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Detailed Solutions

ESE-2024 Mains Test Series

Mechanical Engineering Test No : 3

Section A : Fluid Mechanics and Turbo Machinery [All Topics]
Section B : Heat Transfer-1 + Refrigeration and Air-Conditioning-1 [Part Syllabus]
Thermodynamics-2 + Strength of Materials & Mechanics-2 [Part Syllabus]

Section : A

1. (a)

Consider the figure, $\sin\theta = \dfrac{1.5}{5}$

$\Rightarrow$ $\theta = 17.457°$

Horizontal component of force on the gate,

$$F_H = \rho g A\bar{h}$$

$$= 9.81 \times 10^3 \times (3 \times 1) \times 1.5 = 44.145 \text{ kN}$$

Vertical component of the force on the gate,

$$F_V = \rho g[\text{Sector OMSN} - \Delta\text{OMN}]$$

$$= 10^3 \times 9.81\left[\frac{2 \times 17.457}{360} \times \pi \times 5^2 - \frac{1}{2} \times 3 \times 5\cos 17.457°\right]$$

$$= 4.54 \text{ kN}$$

Resultant force on the gate, $R = \sqrt{F_H^2 + F_V^2} = \sqrt{44.145^2 + 4.54^2} = 44.38$ kN Ans.

If R is inclined at an angle α to the horizontal,

$$\tan\alpha = \frac{F_V}{F_H} = \frac{4.54}{44.145}$$

$\Rightarrow$ $\alpha = 5.87°$ Ans.

Since the surface of the gate is cylindrical, the resultant R passes through the centre of curvature 'O'.

1. (b)

Pulse jet engine figure (a) is having diffuser at intake, combustion chamber and exhaust nozzle, it has mechanically operated flapper valve grids which can allow or stop air flow in the combustion chamber. Thus pulse jet is an intermittent flow, compressor less type of device with minimum number of moving parts.

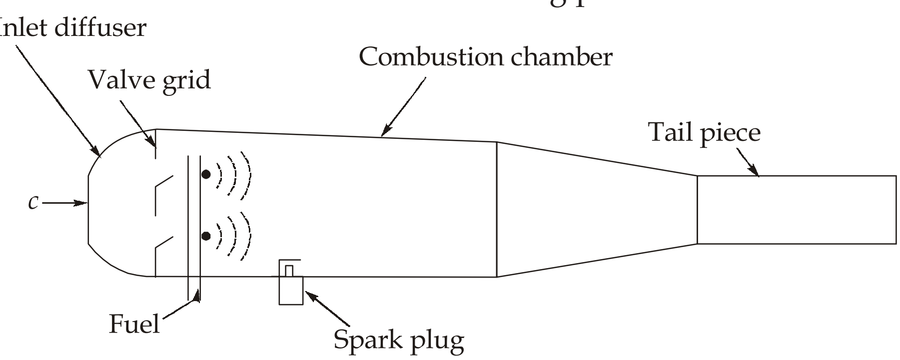


Figure (a) : The pulse jet engine

The basic features of the pulse jet engine are illustrated in figure (a). It consists essentially of the following parts:

(i) a diffuser,

(ii) a valve grid which contains springs that close on their own spring pressure,

(iii) a combustion chamber,

(iv) a spark plug, and

(v) a tail pipe or discharge nozzle

The theoretical and actual P-V diagrams of the pulse jet engine are shown in figure (b) and figure (c) respectively.

The operation of the pulse jet is as follows : During starting compressed air is forced into the inlet which opens the spring loaded flapper valve grid, the air enters combustion chamber into which fuel is injected and burnt with the help of a spark plug. Combustion occurs with a sudden explosion process 2-3 in figure (b), i.e., the combustion is at constant-volume instead of at constant-pressure as in other propulsive devices. The pulse jet cycle is more near to Otto cycle. Ram action can also be used to increase the pressure of the cycle figure (b).

The function of the diffuser is to convert the kinetic energy of the entering air into static pressure rise by slowing down the air velocity. When a certain pressure difference builds up across the valve grid, the valves will open. This makes the fresh air to enter the combustion chamber, where fuel is mixed with the air and combustion starts. To start the combustion initially the spark plug is used. Once the combustion starts it proceeds at constant-volume. Thereby, there is a rapid increase in pressure, which causes the valve to close rapidly. The products of combustion surges towards the nozzle. They expand in the nozzle and escape into the atmosphere with a higher velocity so that the exit velocity is much higher than the inlet velocity.

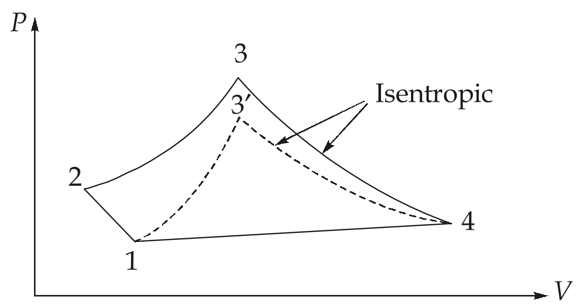


Figure (b) : Theoretical pulse jet cycle on P-V diagram

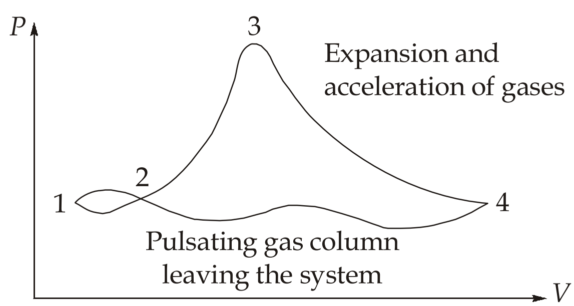


Figure (c) : Actual pulse jet cycle on P-V diagram

Thus, the rate of momentum of the working fluid is changed so as to cause a propulsive thrust.

1. (c)

Given : $\theta = 60°$; $V = 10$ m/s; $d = 8$ cm

In a free jet the pressure is atmospheric throughout the trajectory.

$$V_x = V\cos\theta = \text{constant}$$

$$V_y = V\sin\theta$$

$$x = V_x \cdot t$$

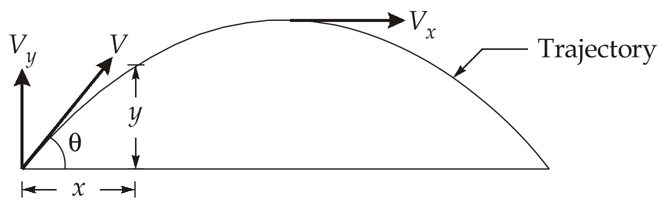
$$\Rightarrow \quad t = \frac{x}{V_x}$$

Now, $$y = V_y \cdot t - \frac{1}{2}gt^2$$

or $$y = V_y \cdot \frac{x}{V_x} - \frac{1}{2}g\left(\frac{x}{V_x}\right)^2$$

∴ $$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

Equation of the trajectory, $$y = x\tan 60 - \frac{9.81x^2}{2\times 10^2 \times \cos^2 60}$$



∴ $$y = 1.732x - 0.1962x^2$$ Ans. (i)

At the point of maximum elevation, $V_y = 0$

∴ $$y_{max} = \frac{V_y^2}{2g} = \frac{(10\sin 60)^2}{2\times 9.81}$$

∴ $$y_{max} = 3.82 \text{ m}$$ Ans. (ii)

If d_x is the diameter of the jet at the point of maximum elevation.

$$V_x\left(\frac{\pi}{4}d_x^2\right) = V\left(\frac{\pi}{4}d^2\right)$$

or $$(10\cos 60°) \times d_x^2 = 10 \times 8^2$$

or $$d_x = 8\times\sqrt{\frac{10}{5}}$$

∴ $$d_x = 11.3 \text{ cm}$$ Ans. (ii)

1. (d)

Case (i) Work done against friction without air vessels

For a single-acting reciprocating pump without any air vessel on the suction and delivery pipes, the velocity of flow in the pipes is given by,

$$V = \frac{A_P}{A}\omega r\sin\theta$$

Loss of head due to friction, $h_f = \dfrac{4fl}{2gd}V^2$

$$= \frac{4fl}{2gd}\left(\frac{A_p}{A}\omega r \sin\theta\right)^2 \quad ...(i)$$

The variation of friction loss h_f during a stroke is parabolic, and area under a parabolic curve is two-thirds of the enclosing rectangle. Therefore, area representing work done by pump per stroke against friction.

$$= \frac{2}{3} \times \text{Base} \times \text{Height}$$

The base of parabola equals the stroke length L, and the height equals friction head h_f at $\theta = -90°$. Hence

$$\text{Area} = \frac{2}{3} \times L \times \frac{4fl}{2gd}\left(\frac{A_p}{A}\omega r\right)^2 \quad (\because \ \sin 90° = 1)$$

Work done or power expended against friction

$$P_1 = \frac{wAN}{60} \times \frac{2}{3} \times L \times \frac{4fl}{2gd}\left(\frac{A_p}{A}\omega r\right)^2$$

$$= W \times \frac{4fl}{3gd}\left(\frac{A_p}{A}\omega r\right)^2 \quad ...(ii)$$

where $W = \dfrac{wALN}{60}$ represents the liquid discharged by the pump per second.

Case (ii) Work done against friction with air vessels of adequate capacity:

When air vessels of adequate capacity are installed on the suction and delivery pipes, the flow velocity in the suction and delivery pipes can be assumed to be uniform and equal to the mean flow velocity.

Discharge = Piston area × Stroke length × Revolutions per second

$$= A_p \times L \times \frac{N}{60} = A_p \times 2r \times \frac{\omega}{2\pi}$$

Constant flow velocity in the pipe $= \dfrac{\text{Discharge}}{\text{Pipe area}} = \dfrac{A_p \times 2r \times \dfrac{\omega}{2\pi}}{A} = \dfrac{A_p \omega r}{A\pi}$

Loss of head due to friction, $h_f = \dfrac{4flV^2}{2gd} = \dfrac{4fl}{2gd}\left(\dfrac{A_p}{A}\dfrac{\omega r}{\pi}\right)^2$ ...(iii)

Evidently with the fitting of air vessels, the head loss due to friction is independent of θ and hence the indicator plot is a rectangle.

Area of indicator plot = Base × Height

$$= L \times \frac{4fl}{2gd}\left(\frac{A_p}{A}\frac{\omega r}{\pi}\right)^2$$

∴ Work done or power expended,

$$P_2 = \frac{wAN}{60} \times L \times \frac{4fl}{2gd}\left(\frac{A_p}{A}\frac{\omega r}{\pi}\right)^2$$

$$= W \times \frac{4fl}{2gd}\left(\frac{A_p}{A}\frac{\omega r}{\pi}\right)^2 \quad \text{...(iv)}$$

$$\text{Ratio } \frac{P_2}{P_1} = \frac{\text{Work done against friction with air vessels}}{\text{Work done against friction without air vessels}}$$

$$= \frac{W \times \frac{4fl}{2gd}\left(\frac{A_p}{A}\frac{\omega r}{\pi}\right)^2}{W \times \frac{4fl}{3gd}\left(\frac{A_p}{A}\times \omega r\right)^2} = \frac{3}{2\pi^2}$$

1. (e)

The essential boundary conditions for a laminar boundary layer are:

- $u = 0 \text{ and } \dfrac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = 0$
- $u = U_0 \text{ at } y = \delta$

(i)

$$\frac{u}{U_0} = 1 + \frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^2$$

$$\frac{\partial u}{\partial y} = U_0\left[\frac{1}{\delta} - \frac{4y}{\delta^2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = U_0\left(-\frac{4}{\delta^2}\right)$$

At $y = 0;$ $u = U_0 \neq 0$ and $\frac{\partial^2 u}{\partial y^2} = -\frac{4U_0}{\delta^2} \neq 0$

At $y = \delta$ $u = 0 \neq U_0$

Obviously, the BCs are not satisfied and hence the given velocity profile is not a proper distribution in a laminar boundary layer.

(ii)
$$\frac{u}{U_0} = \sin\left(\frac{\pi y}{2\delta}\right)$$

$$\frac{\partial u}{\partial y} = U_0 \frac{\pi}{2\delta}\cos\left(\frac{\pi y}{2\delta}\right);$$

$$\frac{\partial^2 u}{\partial y^2} = -\frac{\pi^2 U_0}{4\delta^2}\sin\left(\frac{\pi y}{2\delta}\right)$$

At $y = 0;$ $u = U_0$ and $\frac{\partial^2 u}{\partial y^2} = 0$

At $y = \delta,$ $u = U_0 \sin\left(\frac{\pi}{2}\right) = U_0$

Hence, the essential BCs are satisfied.

2. (a)

Given : Number of nozzles, $n = 4$; Nozzle diameter, $d_j = 52$ mm $= 0.052$ m; Coefficient of velocity, $c_v = 0.98$, Bucket mean diameter, $D = 0.85$ m; Relative reduction factor, $k = (1 - 0.16) = 0.84$; Jet deflection, $\phi = 180° - 165° = 15°$; Mechanical efficiency, $\eta_{mech} = 94\%$

Available head at the nozzle, $H = 300 - 32 = 268$ m

$$\text{Velocity of jet, } V_1 = c_v\sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times 268} = 71.06 \text{ m/s}$$

Bucket speed, $u = 0.46 \times$ Jet velocity

$= 0.46 \times 71.06 = 32.69$ m/s

Total quantity of water flowing through the pipeline,

$$Q_T = n \times A \times V_1$$

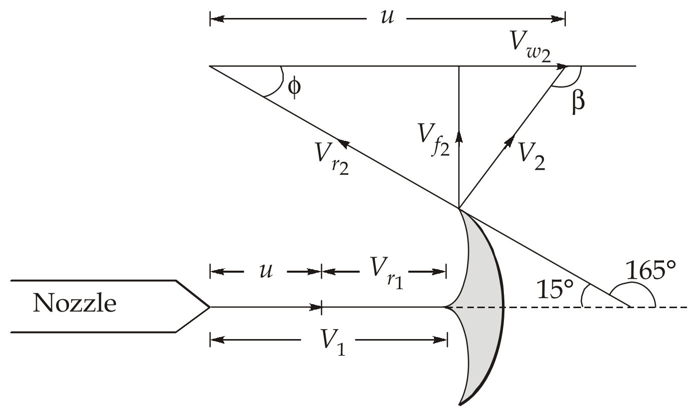
$$= 4 \times \left(\frac{\pi \times 0.052^2}{4}\right) \times 71.06 = 0.6036 \text{ m}^3/\text{s}$$

From Darcy equation for head loss through the pipeline,

$$h_f = \frac{4flV^2}{2gd} = \frac{32flQ^2}{\pi^2 g d^5}$$

$$d = \left[\frac{32 f L Q^2}{\pi^2 g h_f}\right]^{1/5} = \left[\frac{32 \times 0.0058 \times 360 \times 0.6036^2}{\pi^2 \times 9.81 \times 3 \times 32}\right]^{1/5}$$

$\Rightarrow$ $d = 0.38$ m Ans.



At inlet to turbine :
$$V_{w_1} = V_1 = 71.06 \text{ m/s}$$
$$V_{r_1} = V_1 - u = 71.06 - 32.69$$
$$= 38.37 \text{ m/s} \qquad (\because \quad u_1 = u_2 = u)$$

At exit from turbine:

$$V_{r_2} = kV_{r_1} = 0.84 \times 38.37 = 32.23 \text{ m/s}$$
$$V_{r_2}\cos\phi = 32.23 \times \cos 15° = 31.13 \text{ m/s}$$

As $V_{r_2}\cos\phi$ is less than blade speed u_2, the velocity triangle at outlet will be as in above figure ($\beta > 90°$).

$$V_{w_2} = u_2 - V_{r_2}\cos\phi$$
$$= 32.69 - 31.13 = 1.56 \text{ m/s} \qquad [\because \; u_1 = u_2 = u]$$

$\therefore$ Bucket power, $RP = \rho Q(V_{w_1} - V_{w_2}) \cdot u$
$$= 1000 \times 0.6036 \times (71.06 - 1.56) \times 32.69$$
$$= 1371.35 \text{ kW}$$ Ans.

Also, water power, $WP = \rho Q g H$
$$= 1000 \times 0.6036 \times 9.81 \times 268$$
$$= 1586.91 \text{ kW}$$

$\therefore$ Hydraulic efficiency, $\eta_H = \dfrac{\text{Bucket power}}{\text{Water power}} = \dfrac{1371.40}{1586.91}$
$$= 0.8642 \text{ or } 86.42\%$$ Ans.

Overall efficiency, $\eta_0 = \eta_H \cdot \eta_m \cdot \eta_v$

Assuming that there is no leakage and volumetric efficiency equals 100%, then

$$\eta_0 = \eta_m \cdot \eta_H = 0.8642 \times 0.94$$
$$= 0.8123 \text{ or } 81.23\% \quad \text{Ans.}$$

Rotational speed of the wheel, $N = \dfrac{60 \times u}{\pi D} = \dfrac{60 \times 32.69}{\pi \times 0.85} = 734.51 \text{ rpm}$

Shaft power available, P = Bucket power × η_0

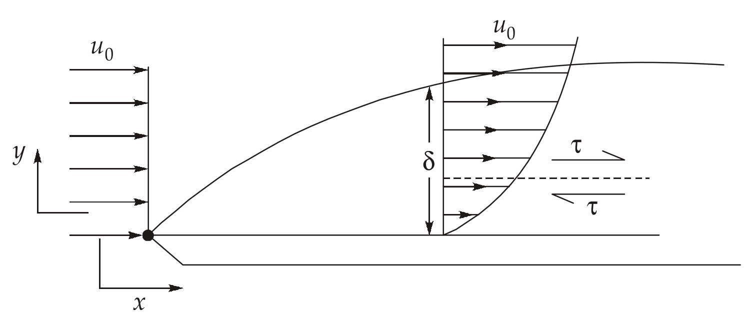
$$= 1371.40 \times 0.8123$$
$$= 1113.988 \text{ kW}$$
$$\simeq 1114 \text{ kW}$$

Unit speed, $N_u = \dfrac{N}{\sqrt{H}} = \dfrac{734.51}{\sqrt{268}} = 44.87$ **Ans.**

Unit power, $P_u = \dfrac{P}{H^{3/2}} = \dfrac{1114}{(268)^{3/2}} = 0.254$ **Ans.**

Specific speed, $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{734.51 \times \sqrt{1114}}{(268)^{5/4}} = 22.61$ **Ans.**

2. (b) (i)



1. **Boundary Layer Separation :** Consider flow over the flat plate in figure. When fluid particles make contact with the surface they attains zero velocity. These particles then act to retard the motion of particles in the next layer, and so on until, at a distance $y = \delta$ from the surface the effect becomes negligible. This retardation of fluid motion is associated with shear stress (τ) acting in planes that are parallel to velocity. With increasing distance y from the surface, the x velocity component of the fluid increases until it approaches the free stream value U_∞. The quantity δ is termed as boundary layer thickness and it is defined as the value of y for which u = $0.99U_\infty$.

2. Methods of controlling the Boundary Layer :

- **Motion of solid boundary :** The formation of the boundary layer is due to the difference between the velocity of the flowing fluid and that of solid boundary. As such it is possible to eliminate the formation of a boundary layer by causing the solid body to move with the fluid e.g. : Rotating Cylinder.
- **Acceleration of the fluid in the Boundary Layer.** This method consist of supplying additional energy to the particles of fluid which are being retarded in the boundary layer. This may be achieved either by injecting fluid into the region of boundary layer from the interior of the body with the help of some suitable device or by directing a portion of the fluid of the main stream from the region of high pressure to the retarded region.
- **Suction of fluid from the Boundary Layer.** In this method the slow moving fluid in the boundary layer is removed by suction through slots or through a porous surface, so that on the down stream of the point of suction a new boundary layer starts developing which is able to withstand an adverse pressure gradient and hence separation is prevented.
- **Streamline of body shapes.** By the use of suitable shaped bodies the point of transition of the boundary layer from laminar to turbulent can be moved down stream which results in the reduction of the skin friction drag. Furthermore by stream lining of body shapes the separation may be eliminated.

2. (b) (ii)

Head loss is given by, $h_f = \dfrac{fLV^2}{2gD} = \dfrac{fLQ^2}{2gD\left(\frac{\pi}{4}D^2\right)^2}$

or
$$h_f = \frac{8fLQ^2}{\pi^2 gD^5}$$

Case I :
$$h_1 = \frac{8fLQ_1^2}{\pi^2 gD_1^5} = \frac{8fLQ_2^2}{\pi^2 gD_2^5}$$

or
$$\left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{D_1}{D_2}\right)^5$$

$\therefore$
$$Q_1 = Q_2\left(\frac{D_1}{D_2}\right)^{5/2}$$

Also,
$$Q_1 + Q_2 = Q$$

or $$Q_2\left[\left(\frac{D_1}{D_2}\right)^{5/2}+1\right] = Q$$

Substituting, $$D_1 = 1.5D_2$$

∴ $$Q = Q_2\left[(1.5)^{5/2}+1\right]$$

or $$Q_2 = \frac{Q}{3.755}$$

∴ $$h_1 = \frac{8fLQ^2}{\pi^2 g D_2^5 \times 3.755^2}$$

∴ $$h_1 = \frac{1}{14.1}\left(\frac{8fLQ^2}{\pi^2 g D_2^2}\right) \quad \text{...(i)}$$

Case II :

$$h_2 = \frac{8fLQ^2}{\pi^2 g D_1^5} + \frac{8fLQ^2}{\pi^2 g D_2^5} = \frac{8fLQ^2}{\pi^2 g}\left[\frac{1}{D_1{}^5}+\frac{1}{D_2{}^5}\right]$$

$$h_2 = \frac{8fLQ^2}{\pi^2 g D_2^5}\left[\frac{1}{1.5^5}+1\right]$$

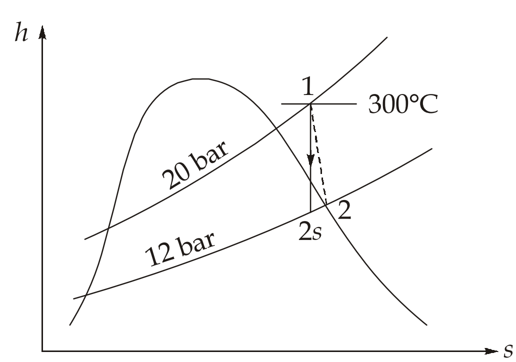
∴ $$h_2 = 1.1317 \times \frac{8fLQ^2}{\pi^2 g D_2^5} \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\frac{h_1}{h_2} = \frac{1}{14.1}\times\frac{1}{1.1317} = 0.0626$$ Ans.

2. (c)

Given : η_{nozzle} = 96%; α = 18°; k = 0.88; $\dot{m}$ = 1440 kg/hr



At 20 bar and 300°C :

$$h_1 = 3024.2 \text{ kJ/kg}$$

The state "2s" is in the superheated region

$$t_{2s} = 240°C; h_{2s} = 2912.7 \text{ kJ/kg}$$

$$V_1 = \sqrt{2000 \times (h_1 - h_{2s}) \times \eta_{nozzle}}$$

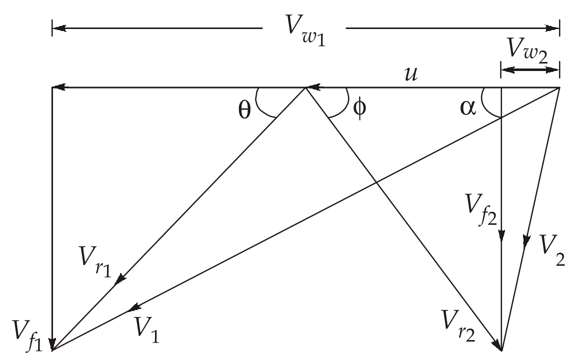
$$= \sqrt{2000 \times (3024.2 - 2912.7) \times 0.96} = 462.69 \text{ m/s}$$

For maximum work output,

$$\frac{u}{V_1} = \frac{\cos\alpha}{2}$$

$$\Rightarrow \quad u = V_1 \times \frac{\cos\alpha}{2}$$

$$\Rightarrow \quad u = 462.69 \times \frac{\cos 18°}{2} = 220.02 \text{ m/s}$$



$$V_{w_1} = V_1 \cos\alpha = 462.69 \cos 18° = 440.04 \text{ m/s}$$

$$V_{f_1} = V_1 \sin\alpha = 462.69 \sin 18° = 142.98 \text{ m/s}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u} = \frac{142.98}{440.04 - 220.02}$$

$$\Rightarrow \quad \theta = 33.02°$$

$$\phi = \theta - 6° = 33.02° - 6° = 27.02°$$

$$\sin\theta = \frac{V_{f_1}}{V_{r_1}}$$

$$\Rightarrow \quad V_{r_1} = \frac{142.98}{\sin 33.02°} = 262.38 \text{ m/s}$$

$$V_{r_2} = k \cdot V_{r_1} = 0.88 \times 262.38 = 230.89 \text{ m/s}$$

$$V_{f_2} = V_{r_2} \times \sin\phi = 230.89 \times \sin 27.02 = 104.89 \text{ m/s}$$

$$\Delta V_w = V_{r_1} \times \cos\theta + V_{r_2} \times \cos\phi$$

$$= 262.38 \times \cos 33.02° + 230.89 \cos 27.02°$$

$$= 425.69 \text{ m/s}$$

$\therefore$ Axial thrust, $F_a = \dot{m} \times \left(V_{F_1} - V_{F_2}\right)$

$$= \frac{1440}{3600} \times (142.98 - 104.89) = 15.24 \text{ N} \qquad \text{Ans.}$$

Diagram power, $P = \dot{m} \cdot u \cdot \Delta V_w$

$$= \frac{1440}{3600} \times 220.02 \times 425.69 \times 10^{-3}$$

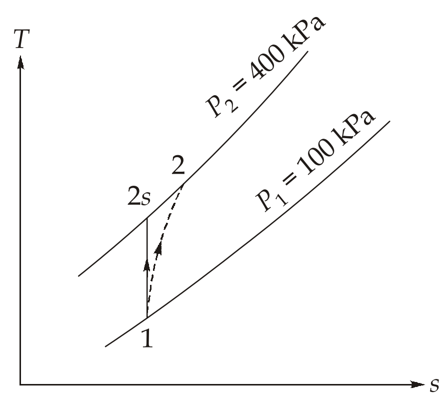
$$= 37.46 \text{ kW} \qquad \text{Ans.}$$

$$\text{Diagram efficiency, } \eta_{\text{dig.}} = \frac{P}{\frac{\dot{m}V_1^2}{2}} = \frac{37.46 \times 10^3}{\frac{1440}{3600} \times \frac{462.69^2}{2}}$$

$$= 0.8749 \text{ or } 87.49\% \qquad \text{Ans.}$$

3. (a)

Given : $N = 10500$ rpm; $FAD = \dot{V}_t = 840 \text{ m}^3/\text{min}$; $P_1 = 100$ kPa; $T_1 = 7°\text{C} = 280$ K; $r_p = 4$; $\eta_{\text{ise}} = 84\%$; $V_{f_1} = V_{f_2} = V_f = 62$ m/s; $\phi_s = 0.92$; $k_1 = 0.86$; $R_2 = 2 \times R_1$



$$\frac{T_{2s}}{T_1} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \quad T_{2s} = 280 \times (4)^{\frac{0.4}{1.4}} = 416.08 \text{ K}$$

$$\eta_{\text{ise}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$T_2 = \frac{T_{2s} - T_1}{\eta_{ise}} + T_1 = \frac{416.08 - 280}{0.84} + 280$$

$$T_2 = 442 \text{ K} \qquad \text{Ans.}$$

$$\text{Mass flow rate of air, } \dot{m} = \frac{P_1\dot{V}_t}{RT_1} = \frac{100 \times 840}{0.287 \times 280 \times 60} = 17.42 \text{ kg/s}$$

$$\text{Power required, } P = \dot{m}c_p(T_2 - T_1)$$

$$= 17.42 \times 1.005 \times (442 - 280)$$

$$= 2836.15 \text{ kW}$$ **Ans.**

$$\text{Power required, } P = \dot{m}\phi_s \times u_2^2$$ [Assuming radial inlet]

$$\Rightarrow \quad 2836.15 \times 10^3 = 17.42 \times 0.92 \times u_2^2$$

$$\Rightarrow \quad u_2 = 420.67 \text{ m/s}$$

$$\Rightarrow \quad \frac{\pi D_2 N}{60} = 420.67$$

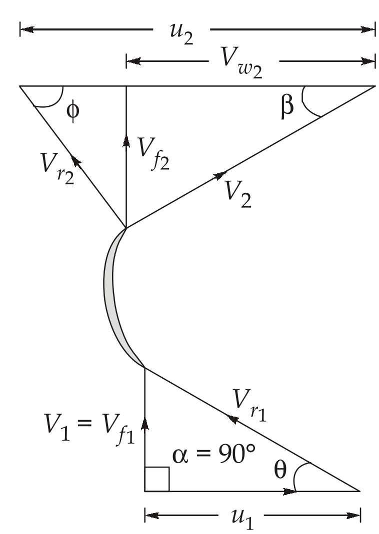
$$\Rightarrow \quad D_2 = \frac{420.67 \times 60}{\pi \times 10500} = 0.765 \text{ m}$$ **Ans.**

$$\Rightarrow \quad D_1 = \frac{D_2}{2} = \frac{0.765}{2} = 0.3826 \text{ m}$$ **Ans.**

Also, $$\dot{V}_t = k_1 \cdot \pi B_1 D_1 \times V_{f_1}$$

$$\Rightarrow \quad \frac{840}{60} = 0.86 \times \pi \times B_1 \times 0.38 \times 62$$

$$\Rightarrow \quad B_1 = 0.22 \text{ m}$$ **Ans.**



$$\tan\theta = \frac{V_{f_1}}{u_1} = \frac{62}{\left(\frac{\pi D_1 N}{60}\right)} = \frac{62 \times 60}{\pi \times 0.3826 \times 10500}$$

$\Rightarrow$ $\theta = 16.42°$ Ans.

$$V_{w_2} = \phi_s \times u_2 = 0.92 \times 420.67 = 387.02 \text{ m/s}$$

$$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = \frac{62}{387.02}$$

$$\beta = 9.10°$$ Ans.

3. (b)

For the cone, $D = 30$ cm; $H = 20$ cm; $s = 0.8$

Let θ = Semi-vertex angle

$\therefore$ $$\tan\theta = \frac{15}{20}$$

$\Rightarrow$ $$\theta = 36.87°$$

Diameter of cone at water surface,

$$d = 2y\tan\theta$$

$$\text{Weight of cone} = \frac{1}{3} \times \frac{\pi D^2}{4} \times H \times \gamma s$$

= Weight of water displaced

$$= \frac{1}{3} \times \frac{\pi d^2}{4} \times \gamma \times y$$

$\therefore$ $$D^2Hs = d^2y$$

or $$y = \frac{D^2Hs}{d^2} = \frac{D^2Hs}{4y^2\tan^2\theta}$$

$\therefore$ $$y^3 = \frac{D^2Hs}{4\tan^2\theta}$$

or $$4y^3\tan^2\theta = D^2Hs$$

Also $$4y^3\left(\frac{D}{2H}\right)^2 = D^2Hs$$

$\therefore$ $$y^3 = H^3s$$

$\Rightarrow$ $$y = H \times s^{1/3}$$

$\Rightarrow$ $$y = 20 \times 0.8^{1/3}$$

$$= 18.566 \text{ cm}$$

if B is the centre of buoyancy,

$$OB = \frac{3}{4}y = 0.75 \times 18.566 = 13.925 \text{ cm}$$

Also, $$OG = \frac{3}{4}H = 0.75 \times 20 = 15 \text{ cm}$$

and $$d = 2y\tan\theta = 2 \times 18.566 \times \frac{15}{20}$$

$\therefore$ $$d = 27.85 \text{ cm}$$

$$BM = \frac{I}{\forall} = \frac{\frac{\pi d^4}{64}}{\frac{1}{3}\left(\frac{\pi d^2}{4}\right)y}$$

or $$BM = \frac{3}{16}\frac{d^2}{y} = \frac{3}{16} \times \frac{27.85^2}{18.566}$$

$\therefore$ $$BM = 7.833 \text{ cm}$$

$$OM = OB + BM$$
$$= 13.925 + 7.833 = 21.758 \text{ cm}$$

$\therefore$ $$MG = OM - OG$$
$$= 21.758 - 15$$
$$= 6.758 \text{ cm}$$

i.e. M is above G by 6.758 cm. Hence the cone is under stable equilibrium.

3. (c)

A draft type has the following functions to perform:

- Decreases the pressure at the runner exit to a value less than atmospheric pressure and there by increase the working head.
- Recover a portion of the exit kinetic energy which otherwise goes waste to the tail race.

These two factors help the turbine to develop more power and thereby the efficiency of the turbine increases. The unit also serves to fix the turbine above the tail race and that ensures proper inspection of the turbine.

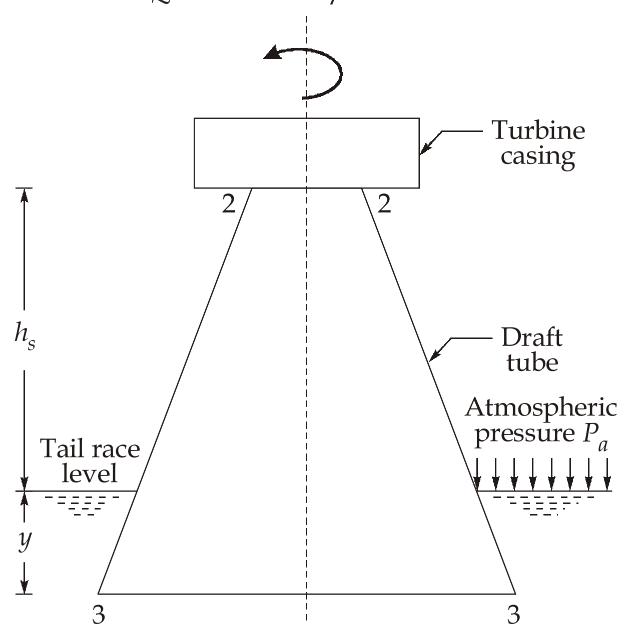
Francis turbine	Kaplan turbine
1. Radially inwards or mixed flow turbine	1. Purely axial flow turbine.
2. Horizontal or vertical disposition of shaft.	2. Only vertical shaft disposition.
3. Runner vanes are not adjustable.	3. Runner vanes are adjustable.
4. Large number of vanes ; 16 to 24 blades.	4. Small number of vanes ; 3 to 8 blades.
5. Large resistance needs to be overcome owing to more vanes and greater area of contact with water.	5. Less resistance as there are fewer vanes and less wetted area.
6. Medium head turbine (60 m to 250 m) and works under medium flow rate.	6. Low head turbine (upto 30 m) and requires very large volumetric flow rates.
7. Specific speed ranges from 50-250.	7. Specific speed ranges from 250-850.
8. Ordinary governor is sufficient for speed controls as the servomotor is of larger size.	8. Heavy duty governor is essential for speed control due to smaller size of the servomotors.

Power available from the shaft,

$$P = \rho Q g H \times \eta_0$$

$$1835 \times 10^3 = 1000 \times 9.81 \times Q \times 7.6 \times 0.88$$

$$Q = 27.97 \text{ m}^3/\text{s}$$



From the above figure,

$$\frac{P_2}{\rho g} = \frac{P_a}{\rho g} - h_s - \left[\frac{V_2^2 - V_3^2}{2g} - h_f\right]$$

Now,
$$V_2 = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{27.97}{\frac{\pi \times 3.1^2}{4}} = 3.70 \text{ m/s}$$

and $$h_f = 0.25 \times \frac{V_3^2}{2g}$$

$\therefore$ $$\frac{P_2}{\rho g} - \frac{P_a}{\rho g} = -h_s - \left[\frac{V_2^2 - V_3^2}{2g} - \frac{0.25V_3^2}{2g}\right]$$

$\Rightarrow$ $$-3.17 = -2.6 - \left[\frac{3.70^2}{2 \times 9.81} - \frac{1.25 \times V_3^2}{2 \times 9.81}\right]$$

$$V_3 = 1.416 \text{ m/s}$$

$\therefore$ $$\frac{V_2^2 - V_3^2}{2g} = \frac{3.70^2 - 1.416^2}{2 \times 9.81} = 0.5955$$

$$\frac{V_2^2}{2g} = \frac{3.70^2}{2 \times 9.81} = 0.6977$$

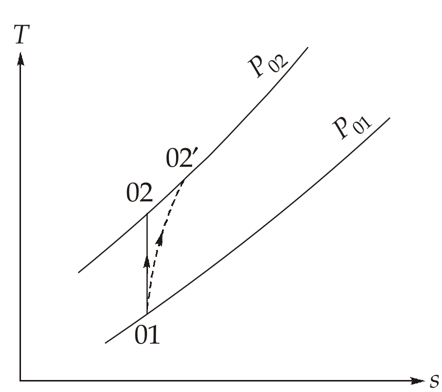
$$h_f = \frac{0.25 \times V_3^2}{2g} = \frac{0.25 \times 1.416^2}{2 \times 9.81} = 0.0255$$

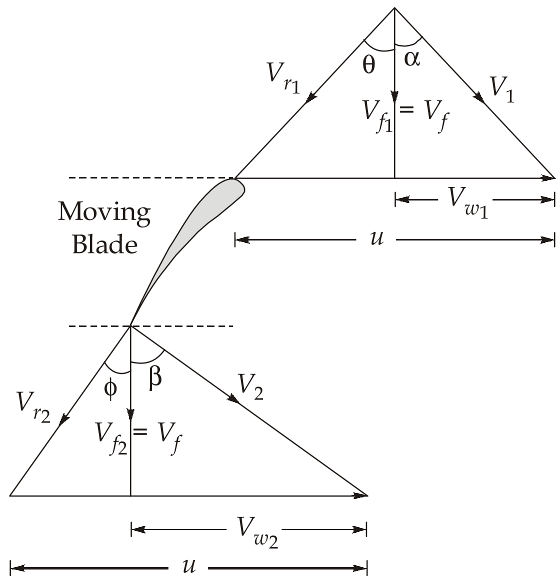
$\therefore$ Draft tube efficiency, $$\eta_{DT} = \frac{\frac{V_2^2 - V_3^2}{2g} - h_f}{\frac{V_2^2}{2g}} = \frac{0.5955 - 0.0255}{0.6977}$$

$$= 0.8169 \text{ or } 81.69\%$$ Ans.

4. (a)

Given : $\dot{m} = 48$ kg/s; $r_p = 5$; $T_{01} = 290$ k; $P_{01} = 1$ bar; $\eta_{isen} = \eta_{stage} = 88\%$; $n = 10$; $V_f = 160$ m/s $= V_{f_1} = V_{f_2}$; $U_1 = U_2 = U = 210$ m/s; R = 0.5 at mean blade height, $\phi_w = 0.86$; $D_h = 0.82 \times D_t$; $(\Delta T_0)_{stage}$ = constant





$$\frac{T_{02}}{T_{01}} = (r_p)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \quad T_{02} = T_{01} \times (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \quad T_{02} = 290 \times (5)^{\frac{0.4}{1.4}} = 459.31 \text{ K}$$

$$\eta_{ise} = \frac{T_{02} - T_{01}}{T_{02'} - T_{01}}$$

$$\Rightarrow \quad T_{02'} = \frac{459.31 - 290}{0.88} + 290$$

$$\Rightarrow \quad T_{02'} = 482.40 \text{ K}$$

$$\therefore \quad \Delta T'_0 = T_{02'} - T_{01} = 482.40 - 290 = 192.4 \text{ K}$$

$$\therefore \quad (\Delta T'_0)_{stage} = \frac{\Delta T_0'}{n} = \frac{192.4}{10} = 19.24 \text{ K}$$

Work done per stage, $W_{stage} = c_p \cdot (\Delta T_0'')_{stage}$

$$= \phi_w \cdot u\, V_f \cdot (\tan\theta - \tan\phi)$$

$$\Rightarrow \quad 1.005 \times 10^3 \times 19.24 = 0.86 \times 210 \times 160 \times (\tan\theta - \tan\phi)$$

$$\Rightarrow \quad \tan\theta - \tan\phi = 0.67 \qquad \text{...(i)}$$

Also degree of reaction, $R = \frac{V_f}{2u}(\tan\theta + \tan\phi)$

$$\Rightarrow \quad 0.5 = \frac{160}{2 \times 210} \times (\tan\theta + \tan\phi)$$

$\Rightarrow \quad \tan\theta + \tan\phi = 1.3125 \quad ...(ii)$

From equation (i) and (ii), we get

$$\theta = 44.75° = \beta \text{ and } \phi = \alpha = 17.81° \quad \text{Ans.}$$

From continuity equation,

$$\dot{m} = \rho A V_f = \rho \frac{\pi}{4}\left(D_t^2 - D_h^2\right) V_f$$

$$\rho = \frac{P_{01}}{RT_{01}} = \frac{100}{0.287 \times 290} = 1.2015 \text{ kg/m}^3$$

$$\Rightarrow \quad 48 = 1.2015 \times \frac{\pi}{4} \times D_t^2 \times \left(1 - 0.82^2\right) \times 160$$

$$\Rightarrow \quad D_t = 0.985 \text{ m}$$

$$\Rightarrow \quad D_h = 0.82 \times D_t = 0.82 \times 0.985 = 0.808 \text{ m} \quad \text{Ans.}$$

$$\Rightarrow \quad \text{Height of the blade, } H = \frac{D_t - D_h}{2} = \frac{0.985 - 0.808}{2} = 0.0885 \text{ m} \quad \text{Ans.}$$

$$\text{Mean diameter, } D_m = \frac{D_h + D_t}{2} = \frac{0.808 + 0.985}{2} = 0.8965 \text{ m}$$

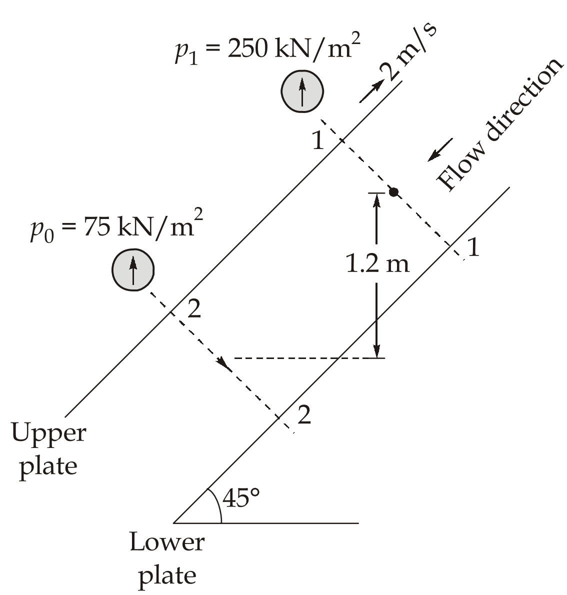
$$U = \frac{\pi D_m N}{60}$$

$$\Rightarrow \quad 210 = \frac{\pi \times 0.8965 \times N}{60}$$

$$\Rightarrow \quad N = 4473.74 \text{ rpm} \quad \text{Ans.}$$

4. (b)

Given : $\mu = 0.8$ kg/ms; $\rho = 1300$ kg/m^3; $b = 15$ mm $= 0.015$ m; $P_1 = 250$ kPa; $P_2 = 75$ kPa



Since the plates are placed uniformly apart, velocity head would be same at the two sections and as such flow directions will be dictated by value of piezometric head. Taking a horizontal line passing through point 2 as datum.

Now, Piezometric heads, $P_1^* = P_1 + \rho g y_1 = 250 + \dfrac{9.81 \times 10^3 \times 1.2}{1000}$

$\therefore$ $P_1^* = 261.77$ kPa

and $P_2^* = P_2 + \rho g y_2 = 75 + \dfrac{9.81 \times 10^3 \times 0}{1000}$

$P_2^* = 75$ kPa

Since $P_1^* > P_2^*$, the direction of flow is from section 1-1 to section 2-2 i.e. downward. Apparently the upper plate is moved up the slope.

$\therefore$ Pressure gradient, $\left(-\dfrac{\partial P}{\partial x}\right) = \dfrac{P_1^* - P_2^*}{l} = \dfrac{261.77 - 75}{1.2/\sin 45°}$

$$\left(-\frac{\partial P}{\partial x}\right) = 110.05 \text{ kPa/m}$$

For Couette flow, $U = \dfrac{Vy}{b} + \dfrac{1}{2\mu}\left(-\dfrac{\partial P}{\partial x}\right)\left(by - y^2\right)$

Here, $V = -2$m/s, $b = 0.015$ m; $\mu = 0.8$ kg/ms

$\therefore$ $u = \dfrac{-2y}{0.015} + \dfrac{1}{2 \times 0.8} \times \left(110.05 \times 10^3\right)\left(0.015y - y^2\right)$

Simplifying we get

$$u = -133.33y + 68781.25(0.015y - y^2)$$

or $u = 898.4y - 68781.25y^2$ **Ans.**

Maximum velocity occurs where $\dfrac{du}{dy} = 0$, i.e.

$$898.4 - 137562.5y = 0$$

or $y = 6.53 \times 10^{-3}$ m

$\therefore$ $V_{max} = 898.4 \times (6.53 \times 10^{-3}) - 68781.25 \times (6.53 \times 10^{-3})^2$

$\therefore$ $V_{max} = 2.93$ m/s **Ans.**

Shear stress, $\tau = \mu \dfrac{du}{dy} = 0.8(898.4 - 137562.5y)$

or $\tau = 718.72 - 110050y$ **Ans.**

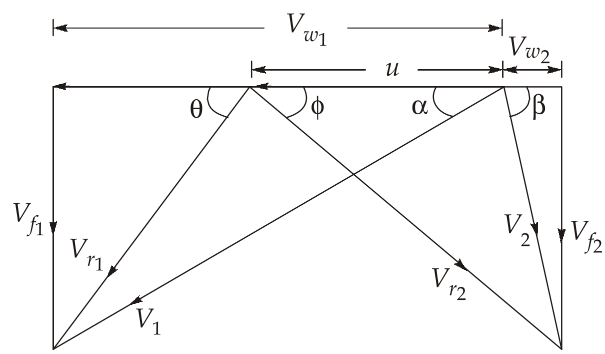
∴ Shear stress at $y = 0.015$

$$\tau = 718.72 - 110050 \times 0.015$$

∴ $\tau = -932.03$ Pa **Ans.**

4. (c)

The velocity diagram for the moving blades of a 50% reaction turbine are shown in figure below.



If equal enthalpy drops occurs in the fixed and moving blades,

i.e. $$\Delta h_{FB} = \Delta h_{MB} = \frac{(\Delta h)_{stage}}{2}, \quad \text{So, } R = \frac{1}{2} \text{ or } 50\%.$$

50% reaction turbines are also called Parson's turbine. For manufacturing advantage, both fixed and moving blades are made similar in shape so that they can be extruded from the same set of dies.

Since, $\Delta h_{FB} = \Delta h_{MB}$, $V_1 = V_{r2}$

Again, for similar geometry and similar velocity triangles,

$$\alpha = \phi, \quad \theta = \beta, \quad V_{r_1} = V_2$$

Since $\Delta V_f = 0$, there is no axial thrust imposed on the blades due to change in axial velocity in a 50% reaction turbine.

$$\text{Blade efficiency, } \eta_b = \frac{u(V_{w_1} + V_{w_2})}{\frac{V_1^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}} = \frac{2 \cdot u \cdot \left(V_{w_1} + V_{w_2}\right)}{V_1^2 + V_{r_2}^2 - V_{r_1}^2}$$

$$\eta_b = \frac{2 \cdot u \cdot \left(V_{w_1} + V_{w_2}\right)}{2V_1^2 - V_{r_1}^2} \quad \text{...(i)} \quad [\because \; V_1 = V_{r_2}]$$

From velocity triangles, $\Delta V_w = V_{w_1} + V_{w_2}$

$$= V_1\cos\alpha + V_{r2}\cos\phi - u = V_1\cos\alpha + V_1\cos\alpha - u$$

$$\Delta V_w = 2V_1\cos\alpha - u \qquad \text{...(ii)}$$

$$V_{r1}^2 = V_{f1}^2 + \left(V_{w_1} - u\right)^2 \qquad \text{...(iii)}$$

$$V_1^2 = V_{f1}^2 + V_{w1}^2 = V_{f1}^2 + \left[\left(V_{w_1} - u\right) + u\right]^2$$

$$V_1^2 = V_{f_1}^2 + \left(V_{w_1} - u\right)^2 + u^2 + 2uV_{w_1} - 2u^2 \qquad \text{...(iv)}$$

From equation (iii) and (iv), we get

$$V_1^2 = V_{r_1}^2 + 2uV_{w_1} - u^2$$

$$\Rightarrow \qquad V_{r1}^2 = V_1^2 - 2V_1 u\cos\alpha + u^2 \qquad \text{...(v)}$$

From equation (i), (iii) and (v), we get

$$\eta_b = \frac{2\cdot u\cdot(2V_1\cos\alpha - u)}{2V_1^2 - V_1^2 + 2V_1 u\cos\alpha - u^2}$$

$$\eta_b = \frac{2\cdot u\cdot(2V_1\cos\alpha - u)}{V_1^2 + 2V_1 u\cos\alpha - u^2}$$

$$\text{Speed ratio, } \rho = \frac{u}{V_1}$$

$$\Rightarrow \qquad u = \rho V_1$$

$$\eta_b = \frac{2\cdot\rho\cdot(2\cos\alpha - \rho)}{1 + 2\rho\cos\alpha - \rho^2} = \frac{2\left(2\rho\cos\alpha - \rho^2\right)}{1 - \rho^2 + 2\rho\cos\alpha} \qquad \text{...(vi)}$$

There is a particular value of ρ for which η_b is a maximum. Differentiating η_b with respect to ρ and equating it to zero.

$$\frac{d\eta_b}{d\rho} = \frac{\left(1-\rho^2+2\rho\cos\alpha\right)2(2\cos\alpha - 2\rho) - 2\rho(2\cos\alpha-\rho)(-2\rho+2\cos\alpha)}{(1-\rho^2+2\rho\cos\alpha)^2}$$

$$\Rightarrow \qquad \frac{4(1-\rho^2+2\rho\cos\alpha)(\cos\alpha-\rho) - 2\rho(2\cos\alpha-\rho)(-2\rho+2\cos\alpha)}{(1-\rho^2+2\rho\cos\alpha)^2} = 0$$

$$\Rightarrow \qquad 4(1-\rho^2+2\rho\cos\alpha)(\cos\alpha-\rho) - 4\rho(2\cos\alpha-\rho)(\cos\alpha-\rho) = 0$$

$$\Rightarrow \qquad 4(\cos\alpha - \rho) = 0$$

$$\Rightarrow \qquad \rho = \cos\alpha$$

$$\therefore \qquad \rho_{optimum} = \cos\alpha$$

$$\Rightarrow \qquad U = V_1 \cos\alpha \qquad \text{...(vii)}$$

Thus, substituting the value of $\rho_{optimum}$ in equation (vi), we get

$$\eta_b = \frac{2\left(2 \cdot \cos^2\alpha - \cos^2\alpha\right)}{1 - \cos^2\alpha + 2\cos^2\alpha}$$

$$(\eta_b)_{max} = \frac{2 \cdot \cos^2\alpha}{1 + \cos^2\alpha}$$

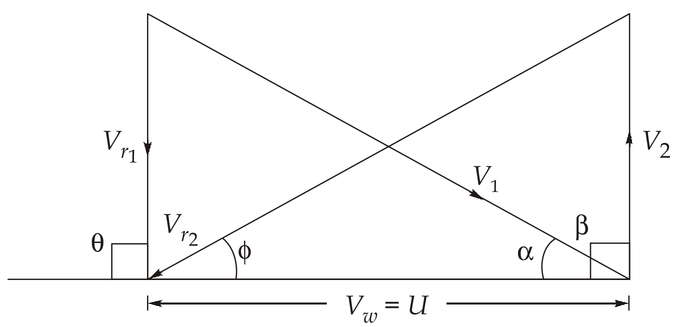
Also, the diagram work per kg of steam,

$$W_D = u \cdot \Delta V_w = u(2V_1 \cos\alpha - u)$$

From equation (vii), we get

$$W_D = u \cdot (2 \cdot u - u) = u^2$$

The velocity diagrams for a 50% reaction turbine operating with maximum blading efficiency is shown below.



$$\Delta V_w = V_1 \cos\alpha = u$$

Section : B

5. (a)

$$\text{Mean film temperature, } T_f = \frac{970 + 330}{2} = 650°\text{C}$$

$$\Rightarrow \qquad \beta = \frac{1}{T_f} = \frac{1}{650 + 273} = 1.0834 \times 10^{-3}$$

$$\text{Pr} = \frac{\mu c_p}{k} = \frac{\left(\frac{3.12}{3600}\right) \times 150.7}{13.02} = 0.01$$

$$\text{Grashof number, Gr} = \frac{g\beta \Delta T L^3}{\nu^2} = \frac{9.81 \times 1.0834 \times 10^{-3} \times (970 - 330) \times 2.4^3}{\left(\frac{3.12}{3600 \times 10^4}\right)^2}$$

$$Gr = 1.252 \times 10^{16}$$

$$GrPr = 1.252 \times 10^{16} \times 0.01 = 1.252 \times 10^{14}$$

$$Nu = \frac{hL}{k} = 0.13(Gr\,Pr)^{1/3}$$

$$\Rightarrow \quad h = \frac{13.02 \times 0.13 \times (1.252 \times 10^{14})^{1/3}}{2.4} = 35281.3 \text{ W/m}^2\text{K}$$

∴ Heat dissipation from both sides of each plate,

$$Q = h \cdot (2A) \cdot \Delta T$$

$$= 35281.3 \times (2 \times 2.4 \times 1.5) \times (970 - 330)$$

$$= 162.57 \text{ MW}$$ Ans.

5. (b)

Given : $D = D_0 = 80$ mm; $\left(\frac{\theta}{l}\right)_{steel} = 0.72 \times \left(\frac{\theta}{l}\right)_{alloy}$; $G_{steel} = 1.8 \times G_{alloy}$; $P = 720$ kW

Angle of twist per unit length,

$$\left(\frac{\theta}{l}\right)_{steel} = 0.72 \times \left(\frac{\theta}{l}\right)_{alloy}$$

$$\Rightarrow \quad \left(\frac{T}{GJ}\right)_{steel} = 0.72\left(\frac{T}{GJ}\right)_{alloy}$$

$$\Rightarrow \quad \frac{32T}{G_{steel} \times \pi\left(D_0^4 - D_i^4\right)} = \frac{0.72 \times 32 \times T}{G_{alloy} \times \pi \times D^4}$$

$$\Rightarrow \quad \frac{1}{1.8 \times G_{alloy} \times \left(80^4 - D_i^4\right)} = \frac{0.72}{G_{alloy} \times 80^4}$$

$$\Rightarrow \quad D_i = 55.30 \text{ mm}$$ Ans.

Also,

$$\left(\frac{\theta}{l}\right)_{steel} = 0.72 \times \left(\frac{\theta}{l}\right)_{alloy}$$

$$\left(\frac{\tau}{GR}\right)_{steel} = 0.72 \times \left(\frac{\tau}{GR}\right)_{alloy}$$

$$\Rightarrow \quad \tau_{steel} = 0.72 \times \frac{G_{steel}}{G_{alloy}} \times \frac{R_{steel}}{R_{alloy}} \times \tau_{alloy}$$

$$\Rightarrow \quad \tau_{steel} = 0.72 \times 1.8 \times \tau_{alloy}$$

$$\Rightarrow \qquad \tau_{steel} = 1.296 \times \tau_{alloy}$$

If the maximum stress in the steel shaft is to be 80 MPa, then the maximum stress in the alloy shaft will be $\frac{80}{1.296} = 61.73$ MPa

$$\therefore \qquad \text{Torque, } T = \tau \cdot \frac{\pi D^3}{16} = 61.73 \times \frac{\pi \times 80^3}{16} = 6205.61 \times 10^3 \text{ Nmm}$$

$$\therefore \qquad \text{Power, } P = \frac{2\pi NT}{60}$$

$$720 = \frac{2\pi \times N \times 6205.61 \times 10^3}{60 \times 10^6}$$

$$\Rightarrow \qquad N = 1107.95 \text{ rpm}$$ Ans.

5. (c)

A thermostatic expansion valve is a throttling device which works automatically, maintaining proper and correct liquid flow as per the requirements of the load on the evaporator.

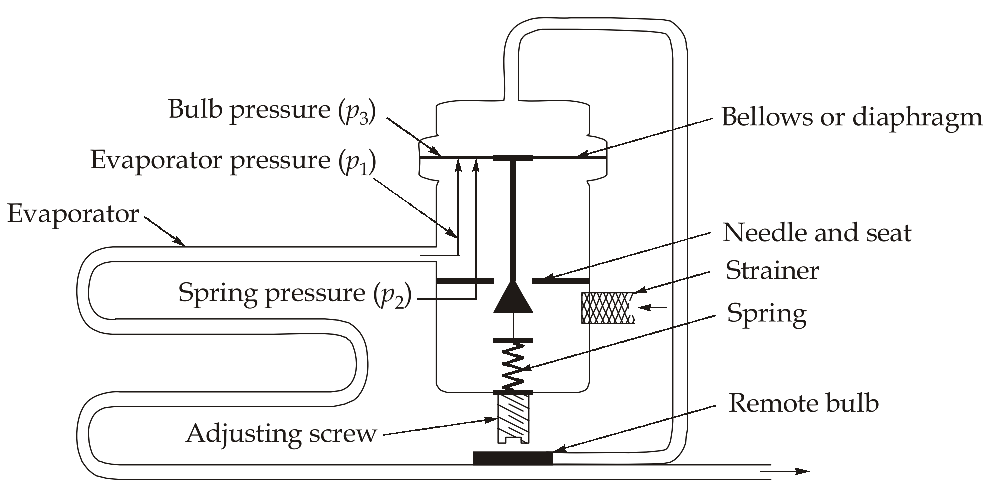
A thermostatic expansion valve performs the following functions:

(i) Reduces the pressure of liquid from the condenser pressure to evaporator pressure.

(ii) Keeps the evaporator fully active.

(iii) Modulates the flow of liquid to the evaporator according to the load requirement of the evaporator so as to prevent flood back of liquid refrigerant to the compressor.

Figure below shows a thermostatic expansion valve.



Thermostatic expansion valve

The following are the important parts of the valve:

1. Power element with a feeler bulb

2. Valve seat and needle
3. Adjustment spring
4. Bellows or diaphragm

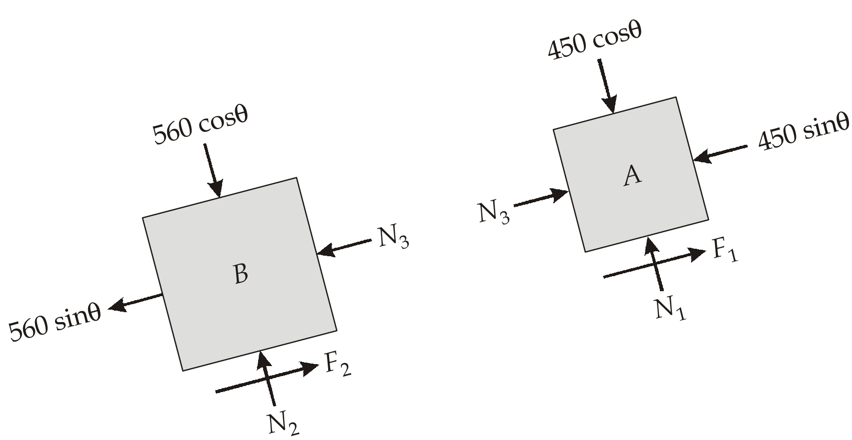
The remote bulb charged with fluid which is open on one side of the diaphragm through a capacity tube is clamped firmly to the evaporator outlet. The temperature of the saturated liquid in vapour mixture is the same as the temperature of the superheat gas leaving the evaporator at the location. The pressure of the liquid in the bulb (p_3) tends to open the valve. This pressure is balanced by pressure due to spring (p_2) plus pressure in the evaporator (p_1).

The performance characteristics of thermostatic-expansion valves are most suitable for application in air-conditioning and refrigerant plants. When the cooling load 'increases', the refrigerant evaporates at a faster rate in the evaporator than the compressor can suck. As a result the pressure and degree of superheat in the evaporator increase. The increase in superheat causes the valve to open more and to allow more refrigerant to enter the evaporator. At the same time, the increase in suction pressure also enables the compressor to deliver increased refrigerating capacity. When the cooling load 'decreases' the refrigerant evaporates at a slower rate than the compressor can suck. As a result, the evaporator pressure drops and the degree of superheat decreases. The valve tends to close and the compressor delivers less refrigerating capacity at a decreased suction pressure. Thus the thermostatic-expansion valve, as opposed to the automatic expansion valve, is capable of meeting varying load requirement.

Most thermostatic expansion valves are set for 5°C superheat and are usually rated in tonnes of refrigeration.

5. (d)

Given : W_A = 450 N; W_B = 560 N; μ_A = 0.25; μ_B = 0.32



FBD of blocks *A* and *B*

Say the blocks are at the point of sliding down the plane. Frictional force on block A is less than friction force on block B, so block *A* wall exert push on block *B*.

For block A : Normal reaction, $N_1 = 450 \times \cos\theta$ ...(i)

Frictional force, $F_1 = \mu_A \cdot N_1 = 0.25 \times 450 \times \cos\theta$

$\Rightarrow \quad F_1 = 112.5 \cos\theta$ ...(ii)

Force in the direction parallel to the plane,

$\Rightarrow \quad N_3 + F_1 - 450\sin\theta = 0$ [from equation (ii)]

$\Rightarrow \quad N_3 = 450\sin\theta - 112.5\cos\theta$ ...(iii)

[where N_3 is the reaction between blocks A and B]

For block B : Normal reaction, $N_2 = 560 \cos\theta$ ...(iv)

Frictional force, $F_2 = \mu_B \cdot N_2 = 0.32 \times 560 \times \cos\theta$

$\Rightarrow \quad F_2 = 179.2 \times \cos\theta$ ...(v)

Force in the direction parallel to the plane,

$$F_2 - N_3 - 560 \sin\theta = 0$$

From equation (v), we get

$$N_3 = 179.2 \times \cos\theta - 560 \times \sin\theta \quad \text{...(vi)}$$

Equating equation (iii) and (vi), we get

$$450 \sin\theta - 112.5 \cos\theta = 179.2\cos\theta - 560\sin\theta$$

$$1010\sin\theta = 291.7\cos\theta$$

$$\tan\theta = \frac{291.7}{1010}$$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{291.7}{1010}\right)$

$\Rightarrow \quad \theta = 16.11°$ Ans.

5. (e)

The maximum work obtained is equal to the decrease in the available energy of the steam.

i.e. $W_{max} = A_1 - A_2$

$$= (U_1 - T_1 s_1) - (U_2 - T_2 s_2)$$

$$W_{max} = m[(U_1 - T_1 s_1) - (U_2 - T_2 s_2)]$$

For steam at 30 bar and 300°C,

$$s_1 = 6.5412 \text{ kJ/kgK}$$

$$U_1 = 2750.8 \text{ kJ/kg}$$

and at 1 bar and 300°C,

$$U_2 = 2810.6 \text{ kJ/kg}; \; s_2 = 8.2172 \text{ kJ/kg}$$

$\therefore$ $$W_{max} = 0.2[(2750.8 - 573 \times 6.5412) - (2810.6 - 573 \times 8.2172)]$$

$$= 0.2[-997.307 - (-1897.85)]$$

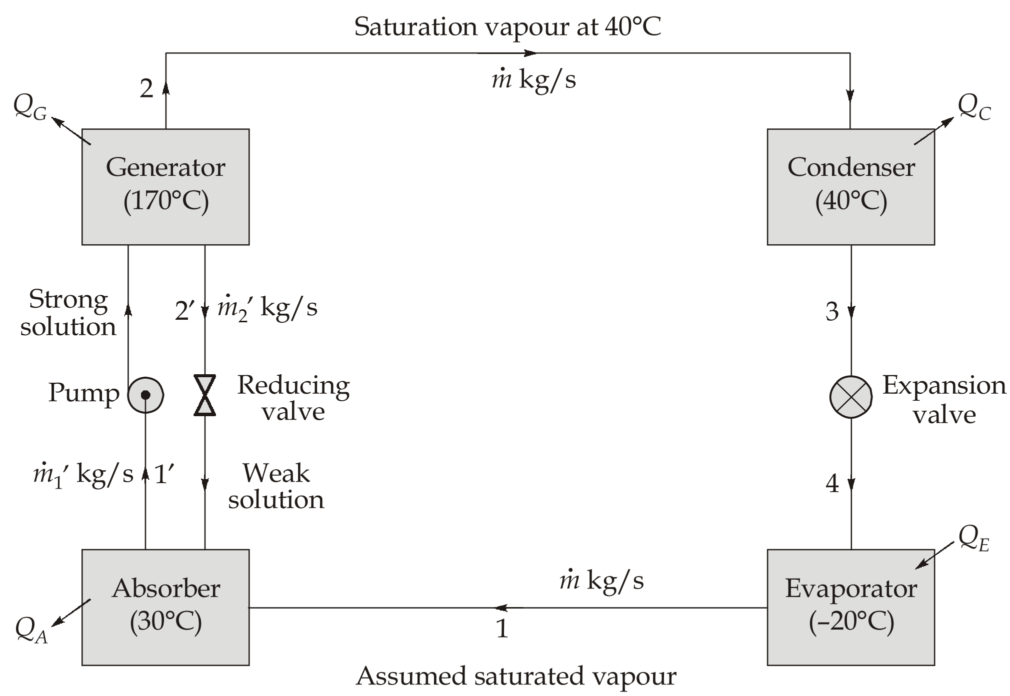
$$W_{max} = 180.1 \text{ kJ}$$ Ans

6. (a)

Given : $x_1' = 0.4$; $h_1' = 25$ kJ/kg; $x_2' = 0.2$; $h_2' = 700$ kJ/kg; $h_2 = 1473.3$ kJ/kg;

$h_3 = 371.9$ kJ/kg; $h_1 = 1420$ kJ/kg

The schematic arrangement of the system is shown below.



Refrigerating effect, R.E. $= h_1 - h_4$

$$\text{R.E.} = 1420 - 371.9$$

$$= 1048.1 \text{ kJ/kg} \qquad (\because \; h_3 = h_4)$$

Now, $$\dot{m} \times R.E. = \text{R.C.}$$

$\therefore$ $$\dot{m} = \frac{10 \times 3.5}{1048.1} = 0.0334 \text{ kg/s}$$ Ans. (i)

Overall mass balance for absorber gives

$$\dot{m} + \dot{m}_2' = \dot{m}_1'$$

Ammonia balance for the absorber gives

$$\dot{m} + 0.2\dot{m}_2' = 0.4\dot{m}_1' = 0.4(\dot{m} + \dot{m}_2')$$

or $$\dot{m} + 0.2\dot{m}_2' = 0.4\dot{m} + 0.4\dot{m}_2'$$

∴ $$\dot{m}_2' = \frac{\dot{m}(1-0.4)}{0.4-0.2} = 0.0334 \times \frac{0.6}{0.2}$$

$$\dot{m}_2' = 0.1002 \text{ kg/s}$$ Ans. (ii)

and $$\dot{m}_1' = \dot{m} + \dot{m}_2' = 0.0334 + 0.1002$$

$$\dot{m}_1' = 0.1336 \text{ kg/s}$$ Ans (ii)

Now, making energy balance for absorber we have,

$$\dot{m}h_1 + \dot{m}_2'h_2' = \dot{m}_1'h_1' + Q_A$$

$$0.0334 \times 1420 + 0.1002 \times 700 = 0.1336 \times 25 + Q_A$$

∴ $$Q_A = 114.23 \text{ kW}$$ Ans. (iii)

Energy balance for generator gives

$$\dot{m}_1'h_1' + Q_4 = m_2'h_2' + mh_2$$

∴ $$0.1336 \times 25 + Q_4 = 0.1002 \times 700 + 0.0334 \times 1473.3$$

∴ $$Q_4 = 116 \text{ kW}$$ Ans. (iii)

Now, for condenser,

$$Q_C = \dot{m}(h_2 - h_3) = 0.0334(1473.3 - 371.9)$$

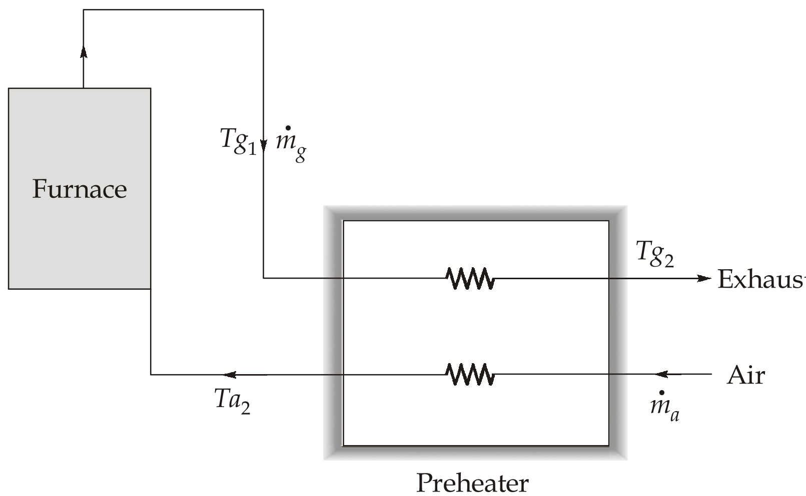
∴ $$Q_C = 36.78 \text{ kW}$$ Ans. (iii)

$$\text{COP} = \frac{R.E.}{Q_4} = \frac{10 \times 3.5}{116}$$

∴ $$\text{COP} = 0.301$$ Ans. (iii)

6. (b)

Given : $\dot{m}_g$ = 12 kg/sec, T_{g1} = 600°C; T_{g2} = 450°C; $\dot{m}_a$ = 8 kg/sec; T_{a1} = 40°C; T_0 = 300 K



Neglecting the change in K.E. and P.E.,

$$\text{Initial availability, } \phi_1 = h_1 - h_0 - T_0(s_1 - s_0)$$

$$= \dot{m}_g c_{pg}\left[\left(T_{1g} - T_0\right) - T_0 \ln\frac{T_{1g}}{T_0}\right]$$

$$= 12 \times 1.09\left[(873 - 300) - 300\ln\frac{873}{300}\right]$$

$$\phi_1 = 3303.4 \text{ kW} \qquad \text{Ans.(i)}$$

$$\text{Similarly, Final availability, } \phi_2 = \dot{m}_g c_{pg}\left[\left(T_{2g} - T_0\right) - T_0 \ln\frac{T_{2g}}{T_0}\right]$$

$$= 12 \times 1.09\left[(723 - 300) - 300\ln\frac{723}{300}\right]$$

$$= 2081.18 \text{ kW} \qquad \text{Ans.(ii)}$$

Now, applying heat balance in preheater

$$Q_{\text{air}} = Q_{\text{combustion}}$$

$$\dot{m}_a c_{pa}\left(T_{2a} - T_{1a}\right) = \dot{m}_a c_{pg}\left(T_{1g} - T_{2g}\right)$$

or $$8 \times 1.005(T_{2a} - 313) = 12 \times 1.09(873 - 723)$$

∴ $$T_{2a} = 557.03 \text{ K}$$

Now, change in entropy,

$$\Delta s_{\text{product}} = \dot{m}_g c_{pg} \ln \frac{T_{2g}}{T_{1g}}$$

$$= 12 \times 1.09 \ln\left(\frac{723}{873}\right) = -2.466 \text{ kW/K}$$

For air,

$$\Delta s_{\text{air}} = \dot{m}_a c_{pa} \ln \frac{T_{2a}}{T_{1a}}$$

$$= 8 \times 1.005 \ln\left(\frac{557.03}{313}\right) = 4.634 \text{ kW/K}$$

$\therefore$ Irreversibility,

$$I = T_0\left(\Delta s_{air} + \Delta s_{product}\right)$$

$$= 300(4.634 - 2.466)$$

$$= 650.4 \text{ kW}$$

Ans.(ii)

Now, for reversible process

$$\Delta s_{\text{univ}} = 0$$

or

$$-\Delta s_{\text{air}} = \Delta s_{\text{product}}$$

$$-\dot{m}_a c_{pa} \ln \frac{T_{2a}}{T_{1a}} = \dot{m}_g c_{pg} \ln \frac{T_{2g}}{T_{1g}}$$

or

$$-8 \times 1.005 \ln \frac{T_{2g}}{313} = 12 \times 1.09 \ln\left(\frac{723}{873}\right)$$

or

$$\ln \frac{T_{2a}}{313} = 0.3067$$

$\therefore$

$$T_{2a} = 425.34 \text{ K}$$

So, heat transfer by products of combustion

$$Q_1 = \dot{m}_g c_{pg}\left(T_{1g} - T_{2g}\right)$$

$$Q_1 = 12 \times 1.09 \times (600 - 450) = 1962 \text{ kW}$$

and for air,

$$Q_2 = \dot{m}_g c_{pa}\left(T_{2a} - T_{1a}\right)$$

$$= 8 \times 1.005 \times (425.34 - 313)$$

$$= 903.21 \text{ kW}$$

$\therefore$ Power developed by engine,

$$W = Q_1 - Q_2$$

$= 1962 - 903.21$

$= 1058.79$ kW Ans. (iii)

6. (c)

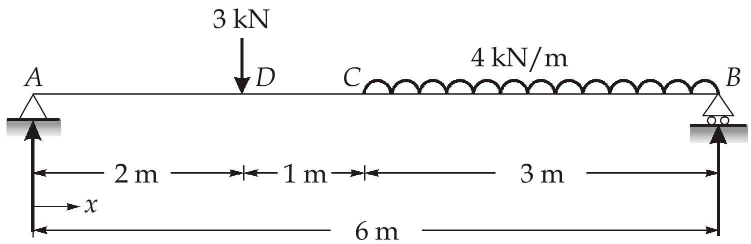
$\Sigma F_V = 0$

$\Rightarrow \quad R_A + R_B = 3 + 4 \times 3 = 15$ kN ...(i)

Taking moment about B, $\Sigma M_B = 0$

$\Rightarrow R_A \times 6 - 3 \times 4 - (4 \times 3) \times 1.5 = 0$

$\Rightarrow \quad R_A = 5$ kN and $R_B = 10$ kN [From equation (i)]



Measuring x from A, we have

$$EI\frac{d^2y}{dx^2} = -5x\Big|+3(x-2)\Big|+\frac{4(x-3)^2}{2}$$

$$EI\frac{d^2y}{dx^2} = -5x\Big|+3(x-2)\Big|+2(x-3)^2 \quad \text{...(ii)}$$

Integrating the above equation, we get

$$EI\frac{dy}{dx} = -\frac{5x^2}{2}+c_1\Bigg|+\frac{3}{2}(x-2)^2\Bigg|+\frac{2(x-3)^3}{3} \quad \text{...(iii)}$$

Again integrating the above equation, we get

$$EI\cdot y = -\frac{5x^3}{6}+c_1x+c_2\Bigg|+\frac{3}{6}(x-2)^3\Bigg|+\frac{2(x-3)^4}{12} \quad \text{...(iv)}$$

Boundary condition:

At $x = 0$, $y = 0 \Rightarrow c_2 = 0$

At $x = 6$ m, $y = 0$

$$\Rightarrow \quad -\frac{5(6)^3}{6}+c_1(6)+\frac{3}{6}\times(6-2)^3+\frac{2}{12}(6-3)^4 = 0$$

$\Rightarrow \quad c_1 = 22.4167$

Hence, the slope and deflection equations are:

$$EI\frac{dy}{dx} = -2.5x^2 + 22.4167\Big|+1.5(x-2)^2\Big| + \frac{2}{3}(x-3)^3 \quad \text{...(v)}$$

and $$EIy = -\frac{5}{6}x^3 + 22.4167x\Big|+0.5(x-2)^3\Big| + \frac{(x-3)^4}{6} \quad \text{...(vi)}$$

For maximum deflection, $\frac{dy}{dx} = 0$, However, the maximum deflection will be very near to the mid-point C, say in the sector DC. Hence, from equation (v), including the terms upto point C, we get

$$0 = -2.5x^2 + 22.4167 + 1.5(x-2)^2$$

$\Rightarrow$ $$x = 3.1169 \text{ m}$$

Hence, from equation (vi), we get

$$EIy_{max} = \frac{5}{6}\times(3.1169)^3 + 22.4167\times 3.1169 + 0.5\times(3.1169-2)^3 + \frac{(3.1169-3)^4}{6}$$

$\Rightarrow$ $$y_{\max} = \frac{45.3332\times 10^3}{2\times 10^5\times 10^6\times 2500\times 10^{-8}} = 9.0666\times 10^{-3} \text{ m} \quad \text{Ans.}$$

For deflection at C, put $x = 3$ in equation (vi) in terms upto point C, we get

$$EI\cdot y_c = -\frac{5}{6}\times 3^3 + 22.4167\times 3 + 0.5\times(3-2)^3$$

$$EI\cdot y_c = 45.2501 \text{ kN-m}^3$$

$\Rightarrow$ $$y_c = \frac{45.2501\times 10^3}{2\times 10^{11}\times 2500\times 10^{-8}} = 9.05002\times 10^{-3}\text{m} \quad \text{Ans.}$$

For slope at point A, putting $x = 0$ in equation (v), we get

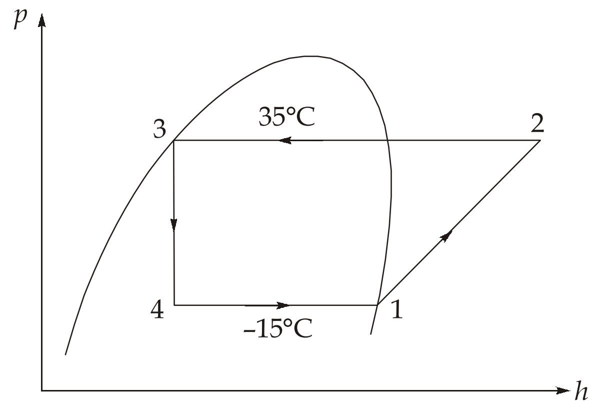
$$EI\left(\frac{dy}{dx}\right)_A = 22.4167 \text{ kN-m}^2$$

$\Rightarrow$ $$\left(\frac{dy}{dx}\right)_A = \theta_A = \frac{22.4167\times 10^3}{2\times 10^{11}\times 2500\times 10^{-8}}$$

$$= 4.4833\times 10^{-3} \text{ radian} \quad \text{Ans.}$$

7. (a)

The cycle is shown on the P-h diagram as shown in figure below.



The required refrigerating capacity of the plant is given by,

$$\text{R.C.} = \frac{10 \times 1000}{24 \times 3600}\left[c_{pw}(T_w - 0) + h_{fg} + c_{p,\,ice}\left(T_f - T_i\right)\right]$$

or $$\text{R.C.} = 0.11574[4.2(27 - 0) + 335 + 1.94 \times 5]$$

∴ $$\text{R.C.} = 53.02 \text{ kW}$$ Ans. (i)

Now, $$\dot{m}_{ref} \times \text{R.E.} = \text{R.C.}$$

∴ $$\dot{m}_{ref} = \frac{53.02}{h_1 - h_4} = \frac{53.02}{1426 - 347.5}$$

∴ $$\dot{m}_{ref} = 0.049 \text{ kg/s}$$ Ans. (ii)

The compression process 1-2 is isentropic

∴ $$s_1 = s_2$$

or $$s_{g1} = s_{g2} + c_{pv}\ln\left(\frac{T_2}{T_{25}}\right)$$

$$5.549 = 4.93 + 2.8\ln\left(\frac{T_2}{308}\right)$$

∴ $$T_2 = 1.247 \times 308 = 384.2 \text{ K}$$

∴ $$h_2 = h_{g2} + c_{pv}(T_2 - T_{2s})$$

$$= 1471 + 2.8(384.2 - 308) = 1684.36 \text{ kJ/kg}$$

∴ Power required to run the compressor,

$$P = \frac{\dot{m}_{ref}(h_2 - h_1)}{\eta_{isen} \times \eta_{mech}}$$

$$P = \frac{0.049(1684.36-1426)}{0.82\times0.94} = 16.424 \text{ kW}$$ Ans.

$$\text{COP}_{\text{act}} = \frac{R.C.}{\dot{P}} = \frac{53.02}{16.424} = 3.228$$

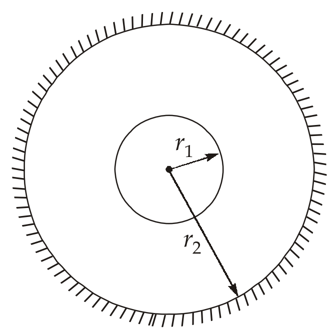
and Ideal COP

$$\text{COP}_{\text{rev}} = \frac{T_L}{T_H - T_L} = \frac{-15+273}{35+15} = 5.16$$

∴ $$\text{Relative COP} = \frac{COP_{act}}{COP_{rev}} = \frac{3.228}{5.16} = 0.625$$ Ans.

7. (b)

Given : r_1 = 4 mm; r_2 = 6 mm; I = 2100 A; T_∞ = 21°C; h = 12500 W/m^{2}K



The governing equation is,

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q_g'''}{k} = 0$$

Integrating the above equation, we get

$$\frac{dT}{dr} + \frac{q_g''' r}{2k} = \frac{c_1}{r} \quad \text{...(i)}$$

Again integrating the above equation, we get

$$T + \frac{q_g''' r^2}{4k} = c_1 \ln(r) + c_2 \quad \text{...(ii)}$$

Boundary condition, $\frac{dT}{dr} = 0$ at $r = r_2$, we get

$$c_1 = \frac{q_g''' r_2^2}{2k} \quad \text{...(iii)}$$

Using another boundary condition,

Heat conducted at r_1 = Heat convected from inner boundary to water.

If 'h' is the heat transfer coefficient and T_∞ is the average temperature of coolant water, the heat balance for unit length is,

$$k \cdot 2\pi r_1 \left(\frac{dT}{dr}\right)_{r=r_1} = 2\pi r_1 h (T_1 - T_\infty)$$

From equation (i), we get

$$k\left[\frac{q_g'''}{2k}\cdot\frac{r_2^2}{r} - \frac{q_g''' r}{2k}\right]_{r=r_1} = h(T_1 - T_\infty)$$

$$\Rightarrow \qquad T_1 = T_\infty + \frac{q_g''' r_1}{2h}\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right] \qquad \text{...(iv)}$$

Now, putting the value of c_1 in equation (ii), we get

$$T + \frac{q_g''' r^2}{4k} = \frac{q_g''' r_2^2}{2k}\ln(r) + c_2 \qquad \text{...(v)}$$

If $T = T_1$ at $r = r_1$, we get

$$T_1 + \frac{q_g''' r_1^2}{4k} = \frac{q_g''' r_2^2}{2k}\ln(r_1) + c_2$$

$$\therefore \qquad c_2 = T_1 + \frac{q_g''' r_1^2}{4k} - \frac{q_g''' r_2^2}{2k}\ln(r_1)$$

Putting the value of c_1 and c_2 in equation (ii), we get

$$T + \frac{q_g''' r^2}{4k} = \frac{q_g''' r_2^2}{2k}\ln(r) + T_1 + \frac{q_g''' r_1^2}{4k} - \frac{q_g''' r_2^2}{2k}\ln(r_1)$$

Since temperature difference is required, hence $T = T_2$ at $r = r_2$

$$\Rightarrow \qquad T_2 - T_1 = \frac{q_g''' r_2^2}{4k}\left[2\ln\left(\frac{r_2}{r_1}\right) + \left(\frac{r_1}{r_2}\right)^2 - 1\right] \qquad \text{...(vi)}$$

The heat generation rate, $q''' = \dfrac{I^2 R}{\text{Volume of wire (m}^3\text{/m)}}$

$$R = \frac{0.11}{\pi\left(r_2^2 - r_1^2\right)} = \frac{0.11}{\pi \times \left(6^2 - 4^2\right)}\Omega$$

$$\therefore \qquad q_g''' = \frac{(2100)^2 \times 0.11 \times 10^6}{\pi(6^2 - 4^2) \times \pi(6^2 - 4^2) \times 1} = 122.877 \times 10^6 \text{ W/m}^3$$

Putting the value of q_g''' in equation (vi), we get

$$T_2 - T_1 = \frac{122.877 \times 10^6 \times 6^2 \times 10^{-6}}{4 \times 18}\left[2\ln\left(\frac{6}{4}\right) + \left(\frac{4}{6}\right)^2 - 1\right]$$

$$= 15.69°C$$

$$T_1 = T_\infty + \frac{q_g''' r_1}{2h}\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right]$$

$$T_1 = 20 + \frac{122.877 \times 10^6 \times 4 \times 10^{-3}}{2 \times 12500}\left[\left(\frac{6}{4}\right)^2 - 1\right]$$

$\Rightarrow$ $$T_1 = 44.57°C$$

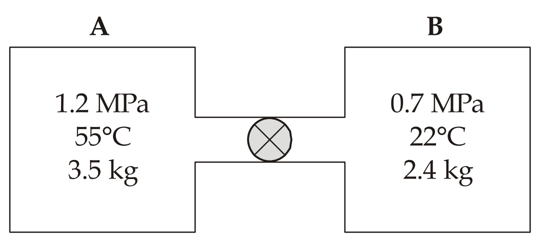
$\therefore$ The heat transfer rate, $q = 2\pi r_1 \times h(T_1 - T_\infty)$

$$= 2\pi \times 4 \times 10^{-3} \times 12500 \times (44.57 - 20)$$

$$= 7718.9 \text{ W/m}$$ Ans.

7. (c)

Refer to figure,



For the gas in vessel 'A'

$$P_A V_A = m_A R T_A$$

$$1.2 \times 10^3 \times V_A = 3.5 \times \frac{8.314}{28} \times 328$$

$\therefore$ $$V_A = 0.284 \text{ m}^3$$

For nitrogen, $$R = \frac{8.314}{28} = 0.297 \text{ kJ/kgK}$$

For the vessel 'B'

$$P_B V_B = m_B R T_B$$

$$0.7 \times 10^3 \times V_B = 2.4 \times 0.297 \times 295$$

$$V_B = 0.3 \text{ m}^3$$

$\therefore$ Total volume of A and B

$$V = V_A + V_B = 0.3 + 0.284$$
$$= 0.584 \text{ m}^3$$

Total mass of gas, $m = 3.5 + 2.4 = 5.9$ kg

Final temperature after mixing,

$$T = 25 + 273 = 298 \text{ K}$$

$\therefore$ Final pressure, $P = \dfrac{mRT}{V} = \dfrac{5.9 \times 0.297 \times 298}{0.584}$

$$P = 894.15 \text{ kPa or } 8.94 \text{ bar}$$ Ans.

Now, $C_v = \dfrac{R}{\gamma - 1} = \dfrac{0.297}{0.4} = 0.743$ kJ/kgK

Since there is no work transfer, the amount of heat transfer

$$Q = U_2 - U_1$$
$$= mC_vT - (m_AC_vT_A + m_BC_vT_B)$$
$$= 5.9 \times 0.743 \times 298 - (3.5 \times 0.743 \times 328 + 2.4 \times 0.743 \times 295)$$
$$Q = -72.66 \text{ kJ or } 72.66 \text{ kJ} \quad \text{(Rejected)}$$

Now, if the vessels were insulated, then

$$U_1 = U_2$$
$$m_AC_vT_A + m_BC_vT_B = mC_vT$$

where T would have been the final temperature

$$T = \frac{m_AT_A + m_BT_B}{m}$$

$$T = \frac{3.5 \times 328 + 2.4 \times 295}{5.9} = 314.57 \text{ K}$$ Ans

The final pressure, $P = \dfrac{mRT}{V} = \dfrac{5.9 \times 0.297 \times 314.57}{0.584}$

$$P = 943.87 \text{ kPa or } 9.43 \text{ bar}$$ Ans.

8. (a)

(i) For beam of constant width

Let the depth at the mid-span be d and that at any section distant x be d_x.

$$\therefore \qquad (Z_{NA})_x = \frac{(I_{NA})_x}{\left(\frac{d_x}{2}\right)} = \frac{\frac{1}{12}bd_x^3}{\left(\frac{d_x}{2}\right)} = \frac{bd_x^2}{6}$$

$\therefore$ Moment of resistance, $M_{Rx} = \sigma_{per} \cdot (Z_{NA})_x$

$$= \sigma_{per} \cdot \frac{b}{6} d_x^2 \qquad \text{...(i)}$$

Also,
$$M_x = \frac{wL}{2}x - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2) \qquad \text{...(ii)}$$

Equating equation (i) and (ii), we get

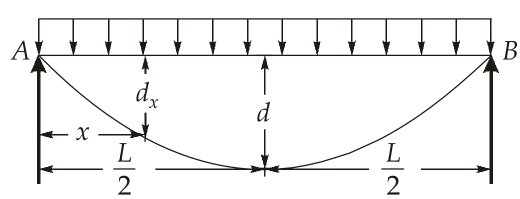
$$\sigma_{per} \cdot \frac{b}{6} d_x^2 = \frac{w}{2}\left(Lx - x^2\right)$$

$$\Rightarrow \qquad d_x = \sqrt{\frac{3w}{\sigma_{per} \cdot b}(Lx - x^2)} \qquad \text{Ans.}$$

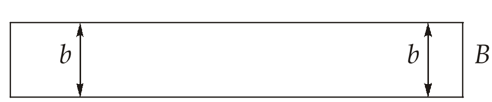
At mid-span, $x = \frac{L}{2}$

$$d = \sqrt{\frac{3w}{\sigma_{per} \cdot b}\left(L \cdot \frac{L}{2} - \left(\frac{L}{2}\right)^2\right)}$$

$$= \sqrt{\frac{3w}{\sigma_{per} \cdot b}} \cdot \frac{L}{2} \qquad \text{Ans.}$$



(a) Longitudinal-section



(b) PLAN

(b) For beam of constant depth

Let the width at any mid-span be b and the width at any other distance x be b_x. Let the depth d be constant.

$$\therefore \qquad (Z_{NA})_x = \frac{(I_{NA})_x}{\left(\frac{d}{2}\right)} = \frac{\frac{1}{12} \times b_x \cdot d^3}{\left(\frac{d}{2}\right)} = \frac{1}{6} b_x d^2 \qquad \text{...(iii)}$$

Moment of resistance, $M_{Rx} = \sigma_{\text{per}}(Z_{NA})_x = \sigma_{per} \cdot \frac{1}{6} b_x d^2$

$$M_x = \frac{wL}{2}x - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2) \qquad \text{...(iv)}$$

Equating equation (iii) and (iv), we get

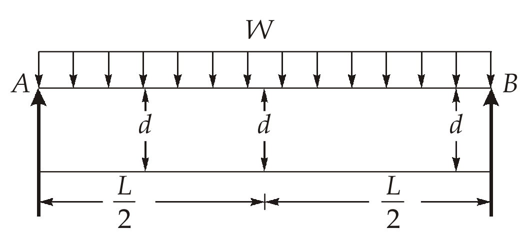
$$\therefore \qquad \sigma_{per} \cdot \frac{1}{6} b_x \cdot d^2 = \frac{w}{2}(Lx - x^2)$$

$$\Rightarrow \qquad b_x = \frac{3w}{\sigma_{per} \cdot d^2}(Lx - x^2) \quad \text{[Parabolic variation]} \qquad \text{Ans.}$$

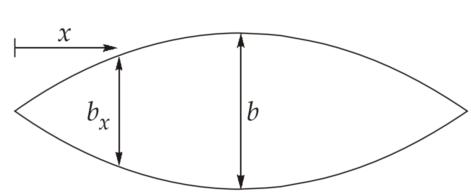
At mid-span, $x = \frac{L}{2}$

$$\therefore \qquad b = \frac{3w}{\sigma_{per} \cdot d^2} \times \left(L \cdot \frac{L}{2} - \left(\frac{L}{2}\right)^2\right)$$

$$b = \frac{3w \cdot L^2}{4\sigma_{per} \cdot d^2} \qquad \text{Ans.}$$



(a) Longitudinal-section



(b) PLAN

8. (b) (i)

Harmful Effect of R-12 Refrigerant: The earth's ozone layer in the upper atmosphere (stratosphere) is needed for the absorption of harmful ultraviolet rays from the sun. These UV rays can cause skin cancer. R-12 is dichlorodifluoromethane and comes under CFCs category of refrigerant. In 1985, as per scientific observations, it was found that there is a gaping hole above Antarctic in the ozone layer which protects earth from UV rays. R-12 refrigerant has been linked to the depletion of this ozone layer. CFCs are having varying degrees of ozone depletion potential (ODP). In addition, they act as greenhouse gases. Hence, CFCs have global warming potential (GWP) as well. According to an international agreement (montreal protocol), the use of fully halogenated CFCs (no hydrogen in the molecule) are considered to high ODP and R-12 falls in this category. The problem with CFC refrigerant is mainly that a single atom of Cl released from CFC reacts taking out 100,000 O_3 (ozone) molecules. That's why use of R-12 refrigerant is detrimental to the environment.

Harmful effect of R-22 Refrigerant: R-22 is Hydro-chlorofluorocarbons refrigerant which contain hydrogen atom alongwith chlorine atom. HCFCs have much lower ODP and GWP as compared to R-12 refrigerant. ODP of R-22 is only 5% of that of R-12. R-22 is considered to be transitional refrigerant and will have to be ultimately phased out by 2030 AD.

Refrigerant	R-12	R-22
Chemical formula	CCl_2F_2	$CHClF_2$
NBP temperature	–29.8°C	–40.8°C

Refrigerant used before year 2000	Substitute Refrigerant	
	Short Term	**Long Term**
R-12	R-134a	R-134a
R-22	R-22 (upto 2030 AD)	HFC 134 a, R 407C, R410A
	R-134a	Other blends of R-32, R-134a and other

8. (b) (ii)

The availability of a given system is defined as the maximum useful work that can be obtained in a process in which the system comes to equilibrium with the surroundings or attains a dead state.

Suppose a system is initially available at temperature T and pressure P while the surroundings are T_0 and P_0. The system can undergo a process in which it reaches a state of thermal equilibrium and mechanical equilibrium with the surroundings and delivers some work. When the system is in equilibrium with the surroundings, there is no possibility of obtaining further work from such a system. In other words, the system attains a dead state. Then the total work delivered by the system will be maximum. A part of the total work delivered by the system is used in pushing the surrounding atmosphere, which will not be available to move a body. Therefore, the maximum useful work that can be obtained from a system is less than the maximum work done by the system during a given process. This maximum useful work delivered by the system is called the availability of the system.

Suppose a system does work while it is interacting with the surroundings at T_0 in such a way that the initial and final temperatures of the system are identical with the surroundings temperature. Then the maximum work that can be obtained is equal to the decrease in the Helmholtz free energy of the system. Suppose the system is initially in the state P_1, V_1 and reaches the final state P_0, V_0 which is in equilibrium with the surroundings. Then

$$W_{max} = A_1 - A_0 = (U_1 - T_1S_1) - (U_1 - T_1S_1) \quad \text{where } T_1 = T_0$$

The change in the volume of the system = $V_0 - V_1$

The work done by the system in pushing the atmosphere = $P_0(V_0 - V_1)$

Then, the maximum useful work or availability is given by,

$$\begin{aligned} \text{Availability} = W_{\text{maximum useful}} &= W_{max} - P_0(V_0 - V_1) \\ &= (U_1 - T_0S_1) - (U_0 - T_0S_0) - P_0(V_0 - V_1) \\ &= (U_1 + P_0V_1 - T_0S_1) - (U_0 + P_0V_0 - T_0S_0) \\ &= \phi_1 - \phi_0 \end{aligned}$$

where $\phi = U + P_0V - T_0S$ = Availability function for non-flow process.

Suppose a system is in the initial state 1, where the availability is $(\phi_1 - \phi_0)$ and reaches the final state 2, where the availability is $(\phi_2 - \phi_0)$ during a given process, then the change in the availability or the change in the maximum useful work associated with the process is

$$(\phi_2 - \phi_0) - (\phi_2 - \phi_0) = \phi_2 - \phi_1$$

8. (c)

Given : $T_\infty = 22°C$; $V_\infty = 4.8$ m/s; $T_s = 58°C$

$$\text{Mean film temperature, } T_m = \frac{T_\infty + T_s}{2} = \frac{22+58}{2} = 40°C$$

At transition point, $$Re_{xc} = \frac{V_\infty \cdot x_c}{\nu}$$

$$\Rightarrow \quad x_c = \frac{5\times10^5 \times 16.96\times10^{-6}}{4.8} = 1.7667 \text{ m}$$

(i) Thickness of hydrodynamic boundary layer

$$\delta = \frac{4.64x}{\sqrt{\text{Re}_{xc}}} = \frac{4.64\times1.7667}{\sqrt{5\times10^5}}$$

$$= 0.01159 \text{ m or } 11.59 \text{ mm} \qquad \text{Ans.}$$

(ii) Thickness of thermal boundary layer

$$\delta_{th} = \frac{\delta}{1.026 \times (Pr)^{1/3}} = \frac{0.01159}{1.026 \times 0.7^{1/3}}$$

$$= 0.01272 \text{ m or } 12.72 \text{ mm}$$ Ans.

(iii)

$$Nu_{x_c} = 0.332 \cdot \left(Re_{x_c}\right)^{0.5} (Pr)^{1/3}$$

$$\frac{(h_x)_c x_c}{k} = 0.332 \cdot \left(5 \times 10^5\right)^{0.5} (0.7)^{1/3}$$

$\Rightarrow$ $$(h_x)_c = \frac{0.02755}{1.7667} \times 208.44 = 3.25 \text{ W/m}^2\text{K}$$

Average heat transfer coefficient,

$$\bar{h} = 2 \times (h_x)_c = 6.50 \text{ W/m}^2\text{K}$$

(iv) Heat transfer from both side of plate per unit width of plate

$$Q = \bar{h} \times (2A) \times \Delta T = 6.5 \times 2 \times 1.7667 \times 1 \times (58 - 22)$$

$$= 826.94 \text{ W/m}$$ Ans.

(v) Mass entrainment in the boundary layer

$$\dot{m} = \frac{5}{8} \rho V_\infty (\delta_2 - \delta_1)$$

$$= \frac{5}{8} \times 1.128 \times 4.8 \times (0.01159 - 0) = 0.0392 \text{ kg/s}$$ Ans.