\frac{\text{MVA}_{\text{new}}}{\text{MVA}_{\text{old}}}\right)\left(\frac{\text{kV}_{\text{old}}}{\text{kV}_{\text{new}}}\right)^2$$

$$(X_{G3})_{\text{new}} = 0.25\left(\frac{50}{30}\right)\left(\frac{11}{15}\right)^2 = 0.224 \text{ pu}$$

Reactance diagram of the network shown below :




**Q.2 (b) Solution:**

Swing equation describes the relative motion of rotor w.r.t. the stator field as a function of time. It is derived by assuming that windage, friction and iron losses are negligible.

Figure below shows the torque, speed and flow of mechanical and electrical power in synchronous machine,


$T_m$ = shaft torque, $T_e$ = electric torque

$\delta$ = torque angle

$T_a$ = accelerating torque

$T_a = T_m - T_e$

$P_a$ = accelerating power = $P_m - P_e$

$P_a = T_a \cdot \omega$

$$\omega = \frac{2\pi N_s}{60} \text{rad/sec}$$

$$P_a = T_a \cdot \omega = I\,\alpha\,\omega = M\,\alpha$$

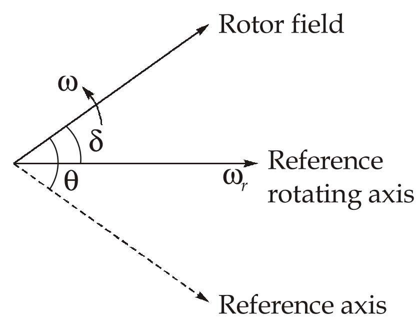
$M$ = Angular momentum in J-S/elec-rad

$\alpha \rightarrow$ acceleration in mechanical rad/sec$^2$

$$\alpha = \frac{d^2\theta}{dt^2}$$

angle $\theta$ changes continuously w.r.t. time when a sudden change occurs in system,

$$\theta = \omega_r \cdot t + \delta$$




$\omega_r$ is angular velocity of the reference synchronously rotating axis and $\delta$ is angular displacement in electrical degrees from synchronously rotating reference axis,

$$M\alpha = \frac{M d^2\theta}{dt^2} = \frac{M d^2\delta}{dt^2} = P_a = P_m - P_e$$

$$\frac{M d^2\delta}{dt^2} = P_m - P_e \qquad \text{...(i)}$$

$$\omega = \frac{2\pi N_s}{60} \text{ rad/sec}$$

Further inertia constant of a machine $H$

$$= \frac{\text{Stored energy in mega Joules}}{\text{Rating in MVA}}, G \rightarrow \text{Rating in MVA}$$

$$\text{Stored energy} = \frac{1}{2}\text{Mw} = \frac{1}{2}M(2\pi f) = M\pi f$$

Then,

$$H = \frac{M\pi f}{G}$$

$$M = \frac{GH}{\pi f} \text{ MJ-sec/radian}$$

$$= \frac{GH}{180 f} \text{ MJ-sec/degree}$$

Putting in equation (i),

$$\frac{GH}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$$

In p.u. system with $G$(MVA) as base

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = (P_m - P_e) \text{ in p.u.} \quad \text{...(ii)}$$

Equation (i) and (ii), are swing equation that describes dynamics for a synchronous machine.

**Q.2 (c) Solution:**

Given :

$$Z = (0.15 + j0.78)\ \Omega/\text{km} = 0.794\angle 79.11°\ \Omega/\text{km}$$

$$Y = j5.0 \times 10^{-6}\ \mho/\text{km} = 5 \times 10^{-6}\angle 90°\ \mho/\text{km}$$

Characteristic impedance,

$$Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{0.794\angle 79.11°}{5 \times 10^{-6}\angle 90°}} = 398.5\angle -5.45°\ \Omega \quad \text{...(1)}$$

Propagation constant,

$$\gamma = \sqrt{YZ}$$

$$= \sqrt{(5 \times 10^{-6})\angle 90° \times 0.794\angle 79.11}$$

$$\gamma = 1.99 \times 10^{-3}\angle 84.56° \quad \text{...(2)}$$

Length of the line = 400 km

$$\gamma l = 1.99 \times 10^{-3} \times 400\angle 84.56°$$

$$= 796 \times 10^{-3}\angle 84.56°$$

$$= 0.07546 + j0.7924$$

Now,

$$\sin h\gamma l = \sin h(0.07546 + j0.7924)$$

$$= \sin h(0.07546)\cos(0.7924) + j\cos h(0.07546)\sin(0.7924)$$

$$= 0.0530 + j0.7140$$

$$= 0.716\angle 85.75°$$

$$\cos h\gamma l = \cos h(0.07546 + j0.7924)$$

$$= \cos h(0.07546).\cos(0.7924) + \text{j}\sin h(0.07546)\sin(0.7924)$$

$$= 0.7041 + j0.05378$$

$$= 0.70615\angle 4.367°$$

Series impedance of 400 km line,

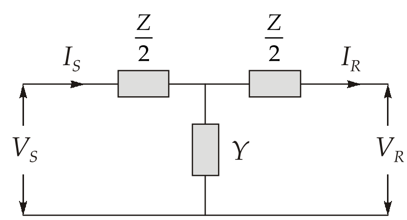
$$Z = Zl = 0.794 \times 400\angle 79.11$$

$$= 317.6\angle 79.11\ \Omega$$

Shunt admittance of 400 km line,

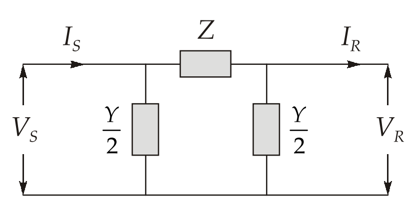
$$Y = Yl = 5 \times 10^{-6} \times 400\angle 90° \,\mho$$
$$= 2 \times 10^{-3}\angle 90° \,\mho$$

**(i) Nominal T-representation :**




$$A = 1 + \frac{YZ}{2}$$
$$= 1 + \frac{(317.6\angle 79.11)(2 \times 10^{-3}\angle 90°)}{2}$$
$$= 0.6907\angle 4.983°$$
$$B = Z\left(1 + \frac{YZ}{4}\right)$$
$$= (317\angle 79.11°)\left[1 + \frac{(317.6\angle 79.11°)(2 \times 10^{-3}\angle 90°)}{4}\right]$$
$$= 268.2426\angle 81.145° \,\Omega$$
$$C = Y = 2 \times 10^{-3}\angle 90° \,\mho$$
$$D = A = 1 + \frac{YZ}{2} = 0.6907\angle 4.983°$$

**(ii) Nominal π-Network :**




$$A = 1 + \frac{YZ}{2} = 0.6907\angle 4.983° \text{ (calculated before)}$$
$$B = Z = 317.6\angle 79.11° \,\Omega$$
$$C = Y\left(1 + \frac{YZ}{4}\right)$$

$$= 2\times10^{-3}\angle 90^\circ\left(1+\frac{(317.6\angle 79.11^\circ)(2\times10^{-3}\angle 90^\circ)}{4}\right)$$

$$C = 1.6891\times10^{-3}\angle 92.035^\circ$$

$$D = A = 0.6907\angle 4.983^\circ$$

**(iii) Exact Representation :**

$$A = \cos h\gamma l = 0.70615\angle 4.367^\circ$$

$$B = Z_c \sin h\gamma l = (398.5\angle -5.45)(0.716\angle 85.75^\circ)$$

$$= 285.326\angle 80.3^\circ$$

$$C = \frac{\sin h\gamma l}{Z_c} = \frac{0.716\angle 85.75^\circ}{398.5\angle -5.45^\circ}$$

$$= 1.796\times10^{-3}\angle 91.2\ \mho$$

$$D = A = \cos h\gamma l = 0.70615\angle 4.367^\circ$$

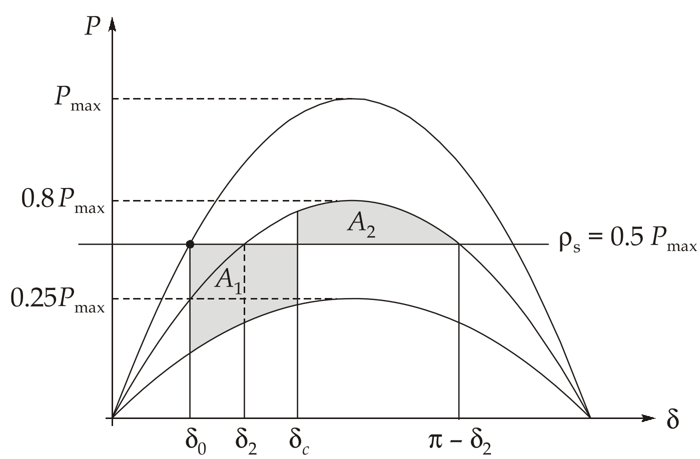
**Q.3 (a) Solution:**

**Before fault :**

$$P_e = P_{max}\sin\delta_o$$

$$0.5P_{max} = P_{max}\sin\delta_o$$

$$\delta_o = 30^\circ = 0.523\text{ rad}$$




**During fault :**

Reactance between generator and infinite bus increased to 400%.

$$P_e = \frac{P_{max}}{4}\sin\delta$$

$$P_e = 0.25P_{max}\sin\delta$$

**After fault :**

$$P_e = 0.8P_{max} \sin \delta$$

At $P_e = P_s - 0.5P_{max}$,

$$0.5P_{max} = 0.8P_{max} \sin \delta_2$$

$$\delta_2 = 38.68°$$

$$= 0.675 \text{ rad}$$

For critical clearing angle,

$$A_1 = A_2$$

$$\int_{\delta_o}^{\delta_c} (0.5P_{max} - 0.25P_{max} \sin\delta)d\delta = \int_{\delta_c}^{\pi-\delta_2} (0.8P_{max} \sin\delta - 0.5P_{max})d\delta$$

$$\int_{\delta_o}^{\pi-\delta_2} 0.5P_{max}d\delta = \left[0.25\int_{\delta_o}^{\delta_c} \sin\delta \cdot d\delta + \int_{\delta_c}^{\pi-\delta_2} 0.8\sin\delta \cdot d\delta\right]P_{max}$$

$$0.5[\pi - \delta_2 - \delta_o] = 0.25[\cos \delta_o - \cos \delta_c] + 0.8[\cos \delta_2 + \cos \delta_c]$$

$$0.5(\pi - 0.675 - 0.523) = 0.25[0.866 - \cos \delta_o] + 0.8[0.78 + \cos \delta_c]$$

$$0.9717 = 0.2165 - 0.25 \cos \delta_c + 0.624 + 0.8 \cos \delta_c$$

$$0.1312 = 0.55 \cos \delta_c$$

$$\delta_c = 76.18°$$

**Q.3 (b) Solution:**

For positive sequence network,

The base voltage on line side of transformer,

$$= 13.8 \times \frac{115}{13.2} = 120.2 \text{ kV}$$

Base voltage on motor side of transformer

$$= 120.2 \times \frac{13.2}{115} = 13.8 \text{ kV}$$

$$\text{Percent reactance of transformer} = 10 \times \left(\frac{13.2}{13.8}\right)^2 \left(\frac{30}{35}\right) = 7.8423\%$$

$$\text{Percent reactance of motor-1} = 20 \times \left(\frac{12.5}{13.8}\right)^2 \left(\frac{30}{20}\right) = 24.6\%$$

$$\text{Percentage rectacne of motor-2} = 10\times\left(\frac{12.5}{13.8}\right)^2\left(\frac{30}{10}\right) = 49.2\%$$

$$\text{Percent reactance of line} = 80\times\left(\frac{30}{120^2}\right)\times 100 = 16.7\%$$

As negative sequence network is identical to positive sequence network (except for sources).

Zero sequence network,

$$\text{Neutral reactance} = 2\times 3\times\frac{30}{(13.8)^2}\times 100 = 94.5\%$$

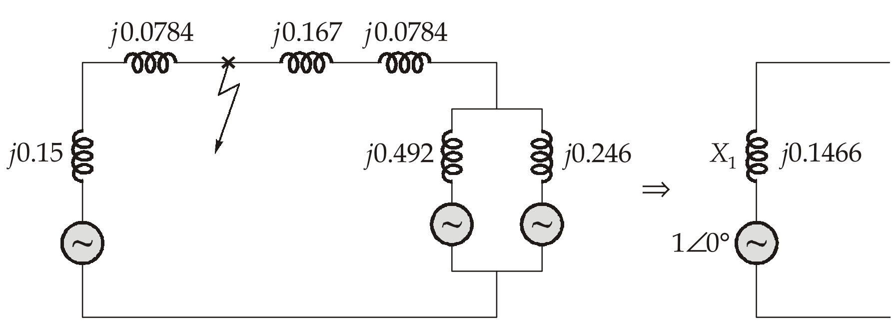
Zero sequence reactance of the line

$$= 200\times\frac{30}{(120)^2}\times 100 = 41.6\%$$

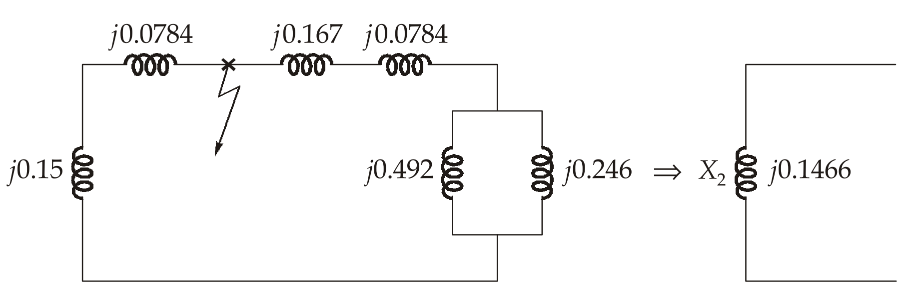
+ve sequence network Thevenin's equivalent from point '*P*'

$$\text{Zero sequence reactance motor-1} = 5\times\left(\frac{30}{20}\right)\times\left(\frac{12.5}{13.8}\right)^2 = 6.15\%$$

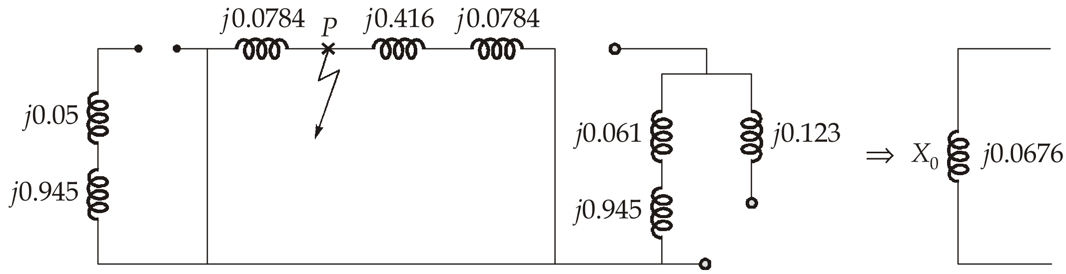
$$\text{Zero sequence reactance motor-2} = 5\times\left(\frac{30}{10}\right)\times\left(\frac{12.5}{13.8}\right)^2 = 12.30\%$$




+ve sequence network,



Zero sequence network,




**(i) L-G fault:**

The sequence network (positive, negative and zero) are connected in series,

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{j0.1466 + j0.1466 + j0.0676} = -j2.77 \text{ p.u.}$$

$$\text{Fault current } (I_f) = 3\, I_{a0}$$

$$= -j8.314 \text{ p.u.}$$

$$\text{Base current} = \frac{30 \times 10^3}{\sqrt{3} \times 13.8} = 1255 \text{ A}$$

On line side:

$$\text{Base current} = \frac{30 \times 10^3}{\sqrt{3} \times 120} = 144.3 \text{ A}$$

$$\text{Fault current} = 144.3 \times 8.314$$

$$= 1199.133 \text{ A}$$

**(ii) L-L fault:**

Only positive and negative sequence are involved,

$$I_{a1} = \frac{1}{Z_1 + Z_2} = \frac{1}{X_1 + X_2} = \frac{1}{j0.1466 + j0.1466} = -j3.41 \text{ p.u.}$$

$$\text{Fault current} = \sqrt{3}|I_{a1}|$$

$$= \sqrt{3} \times 3.41 = 5.90 \text{ p.u.}$$

$$\text{Fault current} = 5.90 \times 144.3 = 855.9 \text{ A}$$

**(iii) LLG fault:**

$$I_{a1} = \frac{1}{j0.1467 + \dfrac{j0.146 \times 0.0676}{j0.146 + j0.0676}} = -j5.2 \text{ p.u.}$$

$$I_{a2} = -I_{a1} \cdot \frac{Z_0}{Z_2 + Z_0} = \frac{(-j5.2)(j0.06767)}{j0.21367} = j1.647 \text{ p.u.}$$

As, $I_{a0} + I_{a1} + I_{a2} = 0$

$$I_{a0} = j3.553$$

$$\text{Fault current} = 3I_{a0} = 3 \times j3.553$$

$$\text{Fault current magnitude} = 3 \times 3.553 \times 144.3$$

$$= 1538 \text{ A}$$

**Q.3 (c) Solution:**

**(i)** Methods to improve string efficiency for an insulator.

1. **By using longer cross arm :** The value of string efficiency depends upon $K$ (ratio of shunt capacitance to mutual capacitance). Lesser $K$ value, greater is string efficiency. Hence longer cross arm will reduce shunt capacitance and maximize the string efficiency.

2. By grading the insulators, such that the top unit has maximum capacitance, increasing progressively as the bottom unit is reached. As voltage is inversely proportional to capacitance, this method tends to equalize potential distribution across the units in the string.

**(ii)** Given, Span length, $l = 375$ m

Weight of conductor/m length,

$$W = 0.865 \text{ kg}$$

$$\text{Conductor dia, } d = 1.96 \text{ cm}$$

$$\text{ice coating in thickness, } t = 1.27 \text{ cm}$$

$$\text{Working tension, } T = \frac{9060}{2} = 4530 \text{ kg}$$

Volume of ice/m (i.e 100 cm) length of conductor

$$= \pi\, t\, (d + t) \times 100 \text{ cm}^3$$

$$= \pi \times (1.27)\,(1.96 + 1.27) \times 100 = 1288 \text{ cm}^3$$

Wt of ice/m length of conductor is

$$Wi = 0.91 \times 1288 = 1172 \text{ gm}$$

Wind force/m length of conductor

$$W_W = (\text{Pressure}) \times (d + 2t) \times 100$$

$$= 3.9 \times (1.96 + 2 \times 1.27) \times 100 \text{ gm} = 1755 \text{ gm}$$

Total weight of conductor per meter length of conductor is

$$W_t = \sqrt{(W + W_i)^2 + (W_W)^2}$$

$$= \sqrt{(0.865 + 1.172)^2 + (1.755)^2} = 2.688 \text{ Kg}$$

$$S_{ag}, S = \frac{W_t \cdot l^2}{8T} = \frac{2.688 \times (375)^2}{4530 \times 8} = 10.430 \text{ m}$$

**Q.4 (a) Solution:**

Let '$r$' be the earthling resistance needed to leave 10% of winding unprotected,

$$V_{ph} = \frac{6.6 \times 10^3}{\sqrt{3}} = 3810.5 \text{ V}$$

Full load current, $$I = \frac{10 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 874.77 \text{ A}$$

Let reactance per phase be '$x$' ohms

$$Z_{base} = \frac{\text{kV}^2}{\text{MVA}} = 4.356\ \Omega$$

$$x = Z_{Base} \times (x)_{p.u.} = 4.356 \times 0.2 = 0.8712\ \Omega$$

Reactance of 20% winding $= 0.2 \times 0.8712 = 0.17424\ \Omega$

Voltage of 20% winding $= V_{ph} \times 0.2 = 3810.5 \times 0.2$

$= 762.1$ V

Impedance offered to fault by 20% winding

$$Z_f = \sqrt{(0.17424)^2 + r^2}$$

Earth fault due to 20% winding $= \dfrac{762.1}{Z_f}$

When this fault current, between 170 A, the relay will trip

$$\frac{762.1}{Z_f} = 170$$

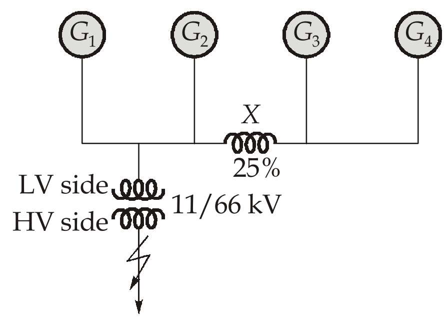
$$\frac{762.1}{\sqrt{(0.1742)^2 + r^2}} = 170$$

$$(0.17424)^2 + r^2 = \left(\frac{762.1}{170}\right)^2$$

$$r = 4.48\ \Omega$$

**Q.4 (b)**

The single line diagram is as shown:




Let the base MVA chosen be 20 and base kV be 11 kV on generator side and 66 kV on feeder side. Per unit reactance of each generator,

$$X_{g\text{ p.u.}} = 0.20 \text{ p.u.}$$

Per unit reactance of transformer on new base,

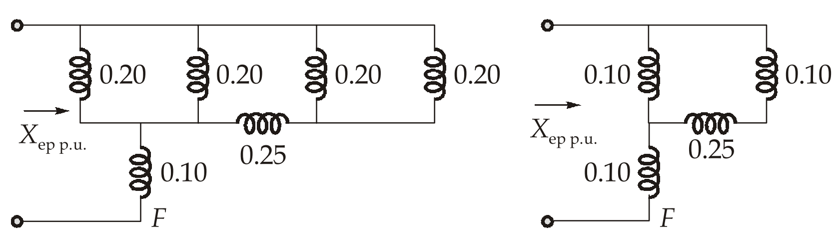
$$X_{t\text{ p.u.}} = 0.075 \times \frac{20}{15} = 0.1 \text{ p.u.}$$

Per unit reactance of the reactor,

$$X_{\text{p.u.}} = 0.25 \text{ p.u.}$$

The fault occurs at point *F* on hv side of the transformer pre-fault voltage at *F*,

$$V_{f\text{ pu}} = \frac{66}{66} = 1.0\angle 0°$$




The equivalent reactance is given by,

$$X_{eq\text{ p.u.}} = 0.1 + \frac{0.10(0.25+0.10)}{(0.10+0.25+0.10)} = 0.1778 \text{ p.u.}$$

$$\text{Thevenin's voltage} = V_{f\text{ p.u.}} = 1.0$$

The per unit fault current, $I_{f\text{ p.u.}} = \dfrac{V_{f\text{ p.u.}}}{X_{eq\text{ p.u.}}} = \dfrac{1.0}{0.1778} = 5.624$ p.u.

Base current on h.v. side, $I_b = \frac{(\text{MVA})_b \times 10^3}{\sqrt{3} \times (\text{kV})_b} = \frac{20 \times 10^3}{\sqrt{3} \times 66} = 175$ A

Actual current, $I_f = I_{f\,\text{p.u.}} \times I_b = 5.624 \times 175 = 984.251$ A

**Q.4 (c) Solution:**

Let $V$ be the operating voltage and $C$ be the capacitance to the ground.

Given, number of insulation unit,

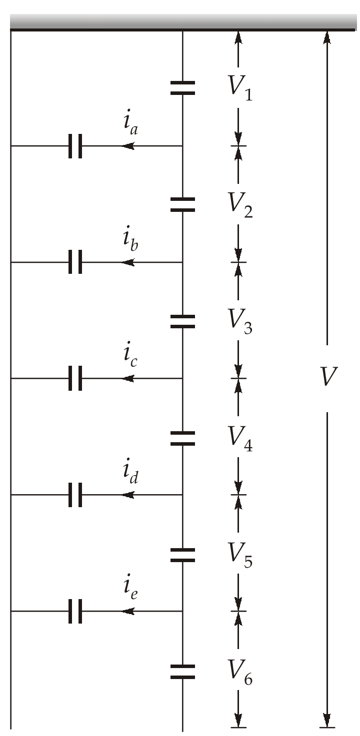
$$n = 6$$

According to question, mutual capacitance

$$= 10 \times \text{Capacitance to ground} = 10C$$

Hence, $$m = \frac{\text{Mutual capacitance}}{\text{Capacitance to the ground}} = \frac{10C}{C} = 10$$

Let $V_1$, $V_2$, $V_3$, $V_4$, $V_5$ and $V_6$ be the voltages across the different units as shown in figure below :




To calculate the voltage across different units, apply the relation :

$$V_{n+1} = \sum_{1}^{n} \frac{V}{m} + V_n \qquad ...(1)$$

where $n = 1, 2, 3, 4, 5$ and 6

Now,
$$V_2 = \frac{V_1}{m} + V_1$$
$$= \frac{V_1}{10} + V_1 = \frac{11V_1}{10} = 1.1V_1 \quad ...(2)$$

The voltage across the third unit is given by :

$$V_3 = \frac{V}{m} + V_2 = \frac{V_1 + V_2}{10} + 1.1V_1 = \frac{V_1 + 1.1V_1}{10} + 1.1V_1$$
$$= 0.21V_1 + 1.1V_1$$
$$V_3 = 1.31V_1 \quad ...(3)$$

Similarly,
$$V_4 = \frac{V}{m} + V_3 = \frac{V_1 + V_2 + V_3}{10} + V_3$$
$$= \frac{V_1 + 1.1V_1 + 1.31V_1}{10} + 1.31V_1$$
$$V_4 = 1.651V_1 \quad ...(4)$$
$$V_5 = \frac{V_1 + V_2 + V_3 + V_4}{m} + V_4$$
$$= \frac{V_1 + 1.1V_1 + 1.31V_1 + 1.651V_1}{10} + 1.651V_1$$
$$V_5 = 2.157V_1 \quad ...(5)$$
$$V_6 = \frac{V_1 + V_2 + V_3 + V_4 + V_5}{m} + V_5$$
$$= \frac{V_1 + 1.1V_1 + 1.31V_1 + 1.651V_1 + 2.157V_1}{10} + 2.157V_1$$
$$V_6 = 2.879V_1$$

Voltage across the string,

$$V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$$
$$= V_1 + 1.1V_1 + 1.31V_1 + 1.651V_1 + 2.157V_1 + 2.879V_1$$
$$V = 10.097V_1 \quad ...(6)$$

From eqn. (6),

$$V_1 = 0.099 \text{ V}$$

Thus, voltage across topmost unit,

$$V_1 = 0.099 \text{ V}$$

Similarly,
$$V_2 = 1.1V_1 = 1.1 \times 0.099 \text{ V}$$
$$V_2 = 0.109 \text{ V}$$
$$V_3 = 1.31V_1 = 1.31 \times 0.099 \text{ V}$$
$$V_3 = 0.130 \text{ V}$$
$$V_4 = 1.651V_1 = 1.651 \times 0.099 \text{ V} = 0.163 \text{ V}$$
$$V_5 = 2.157V_1 = 2.157 \times 0.099 \text{ V} = 0.2135 \text{ V}$$
$$V_6 = 2.879V_1 = 2.879 \times 0.099 \text{ V} = 0.285 \text{ V}$$

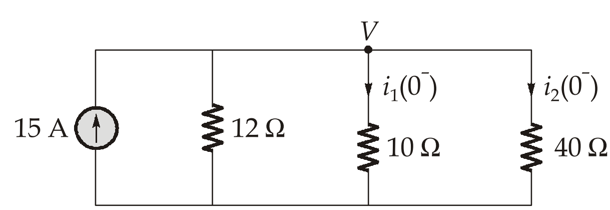
As we know that the voltage across the unit nearest to the conductor is maximum
$$V_6 = \text{Maximum unit voltage} = 0.285 \text{ V}$$

Now, % string efficiency
$$= \frac{\text{Operating voltage}}{n \times \text{maximum voltage across the unit nearest to conductor}} \times 100\%$$
$$= \frac{V}{6 \times 0.285V} \times 100\% = 58.48\%$$

**Section B : Systems and Signal Processing-1 + Microprocessor-1 + Electrical Circuits-2 + Control Systems-2**

**Q.5 (a) Solution:**

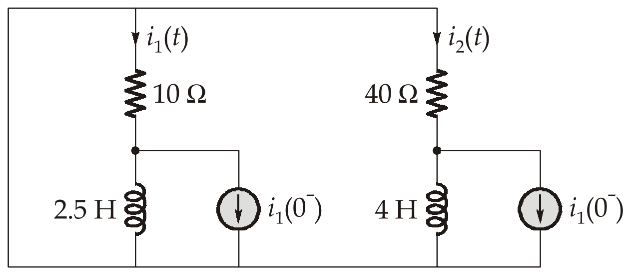
Switch is closed at $t = 0$, for $t < 0$ inductor acts like short circuit. Hence, at $t = 0-$ the equivalent circuit becomes as below :



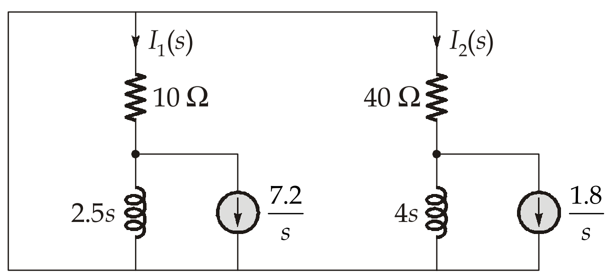

Assuming node voltages as $V$ as shown above. In the above circuit 12 Ω, 10 Ω and 40 Ω are in parallel.
$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{10} + \frac{1}{40}$$
$$R_{eq} = 4.8 \ \Omega$$
$$V = 4.8 \times 15 = 72 \text{ V}$$
$$\therefore \quad i_1(0^-) = \frac{V}{10} = \frac{72}{10} = 7.2 \text{ A}$$

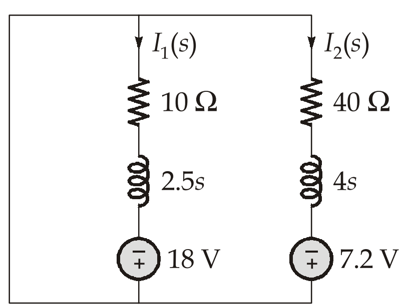
$$i_2(0^-) = \frac{V}{40} = \frac{72}{40} = 1.8 \text{ A}$$

The equivalent of the given circuit for $t > 0$ is :




The above circuit can be given in Laplace domain as shown below :




$$I_1(s) = \frac{0-(-18)}{2.5s+10} = \frac{18}{2.5s+10} = \frac{7.2}{s+4}$$

$$I_2(s) = \frac{0-(-7.2)}{4s+40} = \frac{7.2}{4s+40} = \frac{1.8}{s+10}$$

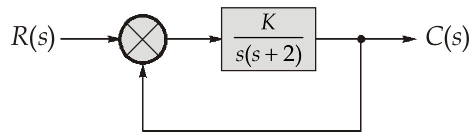
By taking inverse laplace transform of the above two equations, we get

$$i_1(t) = 7.2e^{-4t}u(t) \text{ A}$$

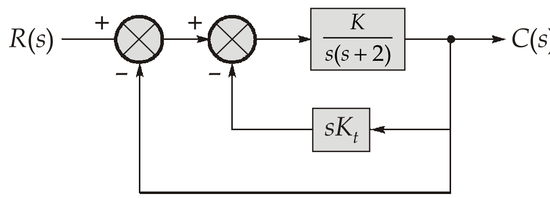
$$i_2(t) = 1.8e^{-10t}u(t) \text{ A}$$

**Q.5 (b) Solution:**

The block diagram of uncompensated system is drawn.




With tachometer feedback the block is redrawn as shown :




The overall transfer function of the compensated system is

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + (2 + KK_t)s + K}$$

Since $M_p = 25\%$

$$\therefore \quad 0.25 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\therefore \quad \ln 0.25 = \frac{-\pi\xi}{\sqrt{1-\xi^2}} \ln e$$

$$\therefore \quad -1.38 = \frac{-\pi\xi}{\sqrt{1-\xi^2}} \times 1$$

$$\therefore \quad \xi = 0.4$$

$$t_p = 1 \text{ sec}$$

$$\because \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\therefore \quad 1 = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\therefore \quad \omega_n\sqrt{1-\xi^2} = \pi$$

$$\omega_n = \frac{\pi}{\sqrt{1-\xi^2}} = \frac{\pi}{\sqrt{1-(0.4)^2}} = 3.42 \text{ rad/sec}$$

The characteristic equation for the compensated system is

$$s^2 + (2 + KK_t)s + K = 0$$

From the characteristic equation, it is noted that

$$\omega_n = \sqrt{K}$$

and $$2\xi\omega_n = 2 + KK_t$$

$\therefore$ $$3.42 = \sqrt{K}$$

$$K = (3.42)^2 = 11.7$$

$$2\xi\omega_n = 2 + KK_t$$

$\therefore$ $$2 \times 0.4 \times 3.42 = (2 + 11.7K_t)$$

$$K_t = 0.065$$

**Q.5 (c) Solution:**

From the given block diagram,

$$G(s) = \frac{4(K_P + K_D s)}{s(s+4)} \text{ and } H(s) = 1$$

The steady state error for unit ramp input is defined as

$$e_{ss} = \frac{1}{K_V} = \frac{1}{\lim_{s\to 0} sG(s)H(s)}$$

$$0.20 = \frac{1}{\lim_{s\to 0} s \times \frac{4(K_P + K_D s)}{s(s+4)}}$$

or $$5 = \frac{4(K_P)}{0+4}$$

$$K_P = 5$$

Now, the closed loop transfer function of the system is

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{4(K_P + K_D s)}{s(s+4) + 4(K_P + K_D s)}$$

For $K_P = 5$ :

$$T(s) = \frac{4(5 + K_D s)}{s(s+4) + 4(5 + K_D s)} = \frac{20 + 4K_D s}{s^2 + 4s + 20 + 4K_D s}$$

$$= \frac{20 + 4K_D s}{s^2 + (4 + 4K_D)s + 20}$$

On comparing the characteristic equation with the standard second order equation, we get

$$\omega_n = \sqrt{20}$$

and

$$2\xi\omega_n = 4 + 4K_D$$

$$2 \times \sqrt{20} \times 0.75 - 4 = 4K_D$$

$$K_D = 0.677$$

$$K_P = 5$$

**Q.5 (d) Solution:**

**(i)**

$$X(\omega) = j\delta\left(\omega - \frac{\pi}{3}\right) + 2\delta\left(\omega - \frac{\pi}{7}\right)$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\left(\frac{\pi}{3}\right)} = 6$$

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\left(\frac{\pi}{7}\right)} = 14$$

Time period, $T = \text{LCM of } (T_1 \text{ and } T_2) = 42$

Fundamental frequency, $\omega_o = \frac{2\pi}{T} = \frac{\pi}{21}$ rad

or $f_o = \frac{1}{T} = \frac{1}{42}$ Hz

**(ii)**

$$\omega_o = \frac{\pi}{21}$$

$X(\omega)$ can be rewritten as,

$$X(\omega) = 2\pi\left\{X(7)\delta\left(\omega - \frac{7\pi}{21}\right) + X(3)\delta\left(\omega - \frac{3\pi}{21}\right)\right\}$$

Comparing with $X(\omega) = 2\pi \sum_{K=-\infty}^{\infty} X_K\, \delta(\omega - K\omega_0)$

We have

$$X[K] = \begin{cases} \frac{j}{2\pi}, & K = 7 \\ \frac{1}{\pi}, & K = 3 \\ 0, & \text{otherwise} \end{cases}$$

**(iii)**
$$x(t) = \sum_{K=-\infty}^{\infty} X[K]e^{jK\omega_o t}$$
$$= \frac{j}{2\pi}e^{\frac{j\pi}{3}t} + \frac{1}{\pi}e^{j\frac{\pi}{7}t}$$

**Q.5 (e) Solution:**

**(i)** **XCHG :** On execution of this instruction the content of HL register pair are exchanged with DE register pair.

It requires 1 byte, 1 machine cycle and 4T states. No flags are effected.

**(ii)** **IN :** Using this instruction, the data from input device having port address given in the instruction will be copied to the accumulator.

- It is a 2-byte instruction.
- It requires 3-machine cycle and 10T states.
- No flags are affected.

**(iii)** **OUT :** Using this instruction the contents of accumulator will be copied to into the output device having port address given in the instruction.

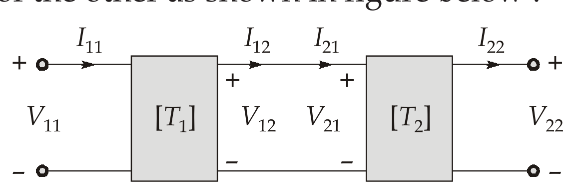
- It is a 2-byte instruction.
- It requires 3-machine cycle and 10T states.
- No flags are affected.

**(iv)** **DAA :** Decimal adjust accumulator. This instruction converts the binary number in accumulator to BCD number.

- It is 1-byte instruction.
- It requires 1 machine cycle and 4T states.
- All flags are affected.

**Q.6 (a) Solution:**

**(i)** Two 2-port network are said to be connected in cascade when the output port of one is the input port of the other as shown in figure below :




By the definition of transmission parameters

For 1st network, $$\begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{12} \\ I_{12} \end{bmatrix} \quad ...(i)$$

For 2[nd] network, $$\begin{bmatrix} V_{21} \\ I_{21} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{22} \\ I_{22} \end{bmatrix} \quad \text{...(ii)}$$

$$\begin{bmatrix} V_{21} \\ I_{21} \end{bmatrix} = \begin{bmatrix} V_{12} \\ I_{12} \end{bmatrix}$$

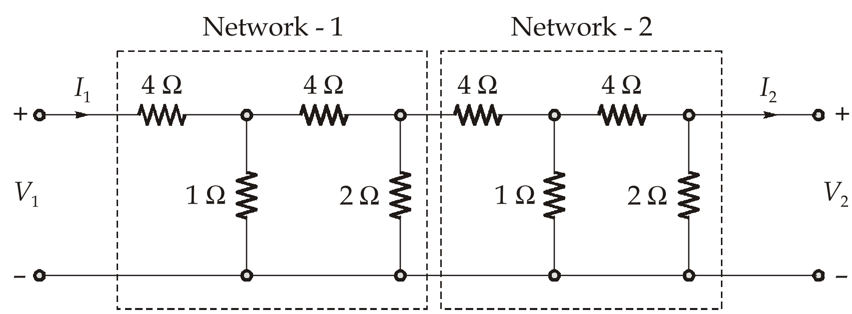
Substituting equation (ii) in (i),

$$\begin{bmatrix} V_{11} \\ I_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{22} \\ I_{22} \end{bmatrix}$$

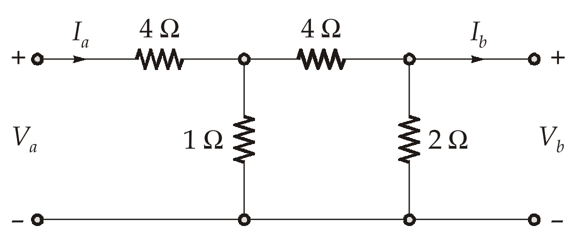
$$[T]_{\text{overall}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Thus, transmission parameters or *ABCD* parameter matrix of the overall two port network is equal to the product of the matrices of *ABCD* parameters of the individual network.

(ii)




Here, network 1 and network 2 are identical and cascaded.

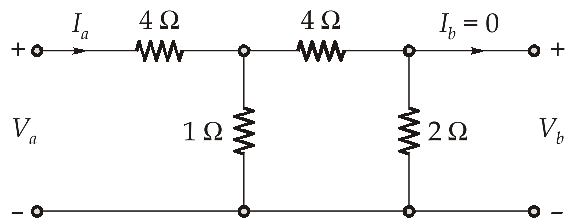



$$\begin{bmatrix} V_a \\ I_a \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_b \\ I_b \end{bmatrix}$$

$$V_a = A_a V_b + B_a I_b$$

$$I_a = C_a V_b + D_a I_b$$

**Case I :** Open circuiting the output port, i.e., $I_b = 0$.




Current Ic through 4 Ω and 2 Ω resistor

$$= I_a \times \frac{1}{1+2+4} = \frac{I_a}{7}$$

$$V_b = \frac{I_a}{7} \times 2 = \frac{2}{7} I_a$$

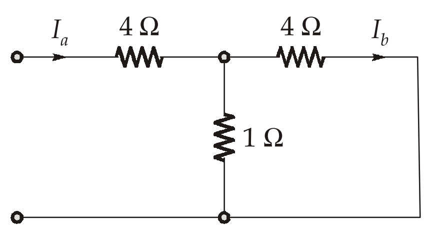
$$C_a = \frac{I_a}{V_b} = 3.5\ \mho$$

$$V_a = 4I_a + I_c(4+2) = 4I_a + \frac{I_a}{7}(6)$$

$$V_a = \frac{34}{7} I_a$$

$$A_a = \frac{V_a}{V_b} = \frac{\frac{34}{7} I_a}{\frac{2}{7} I_a} = 17$$

**Case II :** Short circuiting the output port, i.e., $V_b = 0$.




$$I_b = \frac{I_a \times 1}{1+4} = \frac{I_a}{5}$$

$$V_a = I_a\left(4 + \frac{1 \times 4}{1+4}\right) = 4.8 I_a = 4.8 \times 5 \times I_b$$

$$B_a = \frac{V_a}{I_b} = 24\ \Omega$$

$$D_a = \frac{I_a}{I_b} = 5$$

Since, $T_1$ and $T_2$ are identical networks

$$T_1 = T_2 = \begin{bmatrix} 17 & 24 \\ 3.5 & 5 \end{bmatrix}$$

$$[T] = [T_1][T_2] = \begin{bmatrix} 17 & 24 \\ 3.5 & 5 \end{bmatrix}\begin{bmatrix} 17 & 24 \\ 3.5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 373 & 528 \\ 77 & 109 \end{bmatrix}$$

**Q.6 (b) (i) Solution:**

**1.** Similarities between JUMP and CALL :

1. Both change the sequence of program execution.
2. Both are 3 bytes instructions.
3. They do not have any effect on flags.

Difference between JUMP and CALL :

1. The contents of PC are transferred to stack by CALL but not by JUMP.
2. JUMP requires 10T states while CALL requires 18T states.
3. JUMP is used for branching while CALL is used for subroutines.
4. A CALL instruction has to be followed by a RETURN instruction. There is no such compulsion for JUMP.

**2.** Similarities between STA and STAX :

1. Both transfer contents of accumulator to a memory location.
2. They do not affect any flags.

Difference between STA and STAX :

1. STA uses direct addressing mode while STAX uses indirect addressing mode.
2. STA is 3 bytes while STAX is 1 byte.
3. STA needs 13 T-states while STAX needs 7 T-states.

**Q.6 (b) (ii) Solution:**

Assume 16-bit number at memory locations 2501H (MSB) and 2502H (LSB) and complemented value at 2503H and 2504H.

```
LXI   H, 2501H  ; Load HL pair with an address 2501 H
MOV   A, M      ; Data stored at location indicated by HL pair moved into
                  accumulator
CMA             ; Complement the data stored in accumulator
```

| | | |
|---|---|---|
| STA | 2503 H | ; Store the data of accumulator at address 2503 H |
| INX | H | ; Increment the content of HL pair |
| MOV | A, M | ; Data stored at location indicated by HL pair moved into accumulator. |
| CMA | | ; Complement the data stored in accumulator |
| INR | A | ; Increment the content stored in accumulator |
| STA | 2504 H | ; Store the data of accumulator at address 2504 H |
| JNC | HALF | ; Jump to end of programm in case of no carry |
| INX | H | ; HL pair now points to 2503 H |
| MOV | A, M | ; MSB again brougth into accumulator |
| INR | A | ; Increment accumulator |
| STA | 2503 H | ; stare the data of accumulator of address 2503 H. (MSB) |
| HALT | HLT | ; stop |

**Q.6 (c) Solution:**

Consider the given signal

$$x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}u(2t-n)$$

for $x(t)$ to be periodic,

$$x(t+T) = x(t)$$

$$x(t+T) = \sum_{n=-\infty}^{\infty} e^{-(2(t+T)-n)}u(2(t+T)-n)$$

$$= \sum_{n=-\infty}^{\infty} e^{-(2t+2T-n)}u(2t+2T-n)$$

Let $2T - n = -m$

which also yields, $m = -\infty$

as $n = -\infty$

and $m = \infty$ as $n = \infty$

$$x(t+T) = \sum_{m=-\infty}^{\infty} e^{-(2t-m)}u(2t-m) = x(t)$$

Thus, $x(t)$ is periodic if

$$2T - n = -m$$

$$T = \frac{n-m}{2}$$

Thus, the fundamental time period is

$$T = \frac{1}{2}$$

$$u(2t-n) = 1$$

For $t > \frac{n}{2}$

$$x(t) = \sum_{n=-\infty}^{\infty} e^{-2t}e^{n}u(2t-n)$$

Let us now define $x(t)$ over one time period $0 < t < \frac{1}{2}$

$$x(t) = \sum_{n=-\infty}^{0} e^{-(2t-n)}, 0 < t < \frac{1}{2}$$

$$= e^{-2t}\sum_{n=-\infty}^{0} e^{n} = e^{-2t}\sum_{n=0}^{\infty}(e^{-1})^{n}$$

$$x(t) = \frac{e^{-2t}}{1-e^{-1}}; \quad 0 < t < \frac{1}{2}$$

The average power contained in $x(t)$ is given by

$$P_x = \frac{1}{T}\int_0^T |x(t)|^2\, dt$$

$$= 2\int_0^{1/2}\left[\frac{e^{-2t}}{1-e^{-1}}\right]^2 dt$$

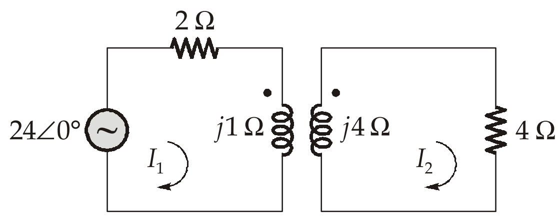
$$= \frac{2}{(1-e^{-1})^2}\int_0^{1/2} e^{-4t}dt$$

$$= \frac{2}{(1-e^{-1})^2}\left|\frac{e^{-4t}}{-4}\right|_0^{1/2} = \frac{2(1-e^{-2})}{4(1-e^{-1})^2}$$

$$= \frac{1}{2}\cdot\frac{(1-e^{-1})(1+e^{-1})}{(1-e^{-1})^2} = \frac{1}{2}\left(\frac{1+e^{-1}}{1-e^{-1}}\right)$$

**Q.7 (a) Solution:**

The frequency domain equivalent is shown below :




$$X_1 = 2\pi f L_1 = 1\ \Omega$$
$$X_2 = 2\pi f L_2 = 4\ \Omega$$

From the data on mutual inductance,

$$M = K\sqrt{L_1L_2} = \sqrt{(3.185)(12.74)} \qquad (\because\ K = 1)$$
$$M = 6.37\ \text{mH}$$
$$X_M = 2\pi f M = 2\ \Omega$$

Mesh equation for network are :

$$(2 + j1)I_1 - j2I_2 = 24\angle 0° \qquad \text{...(i)}$$
$$-2jI_1 + (4 + 4j)I_2 = 0 \qquad \text{...(ii)}$$

Solving (i) and (ii) gives

$$I_1 = 9.41\angle{-11.31°}\ \text{A}$$
$$I_2 = 3.33\angle 33.69°\ \text{A}$$
$$\therefore \quad i_1(t) = 9.41\cos(314t - 11.31°)\ \text{A}$$
$$\therefore \quad i_2(t) = 3.33\cos(314t + 33.69°)\ \text{A}$$

At $t = 5$ msec

$$314t = (314) \times (5 \times 10^{-3}) = 1.57 = 90°$$
$$i_1(t = 5\ \text{msec}) = 9.41\cos(90 - 11.31°) = 1.845\ \text{A}$$
$$i_2(t = 5\ \text{msec}) = 3.33\cos(90 + 33.69°) = -1.847\ \text{A}$$

Therefore, energy stored in mutually coupled inductor,

$$W(t)|_{t\,=\,0.005\ \text{sec}} = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - Mi_1i_2$$
$$= \frac{1}{2}(3.185 \times 10^{-3})(1.845)^2 + \frac{1}{2}(12.74 \times 10^{-3})(-1.847)^2 - (6.37 \times 10^{-3}) \times (1.845)(-1.847)$$
$$= 5.42 \times 10^{-3} + 21.73 \times 10^{-3} + 21.47 \times 10^{-3}$$
$$W(t) = 0.005\ \text{sec}) = 48.62\ \text{mJ}$$

**Q.7 (b) Solution:**

Eigen values :

$$[\lambda I - A] = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & -1 & 0 \\ -3 & \lambda & -2 \\ 12 & 7 & \lambda+6 \end{bmatrix}$$

Characteristic equation is

$$\lambda[\lambda(\lambda + 6) - (7)(-2)] - (-1)[-3(\lambda + 6) - (-2)(12)] = 0$$

$$(\lambda + 1)(\lambda + 2)(\lambda + 3) = 0$$

∴ Eigen values are

$$\lambda = -1, -2, -3$$

Corresponding to $\lambda = -1$, the eigen vector is

$$\begin{bmatrix} -1 & -1 & 0 \\ -3 & -1 & -2 \\ 12 & 7 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

i.e., $$x_1 + x_2 = 0$$

$$x_2 = -x_1$$

$$3x_1 + x_2 + 2x_3 = 0$$

or $$x_3 = -x_1$$

$$12x_1 + 7x_2 + 5x_3 = 0$$

If $x_1 = 1$, then $x_2$ and $x_3$ are –1.

Therefore, eigen vector is $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$.

Corresponding to $\lambda = -2$, the eigen vector is

$$\begin{bmatrix} -2 & -1 & 0 \\ -3 & -2 & -2 \\ 12 & 7 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

i.e., $$2x_1 + x_2 = 0 \quad \text{or } x_2 = -2x_1$$

$$3x_1 + 2x_2 + 2x_3 = 0$$

or $$x_3 = \frac{1}{2}x_1$$

$$12x_1 + 7x_2 + 4x_3 = 0$$

If $x_1 = 2$, then $x_2 = -4$ and $x_3 = 1$

therefore, the eigen vector is $\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = -3$, the eigen vector is

$$\begin{bmatrix} -3 & -1 & 0 \\ -3 & -3 & -2 \\ 12 & 7 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$3x_1 + x_2 = 0$$

or $$x_2 = -3x_1$$

$$3x_1 + 3x_2 + 2x_3 = 0 \quad \text{or} \quad x_3 = 3x_1$$

$$12x_1 + 7x_2 + 3x_3 = 0$$

If $x_1 = 1$, then $x_2 = -3$ and $x_3 = 3$.

Therefore, the eigen vector is $\begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$

Modal matrix is $\begin{bmatrix} 1 & 2 & 1 \\ -1 & -4 & -3 \\ -1 & 1 & 3 \end{bmatrix}$.

$$\text{adj}\,[M] = \begin{bmatrix} +(-12+3) & -(6-1) & +(-6+4) \\ -(-3-3) & +(3+1) & -(-3+1) \\ +(-1-4) & -(1+2) & +(-4+2) \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -5 & -2 \\ 6 & 4 & 2 \\ -5 & -3 & -2 \end{bmatrix}$$

$$|M| = \{(-4-3)-(-3\times 1)\} - 2\{(-1\times 3)-(-1\times -3)\} + 1\{(-1\times -1)-(-4\times -1)\} = -2$$

$$\therefore \quad M^{-1} = -\frac{1}{2}\begin{bmatrix} -9 & -5 & -2 \\ 6 & 4 & 2 \\ -5 & -3 & -2 \end{bmatrix}$$

$$M^{-1}AM = -\frac{1}{2}\begin{bmatrix} -9 & -5 & -2 \\ 6 & 4 & 2 \\ -5 & -3 & -2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ -1 & -4 & -3 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$M^{-1}B = -\frac{1}{2}\begin{bmatrix} -9 & -5 & -2 \\ 6 & 4 & 2 \\ -5 & -3 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} -13 \\ 10 \\ -9 \end{bmatrix} = \begin{bmatrix} \frac{13}{2} \\ -5 \\ \frac{9}{2} \end{bmatrix}$$

Therefore, canonical representation is

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} \frac{13}{2} \\ -5 \\ \frac{9}{2} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

**Q.7 (c) Solution:**

For given system,

Overall closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{\frac{16(1+K_d s)}{s^2+1.6s}}{1+\frac{16(1+K_d s)}{s^2+1.6s}} = \frac{16(1+K_d s)}{s^2+(1.6+16K_d)s+16}$$

Using characteristic equation :

$$1 + G(s)H(s) = 0$$

$$\omega_n^2 = 16$$

$$\omega_n = 4 \text{ rad/sec}$$

Also, $2\,\xi\omega_n = 1.6 + 16\,K_d$ (Given, $\xi = 0.8$)

$$2(0.8)4 = 1.6 + 16\,K_d$$

$$K_d = 0.3$$

If the input is unit step input,

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{s}\cdot\frac{16(1+0.3s)}{s^2+6.4s+16}$$

$$= \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2+6.4s+16}$$ ...(By partial fraction)

$$K_1 = \frac{16}{16} = 1$$

and $16(1 + 0.3s) = (s^2 + 6.4s + 16) + (K_2 s + K_3)s$

$$K_2 + 1 = 0$$

$$K_2 = -1$$

$$K_3 + 6.4 = 4.8$$

$$K_3 = -1.6$$

$\therefore$

$$C(s) = \frac{1}{s} + \frac{-s-1.6}{s^2+6.4s+16} = \frac{1}{s} + \frac{-s-1.6}{(s+3.2)^2+5.76}$$

$$= \frac{1}{s} - \frac{s+3.2}{(s+3.2)^2+5.76} + \frac{1.6}{(s+3.2)^2+5.76}$$

Taking inverse Laplace transform of above equation,

$$c(t) = 1 - \cos 2.4t(e^{-3.2t}) + \frac{1.6}{2.4}\sin 2.4t(e^{-3.2t})$$

$$= 1 - e^{-3.2t}[\cos 2.4t - 0.667 \sin 2.4t]$$

Now differentiating above expression w.r.t. time,

$$\frac{dc(t)}{dt} = 0 - [(e^{-3.2t})(-2.4 \sin 2.4t - 0.667 \times 2.4 \cos 2.4t)$$

$$+ (\cos 2.4t - 0.667 \sin 2.4t)(-3.2\, e^{-3.2t})]$$

$$= -e^{-3.2t}[(-2.4 \sin 2.4t - 1.60 \cos 2.4t) - (3.2 \cos 2.4t - 2.13 \sin 2.4t)]$$

$$= e^{-3.2t}[-0.27 \sin 2.4t - 4.8 \cos 2.4t]$$

For obtaining the peak time,

$$\frac{dc(t)}{dt} = 0$$

$$-0.27 \sin 2.4\, t_p - 4.8 \cos 2.4\, t_p = 0$$

$$\tan 2.4\, t_p = \frac{4.8}{0.27} = -17.07$$

$$t_p = \frac{\tan^{-1}(-17.07)}{2.4} = \frac{\pi - \tan^{-1}(17.07)}{2.4}$$

$$t_p = 0.678 \text{ sec}$$

For maximum value $c(t)_{max}$

$$t = t_p = 0.678$$

$$c(t)_{max} = 1 - e^{-3.2 \times 0.678}[\cos(2.4 \times 0.678) - 0.667 \sin(2.4 \times 0.678)]$$

$$= 1 - 0.114 [\cos 1.6272 - 0.667 \sin 1.6272)]$$

$$= 1 - 0.114 [-0.0563 - 0.667 \times 0.9984]$$

$$= 1 - 0.114 [-0.722] = 1.0823$$

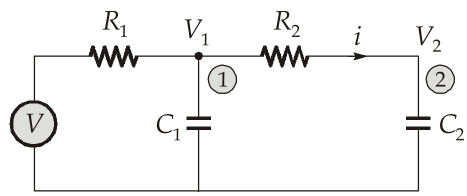
% maximum overshoot, for unit step input

$$= \frac{[c(t)_{max} - c(t)_{final}]}{c(t)_{final}} \times 100$$

$$= \frac{[c(t)_{max} - 1]}{1} \times 100 = 8.23\%$$

**Q.8 (a) Solution:**

**(i)** Given electrical network




Applying Kirchhoff's current law at node 1 and 2,

$$\frac{V_1 - V}{R_1} + C_1\frac{dV_1}{dt} + \frac{V_1 - V_2}{R_2} = 0 \quad \text{...(i)}$$

$$\frac{V_2 - V_1}{R_2} + C_2\frac{dV_2}{dt} = 0 \quad \text{...(ii)}$$

$$i = \frac{V_1 - V_2}{R_2} \quad \text{...(iii)}$$

Rearranging (i), (ii) and (iii),

$$\frac{dV_1}{dt} = \frac{V}{C_1R_1} + \frac{V_2}{C_2R_2} - \frac{V_1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{dV_2}{dt} = \frac{V_1}{C_2R_2} - \frac{V_2}{C_2R_2}$$

$$i = \frac{V_1}{R_2} - \frac{V_2}{R_2}$$

Putting $V_1 = x_1$ and $\frac{dV_1}{dt} = \dot{x}_1$

$V_2 = x_2$ and $\frac{dV_2}{dt} = \dot{x}_2$

Therefore,

$$\dot{x}_1 = -\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)x_1 + \frac{x_2}{C_2R_2} + \frac{V}{C_1R_1}$$

$$\dot{x}_2 = \frac{x_1}{C_2R_2} - \frac{x_2}{C_2R_2}$$

$$y = \frac{1 \cdot x_1}{R_2} - \frac{x_2}{R_2}$$

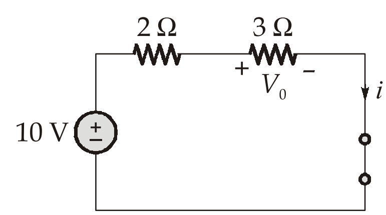
The state model is,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_L}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & \frac{1}{C_2R_2} \\ \frac{1}{C_2R_2} & \frac{-1}{C_2R_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1R_1} \\ 0 \end{bmatrix} V$$

$$y = \begin{bmatrix} \frac{1}{R_2} & \frac{-1}{R_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

**Q.8 (a) (ii) Solution:**

For $t < 0$, the switch is open and inductor acts like a short circuit.

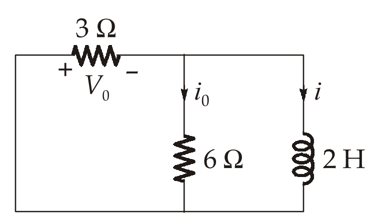



$$i(t) = \frac{10}{2+3} = 2\text{ A},\ t < 0$$

$$v_0(t) = 3\,i(t) = 6\text{ V},\ t < 0$$

Thus, $i(0) = 2\text{ A}$

For $t > 0$, the switch is closed and circuit be




$$R_{Th} = 3\,||\,6 = 2\ \Omega$$

So that time constant is

$$\tau = \frac{L}{R_{Th}} = 1\text{ sec}$$

Hence, $i(t) = i(0)\,e^{-t/\tau} = 2\,e^{-t} A, t > 0$

Since the inductor is in parallel with the 6 Ω and 3 Ω resistors.

$$v_0(t) = -V_L = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t}V;\quad t > 0$$

$$i_0(t) = \frac{V_L}{6} = \frac{-4e^{-t}}{6} = \frac{-2}{3}e^{-t}\text{ A}$$

and

$$i_0(t) = \begin{cases} 0\text{ A}, & t < 0 \\ \dfrac{-2}{3}e^{-t}\text{A}, & t > 0 \end{cases}$$

$$V_0(t) = \begin{cases} 6\text{ V}, & t < 0 \\ 4e^{-t}\text{V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2\text{ A}, & t < 0 \\ 2e^{-t}\text{A}, & t \geq 0 \end{cases}$$

**Q.8 (b) Solution:**

**(i)** To determine whether or not a system is linear, we consider two arbitrary inputs $x_1(n)$ and $x_2(n)$.

For input $x_1(n)$, output,

$$y_1(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k)$$

For input $x_2(n)$, output,

$$y_2(n) = \sum_{k=n-n_0}^{n+n_0} x_2(k)$$

For input $x_3(n)$, output,

$$y_3(n) = \sum_{k=n-n_0}^{n+n_0} x_3(k)$$

Let $x_3(n)$ be a linear combination of $x_1(n)$ and $x_2(n)$. That is,

$$x_3(n) = ax_1(n) + bx_2(n)$$

where, $a$ and $b$ are arbitrary scalars.

If the system is linear, then

$$y_3(n) = ay_1(n) + by_2(n)$$

Consider the LHS of the above equation,

$$y_3(n) = \sum_{k=n-n_0}^{n+n_0} x_3(k) = \sum_{k=n-n_0}^{n+n_0} ax_1(k) + bx_2(k)$$

$$= a \sum_{k=n-n_0}^{n+n_0} x_1(k) + b \sum_{k=n-n_0}^{n+n_0} x_2(k)$$

$$= ay_1(n) + by_2(n) = \text{RHS}$$

Since, LHS = RHS, we conclude that the system is linear.

**(ii)** Let $x_1(n)$ be an arbitrary input to the system,

$$x_1(n) \rightarrow y_1(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k)$$

$$x_2(n) \rightarrow y_2(n) = \sum_{k=n-n_0}^{n+n_0} x_2(k)$$

Let $x_2(n)$ be obtained by shifting $x_1(n)$ in time, i.e.,

$$x_2(n) = x_1(n - m)$$

If the system is time-invariant.

Then, $$y_2(n) = y_1(n - m)$$

Consider the LHS of the above equation,

$$y_2(n) = \sum_{k=n-n_0}^{n+n_0} x_2(k) = \sum_{k=n-n_0}^{n+n_0} x_1(k-m)$$

Now, consider,

$$y_1(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k)$$

$$y_1(n - m) = \sum_{k=n-m-n_0}^{n-m+n_0} x_1(k) = \sum_{k=n-n_0}^{n+n_0} x_1(k-m)$$

Clearly, $$y_2(n) = y_1(n - m)$$

Thus, the system is time-invariant.

**(iii)** $x(n)$ is bounded by a finite integer $B_x$, i.e. $|x(n)| < B_x$ for all, $n$

$$|y(n)| = \left|\sum_{k=n-n_0}^{n+n_0} x(k)\right| = \sum_{k=n-n_0}^{n+n_0} |x(k)| \leq \sum_{k=n-n_0}^{n+n_0} B_x$$

$$\leq B_x[(n + n_0) - (n - n_0) + 1]$$

$$\leq B_x(2n_0 + 1)$$

Hence, $$C \leq (2n_0 + 1) B_x$$

**Q.8 (c) Solution:**

The waveform is periodic with period $T = 2$ and fundamental frequency

$$\omega_o = \frac{2\pi}{T} = \pi$$

The given waveform for one period may be written as

$$x(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

Since the wave is neither even nor odd, the series will contain both sine and cosine terms.

$$a_o = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{2}\left(\int_0^1 t dt + \int_1^2 0 dt\right)$$

$$= \frac{1}{4}$$

Now, we determine $a_n$ and $b_n$ :

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(n\omega_o t)dt$$

$$= \frac{2}{2}\left(\int_0^1 t\cos(n\pi t)dt + \int_1^2 0\cos(n\pi t)dt\right)$$

$$= \int_0^1 t\cos(n\pi t)dt + 0$$

$$= \left[\frac{t\sin(n\pi t)}{n\pi}\right]_0^1 + \left[\frac{\cos(n\pi t)}{(n\pi)^2}\right]_0^1$$

$$= \frac{1}{(n\pi)^2}(\cos(n\pi) - 1)$$

$$a_n = \begin{cases} \dfrac{-2}{(n\pi)^2}, & n = 1,3,5,..... \\ 0, & n = 2,4,6,..... \end{cases}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_o t)dt$$

$$= \frac{2}{2}\left(\int_0^1 t\sin(n\pi t)dt + \int_1^2 0\sin(n\pi t)dt\right)$$

$$= \int_0^1 t\sin(n\pi t)dt + 0 = \left[\frac{-t\cos(n\pi t)}{n\pi}\right]_0^1 + \left[\frac{\sin(n\pi t)}{(n\pi)^2}\right]_0^1$$

$$= \frac{-1}{n\pi}\cos(n\pi)$$

$$b_n = \begin{cases} \frac{1}{n\pi}, & n = 1,3,5,..... \\ \frac{-1}{n\pi}, & n = 2,4,6,..... \end{cases}$$

Substituting the values of $a_o$, $a_n$ and $b_n$

$$x(t) = a_o + \sum_{n=1}^{\infty}[a_n\cos(n\omega_o t) + b_n\sin(n\omega_o t)]$$

$$= \frac{1}{4} - \frac{2}{\pi^2}\cos(\pi t) - \frac{2}{(3\pi)^2}\cos(3\pi t) - \frac{2}{(5\pi)^2}\cos(5\pi t).....$$

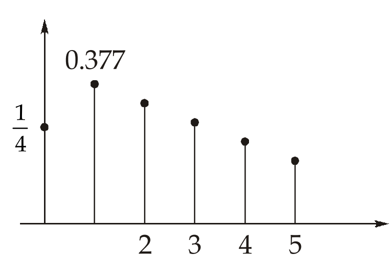
$$+ \frac{1}{\pi}\sin(\pi t) - \frac{1}{2\pi}\sin(2\pi t) + \frac{1}{3\pi}\sin(3\pi t).....$$

The even harmonic amplitudes are given directly by $|b_n|$. Since, there are no even-harmonic cosine terms. However, the odd-harmonic amplitudes must be computed using $C_n = \sqrt{a_n^2 + b_n^2}$, $n \geq 1$ and $C_o = a_o$. Thus,

$$C_1 = \sqrt{\left(\frac{2}{\pi}\right)^2 + \left(\frac{1}{\pi}\right)^2} = 0.377$$

$$C_3 = 0.109$$

$$C_5 = 0.064$$

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