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Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 2

Section A : Signals and Systems + Microprocessors and Microcontroller

Q.1 (a) Solution:

The normalized magnitude of the low-pass butterworth filter transfer function is,

$$\left|\frac{H(j\omega)}{H_0}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}} \quad ...(i)$$

An attenuation of 40 dB corresponds to

$$20\log\left|\frac{H(j\omega)}{H_0}\right| = -40$$

$$\frac{H(j\omega)}{H_0} = 10^{-2}$$

$$\therefore \quad \frac{H(j\omega)}{H_0} = 0.01$$

Substituting in equation (i), we get,

$$(0.01)^2 = \frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}$$

Given, $\qquad \dfrac{\omega}{\omega_0} = 2$

$$(0.01)^2 = \frac{1}{1+(2)^{2n}}$$

$$\Rightarrow \quad 1 + 2^{2n} = 10^4$$

$$2^{2n} = 10^4 - 1$$

Taking logarithm on both sides,

$$\log(2^{2n}) = \log(10^4 - 1)$$

$$2n = \frac{\log(10^4 - 1)}{\log 2}$$

$$\therefore \quad n = 6.64$$

Since the order of the filter must be an integer hence $n = 7$.

Q.1 (b) Solution:

The alternate pattern of 0/1 bits can be provided by loading the register with AA H(10101010) and rotating the pattern once through each loop. Bit D_0 of the output port is used to proved logic 0 and 1. Therefore, all other bits can be masked by ANDing the accumulator with 01H.

Label	Mnemonics	T-states		Comments
	MVI C, AA H		;	Move AA H to register C. Register C is used to store the number to be displayed at output port.
ROTATE:	MOV A, C	4	;	Moves the number from C to A;
	RLC	4	;	Rotates the contents of A to left without carry flag.
	MOV C, A	4	;	Moves the shifted contents of A to C.
	ANI 01 H	7	;	Masks D_1 - D_7 bits
	OUT PORT 0	10	;	Displays contents of A at output device connected at PORT 0.
	MVI B, 36 H	7	:	Load B with 36 H, Register B is used as 8-bit counter for loop delay. It stores the count number.
DELAY:	DCR B	4	;	Decrements the content of B counter by one.
	JNZ DELAY	10/7	;	Jumps to DELAY until B = 00H
	JMP ROTATE	10	;	Jumps to rotate to switch the output.

Time Delay Calculations:

Time delay provided by loop DELAY = 4 + 10 = 14 T-states; when condition is TRUE.

= 4 + 7 = 11 T-states ; when condition is false.

Time delay outside DELAY from: ROTATE : MOV A, C to JMP ROTATE = 46 T-states

$$\text{Time Delay} = (\text{Count-1}) \times (14T) + 11T + 46T$$

$$= (\text{Count-1}) \times (14T) + 57T$$

Given, T = 350 ns and time period of WAVE = 560 μs

Time after which the output switches = 280 μs

$$\Rightarrow \quad 280 \times 10^{-6} = (\text{Count-1}) \times 14 \times 350 \times 10^{-9} + 57 \times 350 \times 10^{-9}$$

$$\text{Count} = \frac{280 \times 10^{-6} - 57 \times 350 \times 10^{-9}}{14 \times 350 \times 10^{-9}} + 1$$

$$\text{Count} = 54.07 \cong 54 = (36)_{16}$$

Q.1 (c) Solution:

(i) There are three registers in 8259 which are:

1. Interrupt request register
2. Interrupt service register
3. Interrupt mask register

1. Interrupt request register (IRR):

It stores the interrupt requests. It keeps information about the interrupt inputs which have requested for interrupt service. It is an 8-bit register with one bit for each interrupt request. When an interrupt request is received, the corresponding bit in IRR is set.

2. Interrupt service register (ISR)

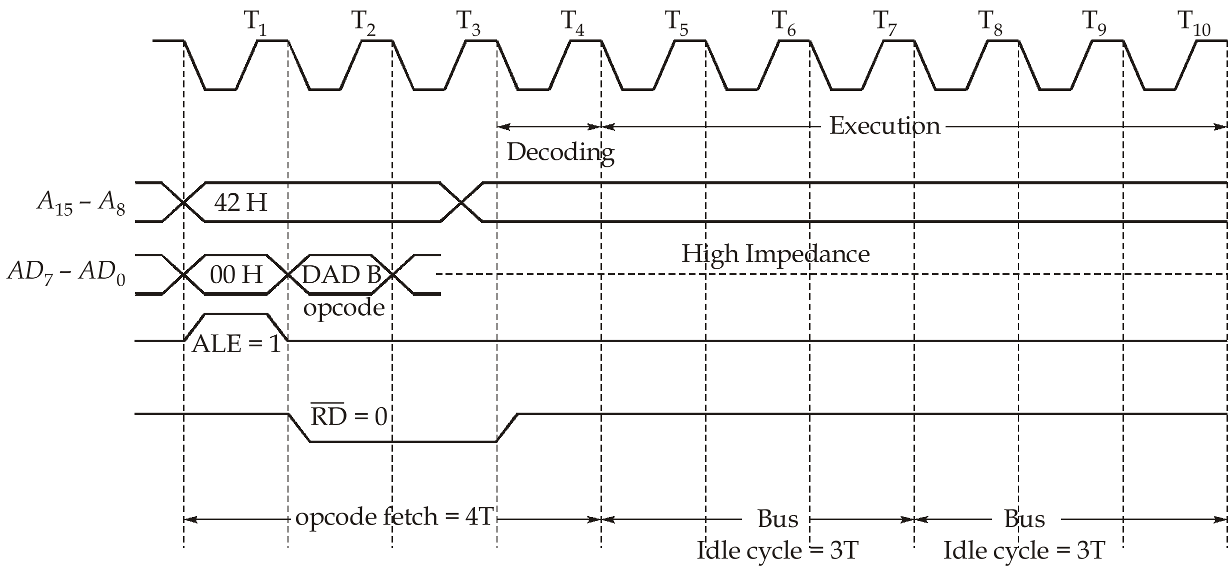
It is an 8-bit register that keeps track of the interrupt requests that are currently being serviced. The priority resolver determines the priority of the bits set in the IRR. If the priority resolver find that the new interrupt has a higher priority than the highest priority interrupt currently being serviced and the new interrupt is not in service, then it will set appropriate bit in the ISR and send the INTA signal to the microprocessor for new interrupt request. The corresponding IRR bit is reset.

3. Interrupt mask register (IMR):

It is an 8-bit register contains a specific bit for each interrupt line used to mask (disable) or enable (unmask) individual interrupt input. An interrupt input can be masked by setting the corresponding bit to 1 in IMR. An interrupt which is masked by software (by programming IMR) is not recognized and serviced even if the corresponding bit is set in the IRR.

(ii) DAD B $\rightarrow$ 10T-states

Let 4200 H is starting address.



Q.1 (d) Solution:

Given,
$$x_1(n) = \begin{cases} x(n/2)\,; & n\text{-even} \\ 0 \quad ; & n\text{-odd} \end{cases}$$

$$x_2(n) = x(2n)$$

From the definition of DTFT,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2}\right)e^{-j\omega n} \quad \text{where, } n \text{ is even}$$

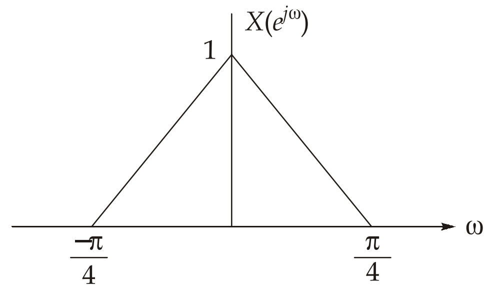
Let $\frac{n}{2} = p \quad \Rightarrow \quad n = 2p$

$$= \sum_{p=-\infty}^{\infty} x(p)e^{-j\omega(2p)}$$

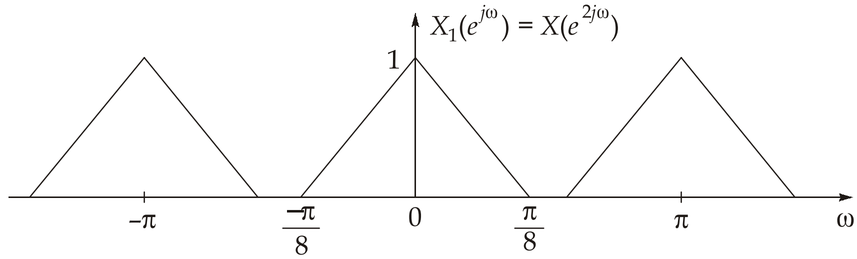
$$= \sum_{p=-\infty}^{\infty} x(p)e^{-j(2\omega)p}$$

$$X_1(e^{j\omega}) = X(e^{j2\omega})$$

Given, $X(e^{j\omega})$



$X_1(e^{j\omega})$ can be obtained as below:



$$X_2(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_2(n)e^{-j\omega n}$$

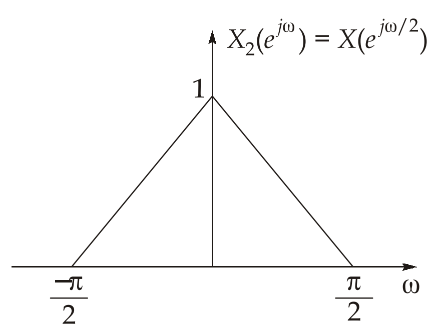
$$= \sum_{n=-\infty}^{\infty} x(2n)e^{-j\omega n}$$

Let $2n = p$, $n = \dfrac{p}{2}$

$$= \sum_{p=-\infty}^{\infty} x(p)e^{-j\left(\frac{\omega}{2}\right)p}$$

$$X_2(e^{j\omega}) = X\left(e^{j\left(\frac{\omega}{2}\right)}\right)$$

$X_2(e^{j\omega})$ can be obtained as below:



Q.1 (e) Solution:

The basic principle used here is that the MSB for a positive number is 0 and that for a negative number is 1.

	MOV AX, 3000 H		
	MOV DS, AX	;	Initialize DS with 3000 H
	MOV CX, 64 H	;	Move the number of data items to CX
	MOV BX, 4000 H	;	Move the starting offset address of the array to BX.
	MOV DH, 00H	;	Initialize DH with 00H to store the number of positive data items.
	MOV DL, 00H	;	Initialize DL with 00H to store the number of negative data items
L2:	MOV AL, [BX]	;	Move a byte data from the array.
	RCL AL, 01	;	Rotate AL left by one. Now the MSB in AL goes to the carry flag and also the carry to LSB of AL.
	JC NEG	;	If the carry is 1, the data is negative, so jump to NEG
	INC DH	;	If the carry is 0, the data is positive, so increment DH.
	JMP L1	;	Jump to L1.
NEG:	INC DL	;	Increment DL
L1:	INC BX	;	Increment BX to point to the next data
	LOOP L2	;	Repeat the process from L2 to check all the data items in array
	MOV DH, [1000H]	;	Store the content of DH (the number of positive data items) at the offset address 1000H.
	MOV DL, [1001H]	;	Store the content of DL (the number of negative data items) at the offset address 1001 H.
	HLT	;	STOP

Q.2 (a) Solution:

(i)

$$y[n] = x[n] * h[n]$$

Taking discrete time Fourier transform,

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$$Y(e^{j\omega}) = \frac{1}{(1-0.8e^{-j\omega})}\cdot\frac{1}{(1-0.2e^{-j\omega})}$$

For $Y(e^{j\pi})$, put $\omega = \pi$ rad/sec.

$$Y(e^{j\pi}) = \frac{1}{[1-0.8(-1)]}\cdot\frac{1}{[1-0.2(-1)]} = \frac{1}{1.8}\cdot\frac{1}{1.2}$$

$$Y(e^{j\pi}) = \frac{100}{18\times 12} = 0.462$$

We have, $$Y(e^{j\omega}) = \frac{(4/3)}{1-0.8e^{-j\omega}} + \frac{(-1/3)}{1-0.2e^{-j\omega}}$$

Taking inverse DTFT, we get

$$y(n) = (4/3)(0.8)^n u(n) - (1/3)(0.2)^n u(n)$$

(ii) $$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y(t) = x(t) - \frac{2dx}{dt}$$

Taking Laplace Transform,

$$(s^2 + s - 2)Y(s) = X(s) - 2s\,X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1-2s}{(s^2+s-2)}$$

$$H(s) = \frac{1-2s}{(s+2)(s-1)}$$

$$H(s) = \frac{-5}{3(s+2)} - \frac{1}{3(s-1)}$$

Possible ROCs, $\sigma > -2, \sigma > 1$ i.e., $\sigma > 1$

$\sigma < -2, \sigma < 1$ i.e., $\sigma < -2$

$\sigma > -2, \sigma < 1$ i.e., $1 < \sigma < -2$

For the system to be stable, ROC should include the $j\omega$ axis.

$$H(s) = \frac{-5}{3(s+2)} - \frac{1}{3(s-1)} \quad \text{ROC: } 1 < \sigma < -2$$

$$h(t) = \frac{-5}{3}e^{-2t}u(t) + \frac{1}{3}e^{t}u(-t)$$

Initial value of impulse response,

$$\lim_{t\to 0} h(t) = \frac{-5}{3} + \frac{1}{3} = \frac{-4}{3} = -1.33$$

Q.2 (b) Solution:

(i) Direct Memory Access (DMA) is a process of data transfer controlled by an external peripheral. In the conditions where microprocessor controlled data transfer is too slow, the DMA is generally used.

The 8085 has two pins available for DMA type of I/O communication:

HOLD and **HLDA**

- HOLD is an active high input signal to 8085 from another master requesting the use of address and data buses. After receiving the HOLD request, the MPU reliquinshes the buses in the following machine cycle. All buses are tri-stated and HLDA (Hold acknowledge) signal is sent out. MPU regains the control of buses after HOLD goes low.
- HLDA is HOLD acknowledge. This is active high output signal indicating that the MPU is reliquinshing the control of the buses. Typically an external peripheral such as DMA controller send a request i.e. a high signal to the HOLD pin. The processor completes the execution of current machine cycle, floats the address, the data and control lines; and sends HLDA signal. The DMA controller takes control of the buses and transfers data directly between source and destination thus bypassing microprocessor.

  At the end, controller terminates the request by sending a low signal to HOLD pin, and MPU regains control of the buses.

(ii) The three 8085 instructions using stack memory location are XTHL, POP and PUSH.

1. XTHL :

(a) It exchanges the contents of two top locations of stack with contents of HL pair.

(b) It is a 1 byte instruction.

(c) It uses register indirect addressing mode.

(d) Five machine cycles and 16 T states.

(e) No flags are affected.

2. POP rp :

(a) It retrieves the contents of two top locations of stack to register pair (BC, DE, or HL).

(b) It is a 1 byte instruction.

(c) It uses register indirect addressing mode as addresses are defined by Stack pointer register.

(d) 3 machine cycles and 10 T states.

(e) No flags are affected.

3. **PUSH rp :**

(a) It stores the contents of register pair (BC, DE, or HL) on two top locations of stack.

(b) It uses 1-byte instruction.

(c) It uses Register indirect addressing mode.

(d) 3 machine cycles and 12-T states.

(e) No flags are affected.

4. **SPHL :**

(a) Move the content of HL pair to SP.

(b) It uses 1 byte instruction.

(c) It uses Register addressing mode.

(d) One machine cycle and 6T-states

(e) No flags are affected.

5. **LXI SP, 16 Bit data :**

(a) LXI SP, 16 Bit data instruction is a special case of LXI rp, 16 bit data. Using this instruction, we can load 16 bit immediate data/address on the stack pointer (SP).

(b) It uses 3 byte instruction.

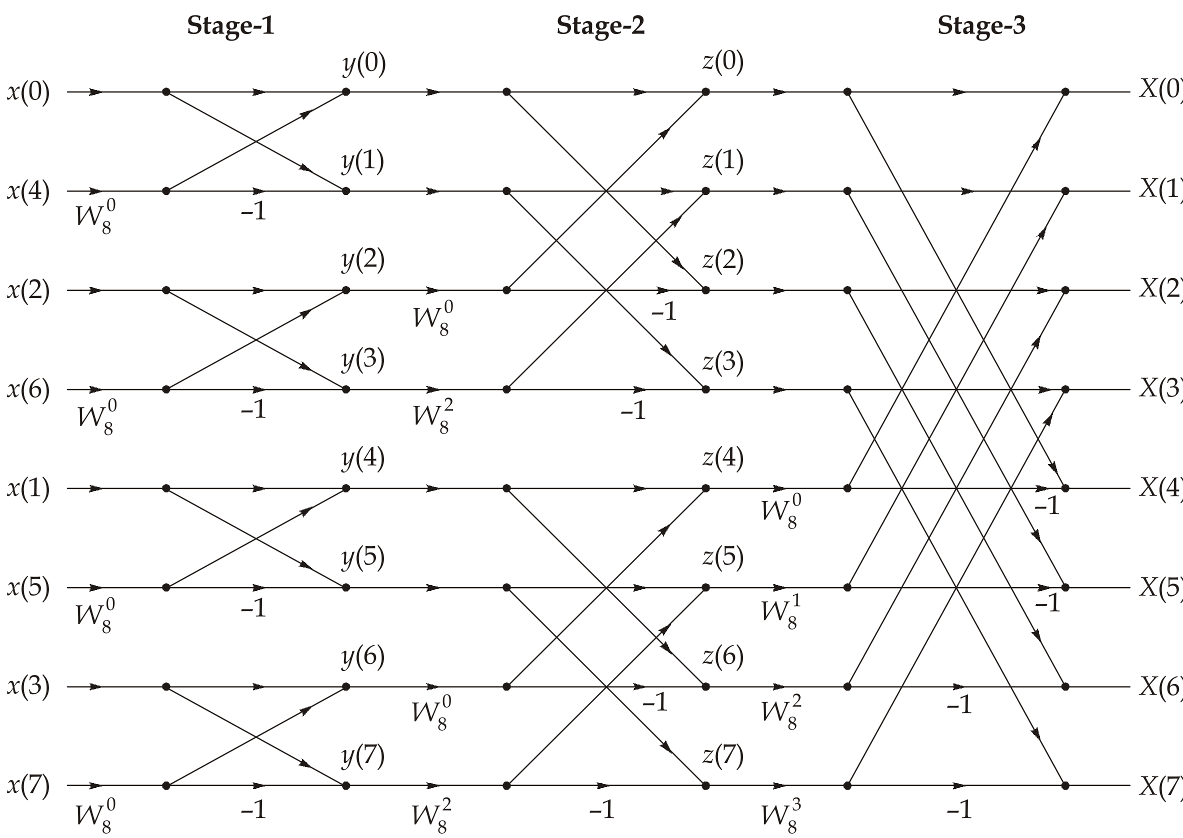
(c) It uses immediate addressing mode.

(d) Three machine cycles and 10T-states.

(e) No flags are affected.

Q.2 (c) Solution

The butterfly diagram of radix-2 DIT-FFT algorithm for $N = 8$ can be given as shown below.



$$W_N^k = e^{-j\frac{2\pi}{N}k}; \quad W_8^k = e^{-j\frac{\pi}{4}k}$$

So, $$W_8^0 = 1, \quad W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}, \quad W_8^2 = -j, \quad W_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

At the output of the stage-1,

$$y(0) = x(0) + x(4)\,W_8^0 = 0 + 4 = 4$$
$$y(1) = x(0) - x(4)\,W_8^0 = 0 - 4 = -4$$
$$y(2) = x(2) + x(6)\,W_8^0 = 2 + 6 = 8$$
$$y(3) = x(2) - x(6)\,W_8^0 = 2 - 6 = -4$$
$$y(4) = x(1) + x(5)\,W_8^0 = 1 + 5 = 6$$
$$y(5) = x(1) - x(5)\,W_8^0 = 1 - 5 = -4$$
$$y(6) = x(3) + x(7)\,W_8^0 = 3 + 7 = 10$$
$$y(7) = x(3) - x(7)\,W_8^0 = 3 - 7 = -4$$

At the output of the stage-2,

$$z(0) = y(0) + y(2)\,W_8^0 = 12$$
$$z(1) = y(1) + y(3)\,W_8^2 = -4 + j4$$
$$z(2) = y(0) - y(2)\,W_8^0 = -4$$
$$z(3) = y(1) - y(3)\,W_8^2 = -4 - j4$$
$$z(4) = y(4) + y(6)\,W_8^0 = 16$$
$$z(5) = y(5) + y(7)\,W_8^2 = -4 + j4$$
$$z(6) = y(4) - y(6)\,W_8^0 = -4$$
$$z(7) = y(5) - y(7)\,W_8^2 = -4 - j4$$

At the output of the stage-3,

$$X(0) = z(0) + z(4)\,W_8^0 = 28$$
$$X(1) = z(1) + z(5)\,W_8^1 = -4 + j9.7$$
$$X(2) = z(2) + z(6)\,W_8^2 = -4 + j4$$
$$X(3) = z(3) + z(7)\,W_8^3 = -4 + j1.7$$
$$X(4) = z(0) - z(4)\,W_8^0 = -4$$
$$X(5) = z(1) - z(5)\,W_8^1 = -4 - j1.7$$
$$X(6) = z(2) - z(6)\,W_8^2 = -4 - j4$$
$$X(7) = z(3) - z(7)\,W_8^3 = -4 - j9.7$$

So, the DFT of the sequence $x(n)$ is,

$$X(k) = \left\{\underset{\uparrow}{28}, -4+j9.7, -4+j4, -4+j1.7, -4, -4-j1.7, -4-j4, -4-j9.7\right\}$$

Q.3 (a) Solution:

Given periodic signal $x(t)$ with Fourier series coefficient a_k.

The fundamental frequency, $\omega_0 = 100$ rad/sec from the expression of $x(t)$.

Let $$Y(\omega) = 2\pi\delta(\omega - \omega_0)$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)e^{j\omega t}d\omega$$

$\therefore$ $$y(t) = e^{j\omega_0 t}$$

If $Y(\omega)$ is of the form of a linear combination of impulses equally spaced in frequency,

i.e. $$Y(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

whose IFT is $y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0' t} = x(t)$ with $\omega_o = 100$

From the above property,

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - 100k) \qquad (\because \omega_0 = 100)$$

Assuming, $y_1(t) = x(t)\cos(\omega_0 t) \longleftrightarrow Y_1(\omega) = \frac{1}{2}[X(\omega-\omega_0) + X(\omega+\omega_0)]$

We have, $Y_1(\omega) = \pi \sum_{k=-\infty}^{\infty} [a_k\delta(\omega - 100k - \omega_0) + a_k\delta(\omega - 100k + \omega_0)]$

$$= \pi \sum_{k=-\infty}^{\infty} \{a_{-k}\delta(\omega + 100k - \omega_0) + a_k\delta(\omega - 100k + \omega_0)\}$$

Given ω_0 = 500 rad/sec, then the above summation with k = 5 becomes

$$Y_1'(\omega) = \pi a_{-5}\delta(\omega) + \pi a_5 \delta(\omega)$$

Since $x(t)$ is real, $a_k = a_{-k}^*$

$\therefore$ $Y_1'(\omega) = 2\pi\, \text{Re}\{a_5\}\delta(\omega)$ which is an impulse at $\omega = 0$.

Let $G_1(\omega) = 2\pi \text{Re}\{a_5\}\delta(\omega) \overset{IFT}{\longleftrightarrow} \text{Re}\{a_5\} = g_1(t)$

Output of LTI system,

$$Y_1(\omega)H(\omega) = G_1(\omega) = 2\pi\text{Re}\{a_5\}\delta(\omega)$$

To obtain $G_1(\omega)$ as above, $H(\omega)$ must be such that it is zero for ω = 100m, for m = ±1, ±2, ... except for ω = 0.

$\therefore$ $H(\omega) = 0$ for ω = 100 m; m = ±1, ±2, ...

Second fact

$$y_2(t) = x(t)\sin(\omega_0 t) \longleftrightarrow Y_2(\omega) = \frac{1}{2j}[X(\omega-\omega_0) - X(\omega+\omega_0)]$$

$$Y_2(\omega) = \frac{\pi}{j}\sum_{k=-\infty}^{\infty} [a_k\delta(\omega - 100k - \omega_0) - a_k\delta(\omega - 100k + \omega_0)]$$

$$= \frac{\pi}{j}\sum_{k=-\infty}^{\infty} [a_{-k}\delta(\omega + 100k - \omega_0) - a_k\delta(\omega - 100k + \omega_0)]$$

If ω_0 = 500 rad/sec, then the above summation with k = 5 becomes,

$$Y_2(\omega) = \frac{\pi}{j} a_{-5}\delta(\omega) - \frac{\pi}{j} a_5\delta(\omega)$$

$$= 2\pi \,\text{Img}\{a_5\}\delta(\omega) \text{ which is impulse at } \omega = 0$$

Hence, $$G_2(\omega) = 2\pi \,\text{Img}\{a_5\}\delta(\omega) \xleftrightarrow{IFT} \text{Img}\{a_5\} = g_2(t)$$

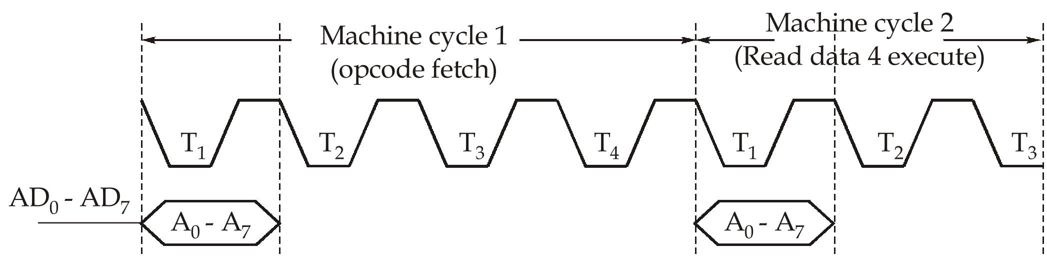
$\therefore$ $$Y_2(\omega)H(\omega) = G_2(\omega) = 2\pi \text{Re}\{a_5\}\delta(\omega)$$

$\therefore$ $$H(\omega) = 0 \text{ for } \omega = 100\,n\,;\, n = \pm 1, \pm 2, \ldots$$

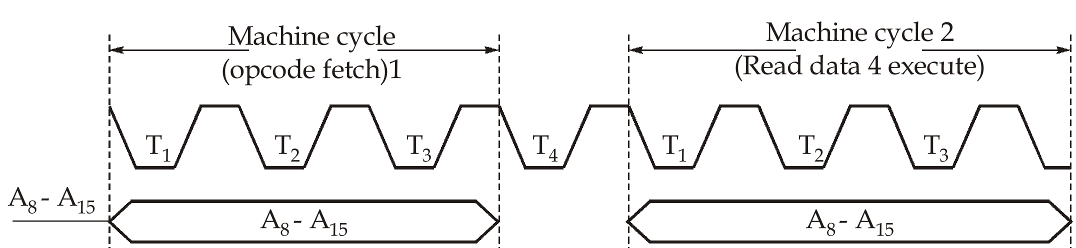
Q.3 (b) Solution:

(i) The lower byte of address (AD_0-AD_7) is available on the multiplexed address\data bus during T_1 state of each machine cycle, except during the bus idle machine cycle.

The higher byte of address (A_8-A_{15}) is available during T_1 to T_3 states of each machine cycle except during the bus idle machine cycle.

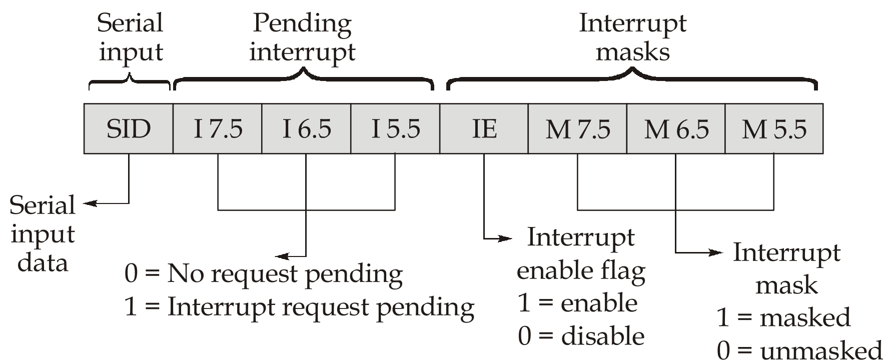


Lower byte address on the multiplexed bus



Higher byte address on A_8-A_{15}

(ii) RIM stands for 'Read Interrupt Mask' The instruction is used to read the status of RST 7.5, RST 6.5 and RST 5.5 interrupts and serial data reception. The contents of the Accumulator after the execution of the RIM instruction provide the information in the format as below:



The RIM instruction format

When RIM instruction is executed , the status of SID, pending RST 7.5, RST6.5 and RST 5.5 interrupts and interrupt masks are loaded in the accumulator. Thus, their status can be monitored. For instance, it may so happen that when one interrupt is being serviced, other interrupt(s) may occur. The status of these pending interrupts can be monitored by RIM instruction.

(iii) Assume 16-bit number at memory locations 2501 H (LSB) and 2502H (MSB)

LXI H, 2501 H	;	Load HL pair with an address 2501 H.
MOV A, M	;	Data stored at location indicated by HL pair (LSB of 16-bit number) moved into accumulator
CMA	;	Complement the data stored in accumulator
INR A	;	Add '1' to the complement of LSB to obtain 2's complement
STA 2503 H	;	Store the data of accumulator (LSB of 2's complement) at address 2503 H
INX H	;	Increment the content of HL pair
MOV A, M	;	Data stored at location indicated by HL pair (MSB of 16-bit number) moved into accumulator
CMA	;	Complement the data stored in accumulator
ADC A	;	Add the carry obtained from LSB
STA 2504 H	;	Store the data of accumulator at (MSB of 2's complement) address 2504 H
HLT	;	Stop

Q.3 (c) Solution:

Addressing Mode:

The CPU can access data in various ways. The data could be in a register or in memory, or be provided as an immediate value. These various ways of accessing data are called addressing modes.

The 8051 provides a total of six distinct addressing modes:

1. Immediate addressing mode:

In this addressing mode, the data is provided in the instruction itself. In immediate addressing mode, as the name implies, when the instruction is assembled, the operand comes immediately after the opcode.

e.g.: MOV A, #84H ; load 84 H into A

MOV R4, #56 ; load the decimal value 56 into R_4

MOV DPTR, #5235H ; DPTR = 5235 H

2. Register addressing mode:

Register addressing mode involves the use of registers to hold the data to be manipulated.

e.g. MOV A, R0 ; Copy the contents of R0 into A

MOV R2, A ; Copy the contents of A into R2.

3. Direct addressing mode:

There is 128 bytes of RAM in 8051. The RAM has been assigned addresses 00 to 7 FH. The details are as below:

(a) RAM locations 00-1F H are assigned to the register banks and stack.

(b) RAM locations 20-2F H are set aside as bit-addressable space to save single-bit data.

(c) RAM locations 30-7F H are available as a place to save byte-sized data.

Although the entire 128 bytes of RAM can be accessed using direct addressing mode, it is most often used to access RAM locations 30-7F H.

In direct addressing mode, the data is in a RAM memory location whose address is given as a part of the instruction. The absence of #sign in the operand distinguishes it from the immediate addressing mode.

e.g. MOV R0, 40 H ; Save content of RAM location 40 H in R0

MOV 56 H, A ; Save content of A in RAM location 56 H

4. **Register Indirect Addressing Mode:** In the register indirect addressing mode, a register is used as a pointer to the data. If the data is inside the CPU, only registers R0 and R1 are used for this purpose.

R2-R7 cannot be used to hold the address of an operand located in RAM when using this addressing mode when R0 and R1 are used as pointers, that is, when they hold the address of RAM locations, they must be preceded by the '@' sign.

In direct addressing mode, the data is in a RAM memory location whose address is given as a part of the instruction. The absence of #sign in the operand distinguishes it from the immediate addressing mode.

e.g.

```
MOV A, @ R0   ; Move contents of RAM location whose
              ; address is held by R0 into A.
MOV @ R1, B   ; Move contents of B into RAM location
              ; whose address is held by R1
```

5. **Indexed addressing mode and on-chip ROM access:**

Indexed addressing mode is widely used in accessing data elements of look-up tables entries located in the program ROM space of the 8051. The instruction used for this purpose is "MOVC A, @ A + DPTR". Because the data elements are stored in the program (code) space ROM of the 8051, the instruction MOVC is used instead of MOV.

The 16 bit register DPTR and register A are used to form the address of the data elements stored in on-chip ROM.

In many applications, the size of program code does not leave any room to share the 64K-byte code space with data.

For this reason the 8051 has another 64 K bytes of memory space set aside exclusively for data storage. This data memory space is referred to as external memory and it is accessed only by the MOV X instruction. In other words, the 8051 has a total of 128 K bytes of memory space since 64 K bytes of code added to 64 K bytes of data space gives us 128 K bytes.

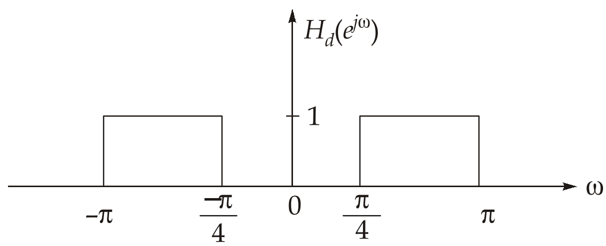
6. **Implied Addressing Mode:** In the implied addressing mode, the operands are specified implicitly in the instruction. For example: RLA is used to rotate the A register content to the Left, SWAPA is used to swap the nibbles in A register.

Q.4 (a) Solution:

(i) 1. Given, $H(e^{j\omega}) = 1 \text{ for } \frac{\pi}{4} \le |\omega| \le \pi$

$$= 0 \text{ for } |\omega| \le \frac{\pi}{4}$$

The frequency response of the digital filter can be drawn as below:



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega n}d\omega + \int_{\pi/4}^{\pi} e^{j\omega n}d\omega\right]$$

$$= \frac{1}{2\pi jn}\left[e^{j\omega n}\Big|_{-\pi}^{-\pi/4} + e^{j\omega n}\Big|_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{\pi n(2j)}\left[e^{-j\pi\frac{n}{4}} - e^{-j\pi n} + e^{j\pi n} - e^{+j\pi\frac{n}{4}}\right]$$

$\therefore$ $$h_d(n) = \frac{1}{\pi n}\left[\sin n\pi - \sin\frac{\pi}{4}n\right]; \quad -\infty \le n \le \infty$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = h_d(n) \text{ for } |n| \le 5$$

$$= 0 \quad \text{otherwise}$$

for $n = 0$, $$h(0) = \underset{n\to 0}{lt}\frac{\sin n\pi}{n\pi} - \underset{n\to 0}{lt}\frac{\sin\frac{n\pi}{4}}{\pi n}$$

$$= \underset{n\to 0}{lt}\frac{\sin n\pi}{n\pi} - \frac{1}{4}\underset{n\to 0}{lt}\frac{\sin\frac{n\pi}{4}}{\frac{\pi}{4}n}$$

$$= 1 - \frac{1}{4}$$

∴ Filter coefficient at $n = 0$;

$$h(0) = 0.75$$

for $n = 1$, $$h(1) = h(-1) = \frac{\sin\pi - \sin\frac{\pi}{4}}{\pi} = -0.225$$

$$h(2) = h(-2) = \frac{\sin 2\pi - \sin\frac{\pi}{2}}{2\pi} = -0.159$$

$$h(3) = h(-3) = \frac{\sin 3\pi - \sin\frac{3\pi}{4}}{3\pi} = -0.075$$

$$h(4) = h(-4) = \frac{\sin 4\pi - \sin\pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin 5\pi - \sin\frac{5\pi}{4}}{5\pi} = 0.045$$

$$\therefore h(n) = \{0.045, 0, -0.075, -0.159, -0.225, \underset{\uparrow}{0.75}, -0.225, -0.159, -0.075, 0, 0.045\}$$

2. The transfer function of the filter,

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^n + z^{-n}\right]$$

$$= 0.75 + \sum_{n=1}^{5} h(n)\left[z^n + z^{-n}\right]$$

$$= 0.75 + h(1)(z + z^{-1}) + h(2)(z^2 + z^{-2}) + h(3)(z^3 + z^{-3})$$
$$+ h(4)(z^4 + z^{-4}) + h(5)(z^5 + z^{-5})$$

$$= 0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2}) - 0.075(z^3 + z^{-3})$$
$$0(z^4 + z^{-4}) + 0.045(z^5 + z^{-5})$$

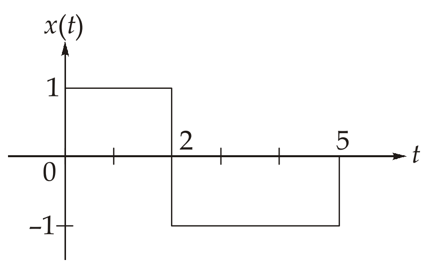
$$\therefore \quad H(z) = 0.75 - 0.225z - 0.225z^{-1} - 0.159z^2 - 0.159z^{-2}$$
$$-0.075z^3 - 0.075z^{-3} + 0.045z^5 + 0.045z^{-5}$$

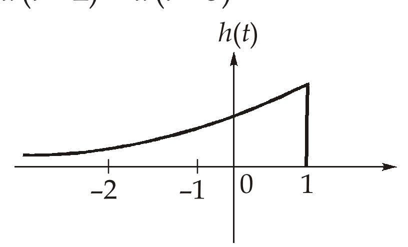
(ii)

FIR filter	IIR filter
1. Impulse response is restricted to finite number of samples.	1. Impulse response has infinite number of samples.
2. FIR filters have precisely linear phase.	2. IIR filters do not have linear phase.
3. Most of the design methods are iterative procedures, requiring powerful computational facilities for their implementation.	3. These filters can be designed using only a panel calculator and tables of analog filter design parameter.
4. In these filters, the poles are fixed at the origin, high selectivity is achieved by using a relatively high order for the transfer function.	4. The poles are placed anywhere inside the unit circle, high selectivity can be achieved with low order transfer function.
5. Always stable.	5. Not always stable.
6. Errors due to round off noise are less severe, mainly because feedback is not used.	6. Errors due to round off noise more severe.

Q.4 (b) Solution:

Given that, $h(t) = e^{2t}\, u(1 - t)$ and $x(t) = u(t) - 2u(t - 2) + u(t - 5)$



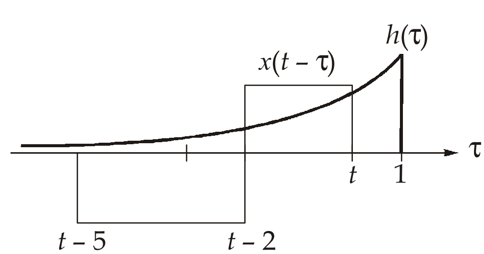


Using convolution integral,

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{2\tau}\, u(1-\tau)[u(t-\tau) - 2u(t-2-\tau) + u(t-5-\tau)]\, d\tau$$

- **For $t \le 1$,**



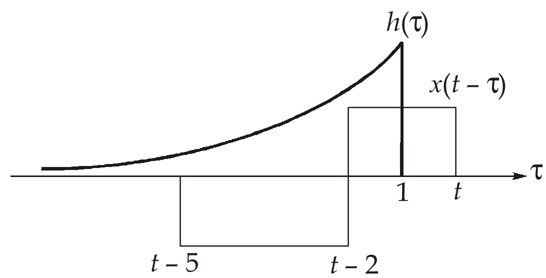
$$y(t) = \int_{t-5}^{t} e^{2\tau}[u(t-\tau)-2u(t-2-\tau)+u(t-5-\tau)]d\tau$$

$$= \int_{t-5}^{t-2} (-1)e^{2\tau}d\tau + \int_{t-2}^{t} e^{2\tau}d\tau$$

$$= -\frac{1}{2}\left[e^{2(t-2)} - e^{2(t-5)}\right] + \frac{1}{2}\left[e^{2t} - e^{2(t-2)}\right]$$

$$y(t) = \frac{1}{2}\left[1 - 2e^{-4} + e^{-10}\right]e^{2t}\ ;\ t \le 1$$

- **For $1 < t \le 3$ or $(t-2) \le 1 < t$,**



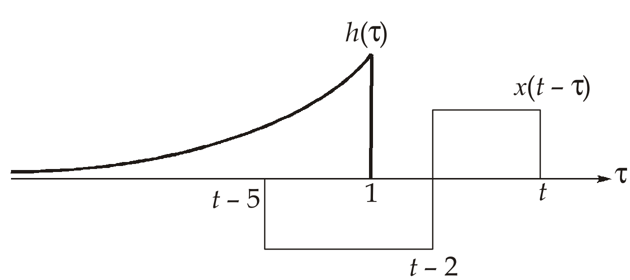
$$y(t) = \int_{t-5}^{t-2} -e^{2\tau}d\tau + \int_{t-2}^{1} e^{2\tau}d\tau$$

$$= -\frac{1}{2}\left[e^{2(t-2)} - e^{2(t-5)}\right] + \frac{1}{2}\left[e^{2} - e^{2(t-2)}\right]$$

$$y(t) = \frac{1}{2}\left[-2e^{-4} + e^{-10}\right]e^{2t} + \frac{1}{2}e^{2}\ ;\ 1 < t \le 3$$

- **For $3 < t \le 6$ or $(t-5) \le 1 < (t-2)$,**

$$y(t) = \int_{t-5}^{1} -e^{2\tau}d\tau = \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right]\ ;\ 3 < t \le 6$$



- **For $t - 5 > 1$ or $t > 6$,**

$$y(t) = 0\ ;\ t > 6$$

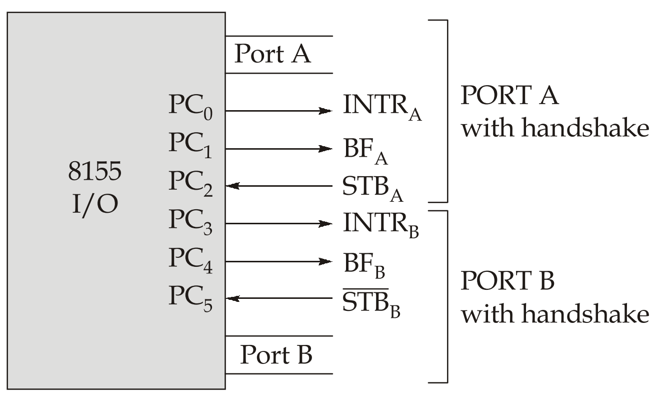
The overall solution is given by,

$$y(t) = \begin{cases} \frac{1}{2}\left[1-2e^{-4}+e^{-10}\right]e^{2t} & ; \quad t \le 1 \\ \frac{1}{2}\left[-2e^{-4}+e^{-10}\right]e^{2t}+\frac{1}{2}e^{2} & ; \quad 1 < t \le 3 \\ \frac{1}{2}\left[e^{2(t-5)}-e^{2}\right] & ; \quad 3 < t \le 6 \\ 0 & ; \quad t > 6 \end{cases}$$

Q.4 (c) Solution:

(i) In handshake mode, data transfer occurs between the MPU and peripherals using control signals called handshake signals. Two I/O ports of 8155, A and B can be configured in the handshake mode; each uses 3 signals from port C as control signals.

When both ports A and B are configured in the handshake mode, port-A uses the lower three signals of port-C (PC_0, PC_1 and PC_2) and port B, uses the upper three signal (PC_3, PC_4, PC_5) as shown below:



The functions of these signals are as follows:

1. **$\overline{STB}$ (Strobe input):** This is an input handshake from a peripheral to the 8155. The low on this signal informs the 8155 that data are strobed into the input port.
2. **BF (Buffer full):** This is an active high signal indicating the presence of a data byte in the port.
3. **INTR (Interrupt request):** This signal is generated by the rising edge of the $\overline{STB}$ signal if the interrupt flip-flop (INTE) is enabled. This signal can be used to interrupt the MPU.

4. **INTE:** This is an internal flip-flop used to enable or disable the interrupt capability of 8155.

The interrupt for port A and B are controlled by bits D_4 and D_5 respectively in control register.

(ii) 1. XTHL: Exchange the content of HL register pair with top of stack.

	Operand	T-states	Flag affected
XTHL	None	16	None

The content of L register are exchanged with stack location pointed out by the content of stack pointer register. The contents of H are exchanged with the contents of next stack pointer location (SP + 1).

2. SHLD: Store H and L register direct.

	Operand	T-states	Flag affected
SHLD	16-bit address	16	None

The content of L register are stored in the memory location specified by 16-bit address in the operand and the contents of H register are stored in the next memory location obtained by incrementing the operand.

3. STAX: Store the accumulator indirect.

	Operand	T-states	Flag affected
STAX	B/D reg-pair	7	None

Content of accumulator are copied into the memory location specified by content of the operand (register pair). The content of accumulator are not altered.

4. PCHL: Load program counter with HL contents.

	Operand	T-states	Flag affected
PCHL	None	6	None

The contents of registers H and L are copied into the program counter. The content of H are placed as high-order byte and of L as low order byte.

5. SPHL: Copy H and L registers to the stack.

	Operand	T-states	Flag affected
SPHL	None	6	None

The instruction loads the contents S of the H and L registers into the stack pointer register. The contents of the H register provide the high-order address and the contents of the L register provide the low-order address.

Section B : Network Theory-1 + Control Systems-1

Q.5 (a) Solution:

Using KVL in loop 1, we get

$$-30 + 250I_1 - V_1 - V_2 = 0$$

$$250I_1 - V_1 - V_2 = 30 \quad \text{...(i)}$$

On applying KVL in loop 2, we get

$$100I_2 + 150(I_2 - I_3) + V_1 = 0$$

$$250I_2 - 150I_3 + V_1 = 0 \quad \text{...(ii)}$$

Similarly on using KVL in loop 3, we get

$$100I_3 + V_2 + 150(I_3 - I_2) = 0$$

$$-150I_2 + 250I_3 + V_2 = 0 \quad \text{...(iii)}$$

Now, from circuit diagram, we have

$$I_1 - I_3 = 0.1 \text{ A} \quad \text{...(iv)}$$

and

$$I_1 - I_2 = 0.04V_x \quad \text{where } V_x = 150(I_3 - I_2)$$

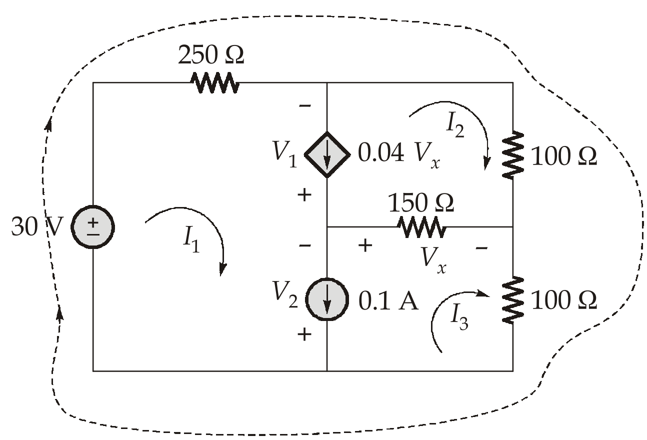
$$I_1 - I_2 = 0.04(150(I_3 - I_2))$$

$$I_1 - I_2 = 6I_3 - 6I_2$$

$$I_1 + 5I_2 - 6I_3 = 0 \quad \text{...(v)}$$

Now,

again apply KVL in bigger loop as shown below:



$$-30 + 250I_1 + 100I_2 + 100I_3 = 0$$

$$250I_1 + 100I_2 + 100I_3 = 30 \quad \text{...(vi)}$$

From equation (iv), (v) and (vi) we get

$$I_1 = \frac{26}{225} \text{A}$$

$$I_2 = -\frac{1}{225}\text{A}$$

$$I_3 = \frac{7}{450}\text{A}$$

Thus,

- Power delivered by 30 V independent voltage source,

$$P_{30\text{ V}} = 30I_1 = 30 \times \frac{26}{225} = 3.5\text{ W}$$

- Power delivered by 0.1 A independent current source,

$$P_{0.1\text{A}} = 0.1V_2$$

where $+V_2 + V_x + 100I_3 = 0$

$$-V_2 = (V_x + 100I_3)$$

$$V_2 = -(V_x + 100I_3) \qquad \because V_x = 150(I_3 - I_2)$$

$$V_2 = -\left(3 + 100 \times \frac{7}{450}\right) \qquad = 150\left(\frac{7}{450} - \left(-\frac{1}{225}\right)\right)$$

$$V_2 = -\frac{41}{9}\text{V} \qquad = 3\text{ V}$$

$\because$
$$P_{0.1\text{A}} = 0.1\left(-\frac{41}{9}\right)$$

$$P_{0.1\text{A}} = -0.46\text{ Watt}$$

- Power delivered by $0.04V_x$ dependent source, $P_{0.04V_x} = V_1(0.04V_x)$

where, $+V_1 + 100I_2 - V_x = 0$

$$V_1 = -100I_2 + V_x$$

$$V_1 = -100\left(-\frac{1}{225}\right) + 3$$

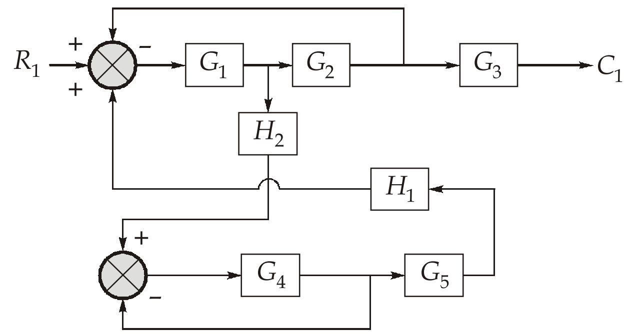
$$V_1 = 3.44\text{ V}$$

$$P_{0.04V_x} = 3.44 \times 0.04 \times 3$$

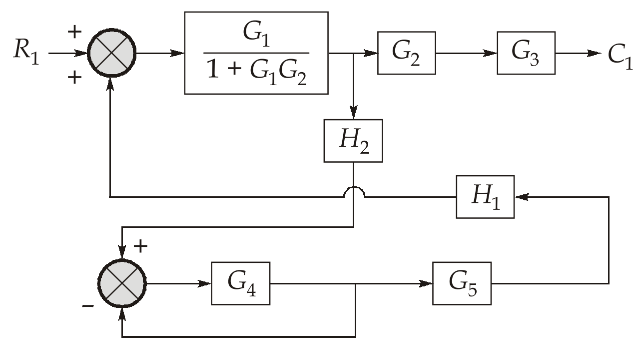
$$= 0.41\text{ W}$$

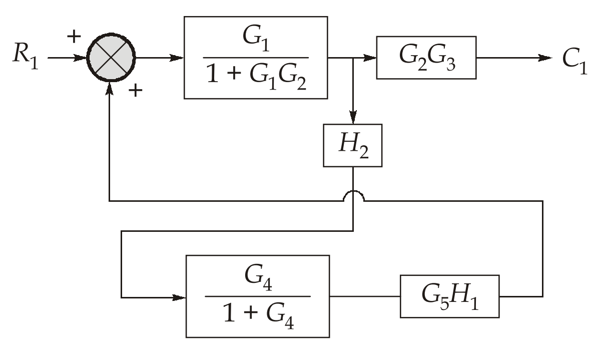
Q.5 (b) Solution:

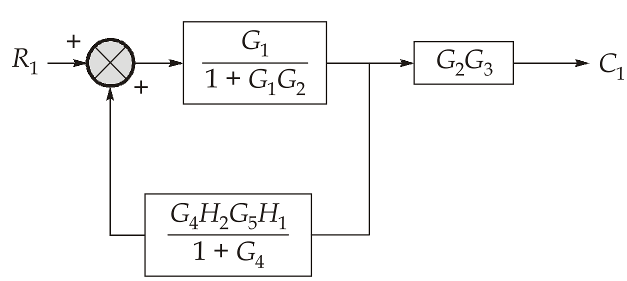
Evaluation of $\frac{C_1}{R_1}$: Assuming $R_2 = 0$, $C_2 = 0$ and Rearranging, we get



Shifting the take-off point before G_2,



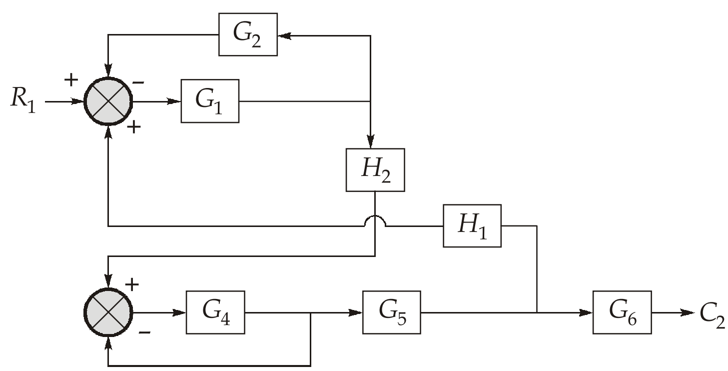


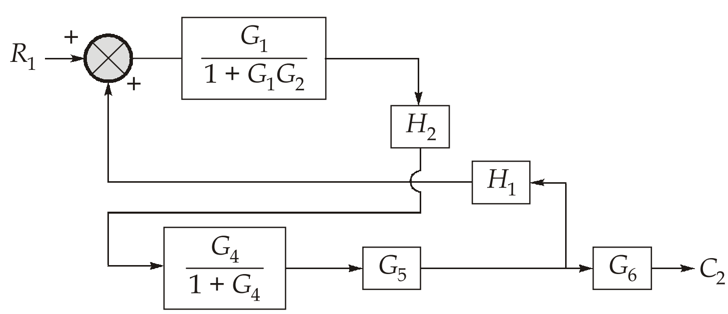


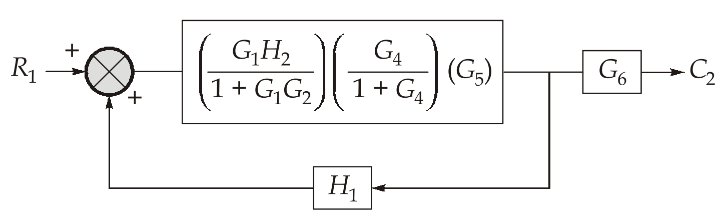
$$R_1 \rightarrow \boxed{\dfrac{G_1(1+G_4)G_2G_3}{(1+G_1G_2)(1+G_4)-(G_1G_4G_5H_1H_2)}} \rightarrow C_1$$

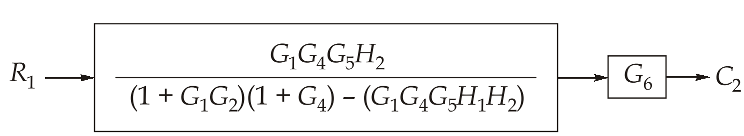
$$\frac{C_1}{R_1} = \frac{G_1G_2G_3(1+G_4)}{(1+G_1G_2)(1+G_4)-G_1G_4G_5H_1H_2}$$

Evaluation of $\dfrac{C_2}{R_1}$: Assuming $R_2 = 0$, $C_1 = 0$ and rearranging we get









$$\frac{C_2}{R_1} = \frac{G_1G_4G_5H_2G_6}{(1+G_1G_2)(1+G_4)-(G_1G_4G_5H_1H_2)}$$

Q.5 (c) Solution:

(i)
$$G(s)H(s) = \frac{K(1+s)}{(1-s)}$$

$$\phi = \tan^{-1}(\omega) - \tan^{-1}\left(\frac{-\omega}{1}\right)$$

$$M = \frac{K\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}} = K$$

when $\omega = 0,\quad \phi = 0°,\quad M = K$

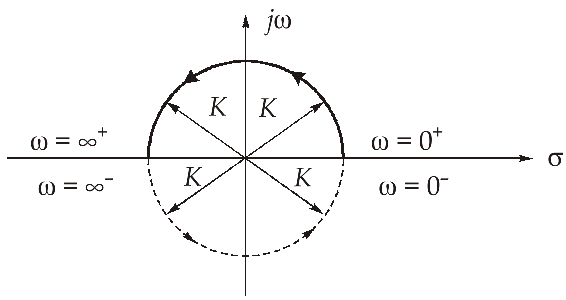
$\omega = 1,\quad \phi = 45° - (-45°),\quad M = K$

$\phi = 90°,$

$\omega = 2,\quad \phi = 63.43° + 63.43° = 126.86°,\quad M = K$

$\omega = \infty,\quad \phi = 90° - (-90°) = 180°,\quad M = K$

The Nyquist plot is shown in figure



Here, $P = 1$ (as $G(s)H(s)$ has one open loop pole on right hand side of the s-plane)

If $K > 1$, N(encirclement about $-1 + j0$ point) = 1

According to Nyquist criterion,

$$Z = P - N = 1 - 1 = 0,$$

i.e. no closed loop pole lie in the right half of s-plane which implies system is stable

If $K < 1, N = 0$

$$Z = P - N = 1 - 0 = 1,$$

i.e. one closed loop pole lie in the right half of s-plane which implies system is unstable.

Therefore, the feedback system is stable for $K > 1$.

(ii) Given the input, $r(t) = 1 + 6t$

On taking Laplace transform of $r(t)$ we get $R(s)$ as

$$R(s) = L\{r(t)\} = L[1 + 6t] = \frac{1}{s} + \frac{6}{s^2}$$

The error signal in s-domain,

$$E(s) = \frac{R(s)}{1 + G(s)H(s)} \qquad \text{Here, } H(s) = 1$$

$$E(s) = \frac{\frac{1}{s} + \frac{6}{s^2}}{1 + \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}}$$

$$E(s) = \frac{\frac{1}{s} + \frac{6}{s^2}}{\frac{s(5s+1)(1+s)^2 + K_1(2s+1)}{s(5s+1)(1+s)^2}}$$

$$= \frac{1}{s}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right] + \frac{6}{s^2}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right]$$

The steady state error e_{ss} can be obtained from final value theorem.

$$e_{ss} = \underset{t\to\infty}{lt}\ e(t) = \underset{s\to 0}{lt}\ sE(s)$$

$$= \underset{s\to 0}{lt}\ s\left\{\frac{1}{s}\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right\}$$

$$+ \underset{s\to 0}{lt}\ s\left\{\frac{6}{s^2}\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right\}$$

$$e_{ss} = 0 + \frac{6}{K_1}$$

Given that, $e_{ss} < 0.1$

$\therefore$ $$0.1 > \frac{6}{K_1} \text{ or } K_1 > \frac{6}{0.1} > 60$$

For steady state error, $e_{ss} < 0.1$, the value of K_1 should be greater than 60. Hence, the minimum value of K_1 should be 60.

Alternate Method:

Given, $$G(s) = \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}$$

The input, $r(t) = 1 + 6t$

On taking Laplace transform of $r(t)$ we get $R(s)$

$$R(s) = L\{r(t)\} = 1\{1 + 6t\} = \frac{1}{s} + \frac{6}{s^2}$$

For unit step input

$$K_p = \underset{s\to 0}{lt}\, G(s) = \underset{s\to 0}{lt} \frac{K_1(2s+1)}{s(5s+1)(1+s)^2} = \infty$$

$$\therefore \quad e_{ss1} = \frac{A}{1+K_P} = \frac{1}{\infty} = 0$$

For $6t$ ramp input

$$K_V = \underset{s\to 0}{lt}\, sG(s) = \underset{s\to 0}{lt} \frac{s \cdot K_1(2s+1)}{s(5s+1)(1+s)^2} = K_1$$

$$\therefore \quad e_{ss2} = \frac{A}{K_V} = \frac{6}{K_1}$$

Given that, $e_{ss} < 0.1$

$$\therefore \quad 0 + \frac{6}{K_1} < 0.1$$

or $60 < K_1$

For steady state error, $e_{ss} < 0.1$, the value of K_1 should be greater than 60. Hence minimum value of K_1 should be 60.

Q.5 (d) Solution:

(i) On applying KVL in both the loops, we get

$$-100 + 5I_1 + j8I_1 + j2I_2 = 0$$

$$(5 + j8)I_1 + j2I_2 = 100 \quad \text{...(i)}$$

$$-V_2 + 2I_2 + j2I_2 + j2I_1 = 0$$

$$j2I_1 + (2 + j2)I_2 = V_2 \quad \text{...(ii)}$$

According to given question,

$$I_1 = 0$$

$$(5 + j8)(0) + j2I_2 = 100$$

$$I_2 = -j50 \text{ A}$$

From equation (ii), we get

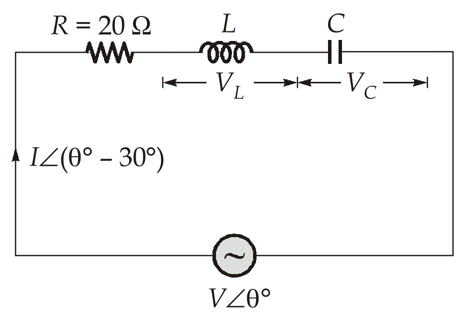
$$(2 + j2)(-j50) = V_2$$

$$V_2 = (100 - 100j) \text{ V}$$

$$V_2 = 141.4\angle -45° \text{ Volt}$$

When $I_1 = 0$, there will be no drop in 5 Ω resistance. Hence 100 ∠ 0° V appear across $j8$ Ω inductive impedance.

(ii) Drawing the circuit according to given data,



$$V_L = 100 \sin 377t \text{ V}$$

$$V_{L(\max)} = 100 \text{ V}$$

$$V_{C(\max)} = \frac{V_{L(\max)}}{2}$$

$$V_{C(\max)} = 50 \text{ V}$$

We know that,

$$V_{L(\max)} = I_{(\max)} \times X_L = 100 \quad \text{...(i)}$$

$$V_{C(\max)} = I_{(\max)} \times X_C = 50 \quad \text{...(ii)}$$

Now, we also know that,

$$\frac{V\angle\theta°}{I\angle(\theta° - 30°)} = Z_{eq} \qquad \therefore \quad Z_{eq} = R + j(X_L - X_C)$$

$$R + j(X_L - X_C) = \frac{V}{I}\angle +30°$$

$$\because \quad \tan 30° = \frac{X_L - X_C}{R}$$

$$0.58 = \frac{\omega L - \dfrac{1}{\omega C}}{20}$$

$$\omega L - \frac{1}{\omega C} = 11.6 \qquad \because \omega = 377 \text{ rad/sec}$$

$$377L - \frac{1}{377C} = 11.6$$

$$(377)^2 LC - 1 = (11.6)(377C) \quad \text{...(iii)}$$

The ratio of equation (i) and (ii) gives

$$2 = \frac{X_L}{X_C} = \frac{\omega L}{\frac{1}{\omega C}}$$

$$2 = \omega^2 LC$$

$$LC = \frac{2}{(377)^2}$$

Put $LC = \frac{2}{(377)^2}$ in equation (iii), we get

$$(377)^2 \times \frac{2}{(377)^2} - 1 = (11.6)(377C)$$

$$\frac{1}{11.6 \times 377} = C$$

$$C = 228.7\ \mu F$$

and $$L = \frac{2}{(377)^2 \times 228.7 \times 10^{-6}} = 61.53\ \text{mH}$$

Q.5 (e) Solution:

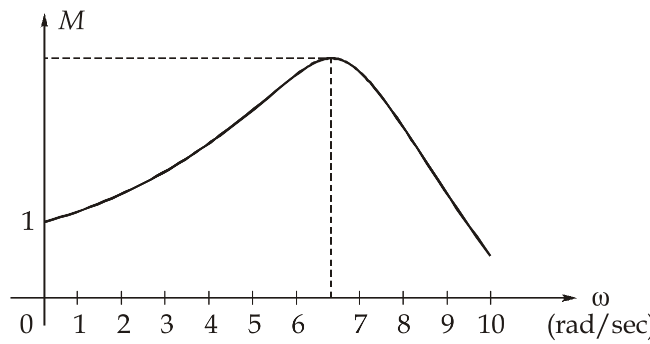
(i) Given: $$T(s) = \frac{1000}{(s+22.5)(s^2+2.45s+44.4)}$$

$$T(j\omega) = \frac{1000}{(j\omega+22.5)((j\omega)^2+2.45j\omega+44.4)}$$

$$T(j\omega) = \frac{1}{(0.044j\omega+1)\left[1-0.0225\omega^2+j0.055\omega\right]}$$

$$M = \frac{1}{\sqrt{1+(0.044\omega)^2}\sqrt{(1-0.0225\omega^2)^2+(0.055\omega)^2}}$$

Sr. No.	ω (rad/sec)	M
1	0	1
2	1	1.02
3	2	1.087
4	4	1.46
5	5	1.89
6	6	2.54
7	6.5	2.664
8	6.7	2.60
9	7	2.40
10	10	0.67



From the curve it is seen that

$$\omega_r = 6.5 \text{ rad/sec and } M_r = 2.664$$

Now, $$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

or, $$2.664 = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

or, $$2\xi\sqrt{1-\xi^2} = 0.375$$

$$\xi = 0.190,\ \xi = 0.9816$$

($\xi = 0.9816$ is neglected because for $\xi > 0.707$ there is no resonant peak)

Also, $$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$6.5 = \omega_n\sqrt{1-2\times(0.190)^2}$$

$$\omega_n = 6.75 \text{ rad/sec}$$

(ii) $$BW = \omega_n\left[\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}\right]$$

Putting $\xi = 0.190$ and $\omega_n = 6.75$ rad/sec

$$BW = 6.75\left[\sqrt{\left[1-2\times(0.190)^2\right]+\sqrt{4(0.190)^4-4(0.190)^2+2}}\right]$$

We get, $$BW = 10.22 \text{ rad/sec}$$

Alternate Method:

Given, $$T(s) = \frac{1000}{(s+22.5)(s^2+2.45s+44.4)}$$

Writing in time constant form

$$T(s) = \frac{1}{\left(\frac{s}{22.5}+1\right)\left(\frac{s^2}{44.4}+\frac{2.45}{44.4}s+1\right)}$$

Since (s + 22.5) is a dominant pole hence, the transfer function can be written as

$$T(s) = \frac{1}{(s^2+2.45s+44.4)}$$

Characteristic equation is $s^2 + 2.45s + 44.4$ on comparing from second order standard equation $s^2+2\xi\omega_n s+\omega_n^2$

We get, $$2\xi\omega_n = 2.45,\ \text{where, } \omega_n^2 = 44.4 \Rightarrow \omega_n = 6.66 \text{ rad/sec}$$

$$2 \times \xi \times 6.66 = 2.45$$

$$\xi = 0.1839$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

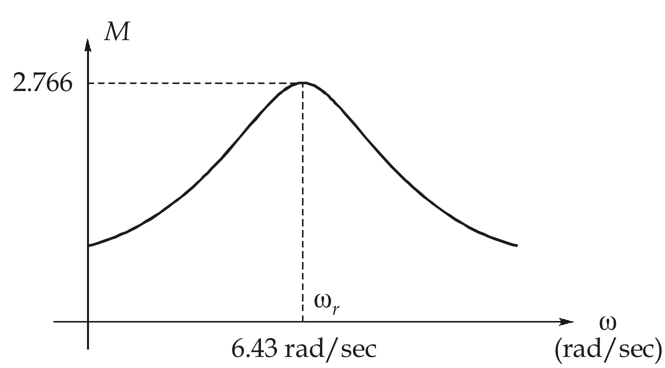
$$M_r = \frac{1}{2\times 0.1839\sqrt{1-(0.1839)^2}}$$

$$M_r = 2.766$$

and, $$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$\omega_r = 6.66\sqrt{1-2(0.1839)^2}$$

$$\omega_r = 6.43 \text{ rad/se}$$



$$BW = \omega_n\left[\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}\right]$$

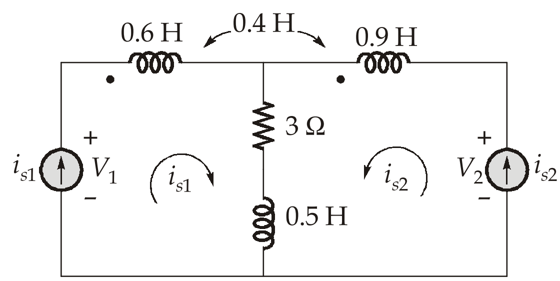
Putting $\xi = 0.1839$ and $\omega_n = 6.66$ rad/sec

$$BW = 6.66\left[\sqrt{\left[1-2\times(0.1839)^2\right]+\sqrt{4(0.1839)^4-4(0.1839)^2+2}}\right]$$

We get, $BW = 11.147$ rad/sec

Q.6 (a) Solution:

(i) Redrawing the given circuit



1. Writing KVL in mesh 1,

$$-V_1(t)+\frac{0.6di_{s1}}{dt}+3(i_{s1}+i_{s2})+\frac{0.5d(i_{s1}+i_{s2})}{dt}-\frac{0.4di_{s2}}{dt}=0$$

$$V_1(t) = \frac{0.6di_{s1}}{dt}+3(i_{s1}+i_{s2})+\frac{0.5d(i_{s1}+i_{s2})}{dt}-\frac{0.4di_{s2}}{dt}$$

$$V_1(t) = 3(i_{s1}+i_{s2})+\frac{1.1di_{s1}}{dt}+\frac{0.1di_{s2}}{dt}$$

Similarly, writing KVL in mesh 2,

$$-V_2(t)+\frac{0.9di_{s2}}{dt}+3(i_{s2}+i_{s1})+\frac{0.5d(i_{s1}+i_{s2})}{dt}-\frac{0.4di_{s1}}{dt}=0$$

$$V_2(t) = 3(i_{s1}+i_{s2})+\frac{1.4di_{s2}}{dt}+\frac{0.1di_{s1}}{dt} \quad \text{...(ii)}$$

Since, $\left.\begin{array}{l} i_{s1}(t) = 10\cos 10t \text{ A} \\ i_{s2}(t) = 6\cos 10t \text{ A}\end{array}\right\}$ ...(iii)

By substituting equation (iii) in equation (ii) and (i), we get

$$V_1(t) = 3(10\cos t + 6\cos 10t)+\frac{1.1d(10\cos 10t))}{dt}+\frac{0.1d(6\cos 10t)}{dt}$$

$$V_1(t) = 30 \cos 10t + 18 \cos 10t + (-110 \sin 10t) + (-6 \sin 10t)$$

$$V_1(t) = (-116 \sin 10t + 48 \cos 10t) \text{ Volt}$$

$$= \sqrt{(116)^2 + (48)^2} \cos\left(10t + \tan^{-1}\left(\frac{+116}{48}\right)\right)$$

$$V_1(t) = 125.539 \cos(10t + 67.52°) \text{ Volt}$$

2. Now, using equation (ii), we get

$$V_2(t) = 3(10\cos 10t + 6\cos 10t) + \frac{1.4d(6\cos 10t)}{dt} + \frac{0.1d(10\cos 10t)}{dt}$$

$$V_2(t) = 48 \cos 10t - 84 \sin 10t - 10 \sin 10t$$

$$V_2(t) = 48 \cos 10t - 94 \sin 10t$$

$$V_2(t) = \sqrt{(48)^2 + (94)^2} \cos\left(10t + \tan^{-1}\left(\frac{94}{48}\right)\right)$$

$$V_2(t) = 105.55 \cos(10t + 62.95°) \text{ volt}$$

3. Average power, supplied by source V_1

$$P_1 = V_{1\text{ rms}} \times i_{s1\text{ rms}} \cos\theta_1$$

$$P_1 = \frac{125.539}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos(67.52°)$$

$$P_1 = 240 \text{ Watt}$$

Average power supplied by source V_2

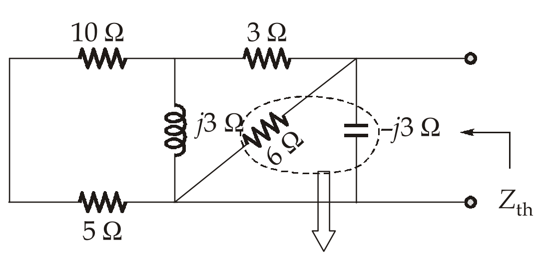
$$P_2 = V_{2\text{ rms}} \times i_{s2\text{ rms}} \times \cos\theta_2$$

$$= \frac{105.55}{\sqrt{2}} \times \frac{6}{\sqrt{2}} \cos(62.95°)$$

$$P_2 = 144 \text{ Watt}$$

(ii) Step-I: Calculation of Z_{Th}:

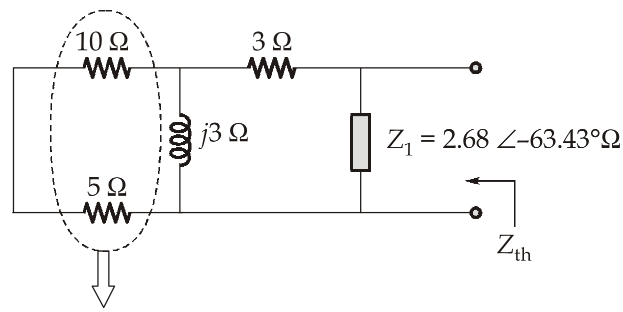
For calculation of equivalent impedance across load impedance, (Z_{Th}) we have to deactivate all the independent voltage and current source i.e., replace voltage source by short circuit and current source by open circuit. We can redraw the circuit as shown below:



6 Ω and $-j3\Omega$ are in parallel. Hence,

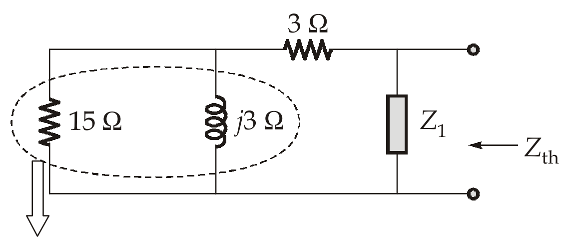
Hence, $$Z_1 = 6\Omega \parallel -j3\Omega = \frac{6\times(-j3)}{6-j3} = 2.68\angle -63.43^\circ\ \Omega$$

The simplified circuit can be drawn as below:



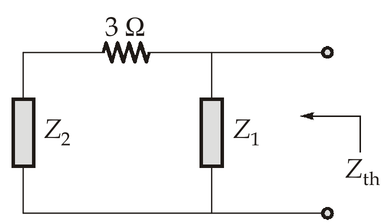
10 Ω and 5Ω are series. Hence,

Hence, $$10 + 5 = 15\ \Omega$$



$$Z_2 = 15 \parallel j3 = \frac{15\times j3}{15 + j3}$$

$$Z_2 = 2.94\angle 78.70^\circ\ \Omega$$



$$Z_{th} = (Z_2 + 3) \parallel Z_1$$

$$Z_{th} = (2.94(\angle 78.70^\circ + 3)) \parallel (2.68\angle -63.43^\circ)$$

$$Z_{th} = (4.60\angle 38.87^\circ) \parallel (2.68\angle -63.43^\circ)$$

$$Z_{th} = \frac{(4.60\angle 38.87^\circ)(2.68\angle -63.43^\circ)}{(4.60\angle 38.87^\circ) + (2.68\angle -63.43^\circ)}$$

$$= \frac{12.328\angle -24.56^\circ}{48\angle 5.85^\circ}$$

$$Z_{th} = 2.57\angle -30.41^\circ\ \Omega$$

Step-II:

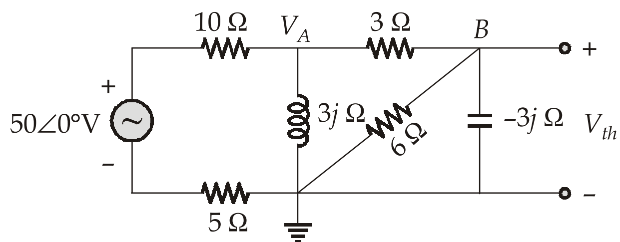
For maximum power transfer, $Z_L = Z_{Th}^*$

$$Z_L = 2.57\angle+30.41°\ \Omega = (2.216 + j1.3)\ \Omega$$

Step-III:

Calculation of V_{th}:

For calculation of V_{th}, assume load impedance to be infinite and calculate voltage across it, i.e.,



Apply KCL at node A,

$$\frac{V_A - 50\angle 0°}{15} + \frac{V_A}{3j} + \frac{V_A - V_{th}}{3} = 0$$

$$V_A\left[\frac{1}{15} + \frac{1}{3j} + \frac{1}{3}\right] + V_{th}\left[\frac{-1}{3}\right] = 3.33$$

$$V_A(0.52\angle{-39.81°}) - V_{th}0.33 = 3.33 \quad \text{...(i)}$$

Similarly, writing KCL at node B,

$$\frac{V_{th} - V_A}{3} + \frac{V_{th}}{(6 \parallel -3j)} = 0 \qquad \because 6 \parallel -3j = Z_1 = 2.68\angle{-63.43°}\ \Omega$$

$$V_{th}\left[\frac{1}{3} + \frac{1}{2.68\angle -63.43°}\right] + V_A\left[-\frac{1}{3}\right] = 0$$

$$V_{th}[0.60\angle 33.70°] - V_A 0.33 = 0 \quad \text{...(ii)}$$

Write equation (i) and (ii) in matrix form,

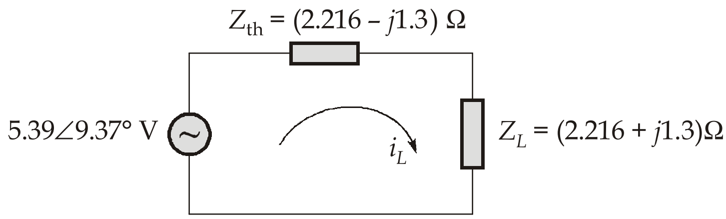
$$\begin{bmatrix} 0.52\angle -39.81° & -0.33 \\ -0.33 & 0.60\angle 33.70° \end{bmatrix}\begin{bmatrix} V_A \\ V_{th} \end{bmatrix} = \begin{bmatrix} 3.33\angle 0° \\ 0 \end{bmatrix}$$

Using Cramer's rule,

$$V_{th} = \frac{\begin{vmatrix} 0.52\angle -39.81° & 3.33\angle 0° \\ -0.33 & 0 \end{vmatrix}}{\begin{vmatrix} 0.52\angle -39.81° & -0.33 \\ -0.33 & 0.60\angle 33.70° \end{vmatrix}} = 5.39\angle 9.37°\ \text{Volt}$$

Step-IV:

Calculation of maximum power, P_{max}



$$P_{max} = I_L^2 R_L \quad , \quad \text{where } I_L = \frac{5.39\angle 9.37°}{4.432}$$

$$P_{max} = (1.22)^2(2.216) \qquad I_L = 1.22\angle 9.37° \text{ A}$$

$$P_{max} = 3.30 \text{ W}$$

Q.6 (b) Solution:

Given, unity feedback system, $G(s) = \frac{K(s+6)}{s(s+2)}$

Any point on Root locus must satisfy the characteristic equation $1 + G(s) = 0$. Therefore,

At any point on RL, $$K = \frac{-(s^2+2s)}{(s+6)}$$

$$\frac{dK}{ds} = \frac{-(s+6)(2s+2)-(s^2+2s)(1)}{(s+6)^2}$$

$$\frac{dK}{ds} = -\left[\frac{2s^2+2s+12s+12-s^2-2s}{(s+6)^2}\right] = -\left[\frac{s^2+12s+12}{(s+6)^2}\right]$$

The value of $\left.\frac{dK}{ds}\right|_{K=50}$ is to be determined and for this, closed loop poles for $K = 50$ are determined using characteristic equation.

Characteristic equation is

$$s^2 + 2s + Ks + 6K = 0$$

$$s^2 + 52s + 300 = 0$$

The closed loop poles are $s_1 = -6.61$, $s_2 = -45.39$

At $s = -6.61$

$$\left.\frac{dK}{ds}\right|_{s=-6.61} = -\left[\frac{(-6.61)^2+12(-6.61)+12}{(-6.61+6)^2}\right] = 63.49$$

Root sensitivity,

$$S_{s,K} = \frac{K}{s}\cdot\frac{ds}{dK} = \frac{50}{-6.61}\times\frac{1}{63.49} = -0.1191$$

For a 10% change in the value of K, change in the location of closed loop poles at $s_1 = -6.61$ is

$$ds_1 = S_{s_1K}\cdot s_1\frac{dK}{K} = -0.1191\times(-6.61)\times 0.10$$

$$ds_1 = 0.0787$$

The closed loop poles will move to the right by 0.079 units for a 10% change in K,

At $$s_2 = -45.39$$

$$\frac{dK}{ds} = -\left[\frac{(-45.39)^2+12(-45.39)+12}{(-45.39+6)^2}\right]$$

$$\frac{dK}{ds} = -0.9845$$

Root sensitivity, $$S_{s_2K} = \frac{K}{s_2}\frac{ds}{dK}$$

$$= \frac{50}{-45.39}\left(\frac{-1}{0.9845}\right) = 1.1189$$

For a 10% change in the value of K, change in location of closed loop poles at $s_2 = -45.39$

$$ds_2 = S_{s_2K}\cdot s_2\cdot\frac{dK}{K}$$

$$ds_2 = -45.39 \times 1.1189 \times 0.1 = -5.08$$

The closed loop poles at $s_2 = -45.39$ will move to the left by 5.08 units for 10% change in K.

Alternate Method:

Given: $$G(s) = \frac{K(s+6)}{s(s+2)}$$

Since, $K = 50$ for a 10% change in the value of K, K could be 45 or 55.

For $K_1 = 45$

$$G(s) = \frac{K(s+6)}{s(s+2)}$$

Any point on root locus must satisfy the characteristic equation $1 + G(s) = 0$.

$$\therefore \quad 1+\frac{K(s+6)}{s(s+2)} = 0$$

$$s(s + 2) + K(s + 6) = 0$$

$$K = \frac{-s(s+2)}{(s+6)}$$

$$45 = \frac{-s_1(s_1+2)}{(s_1+6)}$$

$$45(s_1 + 6) = -s_1^2 - 2s_1$$

$$s_1^2 + 2s_1 + 45s_1 + 270 = 0$$

$$s_1^2 + 47s_1 + 270 = 0$$

$$s_1 = -6.699, -40.30$$

For $K = 55$

$$55(s_2 + 6) = -(s_2^2 + 2s_2)$$

$$s_2^2 + 2s_2 + 55s_2 + 330 = 0$$

$$s_2^2 + 57s_2 + 330 = 0$$

$$s_2 = -6.53, -50.460$$

For $K = 50$

$$50(s + 6) = -(s^2 + 2s)$$

$$s^2 + 2s + 50s + 300 = 0$$

$$s^2 + 52s + 300 = 0$$

$$s = -6.609, -45.390$$

Now, change in location of closed loop pole for 10% change in value of K is

For $K = 45$

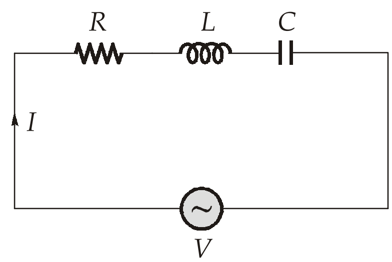
$$s = -0.09, -5.09$$

For $K = 55$

$$s = -0.079, -5.07$$

Q.6 (c) Solution:

(i) Let us assume a series RLC circuit as shown below:



At a given frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad ...(i)$$

At half-power points,

$$I = \frac{I_0}{\sqrt{2}}$$

where $I_0 = \frac{V}{R}$ (at resonance)

$$I = \frac{V}{\sqrt{2}R} \quad ...(ii)$$

From equation (i) and (ii), we get,

$$\frac{V}{\sqrt{2}R} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Squaring both side, we get,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R$$

$$\omega^2 LC - 1 = \pm R\omega C$$

On solving, we get,

$$\omega^2 \pm \frac{R\omega}{L} - \frac{1}{LC} = 0$$

$$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

For low value of R, the term $\left(\frac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\frac{1}{LC}$.

Then ω is given by,

$$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm\frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$$

We know that the resonance frequency of this circuit is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{rad/sec}$$

$$\omega = \omega_0 \pm \frac{R}{2L}$$

Hence,
$$\omega_1 = \omega_0 - \frac{R}{2L}; \; \omega_2 = \omega_0 + \frac{R}{2L}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L} \text{rad/sec}$$

(ii) We have,
$$\text{frequency}, f_0 = 1 \text{ MHz}$$
$$C_1 = 500 \text{ pF}$$
$$C_2 = 600 \text{ pF}$$

As resonance, $C = 500 \text{ pF} = 500 \times 10^{-12}\text{F}$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$10^6 = \frac{1}{2\pi\sqrt{L \times 500 \times 10^{-12}}}$$

$$L = 0.05 \text{ mH}$$

$$X_L = 2\pi f_0 L = 2\pi \times 10^6 \times 0.05 \times 10^{-3}$$

$$X_L = 314.16 \ \Omega$$

When capacitance is 600 pF, the current reduces to one-half of the current at resonance,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^6 \times 600 \times 10^{-12}} = 265.26 \ \Omega$$

According to question,

When C_2 = 600 pF, then

$$I = \frac{I_0}{2}$$

$$\frac{V}{Z} = \frac{V}{2R}$$

$$Z = 2R$$

$$\sqrt{R^2 + (X_L - X_C)^2} = 2R$$

$$R^2 + (314.16 - 265.26)^2 = 4R^2$$

$$R = 28.23\ \Omega$$

$$\text{Quality factor, } Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{28.23}\sqrt{\frac{0.05 \times 10^{-3}}{500 \times 10^{-12}}} = 11.2$$

Q.7 (a) Solution:

In the Bode plot, one pole offers the slope of –20 dB/decade and a zero offers the slope of +20 dB/decade. The initial slope of the Bode plot defines the type of the system. From the given Bode plot, we can say

(i) Type of the system = 0 as the initial slope is 0 dB/decade

(ii) System has 1 plot at s = 5, two poles at s = 40 and one pole at s = 100

The open loop transfer function $G(s)$ is given as

$$G(s) = \frac{K}{\left(1+\frac{s}{5}\right)\left(1+\frac{s}{40}\right)^2\left(1+\frac{s}{100}\right)}$$

Since, there is no cutoff frequency less than 1 rad/sec. Hence,

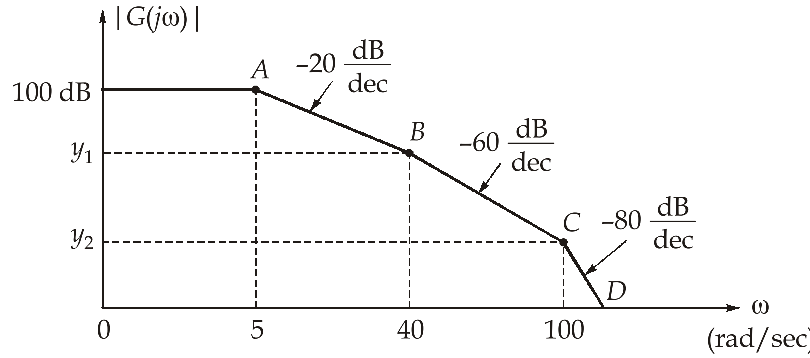
$$20 \log_{10} K = 100$$

$$K = 10^5$$

$$G(s) = \frac{10^5}{\left(1+\frac{s}{5}\right)^1\left(1+\frac{s}{40}\right)^2\left(1+\frac{s}{100}\right)^1}$$

The corner frequencies are ω = 5, ω = 40 and ω = 100 rad/sec.

At ω = 5 rad/sec, $|G(j\omega)|$ = 100 dB



From equation of line *AB*

$$-20 = \frac{100 - y_1}{\log_{10} 5 - \log_{10} 40}$$

$$-20\log_{10}\left(\frac{1}{8}\right) = 100 - y_1$$

$$y_1 = 81.94 \text{ dB}$$

Similarly, from equation of line BC,

$$-60 = \frac{y_1 - y_2}{\log_{10} 40 - \log_{10} 100}$$

$$-60\log_{10}\left(\frac{2}{5}\right) = 81.94 - y_2$$

$$y_2 = 58.06 \text{ dB}$$

Hence, corner frequency at ω = 40 rad/sec

$$|G(j\omega)| = 81.94 \text{ dB}$$

At corner frequency, $\omega = 100$ rad/sec

$$|G(j\omega)| = 58.06 \text{ dB}$$

Now, the phase margin of the system is given as

$$\text{PM} = 180° + \angle G(j\omega_{gc})H(j\omega_{gc})$$

Since, $H(j\omega) = 1$ for unity feedback

At $\omega = \omega_{gc}$ $|G(j\omega)| = 0 \text{ dB}$

From equation of the line CD,

$$-80 = \frac{y_2 - 0}{\log_{10} 100 - \log_{10} \omega_{gc}}$$

$$\log_{10}\left(\frac{100}{\omega_{gc}}\right) = \frac{-58.06}{80} = -0.72575$$

$$\log_{10}(100) - \log_{10}(\omega_{gc}) = -0.72575$$

$$\omega_{gc} = 531.8 \text{ rad/sec}$$

The phase of the system at ω_{gc} can be calculated as

$$\angle G(j\omega_{gc}) = -\left[\tan^{-1}\left(\frac{\omega_{gc}}{5}\right) + 2\tan^{-1}\left(\frac{\omega_{gc}}{40}\right) + \tan^{-1}\left(\frac{\omega_{gc}}{100}\right)\right]$$

Substituting $\omega_{gc} = 531.8$ rad/sec

$$\angle G(j\omega_{gc}) = -\left[\tan^{-1}\left(\frac{531.8}{5}\right) + 2\tan^{-1}\left(\frac{531.8}{40}\right) + \tan^{-1}\left(\frac{531.8}{100}\right)\right]$$

$$= -340.2°$$

Hence,

$$PM = 180° + \angle G(j\omega_{gc})$$

$$PM = 180° - 340.2°$$

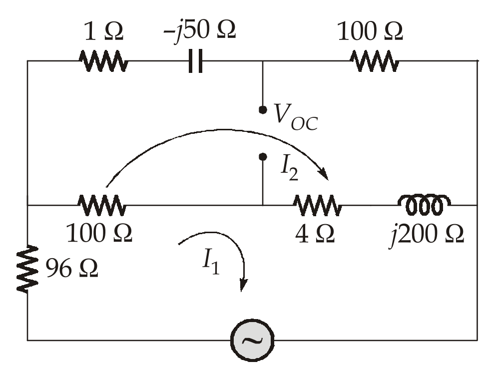
$$PM = -160.2°$$

Q.7 (b) Solution:

(i) Thevenin's Theorem:

Calculation of V_{th}:

⇒ For the calculation of V_{th}, take load impedance as infinite and calculate voltage across it.



On applying KVL in mesh 1, we get

$$+96I_1 + 100(I_1 - I_2) + (4 + j200)(I_1 - I_2) = 10$$

$$(200 + j200)I_1 + (-104 - j200)I_2 = 10 \quad \text{...(i)}$$

Applying KVL to mesh 2, we get

$(205 + j150)I_2 - (104 + j200)I_1 = 0$ ...(ii)

Writing equations in matrix form,

$$\begin{bmatrix} 200 + j200 & -(104 + j200) \\ -(104 + j200) & (205 + j150) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule, we get

$$I_1 = \frac{\begin{vmatrix} 10\angle 0° & -(104 + j200) \\ 0 & (205 + j150) \end{vmatrix}}{\begin{vmatrix} 200 + j200 & -(104 + j200) \\ -(104 + j200) & (205 + j150) \end{vmatrix}} = 0.05\angle 0.00272° \text{A}$$

$$I_2 = \frac{\begin{vmatrix} 200 + j200 & 10\angle 0° \\ -(104 + j200) & 0 \end{vmatrix}}{\begin{vmatrix} 200 + j200 & -(104 + j200) \\ -(104 + j200) & (205 + j150) \end{vmatrix}} = 0.045\angle 26.34°$$

$V_{OC} = 100I_2 - (4 + j200)(I_1 - I_2)$

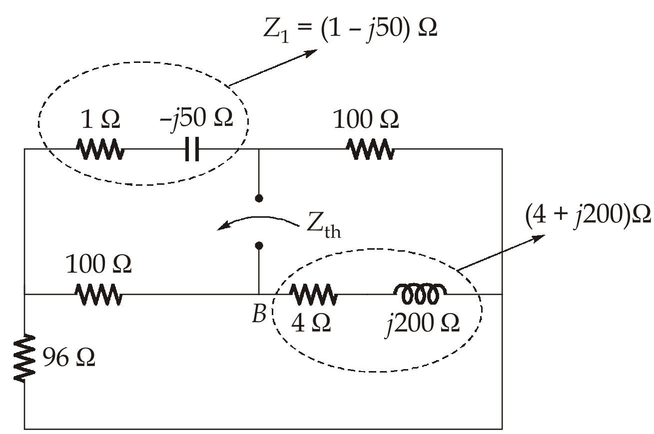
$V_{OC} = 100(0.045\angle 26.34°) - (4 + j200)(0.05\angle 0.00272° - 0.045\angle 26.34°)$

$$V_{OC} = 0.14\angle 89.47°$$

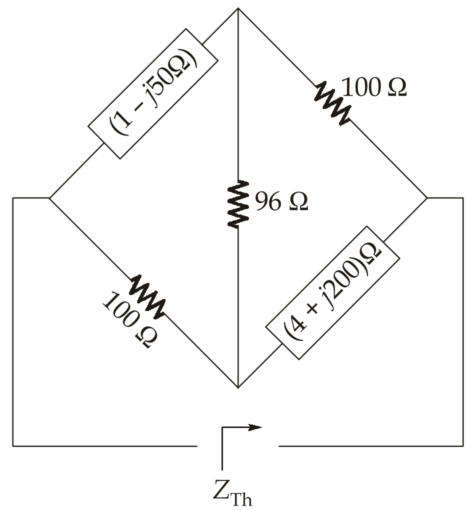
Hence, $V_{OC} = V_{th} = 0.14\angle 89.47°$

Calculation of Z_{th}:

⇒ For the calculation of Z_{th}, replace the voltage source with short circuit and current source with open circuit and calculate equivalent impedance across the load resistance.



We can redraw the minimized circuit as shown below:

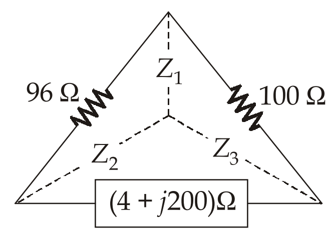


Now, using star to Delta conversion,

$$Z_1 = \frac{96 \times 100}{96 + 100 + 4 + j200} = (24 - 24j)\Omega$$

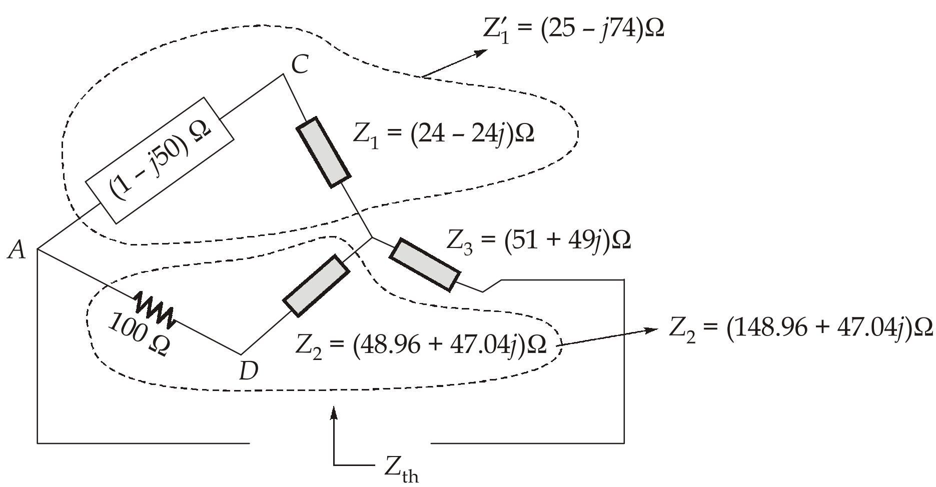
$$Z_2 = \frac{(96)(4 + j200)}{96 + 100 + 4 + j200} = (48.96 + 47.04j)\Omega$$

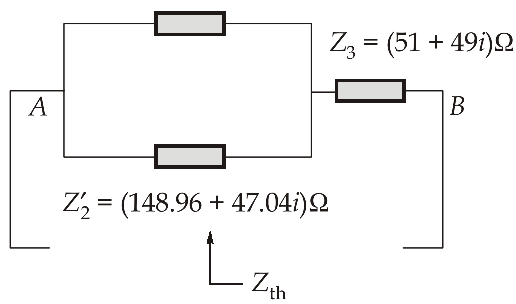
$$Z_3 = \frac{(100)(4 + j200)}{96 + 100 + 4 + j200} = (51 + 49j)\Omega$$



Now,

We have,





$$Z_{th} = (Z_1' \parallel Z_2') + Z_3$$

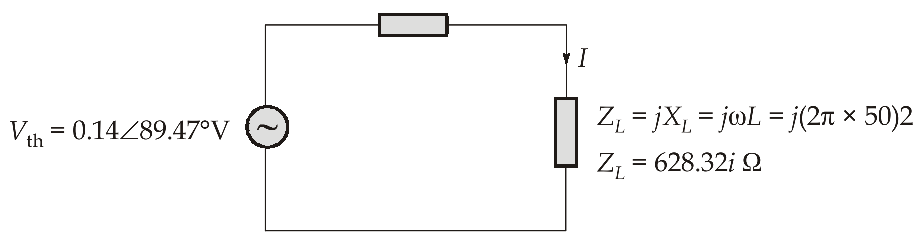
$$\therefore \quad Z_1' \parallel Z_2' = \frac{(25-74i)(148.96+47.04i)}{148.96+47.04i+25-74i}$$

$$Z_1' \parallel Z_2' = 49.01 - 49i$$

$$Z_{th} = (49.01 - 49i) + (51 + 49i)$$

$$Z_{th} \cong 100\ \Omega$$

Now, we can draw the Thevenin equivalent circuit as shown below:



$$I = \frac{V_{th}}{Z_{th}+Z_L} = \frac{0.14\angle 89.47^\circ}{100+628.32i}$$

$$I = 0.22\angle 8.51^\circ \text{ mA}$$

(ii) Using Norton's Theorem:

Calculation of $I_{s.c}$ (short circuit current):

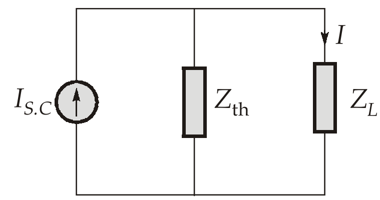
To calculate $I_{s.c}$, we have to assume the load impedance as zero ohm and calculate current across it.

We can calculate $I_{s.c}$ as,

$$I_{s.c} = \frac{V_{Th}}{Z_{Th}} = \frac{0.14\angle 89.47^\circ}{100}$$

$$I_{s.c} = 1.4\angle 89.47^\circ \text{ mA}$$

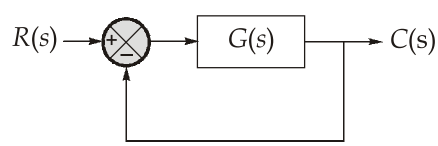
Now, Draw the Norton equivalent circuit,



Q.7 (c) Solution:

(i) Consider the unity feedback system as shown in figure with

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{G(s)}{1+G(s)}$$



Therefore, $$G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\xi\omega_n)}$$

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + j2\xi\omega \cdot \omega_n}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\omega\sqrt{\omega^2 + 4\xi^2\omega_n^2}}$$

and $$\angle G(j\omega) = -90° - \tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right)$$

At phase crossover frequency, ω_{pc}

$$\angle G(j\omega_{pc}) = -180°$$

$$-90° - \tan^{-1}\left(\frac{\omega_{pc}}{2\xi\omega_n}\right) = -180°$$

$$\tan^{-1}\left(\frac{\omega_{pc}}{2\xi\omega_n}\right) = 90°$$

$$\omega_{pc} = \infty \text{ rad/sec}$$

$$\therefore \qquad GM = \frac{1}{|G(j\omega_{pc})|} = \frac{\omega_{pc}\sqrt{\omega_{pc}^2 + 4\xi^2\omega_n^2}}{\omega_n^2} = \infty$$

At gain crossover frequency ω_{gc},

$$|G(j\omega_{gc})| = 1$$

$$\frac{\omega_n^2}{\omega_{gc}\sqrt{\omega_{gc}^2 + 4\xi^2\omega_n^2}} = 1$$

$\Rightarrow$ On solving,

$$\omega_{gc} = \omega_n\sqrt{-2\xi^2 + \sqrt{4\xi^4 + 1}}$$

Hence, phase margin

$$PM = 180° + \angle G(j\omega_{gc})$$

$$PM = 90° - \tan^{-1}\left[\frac{\sqrt{-2\xi^2 + \sqrt{4\xi^4 + 1}}}{2\xi}\right]$$

(ii) We have,

$$G(s) = \frac{1+4s}{s(1+s)(1+2s)}$$

Put $s = j\omega$,

$$G(j\omega) = \frac{1+4j\omega}{j\omega(1+j\omega)(1+2j\omega)}$$

At $\omega \to 0$,

$$G(j0) = \frac{1+j0}{j0(1+j0)(1+j0)} = \infty\angle -90°$$

At $\omega \to \infty$,

$$G(j\infty) = \frac{1+j\infty}{j\infty(1+j\infty)(1+j\infty)} = 0\angle -180°$$

Rationalise the transfer function,

$$G(j\omega) = \frac{(1+4j\omega)(1-j\omega)(1-2j\omega)}{j\omega(1+j\omega)(1+2j\omega)(1-j\omega)(1-2j\omega)}$$

$$G(j\omega) = \frac{(1+4j\omega)(1-2\omega^2-j3\omega)}{j\omega(1+\omega^2)(1+4\omega^2)}$$

$$G(j\omega) = \frac{(1+10\omega^2)+j\omega(1-8\omega^2)}{j\omega(1+\omega^2)(1+4\omega^2)}$$

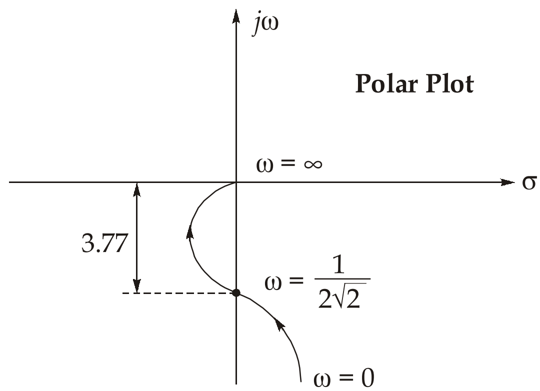
$$G(j\omega) = \frac{(1-8\omega^2)}{(1+\omega^2)(1+4\omega^2)} - \frac{j(1+10\omega^2)}{\omega(1+\omega^2)(1+4\omega^2)}$$

The polar plot does not cross the real axis. For intersection with imaginary axis,

$$1 - 8\omega^2 = 0$$

$$1 = 8\omega^2$$

$$\omega = \frac{1}{2\sqrt{2}} \text{ rad/sec}$$



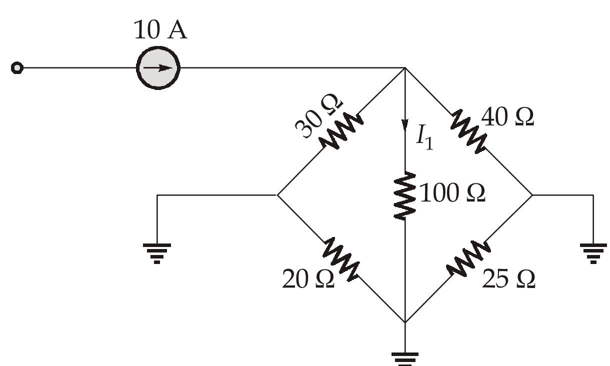
The magnitude of $G(j\omega)$ at $\omega = \frac{1}{2\sqrt{2}}$ rad/sec is calculated as

$$|G(j\omega)|_{\omega=\frac{1}{\sqrt{2}}} = \left.\frac{1+10\omega^2}{\omega(1+\omega^2)(1+4\omega^2)}\right|_{\omega=\frac{1}{2\sqrt{2}}} = 3.77$$

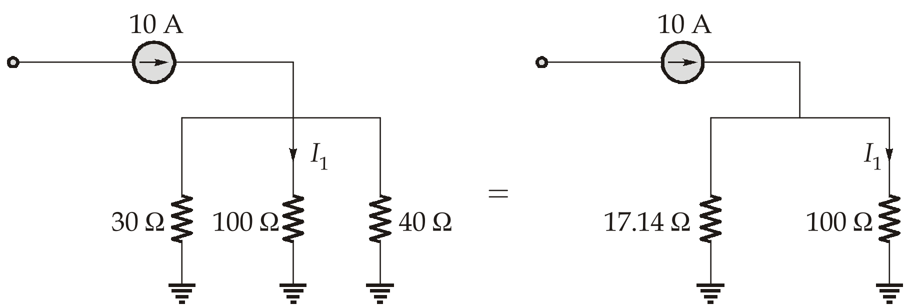
Q.8 (a) Solution:

(i) Using Superposition Theorem:

⇒ **Case-I:** When 10 A current source is activated.



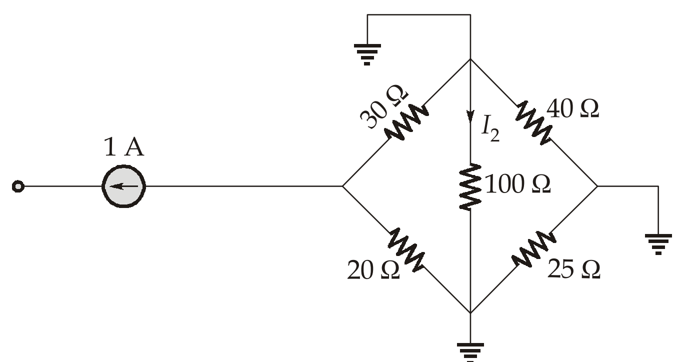
Both the terminal of 20 Ω and 25 Ω are connected to ground. Therefore, it is considered as short circuit element. The simplified circuit can be drawn as below:



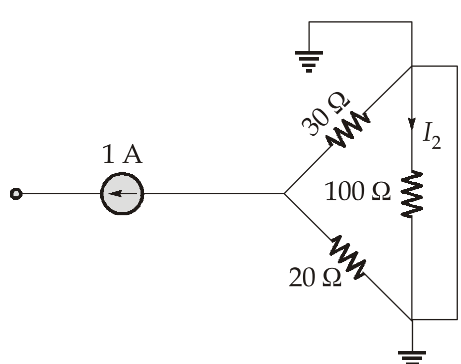
Using current division rule, we get

$$I_1 = \frac{10 \times 17.14}{100 + 17.14} = 1.46 \text{ A}$$

Case-II: When 1 A current source is activated and other terminals are assumed to be grounded then, the circuit can be drawn as below:

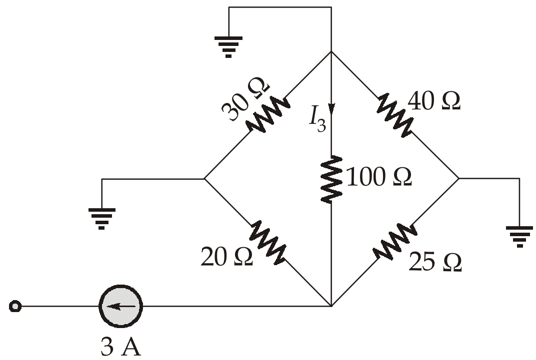


In this case 40 Ω and 25 Ω behaves as short circuit element. The simplified circuit, therefore, can be drawn as below:

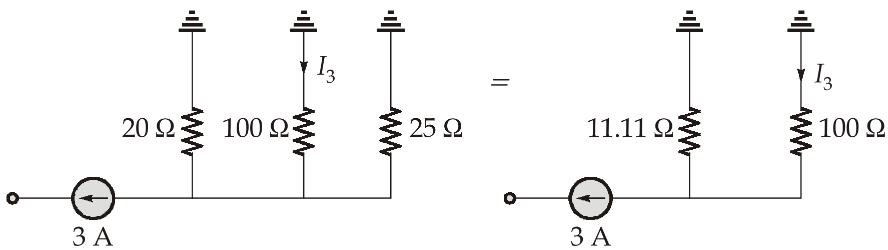


Hence $I_2 = 0$ A

Case-III: When 3A current source is activated and others are assumed to be grounded then, the circuit can be drawn as below:



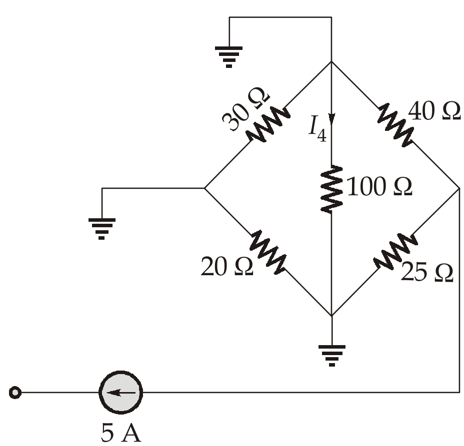
In this case, 30 Ω and 40 Ω behaves as short circuit element. The simplified circuit, therefore, can be drawn as below:



Using current division rule, we get

$$I_3 = \frac{3 \times 11.11}{100 + 11.11} = 0.3 \text{ A}$$

Case-IV: When 5A current source is activated and others are assumed to be grounded then, the circuit can be drawn as below:



As both the terminals of 100 Ω resistance are connected to ground only hence no current passes through it. Hence $I_4 = 0$ A.

From superposition theorem, current flowing through 100 Ω resistor can be calculated as

$$I_T = I_1 + I_2 + I_3 + I_4$$
$$I_T = 1.46 + 0 + 0.3 + 0$$
$$I_T = 1.76 \text{ A}$$

(ii) The values at resonance are given as,

Resonant frequency, $f_0 = 200$ Hz

Quality factor, $Q_0 = 7.5$

Inductive reactance, $X_L = 250\ \Omega$

1. We know that,

$$X_L = \omega_0 L = 250\ \Omega$$
$$L = \frac{250}{2\pi \times 200} = 0.2 \text{ Henry}$$

Since,
$$\omega_0^2 = \frac{1}{LC}$$
$$C = \frac{1}{\omega_0^2 L}$$
$$C = \frac{1}{(2\pi \times 200)^2 \times 0.2}$$
$$C = 3.16\ \mu\text{F}$$

Also,
$$Q_0 = \frac{\omega_0 L}{R}$$
$$R = \frac{\omega_0 L}{Q_0} = \frac{2\pi \times 200 \times 0.2}{7.5}$$
$$R = 33.51\ \Omega$$

2. For series RLC resonant circuit,

$$V_C = \frac{V_S}{Z} \times (-jX_C);\ \text{where } Z = R + j\omega L - \frac{j}{\omega C}$$

for input source voltage, $V_S = 5\angle 45°$ V

$$V_C = \frac{(5\angle 45°)}{\left(R + j\omega L - \frac{j}{\omega C}\right)} \times \frac{1}{j\omega C}$$
$$V_C = \frac{(5\angle 45°)}{(1 - \omega^2 LC) + j\omega CR}$$
$$|V_C| = \frac{5}{\sqrt{(1 - \omega^2 LC)^2 + (\omega CR)^2}}$$

Put all the values i.e.,

$$\omega = (2\pi \times 300)\text{Hz}$$
$$L = 0.2\text{ H}$$
$$C = 3.16\ \mu\text{F}$$
$$R = 33.51\ \Omega$$

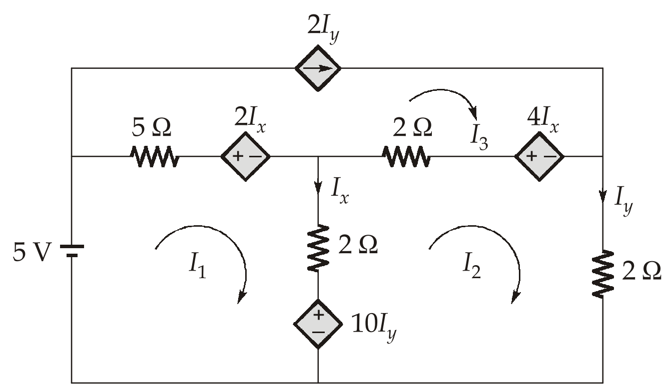
We get,

$$|V_C| = \frac{5}{\sqrt{(1-(2\pi\times 300)^2(0.2)(3.16\times 10^{-6}))^2 + ((2\pi\times 300)(3.16\times 10^{-6})(33.51))^2}}$$

$$|V_C| = 4.01\text{ Volt}$$

Q.8 (b) Solution:

(i) We have the circuit,



On applying KVL in loop 1 we get,

$$-5 + 5(I_1 - I_3) + 2I_x + 2I_x + 10I_y = 0$$

$$4I_x + 10I_y + 5I_1 - 5I_3 = 5 \quad \text{...(i)}$$

On applying KVL in loop 2, we get

$$2(I_2 - I_3) + 4I_x + 2I_y - 10I_y - 2I_x = 0$$

$$2I_x - 8I_y + 2I_2 - 2I_3 = 0 \quad \text{...(ii)}$$

Since,

$$I_2 = I_y \quad \text{...(iii)}$$

$$I_1 - I_2 = I_x \Rightarrow I_1 = I_2 + I_x$$

$$I_1 = I_x + I_y \quad \text{...(iv)}$$

$$I_3 = 2I_y \quad \text{...(v)}$$

Using equation (iv) and (v) in equation (i) we get,

$$4I_x + 10I_y + 5(I_x + I_y) - 5(2I_y) = 5$$

$$9I_x + 5I_y = 5 \quad \text{...(vi)}$$

Similarly, put equation (iii) and (v) in equation (ii) we get,

$$2I_x - 8I_y + 2I_y - 2(2I_y) = 0$$

$$2I_x - 10I_y = 0 \quad \text{...(vii)}$$

On solving equation (vi) and (vii), we get,

$$I_x = 0.5 \text{ A}$$

$$I_y = 0.1 \text{ A}$$

We have, $I_2 = I_y$; $I_1 = I_x + I_y$; $I_3 = 2I_y$

$I_2 = 0.1$ A ; $I_1 = 0.5 + 0.1$; $I_3 = 2(0.1)$

$I_1 = 0.6$A ; $I_3 = 0.2$ A

(ii) To find the maximum power that can be transferred to the load, we first obtain the Thevenin equivalent of the circuit.

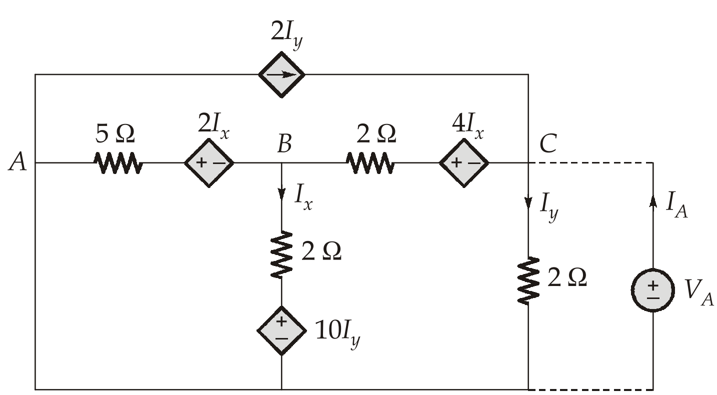
Step-I: Calculation of V_{Th}

⇒ We have $I_y = 0.1$ A

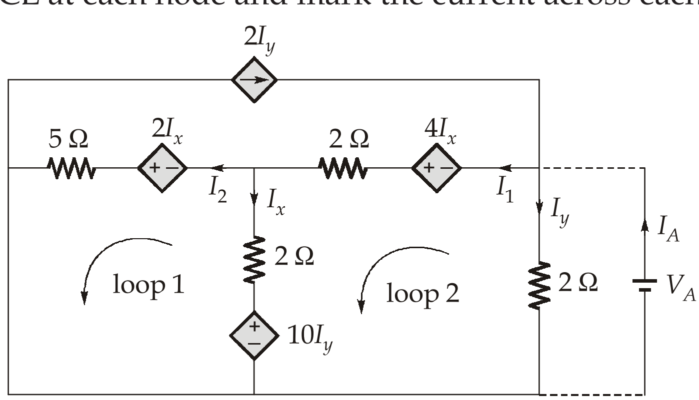
Therefore $V_{Th} = 0.1 \times 2 = 0.2$ Volt

Step-II: Calculation of Z_{th}

⇒ Deactivate the 5 V voltage source and redraw the circuit assuming a voltage source V_A as shown below:



Now apply KCL at each node and mark the current across each element i.e.,



where, $I_1 = I_A + I_y; \quad I_2 = I_A + I_y - I_x$

On applying KVL in loop 1, we get

$$-10I_y - 2I_x - 2I_x + 5I_2 = 0 \qquad \because I_2 = I_A + I_y - I_x$$

$$-10I_y - 2I_x - 2I_x + 5(I_A + I_y - I_x) = 0$$

$$5I_A - 5I_y - 9I_x = 0 \qquad ...(i)$$

Similarly KVL in loop 2 gives us,

$$-2I_y - 4I_x + 2I_1 + 2I_x + 10I_y = 0 \qquad \because I_1 = I_A + I_y$$

$$-2I_y - 4I_x + 2(I_A + I_y) + 2I_x + 10I_y = 0$$

$$2I_A - 2I_x + 10I_y = 0 \qquad ...(ii)$$

Since, $$I_y = \frac{V_A}{2} \qquad ...(iii)$$

Put equation (iii) in equation (ii) we get,

$$2I_A - 2I_x + 10\left(\frac{V_A}{2}\right) = 0$$

$$2I_A + 5V_A - 2I_x = 0$$

$$I_x = I_A + \frac{5V_A}{2} \qquad ...(iv)$$

Put equation (iii) and (iv) in equation (i) we get,

$$5I_A - 5\left(\frac{V_A}{2}\right) - 9\left(I_A + \frac{5V_A}{2}\right) = 0$$

$$5I_A - \frac{5V_A}{2} - 9I_A - \frac{45V_A}{2} = 0$$

$$-4I_A - 25V_A = 0$$

$$-4I_A = 25V_A$$

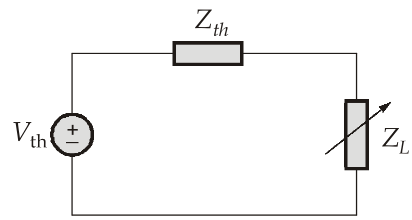
$$\frac{V_A}{I_A} = \frac{-4}{25}$$

$$\frac{V_A}{I_A} = -160 \text{ m}\Omega = Z_{Th}$$

Note :

- Whenever there is dependent source present in the circuit there is possibility to get negative resistance.
- Negative resistance show it is delivering power.

Step-III:



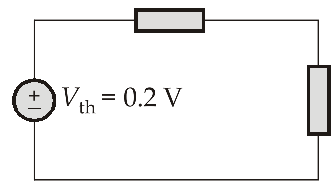
To get maximum power,

$$Z_L = Z_{th}^*$$

$$Z_L = -160 \text{ m}\Omega$$

and we have, $V_{th} = 0.2$ Volt

Redraw the circuit,



Here,
$$P_{max} = \frac{V_{th}^2}{4Z_{th}} = \frac{(0.2)^2}{4\times(-160\times10^{-3})} \text{W}$$

$$P_{max} = -0.0625 \text{ Watt}$$

Here, negative sign indicate, the load element is an active element delivering power.

Q.8 (c) Solution:

(i)
$$G(s)H(s) = \frac{10e^{-T_1 s}}{s(s+10)}$$

$$G(j\omega)H(j\omega) = \frac{10e^{-j\omega T_1}}{j\omega(j\omega+10)}$$

$$= \frac{10}{\omega\sqrt{\omega^2+100}} \angle -\omega T_1 - 90° - \tan^{-1}\left(\frac{\omega}{10}\right)$$

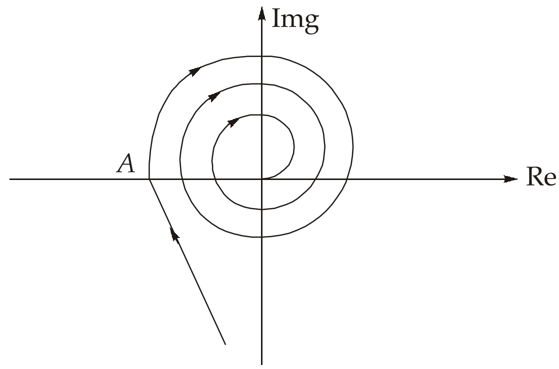
when,
$$\omega \to 0,\ G(j\omega)H(j\omega) = \infty \angle -90°$$

$$\omega = 1,\ G(j\omega)H(j\omega) = 0.995 \angle -T_1 - 95.71°$$

$$\omega = 10,\ G(j\omega)H(j\omega) = 0.0707 \angle -10T_1 - 135°$$

$$\omega \to \infty,\ G(j\omega)H(j\omega) = 0\angle -360°$$

For the given transfer function, a spiral polar plot is obtained as below:



The polar plot cuts the real axis at phase crossover frequency (ω_{pc}), where

$$-\omega T_1 - 90° - \tan^{-1}\left(\frac{\omega}{10}\right) = -180°$$

$$\omega T_1 = 90° - \tan^{-1}\left(\frac{\omega}{10}\right) \Rightarrow \tan(\omega T_1) = \frac{10}{\omega}$$

or $$\omega = 10 \cot \omega T_1 \quad \text{...(i)}$$

For the system to be marginaly stable, $\omega_{gc} = \omega_{pc}$. Hence, the polar plot must intersect at A, which must be ($-1 + j0$). At this point the magnitude of $G(j\omega)H(j\omega)$ is 1 i.e.

$$\frac{10}{(10\cot\omega T_1)\sqrt{100+100\cot^2\omega T_1}} = 1$$

$$100 \cot^2\omega T_1(100 + 100 \cot^2\omega T_1) = 100$$

$$\cot^4\omega T_1 + \cot^2\omega T_1 = 0.01$$

$$\cot^2\omega T_1 = \frac{-1}{2} \pm \sqrt{\frac{1}{4} + 0.01} = 0.01$$

$$\cot \omega T_1 = 0.1$$

$$\omega T_1 = \cot^{-1}(0.1)$$

$$\omega T_1 = 1.471 \text{ rad}$$

Using equation (i), $$T_1 = \frac{1.471}{10\cot\omega T_1} = \frac{1.471}{10 \times 0.1} = 1.471 \text{ sec}$$

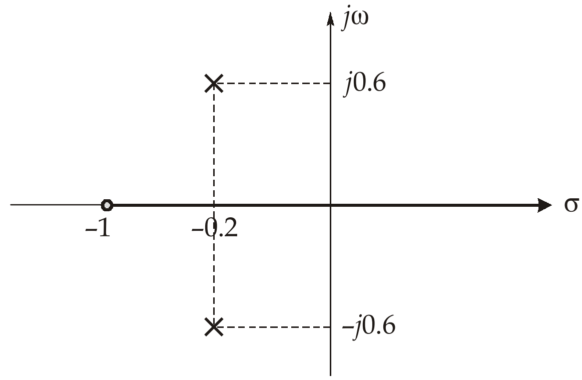
(ii) The open loop transfer function of the system is

$$G(s)H(s) = \frac{K(s+1)}{s^2 + 0.4s + 0.4}$$

Open loop zeros: $s = -1$

Open loop poles: $s = -0.2 \pm j0.6$

1. For positive feedback system, root locus exists on real axis where the number of open loop poles and zeros are even to the right of it.



The root locus exists on real axis from

$$s = -1 \text{ to } s = \infty$$

2. Number of asymptotes = $P - Z = 2 - 1 = 1$

$$\text{Centroid, } \sigma = \frac{\Sigma P - \Sigma Z}{P - Z} = \frac{-0.2 - 0.2 + 1}{1} = 1 - 0.4 = 0.6$$

$$\text{Angle of asymptote} = \frac{2K\pi}{P - Z};\ K = 0, 1, ...(P - Z - 1)$$

$$= 0$$

3. The characteristic equation for the system is

$$1 - \frac{K(s+1)}{s^2 + 0.4s + 0.4} = 0$$

$$s^2 + 0.4s + 0.4 - Ks - K = 0$$

$$K = +\frac{(s^2 + 0.4s + 0.4)}{(s+1)}$$

To find break point,

$$\frac{dK}{ds} = 0$$

$$(s + 1)(2s + 0.4) - (s^2 + 0.4s + 0.4) \times 1 = 0$$

$$2s^2 + 0.4s + 2s + 0.4 - s^2 - 0.4s - 0.4 = 0$$

$$s^2 + 2s = 0$$

$$s(s + 2) = 0$$

$$s = 0, -2$$

Since, the root locus exists on real axis from $s = -1$ to $s = \infty$. Therefore, $s = -2$ is not a valid breakpoint.

4. Intersection with imaginary axis:

Characteristic equation is $1 - G(s)H(s) = 0$

$s^2 + 0.4s + 0.4 - K(s + 1) = 0$

$s^2 + (0.4 - K)s + (0.4 - K) = 0$

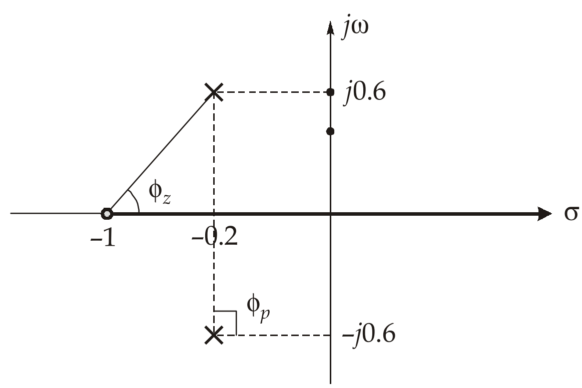
$$\begin{array}{c|cc} s^2 & 1 & 0.4-K \\ s^1 & 0.4-K & 0 \\ s^0 & \dfrac{(0.4-K)^2 - 0}{(0.4-K)} & \end{array}$$

For intersection with $j\omega$ axis,

$$0.4 - K = 0$$
$$K = 0.4$$

Auxiliary equation is

$$s^2 + (0.4 - K) = 0$$
$$s^2 = 0$$
$$s = 0$$



Angle of departure at $s = -0.2 + j0.6$

$$\phi_d = \phi_z - \phi_p$$

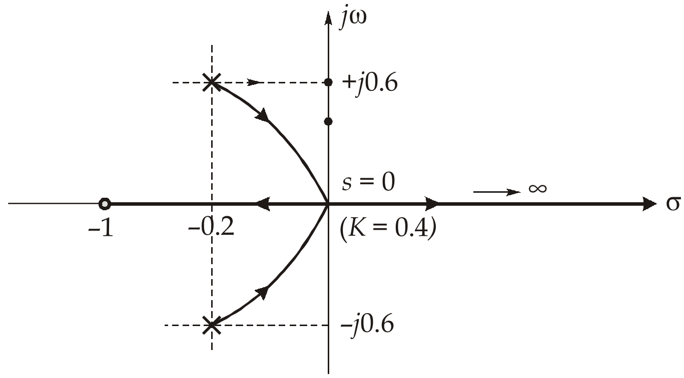
$$\phi_z = \tan^{-1}\left(\frac{0.6}{0.8}\right) = 36.87°$$

$$\phi_p = 90°$$

$$\phi_d = 36.87° - 90° = -53.13°$$

Angle of departure at $s = -0.2 - j0.6 = 53.13°$

Hence, the root locus can be plotted as below:



From, the root locus, it is seen that poles of the closed loop system exist on the left side of s plane only for $K < 0.4$. Hence, the system is stable for $0 < K < 0.4$.

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