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Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 3

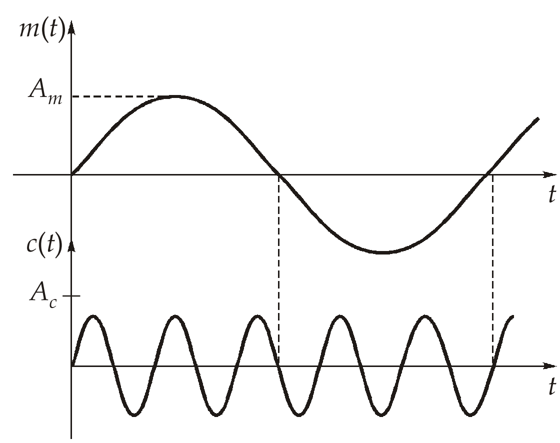
Section A : Analog and Digital Communication Systems

Q.1 (a) Solution:

Let us assume a sinusoidal message signal, $m(t) = A_m \cos \omega_m t$ and a carrier signal, $c(t) = A_c \cos \omega_c t$.

As, the frequency of carrier signal is very high. Therefore, $\omega_c >> \omega_m$.

Graphically,



In AM, the instantaneous amplitude of carrier changes in proportion with the modulating signal as shown below.

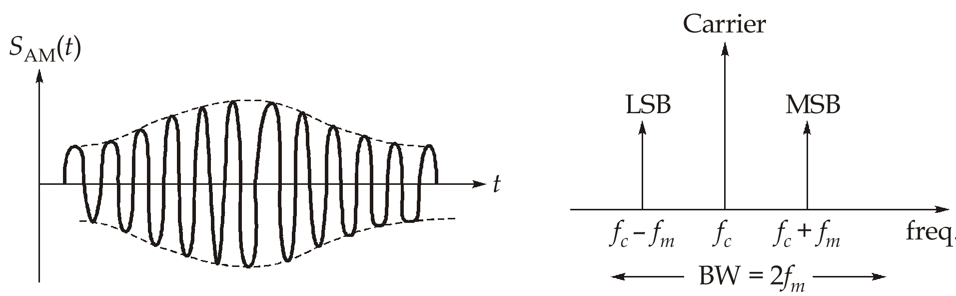
For amplitude modulation, we use

$$S_{AM}(t) = c(t) + m(t)c(t)$$

$$S_{AM}(t) = A_c \cos \omega_c t + A_m \cos \omega_m t \cdot A_c \cos \omega_c t$$

$$S_{AM}(t) = A_c \cos\omega_c t + \frac{A_m A_c}{2}\cos((\omega_c - \omega_m)t) + \frac{A_m A_c}{2}\cos((\omega_c + \omega_m)t)$$

When we draw the waveform and frequency spectrum for above modulated signal, we get



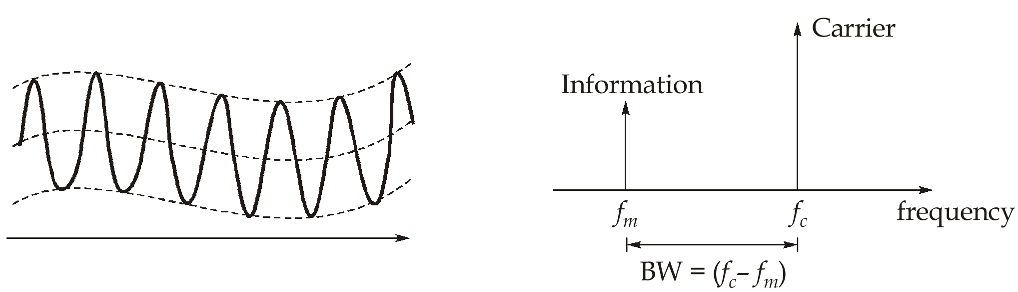
- Due to AM, the information signal gets translated to a higher frequency spectrum as shown above in figure. The upper and lower sidebands are produced symmetric to the carrier frequency f_c carrying the information of modulating signal. Both LSB and USB carry the same information..
- The result of linear addition of a carrier and modulating signal is shown below. The carrier is superimposed on the modulating signal.

  For Linear Addition,

$$S_L(t) = c(t) + m(t)$$

$$S_L(t) = A_c \cos \omega_c t + A_m \cos \omega_m t$$

The graphical representation and the frequency spectrum of the linealy superimposed signal can be drawn as below:



The spectrum of the linear addition as shown in fig. above. It indicates that there is no freq. translation of the modulating signal and the information is contained in the low frequency spectrum.

Q.1 (b) Solution:

(i) An FM signal is generated by NBFM with carrier frequency f_1 and frequency deviation Δf_1. The output frequency deviation can be given by combining all the multipliers.

$$\Delta f = (\Delta f_1) n_1 \, n_2$$
$$\Delta f = (50 \times 16 \times 32) \text{ Hz}$$
$$\Delta f = 25.6 \text{ kHz}$$

Note: The mixer has no effect on the frequency deviation of the signal.

(ii) We have,
$$f_2 = n_1 f_1$$
$$f_2 = 16 \times 175 \text{ kHz}$$
$$f_2 = 2.8 \text{ MHz}$$

Now, carrier frequency of output FM signal,
$$f_3 = f_2 \pm f_{LO}$$
$$= [2.8 \pm 1] \text{ MHz}$$
$$f_3 = 3.8 \text{ MHz or } 1.8 \text{ MHz}$$

(iii) Now, if $f_3 = 3.8$ MHz, then
$$f_c = n_2 \, f_3$$
$$f_c = 32 \times 3.8$$
$$f_c = 121.6 \text{ MHz}$$

and when $f_3 = 1.8$ MHz
$$f_c = n_2 f_3$$
$$f_c = 32 \times 1.8$$
$$f_c = 57.6 \text{ MHz}$$

Q.1 (c) Solution:

(i) We have,

$$\text{Carrier signal } c(t) = A \cos 2\pi \times 10^6 t$$
$$\text{Modulation index, } \beta_{\text{PM}} = 9$$
$$\beta_{\text{FM}} = 4.5$$
$$(\text{BW})_{\text{PM}} = 8.25 \text{ kHz}$$
$$(\text{BW})_{\text{FM}} = 15 \text{ kHz}$$
$$K_p = 3 \text{ rad/V}$$

We know that,
$$\beta_{\text{PM}} = K_p A_m = 9 \quad \text{...(i)}$$

Since,
$$K_p = 3 \text{ rad/V}$$

$$A_m = \frac{9}{3} = 3 \text{ V}$$

Also, $$\beta_{FM} = \frac{K_f A_m}{f_m} = 4.5$$

$$= \frac{K_f \times 3}{f_m} = 4.5$$

$$1.5 = \frac{K_f}{f_m} \quad \text{...(ii)}$$

$$(BW)_{PM} = 2(\beta_{PM} + 1)f_m = 8.25 \text{ kHz}$$

$$f_m = 412.5 \text{ Hz}$$

Now from equation (ii) we get

$$K_f = 1.5 \times 412.5$$

$$K_f = 618.75 \text{ Hz/V}$$

Now, $$m(t) = A_m \cos 2\pi f_m t = 3 \cos (2\pi \times 412.5)t$$

We get, $$m(t) = 3 \cos 825 \pi t$$

and $$K_f = 618.75 \text{ Hz/V}$$

(ii) We know that,

For PM signal, $$S_{PM}(t) = A_c \cos(2\pi f_c t + K_p m(t))$$

$$S_{PM}(t) = A_c \cos (2\pi \times 10^6 t + 9 \cos 825\pi t)$$

and for FM signal, $$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int m(t)dt\right)$$

$$S_{FM}(t) = A_c \cos\left(2\pi \times 10^6 t + 2\pi(618.75)\int 3\cos 825\pi t dt\right)$$

$$S_{FM}(t) = A_c \cos(2\pi \times 10^6 t + 4.5 \sin 825\pi\, t)$$

(iii) According to question,

$$A'_m = \frac{A_m}{2}$$

$$A'_m = \frac{3}{2} = 1.5 \text{ V}$$

$$\beta'_{PM} = K_p A'_m = 3 \times 1.5 = 4.5$$

$$\beta'_{FM} = \frac{K_f A'_m}{f_m} = \frac{618.75 \times 1.5}{412.5}$$

$$\beta'_{FM} = 2.25$$

If the amplitude of $m(t)$ is decreased by factor of two, then modulation index 'β' of both PM and FM signal also decreases by the factor of two.

Q.1 (d) Solution:

If no coding is employed, we have

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2P}{RN_0}}\right)$$

Given:

$$P = 10^{-6} \text{ W}$$

$$\frac{N_0}{2} = 10^{-11} \text{ W/Hz}$$

$$R = 10^4 \text{ bits/sec}$$

Probability of error in single bit

$$P_e = Q\left(\sqrt{\frac{2\times 10^{-6}}{10^4 \times 2\times 10^{-11}}}\right) = Q\left(\sqrt{10}\right)$$

$$P_e = Q(3.162) = 7.86 \times 10^{-4}$$

The error probability for four bits will be

$$P_{\text{error in 4 bits}} = 1 - (1 - P_e)^4 = 3.14 \times 10^{-3}$$

If coding is employed, we have $d_{min} = 3$ and

$$\frac{E}{N_0} = R_C \frac{E_b}{N_0} = R_C \frac{P}{RN_0}$$

where,

$$R_C = \frac{k}{n} = \frac{4}{7}$$

$$\frac{E}{N_0} = \frac{4}{7} \times \frac{10^{-6}}{10^4 \times 2 \times 10^{-11}} = \frac{20}{7}$$

Therefore, the message error probability is given by

$$P_e \leq (M-1)Q\left(\sqrt{\frac{2d_{\min}E}{N_0}}\right)$$

where,

$$M = 2^k$$

$$M = 2^4 = 16$$

$$P_e \leq 15Q\left(\sqrt{\frac{2\times 3\times 20}{7}}\right) = 15Q(4.14)$$

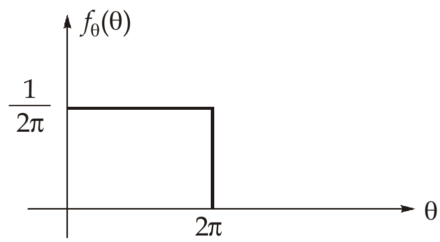
$$P_e \cong 2.6 \times 10^{-4}$$

It is seen that using this simple code, error probability is decreased by a factor of 12, of course, the price that has been paid is an increase in the bandwidth required for transmission of the message. The bandwidth expansion ratio is given by

$$\frac{(BW)_{\text{coded}}}{(BW)_{\text{uncoded}}} = \frac{1}{R_c} = \frac{7}{4} = 1.75$$

Q.1 (e) Solution:

Given that, θ is uniformly distributed in the range [0, 2π]. Hence, the probability density function $f_\theta(\theta)$ is given as below:



Mean value of X,

$$E[X] = \int_{-\infty}^{\infty} f_\theta(\theta)\sin\theta\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\sin\theta d\theta = 0$$

Mean value of Y,

$$E(Y) = \int_{-\infty}^{\infty} f_\theta(\theta)\cos\theta d\theta = \frac{1}{2\pi}\int_0^{2\pi}\cos\theta d\theta = \frac{1}{2\pi}\times 0 = 0$$

Correlation of X and Y:

$$E[XY] = \int_{-\infty}^{\infty} f_\theta(\theta)\sin\theta\cos\theta\, d\theta$$

$$= \frac{1}{4\pi}\int_0^{2\pi} 2\sin\theta\cos\theta d\theta$$

$$= \frac{1}{4\pi}\int_0^{2\pi}\sin 2\theta d\theta = \frac{1}{4\pi}\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} = 0$$

∴ X and Y are orthogonal

Covariance of X and Y:

$$\sigma_{XY} = E[XY] - E[X]E[Y]$$

$$= 0$$

∴ X and Y are uncorrelated.

Now, check for statistical independency.

X and Y are said to be statistically independent if $E[XY] = E[X]E[Y]$.

But in this problem, $E[X] = E[Y] = E[XY] = 0$

So, we cannot check for independency using the above mentioned relation.

Now, we can use $E[X^2]$ and $E[Y^2]$ to check for independency.

$$E[X^2] = \int_{-\infty}^{\infty} f_\theta(\theta)\sin^2\theta d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\sin^2\theta d\theta = \frac{1}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)d\theta = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$E[Y^2] = \int_{-\infty}^{\infty} f_\theta(\theta)\cos^2\theta d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta d\theta = \frac{1}{4\pi}\int_0^{2\pi}(1+\cos 2\theta)d\theta = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$E[X^2Y^2] = \int_{-\infty}^{\infty} f_\theta(\theta)\sin^2\theta\cos^2\theta d\theta = \frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta\sin^2\theta d\theta$$

$$= \frac{1}{8\pi}\int_0^{2\pi}\sin^2 2\theta d\theta = \frac{1}{16\pi}\int_0^{2\pi}(1-\cos 4\theta)d\theta = \frac{2\pi}{16\pi} = \frac{1}{8}$$

$$E[X^2Y^2] = \frac{1}{8} \neq E[X^2]E[Y^2]$$

Hence, X and Y are not independent.

∴ X and Y are orthogonal, uncorrelated but not independent.

Q.2 (a) Solution:

(i) The spectrum of the AM signal $u(t)$ contains the impulses at different frequency components. Thus, we can write frequency spectrum of $u(t)$ as

$$\begin{aligned} U(f) = {} & 0.25[\delta(f-(f_c+1.5k)] + 0.25[\delta(f-(f_c-1.5k)] \\ & + 0.25[\delta(f+(f_c-1.5k)] + 0.25[\delta(f+(f_c+1.5k)] \\ & + 2.5[\delta(f-(f_c+3k)] + 2.5[\delta(f-(f_c-3k)] \\ & + 2.5[\delta(f+(f_c-3k)] + 2.5[\delta(f+(f_c+3k)] \end{aligned}$$

And we know that,

$$A\cos 2\pi f_0 t \xleftrightarrow{\text{F.T.}} \frac{A}{2}[\delta(f-f_0)+\delta(f+f_0)]$$

Using the above relation, we can write

$$u(t) = 0.5\cos(2\pi(f_c + 1500)t) + 0.5\cos(2\pi(f_c - 1500)t) + 5\cos(2\pi(f_c + 3000)t) + 5\cos(2\pi(f_c - 3000)t) \quad ...(i)$$

Since, $\cos(a + b) + \cos(a - b) = 2\cos a \cos b$

$$u(t) = \cos 3000\pi t \times \cos 2\pi f_c t + 10\cos 6000\pi t \cdot \cos 2\pi f_c t \quad (\because A_c = 10)$$

$$u(t) = 10(0.1\cos 3000\pi t + \cos 6000\pi t)\cos 2\pi f_c t \quad ...(ii)$$

Comparing with the standard expression of DSB-SC AM signal $s(t) = m(t).c(t)$. We get,

$$m(t) = (0.1\cos 3000\,\pi t + \cos 6000\,\pi t)$$

$$c(t) = 10\cos 2\pi f_c t$$

(ii) From, the given spectrum we observe that the spectrum has both the sidebands, LSB and USB. The carrier wave is not present.

Therefore, we conclude that here Double sideband-suppressed carrier modulation scheme (DSB-SC) is used.

Main Advantage of DSB-SC over DSB-FC is that, in DSB-SC power consumption is less than DSB-FC. DSB-SC is an amplitude modulated wave transmission scheme in which only sidebands are transmitted and the Carrier is not transmitted as it gets suppressed. Whereas, in DSB-FC sidebands are transmitted along with the carrier. The carrier does not contain any information and its transmission results in loss of power.

Percentage of power saved in DSB-SC as compared to DSB-FC is given as,

$$\%\text{ Power saved} = \frac{P_c}{P_c\left[1 + \frac{\mu^2}{2}\right]} = \frac{2}{2 + \mu^2} \times 100\%$$

From equation (ii) we get

$$\mu_1 = 0.1$$

$$\mu_2 = 1$$

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.1)^2 + 1^2}$$

$$\mu_t \approx 1$$

$$\text{Power saved} = \frac{2}{2 + 1} = 0.667$$

% power saved in DSB-SC w.r.t DSB-FC = 66.7%

(iii) The power content at the frequency, P_{f_c+1500} is the same as the power content at the frequency P_{f_c-1500} and equal to $(0.25)^2 = 0.0625$ Watt.

Similarly,

$$P_{f_c+3000} = P_{f_c-3000} = (2.5)^2 = 6.25 \text{ Watt}$$

(iv) Using equation (i),

Total power dissipated, $$P_t = \frac{P_c\mu_t^2}{2} = \left(\frac{(0.5)^2}{2}\times 2\right) + \left(\frac{5\times 5}{2}\times 2\right)$$

$$P_t = 25.25 \text{ Watt}$$

Thus, $$\frac{P_c\mu_t^2}{2} = 25.25$$

$$\mu_t^2 = \frac{50.5}{P_c} \quad \text{...(iii)}$$

Since, $$P_c = \frac{A_c^2}{2} = \frac{100}{2} = 50 \text{ W}$$

From equation (iii), $$\mu_t^2 = \frac{50.5}{50} = 1.01$$

$$\mu_t \approx 1.005$$

Bandwidth of DSB-SC = $2f_{max}$, where f_{max} is the maximum frequency in the modulating signal

We have, $$m_1(t) = 0.1\cos 3000\pi t \Rightarrow f_{m1} = 1500 \text{ Hz}$$

$$m_2(t) = \cos 6000\,\pi t \Rightarrow f_{m2} = 3000 \text{ Hz}$$

Thus, $$\text{BW} = 2f_{max} = 2f_{m2}$$

$$= 2\times 3000$$

$$\text{BW} = 6 \text{ kHz}$$

Q.2 (b) Solution:

(i) $$m(t) = 5\cos(2000\pi t) + 2\cos(4000\pi t)$$

$$= m_1(t) + m_2(t)$$

To avoid slope overload distortion, we have

$$\left|\frac{dm_1(t)}{dt}\right|_{max} \le \Delta_1 f_s$$

where, Δ_1 is step size and f_s is sampling rate

$$\Delta_1 f_s \ge \left|\frac{dm_1(t)}{dt}\right|_{max}$$

$$\Delta_{1,\,\min} \text{ (minimum value of step size)} = \left|\frac{dm_1(t)}{dt}\right|_{\max} \times \frac{1}{f_s}, \text{ where } f_s = 56000 \text{ Hz}$$

$$= \frac{5(2000\pi)}{56000} = 0.56 \text{ V}$$

Also,
$$\Delta_2 f_s \geq \left|\frac{dm_2(t)}{dt}\right|_{\max}$$

$$\Delta_{2,\,\min} = \frac{8000\pi}{56000} = \frac{\pi}{7} = 0.45 \text{ V}$$

Hence, the larger step size out of two should be the minimum value of the step size i.e., $\Delta_{\min}$ = 0.56 V. If a value larger than minimum will be chosen, then granular noise will occur.

(ii) 1. Data word (message bits) = 1110

Generator polynomial (divisor) = $p^3 + p + 1$

Divisor in binary form = 1011

The divisor is $(q + 1)$ bits long.

Here, $q = 3$, hence we append three 0's to the data word.

Now, the data sequence is 1110000. We divide this data by the divisor of 1011 to obtain the code word to be transmitted:

```
          1 1 0 0
1 0 1 1 ) 1 1 1 0 0 0 0
          1 0 1 1
          -------------
          ø 1 0 1 0
            1 0 1 1
          -------------
            ø ø ø 1 0 0
                  -----
                Remainder
```

The remainder obtained from division is 100. Thus, the transmitted code word is 1110100.

2. Advantage of cyclic codes:

Cyclic codes have several advantages that make them particularly useful in digital communications and storage systems.

(a) Efficient Encoding and Decoding:

Cyclic codes can be encoded and decoded using efficient algorithms, such as the polynomial division method. This simplifies the implementation of encoding and decoding circuits in hardware and reduces computational complexity in software.

(b) Cyclic Redundancy Check (CRC):

Cyclic redundancy check is a widely used error-detecting technique based on cyclic codes. CRC codes can detect a variety of errors, including burst errors, and are extensively used in network protocols, storage systems, and communication interfaces.

(c) Linear Block Code Structure:

Cyclic codes are a subclass of linear block codes, which means they can be easily represented using generator and parity-check matrices. This facilitates the design and analysis of cyclic codes and enable systematic encoding and decoding procedures.

(d) Shift-invariance Property:

Cyclic codes exhibit a shift-invariance property, meaning that cyclically shifting a codeword results in another valid codeword. This property simplifies error correction algorithms and enables efficient decoding techniques based on syndrome calculation.

(e) Low Redundancy Overhead:

Cyclic codes achieve error detection and correction with relatively low redundancy overhead compared to other coding schemes. This is particularly advantageous in bandwidth limited and storage-constrained systems where minimizing overhead is crucial.

(f) Robustness to Burst Errors:

Cyclic codes are well-suited for applications where burst errors are common, such as magnetic storage systems and communication channels affected by noise or interference. Their ability to detect and correct burst errors makes them highly reliable in such environments.

Overall, the efficient encoding and decoding algorithms, along with their error detection and correction capabilities, makes cyclic codes a preferred choice for various digital communication and storage applications.

Q.2 (c) Solution:

(i) Selectivity : The ability of a receiver to accept a given band of frequency and reject all other is called as selectivity. The selectivity of a receiver is measured by measuring the bandwidth of the receiver at its 3-dB point. α-factor known as shape factor is then calculated. The shape factor is defined as the ratio between the bandwidth of the receiver at –3 dB and –60 dB points. Ideally this factor should be equal to 1. This value is not achieved, however a factor of 2 is generally achieved in practical.

(ii) Sensitivity : The sensitivity of a receiver is the minimum RF signal level that can be detected at the input to the receiver and still produce a valid information signal after demodulation. Generally, the signal to noise ratio and the power of the signal at the output of the audio selection are used to determine the quality of the signal received. The minimum acceptable signal to noise ratio value for a broadband microwave receiver is about 40 dB with 5 mW of signal power.

(iii) Dynamic range : The dynamic range of the receiver is defined as the difference in decibels between the minimum input level necessary to discern a signal and the input level that will overdrive the receiver and produce distortion, i.e., the dynamic range is the input power range over which the receiver is useful. The minimum receiver level is a function of front end noise, noise figure, and the desired signal quality. The input signal level that will produce over load distortion is a function of the net gain of the receiver. A dynamic range of 100 dB is considered safe. A low dynamic range results in severe intermodulation distortion of the weaker input signals.

(iv) Fidelity : Fidelity is defined as the ability of a receiver to produce all the frequency components of the original source of information without any amplitude, phase, and frequency distortion. Phase distortion is not so serious as compared to other distortion for voice transmission, but it affects picture, i.e., video transmission, and data. Improper filtering causes phase distortion, which is due to varying phase shift undergone by various frequencies at the break point. This can be taken care of by increasing the bandwidth of the filter beyond the minimum value necessary to pass the highest frequency. If all frequencies experience the same phase delay, it causes only a delay. But if the frequencies experience different phase delays, the received signal will have what is known as phase distortion.

Amplitude distortion is caused by an unequal gain of the receiver for various frequency components. This can be avoided by not overdriving the receiver.

Frequency distortion occurs when frequencies other than the one present in the original source of information are present at the received signal. It is due to harmonic and intermodulation distortion and caused by non-linear amplification. Second-order products $2f_1$, $2f_2$, $f_1 \pm f_2$, and so on are only a problem in a broadband network since they fall outside the narrowband system.

Q.3 (a) Solution:

(i) Given, number of brightness levels = 12

Number of picture elements per frame = 2.5×10^6

$$\left(\frac{S}{N}\right)_{dB} = 30 \text{ dB} = 10^3$$

Given that one picture frame is transmitted every three minutes.

i.e., rate of transmission = $r_s = \frac{1}{3 \times 60} = \frac{1}{180} \frac{\text{frames}}{\text{sec}}$

Each of the 2.5×10^6 picture elements can take any of the 12 equiprobable brightness level. Hence, number of different picture possible = $12^{2.5 \times 10^6}$ each having the same probability.

Entropy of the source, $H = \log_2 12^{2.5 \times 10^6}$

$$H = 2.5 \times 10^6 \log_2 12$$

$$H = 8.9625 \times 10^6 \text{ bits/frame}$$

Information Rate, $R = r_s H$

$$C = \frac{1}{180} \times 8.9625 \times 10^6 = 49.79 \frac{\text{k bits}}{\text{sec}}$$

The channel capacity, C should be greater than or equal to the information rate, R. We assume, $C = R$. From Shanon-Hartley Theorem,

$$C = B \log_2\left(1 + \frac{S}{N}\right) \frac{\text{bits}}{\text{sec}},$$

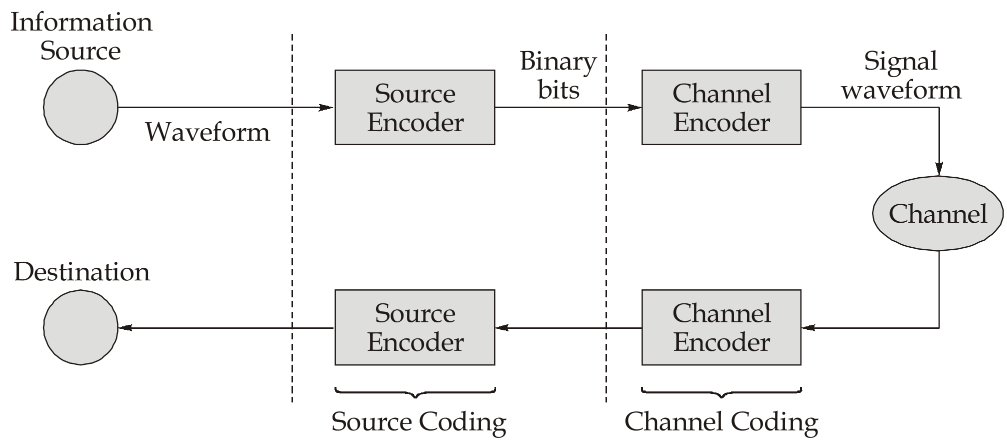
where B is the Bandwidth of the channel

$$49.79 \times 10^3 = B \log_2(1 + 10^3)$$

$$B = 4.995 \text{ kHz} \cong 5 \text{ kHz}$$

(ii) 1. Source coding, is the process of efficiently converting the output of either an analog or a digital source, with as little or no redundancy, into a sequence of binary digits.

Source coding is ubiquitous in modern technology and is used in various applications. The channel encoder converts bits to signal waveform, while the decoder converts received waveformback to bits. The block diagram of source coding is as shown below:



2. Digital modulators and demodulators (modems) serve essential functions in modern communication systems by converting digital data into analog signals for transmission and vice versa.

   (a) Digital Modulator:

   Conversion to Analog signals:

   Modulators convert digital data, typically represented as binary digits (bits), into analog signals suitable for transmission over communication channels. This conversion process involves encoding the digital data onto carrier signals.

   Efficient use of bandwidth: By modulating digital data onto carrier signals, modulators enable the transmission on multiple data streams simultaneously within a given bandwidth. This efficient use of bandwidth maximizes the capacity of communication channels including.

   - Digital media compression: Audio, image and video files are compressed to reduce storage requirements and transmission bandwidth.
   - Data transmission: Compressed data requires less bandwidth, making it more efficient for transmission over networks.
   - Archiving and backup: Compressed data takes up less space, making it easier to store and manage image datasets.
   - Communication systems: Source coding is essential for efficient data transmission in telecommunication, satellite communication and wireless networks.

     The channel encoder, introduces, in a controlled manner, certain amount of redundancy that can be used at the receiver to overcome the effects of noise and serves to increase the reliability of the received data and improves the quality of the received signal. The channel decoder attempts to reconstruct the original information.

Noise Immunity:

Digital modulation schemes are designed to be robust against noise and interference, ensuring reliable communication even in noisy environments.

(b) Digital Demodulator (Demodulator or Modem:

Recovery of Digital Data:

Demodulators extract digital data from analog signals received over communication channels. They perform the inverse operation of modulators, decoding the modulated signals to recover the original digital data.

Error Detection and Correction:

Demodulators may incorporate error detection and correction technique to ensure the accuracy of the received data. Error correction codes embedded in the transmitted signal allow demodulators to detect and correct errors introduced during transmission, improving the reliability of the communication link.

Q.3 (b) Solution:

We have,

$$\text{Carrier frequency}, f_c = 10^6 \text{ Hz}$$

$$\text{Angle modulated signal}, \; S_{EM}(t) = 5\cos(\omega_c t + 20\sin 1000\pi t + 10\sin 2000\pi t)$$

Comparing the given expression with the standard equation of the multi-tone angle modulated signal,

$$S(t) = A_c \cos(2\pi f_c t + \beta_1 \sin 2\pi f_{m1} t + \beta_2 \sin 2\pi f_{m2} t)$$

We get,

$$\text{Carrier amplitude}, A_C = 5 \text{ V}$$

$$\text{message frequency}, f_{m1} = 500 \text{ Hz}$$

$$f_{m2} = 1000 \text{ Hz}$$

(i) Power of the modulated signal:

The power of the angle modulated signal is same as the power of the unmodulated carrier. Hence,

$$P = \frac{A_C^2}{2R_L} \qquad \text{Assume } R_L = 1\ \Omega$$

$$\therefore \quad P = \frac{(5)^2}{2} = 12.5 \text{ W}$$

(ii) The maximum frequency deviation:

We know that

$$\Delta f|_{max} = \left|\frac{1}{2\pi}\frac{d\theta}{dt}\right|_{max}$$

$$\Delta f|_{max} = \left|\frac{1}{2\pi}\frac{d(20\sin 1000\pi t + 10\sin 2000\pi t)}{dt}\right|_{max}$$

$$\Delta f|_{max} = \left|\frac{1}{2\pi}\times(20\times 1000\pi\cos 1000\pi t + 10\times 2000\pi\cos 2000\pi t)\right|_{max}$$

$$\Delta f|_{max} = \left|(10^4\cos 1000\pi t + 10^4\cos 2000\pi t)\right|_{max}$$

$$\Delta f|_{max} = 20 \text{ kHz}$$

(iii) The maximum phase deviation:

$$\Delta\phi|_{max} = |\theta_i|_{max}$$

$$\Delta\phi|_{max} = |(20\sin 1000\pi t + 10\sin 2000\pi t)|_{max}$$

$$\Delta\phi|_{max} = 20 + 10$$

$$\Delta\phi|_{max} = 30 \text{ radian}$$

(iv) Estimated Bandwidth:

Using Carson's rule, $\text{BW} = (\beta + 1)2f_m$

$$\text{BW} = \left(\frac{(\Delta f)_{max}}{f_{max}} + 1\right)2f_{max}$$

$$\text{BW} = \left(\frac{20\times 10^3}{10^3} + 1\right)2\times 1000$$

$$\text{BW} = 42 \text{ kHz}$$

(v) Modulation index:

$$\beta = \frac{(\Delta f)_{max}}{f_{max}}$$

$$\beta = \frac{20\times 10^3}{10^3} = 20$$

Q.3 (c) Solution:

Given,

$$H = [I_{n-k} : P^T] = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

Here, $n = 7$ and $k = 4$ for a (7,4) linear block code

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(i) Generator matrix:

$$G = [P : I_k] = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

(ii)

Message Word (d)	Codeword (C = d.G)	Weight of codeword
0 0 0 0	0 0 0 0 0 0 0	0
0 0 0 1	1 0 1 0 0 0 1	3
0 0 1 0	1 1 1 0 0 1 0	4
0 0 1 1	0 1 0 0 0 1 1	3
0 1 0 0	0 1 1 0 1 0 0	3
0 1 0 1	1 1 0 0 1 0 1	4
0 1 1 0	1 0 0 0 1 1 0	3
0 1 1 1	0 0 1 0 1 1 1	4
1 0 0 0	1 1 0 1 0 0 0	3
1 0 0 1	0 1 1 1 0 0 1	4
1 0 1 0	0 0 1 1 0 1 0	3
1 0 1 1	1 0 0 1 0 1 1	4
1 1 0 0	1 0 1 1 1 0 0	4
1 1 0 1	0 0 0 1 1 0 1	3
1 1 1 0	0 1 0 1 1 1 0	4
1 1 1 1	1 1 1 1 1 1 1	7

(iii) minimum distance

$$d_{min} = \text{Smallest hamming weight of non-zero codeword}$$
$$d_{min} = 3$$

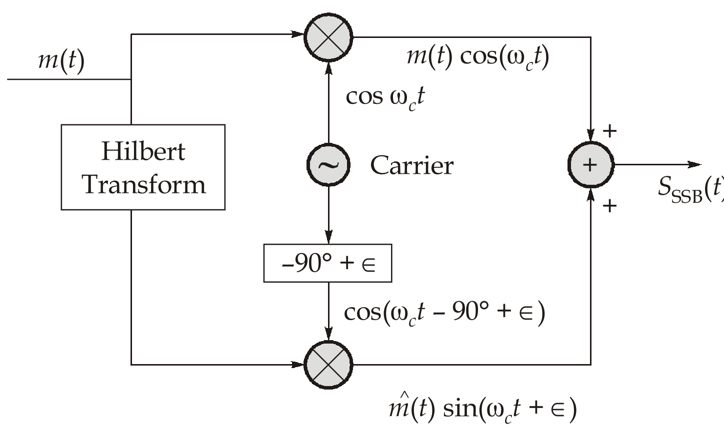
(iv) If 'C' is a valid codeword, then $CH^T = 0$

$$\therefore \quad [0\,1\,0\,0\,0\,1\,1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [0 \quad 0 \quad 0]$$

Hence, [0 1 0 0 0 1 1] is a valid code word.

Q.4 (a) Solution:

(i) Block diagram for SSB-signal generation:



(ii) Here, $m(t) = \cos \omega_m t$.

It's Hilbert Transform is $\hat{m}(t) = \sin \omega_m t$.

The LSB-SSB signal form the above block diagram is given by

$$S_{SSB}(t) = m(t)\cos \omega_c t + \hat{m}(t)\sin(\omega_c t + \in)$$

Since, $m(t) = \cos \omega_m t$

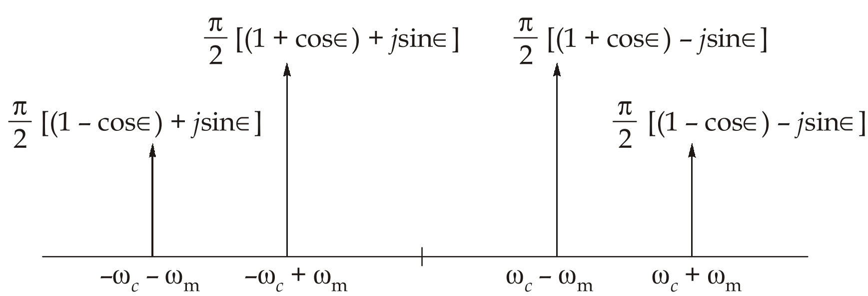
$$S_{SSB}(t) = \left(\frac{1}{2}\right)\left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t\right] + \frac{1}{2}\left[\cos((\omega_c - \omega_m)t + \in) - \cos\left((\omega_c + \omega_m)t + \in\right)\right]$$

$$S_{SSB}(t) = \left(\frac{1}{2}\right)[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\cos\in -$$
$$\sin(\omega_c - \omega_m)t\sin\in - \cos(\omega_c + \omega_m)t\cos\in$$
$$+\sin(\omega_c + \omega_m)t\sin\in]$$

$$S_{SSB}(t) = \frac{1}{2}[(1+\cos\in)\cos(\omega_c - \omega_m)t + (1-\cos\in)\cos(\omega_c + \omega_m)t$$
$$+\sin\in\sin(\omega_c + \omega_m)t + \sin\in\sin(\omega_c - \omega_m)t] \quad ...(i)$$

The corresponding frequency spectrum is

$$S(\omega) = \frac{\pi}{2}\begin{bmatrix} (1+\cos\in)[\delta(\omega-(\omega_c-\omega_m))+\delta(\omega+(\omega_c-\omega_m))]+ \\ (1-\cos\in)[\delta(\omega-(\omega_c+\omega_m))+\delta(\omega+(\omega_c+\omega_m))]+ \\ (-j)\sin\in[\delta(\omega-(\omega_c+\omega_m))-\delta(\omega+(\omega_c+\omega_m))]+ \\ (-j)\sin\in[\delta(\omega-(\omega_c-\omega_m))-\delta(\omega+(\omega_c-\omega_m))] \end{bmatrix}$$



(iii) From equation (i), the power in the desired LSB component at $(\omega_c - \omega_m)$ is

$$P_{desired} = \left[\left(\frac{1+\cos\in}{2\sqrt{2}}\right)^2 + \left(\frac{\sin\in}{2\sqrt{2}}\right)^2\right] = \frac{1}{4}(\cos\in + 1)$$

and the power in the undesired sideband at $(\omega_c + \omega_m)$ is

$$P_{undesired} = \left[\left(\frac{1-\cos\in}{2\sqrt{2}}\right)^2 + \left(\frac{\sin\in}{2\sqrt{2}}\right)^2\right]$$

$$P_{undesired} = \frac{1}{4}[(1-\cos\in)]$$

Hence, $$\frac{P_{desired}}{P_{undesired}} = \frac{1+\cos\in}{1-\cos\in}$$

(iv) We get, $\dfrac{P_{\text{desired}}}{P_{\text{undesired}}} = \dfrac{1+\cos\epsilon}{1-\cos\epsilon}$

If $\epsilon = 15°$

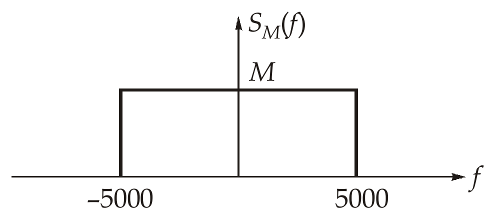
$$\frac{P_{\text{desired}}}{P_{\text{undesired}}} = \frac{1+\cos 15°}{1-\cos 15°} = 57.7$$

Q.4 (b) Solution:

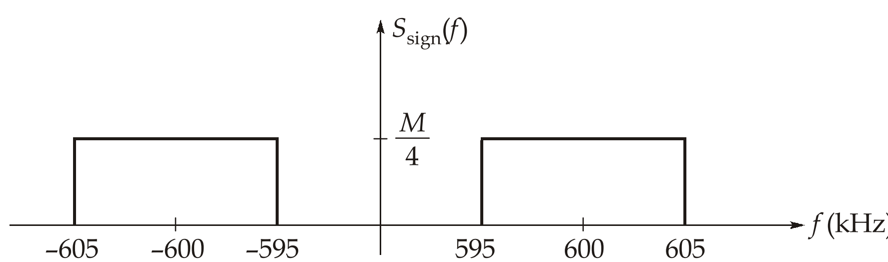
Given, $f_c = 600$ kHz

$$S_n(\omega) = \frac{1}{\omega^2 + a^2}$$

The PSD of message signal can be drawn as below:



After modulation:

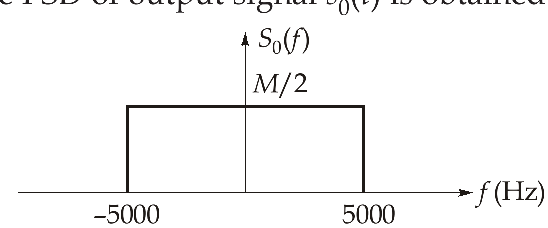


Signal power received at the input of the receiver = 1×10^{-6} watt

$$\left(\frac{M}{4}\right) \times (2 \times 10 \times 10^3) = 10^{-6}$$

$$M = 2 \times 10^{-10} \text{ W/Hz}$$

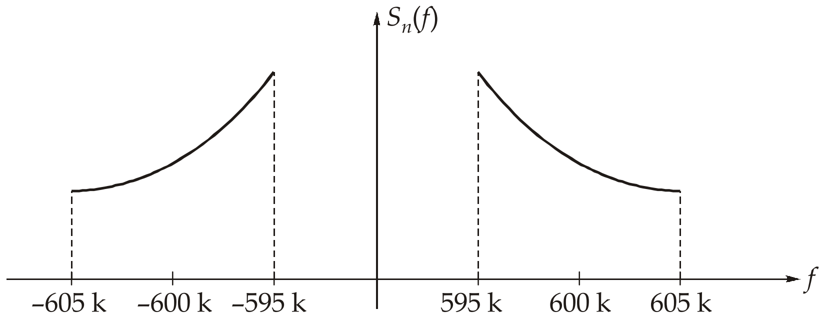
After demodulation, the PSD of output signal $s_0(t)$ is obtained as below:



Output signal power, $S_0 = M/2 \times 2 \times 5000$

$$= \frac{2 \times 10^{-10}}{2} \times 10 \times 10^3 = 10^{-6} \text{ W}$$

Channel noise at the receiver:



$$N_0 = 2\int_{595\times10^3}^{605\times10^3} \frac{1}{(4\pi^2 f^2 + a^2)} df$$

$$= \frac{2}{a^2}\int_{595\times10^3}^{605\times10^3} \frac{1}{\left(\frac{4\pi^2}{a^2}f^2 + 1\right)} df \text{, where } a = 10^6\pi$$

$$= \frac{2}{a^2}\int_{595\times10^3}^{605\times10^3} \frac{1}{(2\times10^{-6} f)^2 + 1} df$$

$$= \frac{2}{a^2}\left[\tan^{-1}(2\times10^{-6} f)\times\frac{1}{(2\times10^{-6})}\Bigg|_{595\times10^3}^{605\times10^3}\right]$$

$$= \frac{2}{(10^6\pi)^2}\times\frac{1}{2\times10^{-6}}\Big[\tan^{-1}(2\times10^{-6}\times605\times10^3)$$

$$-\tan^{-1}(2\times10^{-6}\times595\times10^3)\Big]$$

$$= (1.01 \times 10^{-7})(8.2 \times 10^{-3})$$

$$N_0 = 8.282 \times 10^{-10} \text{ W}$$

We will get same power at the output of demodulator

Hence, $S_0 = (10^{-6}) + (8.282 \times 10^{-10})$

$= 1.0008\ \mu$W

$$\text{Output SNR} = \frac{S_0}{N_0} = \frac{1\times10^{-6}}{8.282\times10^{-10}} = 1207.43$$

Q.4 (c) Solution:

(i) Given,

$$P\left(\frac{Y}{X}\right) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$P(X_1) = \frac{3}{4} \text{ and } P(X_2) = \frac{1}{4}$$

$$P(X, Y) = P(X)\, P\left(\frac{Y}{X}\right)$$

$$P(X, Y) = \begin{bmatrix} \frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{12} & \frac{1}{6} \end{bmatrix}$$

From $P(X, Y)$, we obtain

Since, sum of column of $P_n(X, Y) = P_n(Y)$

$$P(Y_1) = \frac{7}{12} \qquad P(Y_2) = \frac{5}{12}$$

$$H(X) = -\sum_{i=1}^{n} P(X_i)\log_2 P(X_i)$$

We have,

$$H(X) = \frac{-3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} = 0.81127\frac{\text{bits}}{\text{symbol}}$$

$$H(Y) = -\sum_{j=1}^{n} P(Y_j)\log_2 P(Y_j)$$

$$H(Y) = \frac{-7}{12}\log_2\frac{7}{12} - \frac{5}{12}\log_2\frac{5}{12} = 0.9798\frac{\text{bits}}{\text{symbol}}$$

$$H(X, Y) = -\sum_{i=1}^{n}\sum_{j=1}^{n} P(X,Y)\log_2 P(X,Y)$$

$$H(X, Y) = \frac{-1}{2}\log_2\frac{1}{2} - \frac{1}{4}\log_2\frac{1}{4} - \frac{1}{12}\log_2\frac{1}{12} - \frac{1}{6}\log_2\frac{1}{6}$$

$$= 1.7295\frac{\text{bits}}{\text{symbol}}$$

$$H\left(\frac{Y}{X}\right) = -\sum_{i=1}^{n}\sum_{j=1}^{n} P(X,Y)\log_2 P\left(\frac{Y}{X}\right)$$

$$H\left(\frac{Y}{X}\right) = \frac{-1}{2}\log_2\frac{2}{3} - \frac{1}{4}\log_2\frac{1}{3} - \frac{1}{12}\log_2\frac{1}{3} - \frac{1}{6}\log_2\frac{2}{3}$$

$$= 0.9183\,\frac{\text{bits}}{\text{symbol}}$$

$$\text{Mutual information} = I(X,Y) = H(X) - H\left(\frac{X}{Y}\right)$$

$$P\left(\frac{X}{Y}\right) = \frac{P(X,Y)}{P(Y)}$$

$$P\left(\frac{X}{Y}\right) = \frac{1}{P(Y)}\begin{bmatrix}\frac{1}{2} & \frac{1}{4}\\ \frac{1}{12} & \frac{1}{6}\end{bmatrix}$$

$$P\left(\frac{X}{Y}\right) = \begin{bmatrix}\frac{6}{7} & \frac{3}{5}\\ \frac{1}{7} & \frac{2}{5}\end{bmatrix}$$

$$H\left(\frac{X}{Y}\right) = -\sum_{i=1}^{n}\sum_{j=1}^{n} P(X,Y)\log_2 P\left(\frac{X}{Y}\right)$$

$$H\left(\frac{X}{Y}\right) = \frac{-1}{2}\log_2\frac{6}{7} - \frac{1}{4}\log_2\frac{3}{5} - \frac{1}{12}\log\frac{1}{7} - \frac{1}{6}\log_2\frac{2}{5}$$

$$H\left(\frac{X}{Y}\right) = 0.7497 \text{ bits/symbol}$$

$$\text{Mutual Information, } I(X, Y) = 0.8113 - 0.7497$$

$$= 0.0616\,\frac{\text{bits}}{\text{symbol}}$$

(ii) Channel capacity

$$C = \log_2 n - H\left(\frac{Y}{X}\right)$$

where,

$$C = \log_2 2 - 0.9183$$

$$C = 0.0817\,\frac{\text{bits}}{\text{symbol}}$$

$$\text{Efficiency, } \eta = \frac{I(X,Y)}{C}$$

$$\eta = \frac{0.0616}{0.0817}$$

$$\eta = 0.754$$

$$\%\eta = 75.4\%$$

$$\text{Redundancy} = 1 - \eta = 1 - 0.754$$

$$= 0.246 = 24.6\%$$

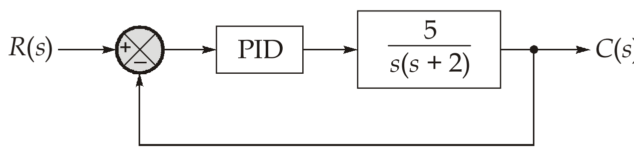
Section B : Signals and Systems-1 + Microprocessors and Microcontroller-1 + Network Theory-2 + Control Systems-2

Q.5 (a) Solution:

(i) Let PID controller be

$$G_c(s) = K_P + K_D \cdot s + \frac{K_I}{s}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{\frac{5}{s(s+2)}\left[K_P + K_D s + \frac{K_I}{s}\right]}{1 + \frac{5}{s(s+2)}\left[K_P + K_D s + \frac{K_I}{s}\right]}$$



$$T(s) = \frac{5\left[sK_P + s^2K_D + K_I\right]}{s^2(s+2) + 5sK_P + s^2 \times 5 \times K_D + 5K_I}$$

$$T(s) = \frac{5\left[s^2K_D + sK_P + K_I\right]}{s^3 + s^2\left[2 + 5K_D\right] + 5sK_P + 5K_I}$$

Desired location of poles are:

$$s = -6 \text{ and } s = -2 \pm 3j$$

Characteristic equation:

$s^3 + s^2[2 + 5K_D] + s5K_p + 5K_I = 0$ ...(i)

Characteristic equation from the given poles:

$$(s + 6)[s + 2 + 3j][s + 2 - 3j] = 0$$

On solving, we get

$s^3 + 10s^2 + 37s + 78 = 0$ ...(ii)

On comparing (i) and (ii), we get

$$2 + 5K_D = 10$$

$$K_D = \frac{8}{5}$$

$$5K_P = 37$$

$$K_P = \frac{37}{5}$$

$$5K_I = 78$$

$$K_I = \frac{78}{5}$$

The transfer function of PID controller is given by

$$G_C(s) = 7.4 + \frac{15.6}{s} + 1.6s$$

(ii)

$$G(s)H(s) = \frac{1}{s(s+1)}$$

Assume the PD controller is defined by $G_C(s) = K_P + sK_D$

Hence,
$$G(s)H(s)G_C(s) = \frac{K_P + sK_D}{s(s+1)}$$

At ω = 2 rad/sec, the phase margin = 40°

Hence, $\angle G(j2)H(j2)G_C(j2) = -140°$

$$-90° - \tan^{-1}\left(\frac{\omega}{1}\right) + \tan^{-1}\left(\frac{\omega K_D}{K_P}\right) = -140°$$

$$-90° - \tan^{-1}(2) + \tan^{-1}\left(\frac{2K_D}{K_P}\right) = -140°$$

$$\tan^{-1}\left(\frac{2K_D}{K_P}\right) - \tan^{-1}(2) = -50°$$

$$\tan^{-1}\left(\frac{2K_D}{K_P}\right) = -50° + 63.43° = 13.43°$$

$$\frac{2K_D}{K_P} = 0.2388 \qquad \text{...(1)}$$

Also,
$$|G(j2)H(j2)G_C(j2)| = \left|\frac{K_P + j2K_D}{j2(1+2j)}\right| = 1$$

$$\frac{\sqrt{K_P^2 + 4K_D^2}}{2\sqrt{5}} = 1$$

$$K_P^2 + 4K_D^2 = 20 \qquad \text{...(2)}$$

$$4K_D^2 = 0.057K_P^2 \text{ (from equation (1))}$$

from equation (1) and (2)

$$K_P = 4.35$$

and $$K_D = 0.5194$$

Hence, the required PD controller,

$$G_c(s) = K_P + K_D s = 4.35 + 0.5194s$$

Q.5 (b) Solution:

There are five hardware interrupt pins in 8085: TRAP, RST 7.5, RST 6.5, RST 5.5 and INTR.

(i) Maskable and Non-maskable interrupts:

Maskable interrupts are those interrupts which can be enabled or disabled.

Enabling and disabling of interrupts can be done by software instructions like EI, DI and SIM. The interrupts RST 7.5, RST 6.5, RST 5.5 and INTR are maskable i.e. interrupt can be enabled or disabled.

The interrupts which can not be disabled are called non-maskable interrupts. These interrupts can never be disabled by any software instruction. TRAP is non-maskable interrupt. Such interrupts are normally used for emergency cases like fire alarming, fire extinguisher system, intruder detector etc.

(ii) Vectored and non-vectored interrupts: Vectored interrupts are those interrupts which have particular memory location where program control is transferred when interrupt occur. Each vectored interrupt points to the particular location in memory. RST 7.5, RST 6.5, RST 5.5 and TRAP are vectored interrupts.

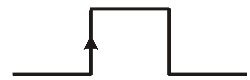
The address to which program control is transferred are:

Name	Vectored Address
RST 7.5	00 3C H
RST 6.5	00 34 H
RST 5.5	00 2C H
TRAP	00 24 H

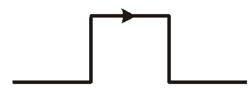
Non-vectored interrupts do not have fixed memory location for transfer of program control.

The address of the memory location is given by interrupting device to the processor along with the interrupt. INTR is a non-vectored interrupt.

(iii) Edge triggered and level triggered interrupts: The interrupts that are triggered at leading or trailing edge are called edge triggered interrupts. RST 7.5 is an edge triggered interrupt. It is triggered during the leading edge.



The interrupts which are triggered at high or low level are called level triggered interrupts. RST 6.5, RST 5.5 and INTR are level triggered interrupts.



TRAP is both edge and level triggered interrupt.

(iv) Priority based interrupts: When there is a simultaneous interrupt request at two or more interrupt pins, then the microprocessor will service the interrupt with a higher priority. To avoid confusion in such cases, all microprocessor assigns priority level to each interrupt pins. Priority is considered by microprocessor only when there are simultaneous requests.

The priority of 8085 interrupts are:

Interrupt	Priority
TRAP	1
RST 7.5	2
RST 6.5	3
RST 5.5	4
INTR	5

Q.5 (c) Solution:

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \quad \text{...(i)}$$

$$g(t) = x(3t) * h(3t)$$

$$=` \int_{-\infty}^{\infty} x(3\tau)h[3(t-\tau)]\,d\tau = \int_{-\infty}^{\infty} x(3\tau)h[(3t-3\tau)]\,d\tau$$

Let, $u = 3\tau \Rightarrow d\tau = \frac{1}{3}du$

So,

$$g(t) = \frac{1}{3}\int_{-\infty}^{\infty} x(u)h(3t-u)\,du \quad \text{...(ii)}$$

By comparing equations (i) and (ii), we get,

$$g(t) = \frac{1}{3}y(3t) = ay(bt)$$

So, $a = \frac{1}{3}$ and $b = 3$.

Q.5 (d) Solution:

The 8086 microprocessor contains the following registers:

(i) General Purpose Registers

There are four 16-bit general purpose registers i.e., AX, BX, CD, DX Each of these can be split into two 8-bit registers. For instance AX can be split into AH and AL, i.e.

AX ≡ AH AL

AH ↓ 8-bit high order register

AL: 8-bit low order register

In addition to general purpose, these registers are also used for some special purposes:

- AX also serves the purpose of accumulator.
- BX also serves as a base register for the computation of address.
- CX also serves the purpose of counter.
- DX is also used when data is transferred between I/O port and memory using certain I/O instruction.

(ii) Pointers and Index Registers

These registers contain the offset of data and instructions. The term offset refers to the distance of a variable, label, or instruction from its base segment. The pointers will always store some address or memory location. The following registers are in the group of pointer and index register:

1. Stack pointer (SP) – Points the top of stack.
2. Base pointer (BP) – Contains base address of the memory.
3. Source Index (SI) – Stores the offset address of source.
4. Destination index (DI) – Stores the offset address of destination.
5. IP – Instruction pointer which stores the address of the next instruction that is to be executed.

(iii) Segment Register:

In an 8086 based system, memory is divided into four segments i.e., code segment, data segment; stack segment and extra segment.

A segment register points to the starting address of a memory segment currently being used. The maximum capacity of any segment may be upto 64 kB.

There are four segment registers:

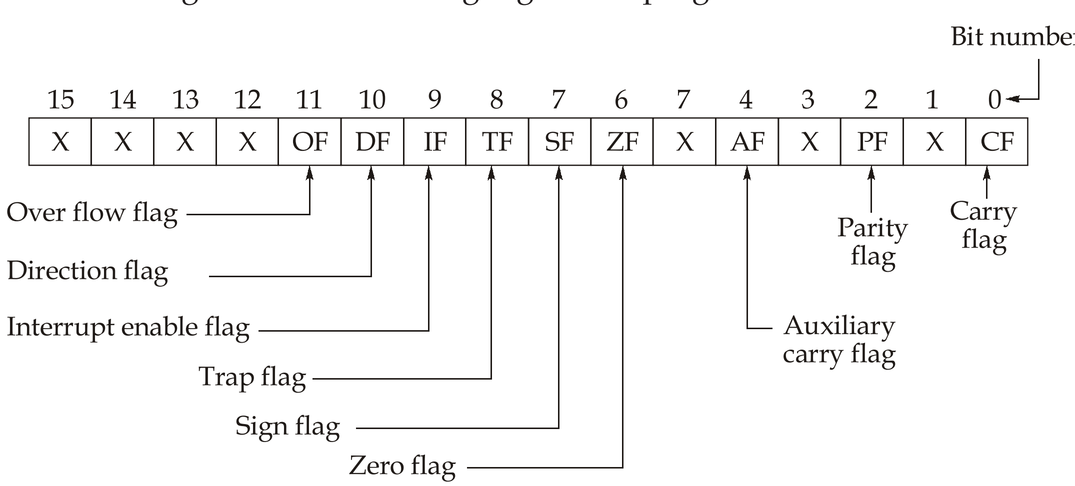
1. Code segment registers (CS) – used for addressing memory location in the code segment of the memory, where the executable program is stored.
2. Data segment registers (DS) – points to the data segment of the memory where the data is stored.
3. Stack segment registers (SS) – used for addressing stack segment of the memory.
4. Extra segment registers (ES) – points to a segment in the memory which is another data segment in the memory. e.g. it is used in string instructions.

Instruction pointer

The instruction pointer points to the address of next instruction to be fetched. It is similar to Program Counter of INTEL 8085. The content of IP and Code segment register are used to compute the memory address of the instruction to the fetched.

Status/Flag Register:

It is 16-bit register also called flag register or program status word.



There are nine status flags as shown in figure. Out of the 9 flags, 6 flags are condition flags and remaining 3 flags are control flags. The condition flags are similar to that of 8085 as carry, parity, auxiliary carry, zero, sign and over flow flags. The control flags are direction, trap and interrupt flags.

Carry Flag (CY): CY holds the carry after an 8-bit or 16-bit addition or the borrow after an 8-bit or 16-bit subtraction operation.

Parity Flag (PF): If the lower eight bits of the result have an odd parity (i.e., odd number of 1s), PF is set to 0, otherwise, it is set to 1.

Auxiliary Carry Flag (AF): AF holds the carry after addition or the borrow after subtraction, from the lower nibble to higher nibble i.e. from bit 3 to bit 4. This flag is used by the DAA or the DAS instruction to adjust the value in AL after a BCD addition or substraction respectively.

Zero flag (ZF): ZF indicates that the result of an arithmetic or logic operation is zero. If ZF = 1, the result is zero and if ZF = 0 the result is not zero.

Sign Flag (SF): SF holds the arithmetic sign of the result after an arithmetic or logic instruction is executed. If SF = 0, the result is positive.

Trap Flag (TF): TF is used to debug a program using the single-step execution. If it is set (i.e., TF = 1) the 8086 get interrupted (trap or single-step interrupt) after the execution of each instruction in the program. If TF is cleared (i.e., TF = 0) the trapping or debugging feature is disabled.

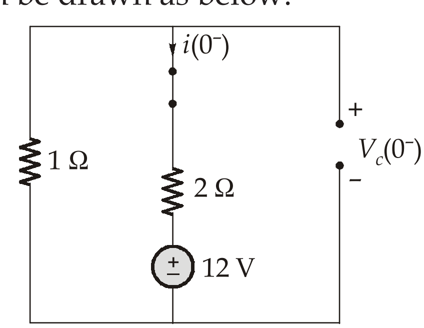
Direction Flag (DF): DF selects either the increment or decrement mode for the DI and/or SI register during the execution of string instructions. If DF = 0, the registers are automatically incremented and if DF = 1, the registers are automatically decremented. This flag can be set and cleared using the STD and CLD instructions, respectively.

Interrupt Flag (IF): If controls the operation of the INTR interrupt pin of the 8086. If IF = 0 the INTR pin is disabled and if IF = 1, the INTR pin is enabled. This flag can be set and cleared using the STI and CLI instructions respectively.

Overflow Flag (OF): It shows that the result is out of range when OF = 1.

Q.5 (e) Solution:

For $t < 0$: At steady state, the inductor acts as short circuit and capacitor acts as open circuit. Thus, the circuit can be drawn as below:

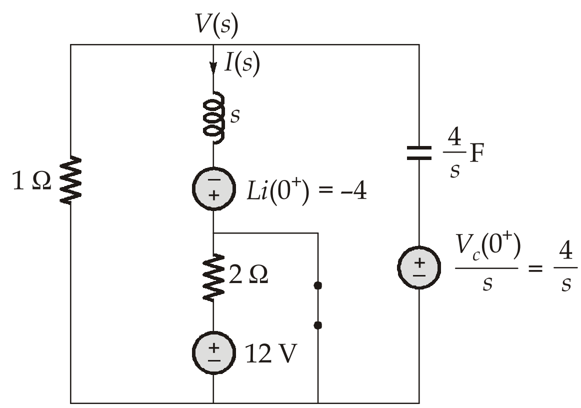


Since, the inductor current and capacitor voltage cannot change instantaneously, we have

$$i(0^+) = i(0^-) = -\frac{12}{3} = -4\text{ A}$$

$$V_c(0^-) = V_c(0^+) = 12 - 8 = 4\text{ V}$$

For $t > 0$: The circuit in s-domain can be drawn as below:



Applying KCL at node V,

$$\frac{V(s) - 0}{1} + \frac{V_s - 4}{s} + \frac{V_s - \frac{4}{s}}{\frac{4}{s}} = 0$$

$$V(s)\left[1 + \frac{1}{s} + \frac{s}{4}\right] = \frac{4}{s} + 1$$

$$V(s)\left[\frac{4s + 4 + s^2}{4s}\right] = \frac{4 + s}{s}$$

$$V(s) = \frac{4(s+4)}{s^2 + 4s + 4}$$

$$V(s) = \frac{4(s+4)}{(s+2)^2}$$

$$V(s) = \frac{A}{s+2} + \frac{B}{(s+2)^2}$$

$$B|_{s=-2} = \frac{4(-2+4)}{1} = 8$$

$$A|_{s=-2} = \frac{d}{dt}4(s+4) = 4[1] = 4$$

$$V(s) = \frac{4}{s+2} - \frac{8}{(s+2)^2}$$

$$V(t) = 4e^{-2t}u(t) + 8te^{-2t}\,u(t)$$

$$V(t) = 4e^{-2t}[1 + 2t]u(t)$$

$$I(s) = \frac{V(s)-4}{s}$$

$$I(s) = \frac{\frac{4(s+4)}{(s+2)^2} - 4}{s}$$

$$I(s) = \frac{-4(s+3)}{(s+2)^2}$$

$$I(s) = \frac{C}{s+2} + \frac{D}{(s+2)^2}$$

$$C = \frac{d}{dt}\left[-4(s+3)\right] = -4$$

$$d\big|_{s=-2} = \frac{-4(1)}{1} = -4$$

$$I(s) = \frac{-4}{s+2} - \frac{4}{(s+2)^2}$$

$$i(t) = [-4e^{-2t} - 4te^{-2t}]u(t)$$

$$i(t) = -4e^{-2t}[1+t]u(t)$$

Q.6 (a) Solution:

For the given system assuming state transition matrix;

$$\phi(t) = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix}$$

We know,

$$x(t) = \phi(t)\,x(0)$$

$$\begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix} = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$e^{-t} = \phi_{11}(t) - 2\phi_{12}(t) \quad ...(1)$$

$$-2e^{-t} = \phi_{21}(t) - 2\phi_{22}(t) \quad ...(2)$$

Also,
$$\begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix} = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$e^{-2t} = \phi_{11}(t) - \phi_{12}(t) \quad ...(3)$$

$$-e^{-2t} = \phi_{21}(t) - \phi_{22}(t) \quad ...(4)$$

Considering equation (1) and (3),

$$e^{-t} = \phi_{11}(t) - 2\phi_{12}(t)$$

$$e^{-2t} = \phi_{11}(t) - \phi_{12}(t)$$

$$\underline{- \qquad - \qquad +}$$

$$e^{-t} - e^{-2t} = -2\phi_{12}(t) + \phi_{12}(t)$$

$$\phi_{12}(t) = e^{-2t} - e^{-t}$$

and
$$\phi_{11}(t) = 2e^{-2t} - e^{-t}$$

Now, considering equation (2) and (4), we get

$$-e^{-2t} = \phi_{21}(t) - \phi_{22}(t)$$

$$-2e^{-t} = \phi_{21}(t) - 2\phi_{22}(t)$$

$$\underline{+ \qquad - \qquad +}$$

$$2e^{-t} - e^{-2t} = \phi_{22}(t)$$

$\therefore$
$$\phi_{22}(t) = 2e^{-t} - e^{-2t}$$

$$\phi_{21}(t) = 2e^{-t} - 2e^{-2t}$$

$$\phi(t) = \begin{bmatrix} 2e^{-2t} - e^{-t} & e^{-2t} - e^{-t} \\ 2e^{-t} - 2e^{-2t} & 2e^{-t} - e^{-2t} \end{bmatrix}$$

On taking Laplace transform,

$$\phi(s) = \begin{bmatrix} \dfrac{2}{s+2} - \dfrac{1}{(s+1)} & \dfrac{1}{(s+2)} - \dfrac{1}{(s+1)} \\ \dfrac{2}{(s+1)} - \dfrac{2}{(s+2)} & \dfrac{2}{(s+1)} - \dfrac{1}{(s+2)} \end{bmatrix}$$

$$\phi(s) = \begin{bmatrix} \dfrac{2(s+1)-(s+2)}{(s+1)(s+2)} & \dfrac{(s+1)-(s+2)}{(s+2)(s+1)} \\ \dfrac{2(s+2)-2(s+1)}{(s+1)(s+2)} & \dfrac{2(s+2)-(s+1)}{(s+1)(s+2)} \end{bmatrix}$$

$$\phi(s) = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$\phi(s) = \frac{1}{\Delta}\begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix} = [sI - A]^{-1}$$

Matrix $[sI - A]$ can be identified as shown below using above result.

$$[sI - A] = \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$[sI] - [A] = \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} = [A]$$

$$\therefore \quad [A] = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$$

Q.6 (b) Solution:

(i) $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 2x(n)$

Taking the DTFT of the above equation, we get,

$$Y(e^{j\omega}) - \frac{3}{4}Y(e^{j\omega})e^{-j\omega} + \frac{1}{8}Y(e^{j\omega})e^{-j2\omega} = 2X(e^{j\omega})$$

$$Y(e^{j\omega})\left(1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}\right) = 2X(e^{j\omega})$$

The frequency response of the system is,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}$$

$$= \frac{2}{\left(1 - \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

Using the partial fraction expansion, we get,

$$H(e^{j\omega}) = \frac{4}{\left(1 - \frac{1}{2}e^{-j\omega}\right)} - \frac{2}{\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

By taking the inverse DTFT of the above equation, the impulse response of the system can be given by,

$$h(n) = 4\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{4}\right)^n u(n)$$

(ii) The DTFT of the input $x(n) = \left(\frac{1}{4}\right)^n u(n)$ can be given by,

$$X(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

$$Y(e^{j\omega}) = H(e^{j\omega})\, X(e^{j\omega})$$

Using the partial fraction expansion, we get

$$Y(e^{j\omega}) = \frac{4}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)} - \frac{2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

$$Y(e^{j\omega}) = \frac{8}{\left(1-\frac{1}{2}e^{-j\omega}\right)} - \frac{4}{\left(1-\frac{1}{4}e^{-j\omega}\right)} - \frac{2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

Considering the following DTFT pairs,

$$a^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-ae^{-j\omega}}$$

$$(n+1)\, a^n u(n) \xleftrightarrow{DTFT} \frac{1}{(1-ae^{-j\omega})^2}$$

So, by taking the inverse DTFT of $Y(e^{j\omega})$, the response of the system for the given input will be,

$$y(n) = 8\left(\frac{1}{2}\right)^n u(n) - 4\left(\frac{1}{4}\right)^n u(n) - 2(n+1)\left(\frac{1}{4}\right)^n u(n)$$

$$= 8\left(\frac{1}{2}\right)^n u(n) - 2(n+3)\left(\frac{1}{4}\right)^n u(n)$$

Q.6 (c) Solution:

(i) At $t = 0^-$, the capacitor is uncharged. We have,

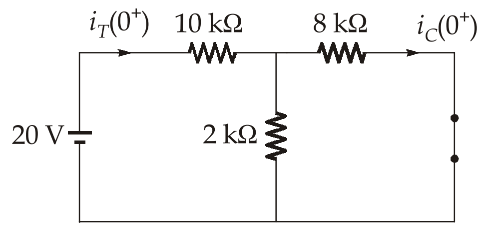
$$V_C(0^-) = 0$$

$$i_c(0^-) = 0$$

Since voltage across the capacitor cannot change instantaneously,

$$V_C(0^+) = V_C(0^-) = 0$$

At $t = 0^+$, the circuit can be drawn as below:



$$i_T(0^+) = \left[\frac{20}{10+(8\parallel 2)}\right] = \frac{20}{10+\frac{8\times 2}{10}} = \frac{20}{10+1.6}$$

$$= 1.724 \text{ mA}$$

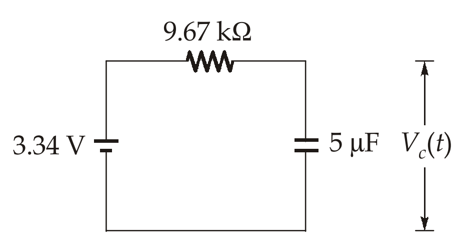
$$i_c(0^+) = 1.724\times\frac{2}{8+2} = 1.724\times\frac{2}{10} = 0.3448 \text{ mA}$$

For $t > 0$, the Thevenin equivalent circuit across the capacitor can be obtained as below:

$$V_{\text{th}} = 20\times\frac{2}{10+2} = 3.34 \text{ V}$$

$$R_{\text{th}} = (10 \parallel 2) + 8 = 9.67 \text{ k}\Omega$$

Writing KCL equation for $t > 0$.



$$5\times 10^{-6}\frac{dV_c}{dt}+\frac{V_c-3.34}{9.67\times 10^3} = 0$$

$$\frac{dV_c}{dt}+20.68\,V_c = 69.07$$

The solution of the above linear differential equation is given by

$$V_c(t) = \frac{Q}{P}+Ke^{-pt}, \text{ where } Q = 69.07 \text{ and } P = 20.68$$

$$V_c(t) = \frac{69.07}{20.682}+Ke^{-20.682t}$$

At $t = 0$; $V_c(0) = 0$

$$0 = 3.339 + K$$

$$K = -3.339$$

$$V_c(t) = 3.339(1 - e^{-20.682t})$$

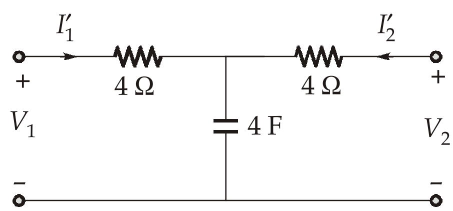
$$i_c(t) = C\frac{dV_c(t)}{dt}$$

$$= 5 \times 10^{-6} \times 3.339 \times 20.682\, e^{-20.682t}$$

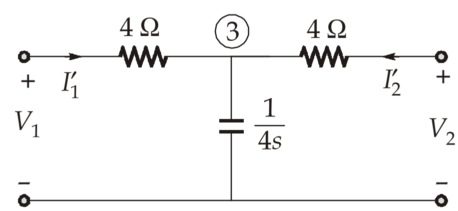
$$i_c(t) = 345.285 \times 10^{-6}\, e^{-20.682t}$$

$$i_c(t) = 345.285\, e^{-20.682t}\ \mu\text{A}$$

(ii) The above network can be considered as a parallel connection of two networks, N_1 and N_2.



(a)



(b)

For the network N_1,

Applying KCL at Node 3,

$$I_1' + I_2' = 4s(V_3) \quad \text{...(i)}$$

From figure,

$$I_1' = \frac{V_1 - V_3}{4} = \frac{1}{4}V_1 - \frac{1}{4}V_3 \quad \text{...(ii)}$$

$$I_2' = \frac{V_2 - V_3}{4} = \frac{1}{4}V_2 - \frac{1}{4}V_3 \quad \text{...(iii)}$$

Substituting the equation (ii) and equation (iii), in the equation (i)

$$\frac{V_1}{4} - \frac{V_3}{4} + \frac{V_2}{4} - \frac{V_3}{4} = (4s)V_3$$

$$(8s+1)V_3 = \frac{V_1}{2} + \frac{V_2}{2}$$

$$V_3 = \frac{1}{2(8s+1)}V_1 + \frac{1}{2(8s+1)}V_2 \quad \text{...(iv)}$$

Substituting the equation (iv) in the equation (ii),

$$I_1' = \frac{V_1}{4} - \frac{1}{4}\left[\frac{1}{2(8s+1)}V_1 + \frac{1}{2(8s+1)}V_2\right]$$

$$= \left[\frac{1}{4} - \frac{1}{8(8s+1)}\right]V_1 - \frac{1}{8(8s+1)}V_2$$

$$= \frac{[2(8s+1)-1]}{8(8s+1)}V_1 - \frac{1}{8(8s+1)}V_2$$

$$I_1' = \frac{16s+1}{8(8s+1)}V_1 - \frac{1}{8(8s+1)}V_2 \quad \text{...(v)}$$

Substituting the equation (iv) in equation (iii),

$$I_2' = \frac{V_2}{4} - \frac{1}{4}\left[\frac{1}{2(8s+1)}V_1 + \frac{1}{2(8s+1)}V_2\right]$$

$$= -\frac{1 \times V_1}{8(8s+1)} + \frac{V_2}{4} - \frac{1}{8(8s+1)}V_2$$

$$= -\frac{V_1}{8(8s+1)} + \frac{2(8s+1)-1}{8(8s+1)}V_2$$

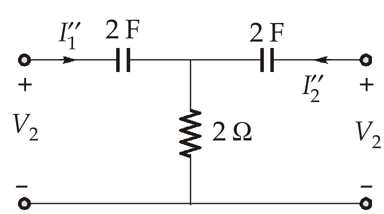
$$= -\frac{V_1}{8(8s+1)} + \frac{(16s+2-1)}{8(8s+1)}V_2$$

$$I_2' = -\frac{V_1}{8(8s+1)} + \frac{16s+1}{8(8s+1)}V_2 \quad \text{...(vi)}$$

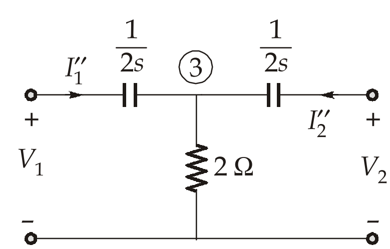
Comparing equation (v) and (vi) with Y-parameter equations, we get,

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} \frac{16s+1}{8(8s+1)} & -\frac{1}{8(8s+1)} \\ \frac{-1}{8(8s+1)} & \frac{16s+1}{8(8s+1)} \end{bmatrix}$$

For the network N_2.



(a)



(b)

Applying KCL at Node 3,

$$I_1'' + I_2'' = \frac{V_3}{2} \quad \text{...(i)}$$

From figure,

$$I_1'' = \frac{V_1 - V_3}{\frac{1}{2s}} = 2sV_1 - 2sV_3 \quad \text{...(ii)}$$

$$I_2'' = \frac{V_2 - V_3}{\frac{1}{2s}} = 2sV_2 - 2sV_3 \quad ...(iii)$$

Substituting the equations (ii) and (iii) in the equation (i),

$$2sV_1 - 2sV_3 + 2sV_2 - 2sV_3 = \frac{V_3}{2}$$

$$\frac{(8s+1)V_3}{2} = 2sV_1 + 2sV_2$$

$$V_3 = \frac{4s}{8s+1}V_1 + \frac{4s}{8s+1}V_2 \quad ...(iv)$$

Substituting the equation (iv) in the equation (ii),

$$I_1'' = 2sV_1 - 2s\left[\frac{4s}{8s+1}V_1 + \frac{4s}{8s+1}V_2\right]$$

$$= \left(2s - \frac{8s^2}{8s+1}\right)V_1 - \frac{8s^2}{8s+1}V_2$$

$$= \frac{16s^2 + 2s - 8s^2}{8s+1}V_1 - \frac{8s^2}{8s+1}V_2$$

$$I_1'' = \frac{2s(4s+1)}{8s+1}V_1 - \frac{8s^2}{8s+1}V_2 \quad ...(v)$$

Substituting the equation (iv) in the equation (iii),

$$I_2'' = 2sV_2 - 2s\left[\frac{4s}{8s+1}V_1 + \frac{4s}{8s+1}V_2\right]$$

$$I_2'' = -\frac{8s^2}{8s+1}V_1 + \left[2s - \frac{8s^2}{8s+1}\right]V_2$$

$$= -\frac{8s^2}{8s+1}V_1 + \frac{(16s^2 + 2s - 8s^2)}{8s+1}V_2$$

$$I_2'' = \frac{-8s^2}{8s+1}V_1 + \frac{2s(4s+1)}{8s+1}V_2 \quad ...(vi)$$

Comparing equations (v) and (vi) with Y-parameter equations, we get

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} \frac{2s(4s+1)}{8s+1} & \frac{-8s^2}{8s+1} \\ \frac{-8s^2}{8s+1} & \frac{2s(4s+1)}{8s+1} \end{bmatrix}$$

Therefore, the overall Y-parameters of the network are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y'_{11} + Y''_{11} & Y'_{12} + Y''_{12} \\ Y'_{21} + Y''_{21} & Y'_{22} + Y''_{22} \end{bmatrix}$$

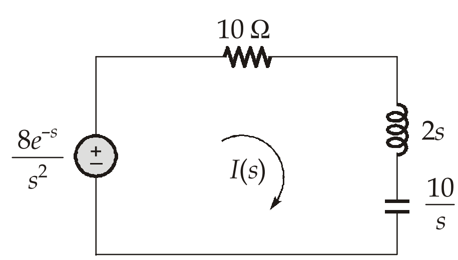
$$= \begin{bmatrix} \dfrac{16s+1}{8(8s+1)} + \dfrac{2s(4s+1)}{8s+1} & -\left[\dfrac{1}{8(8s+1)} + \dfrac{8s^2}{8s+1}\right] \\ \dfrac{-1}{8(8s+1)} - \dfrac{8s^2}{8s+1} & \dfrac{16s+1}{8(8s+1)} + \dfrac{2s(4s+1)}{8s+1} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{16s+1+64s^2+16s}{8(8s+1)} & -\dfrac{(1+64s^2)}{8(8s+1)} \\ \dfrac{-(64s^2+1)}{8(8s+1)} & \dfrac{64s^2+32s+1}{8(8s+1)} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{64s^2+32s+1}{8(8s+1)} & \dfrac{-(64s^2+1)}{8(8s+1)} \\ \dfrac{-(64s^2+1)}{8(8s+1)} & \dfrac{64s^2+32s+1}{8(8s+1)} \end{bmatrix}$$

Q.7 (a) Solution:

(i) The transformed network in s-domain is shown in figure



Applying KVL to the Mesh for $t > 0$,

$$\frac{8e^{-s}}{s^2} = 10I(s) + \left(2s + \frac{10}{s}\right) I(s)$$

$$I(s) = \frac{8e^{-s} \times s}{s^2(2s^2+10s+10)}$$

$$I(s) = \frac{4e^{-s}}{s(s^2+5s+5)} = \left[\frac{A}{s} + \frac{B}{(s+3.618)} + \frac{C}{(s+1.38)}\right] 4e^{-s}$$

$$A = \left.\frac{4}{(s+1.38)(s+3.618)}\right|_{s=0} = \frac{4}{1.38 \times 3.618} = 0.8$$

$$C = \frac{4}{(-1.38)(-1.38+3.618)} = -1.295$$

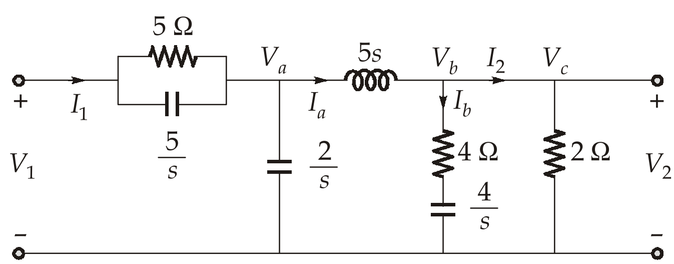
$$B = \frac{4}{(-3.618)(-3.618+1.38)} = 0.494$$

$$I(s) = e^{-s}\left[\frac{0.8}{s} - \frac{1.295}{s+1.38} + \frac{0.494}{s+3.618}\right]$$

Taking inverse Laplace transform,

$$i(t) = 0.8u(t-1) - 1.295e^{-1.38(t-1)}u(t-1) + 0.494\,e^{-3.618(t-1)}\,u(t-1);\ t > 0$$

(ii) Transformed network in s-domain is shown in figure:



$$V_c = V_b = V_2$$

$$I_2 = \frac{V_c}{2} = \frac{V_2}{2}$$

$$I_a = I_b + I_2 = \frac{V_2}{4+\frac{4}{s}} + \frac{V_2}{2}$$

$$= \left(\frac{s}{4s+4} + \frac{1}{2}\right)V_2 = \left(\frac{s+2s+2}{4(s+1)}\right)V_2 = \left(\frac{3s+2}{4(s+1)}\right)V_2$$

$$I_a = \left(\frac{3s+2}{4(s+1)}\right)V_2 \qquad \text{...(i)}$$

$$I_a = \frac{V_a - V_2}{5s} \qquad \text{...(ii)}$$

Equate (i) and (ii) equation

$$\frac{(3s+2)}{4(s+1)}V_2 = \frac{V_a - V_2}{5s}$$

$$V_a = \frac{15s^2 + 14s + 4}{4(s+1)}V_2 \qquad \text{...(iii)}$$

$$I_1 = \frac{V_1 - V_a}{\dfrac{5 \times \dfrac{5}{s}}{5 + \dfrac{5}{s}}} = \frac{V_1 - V_a}{\dfrac{25}{5(s+1)}} = \frac{(V_1 - V_a)(s+1)}{5}$$

$$V_a = \frac{(s+1)V_1 - 5I_1}{s+1} \quad \text{...(iv)}$$

We have,

$$I_1 = \frac{V_a}{(2/s)} + \frac{V_a - V_2}{5s}$$

$\Rightarrow$

$$I_1 = \left[\frac{s}{2} + \frac{1}{5s}\right]V_a - \frac{V_2}{5s}$$

Using equation (iii),

$$I_1 = \left(\frac{s}{2} + \frac{1}{5s}\right)V_a - \left[\frac{4(s+1)}{5s(15s^2 + 14s + 4)}\right]V_a$$

$$I_1 = \frac{5s^2 + 2}{10s} - \frac{4(s+1)}{5s(15s^2 + 14s + 4)}$$

$$I_1 = \left[\frac{75s^4 + 70s^3 + 50s^2 + 20s}{10s(15s^2 + 14s + 4)}\right]V_a$$

$$I_1 = \left[\frac{15s^3 + 14s^2 + 10s + 4}{2(15s^2 + 14s + 4)}\right]V_a$$

Using equation (iv),

$$I_1 = \left(\frac{15s^3 + 14s^2 + 10s + 4}{2(15s^2 + 14s + 4)}\right)\left[V_1 - \frac{5I_1}{s+1}\right]$$

$\Rightarrow$

$$V_1 = \frac{5I_1}{s+1} + \frac{2(15s^2 + 14s + 4)}{15s^3 + 14s^2 + 10s + 4} \cdot I_1$$

$\Rightarrow$

$$\frac{V_1}{I_1} = \frac{105s^3 + 128s^2 + 86s + 28}{(s+1)(15s^3 + 14s^2 + 10s + 4)} \quad \text{...(v)}$$

From equation (iv) and (v),

$$V_a = V_1 - \frac{5}{s+1} I_1$$

$$\Rightarrow \quad V_a = \left[\frac{1 - 5(15s^3 + 14s^2 + 10s + 4)}{105s^3 + 128s^2 + 86s + 28}\right] \cdot V_1$$

$$\Rightarrow \quad V_a = \left(\frac{30s^3 + 58s^2 + 36s + 8}{105s^3 + 128s^2 + 86s + 28}\right) V_1$$

Substituting in equation (iii),

$$\left(\frac{30s^3 + 58s^2 + 36s + 8}{105s^3 + 128s^2 + 86s + 28}\right) V_1 = \left[\frac{15s^2 + 14s + 4}{4(s+1)}\right] V_2$$

$$\Rightarrow \quad \frac{V_2}{V_1} = \frac{4(s+1)(30s^3 + 58s^2 + 36s + 8)}{(105s^3 + 128s^2 + 86s + 28)(15s^2 + 14s + 4)}$$

Using $V_2 = 2I_2$ and substituting V_1 from equation (v), we get

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{2(30s^3 + 58s^2 + 36s + 8)}{(15s^2 + 14s + 4)(15s^3 + 14s^2 + 10s + 4)}$$

Since, $I_2 = V_2/2$, we have

$$\frac{V_2}{I_1} = \frac{4(30s^3 + 58s^2 + 36s + 8)}{(15s^2 + 14s + 4)(15s^3 + 14s^2 + 10s + 4)}$$

Q.7 (b) Solution:

(i) Let $Y(\omega) = \dfrac{2\sin(\omega/4)}{\omega}$ and its inverse Fourier transform be $y(t)$. Then,

$$y(t) = \text{rect}(2t) = \begin{cases} 1, & |t| < 1/4 \\ 0, & \text{otherwise} \end{cases}$$

Let $Y_1(\omega) = 2\sin(4\omega)Y(\omega)$ and its inverse Fourier transform be $y_1(t)$. Then,

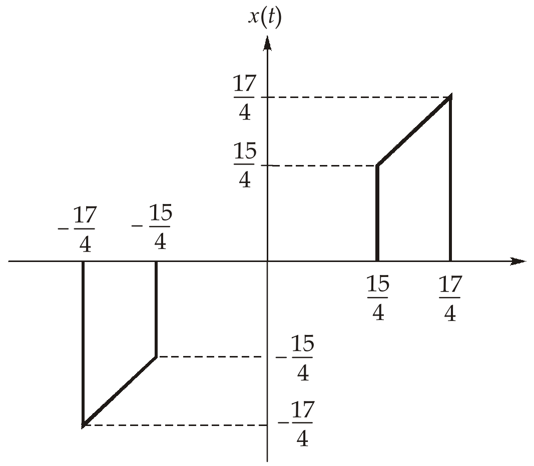
$$y_1(t) = -j[y(t+4) - y(t-4)]$$

Now, $$X(\omega) = \frac{d}{d\omega}[Y_1(\omega)]$$

Using the multiplication by 't' property of Fourier Transform, we get

$$x(t) = -jty_1(t) = -t\,\text{rect}\big(2(t+4)\big) + t\,\text{rect}\big(2(t-4)\big)$$

The signal $x(t)$ can be plotted as shown below.



(ii) Given that, $x(t) \xleftrightarrow{\text{CTFT}} X(\omega)$

To determine the CTFT of $x_1(t)$:

Using the time-shifting property, we get,

$$x(t+1) \xleftrightarrow{\text{CTFT}} X(\omega)e^{j\omega}$$

$$x(t-1) \xleftrightarrow{\text{CTFT}} X(\omega)e^{-j\omega}$$

Now, using the time-reversal property, we get,

$$x(-t+1) \xleftrightarrow{\text{CTFT}} X(-\omega)e^{-j\omega}$$

$$x(-t-1) \xleftrightarrow{\text{CTFT}} X(-\omega)e^{j\omega}$$

Consider $x_1(t) = x(-t+1) + x(-t-1) \xleftrightarrow{\text{CTFT}} X_1(\omega)$

So, $X_1(\omega) = X(-\omega)[e^{j\omega} + e^{-j\omega}] = 2X(-\omega)\cos(\omega)$

To determine the CTFT of $x_2(t)$:

Using the time-shifting property, we get,

$$x(t-6) \xleftrightarrow{\text{CTFT}} X(\omega)e^{-j6\omega}$$

Now, using the time-scaling property, we get,

$$x(3t-6) \xleftrightarrow{\text{CTFT}} \frac{1}{3}X\left(\frac{\omega}{3}\right)e^{-j2\omega}$$

Consider $$x_2(t) \xleftrightarrow{\text{CTFT}} X_2(\omega)$$

So, $$X_2(\omega) = \frac{1}{3}X\left(\frac{\omega}{3}\right)e^{-j2\omega}$$

To determine the CTFT of $x_3(t)$:

Using the differentiation in time-domain property, we get,

$$\frac{dx(t)}{dt} \xleftrightarrow{\text{CTFT}} j\omega X(\omega)$$

$$\frac{d^2x(t)}{dt^2} \xleftrightarrow{\text{CTFT}} (j\omega)^2 X(\omega) = -\omega^2 X(\omega)$$

Now, using the time-shifting property, we get,

$$\frac{d^2x(t-1)}{dt^2} \xleftrightarrow{\text{CTFT}} -\omega^2 X(\omega)e^{-j\omega}$$

Consider $$x_3(t) \xleftrightarrow{\text{CTFT}} X_3(\omega)$$

So, $$X_3(\omega) = -\omega^2 X(\omega)e^{-j\omega}$$

Q.7 (c) Solution:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{(s+3)}{(s+1)(s+2)^2}$$

$$= \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

where, $$A|_{s=-1} = \frac{2}{1} = 2$$

$$C|_{s=-2} = \frac{1}{(-1)} = -1$$

$$B = \frac{d}{ds}\left[\frac{(s+3)}{(s+1)}\right]\Bigg|_{s=-2}$$

$$= \frac{(s+1)\times 1 - (s+3)\times 1}{(s+1)^2}\Bigg|_{S=-2} = \frac{-2}{1} = -2$$

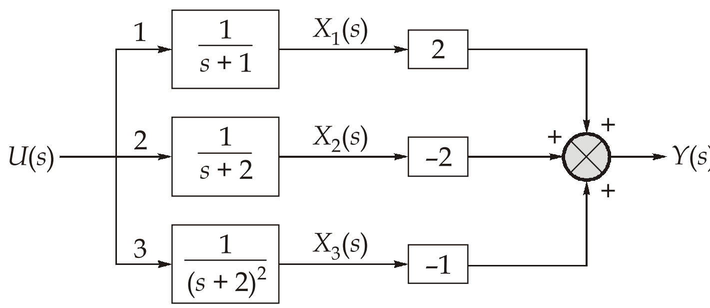
$$G(s) = \frac{Y(s)}{U(s)} = \frac{2}{s+1} - \frac{2}{(s+2)} - \frac{1}{(s+2)^2}$$

Let $X_1(s) = \dfrac{U(s)}{(s+1)}$; $X_2(s) = \dfrac{U(s)}{(s+2)}$ and

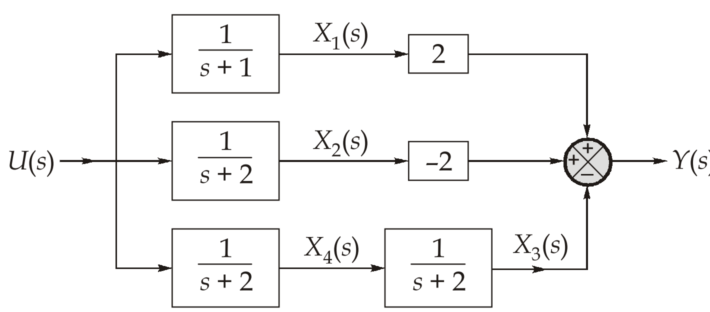
$$X_3(s) = \frac{U(s)}{(s+2)^2}$$

So, $Y(s) = 2X_1(s) - 2X_2(s) - X_3(s)$

The above can be depicted by four number of integrators as shown below:



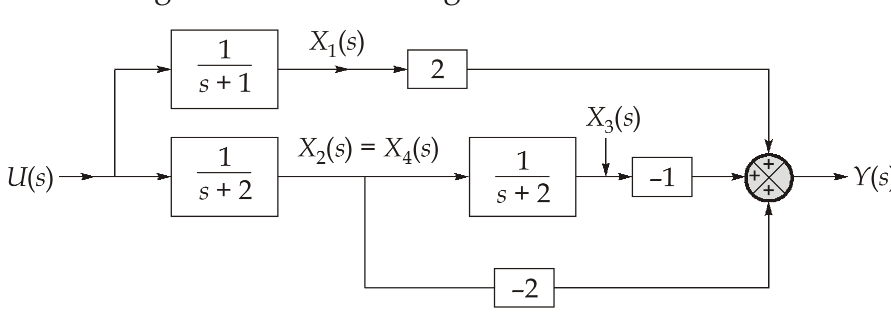
The blocks in branch 1 and 2 can be depicted by one integrator each. However, the block in branch 3 has a squared term and hence, requires two integrators as shown below:

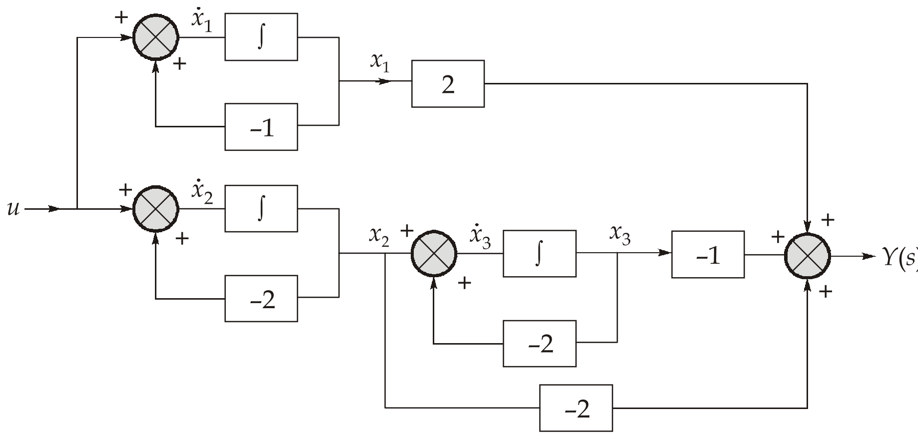


where, $X_3(s) = \dfrac{X_4(s)}{s+2}$ and $X_4(s) = \dfrac{U(s)}{(s+2)}$

Also, $X_2(s) = \dfrac{U(s)}{s+2}$

Thus, it is seen that $X_2(s)$ and $X_4(s)$ are similar and hence we can get rid of one integrator and draw the state diagram with three integrators as shown below:





State Model/State Equation:

$$X_1(s) = \frac{U(s)}{s+1}$$

i.e., $X_1(s)(s+1) = U(s)$

We get, $\dot{x}_1 + x_1 = u$

or $\dot{x}_1 = -x_1 + u$

similarly, $\dot{x}_2 = -2x_2 + u$

and, $\dot{x}_3 = -2x_3 + x_2$

Also, $y = 2x_1 - 2x_2 - x_3$

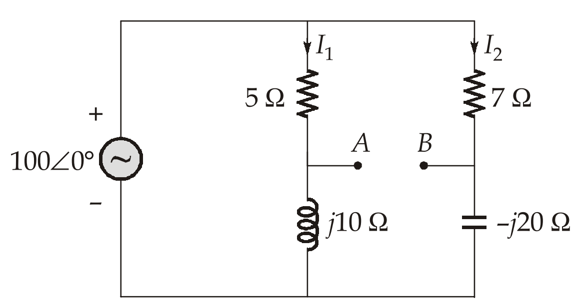
Hence, the state model in Jordon Canonical form is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q.8 (a) Solution:

To calculate the Thevenin equivalent circuit across the load impedance Z_L, the circuit can be drawn as below:



Calculation of V_{th},

$$I_1 = \frac{100\angle 0^\circ}{(5+j10)} = (4-8j)\text{A}$$

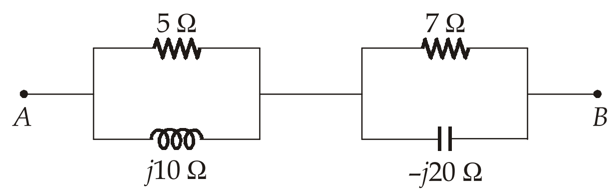
$$I_2 = \frac{100\angle 0^\circ}{7-20j} = (1.56+4.45j)\text{A}$$

$$V_{th} = V_A - V_B$$

$$= (4-8j)(j10) - (1.56+4.45j)(-j20) = -9+71.2j$$

$$= 71.76\angle 97.32^\circ\text{V}$$

Calculation of Z_{th}: To calculate Z_{th}, the voltage source is considered as short circuit.



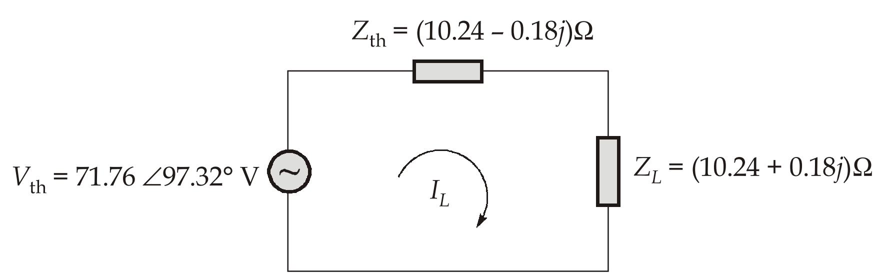
$$Z_{th} = (5 \parallel j10) + (7 \parallel -j20)$$

$$Z_{th} = \frac{5(j10)}{5+j10} + \frac{7(-j20)}{7-j20}$$

$$Z_{th} = (10.24 - 0.18j)\Omega$$

For maximum power transfer, the load impedance should be complex conjugate of the Thevenin impedance,

$$Z_L = Z_{th}^* = (10.24 + j0.18)\Omega$$



$$I_L = \frac{V_{th}}{Z_{th} + Z_L}$$

$$I_L = \frac{71.76\angle 97.32^\circ}{10.24 - 0.18j + 10.24 + 0.18j}$$

$$I_L = 3.50\angle 97.32^\circ\text{ A}$$

Maximum power delivered to the load,

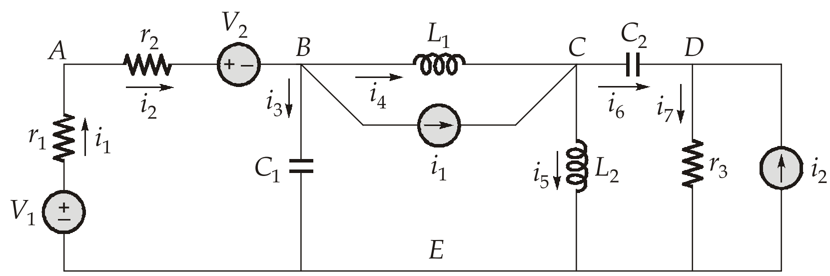
$$P_{max} = I_L^2 R_L$$

$$= (3.50)^2 \times 10.24$$

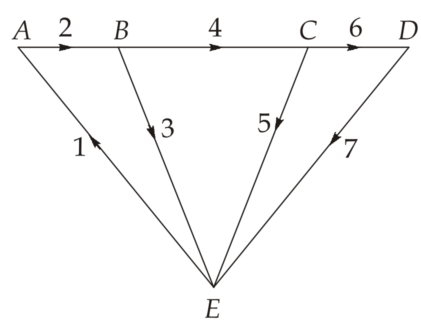
$$= 125.44 \text{ W}$$

Q.8 (b) Solution:

(i) Assume different branch currents in the given network and the nodes as shown below:



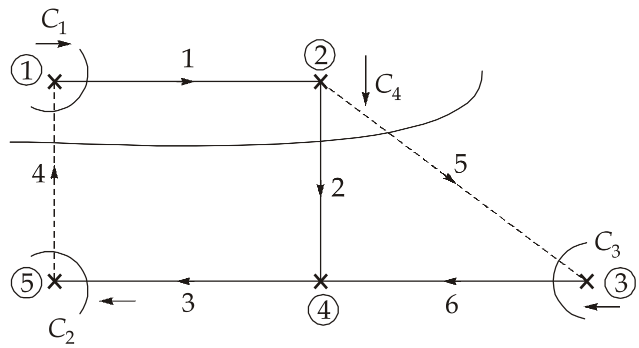
While drawing oriented graph, voltage sources are short circuited while current sources are open circuit. The oriented graph with node E as reference node, and branch currents indicating branch numbers is as shown below.



Incidence Matrix:

Nodes \ Branches	1	2	3	4	5	6	7
A	–1	1	0	0	0	0	0
B	0	–1	1	1	0	0	0
C	0	0	0	–1	1	1	0
D	0	0	0	0	0	–1	1
E	1	0	–1	0	–1	0	–1

(ii) Redraw the graph.



Step 1: Number of cut sets = Number of twigs = 4

Step 2: Let C_1, C_2, C_3, C_4 be the 4 cutsets assigned to twigs 1, 3, 2 and 6 respectively.

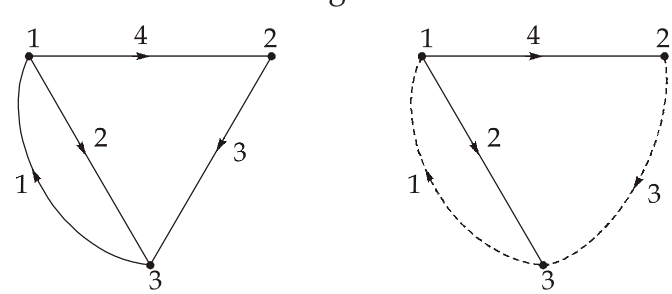
Cut-set Matrix:

Cut sets	1	2	3	4	5	6
C_1	1	0	0	–1	0	0
C_2	0	0	+1	–1	0	0
C_3	0	0	0	0	–1	1
C_4	0	1	0	–1	1	0

(Branches: 1–6)

Direction of cutset is the same as direction of twig.

(iii) The oriented graph and its selected tree are shown in figure. Since voltage v is to be determined, branch 2 is chosen as twig.



Twigs : {2, 4}

f-cutset 2 : {2, 1, 3}

f-cutset 3 : {4, 3}

Cutset matrix,

$$Q = \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \atop \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

The KCL equation in matrix form is given by

$$QY_bQ^TV_t = QI_s - QY_b V_s$$

$$Y_b = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

$$Q^T = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad I_S = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -4 \end{bmatrix}$$

$$V_S = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$QY_b = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

$$= \begin{bmatrix} -0.25 & 0.25 & 0.25 & 0 \\ 0 & 0 & -0.25 & 0.25 \end{bmatrix}$$

$$QY_bQ^T = \begin{bmatrix} -0.25 & 0.25 & 0.25 & 0 \\ 0 & 0 & -0.25 & 0.25 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.5 \end{bmatrix}$$

$$QI_S = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$QY_bV_s = \begin{bmatrix} -0.25 & 0.25 & 0.25 & 0 \\ 0 & 0 & -0.25 & 0.25 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$QI_s - QY_bV_s = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

Hence, the KCL equation can be written as

$$QY_bQ_TV_t = QI_s - QY_bV_s$$

$$\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.5 \end{bmatrix}\begin{bmatrix} V_{t2} \\ V_{t4} \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

From the figure, $V_{t2} = v$

Solving this matrix equation, we get,

$$V_{t2} = -1.6 \text{ V}$$

$$V_{t4} = -8.8 \text{ V}$$

Hence, $v = V_{12} = -1.6$ V

Q.8 (c) Solution:

(i) PSW: PSW stands for Program Status word. It is a 16-bit word, a combination of contents of the 8-bit flag register and the contents of an 8-bit accumulator register.

Accumulator	Flag register

8085 program:

PUSH PSW	;	Store PSW at stack top.
POP B	;	Content of accumulator in *B* and content of flag register in C.
MOV A, C	;	Content of flag register is moved to accumulator (A) from C.
STA 3000 H	;	Content of flag register is stored at memory location
HLT		3000 H from A.

(ii)

	MVI C, 96 H	;	Load counter with 150 = 96 H
	MVI A, 00 H	;	Load accumulator with 00 H
	LXI H, 2400 H	;	Load HL pair with 2400 H
NEXT:	MOV M, A	;	Move 00H from accumulator to the address location whose address is given by HL pair.
	INX H	;	Increase the contents of HL pair by "1"
	DCR C	;	Decrement counter after clearing data in every address location starting from 2400 H.

JNZ NEXT ; Jump to the instruction stored at "NEXT", till the counter value is non zero.

HLT ; Halt the execution

(iii) 1. SBI: Subtract Immediate with Borrow.

SBI 8-bit data e.g. SBI 45H

The 8-bit data and the borrow are subtracted from the contents of the accumulator, and the results are placed in the accumulator. All flags are affected to reflect the result of the operation.

2. SHLD: Store H and L Register Direct

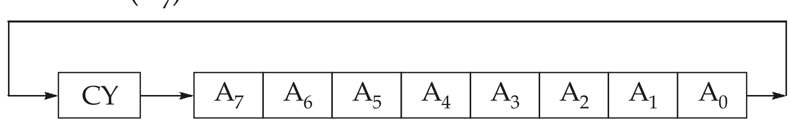
SHLD 16-bit (address) eg. SHLD 3000H

The contents of register L are stored in the memory location specified by the 16-bit address in the operand, and the contents of H register are stored in the next memory location by incrementing the operand. The contents of register pair HL are not altered. This is a 3-byte instruction. No flags are affected.

3. RAR: Rotate Accumulator Right through carry. Symbolically, it can be represented as

$$[A_n] \leftarrow [A_{n+1}]$$
$$[\text{CY}] \leftarrow [A_0]$$
$$[A_7] \leftarrow \text{CY}$$

The content of the accumulator is rotated right, one bit through carry. The zero bit of the accumulator (A_0) is moved to carry and the carry bit to the seventh bit of accumulator (A_7).



4. SPHL: Copy H and L Registers to the Stack Pointer.

The instruction loads the contents of H and L registers into the stack pointer register; the content of H register provide the high order address, and the content of the L register provide the low-order address. The contents of the H and L registers are not altered.

5. DAD: Add Register pair to H and L Registers.

DAD Register Pair e.g. DAD B

The 16-bit contents of the specified register pair are added to the contents of the HL register and the sum is stored in the HL register. The contents of the source register pair are not altered. If the result is larger than 16 bits, the CY flag is set.

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