MADE EASY

India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2024 Mains Test Series

Civil Engineering Test No : 3

Section A : Strength of Materials

Q.1 (a) Solution:

(i)

$$\text{Longitudinal stress, } \sigma = \frac{P}{A} = \frac{85 \times 10^3}{\frac{\pi}{4}(30)^2} \text{MPa} = 120.25 \text{ MPa}$$

$$\text{Axial strain, } \varepsilon = \frac{\sigma}{E} = \frac{120.25}{70 \times 10^3} = 1.72 \times 10^{-3}$$

$$\text{Lateral strain, } \varepsilon_c = -\mu E = -\frac{1}{3} \times 1.72 \times 10^{-3} = -0.57 \times 10^{-3}$$

$$\therefore \text{Decrease in diameter of bar} = \varepsilon_{lateral} \times d = 0.57 \times 10^{-3} \times 30 = 0.0171 \text{ mm}$$

$$\text{Change in the volume of bar, } \Delta V = V_0 \times \epsilon_{axial} \times (1 - 2\mu)$$

$$= \frac{\pi}{4} \times 30^2 \times 3000 \times 1.72 \times 10^{-3} \times \left(1 - 2 \times \frac{1}{3}\right)$$

$$= 1215.8 \text{ mm}^3$$

(ii)

$$W = mg = 40 \text{ kg} \times 9.81 \text{ m/s}^2 = 392.4 \text{ N}$$

$$A = 50 \text{ mm}^2$$

$$E = 130 \text{ GPa} = 130 \times 10^3 \text{ MPa}$$

$$h = 1.2 \text{ m}$$

$$\sigma_{allowable} = \sigma_{max} = 700 \text{ MPa}$$

$$\text{Static stress } (\sigma_{st}) = \frac{W}{A} = \frac{392.4}{50} = 7.848 \text{ MPa}$$

We know,
$$\sigma_{max} = \sigma_{st}\left[1+\sqrt{1+\frac{2h}{\delta_{st}}}\right]$$

$\Rightarrow$
$$\sigma_{max} = \sigma_{st}\left[1+\sqrt{1+\frac{2hE}{L\sigma_{st}}}\right]$$

$\Rightarrow$
$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \sqrt{1+\frac{2hE}{L\sigma_{st}}}$$

On squaring both sides,
$$\left(\frac{\sigma_{max}}{\sigma_{st}} - 1\right)^2 = 1+\frac{2hE}{L\sigma_{st}}$$

On substituting values,
$$\left(\frac{700}{7.848} - 1\right)^2 = 1+\frac{2\times 1.2\times 1000\times 130\times 10^3}{L\times 7.848}$$

$\Rightarrow$
$$L = L_{min} = 5111.713 \text{ mm} = 5.11 \text{ m}$$

So, minimum permissible length of cable is 5.11 m.

Q.1 (b) Solution:

Let the loads shared by steel and concrete be P_s and P_c respectively.

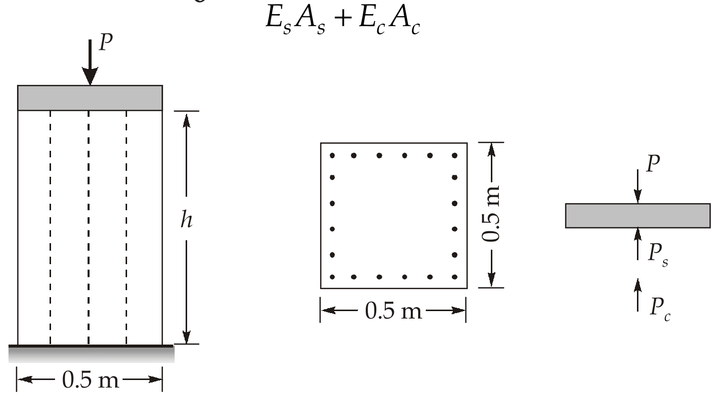
$\therefore$
$$P_s = \frac{E_sA_s\delta}{h} \text{ and } P_c = \frac{E_cA_c\delta}{h}$$

where δ is vertical deflection

Now,
$$P_s + P_c = P \qquad ...(1)$$

$\Rightarrow$
$$\frac{E_sA_s\delta}{h} + \frac{E_cA_c\delta}{h} = P$$

$\Rightarrow$
$$\delta = \frac{Ph}{E_sA_s + E_cA_c}$$



Substituting the value of δ in eq. (1)

$$P_s = \frac{E_s A_s}{E_s A_s + E_c A_c} \times P$$

$$P_c = \frac{E_c A_c}{E_s A_s + E_c A_c} \times P$$

Now,
$$\sigma_s = \frac{P_s}{A_s} = \frac{E_s P}{E_s A_s + E_c A_c}$$

and
$$\sigma_c = \frac{P_c}{A_c} = \frac{E_c P}{E_s A_s + E_c A_c}$$

$$P = \left(A_s + \frac{E_c}{E_s} A_c\right)\sigma_s$$

or
$$P = \left(A_c + \frac{E_s}{E_c} A_s\right)\sigma_c$$

Now,
$$A_s = 12 \times \frac{\pi \times (25)^2}{4} = 5890.5 \text{ mm}^2$$

$$A_c = (500)^2 - 5890.5 = 244109.5 \text{ mm}^2$$

Also
$$\frac{E_s}{E_c} = \frac{200}{25} = 8$$

Substituting the values, we get

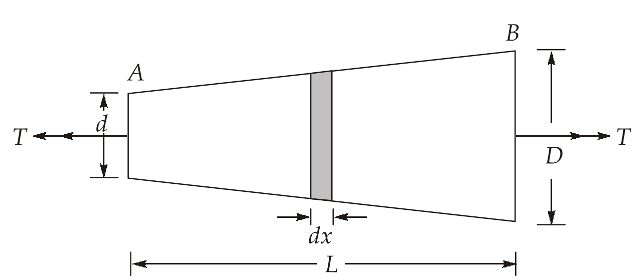
$$P = \left(5890.5 + \frac{244109.5}{8}\right) \times \frac{70}{1000} \text{ kN} = 2548.29 \text{ kN}$$

or
$$P = (244109.5 + 8 \times 5890.5) \times \frac{8}{1000} = 2329.87 \text{ kN}$$

∴ Allowable load = 2329.87 kN

Q.1 (c) Solution:

Consider an element of length dx at a distance x from A as shown in figure.



Now, diameter of element, $d_x = d + \frac{D-d}{L}x$

Polar moment of inertia,

$$I_{px} = \frac{\pi d_x^4}{32} = \frac{\pi}{32}\left(d + \frac{D-d}{L}x\right)^4$$

The torque T_x is constant and is equal to 'T' applied at ends. Therefore, the expression for angle of twist becomes,

$$\text{Angle of twist, } \phi = \int_0^L \frac{Tdx}{GI_P} \quad \text{...(i)}$$

Putting the value of I_P in eq. (i), we get

$$\phi = \int_0^L \frac{Tdx}{G \times \frac{\pi}{32}\left(d + \frac{D-d}{L}x\right)^4}$$

$$= \frac{32T}{\pi G}\int_0^L \frac{dx}{\left(d + \frac{D-d}{L}x\right)^4}$$

$$= \frac{32TL}{3\pi G(D-d)} \times \frac{(D^3 - d^3)}{D^3 \cdot d^3}$$

$$= \frac{32TL}{3\pi G} \times \frac{(D^2 + d^2 + Dd)}{D^3 \cdot d^3}$$

Q.1 (d) Solution:

A material is said to be failed if stress exceeds its permissible value or permanent deformation takes place. In tensile test, yield stress can be easily determined but if a member is subjected to various complex loadings, then it is very difficult to know the point of yielding or fracture. So, to improve the design of machine component, various theories of failure have been developed considering the physical behaviour of material.

Maximum strain energy theory of failure states that failure occurs when maximum strain energy in complex stress system equals to strain energy developed at yield stress in uniaxial loading.

As per this theory,

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_2^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right] \le \frac{\sigma_y^2}{2E}$$

where symbols have their usual standard meanings.

Limitations:

- It is suitable for ductile materials but cannot be applied for brittle materials.
- In case of pure shear, results are unsafe for ductile materials.

Maximum shear strain energy theory proposes that failure will occur when the maximum energy of distortion reaches the equivalent value at yielding in simple tension.

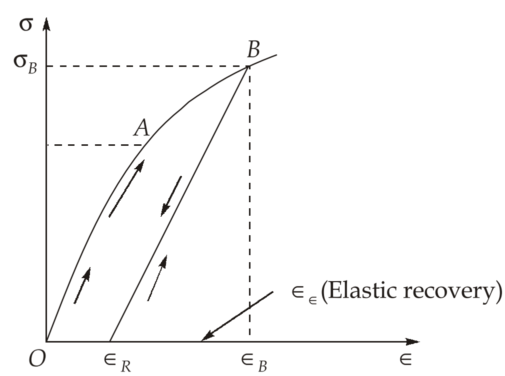
As per this theory,

$$\frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2\right] = \frac{1}{12G}\times 2\sigma_y^2$$

Limitations:

- This theory cannot be applied for brittle materials.
- It cannot be applied for materials under hydrostatic loadings.

Q.1 (e) Solution:



$$L = 120 \text{ cm} = 1.2 \text{ m}$$

$$d = 2.5 \text{ cm} = 0.025 \text{ m}$$

$$P = 250 \text{ kN} = 250 \times 10^3 \text{ N}$$

$$E = 6.89 \times 10^{10} \text{ N/m}^2$$

Stress and strain at point B,

$$\sigma_B = \frac{250\times10^3}{\frac{\pi}{4}\times(0.025)^2} = 5.09\times10^8 \text{ N/m}^2$$

Given: $\in_B = 0.045$

$$\text{Elastic recovery, } \in_E = \frac{\sigma_B}{\text{Slope}}$$

$$= \frac{5.09 \times 10^8}{6.89 \times 10^{10}} = 0.0074$$

$\therefore$ Residual strain, $\in_R = \in_B - \in_E$

$$= 0.045 - 0.0074 = 0.0376$$

(a) Permanent set, $\in_R L = 0.0376 \times 120 \text{ cm} = 4.512 \text{ cm}$

(b) Now proportional limit when reloaded will be σ_B and strain corresponding to that will be $\in_E$.

$\therefore$ Modulus of resilience after load application.

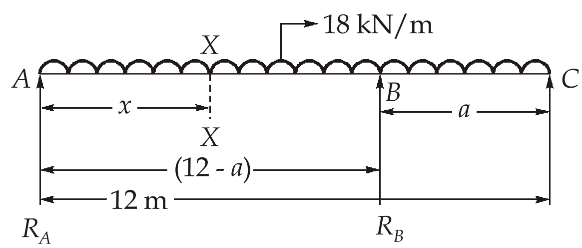
$$= \frac{1}{2}\sigma_B \in_E = \frac{1}{2} \times 5.09 \times 10^8 \times 0.0074$$

$$= 1.88 \times 10^6 \text{ J/m}^3 = 1.88 \text{ MJ/m}^3$$

Q.2 (a) Solution:

(i)

Let us assume that the second support be at B, located at 'a' meters from C



Taking moments about end A,

$\Rightarrow \quad R_B(12 - a) - 18 \times 12 \times 6 = 0$

$$\Rightarrow \quad R_B = \frac{1296}{(12-a)} \quad \text{...(i)}$$

Hence,
$$R_A = 18 \times 12 - \frac{1296}{(12-a)}$$

$$\Rightarrow \quad R_A = \frac{1296 - 216a}{(12-a)} \quad \text{...(ii)}$$

Bending moment at support B,

$$M_B = -18 \times a \times \frac{a}{2} = -9a^2 \quad \text{...(iii)}$$

Maximum positive bending moment occurs at section X-X between supports A and B.

Suppose, this section is at distance x from support A.

For maximum BM at section X-X

$$(SF)_{XX} = 0$$

$$\Rightarrow \quad R_A - 18 \times x = 0$$

$$\Rightarrow \quad \frac{1296 - 216a}{(12 - a)} - 18x = 0$$

$$\Rightarrow \quad x = \frac{1296 - 216a}{18(12 - a)} \quad \text{...(iv)}$$

Hence, maximum positive bending moment at section X-X is,

$$M_{XX} = R_A \cdot x - 18 \times \frac{x^2}{2}$$

$$= \frac{(1296 - 216a)}{(12 - a)} \times \frac{(1296 - 216a)}{18(12 - a)} - 9\left(\frac{1296 - 216a}{18(12 - a)}\right)^2$$

$$= \left[\frac{(1296 - 216a)}{(12 - a)}\right]^2 \left[\frac{1}{18} - \frac{9}{18^2}\right]$$

$$M_{XX} = \frac{1}{36}\left[\frac{1296 - 216a}{12 - a}\right]^2 \quad \text{...(v)}$$

For the condition that maximum bending moment will be as small as possible, the maximum sagging and hogging moment over the support should be numerically equal.

Equating equation (iii) and (v),

$$\Rightarrow \quad \frac{1}{36}\left[\frac{1296 - 216a}{12 - a}\right]^2 = 9a^2$$

$$\Rightarrow \quad \left[\frac{1296 - 216a}{12 - a}\right]^2 = 9 \times 36a^2$$

$$\Rightarrow \quad \frac{1296 - 216a}{12 - a} = 18a$$

$$\text{and} \quad \frac{1296 - 216a}{12 - a} = -18a$$

$$\Rightarrow \quad 1296 - 216a = 216a - 18a^2$$

$$\text{and} \quad 1296 - 216a = -216a + 18a^2$$

$$\Rightarrow \quad 18a^2 - 432a + 1296 = 0$$

and $18a^2 = 1296$

On solving, we get

$\Rightarrow$ $a = 3.5147$ and 20.485

Hence, $a = 3.5147 \approx 3.515$ m

(as a can not be greater than 12 and adopting a lower value of 'a' so that bending moment is smaller)

Position of second support should be at 3.515 m from right end.

Now,

Maximum bending moment $= 9a^2 = 9(3.515)^2 = 111.197$ kNm

(ii)

Castigilano's first theorem: It states that the first partial derivative of the total internal energy (strain energy) in a structure with respect to any particular deflection component is equal to the force applied at that point in the direction of deflection.

$$\frac{\partial U}{\partial \delta} = P$$

Maxwell's reciprocal theorem: According to this theorem, displacement at one point of beam due to load at another point is equal to the displacement at another point when same load is applied at the first point of the beam i.e.

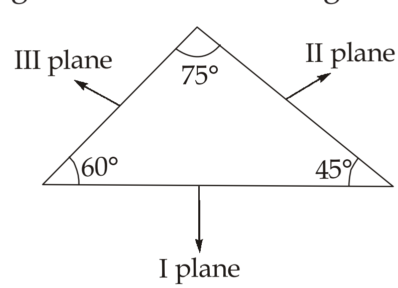
$$\delta_{12} = \delta_{21}$$

where δ_{21} = Deflection at point 2 due to load at point 1.

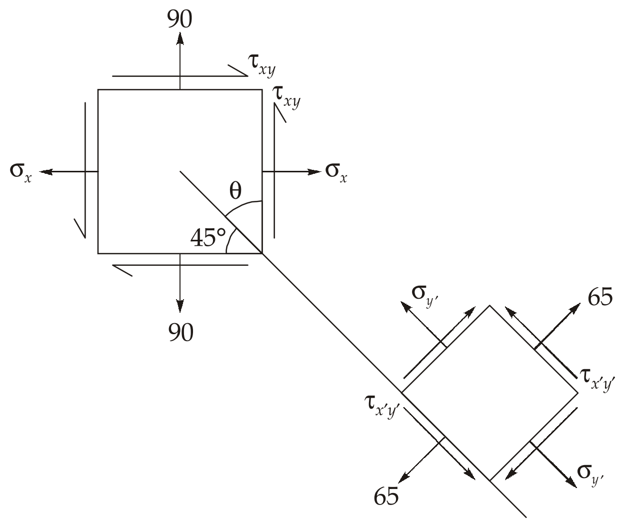
δ_{12} = Deflection at point 1 due to load at point 2.

Q.2 (b) Solution:

Let us assume the plane arrangement as shown in figure,



For II plane:



Now, $\sigma_y = 90$ MPa

$\theta = 90° - 45° = 45°$

$\sigma_x' = 65$ MPa

As we know, $\sigma_x' = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$

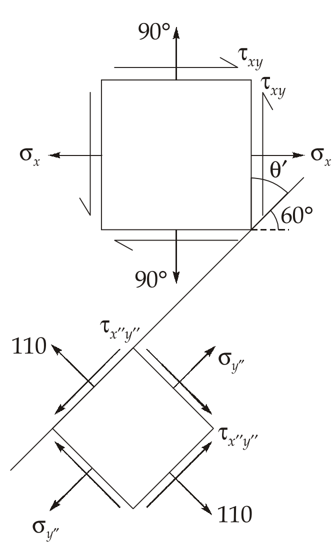
$\Rightarrow$ $65 = \cos^2 45° + 90\sin^2 45° + 2\tau_{xy} \sin 45° \cdot \cos 45°$

$\Rightarrow$ $65 = \frac{\sigma_x}{2} + \frac{90}{2} + \tau_{xy} \times 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\Rightarrow$ $2 \times 65 = \sigma_x + 90 + 2\tau_{xy}$

$\Rightarrow$ $40 = \sigma_x + 2\tau_{xy}$ ...(i)

For III plane:



Here, $\sigma_x'' = 110$ MPa

$$\theta' = -30°$$

Now, $\sigma_x'' = \sigma_x \cos^2\theta' + \sigma_y \sin^2\theta' + 2\tau_{xy} \sin\theta' \cdot \cos\theta'$

$\Rightarrow$ $110 =$

$\sigma_x \cos^2(-30°) + 90(\sin^2(-30°)) + 2\tau_{xy} \sin(-30°)\cos(-30°)$

$\Rightarrow$ $110 = \frac{3}{4} \cdot \sigma_x + \frac{90}{4} - \frac{\sqrt{3}}{2} \cdot \tau_{xy}$

$\Rightarrow$ $440 = 3\sigma_x + 90 - 2\sqrt{3}\tau_{xy}$

$\Rightarrow$ $350 = 3\sigma_x - 2\sqrt{3}\tau_{xy}$ ...(ii)

On solving (i) and (ii), we get

$$\sigma_x = 88.6 \text{ MPa}$$

$$\tau_{xy} = -24.30 \text{ MPa}$$

[–ve sign indicates that assumed direction is wrong]

Similarly for II plane, $\theta = 45°$

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\cos\theta \cdot \sin\theta + \tau_{xy}\left[\cos^2\theta - \sin^2\theta\right]$$

$$= [90 - 88.6]\cos 45° \cdot \sin 45° + (-24.3)\left[\cos^2 45° - \sin^2 45°\right]$$

$$= 0.7 \text{ MPa}$$

Similarly for III plane, $\theta = -30°$

$\tau_{x''y''} =$

$[90 - 88.6]\cos(-30°)\sin(-30°) + (-24.3)\left[\cos^2(-30°) - \sin^2(-30°)\right]$

$$= -12.756 \text{ MPa}$$

Now, $\sigma_x = 88.6$ MPa

$$\sigma_y = 90 \text{ MPa}$$

$$\tau_{xy} = -24.3 \text{ MPa}$$

Principal stresses are,

$$p_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau xy^2}$$

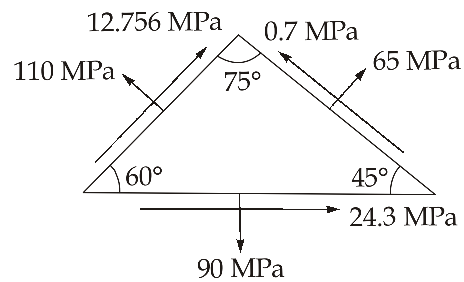
$$= \frac{88.6 + 90}{2} \pm \frac{1}{2}\sqrt{[88.6 - 90]^2 + 4 \times (-24.3)^2}$$

$$= 89.3 \pm 24.31$$

Now, $p_1 = 113.61$ MPa

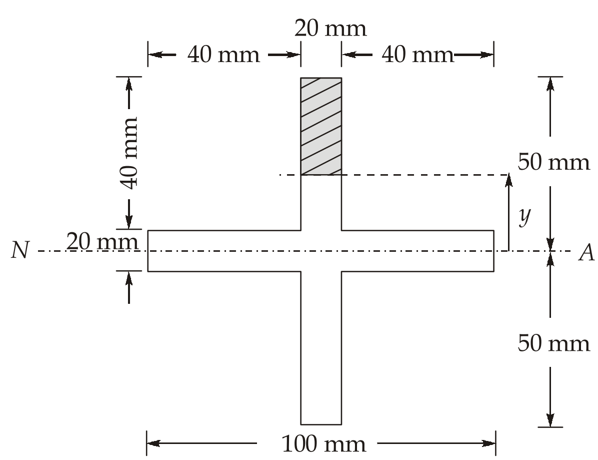
and $p_2 = 64.99$ MPa

The final stress element is as shown below,



Q.2 (c) Solution:

(i)



For sections at a distance y from NA such that 10 mm < $y \le 50$ mm, the shear stress is given by,

$$\tau = \frac{VA\bar{y}}{Ib}$$

$$= \frac{100\times10^3 \times 20\times(50-y)\times\left[y+\frac{50-y}{2}\right]}{I\times20}$$

Moment of inertia of given cross section about centroidal axis (NA),

$$I = \frac{20\times100^3}{12}+\frac{(100-20)\times20^3}{12}$$

$$= 1.72 \times 10^6 \text{ mm}^4$$

$$\therefore \qquad \tau = \frac{5\times10^4\times(50^2-y^2)}{1.72\times10^6}$$

$\therefore$ At $y = 10$ mm $\qquad \tau = 69.8$ MPa

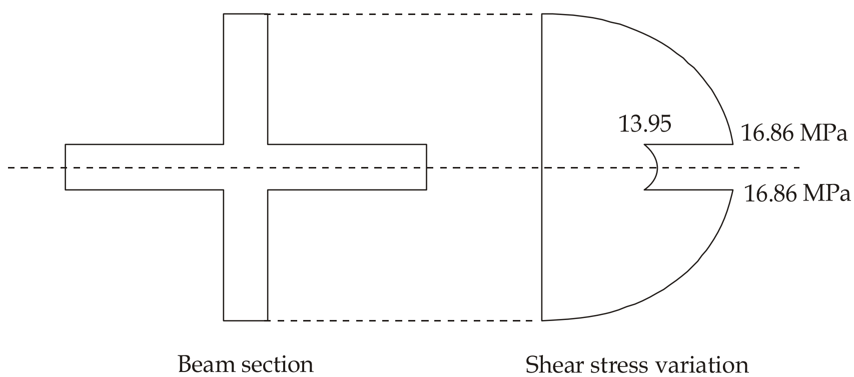
Now, at a distance $y \le 10$ mm from NA,

$$\text{Shear stress, } \tau = \frac{100\times10^3\times\left[(10-y)\times100\times\left(y+\frac{10-y}{2}\right)+20\times40\times(30)\right]}{1.72\times10^6\times100}$$

$$= \frac{1}{1.72\times10^3}\times\left[24000+50\times(100-y^2)\right]$$

$\therefore$ At $y = 0$, $\tau = 16.86$ MPa

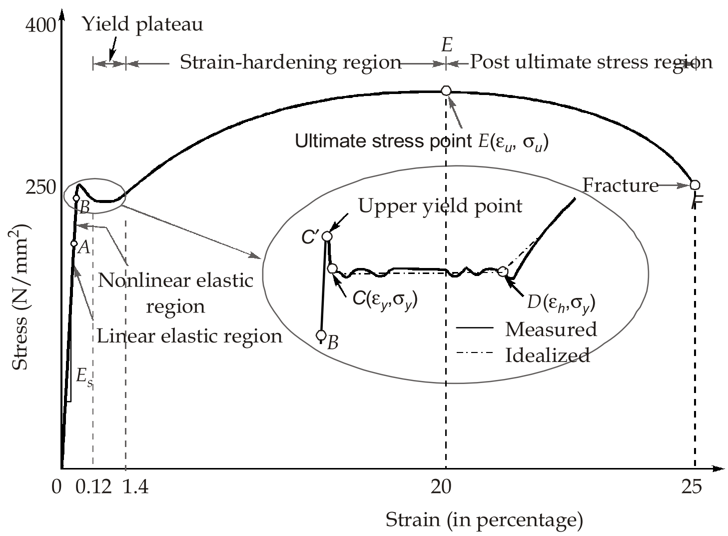
and At $y = 10$ mm, $\tau = 13.95$ MPa



(ii)

Stress strain diagram for mild steel in tension

- ***A* is limit of proportionality:** Beyond this, linear variation of strain w.r.t. stress ceases. Hooke's law is valid in region OA.
- ***B* is elastic limit:** The maximum stress upto which a specimen regains its original length on removal of applied load. For mild steel, *B* is very near to *A*. However, for other materials *B* may be far away from *A* on this curve.



Ideal Tensile stress-strain diagram for Mild Steel

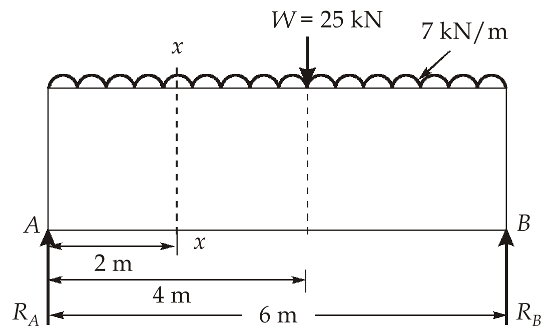
- ***C′* is upper yield point:** The magnitude of the stress corresponding to *C′* depends on the cross-sectional area, shape of the specimen and the type of the equipment used to perform the test. It has no practical significance.
- ***C* is lower yield point:** This is also called actual yield point. The stress at *C* is the yield stress (σ_y) with a typical value of σ_y = 250 N/mm^2 (for mild steel). The yielding begins at this stress.
- ***CD* represents perfectly plastic region:** It is the strain which occurs after the yielding point *C*, without any increase in stress. The strain corresponding to point *D* is about 1.4% and corresponding to *C* is about 0.12% for mild steel. Hence, plastic strain is 10 to 15 times of elastic strain.
- ***DE* represents strain hardening region:** In this range further addition of stress gives additional strain. However, strain increases at faster rate in this region. The material in this range undergoes change in its crystalline structure, resulting in increased resistance to further deformation. This portion is not used for structural design.
- ***E* is ultimate point:** The stress corresponding to this point is ultimate stress (f_u) and the corresponding strain is about 20% for mild steel.
- ***F* is fracture point:** Stress corresponding to this is called breaking stress and strain is called fracture strain. It is about 25% for mild steel.
- ***EF* post ultimate stress region:** In this range, necking occurs, i.e. area of cross-section gets drastically reduced.

Q.3 (a) Solution:

Let the reactions at end supports *A* and *B* be R_A and R_B respectively.

Step 1: Bending moment (M)

Taking moments about A,



$$R_B \times 6 = 25 \times 4 + 7 \times 6 \times \frac{6}{2}$$

$$\Rightarrow \quad R_B = 37.67 \text{ kN}$$

Now, $$R_A + R_B = 25 + 7(6) = 67 \text{ kN}$$

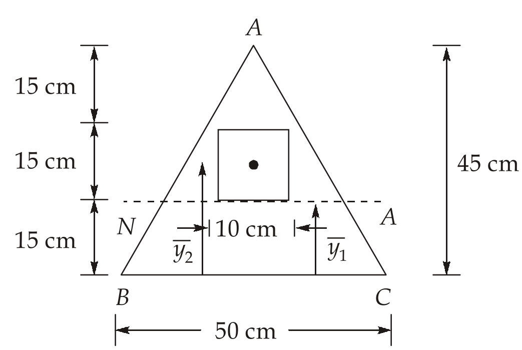
$$\therefore \quad R_A = 67 - 37.67 = 29.33 \text{ kN}$$

So, bending moment at section x-x is,

$$M = R_A \times 2 - 7 \times 2 \times \frac{2}{2}$$

$$= 29.33 \times 2 - 14 = 44.66 \text{ kNm}$$

Step 2: Location of neutral axis.



$$\text{Area of triangle, } A_1 = \frac{1}{2} \times 50 \times 45 = 1125 \text{ cm}^2$$

Distance of CG of area A_1 from bottom,

$$\bar{y}_1 = \frac{1}{3} \times 45 = 15 \text{ cm}$$

$$\text{Area of rectangular hole, } A_2 = 10 \times 15 = 150 \text{ cm}^2$$

Distance of CG of area A_2 from bottom is,

$$\therefore \quad \bar{y}_2 = 15 + \frac{15}{2} = 22.5 \text{ cm}$$

Now, distance of CG of section from bottom,

$$\therefore \quad \bar{y} = \frac{A_1\bar{y}_1 - A_2\bar{y}_2}{A_1 - A_2}$$

$$= \frac{1125 \times 15 - 150 \times 22.5}{1125 - 150} = 13.85 \text{ cm}$$

Step 3: Moment of inertia about neutral axis.

$$I_{NA} = I_1 - I_2$$

$$= \left[(I_1)_{\text{self}} + A_1(15 - 13.85)^2\right] - \left[(I_2)_{\text{self}} + A_2(22.5 - 13.85)^2\right]$$

$$= \left[\frac{50 \times 45^3}{36} + 1125 \times 1.15^2\right] - \left[\frac{10 \times 15^3}{12} + 150 \times 8.65^2\right]$$

$$= 128050.3125 - 14035.875$$

$$= 114014.4375 \simeq 1.14 \times 10^5 \text{ cm}^4$$

Step 4: Maximum bending compressive stress (σ) at *x*-*x* will occur above neutral axis.

Using,

$$\frac{\sigma}{y} = \frac{M}{I_{NA}}$$

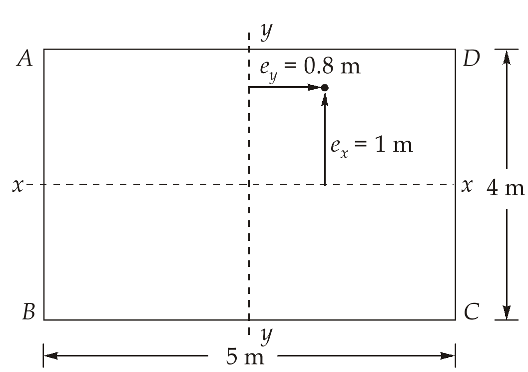
$$\Rightarrow \quad \sigma = \frac{M}{I_{NA}} \times y_{\max}$$

$$= \frac{44.66 \times 10^5}{1.14 \times 10^5} \times (45 - 13.85)$$

$$= 1220.31 \text{ N/cm}^2 = 12.2 \text{ N/mm}^2$$

Q.3 (b) Solution:

(i)



$$\text{Area, } A = 5 \times 4 = 20 \text{ m}^2$$

$$\text{Load, } W = 60 \text{ kN}$$

Now,

$$I_{xx} = \frac{5 \times 4^3}{12} = 26.67 \text{ m}^4$$

$$I_{yy} = \frac{4 \times 5^3}{12} = 41.67 \text{ m}^4$$

$$e_x = 1 \text{ m}, \quad e_y = 0.8 \text{ m}$$

1. Converting eccentric load to an axial load and moments about axes xx, and yy, we have,

$$M_x = 60 \times 1 = 60 \text{ kN-m}$$

$$M_y = 60 \times 0.8 = 48 \text{ kN-m}$$

Now, maximum distance of corners from x and y axes are

$$x = 2.5 \text{ m and } y = 2 \text{ m}$$

Now, stresses developed at each corner of column section are:

$$\sigma_A = \frac{W}{A} + \frac{M_x \cdot y}{I_{xx}} - \frac{M_y \cdot x}{I_{yy}}$$

$$= \frac{60}{20} + \frac{60 \times 2}{26.67} - \frac{48 \times 2.5}{41.67}$$

$$= 3 + 4.5 - 2.88 = 4.62 \text{ kN/m}^2$$

Similarly,

$$\sigma_B = \frac{W}{A} - \frac{M_x \cdot y}{I_{xx}} - \frac{M_y \cdot x}{I_{yy}}$$

$$= 3 - 4.5 - 2.88 = -4.38 \text{ kN/m}^2$$

$$\sigma_C = \frac{W}{A} - \frac{M_x \cdot y}{I_{xx}} + \frac{M_y \cdot x}{I_{yy}}$$

$$= 3 - 4.5 + 2.88 = 1.38 \text{ kN/m}^2$$

$$\sigma_D = \frac{W}{A} + \frac{M_x \cdot y}{I_{xx}} + \frac{M_y \cdot x}{I_{yy}}$$

$$= 3 + 4.5 + 2.88 = 10.38 \text{ kN/m}^2$$

2. Let, additional load at centre required for no tension is W'

Now, direct stress due to axial load = $\frac{W'}{20}$ kN/m^2

For no tension, direct stress due to this W' load must be equal to maximum tesion getting developed at B,

$$\therefore \quad \frac{W'}{20} = 4.38$$

$$\Rightarrow \quad W' = 87.6 \text{ kN}$$

(ii)

Here,

Strain in x-direction, $\varepsilon_x = \frac{0.05}{100} = 0.0005$ mm/mm

and strain in y-direction, $\varepsilon_y = \frac{0.03}{100} = 0.0003$ mm/mm

For plane stress condition,

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y)$$

$$= \frac{200\times10^3}{1-0.3^2}(0.0005+0.3\times0.0003) = 129.67 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x)$$

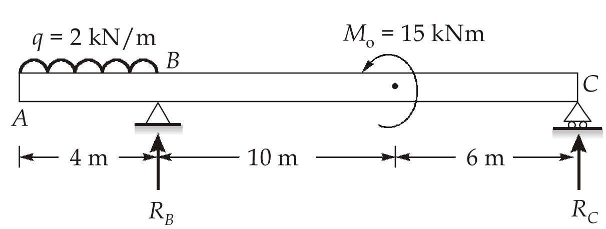
$$= \frac{200\times10^3}{1-0.3^2}(0.0003+0.3\times0.0005)$$

$$= 98.90 \text{ MPa}$$

Q.3 (c) Solution:

(i)

Let reaction at supports B and C be R_B and R_C respectively.



$$\Sigma M_B = 0,$$

$$\Rightarrow \quad 2\times4\times\frac{4}{2}+15+R_C(16) = 0$$

$$\Rightarrow \quad R_C = -1.9375 \simeq -1.94 \text{ kN}$$

Also, $\Sigma F_y = 0 \Rightarrow \quad R_B + R_C = 2 \times 4 = 8$

$$\Rightarrow \quad R_B = 9.94 \text{ kN}$$

(i) SF diagram between A and B,

$$V_A = 0$$

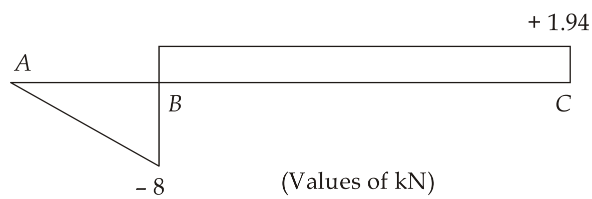
$$\text{Just left of } B,\ V_B = -2(4) = -8 \text{ kN}$$

Between B and C,

$$\text{Just right of } B,\ V_B = -8 + 9.94 = 1.94 \text{ kN}$$

$$V_C = -R_C = 1.94 \text{ kN}$$

Shear force diagram is shown below.



(ii) BM diagram

$$\text{BM at } A = 0$$

$$\text{BM at } B = -2\times4\times\frac{4}{2} = -16 \text{ kN-m}$$

$$\text{BM at } C = 0$$

BM just to the left of couple M_o,

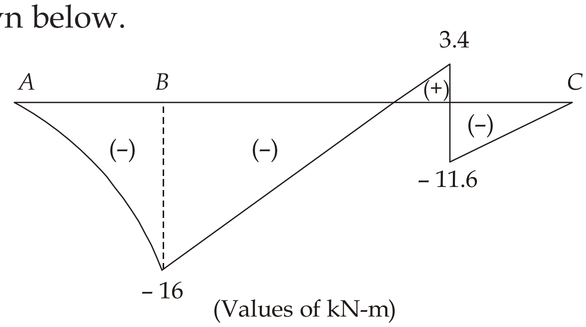
$$= -8 \times 12 + 9.94(10)$$

$$= 3.4 \text{ kN-m}$$

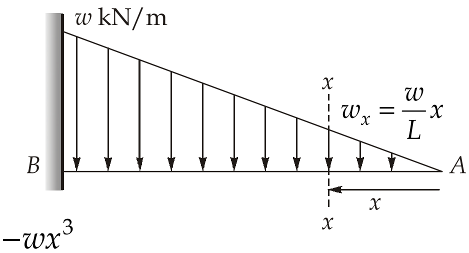
BM just to the right of couple M_o,

$$= 3.4 - 15 = -11.6 \text{ kN-m}$$

∴ BM diagram is shown below.



(ii)



Consider a section x-x at a distance x from end A,

$$\therefore \quad M_x = -\left[\frac{1}{2}\times x \times w_x\right]\frac{x}{3}$$

$$= \left[\frac{1}{2}\times x \times \frac{w}{L}x\right]\frac{x}{3} = \frac{-wx^3}{6L}$$

Using double integration method, we get

$$EI\frac{d^2y}{dx^2} = M_x$$

$$\therefore \quad EI\frac{d^2y}{dx^2} = -\frac{wx^3}{6L} \quad \text{...(i)}$$

Integrating equation (i), we get

$$EI\frac{dy}{dx} = \frac{-wx^4}{24L} + c_1 \quad \text{...(ii)}$$

Integrating (ii), we get

$$EIy = -\frac{wx^5}{120L} + c_1x + c_2 \quad \text{...(iii)}$$

End conditions:

At $x = L$, $\left(\frac{dy}{dx}\right)_B = \theta_B = 0$

From equation (ii), $c_1 = \frac{wL^3}{24}$

Also, at $x = L$, $(y_B) = \delta_B = 0$

$\therefore$ From equation (iii), $c_2 = \frac{wL^4}{120} - \frac{wL^4}{24} = -\frac{wL^4}{30}$

Hence, $EI\frac{dy}{dx} = -\frac{wx^4}{24L} + \frac{wL^3}{24}$

and $EIy = -\frac{wx^5}{120L} + \frac{wL^3x}{24} - \frac{wL^4}{30}$

$\therefore$ At free end A, $x = 0$

$$\therefore \quad \text{Slope at } A, \left(\frac{dy}{dx}\right)_A = \frac{-wL^3}{24EI} = \frac{wL^3}{24EI} \; (\circlearrowleft)$$

$$\text{Deflection at } A, y = \frac{-wL^4}{30EI} = \frac{wL^4}{30EI} \text{ (downwards)}$$

Q.4 (a) Solution:

(i)

Moment area method is used for determising the deflections and angles of rotation of beams. It is based upon two theorems related to the area of the bending moment diagram. It is valid only for linearly elastic beams with small slopes.

- **First moment area theorem**: For two points A and B on a deflection curve, the angle $\theta_{B/A}$ between the tangents to the deflection curve at two points A and B is equal to the area of M/EI diagram between those points.

$$\theta_{B/A} = \int_A^B \frac{Mdx}{EI}$$

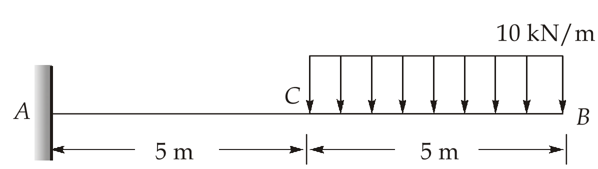
$$= \text{Area of the } \frac{M}{EI} \text{ diagram between points } A \text{ and } B.$$

- **Second moment area theorem**: The tangential deviation $t_{B/A}$ of point B from the tangent at point A is equal to the first moment of the area of the M/EI diagram between A and B, evaluated with respect to B.

$$t_{B/A} = \int_A^B x_1 \frac{Mdx}{EI}$$

= First moment of area of the $\frac{M}{EI}$ diagram between A and B, evaluated with respect to B.

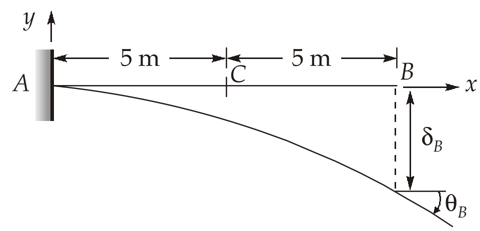
(ii)

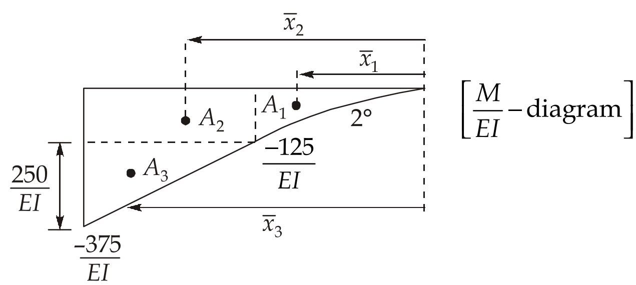


Bending moment,

At C, $$M_C = -10 \times 5 \times \frac{5}{2} = -125 \text{ kN-m}$$

At A, $$M_A = -10\times5\times\left(5+\frac{5}{2}\right) = -375 \text{ kN-m}$$





Angle of rotation:

According to first moment area theorem,

$$\theta_{B/A} = \theta_B - \theta_A = \text{Area of } \frac{M}{EI} \text{ diagram between } A \text{ and } B.$$

$$\theta_B = A_1 + A_2 + A_3 \qquad [\because \theta_A = 0]$$

$$= -\left[\frac{1}{3}\left[5\times\left(\frac{125}{EI}\right)\right]+\frac{125}{EI}\times5+\frac{1}{2}\times\frac{250}{EI}\times5\right]$$

$$= \frac{-1458.33}{EI}$$

Deflection:

According to second moment area theorem,

$$\delta_B = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3$$

where $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_3$ are the distances from point B to the centroids of the respective areas.

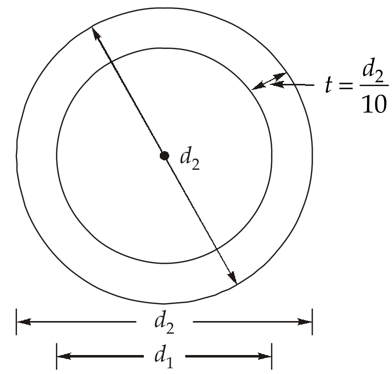
$$\therefore \qquad \delta_B = \frac{1}{3}\times5\times\frac{125}{EI}\times\frac{3}{4}\times5+\frac{125}{EI}\times5\times\left(5+\frac{5}{2}\right)+ \frac{1}{2}\times\frac{250}{EI}\times5\times\left(5+\frac{2}{3}(5)\right)$$

$$= \frac{3125}{4EI}+\frac{9375}{2EI}+\frac{15625}{3EI} = \frac{10677.08}{EI}$$

Q.4 (b) Solution:

(i)

Let the outer diameter of shaft be d_2



$$\therefore \quad \text{Thickness of tube, } t = \frac{d_2}{10}$$

$$\text{Inner diameter, } d_1 = d_2 - 2t$$

$$= d_2 - \frac{2d_2}{10}$$

$$= 0.8d_2$$

Polar moment of inertia for the shaft,

$$I_p = \frac{\pi}{32}\left(d_2^4 - d_1^4\right)$$

$$= \frac{\pi}{32}\left(d_2^4 - (0.8d_2)^4\right)$$

$$= \frac{\pi d_2^4}{32} \times 0.5904 = 0.05796\ d_2^4$$

For the case of allowable shear stress,

$$\text{From torsion formula,} \quad \tau_{max} = \frac{TR}{I_p}$$

$$= \frac{T\left(\frac{d_2}{2}\right)}{0.05796 d_2^4}$$

$$\Rightarrow \quad \tau_{max} = \frac{T}{0.1159 d_2^3} \qquad \left[\because R = \frac{d_2}{2}\right]$$

$$\Rightarrow \quad d_2^3 = \frac{T}{0.1159 \times \tau_{max}} = \frac{1250 \times 10^3}{0.1159 \times 40}$$

$$\Rightarrow \quad d_2 = 64.6 \text{ mm}$$

For allowable rate of twist,

$$\theta_{allow} = \frac{T}{G\left(0.05796 d_2^4\right)}$$

$$\Rightarrow \quad d_2^4 = \frac{1250 \times 10^3}{78 \times 10^3 \times 0.05796 \times \frac{0.75}{1000} \times \frac{\pi}{180}}$$

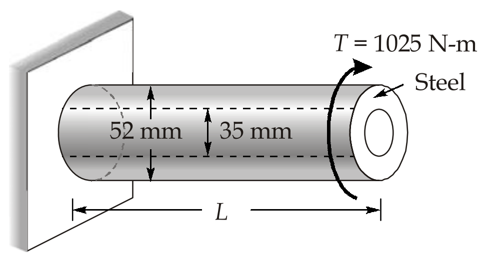
$$\Rightarrow \quad d_2 = 67.79 \text{ mm}$$

Now, diameter of shaft, d = max(64.6, 67.79) = 67.79 m

$$\therefore \quad d_1 = 0.8 d_2 = 0.8 \times 67.79$$

$$= 54.23 \text{ mm}$$

(ii)



Let torque resisted by aluminum is T_A and that by steel is T_s,

$$\therefore \quad \text{Total torque applied} = T_A + T_S$$

$$\Rightarrow \quad T_A + T_S = 1025 \quad \text{...(i)}$$

Both bars are firmly jointed,

$$\therefore \quad \theta_S = \theta_A$$

$$\Rightarrow \quad \frac{T_s L}{G_s I_{pS}} = \frac{T_A L}{G_A I_{PA}}$$

$$\Rightarrow \quad \frac{T_S}{T_A} = \frac{G_S I_{PS}}{G_A I_{PA}} = \frac{80 \times 10^9 \times \frac{\pi}{32} \times (52^4 - 35^4)}{70 \times 10^9 \times \frac{\pi}{32} \times 35^4} = 4.426$$

$$\Rightarrow \quad T_s = 4.426\, T_A \quad \text{...(2)}$$

From eq. (1) and (2)

$$T_A = 188.91 \text{ N-m}$$

$$T_s = 836.12 \text{ N-m}$$

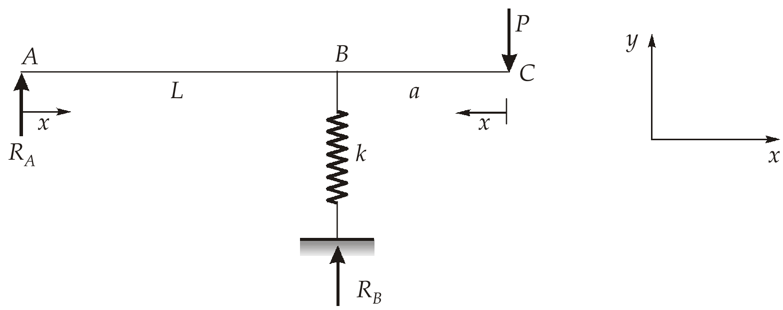
Now, maximum stresses in shafts,

$$\tau_{max} \text{ in steel} = \frac{T_s}{(Z_p)_s} = \frac{836.12 \times 10^3}{\frac{\pi}{16} \times \frac{(52^4 - 35^4)}{52}} = 38.11 \text{ N/mm}^2$$

$$\tau_{max} \text{ in aluminium} = \frac{T_A}{(Z_P)_A} = \frac{188.91\times10^3}{\frac{\pi}{16}\times35^3} = 22.44 \text{ N/mm}^2$$

Q.4 (c) Solution:

(i)



Let the reactions be R_A and R_B at A and B respectively.

Now, $\Sigma F_y = 0$

$\Rightarrow$ $R_A + R_B = P$ ...(1)

Also, $\Sigma M_A = 0$

$\Rightarrow$ $R_B(L) = P(L + a)$

From (1) and (2), $R_B = \dfrac{P(L+a)}{L}$

$$R_A = \frac{-Pa}{L}$$

Now, in span AB,

$$M_x = R_A \cdot x \qquad (0 \le x \le L)$$

$$= \frac{-P_{ax}}{L}$$

$\therefore$ $\dfrac{dM_x}{dP} = \dfrac{-ax}{L}$

Now, in span BC,

$$M_x = -Px \qquad (0 \le x \le a)$$

$\therefore$ $\dfrac{dM_x}{dP} = -x$

Strain energy of the spring,

$$U_s = \frac{R_B^2}{2k} = \frac{P^2(L+a)^2}{2L^2k}$$

$$\therefore \quad \frac{\partial U_s}{\partial P} = \frac{P(L+a)^2}{L^2k}$$

Now, total strain energy, $U = U_{beam} + U_{spring}$

$$= \int \frac{M_x^2 dx}{2EI} + \frac{P^2(L+a)^2}{2L^2k}$$

Using Castigilano's theorem,

$$\delta_c = \frac{dU}{dP}$$

$$= \int \frac{M}{EI}\left(\frac{dM_x}{dP}\right)dx + \frac{dU_{spring}}{dP}$$

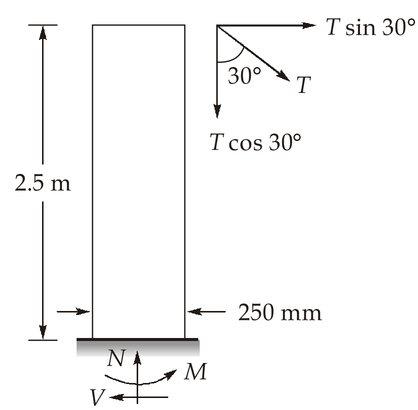
$$= \int_0^L \frac{1}{EI}\left(\frac{-Pax}{L}\right)\left(\frac{-ax}{L}\right)dx + \int_0^a \frac{1}{EI}(-Px)(-x)\,dx + \frac{P(L+a)^2}{L^2k}$$

$$= \frac{Pa^2}{EIL^2}\int_0^L x^2 dx + \frac{P}{EI}\int_0^a x^2 dx + \frac{P(L+a)^2}{L^2k}$$

$$= \frac{Pa^2}{3EIL^2}[x^3]_0^L + \frac{P}{3EI}[x^3]_0^a + \frac{P(L+a)^2}{kL^2}$$

$$= \frac{Pa^2(L+a)}{3EI} + \frac{P(L+a)^2}{kL^2}$$

(ii)



The maximum compressive stress will occur at the base of the column due to combined effect of bending and direct stress.

Moment of inertia, $I = \frac{\pi}{64}\times(250^4 - 200^4)$

$= 1.132 \times 10^8 \text{ mm}^4$

$= 1.132 \times 10^{-4} \text{ m}^4$

At the base of the pole, $N = T\cos 30°$

$$M = (T\cos 30°)\left(\frac{0.25}{2}\right) + (T\sin 30°)(2.5)$$

$$= T\times\frac{\sqrt{3}}{2}\times\frac{0.25}{2}+\frac{T\times 2.5}{2} = 1.358\text{ T}$$

Now, compressive stress at bottom,

$$\sigma_c = \frac{N}{A}+\frac{M}{I}\times\frac{0.25}{2}$$

$$\Rightarrow \quad 100\times 10^6 = \frac{T\cos 30°}{\frac{\pi}{4}\times(0.25^2 - 0.20^2)}+\frac{1.358T}{1.132\times 10^{-4}}\times 0.125$$

$$100 \times 10^6 = 1548.56\text{ T}$$

$$\Rightarrow \quad T = 6457.12\text{ N} = 64.6\text{ kN}$$

Section B : Highway Engineering - 1 + Surveying and Geology-1 + Geo-technical and Foundation Engineering - 2 + Environmental Engineering - 2

Q.5 (a) Solution:

The various factors which control the highway alignment in general may be stated as below:

(a) Obligatory points: These are control points governing the alignment of the highways. These control points may be divided broadly into following two categories:

(i) Obligatory points through which the road alignment has to pass and may often cause the alignment to deviate from the shortest or easiest path. The various examples of this category may be bridge site, intermediate town, a mountain pass or a quarry.

(ii) Obligatory points through which the road should not pass also make it necessary to deviate from the proposed shortest alignment.

(b) Traffic : The alignment should suit traffic requirements. Origin and destination study should be carried out in the area and the desire lines be drawn showing the trends of traffic flow. The new road to be aligned should keep in view the desire lines, traffic flow patterns and future trends.

(c) **Geometric design:** Geometric design factors such as gradient road, radius of curve and sight distance also would govern the final alignment of the highway. As far as possible while aligning a new road, the gradient should be flat and less than the ruling or design gradient. It may be necessary to make adjustment in the horizontal alignment of roads keeping in view the minimum radius of curve and the transition curves. Alignment should be finalized in such a way that the obstructions to visibility do not cause restrictions to the sight distance requirements.

(d) **Economy :** The alignment finalized based on the above factors should also be economical. The initial cost of construction can be decreased if high embankment and deep cuttings are avoided and the alignment is chosen in a manner to balance the cutting and filling.

(e) **Other considerations :** Various other factors which may govern the alignment are drainage considerations, hydrological factors, sight distance considerations and monotony.

Alignment of hill roads:

The hill road alignment should link up the obligatory and control points fitting well in the landscape and satisfying the geometric requirements. The best alignment for a hill road is the one wherein the total sum of the ascends and descends between the extreme points is the least.

In hill roads additional care has to be given for:

(i) **Stability:** While aligning hill roads, special care should be taken to align the road along the side of the hill which is stable.

(ii) **Drainage:** Numerous hill side drains should be provided for adequate drainage facility across the roads.

(iii) **Geometric standards of hill roads:** Different sets of geometric standards are followed in hill roads with reference to gradient, curves and speed and this to consequently influence the sight distance, radius of curve and other related features.

(iv) **Resisting length:** The resisting length of a road may be calculated from the total work to be done to move the loads along the route taking the horizontal length, the actual difference in levels between the two stations and the sum of ineffective rise and fall in excess of floating gradient. In brief, the resisting length of the alignment should be kept minimum.

Q.5 (b) Solution:

(i)

Classification of survey based on purpose:

1. **Geological Survey:** In this type of survey, information about both the surface and sub-surface is acquired for assessing the extent of different reserves like the minerals, rocks etc. It is also used for locating the faults, folds and other unconformities in the ground. This survey helps in determining the type of foundation, soil treatment required etc.
2. **Geographical Survey:** This survey is done for preparation of geographical maps depicting the land use efficiency, irrigation intensity, surface drainage, slope profile, contours, national boundaries etc.
3. **Engineering Survey:** This survey is required to be done for acquiring information for the planning and design of engineering projects like the highways, dams, railway line, water supply design, reservoirs, bridges etc. It involves topographic survey of the area, earthwork measurement etc.
4. **Cadastral Survey:** It is done to establish boundary of properties for legal purposes. It is also called public land survey.
5. **Defence Survey:** Such surveys are done for military purposes. They provide strategic information for deciding the future course of action. Aerial and topographical maps of the area are prepared which gives crucial information about the existing roads, airports, ordnance depots etc.
6. **Mine Survey:** This requires both the surface and the underground surveys. It involves making the surface map and doing the underground survey for locating the reserves of minerals.
7. **Route Survey:** It is a sort of linear survey for deciding the alignment of a highway or a railway or canal etc.
8. **Archaeological Survey:** This is done to gather information about the ancient monuments, towns, villages, kingdoms, past civilizations, temples, forts etc. buried underground due to natural forces like earthquakes, landslides, floods etc. It gives an idea about the past history, culture and development of the civilization that existed in the past. These provide vital links on understanding the evolution of the present civilisation as well as human beings.

(ii)

Correct length of the chain at commencement of work = 30 m

Length of the chain after chaining 1650 m = 30.05 m

Mean length of the chain while measuring the distance upto 1650 m = $\frac{30+30.05}{2}$

$$= 30.025 \text{ m}$$

True distance for the wrong chainage of 1650 m

$$= \frac{30.025}{30} \times 1650 = 1651.375 \text{ m}$$

Now, remaining distance = 3125 – 1650 = 1475 m

Mean length of chain for measuring distance between 1650 m to 3125 m

$$= \frac{30.05+30.08}{2} = 30.065 \text{ m}$$

True length for remaining 1475 m

$$= \frac{30.065}{30} \times 1475 = 1478.196 \text{ m}$$

Hence, total true length = 1651.375 + 1478.196 = 3129.571 m

Q.5 (c) Solution:

The two commonly used geophysical methods of soil exploration are the *seismic refraction method* and the *electrical resistivity method*.

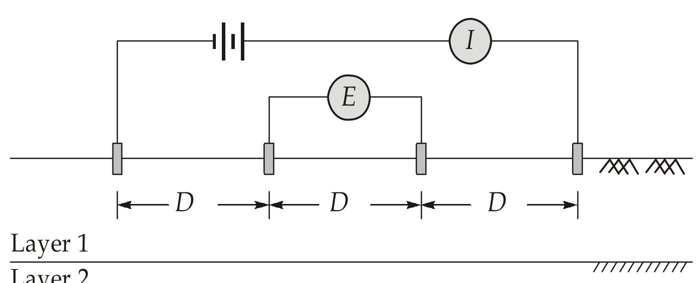
1. **Seismic refraction method**: This method is based on the fact that seismic waves have different velocities in different types of soil or rock. Further, the waves are refracted when they cross the boundary between different types of soil/rock and the approximate depth of boundaries of strata or the bedrock is determined.

   The method consists of inducing impact (by striking a plate on the soil with a hammer) or generating shock by exploding a small charge at or near the ground surface. The radiating shockwaves are recorded by a device called geophone which records the time of travel of the waves. The gasphones are installed at suitable known distances on the ground in a line from the source of shock or the same is moved away from the geophone to produce shock waves at given intervals. Some of the waves, termed direct or primary waves travel directly from the shock source along the ground surface in the direction of the geophones. Other waves travel in a downward direction at various angles to the horizontal and will be refracted if they pass into a stratum of different seismic velocity. If the underlying layer is denser, the refracted waves travel much faster. As the distance between the shock source and the geophones '*d*' increases, the refracted waves reach the geophone earlier than the direct waves. The arrival time is plotted against the distance between the source and the geophone. If the source-geophone spacing is less than *d*, the direct wave reaches the geophone earlier than the refracted wave. On the other hand, if the source spacing is greater than *d*, the refracted wave arrives earlier than the direct wave. The general types of soil or rock can be determined from a knowledge of these velocities.

This method is quick and reliable in establishing profiles of different strata provided the deeper layers have greater densities and hence higher velocities. However, the method cannot be used to justify the exact type of strata. For this purpose, borings sampling are necessary.

2. **Electrical resistivity method**: The electrical resistivity method is based on the measurement and recording of changes in the mean resistivity or apparent specific resistance of various soils. The resistivity, ρ is usually defined as resistance between the opposite faces of a unit cube of materials. Significant variations in resistivity can be detected between different types of soil strata; above and below the water table; between unfissured rocks and soils; between voids and soil/rock.

The test is carried out by driving four metal spikes to serve as electrodes into the ground along a straight line at equal distances. Current (I) from a battery, flows through the soil between the two outer electrodes, producing an electrical field within the soil. The potential difference *E* between the two inner electrodes is then measured.



Electrical resistivity method

The apparent resistivity is the weighted average of true resistivity upto a depth *D* in a large volume of soil, the soil close to the surface being more heavily weighted than the soil at greater depths. If a stratum of low resistivity overlies a stratum of higher resistivity, the current is forced to flow closer to the ground surface, resulting in a higher voltage drop and hence a higher value of apparent resistivity. It would be the opposite if a stratum of high resistivity lies above a stratum of low resistivity.

This method provides rough estimates of the types and depth of strata. The electrical resistivity method is not as reliable as the seismic refraction method. The apparent resistivity of a particular soil or rock can vary widely over a wide range of values. Hence the results of the tests have to be correlated with the bore hole data.

Q.5 (d) Solution:

Speed of overtaking vehicle, $V = 80$ kmph

Speed of overtaken vehicle, $V_b = 50$ kmph

Average acceleration, $a = 0.99\ \text{m/sec}^2$

Assume, reaction time of driver as 2.0 sec. and length of vehicle as 6.0 m.

(i) Now, safe overtaking sight distance (OSD) is given as

$$\text{OSD} = d_1 + d_2 + d_3$$

$$= 0.278\, V_b \cdot t_r + (2S + 0.278\, V_b T) + 0.278\, VT$$

where 'T' is the total time during overtaking operation

$$\therefore \quad V_b = 50 \text{ kmph}$$

$$t_r = 2 \text{ sec.}$$

$$S = 0.2\, V_b + L = 0.2 \times 50 + 6 = 16 \text{ m}$$

$$\therefore \quad T = \sqrt{\frac{4S}{a}} = \sqrt{\frac{4 \times 16}{0.99}} = 8.04 \text{ sec.}$$

$$\therefore \quad \text{OSD} = 0.278 \times 50 \times 2 + 2 \times 16 + 0.278 \times 50 \times 8.04 + 0.278 \times 80 \times 8.04$$

$$= 350.37 \text{ m} \simeq 351 \text{ m (say)}$$

Hence, safe overtaking sight distance is 351 m.

(ii) Minimum length of overtaking zone = 3(OSD) = 3(351) = 1053 m

Desirable length of overtaking zone = 5(OSD) = 5(351) = 1755 m

Q.5 (e) Solution:

- **Stratification of lakes:** The difference in temperature between various layers of lake is referred to as stratification in lake. The water of a lake gets stratified during summers and winters. During summer season, the surface water of a lake gets heated up by sunlight and warm air. This warm water being lighter, remains in upper layer near the surface, until mixed downward by turbulence from winds, waves, boats and other forces. Since such turbulence extends only to a limited depth from below the water surface, the top layers of water in the lake become well mixed and is aerobic. This warmer, well mixed and aerobic depth of water is called" epilimnion zone". The lower depth, which remains cooler, poorly mixed and anaerobic is called the "hypolimnion zone".

  There may also exist an intermediate zone or a dividing line called "thermocline or metallimnion". In winter seasons, the epilimnion zone cools, until it becomes more dense than the hypolimnion. The surface water then sinks causing overturning. The water of the hypolimnion rises to the surface where it cools and again sinks. The lake, thus becomes completely mixed, making it quite aerobic. The lakes in regions of temperate climate will, therefore, have at least one, if not two, cycles of statification and turn over every year.

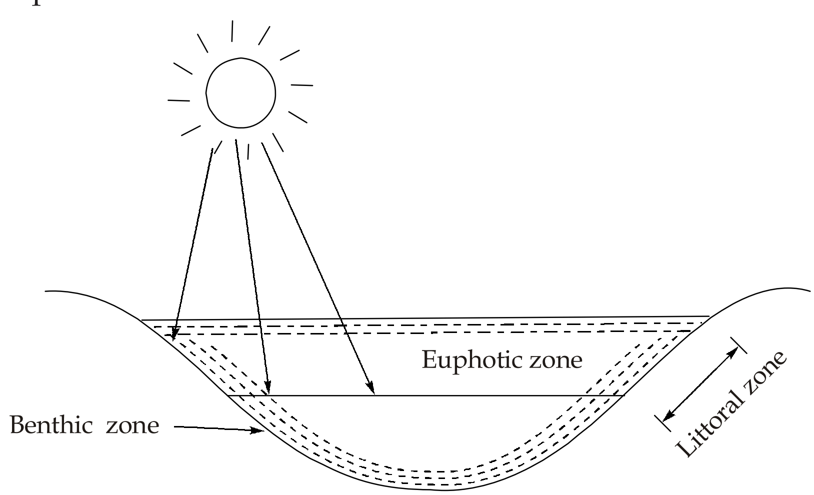
- **Biological zones in lakes**: Lakes have been found to exhibit distinct zones of biological activity, largely determined by the availability of light and oxygen. The most important biological zones are:

(i) Euphotic zone (ii) Littoral zone (iii) Benthic zone

(i) Euphotic zone: The upper layer of lake water through which sunlight can penetrate, is called the euphotic zone. All plant growth occurs in this zone. The depth of the euphotic zone is reduced by the turbidity which blocks sunlight penetration. The depth of the euphotic zone can be measured by a device called the "Secchi disk". The bottom of euphotic zone only rarely concides with the thermocline.

(ii) Littoral zone: The shallow water near the shore, in which rooted plants grow, is called the littoral zone. The extent of the littoral zone depends on the slope of the lake bottom and the depth of the euphotic zone. The littoral zone cannot extend deeper than the euphotic zone.

(iii) Benthic zone: The bottom sediments in a lake comprises the benthic zone. Living organisms after death settle down to the bottom of lake which are decomposed by bacteria present in benthic zone.



Biological zones in a lake

Q.6 (a) Solution:

(i)

Ruling design speed of national highway on plain terrain, V = 100 kmph.

The various geometric elements of the horizontal curve to be designed are as below:

1. Ruling minimum radius, $R_{\text{ruling}} = \dfrac{V^2}{127(e+f)} = \dfrac{100^2}{127[0.07+0.15]}$

$$= 357.91 \text{ m} \simeq 358 \text{ m (say)}$$

2. Superelevation, e (for mixed traffic)

$$e = \frac{V^2}{225R}$$

$$= \frac{100^2}{225\times 358} = 0.124$$

As the value of 0.124 is higher than the maximum superelevation of 0.07, limit the value of e to 0.07. The curve should be safe for the full speed of 100 kmph as the ruling minimum radius has been adopted. However check for the transverse skid resistance.

$$f = \frac{V^2}{127R} - e$$

$$= \frac{100^2}{127\times 358} - 0.07 = 0.1499 < 0.15 \text{ (OK)}$$

As this value of lateral friction coefficient, f developed is less than 0.15, the superelevation value of 0.07 is safe.

3. Extra widening of pavement, w_e

Assume two lane pavement, i.e. w = 7.0 m, number of lanes, n = 2 and wheel base, l = 6 m.

Extra widening of pavement, $w_e = \dfrac{nl^2}{2R} + \dfrac{V(\text{in kmph})}{9.5\sqrt{R}}$

$$= \frac{2\times 6^2}{2\times 358} + \frac{100}{9.5\sqrt{358}} = 0.657 \text{ m}$$

Provide an extra width of 0.657 m and a total width of pavement, $B = w + w_e$

$$= 7 + 0.657 = 7.657 \text{ m}$$

4. Length of transition curve, L_s:

Transition curve length, L_s is to be designed considering:

- Rate of change of centrifugal acceleration.
- Rate of introduction of the amount of superelevation E and
- Minimum length of transition curve formula.

The highest of three values is adopted at the design length L_s

1. Based on rate of change of centrifugal acceleration, C.

$$C = \frac{80}{75+V} = \frac{80}{75+100} = 0.457 \not< 0.5$$

Hence adopt, $C = 0.5$

Now, $$L_s = \frac{0.0215V^3}{C \cdot R} = \frac{0.0215 \times 100^3}{0.5 \times 358} = 120.11 \text{ m} \simeq 121 \text{ m (say)}$$

2. Based on rate of introduction of the amount of superelevation.

$$L_s = e[w + w_e] \text{ N}$$

[N = 1 in 150 and pavement to be rotated about inner edge]

$$L_s = 0.07[7 + 0.657] \times 150 = 80.39 \text{ m} \simeq 81 \text{ m (say)}$$

3. Minimum value of L_s, as per IRC is given by

$$L_s = \frac{2.7V^2}{R}$$

$$= \frac{2.7 \times 100^2}{358} = 75.42 \text{ m} \simeq 76 \text{ m (say)}$$

Adopting the highest of three values, design length of transition curve = 121 m.

(ii)

Intermediate sight distance is twice the stopping sight distance.

Stopping sight distance, SSD is given by

$$\text{SSD} = 0.278Vt_r + \frac{V^2}{254f}$$

Assume, reaction time, t_r = 2.5 sec

Coefficient of longitudinal friction,

$$f = 0.35$$

$$\therefore \quad SSD = 0.278 \times 100 \times 2.5 + \frac{100^2}{254 \times 0.35}$$

$$= 181.99 \text{ m} \simeq 182 \text{ m (say)}$$

Now intermediate sight distance = 2 × (SSD)

$$= 2 \times (182) = 364 \text{ m}$$

∴ Safest intermediate sight distance of 364 m is provided for the given national highway.

Q.6 (b) Solution:

(i)

Length	Latitude (m)	Departure (m)	Correct Latitude (m)	Correct Departure (m)
AB	0.00	183.79	–0.0565	183.86
BC	128.72	98.05	128.67	98.11
CD	177.76	–140.85	177.69	–140.764
DE	–76.66	–154.44	–76.713	–154.375
EF	–177.09	0.00	–177.14	0.0671
FA	–52.43	13.08	–52.45	13.1
	$\Sigma Lat = 0.3$	$\Sigma Dep = -0.37$		

$$\text{Bearing of } AB = \tan^{-1}\left(\frac{D}{L}\right) = \tan^{-1}\left(\frac{183.79}{0}\right) = 90°$$

$$\text{Bearing of } BC = \tan^{-1}\left(\frac{98.05}{128.72}\right) = 37°17'51.52''$$

$$\text{Bearing of } CD = \tan^{-1}\left(-\frac{140.85}{177.76}\right) = 321°36'29.11''$$

$$\text{Bearing of } DE = \tan^{-1}\left(-\frac{154.44}{-76.66}\right) = 243°36'5.02''$$

$$\text{Bearing of } EF = \tan^{-1}\left(\frac{0.00}{-177.09}\right) = 180°$$

$$\text{Bearing of } FA = \tan^{-1}\left(\frac{13.08}{-52.43}\right) = 165°59'31.36''$$

Alternatively, length can be calculated as:

$$L = \sqrt{(\text{Lat})^2 + (\text{Dep})^2}$$

Length calculation:

$$L_{AB} = 183.79 \text{ m}$$

$$L_{BC} = \frac{98.05}{\sin 37°17'51.52''} = 161.81 \text{ m}$$

$$L_{CD} = \frac{-140.85}{\sin(321°36'29.11'')} = 226.798 \text{ m}$$

$$L_{DE} = \frac{-154.44}{\sin(243°36'5.02'')} = 172.42 \text{ m}$$

$$L_{EF} = 177.09 \text{ m}$$

$$L_{FA} = \frac{13.08}{\sin 165°59'31.36''} = 54.0369 \text{ m}$$

∴ ΣL = Length of perimeter = 975.94 m

Length	Correction in latitude $C_L = \Sigma Lat \times \frac{L}{\Sigma L}$	Correction in departure $C_D = \Sigma Dep \times \frac{L}{\Sigma L}$
AB	–0.0565	0.0697
BC	–0.0497	0.0613
CD	–0.0692	0.08598
DE	–0.053	0.0654
EF	–0.0544	0.0671
FA	–0.0166	0.0205

(ii)

1. Compound Curve

- A compound curve is the combination of two or more simple circular curves of different radii. Because of two different radii, the two centers will also be different.
- Due to different length of tangents, compound curve allows the fitting of location of a topography with enhanced refinement as compared to simple circular curve.
- As far as possible, where simple circular curve can be used, there compound curve should not be used.

2. Reverse Curve: Reverse curve consists of two curves with their centres on opposite side of the common tangent at the Point of Reverse Curvature (PRC). The radii of the two curves may be same or different.

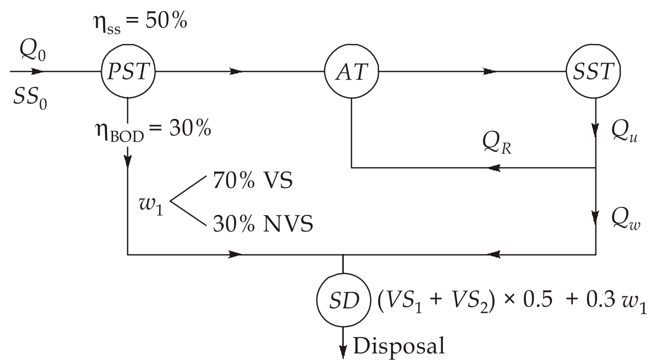
Uses: Reverse curve is used in the following situations:

(a) When the two straight lines are parallel to each other.

(b) When the angle between the two straight lines is very small.

Q.6 (c) Solution:

(i)



Weight of solids in sludge from PST = $Q \times SS_0 \times \eta_{SS0} \times 10^{-6}$ kg

$= 1000 \times 10^3 \times 225 \times 0.5 \times 10^{-6}$

$= 112.5$ kg/day

As 30% of BOD is removed in primary sedimentation tank, therefore 70% BOD is entering the aeration tank and excess activated sludge = 0.4 gm volatile solids produced per gm of BOD applied.

Now,BOD produced in from SST = $Q \times BOD_5 \times \eta_{v.s.}$

$= 1000 \times 10^3 \times 190 \times 0.7 \times 10^{-6} = 133$ kg/day

Excess activated sludge = $0.4 \times 133 = 53.2$ kg/day

Total volatile solids applied to digester = Volatile solids from PST + Volatile solids from SST

[70% of solids in PST sludge are volatile and remaining are non-volatile]

∴ Total volatile soils applied to digester = $112.5 \times 0.7 + 53.2 = 131.95$ kg/day

(b) **Weight of digested sludge**: Volatile solids from digested sludge + Non volatile solids from PST + Non-volatile solids from SST

As, 50% of volatile solids are digested in digestor.

Volatile solids from digested sludge = $0.5 \times 131.95 = 65.975$ kg/d

Non-volatile solids from PST are 30% of total solids removed.

Non-volatile solids from PST = $112.5 \times 0.3 = 33.75$ kg/day

All solids produced from SST are volatile in nature.

Non-volatile solids from SST = 0

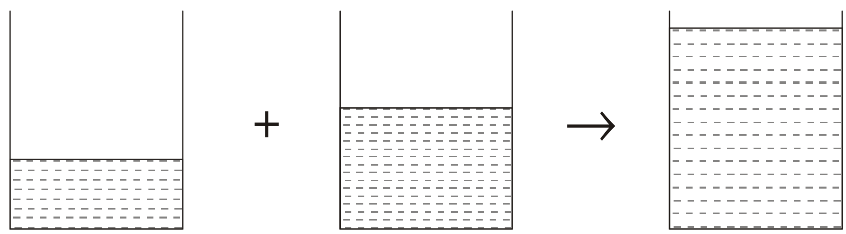
Weight of digested sludge = $65.975 + 33.75 + 0 = 99.725$ kg/day

$$\text{Volume of digested sludge} = \frac{100}{(100-P)\rho} \times w$$

$$= \frac{100}{6} \times \frac{99.725}{1 \times 10^3} = 1.662 \text{ m}^3/\text{day}$$

$$\text{Area of farmland} = \frac{1.662}{2} = 0.831 \text{ ha}$$

(ii)



BOD of seeded water, $(BOD)_{SW}$ = Drop in DO

= 1.5 mg/l

Now, $$(BOD)_{DS} = \frac{(BOD)_{TS} \times V_{TS} + (BOD)_{SW} \times V_{SW}}{V_{TS} + V_{SW}}$$

where $(BOD)_{TS}$ is BOD of test sample

and $(BOD)_{DS}$ = Drop in DO

= 6.8 mg/l

$$\therefore \quad 6.8 = \frac{(BOD)_{TS} \times 20 + 1.5 \times 280}{20 + 280}$$

$$\Rightarrow \quad (BOD)_{TS} = \frac{6.8 \times 300 - 1.5 \times 280}{20}$$

$$\Rightarrow \quad (BOD)_{TS} = 81 \text{ mg/l}$$

Q.7 (a) Solution:

Instrument P at station A and staff held vertical at B

$$s = s_3 - s_1$$

$$\Rightarrow \quad s = 1.795 - 1.090 = 0.705 \text{ m}$$

$$\theta = 5°44'$$

$$\therefore \quad AB = \text{Distance between A and B}$$

$$= D \cos \theta$$

$$= (Ks\cos\theta + C)\cos\theta = Ks\cos^2\theta + C\cos\theta$$
$$= 100 \times 0.705\cos^2 5°44' + 0.3\cos 5°44'$$
$$= 70.095 \text{ m}$$

Now, $V = Ks\cos\theta\sin\theta + C\sin\theta$
$$= 100 \times 0.705 \times \cos 5°44' \times \sin 5°44' + 0.3 \times \sin 5°44'$$
$$= 7.038 \text{ m}$$

$\therefore$ RL of B = RL of A + HI + V – s_2
$$= 100.00 + 1.400 + 7.038 - 1.440$$

$\Rightarrow$ RL of B = 106.998 m

Instrument Q at station A and staff held normal at B

$$AB = (Ks + C)\cos\theta + s_2\sin\theta$$
$$70.095 = (95s + 0.45)\cos 5°44' + s_2\sin 5°44'$$
$$\Rightarrow \quad 69.647 = 94.525s + 0.0999s_2 \quad \text{...(i)}$$

Now, $V = (Ks + C)\sin\theta$
$$= (95s + 0.45)\sin 5°44'$$

$\therefore$ RL of B = RL of A + HI + V – $s_2\cos\theta$
$$\Rightarrow \quad 106.998 = 100 + 1.450 + (95s + 0.45)\sin 5°44' - s_2\cos 5°44'$$
$$\Rightarrow \quad 9.4904s - 0.995s_2 = 5.50305 \quad \text{...(ii)}$$

From equation (i) and (ii),
$$s = 0.7352 \text{ m}$$
$$s_2 = 1.4821 \text{ m}$$

$\therefore$ Lower stadia wire reading $= s_2 - \frac{s}{2} = 1.4821 - \frac{0.7352}{2} = 1.1145$ m

Upper stadia wire reading $= s_2 + \frac{s}{2} = 1.4821 + \frac{0.7352}{2} = 1.8497$ m

$\therefore$ Staff readings with instrument Q are 1.1145, 1.4821 and 1.8497.

Q.7 (b) Solution:

(i)

The geometric design of hill roads is influenced by several factors including:

1. **Terrain**: The topography of the area, including the steepness of the hills and the presene of obstacles like cliffs or rivers, directly affects the roads alignment and slope.

2. **Traffic volume and composition**: The anticipated volume and types of vehicles using the road influence design considerations such as lane width, curvature and sight distance.
3. **Speed**: Desired operating speeds of vehicles on the road impact decisions regarding overtaking opportunities.
4. **Safety**: Safety concerns such as visibility around curves, adequate stopping sight distance, and the prevention of vehicle rollovers are crucial factors guiding the design of road.
5. **Environmental impact**: Minimizing the environmental impact of the road construction and operation, including considerations for erosion, habitat disruption, and runoff management is quite important.
6. **Cost and Budget**: Budget constraints and cost considerations influence decisions regarding the extent of earthwork, retaining structures, and the use of engineering solutions like tunnels or bridges.

   By considering these factors holistically, engineers can develop geometric designs, for hill roads that incorporates safety, functionality, environmental sustainability and cost effectiveness.

(ii)

The integration of psychological widening alongside mechanical widening can enhance the overall effectiveness. By considering both technical aspects (mechanical widening) and the human elements (psychological widening), you create a more holistic and adoptable approach, addressing not only functional requirements but also user experience, acceptance, and potential behavioral factors.

At horizontal curves drivers have a tendency to maintain a greater clearance between the vehicles than on straight stretches of road. Therefore an extra width of pavement is provided for psychological reasons for greater maneuverability of steering at higher speeds and to allow for the extra space requirements for the overhangs of vehicles.

An empirical formula has been recommended by the IRC for deciding the additional psychological widening 'W_{ps}' which is dependent on the design speed, V of the vehicle and the radius, R of the curve.

The psychological widening is given by the formula.

$$W_{ps} = \frac{V(\text{in kmph})}{9.5\sqrt{R}}$$

Q.7 (c) Solution:

(i)

$$\text{Cohesion, } C = \frac{q_u}{2} = \frac{65}{2} = 32.5 \text{ kN/m}^2$$

$$\text{F.O.S.} = 3$$

$$\Rightarrow \text{Mobilised value of cohesion, } C_m = \frac{32.5}{3} = 10.833 \text{ kN/m}^2$$

Let the length of the pile be 15 m and diameter be 0.5 m.

$$\text{Spacing, } S = 3d = 1500 \text{ mm (say)}$$

Let the number of piles = n, considering the piles to act individually, the load at failure is given by

$$(Q_u)_n = Q_u = nc\pi dL$$

$$\Rightarrow \quad 3500 = n \times 10.833 \times \pi \times 0.5 \times 15$$

$$\Rightarrow \quad n = 13.71$$

Let us take 16 number of piles for square arrangement.

Check for group action

$$B = 3S + d = 3 \times 1500 + 50 = 5000 \text{ mm} = 5 \text{ m}$$

$\therefore$ Load taken by group action $(Q_u)_g = 4BL \times C_m + A_b \times C_m N_c$

(taking into account the end bearing)

$$A_b = B \times B = (5)^2 = 25 \text{ m}^2, N_c = 9$$

$$(Q_u)_g = (4 \times 5 \times 15 \times 10.833) + (25 \times 10.833 \times 9)$$

$$= 5687.325 \text{ kN}$$

This is greater than 3500 kN. Hence safe.

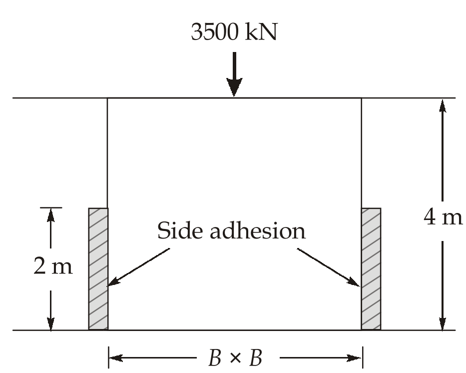
(ii)

$$\phi = 0°, N_c = 5.7, N_q = 1, N_\gamma = 0$$

$$C = 0.15 \text{ N/mm}^2 = 0.15 \times 10^3 \text{ kN/m}^2,$$

$$= 150 \text{ kN/m}^2$$

Given square footing



Ultimate bearing capacity, $q_{f1} = 1.3CN_c + qN_q + 0.4B\gamma N_\gamma$

$= (1.3 \times 150 \times 5.7) + (21 \times 4 \times 1) + (0.4 \times B \times 21 \times 0)$

$= 1111.5 + 84 = 1195.5 \text{ kN/m}^2$

Net ultimate bearing capacity,

$$q_{nf1} = 1195.5 - 21 \times 4 = 1111.5 \text{ kN/m}^2$$

Safe bearing capacity, $q_s = \dfrac{q_{nf_1}}{FOS} + \gamma D_f = \dfrac{1111.5}{3} + 84 = 454.5 \text{ kN/m}^2$

$\therefore$ Safe load, $Q_{S1} = q_s \times \text{base area} = 454.5 \times B^2$ kN

Additional load carrying capacity by side adhesion = $Q_{f2} = 4 \times B \times 2 \times 30$

$= 240$ B kN

Additional safe laod carrying capacity, $Q_{s2} = \dfrac{Q_{f2}}{FOS} = \dfrac{240B}{3} = 80B$

Total safe load carrying capacity, $Q_s = Q_{s1} + Q_{s2} = 454.5\,B^2 + 80B$ ...(1)

Actual load $= 3500 + 4 \times B^2 \times 24 = 3500 + 96B^2$ ...(2)

Equating (1) and (2),

$$454.5B^2 + 80B = 3500 + 96B^2$$

$\Rightarrow$ $B = 3.015$ m

Hence provide 3.1 m × 3.1 m size of square footing.

Q.8 (a) Solution:

(i)

Gradient: It is defined as rate of rise or fall along the length of the road with respect to the horizontal. It can be expressed as

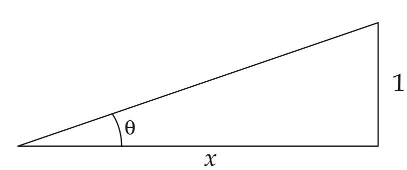
(a) Ratio of 1 in x(1 vertical unit to x horizontal unit)

(b) Percentage such as n% (n vertical unit to 100 horizontal unit)

If gradient of road is expressed as 1 in x and angle of gradient or slope angle is θ, then gradient may be expressed as tan θ as shown in figure below.

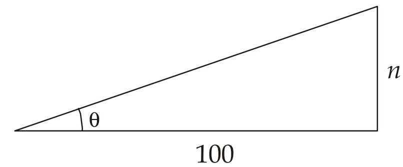
$$\tan\theta = \frac{1}{x}$$

$\Rightarrow$ $\theta \simeq \dfrac{1}{x}$ [∵ Angle θ small]



Gradient expressed as 1 in x

In common practice, values of gradient are very small. Therefore when gradient is expressed as $n\%$ it is understood that this $n\%$ is value of tangent of angle made by gradient with horizontal, i.e., gradient of $n\% = \tan\theta$ as shown in figure below.



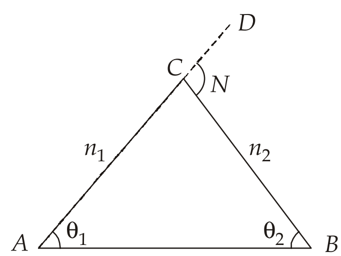
Gradient expressed as n %

Deviation Angle:

It is defined as measurement of change of direction of line at intersection of two grades. It is represented by N. The ascending gradients are given positive sign such as $+n_1\%$, $+n_2\%$ etc. while descending gradients are given negative sign such as $-n_1\%$, $-n_2\%$ etc.

In figure,

$$N = \angle DCB$$
$$= \angle BAC + \angle ABC$$
$$= +n_1 + n_2$$
$$= n_1 - (-n_2)$$



Deviation angle

There are different types of gradient:

1. **Ruling Gradient:** It is the maximum gradient within which designer wants to design the vertical profile of the road. Hence, it is also known as design gradients. It depends upon topography, length of the grade, design speed, pulling power of vehicle and presence of horizontal curves.
2. **Limiting Gradient:** It is steeper than ruling gradient and it is provided only when there is an enormous increase of cost of construction with ruling gradient. On rolling and hilly terrain, limiting gradient may be frequently adopted but the length of limiting gradient stretch should be restricted.
3. **Exceptional Gradient:** It is steeper than ruling gradient and limiting gradient and provided only, if the situation is unavoidable. Length of exceptional gradient stretch should not be more than 100 m.

4. **Minimum Gradient:** It is provided along the length of road for drainage purpose. Minimum gradient for cement concrete drain is 1 in 500, but on soil drains, stepper slopes upto 1 in 100 may be needed.

Gradients for different terrain types - Indian practice				
S.No.	**Terrain type**	**Ruling Gradient (%)**	**Limiting Gradient (%)**	**Exceptional Gradient (%)**
1.	Plain	3.3 (1 in 30)	5 (1 in 20)	6.7 (1 in 15) (for short distance not exceeding 100 m at a stretch)
2.	Rolling	3.3 (1 in 30)	5 (1 in 20)	6.7 (1 in 15) (for short distance not exceeding 100 m at a stretch)
3.	Mountaneous	5 (1 in 20)	6 (1 in 16.7)	7 (1 in 14.3) (for a distance not more than 100 m at a stretch)
4.	Steep (i) Upto 3000m height above mean sea level. (ii) Above 3000m height above mean sea level.	5 (1 in 20) 6 (1 in 16.7)	6 (1 in 16.7) 7 (1 in 14.3)	7 (1 in 14.3) (for a distance not more than 100 m at a stretch) 8 (1 in 12.5) (for a distance not more than 100 m at a stretch)

(ii)

∵ The stopping sight distance (SSD) of 100 m is less than the circular curve length of 250 m.

Now,
$$\frac{S}{\alpha} = \frac{2\pi(R-d)}{360°}$$

$$\Rightarrow \quad \frac{100}{\alpha} = \frac{2\pi(500-1.9)}{360°}$$

$$\Rightarrow \quad \alpha = 11.503°$$

Now setback distance, m is given by

$$m = R-(R-d)\cos\frac{\alpha}{2}$$

$$= 500-(500-1.9)\cos\left(\frac{11.503°}{2}\right)$$

$$= 4.407 \text{ m}$$

Q.8 (b) (i) Solution:

The total disturbing moment is caused by the tangential disturbing force, given as 850 kN/m

The disturbing moment, M_d = 850 × r = 850 × 10 = 8500 kN-m per m length of slope.

The resisting moment is provided by the shear strength developed along the failure arc AB.

$$\text{Resisting moment, } M_r = C\bar{L} + N\tan\phi$$

Normal effective pressure along AB = 225 kN/m^2

Total normal effective force, $N = 225 \times \overline{AB}$

But $$\overset{\frown}{AB} = 2\pi r\frac{\theta°}{360°} = 2\pi\times 10\times\frac{45°}{360°} = 2.5\pi$$

∴ $$N = 225 \times 2.5\pi$$
$$= 1767.146 \text{ kN/m length of slope}$$

$$\text{Shearing resistance} = (55 \text{ kN/m}^2 \times 2.5\pi) + (1767.146 \text{ kN/m} \times \tan 15°)$$
$$= 905.47 \text{ kN per m length of slope}$$

$$\text{Resisting moment} = 905.47 \times r = 905.47 \times 10$$
$$= 9054.7 \text{ kN-m/m run}$$

1. FOS w.r.t. shear strength $= \dfrac{M_r}{M_d} = \dfrac{9054.7}{8500} = 1.065$

2. To calculate FOS with respect of height of cut, we have to find the maximum depth of cut which will be self supporting. This height is given as $2z_0$.

where $$z_0 = \frac{2C}{\gamma\sqrt{k_a}}$$

Maximum depth of unsupported cut = $2z_0 = \dfrac{2\times 2\times 55}{17\times\sqrt{k_a}}$

where $$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 15°}{1+\sin 15°} = 0.589$$

$$\text{Maximum depth of cut} = \frac{4\times 55}{17\times\sqrt{0.589}} = 16.86 \text{ m}$$

∴ Since the depth of cut is 6 m.

∴ Factor of safety w.r.t. height of cut = $\dfrac{16.86}{6} = 2.81$

(ii)

Sludge Treatment Process: A variety of sludge treatment processes have been developed and applied to wastewater treatment operations. Sludge treatment process normally comprises of the following five major stages:

(i) Concentration or thickening
(ii) Stabilisation
(iii) Conditioning
(iv) Dewatering
(v) Reduction or drying

The sludge processing stages are briefly described as follows:

(a) **Thickening :** Separating solids as much as possible by centrifuges, gravity flotation and clarifier.

(b) **Stabilisation or Digestion :** Converting the organic solids to more refractory (inert) forms so that they can be handled or disposed off or used as a soil conditioner without causing nuisance or health hazard.

(c) **Conditioning :** Treating the sludge with chemicals or heat so that the water can be readily separated.

(d) **Dewatering :** Separating water by subjecting the sludge to vacuum, pressure or drying.

(e) **Reduction :** Converting the sludge to a stable form by wet oxidation or incineration.

- Wastewater and sludge treatment and disposal system must be considered together to ensure the most efficient utilization of the resources in meeting treatment requirements. Some of the pertinent technical and nontechnical factors influencing and determining the system suitability and choice include:

  (a) Effectiveness of system
  (b) Impenetrability
  (c) Comparability
  (d) Economic aspects
  (e) Environmental impacts
  (f) Energy requirement
  (g) Administrative factors

Q.8 (c) Solution:

(i)

Let α is the maximum permissible angular error

l = length of the offset = 25 m

Displacement of the point due to incorrect direction.

$$= l \sin \alpha$$

$$= 25 \sin\alpha$$

Maximum error in the length of offset = 0.5 m.

Displacement of the point due to both errors on paper

$$= \frac{\sqrt{(25\sin\alpha)^2 + (0.5)^2}}{75}$$

But maximum displacement is not to exceed 0.025 cm i.e.,

$$\frac{\sqrt{(25\sin\alpha)^2 + (0.5)^2}}{75} = 0.025$$

$$\Rightarrow \quad 625 \sin^2\alpha = (0.025 \times 75)^2 - (0.5)^2$$

$$\Rightarrow \quad 25 \sin\alpha = 1.807$$

$$\therefore \quad \alpha = 4.15°$$

(ii)

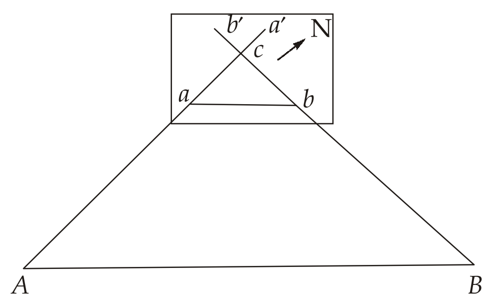
Radiation: In the radiation method, the points are located by drawing radial lines from the plane table station. The distances of these points are measured and scaled off on the respective radial lines to locate the points. The radiation method is quite suitable for survey of small areas which can be controlled from a central station using the plane table. The scope of this method is increased when the distances are measured with the help of a tacheometer.

Traversing: This method is similar to compass or theodolite traversing. A plane table traverse is a very rough type of traverse and is used generally for depicting the topographical details directly on the plane table. Traverse consists of a series of straight lines connected together. In plane table traversing, the angles are not measured but are in fact plotted directly. It is run between the stations whose positions have previously been fixed by some other precise methods like the theodolite traverse or the triangulation. The plane table is set successively on the traverse stations and back sight is taken on the preceding station followed by foresight on the following station. The measured traverse lines are plotted directly on the paper to some suitable scale.

Intersection: In this method, two stations are selected in such a away that all other stations to be plotted are visible from these stations. A line joining these two stations is called base line. The length of this base line is measured very accurately. Rays are drawn from these stations to the station to be plotted. The intersection of the rays from these two stations gives the position of the station to be plotted on the drawing sheet. Sometimes, this method is also called as graphical triangulation.

Resection: In resection, the position of the station that is being occupied by the instrument (i.e. plane table) is located with respect to the stations whose locations have already been plotted. As shown in figure, plane table occupies the station *C* whose position has not been plotted on the paper when the table occupied other stations. For locating the position of station *C* on the paper, the following procedure is adopted:

- Let there be two stations *A* and *B* whose positions '*a*' and '*b*' has been marked on the paper and both the stations are visible from station *C*. In case the orientation of plane table at station *C* is correct then the intersection of the rays from '*a*' and '*b*' will give the location of station *C* i.e. '*c*'. Thus the main problem reduces to that of obtaining the correct orientation at *C*.



Resection method of orienting the plane table

(iii)

1. **Work from Whole to Part**

- It is the first principle of surveying. This principle means that the surveyor should first establish the large frame work consisting of main control points, accurately. In between the large frame work so established, subsidiary small frame works can be established by a relatively less accurate survey. By doing so, the errors in small frame work get localized and are not magnified and thus the accumulation of errors gets confined. In the reverse process of working from part to whole, small errors get magnified due to accumulation of errors from small frame work to large frame work.

2. **Locate a point by atleast two measurements**

- According to this principle, the new point (station) should always be fixed by atleast two measurements (linear or angular) from a fixed reference point.As per this principle,a point is plotted with the help of two other measurements.

■■■■