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Detailed Solutions

ESE-2024 Mains Test Series

E & T Engineering Test No : 1

Section A : Network Theory

Q.1 (a) Solution:

Given: $Z_{11} = Ks$, $Z_{12} = Z_{21} = 10\ Ks$, $Z_{22} = 100\ Ks$

The transmission parameters for the given two-port network N can be obtained as

$$A = \frac{Z_{11}}{Z_{21}} = \frac{Ks}{10\ Ks} = 0.1$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{100K^2s^2 - 100K^2s^2}{10\ Ks} = 0$$

$$C = \frac{1}{Z_{21}} = \frac{1}{10\ Ks}$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{100\ Ks}{10\ Ks} = 10$$

$\therefore \quad A = 0.1, B = 0, C = \frac{1}{10\ Ks}, D = 10$

T parameters are defined by:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$\therefore \quad V_1 = 0.1\ V_2$

and $$I_1 = \frac{1}{10Ks}V_2 - 10I_2$$

We have, $$V_2 = -Z_L \times I_2 = -I_2 \qquad [\text{as } R = Z_L = 1\,\Omega]$$

$$V_1 = -0.1I_2$$

$\therefore$ $$I_1 = \frac{1}{10Ks}(-I_2) - 10I_2$$

$$I_1 = -I_2\left[\frac{1+100\,Ks}{10Ks}\right]$$

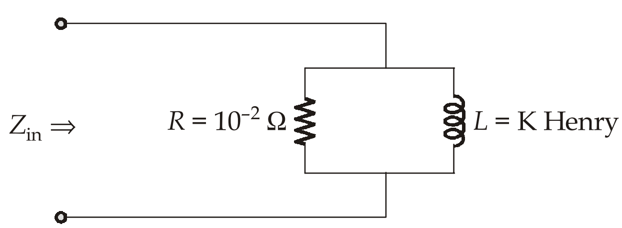
$\therefore$ $$Z_{in} = \frac{V_1}{I_1} = \frac{-0.1I_2}{-I_2\left(\frac{1+100\,Ks}{10Ks}\right)}$$

or $$Z_{in} = \frac{0.1\times 10\,Ks}{(1+100\,Ks)}$$

or, $$Z_{in} = \frac{Ks}{1+100Ks}$$

or, $$Z_{in} = \frac{1}{100+\frac{1}{Ks}} = \frac{1}{\frac{1}{R}+\frac{1}{Ls}}$$

Thus, the equivalent circuit is a parallel combination of a resistor and inductor as shown below:



For $K = 1$, $R = \frac{1}{100}\ \Omega$, $L = 1$ H

For $K = 10^6$, $R = \frac{1}{100}\ \Omega$, $L = 10^6$ H

Q.1 (b) Solution:

(i) For a magnetically coupled circuit.

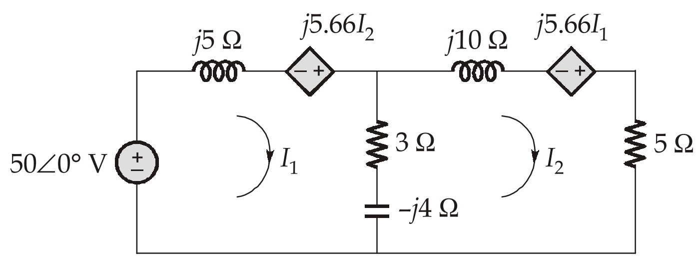
$$X_m = K\sqrt{X_{L1}X_{L2}}$$

$$X_m = 0.8\sqrt{5\times 10}$$

$$X_m = 5.66\ \Omega$$

The equivalent circuit in terms of dependent sources can be drawn as below using the dot convention:

- If a current enters a dotted terminal in one coil, then mutually induced voltage in other coil is positive at the dotted end.
- If a current leaves a dotted terminal in one coil, then mutually induced voltage in other coil is negative at the dotted end.



Applying KVL to Mesh 1,

$$50\angle 0° - j5I_1 - 3(I_1 - I_2) + j4(I_1 - I_2) + j5.66I_2 = 0$$

$$\therefore \quad 50\angle 0° = (3 + j)I_1 - (3 + j1.66)I_2$$

$$(3 + j1)I_1 + (-3 - j1.66)I_2 = 50\angle 0° \qquad ...(1)$$

Applying KVL to Mesh 2,

$$j4(I_2 - I_1) - 3(I_2 - I_1) - j10I_2 + j5.66I_1 - 5I_2 = 0$$

$$j4I_2 - j4I_1 - 3I_2 + 3I_1 - j10I_2 + j5.66I_1 - 5I_2 = 0$$

$$-j4I_2 + j4I_1 + 3I_2 - 3I_1 + j10I_2 - j5.66I_1 + 5I_2 = 0$$

$$(-3 - j1.66)I_1 + (8 + j6)I_2 = 0 \qquad ...(2)$$

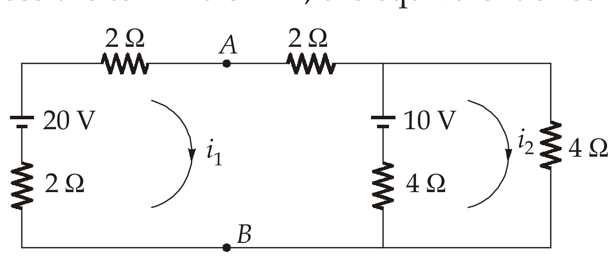
From equation (1) and (2), using Cramer's rule, we get

$$I_2 = \frac{\begin{vmatrix} 3 + j1 & 50\angle 0° \\ -3 - j1.66 & 0 \end{vmatrix}}{\begin{vmatrix} 3 + j1 & -3 - j1.66 \\ -3 - j1.66 & 8 + j6 \end{vmatrix}} = 8.62\angle -24.79 \text{ A}$$

The voltage across 5 Ω resistor is, thus

$$V = 5I_2 = 5(8.62\angle -24.79°) = 43.1\angle -24.79° \text{ V}$$

(ii) To obtain V_{th} across the terminals A-B, the equivalent circuit is given as



$$20 - 2i_1 - 2i_1 - 10 - 4(i_1 - i_2) - 2i_1 = 0$$
$$10 = 4i_1 + 4(i_1 - i_2) + 2i_1$$
$$10 = 10i_1 - 4i_2 \quad ...(1)$$
$$10 - 4(i_2 - i_1) - 4i_2 = 0$$
$$10 = 4(i_2 - i_1) + 4i_2$$
$$10 = 8i_2 - 4i_1 \quad ...(2)$$

On solving, we get $i_1 = 1.875 \text{ A}, i_2 = 2.1875 \text{ A}$

Now, $20 - 2i_1 - V_{AB} - 2i_1 = 0$

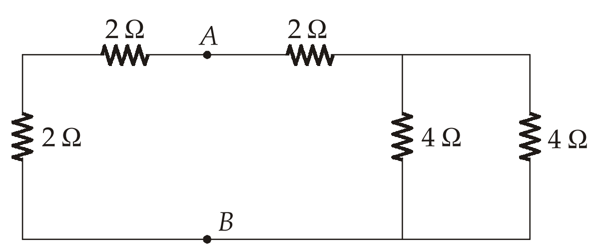
$$20 - 4i_1 = V_{AB}$$

$\therefore$ $V_{AB} = 20 - 4(1.875)$

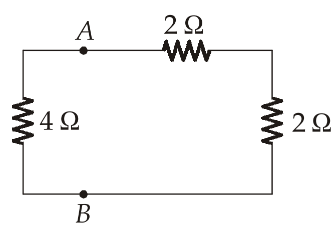
$$V_{AB} = 12.5 \text{ V}$$

Therefore, $V_{\text{th}} = 12.5 \text{ V}$

To calculate R_{th} or R_{eq}

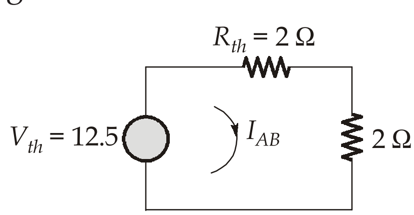


Equivalent circuit:



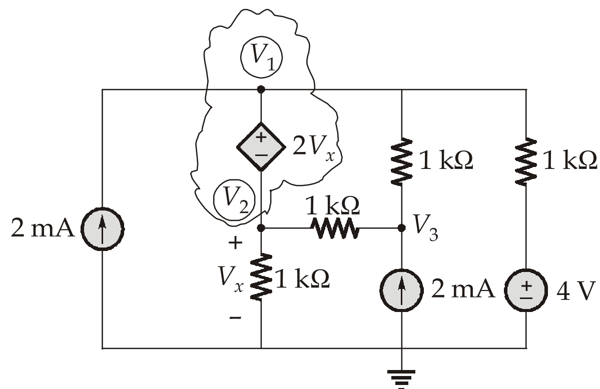
$\Rightarrow$ $R_{\text{th}} = 4 \parallel 4 = 2\ \Omega$

Therefore, current through the *AB* branch



$$I_{AB} = \frac{12.5}{4} = 3.125 \text{ A}$$

Q.1 (c) Solution:



Here, an ideal voltage source is located between two non-reference nodes, so it is considered to be supernode and KCL can be applied on the supernode. We have,

$$V_1 - V_2 = 2V_x$$

where, $$V_2 = V_x$$

and thus, $$V_1 = 3V_x$$

Applying KCL both nodes 1 and 2 (supernode) at a time,

$$2 \times 10^{-3} = \frac{V_1 - V_3}{1k} + \frac{V_1 - 4}{1k} + \frac{V_2 - V_3}{1k} + \frac{V_2}{1k}$$

$$4 + 2 = 2V_1 + 2V_2 - 2V_3$$

$$V_1 + V_2 - V_3 = 3$$

We have, $$V_1 = 3V_x,\ V_2 = V_x$$

Substituting in the above equation, we get

$$3V_x + V_x - V_3 = 3$$

$$4V_x - V_3 = 3 \quad \text{...(1)}$$

Applying KCL at node 3,

$$\frac{V_3 - V_1}{1k} + \frac{V_3 - V_2}{1k} = 2\text{ mA}$$

$$-V_1 + 2V_3 - V_2 = 2$$

Substitute $V_1 = 3V_x$; $V_2 = V_x$ in above eq.

$$-3V_x - V_x + 2V_3 = 2$$

$$-4V_x + 2V_3 = 2$$

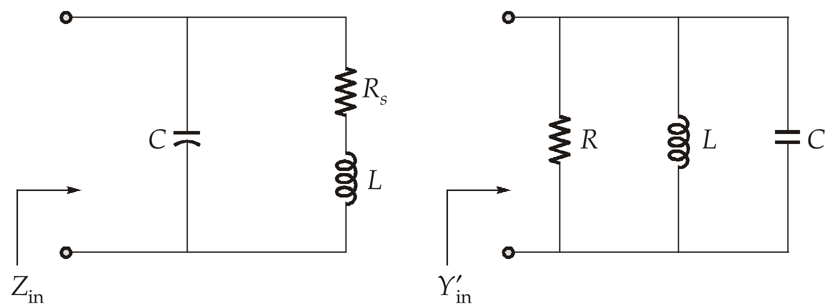
$$-2V_x + V_3 = 1 \quad \text{...(2)}$$

Solving the equation (1) and (2), we obtain

$$V_x = 2\text{ V};\ V_3 = 5\text{ V}$$
$$V_1 = 3V_x = 6\text{ V}$$
$$V_2 = V_x = 2\text{ V}$$

Q.1 (d) Solution:

Given circuit-I as



High Q-Coil Resonant Circuit   Approximated Circuit

$$Y'_{in} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$
$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

The frequency ω_0 at which resonance occurs is

$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

We get, $$Z'_{in} = \frac{j\omega L}{1 + j\omega L / R - \omega^2 LC}$$

For the original circuit, $$Z_{in} = \frac{(R_s + j\omega L)\left(\frac{1}{j\omega C}\right)}{R_s + j\omega L + \frac{1}{j\omega C}} = \frac{R_s + j\omega L}{1 + j\omega R_s C - \omega^2 LC}$$

$$= \omega L \times \left[\frac{\frac{R_s}{\omega L} + j}{1 + j\omega R_s C - \omega^2 LC}\right] \quad \text{...(i)}$$

$$Q\text{-of the coil,} = \frac{\omega L}{R_s}$$

therefore, equation (i) becomes,

$$Z_{in} = \omega L \times \left[\frac{\frac{1}{Q} + j}{1 + j\omega R_s C - \omega^2 LC}\right]$$

Since Q-is very high,

$$Z_{in} = \frac{j\omega L}{1 + j\omega R_s C - \omega^2 LC}$$

$$= \frac{1}{\frac{R_s C}{L} + j\left(\omega C - \frac{1}{\omega L}\right)} = \frac{\frac{R_s C}{L} - j\left(\omega C - \frac{1}{\omega L}\right)}{\left[\left(\frac{R_s C}{L}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2\right]}$$

At resonance frequency ω_0, imaginary part is zero.

$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$\therefore \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Since resonant frequency ω_0 of the circuit (2) for high value of Q is same as that of circuit (1). Hence, circuit (2) can be approximated as circuit (1) for high Q with

$$R = \frac{L}{R_s C}$$

For the given circuit, $$Q = \frac{\omega_0 L}{R}$$

where $$\omega_0 = \frac{1}{\sqrt{LC}} \text{ for high } Q$$

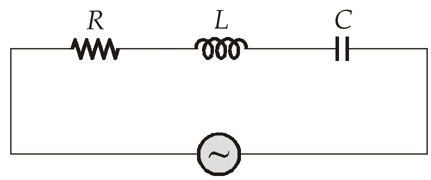
$$\omega_0^2 = \frac{1}{LC}$$

Hence, $$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi \times 10^6)^2 \times 25 \times 10^{-3}} = 1.013 \text{ pF}$$

and $$Q = \frac{\omega_0 L}{R} = \frac{2\pi \times 10^6 \times 25 \times 10^{-3}}{10} \approx 15707$$

Q.1 (e) Solution:

Given, $R = 25\ \Omega;\quad L = 2\text{ H};\quad C = 30\ \mu\text{F}$



Impedance of the above circuit,

$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\text{Phase angle, } \theta = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

(i) For phase angle 45° lagging,

$$\theta = 45^\circ \text{ (current lags voltage by 45°)}$$

$$\tan(45^\circ) = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} = \frac{2\omega - \frac{10^6}{30\omega}}{25}$$

$$25 = 2\omega - \frac{10^6}{30\omega}$$

$$750\ \omega = 60\ \omega^2 - 10^6$$

$$\therefore \quad 60\omega^2 - 750\ \omega - 10^6 = 0$$

On solving for roots,

$$\omega_1 = -123 \text{ rad/sec}$$

$$\omega_2 = 135.5 \text{ rad/sec}$$

Consider only positive value,

$$\therefore \quad f_2 = \frac{135.5}{2\pi} = 21.56 \text{ Hz}$$

$\therefore$ for frequency of 21.56 Hz, phase angle of the circuit is 45° lagging.

(ii) For phase angle leading (or) $\theta = -45°$ (current leads voltage by 45°)

$$\tan(-45°) = \frac{2\omega - \dfrac{1}{\omega \times 30 \times 10^{-6}}}{25}$$

$$-25 = 2\omega - \frac{1}{\omega \times 30 \times 10^{-6}}$$

$$-25 \times \omega \times 30 = 2 \times 30 \times \omega^2 - 10^6$$

$$60\,\omega^2 + 750\,\omega - 10^6 = 0$$

On solving for the roots,

$$\omega_1 = 123 \text{ rad/sec}$$

$$\omega_2 = -135.5 \text{ rad/sec}$$

Consider only positive value,

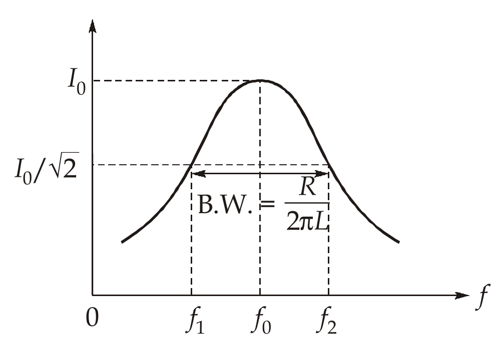
$$\therefore \quad f = \frac{123}{2\pi} = 19.57 \text{ Hz}$$

$\therefore$ For frequency of 19.57 Hz, the phase angle of the circuit is 45° leading.

Alternate Solution

The phase angle at half power-points is 45°. The power factor is leading at the lower half-power point of frequency f_1 and is lagging at the upper half-power point of frequency f_2.

We have,



Resonant frequency,
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2 \times 30 \times 10^{-6}}}$$

$$f_0 = 20.55 \text{ Hz}$$

(i) The phase angle of the circuit will be 45° lagging at

$$f_2 = f_0 + \frac{B.W}{2} = f_0 + \frac{R}{4\pi L}$$

$$\Rightarrow \qquad f_2 = 20.55 + \frac{25}{4\pi \times 2}$$

$$\Rightarrow \qquad f_2 = 21.55 \text{ Hz}$$

(ii) The phase angle of the circuit will be 45° leading at

$$f_1 = f_0 - \frac{B.W}{2} = f_0 - \frac{R}{4\pi L}$$

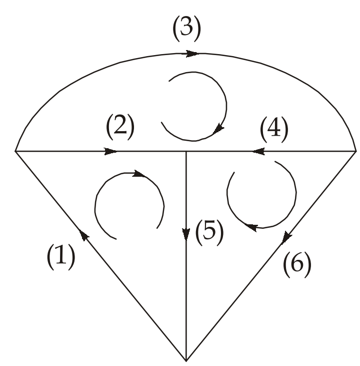
$$f_1 = 20.55 - \frac{25}{4\pi \times 2}$$

$$f_1 = 19.56 \text{ Hz}$$

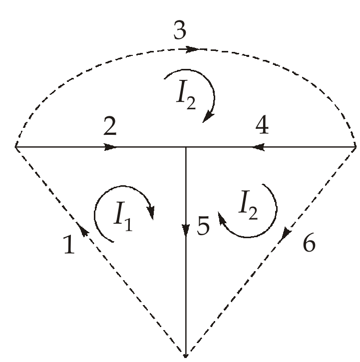
Q.2 (a) Solution:

Graph of the network:

Considering the voltage source as short-circuit and the current source as open-circuit, the graph of the given circuit can be drawn as below:



Selecting the branches 2, 4, 5, the tree of the graph is obtained as below:



Taking one link at a time and thus, forming a loop and taking the orientation of the link as the orientation of the loop we have B_f as follows:

Tie Set Matrix [B_f]:

f tie set	Branches					
	1	**2**	**3**	**4**	**5**	**6**
[1, 2, 5]	1	1	0	0	1	0
[2, 3, 4]	0	–1	1	1	0	0
[4, 5, 6]	0	0	0	–1	–1	1

Loops Branches

$$B_f = \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

Now, branch impedance matrix Z_b:

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix}$$

$$Z_b = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Hence, the loop impedance matrix,

$$Z = B_f Z_b B_f^T$$

$$Z = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Z = \begin{bmatrix} 2 & 5 & 0 & 0 & -j4 & 0 \\ 0 & -5 & 5 & j5 & 0 & 0 \\ 0 & 0 & 0 & -j5 & j4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Z = \begin{bmatrix} 7-j4 & -5 & j4 \\ -5 & 10+j5 & -j5 \\ j4 & -j5 & 2+j \end{bmatrix} = B_f Z_b B_f^T$$

The equilibrium mesh equation in matrix form is given by

$$[B_f][Z_b][B_f]^T[I_l] = [B_f][[V_S]-[Z_b][I_S]] \quad \text{...(1)}$$

where,
$[B_f]$ = Tie set matrix
$[Z_b]$ = branch impedance matrix
$[I_l]$ = link current matrix = $\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$
$[I_s]$ = Source current matrix
$[V_s]$ = Source voltage matrix

We have, $Z = [B_f][Z_b][B_f]^T$

$$Z = \begin{bmatrix} 7-j4 & -5 & j4 \\ -5 & 10+j5 & -j5 \\ j4 & -j5 & 2+j \end{bmatrix}$$

The overall network equation will be

$$ZI_l = E \quad \text{where} \quad E = B_f[V_s - Z_b I_s]$$

$$E = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix} \left[\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right]$$

$$E = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$$

Hence, the final equations are:

$$\begin{bmatrix} 7-j4 & -5 & j4 \\ -5 & 10+j5 & -j5 \\ j4 & -j5 & 2+j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$$

$$(7-4j)I_1 - 5I_2 + j4I_3 = 10 \quad ...(2)$$

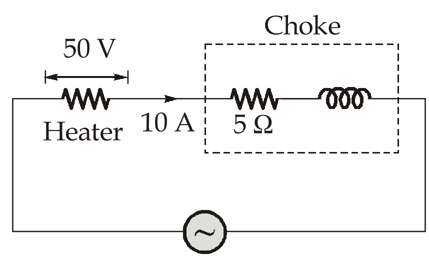
$$-5I_1 + (10+j5)I_2 - j5I_3 = 10 \quad ...(3)$$

$$j4I_1 - j5I_2 + (2+j1)I_3 = 0 \quad ...(4)$$

2, 3 and 4 are the loop equations.

Q.2 (b) Solution:

(i) It is given that $I = 10$ A, V_H (drop across heater) = 50 V, R_C (resistance of choke) = 5 Ω and V_S = 200 V.



$\therefore$ R_H (Resistance of heater) = $\dfrac{50}{10} = 5\ \Omega$

Hence, net resistance of the circuit is

$$R = R_H + R_C = 5 + 5 = 10\ \Omega$$

Obviously, V_{R_H} (drop across R_H) = $R_H \times I$ = 50 V

and $$V_{R_C} = R_C \times I = 5 \times 10 = 50 \text{ V}$$

Also, V_{R_L} (drop across X_L, the inductive reactance of the choke)

$$V_L = IX_L = 10X_L \text{ V}$$

However, the supply voltage being the vector sum of the drops V_R and V_L, we have

$$V = (V_{R_C} + V_{R_H}) + jV_L$$

$\therefore$ $$(200)^2 = (50+50)^2 + V_L^2$$

$\therefore$ $$V_L^2 = (200)^2 - (100)^2 \Rightarrow V_L = 173.21 \text{ V}$$

or, $$X_L = \frac{173.21}{I} = \frac{173.21}{10} = 17.32\ \Omega$$

This gives the impedance of the choke as

$$Z = \sqrt{R_C^2 + X_L^2}$$

$$= \sqrt{(5)^2 + (17.32)^2} = 18.027\ \Omega$$

Thus, the impedance of the coil = 18.02 Ω

∴ The impedance of the whole circuit

$$= \sqrt{(R_C + R_H)^2 + X_L^2}$$

$$= \sqrt{(10)^2 + (17.32)^2} = 20\ \Omega$$

$$\text{Power factor} = \frac{R}{Z} = \frac{10}{20} = 0.5$$

Thus, for the given problem

Impedance of the choke = 18 Ω, P.F = 0.5

(ii) Maximum power transfer theorem:

A resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.

Let R_L be the resistance that is to be connected across *x*-*y* for maximum power transfer from source to load. As per maximum power transfer theorem, R_L should be equal to the Thevenin resistance (R_{th}) across the terminals *x*-*y*. To find R_{th}, all the sources are deactivated i.e. the current source is open-circuited and the voltage source is short-circuited.

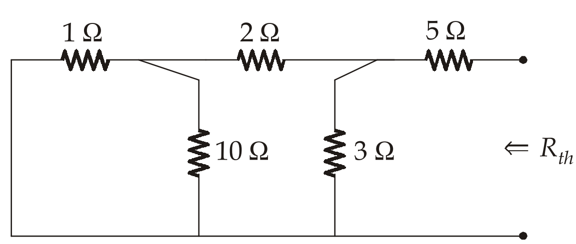


fig. (b)

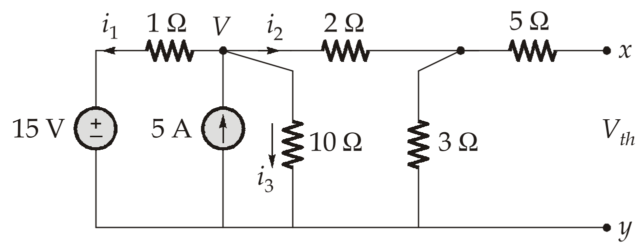
Here, $$R_{th} = \{[(1 \parallel 10) + 2] \parallel 3\} + 5$$

$$R_{th} = 6.48\ \Omega$$

Thus, the load resistance (R_L) must be having a value of 6.48 Ω for the maximum power transfer.

Next, the open circuit voltage (Thevenin voltage) across *x*-*y* in fig. (a) is to be calculated. With reference to fig. (c), KCL at node (1), gives

$$i_1 + i_2 + i_3 = 5$$



Assuming voltage at node (1) to be V.

or, $$\frac{V-15}{1} + \frac{V}{10} + \frac{V-V_{th}}{2} = 5 \quad ...(1)$$

also, $$\frac{V_{th}}{3} + \frac{V_{th}-V}{2} = 0 \quad ...(2)$$

On solving equations (1) and (2), we get

$$V_{th} = 9.23 \text{ V}$$

Amount of maximum power transfer is given by

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(9.23)^2}{4\times(6.48)} = 3.29 \text{ W}$$

Hence, for the given problem,

Resistance to be connected between terminal *x*-*y* for maximum power transfer is 6.48 Ω

Amount of maximum power is 3.29 W.

Q.2 (c) Solution:

(i) **1.** Given that $Pf = \cos\theta = 0.856$,

We obtain the power angle as

$$\theta = \cos^{-1}(0.856) = 31.12°$$

Given the apparent power $S = 12$ kVA,

then the average power or real power is

$$P = S\cos\theta$$

$$P = 12\times10^3\times0.856$$

$$P = 10.272 \text{ kW}$$

While the reactive power is

$$Q = S\sin\theta = 12\times10^3\times0.517$$
$$Q = 6.202\text{ kVAR}$$

2. Since the *Pf* is lagging, the complex power is

$$S = P + jQ$$
$$S = 10.272 + j6.202\text{ kVA}$$

from, $S = V_{rms}I^*_{rms}$, we obtain

$$I^*_{rms} = \frac{S}{V_{rms}} = \frac{(10.272 + j6.202)\text{ kVA}}{120\angle0^\circ}$$
$$I^*_{rms} = 85.6 + j51.7\text{ A} = 100\angle31.13^\circ\text{ A}$$
$$I_{\text{rms}} = 100\angle{-31.13^\circ}\text{ A}$$

Thus, peak current, $I_m = \sqrt{2}\,I_{rms}$

$$I_m = \sqrt{2}(100)$$
$$I_m = 141.42\text{ A}$$

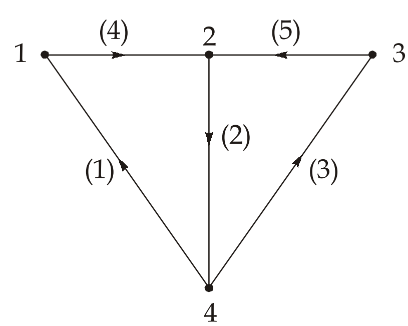
3. the load impedance

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{120\angle0^\circ}{100\angle-31.13^\circ} = 1.2\angle31.13^\circ\ \Omega$$

which is an inductive impedance.

(ii) 1. Graph of the network:

Considering the current sources as open circuit, the graph of the given circuit can be drawn as below:

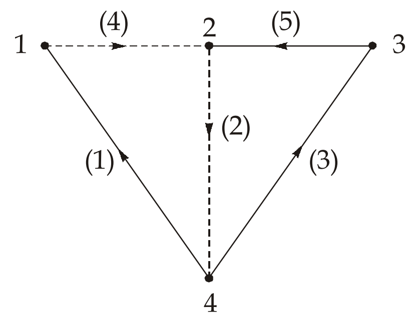


The graph has 4 nodes and 5 branches. Assume the orientations of the branches as shown in the graph.

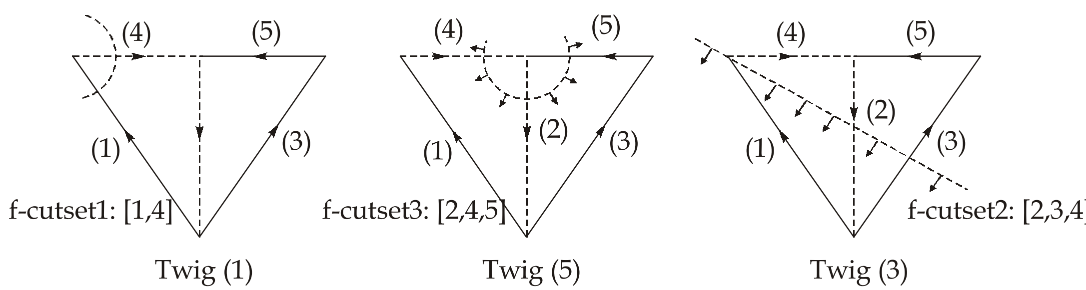
The incidence matrix is given by

$$A = \begin{array}{c} \text{Nodes} \\ \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \text{Branches} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Let us select a tree as shown below with branches 1, 3 and 5 as twigs.



For each twig, the cut--sets are obtained as below:



The cut-set matrix is obtained as follows:

$$Q_f = \begin{array}{c} \text{f-cutset} \\ \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \text{Branches} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

2. The reduced incidence matrix can be written as

$$[A_a] = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

The number of possible trees is given by

$$N = \det\{[A_a]\,[A_a]^T\}$$

$$N = \det\left\{\begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\right\}$$

$$N = \det\left\{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}\right\} = 8$$

$\therefore$ The number of possible trees is 8.

Q.3 (a) Solution:

Given: $$Z = \begin{bmatrix} 4 & 1.5 \\ 10 & 3 \end{bmatrix}$$

$\therefore$ $$V_1 = 4I_1 + 1.5I_2 \quad ...(1)$$

$$V_2 = 10I_1 + 3I_2 \quad ...(2)$$

also, $$V_s - 5I_1 - V_1 = 0$$

$$V_s = 5I_1 + V_1 \quad ...(3)$$

$$V_2 = -2I_2 \quad ...(4)$$

From equation (2) and (4), we get

$$I_2 = -2I_1 \quad ...(5)$$

$\therefore$ $$\frac{I_2}{I_1} = G_I = -2$$

Divide equation (2) by (1)

i.e., $$\frac{V_2}{V_1} = G_V = \frac{10I_1 + 3I_2}{4I_1 + 1.5I_2}$$

$$G_V = \frac{10I_1 + 3(-2I_1)}{4I_1 + 1.5(-2I_1)} \quad \text{.. using equation (5)}$$

$$= \frac{10-6}{4-3} = 4 \quad ...(6)$$

from equation (5) and (6), we get

$$G_P = |G_V \times G_I|$$

$$G_P = |4 \times (-2)| = 8$$

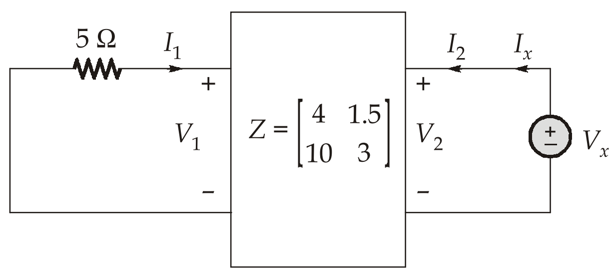
Input Impedance, $$Z_{in} = \frac{V_1}{I_1} = \frac{4I_1 + 1.5I_2}{I_1} = \frac{4I_1 + 1.5(-2)I_1}{I_1}$$

[using equation (5)]

$$Z_{in} = \frac{(4-3)I_1}{I_1} = 1\ \Omega$$

Now, to calculate Z_{out}, short circuit V_s and remove 2 Ω resistor and apply a voltage source V_x providing current I_x. We get, $Z_{out} = \frac{V_x}{I_x}$.

The circuit diagram becomes



$$V_1 = -5I_1 \quad ...(7)$$

$$I_2 = I_x \quad ...(8)$$

and, $$V_2 = V_x \quad ...(9)$$

from equation (1),

$$V_1 = 4I_1 + 1.5I_2$$

$$-5I_1 = 4I_1 + 1.5I_x \quad \text{[using equation (7)]}$$

$$-9I_1 = 1.5I_x$$

∴ $$I_1 = -\frac{1.5}{9}I_x \quad ...(10)$$

from equation (2),

$$V_2 = 10I_1 + 3I_2$$

$$V_x = 10\left(\frac{-1.5I_x}{9}\right) + 3I_x$$

$$V_x = \frac{-15}{9}I_x + 3I_x$$

$$V_x = \frac{-5}{3}I_x + 3I_x = \frac{4I_x}{3}$$

∴ $$\frac{V_x}{I_x} = Z_{out} = 1.33\ \Omega$$

Q.3 (b) Solution:

(i) Let us redraw the circuit

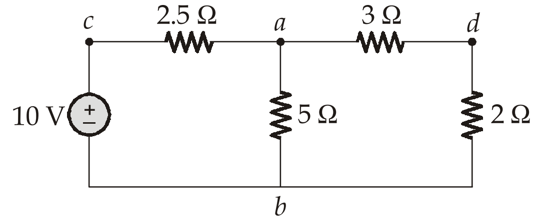


fig (a)

The Thevenin equivalent circuit across the terminals a-b can be drawn as below:

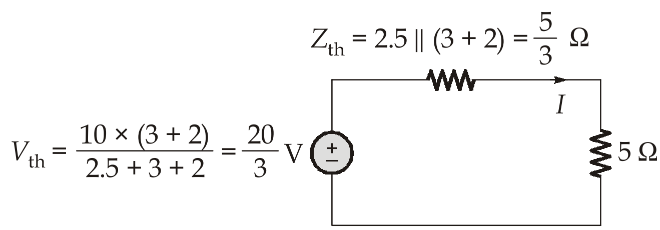


fig (b)

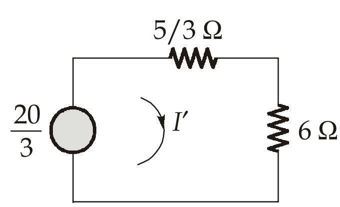


fig. (c)

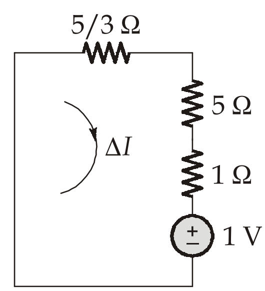


fig. (d)

The current in 5 Ω resistor before the change takes place is

$$I = \frac{\frac{20}{3}}{\frac{5}{3} + 5} = 1\text{ A}$$

Now, when the resistance changes to 6 Ω,

$$I' = \frac{\frac{20}{3}}{\frac{5}{3} + 6} = \frac{\frac{20}{3}}{\frac{23}{3}} = \frac{20}{23}\text{A}$$

$$\Delta I = I' - I = \frac{-3}{23}\text{A}$$

Therefore compensation source voltage is $I\ \Delta Z$

$$= 1 \times 1 = 1 \text{ Volt}$$

The polarity of this source is as shown, as ΔI is negative. If ΔI were positive, the polarity of the source will be opposite to what is given in figure.

Now let us superimpose the current due to compensation source on various branches of the network and find out the branch current and verify the result by solving the original network with 6 Ω resistance rather than 5 Ω. Initially the current through 5 Ω branch is from *a* to *b* and with the current ΔI, from *b* to *a*, the new current in branch *ab* is $1 - \frac{3}{23} = \frac{20}{23}$ A.

Now ΔI current to be distributed among the parallel combination of 2.5 Ω and 5 Ω.

Current through 2.5 Ω branch = $\frac{3}{23} \times \frac{5}{7.5} = \frac{2}{23}$ A

From *a* to *c* net current,

$$2 - \frac{2}{23} = \frac{44}{23}\text{ A}$$

Current through (3 + 2) Ω branch,

$$\frac{3}{23} \times \frac{2.5}{7.5} = \frac{1}{23}\text{ A}$$

With direction *a* to *d* i.e., the net current

$$1 + \frac{1}{23} = \frac{24}{23}\text{ A}$$

The obtained results can be verified by taking 6 Ω resistance in the original network across branch *ab*.

$$\text{Net input resistance} = \frac{5}{2} + \frac{5 \times 6}{11}$$

$$= \frac{5}{2} + \frac{30}{11} = \frac{115}{22}\,\Omega$$

Current through 2.5 Ω resistance = $\frac{10 \times 22}{115} = \frac{44}{23}$ A

Current through 6 Ω branch $\frac{44}{23} \times \frac{5}{11} = \frac{20}{23}$ A

Current through (3 + 2) branch $\frac{44}{23} \times \frac{6}{11} = \frac{24}{23}$ A

Hence, the results are verified.

(ii) 1. The voltage drop across r_1 is given by

$$V_{r1} = Ir_1 = 10 \times 8.2 = 82 \text{ V}$$

Voltage drop across L_1 is given by

$$V_{L1} = IX_{L1} = 10 \times (2\pi \times 25) \times 0.01$$

$$V_{L1} = 15.70 \text{ V}$$

Voltage drop across r_2 is given by

$$V_{r2} = Ir_2 = 10 \times 2.7 = 27 \text{ V}$$

Voltage drop across L_2

$$V_{L2} = IX_{L2} = 10(2\pi \times 25) \times 0.03$$

$$V_{L2} = 47.12 \text{ V}$$

Voltage drop across r_1 and L_1 (branch-1)

$$= V_1 = I\sqrt{r_1^2 + X_{L1}^2}$$

$$= 10\sqrt{(8.2)^2 + (2\pi \times 25 \times 0.01)^2}$$

$$= 83.5 \text{ V}$$

Similarly, voltage drop across r_2 and L_2 (branch-2)

$$= I\sqrt{r_2^2 + X_{L2}^2}$$

$$= 10\sqrt{(2.7)^2 + (2\pi \times 25 \times 0.03)^2}$$

$$= 54.31 \text{ V}$$

2. Total resistive voltage drop

$$V_R = V_{r1} + V_{r2} = 82 + 27 = 109 \text{ V}$$

Total inductive voltage drop

$$V_L = V_{L1} + V_{L2} = 15.7 + 47.1 = 62.8 \text{ V}$$

3. Supply voltage

$$V_0 = \sqrt{V_R^2 + V_L^2}$$

$$V_0 = \sqrt{(109)^2 + (62.8)^2}$$

$$V_0 = 125.8 \text{ V}$$

4. θ_1 = impedance angle of branch-1

$$= \tan^{-1}\left(\frac{\omega L_1}{r_1}\right) = 10°50' = 10.83°$$

$$\theta_2 = \text{impedance angle of branch-2}$$

$$= \tan^{-1}\left(\frac{\omega L_2}{r_2}\right) = 60°11' = 60.19°$$

and

$$\text{P.F angle of the circuit} = \theta_0 = \tan^{-1}\left(\frac{X_{L1} + X_{L2}}{r_1 + r_2}\right) = 30°$$

$$\text{P.F} = \cos 30° = 0.866$$

Q.3 (c) Solution:

(i) Applying KVL in the left most loop,

$$V_0 - 100\, I_0 - 20(I_0 - I_1) = 0$$

$$V_0 = 100 I_0 + 20(I_0 - I_1)$$

where, $I_1 = -10 I_0$

$$V_0 = 100 I_0 + 20(I_0 + 10 I_0)$$

$$V_0 = 100\, I_0 + 220\, I_0$$

$$I_0 = \frac{V_0}{320}\,\text{A}$$

Also, $I_1 = -10 I_0 = \dfrac{-10 V_0}{320} = \dfrac{-V_0}{32}$

From the theory of ideal transformer,

$$I_2 = \frac{-I_1}{n} = \frac{-I_1}{\left(\frac{1}{10}\right)} = +\frac{V_0}{32}(10) = \frac{10 V_0}{32}$$

Thus, $V_2 = -R_L I_2 = -10 \times \left(\dfrac{10 V_0}{32}\right) = \dfrac{-100}{32} V_0$

or $\text{Voltage gain} = \dfrac{V_2}{V_0} = \dfrac{-100}{32} = -3.125$

To find power gain, first let us calculate the power supplied (instantaneous) by the independent voltage source (V_0)

$\therefore$ $P_0 = V_0 I_0 = V_0 \times \dfrac{V_0}{320} = \dfrac{V_0^2}{320}$

Power absorbed by the load resistor (R_L) is

$$P_2 = \frac{V_2^2}{10} = \frac{\left(-\frac{100}{32}V_0\right)^2}{10} = \frac{10^3 V_0^2}{1024}$$

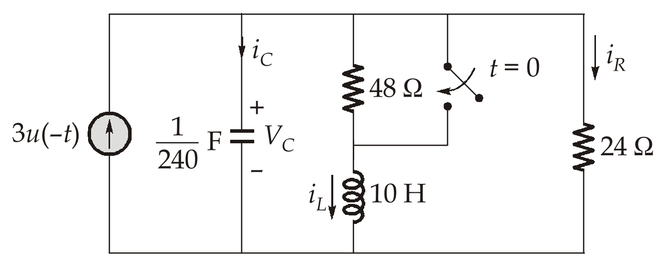
$$\therefore \qquad \text{Power gain} = \frac{P_2}{P_0} = \frac{10^3}{1024} \times 320 = 312.5$$

So, for the given problem

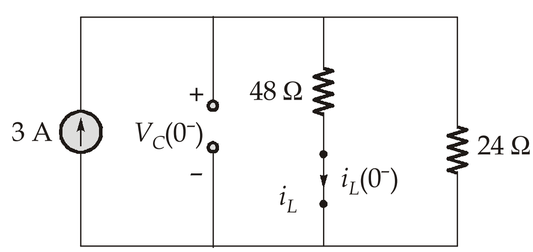
Voltage gain = –3.125

and Power gain = 312.5

(ii) Given circuit



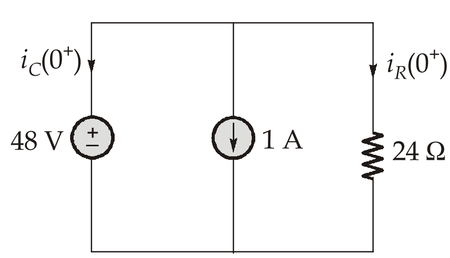
Before $t = 0$, a current source of 3A is connected and the switch is open. At steady state i.e. at $t = 0^-$, capacitor acts as open circuit and inductor acts as short circuit. The circuit in the steady state for $t < 0$ can be drawn as below:



1. $i_L(0^-) = \frac{3 \times 24}{48 + 24} = 1\text{ A} = i_L(0^+)$

2. $V_C(0^-) = 48 \times i_L(0^-) = 48 \times 1 = 48\text{ V} = V_C(0^+)$

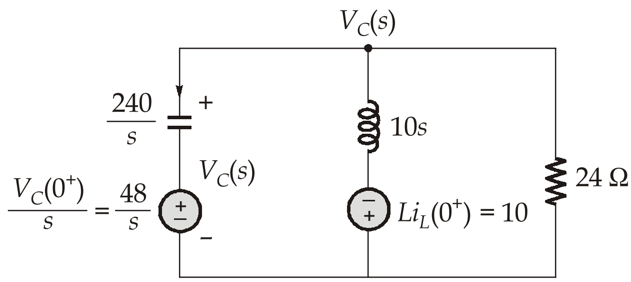
At $t > 0$, the switch is closed and the step current source is deactivated (open). The circuit at $t = 0^+$ can be drawn as below:



3. $i_R(0^+) = \dfrac{48}{24} = 2\text{ A}$

4. Using KCL, $i_C(0^+) = -1 - 2 = -3\text{ A}$

5. To calculate $V_C(t)$, the circuit in s-domain for $t > 0$ is drawn as below:



by nodal analysis,

$$\frac{V_C(s) - \frac{48}{s}}{\frac{240}{s}} + \frac{V_C(s) + 10}{10s} + \frac{V_C(s)}{24} = 0$$

$$V_C(s)\left[\frac{s}{240} + \frac{1}{10s} + \frac{1}{24}\right] = \frac{1}{5} - \frac{1}{s}$$

$$V_C(s)\left[\frac{s^2 + 24 + 10s}{240s}\right] = \frac{s-5}{5s}$$

$$V_C(s) = \frac{48s - 240}{s^2 + 10s + 24} = \frac{48(s-5)}{(s+4)(s+6)}$$

By using partial fraction expansion

$$\therefore \quad \frac{48(s-5)}{(s+4)(s+6)} = \frac{A}{s+4} + \frac{B}{s+6}$$

$$A = \left.\frac{48(s-5)}{s+6}\right|_{s=-4} = \frac{48(-4-5)}{2} = -216$$

$$B = \left.\frac{48(s-5)}{s+4}\right|_{s=-6} = \frac{48(-6-5)}{-2} = 264$$

$$\therefore \quad V_C(t) = -216e^{-4t} + 264e^{-6t}\ ; \quad t > 0$$

at $t = 0.2$,

$$V_C(0.2) = -216e^{-0.8} + 264\, e^{-1.2}$$

$$\therefore \quad V_C(0.2) = -17.54\text{ V}$$

Q.4 (a) Solution:

Given; $$Y(s) = \frac{(s^2+1)(s^2+5)}{s(s^2+3)}$$

Cauer-I form: The Cauer-I form is obtained by continued fraction expansion of the network function about the pole at infinity. Hence, the continued fractione expansion is performed on $Y(s)$. We have,

$$Z(s) = \frac{1}{Y(s)}$$

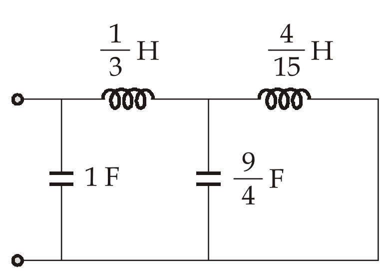
$$Z(s) = \frac{s(s^2+3)}{(s^2+1)(s^2+5)} = 0 + \frac{1}{\left(\dfrac{s^4+6s^2+5}{s^3+3s}\right)}$$

$$s^3+3s\,\big)\,s^4+6s^2+5\,\big(\,s = Y_2$$
$$\underline{-\,s^4 - 3s^2}$$
$$3s^2+5\,\big)\,s^3+3s\,\big(\,\frac{s}{3} = Z_3$$
$$\underline{-\,s^3 - \frac{5}{3}s}$$
$$\frac{4}{3}s\,\big)\,3s^2+5\,\big(\,\frac{9}{4}s = Y_4$$
$$\underline{-\,3s^2}$$
$$5\,\big)\,\frac{4}{3}s\,\big(\,\frac{4}{15}s = Z_5$$
$$\underline{\frac{4}{3}s}$$
$$0$$

$\therefore$
$$Z(s) = 0 + \cfrac{1}{s + \cfrac{1}{\cfrac{s}{3} + \cfrac{1}{\cfrac{9}{4}s + \cfrac{1}{\cfrac{4}{15}s}}}}$$

where $Z_1 = 0$, $Y_2 = s$, $Z_3 = \frac{s}{3}$, $Y_4 = \frac{9}{4}s$, $Z_5 = \frac{4}{15}s$.

$\therefore$ The Cauer-I form:



Cauer-II form: To obtain Cauer-II Form, the polynomials are arranged in ascending power of s. The continued fraction expansion is done about the pole at the origin. $Z(s)$ has no pole at origin. So, we consider $Y(s)$.

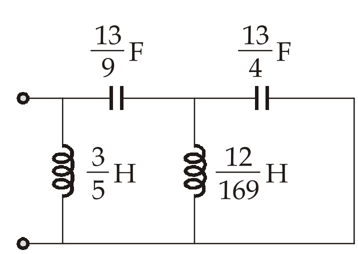
Given,
$$Z(s) = \frac{3s+s^3}{5+6s^2+s^4} = \frac{1}{\dfrac{5+6s^2+s^4}{3s+s^3}} = \frac{1}{Y(s)}$$

Performing continued fraction expansion on $Y(s)$,

$$3s+s^3\,\overline{)\,5+6s^2+s^4\,}\left(\frac{5}{3s} = Y_2\right.$$
$$\underline{5+\frac{5}{3}s^2}$$
$$\frac{13}{3}s^2+s^4\,\overline{)\,3s+s^3\,}\left(\frac{9}{13s} = Z_3\right.$$
$$\underline{3s+\frac{9}{13}s^3}$$
$$\frac{4}{13}s^3\,\overline{)\,\frac{13}{3}s^2+s^4\,}\left(\frac{169}{12s} = Y_4\right.$$
$$\underline{\frac{13}{3}s^2}$$
$$s^4\,\overline{)\,\frac{4}{13}s^3\,}\left(\frac{4}{13s} = Z_5\right.$$
$$\underline{\frac{4}{13}s^3}$$
$$\underline{0}$$

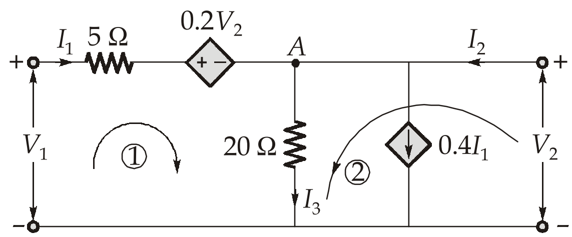
We obtain,
$$Z(s) = 0 + \cfrac{1}{\cfrac{5}{3s} + \cfrac{1}{\cfrac{9}{13s} + \cfrac{1}{\cfrac{169}{12s} + \cfrac{1}{\cfrac{4}{13s}}}}}$$
where $Z_1 = 0$, $Y_2 = \frac{5}{3s}$, $Z_3 = \frac{9}{13s}$, $Y_4 = \frac{169}{12s}$, $Z_5 = \frac{4}{13s}$.

The Cauer-II form,



Q.4 (b) Solution:

(i)



KCL at Node A,

$$I_3 = I_1 + I_2 - 0.4I_1 = 0.6I_1 + I_2 \quad ...(1)$$

Now, KVL in loop (2)

$$-V_2 + 20I_3 = 0$$
$$V_2 = 20(0.6I_1 + I_2)$$
$$V_2 = 12I_1 + 20I_2 \quad ...(2)$$

KVL in loop (1)

$$V_1 = 5I_1 + 0.2V_2 + V_2$$
$$V_1 = 5I_1 + 1.2V_2$$
$$5I_1 = V_1 - 1.2V_2$$
$$I_1 = 0.2V_1 - 0.24V_2 \quad ...(3)$$

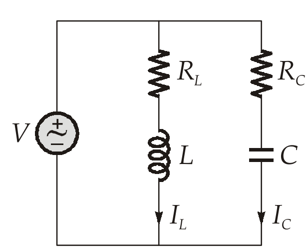
Putting I_1 in equation (2),

$$V_2 = 12(0.2V_1 - 0.24V_2) + 20I_2$$
$$20I_2 = -2.4V_1 + 3.88V_2$$
$$I_2 = -0.12V_1 + 0.194V_2 \quad ...(4)$$

So, comparing eqn. (3) and (4) with standard y-parameter equation

$$[y] = \begin{bmatrix} 0.2 & -0.24 \\ -0.12 & 0.194 \end{bmatrix}$$

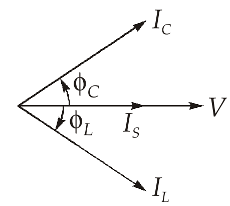
(ii) For this resonant circuit, the phasor diagram is shown in figure,



In the inductive branch, the current I_L lags the voltage V and in the capacitive branch, the current I_C leads the voltage V

Phase angle of the inductive branch,

$$\phi_L = \tan^{-1}\left(\frac{\omega L}{R_L}\right)$$



Phase angle of the capacitive branch,

$$\phi_C = \tan^{-1}\left(\frac{1}{\omega R_C C}\right)$$

For the two currents to be in quadrature, the below condition must hold true

$$\phi_L + \phi_C = 90°$$

$$\tan^{-1}\left(\frac{\omega L}{R_L}\right) + \tan^{-1}\left(\frac{1}{\omega R_C C}\right) = 90°$$

$$\tan^{-1}\left[\frac{\frac{\omega L}{R_L} + \frac{1}{\omega R_C C}}{1 - \frac{\omega L}{R_L} \times \frac{1}{\omega R_C C}}\right] = 90°$$

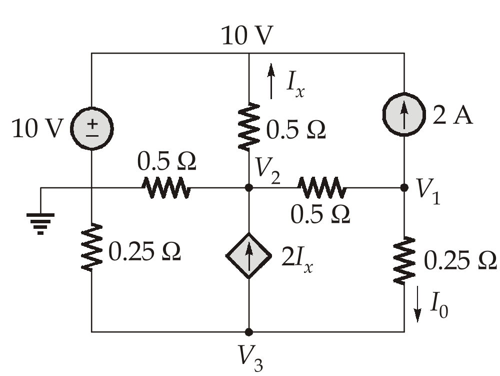
$$\frac{\frac{\omega L}{R_L} + \frac{1}{\omega R_C C}}{1 - \frac{L}{R_L R_C C}} = \tan 90° = \infty$$

$$1 - \frac{L}{R_L R_C C} = 0$$

$$R_L R_C = \frac{L}{C}$$

Q.4 (c) Solution:

(i)



Apply KCL at node (1)

$$\frac{V_1 - V_3}{0.25} + \frac{V_1 - V_2}{0.5} + 2 = 0$$

$$6V_1 - 2V_2 - 4V_3 = -2$$

$$3V_1 - V_2 - 2V_3 = -1 \quad \text{...(1)}$$

Apply KCL at node (2),

$$2I_x = I_x + \frac{V_2 - V_1}{0.5} + \frac{V_2}{0.5}$$

$$I_x = -2V_1 + 4V_2$$

where, $I_x = \frac{V_2 - 10}{0.5} \Rightarrow I_x = 2V_2 - 20$

$$2V_2 - 20 = -2V_1 + 4V_2$$

$$-2V_1 + 2V_2 = -20$$

$$-V_1 + V_2 = -10 \quad \text{...(2)}$$

Apply KCL at node (3)

$$\frac{V_3}{0.25} + 2I_x + \frac{V_3 - V_1}{0.25} = 0$$

$$8V_3 - 4V_1 + 2(2V_2 - 20) = 0$$

$$-4V_1 + 4V_2 + 8V_3 = 40$$

$$-V_1 + V_2 + 2V_3 = 10 \quad \text{...(3)}$$

The equations (1), (2) and (3) can be written in the matrix form as below:

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -10 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 1 & 0 \\ -1 & 1 & 2 \end{vmatrix} \Rightarrow \Delta = 3(2) + 1(-2) - 2(0) = 4$$

$$\Delta_1 = \begin{vmatrix} -1 & -1 & -2 \\ -10 & 1 & 0 \\ 10 & 1 & 2 \end{vmatrix} \Rightarrow \Delta_1 = -1(2) + 1(-20) - 2(-20) = 18$$

$$\Delta_2 = \begin{vmatrix} 3 & -1 & -2 \\ -1 & -10 & 0 \\ -1 & 10 & 2 \end{vmatrix} \Rightarrow \Delta_2 = 3(-20) + 1(-2) - 2(-20) = -22$$

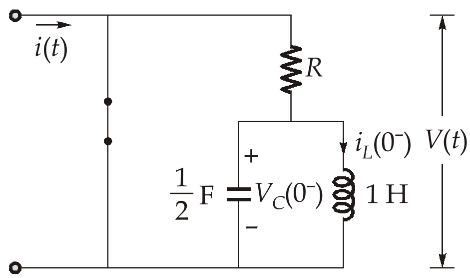
$$\Delta_3 = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 1 & -10 \\ -1 & 1 & 10 \end{vmatrix} \Rightarrow \Delta_3 = 3(20) + 1(-20) - 1(-1+1) = 40$$

Using Cramer's rule,

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{18}{4} = 4.5 \text{ V}, V_3 = \frac{\Delta_3}{\Delta} = \frac{40}{4} = 10 \text{ V}$$

$$I_0 = \frac{V_1 - V_3}{0.25} = \frac{4.5 - 10}{0.25} = -22 \text{ A}$$

(ii) Given, circuit has zero initial energy. At $t = 0^-$, switch is closed.



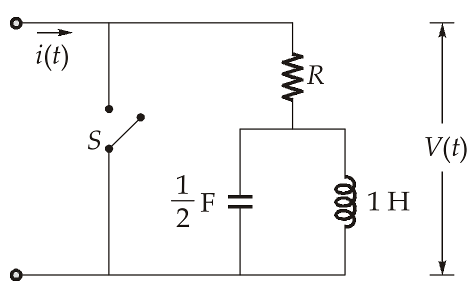
$\therefore$

$$i_L(0^-) = 0$$
$$V_C(0^-) = 0$$

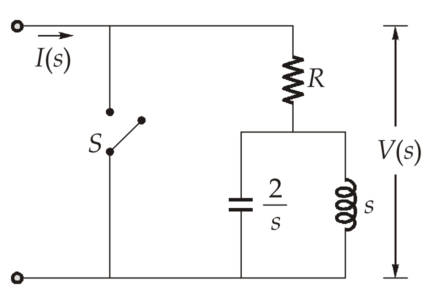
Since inductor current and capacitor voltage cannot change instantaneously.

$$i_L(0^-) = 0 = i_L(0^+)$$
$$V_C(0^-) = V_C(0^+) = 0$$

At $t > 0$, switch is opened.



by transforming above circuit into Laplace domain,



$$V(s) = I(s) \times Z(s)$$

where, $$Z(s) = R + s \parallel \frac{2}{s} = R + \frac{s \times \frac{2}{s}}{s + \frac{2}{s}} = R + \frac{2s}{s^2 + 2}$$

$$\frac{V(s)}{I(s)} = R + \frac{2s}{s^2 + 2}$$

where, $$V(s) = \frac{0.5 \times (\sqrt{2})}{s^2 + (\sqrt{2})^2} = \frac{0.5\sqrt{2}}{s^2 + 2} \qquad \text{as } V(t) = 0.5 \sin \sqrt{2}t\, u(t)$$

$$I(s) = \frac{1}{(s + \sqrt{2})^2} = \frac{1}{s^2 + 2\sqrt{2}s + 2} \qquad \text{as } i(t) = te^{-\sqrt{2}t} u(t)$$

∴ $$\frac{V(s)}{I(s)} = \frac{\frac{0.5\sqrt{2}}{s^2 + 2}}{\frac{1}{s^2 + 2\sqrt{2}s + 2}} = R + \frac{2s}{s^2 + 2}$$

$$0.5\sqrt{2}(s^2 + 2\sqrt{2}s + 2) = s^2R + 2s + 2R$$

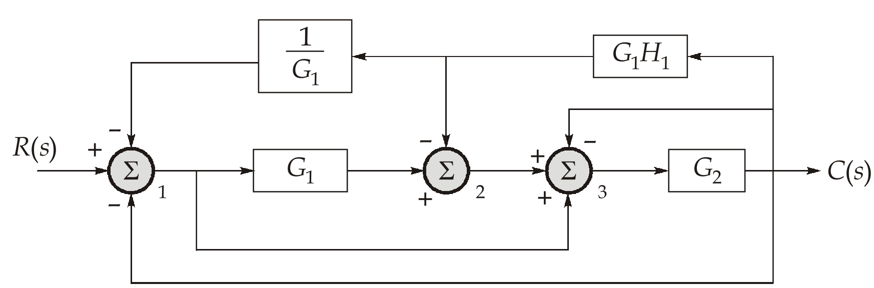
$$0.5\sqrt{2}s^2 + 2s + \sqrt{2} = s^2R + 2s + 2R$$

On comparing, $$R = 0.707\ \Omega$$

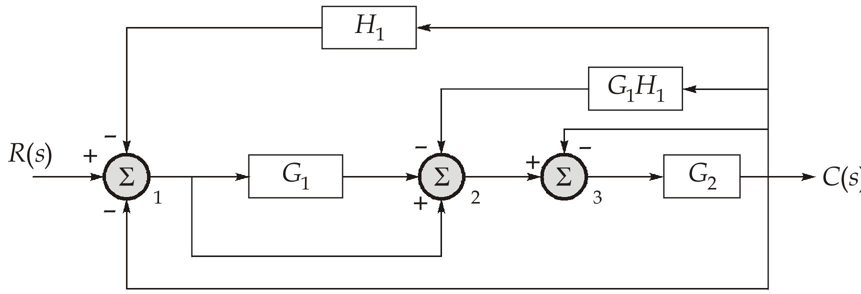
Section B : Control Systems

Q.5 (a) Solution:

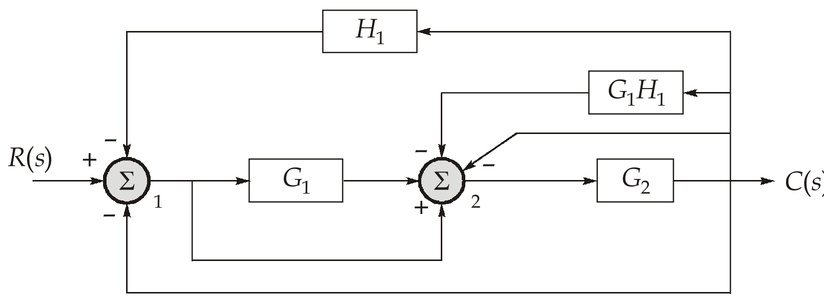
Step-I: Shifting the summing point 2 to the right of 'G_1' as shown below:



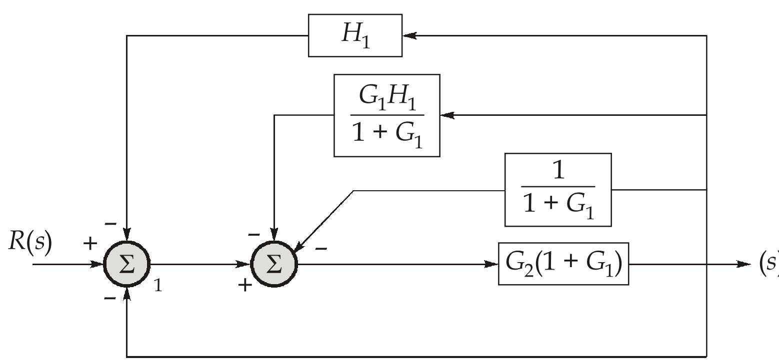
Step-II: On splitting the upper combined feedback path and replacing it with individual one, we get



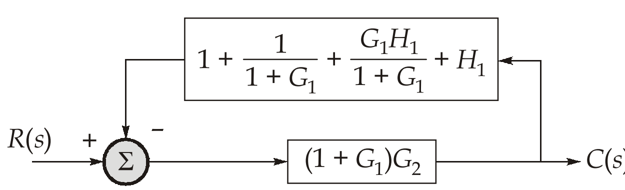
Step-III: On combining summing point 2 and 3, we get



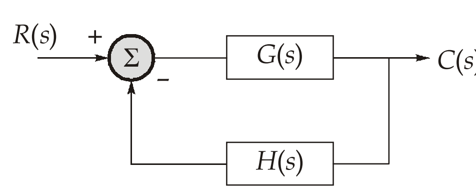
Step-IV: Now, shift $(1 + G_1)$ right of summing point 2 and redraw the block diagram as follow:



Step-V: Now, adding all the parallel feedback paths we get



By using negative feedback formula,



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Here, $G(s) = (1 + G_1)G_2$

$$H(s) = 1+\frac{1}{1+G_1}+\frac{G_1H_1}{1+G_1}+H_1$$

$$\frac{C(s)}{R(s)} = \frac{(1+G_1)G_2}{1+\left[(1+G_1)(G_2)\left(1+\frac{1}{1+G_1}+\frac{G_1H_1}{1+G_1}+H_1\right)\right]}$$

$$\frac{C(s)}{R(s)} = \frac{(1+G_1)G_2}{1+\left[(1+G_1)G_2+G_2+G_1G_2H_1+(1+G_1)G_2H_1\right]}$$

Q.5 (b) Solution:

We have,

Closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{10}{s^3+0.1s^2+10}$$

$$\text{input, } r(t) = 5+10t+4t^2$$

We know that,

Closed loop transfer function is given as

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Now rearrange the given transfer function to obtain the open loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{10}{1+\frac{10}{s^3+0.1s^2}} = \frac{10}{1+\frac{10}{s^2(s+0.1)}}$$

On comparison, we get

open loop transfer function,

$$G(s)H(s) = \frac{10}{s^2(s+0.1)}$$

We have input $r(t) = 5 + 10t + 4t^2 = 5+10t+\frac{8}{2}t^2$

The error coefficients can be obtained as:

$$k_p = \lim_{s\to 0} G(s)H(s)$$

$$= \lim_{s\to 0}\frac{10}{s^2(s+0.1)} = \infty$$

$$k_v = \lim_{s\to 0} s\times\frac{10}{s^2(s+0.1)} = \infty$$

$$k_a = \lim_{s\to 0} s^2 \times \frac{10}{s^2(s+0.1)} = 100$$

Now, steady state error e_{ss} is given as

$$e_{ss} = e_{ss1} + e_{ss2} + e_{ss3}$$

where,

e_{ss1} is due to step input ($A_1 = 5$)

e_{ss2} is due to ramp input ($A_2 = 10$)

e_{ss3} is due to parabolic input ($A_3 = 8$)

$$e_{ss} = \frac{A_1}{1+k_p} + \frac{A_2}{k_v} + \frac{A_3}{k_a}$$

$$e_{ss} = \frac{5}{1+\infty} + \frac{10}{\infty} + \frac{8}{100}$$

$$e_{ss} = 0.08$$

Q.5 (c) Solution:

(i) We have,

open loop transfer function, $G(s) = \dfrac{K}{s(1+sT)}$

Let the value of damping ratio is ξ_1 when the peak overshoot is 75% and ξ_2 when peak overshoot is 20%.

We know that, peak overshoot is given as

$$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

for 75% of peak overshoot,

$$\frac{75}{100} = e^{-\frac{\pi\xi_1}{\sqrt{1-\xi_1^2}}}$$

$$-0.29 = \frac{-\pi\xi_1}{\sqrt{1-\xi_1^2}}$$

On squaring both sides, we get,

$$(-0.29)^2 = \frac{(-\pi\xi_1)^2}{\left(\sqrt{1-\xi_1^2}\right)^2}$$

$$0.0841 = \frac{\pi^2 \xi_1^2}{\left(1 - \xi_1^2\right)}$$

$$0.0841 - 0.0841\xi_1^2 = \pi^2 \xi_1^2$$

$$\xi_1^2 = 8.45 \times 10^{-3}$$

$$\xi_1 = 0.092 \qquad \text{...(i)}$$

Similarly, for peak overshoot of 20%, damping ratio is ξ_2

$$0.20 = e^{\frac{-\pi\xi_2}{\sqrt{1-\xi_2^2}}}$$

$$-1.61 = \frac{-\pi\xi_2}{\sqrt{1 - \xi_2^2}}$$

On squaring both sides, we get

$$2.60 = \frac{\pi^2 \xi_2^2}{1 - \xi_2^2}$$

$$2.60 - 2.60\xi_2^2 = \pi^2 \xi_2^2$$

$$\xi_2^2 = 0.209$$

$$\xi_2 = 0.46 \qquad \text{...(ii)}$$

The open loop transfer function of standard 2nd order system is given as $\dfrac{\omega_n^2}{s(s + 2\xi\omega_n)}$

and we have, $$G(s) = \frac{(K/T)}{s\left(\frac{1}{T} + s\right)}$$

On comparison, we get $$\omega_n^2 = \frac{K}{T}$$

$$\omega_n = \sqrt{\frac{K}{T}}$$

and $$2\xi\omega_n = \frac{1}{T}$$

$$\xi = \frac{1}{2T\omega_n}$$

$$\xi = \frac{1}{2T}\sqrt{\frac{T}{K}}$$

$$\xi = \frac{1}{2\sqrt{TK}}$$

From (i) and (ii) we get,

$$\frac{\xi_1}{\xi_2} = \frac{\sqrt{K_2}}{\sqrt{K_1}} = \frac{0.092}{0.46}$$

$$\frac{K_2}{K_1} = 0.04$$

$$K_2 = 0.04\ K_1$$

or, the amplifier gain has to be reduced by a factor $\frac{1}{0.04} = 25$.

(ii) Let, $\xi_1 = 0.2$, when gain is K_1, and

$\xi_2 = 0.6$, when gain is K_2

$\therefore$
$$\frac{\xi_1}{\xi_2} = \sqrt{\frac{K_2}{K_1}}$$

$$\frac{0.2}{0.6} = \sqrt{\frac{K_2}{K_1}}$$

$$\frac{1}{9} = \frac{K_2}{K_1}$$

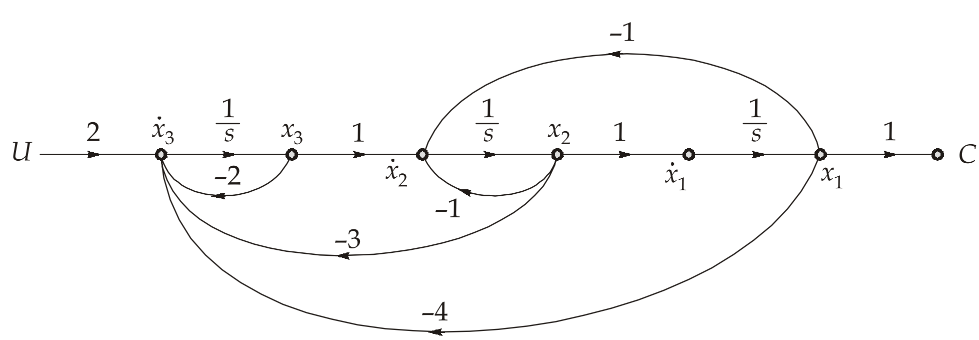
$$K_1 = 9K_2$$

$$K_2 = \frac{K_1}{9}$$

$\therefore$ the amplifier gain should be reduced by a factor of 9.

Q.5 (d) Solution:

(i) The signal flow graph representing state variables is as below:



The state equations can be written as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3 - x_1 - x_2$$

$$\dot{x}_3 = -2x_3 - 3x_2 - 4x_1 + 2u$$

Writing in Matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ -4 & -3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u = Ax + Bu$$

and $y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x = Cx$

(ii) The transfer function of the system is given as:

$$H(s) = \frac{C(s)}{U(s)} = C[sI - A]^{-1} B$$

We have, $[sI - A] = \begin{bmatrix} s & -1 & 0 \\ 1 & s+1 & -1 \\ 4 & 3 & s+2 \end{bmatrix}$

$$[sI - A]^{-1} = \frac{1}{s(s^2+3s+5)+1(s+6)} \begin{bmatrix} s^2+3s+5 & -(s+6) & -(4s+1) \\ s+2 & s(s+2) & -(3s+4) \\ 1 & s & s^2+s+1 \end{bmatrix}^T$$

$$= \frac{1}{s^3+3s^2+6s+6} \begin{bmatrix} s^2+3s+5 & s+2 & 1 \\ -(s+6) & s(s+2) & s \\ -(4s+1) & -(3s+4) & s^2+s+1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s^2+3s+5}{s^3+3s^2+6s+6} & \frac{s+2}{s^3+3s^2+6s+6} & \frac{1}{s^3+3s^2+6s+6} \\ \frac{-(s+6)}{s^3+3s^2+6s+6} & \frac{s(s+2)}{s^3+3s^2+6s+6} & \frac{s}{s^3+3s^2+6s+6} \\ \frac{-(4s+1)}{s^3+3s^2+6s+6} & \frac{-(3s+4)}{s^3+3s^2+6s+6} & \frac{s^2+s+1}{s^3+3s^2+6s+6} \end{bmatrix}$$

Hence,
$$H(s) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}[sI - A]^{-1}\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dfrac{2}{s^3 + 3s^2 + 6s + 6} \\ \dfrac{2s}{s^3 + 3s^2 + 6s + 6} \\ \dfrac{2(s^2 + s + 1)}{s^3 + 3s^2 + 6s + 6} \end{bmatrix}$$

$$H(s) = \frac{2}{s^3 + 3s^2 + 6s + 6}$$

Q.5 (e) Solution:

(i) The closed-loop transfer function of the system shown in figure is

$$\frac{C(s)}{R(s)} = \frac{\dfrac{k}{s(s+b)}}{1 + \dfrac{k}{s(s+b)}} = \frac{k}{s^2 + bs + k}$$

Comparing it with the standard form of the closed-loop transfer function of a second-order system,

$$\frac{k}{s^2 + bs + k} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

We have,
$$\omega_n^2 = k$$

or
$$\omega_n = \sqrt{k}$$

$$2\xi\omega_n = b$$

$$\xi = \frac{b}{2\omega_n} = \frac{b}{2\sqrt{k}}$$

i.e.,
$$\xi^2 = \frac{b^2}{4k}$$

we know,
$$M_r = \frac{1}{2\xi\sqrt{1 - \xi^2}} = 1.6 \text{ [Given]}$$

$$\frac{1}{4\xi^2(1-\xi^2)} = 2.56 = \frac{1}{\frac{4b^2}{4k}\left(1-\frac{b^2}{4k}\right)} = \frac{1}{\frac{b^2}{k}\left(1-\frac{b^2}{4k}\right)} \quad ...(i)$$

We have, $\omega_r = \omega_n\sqrt{1-2\xi^2} = 15$ [Given]

$$225 = \omega_n^2(1-2\xi^2) = k\left(1-2\frac{b^2}{4k}\right)$$

or
$$225 = k - \frac{b^2}{2}$$

$$b^2 = 2k - 450 \quad ...(ii)$$

From equation (1)

$$\frac{b^2}{k}\left(1-\frac{b^2}{4k}\right) = \frac{1}{2.56}$$

$$b^2(4k - b^2) = \frac{4k^2}{2.56} \quad ...(iii)$$

Put value of b^2 in equation (iii)

$$(2k-450)(4k-2k+450) = \frac{4k^2}{2.56}$$

$$(2k-450)(2k+450) = \frac{4k^2}{2.56}$$

$$\Rightarrow \quad 4k^2 - 202500 = \frac{4k^2}{2.56}$$

$$10.24k^2 - 4k^2 - 518400 = 0$$

$$k = \sqrt{\frac{518400}{6.24}} = 288.23$$

Therefore, $b = \sqrt{2k-450} = 11.245$

$$\omega_n = \sqrt{k} = \sqrt{288.23} = 16.97 \text{ rad/sec}$$

$$\xi = \frac{b}{2\sqrt{k}} = \frac{11.245}{2\times 16.97} = 0.331$$

(ii) Settling time

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{5.617} = 0.712 \text{ sec} \quad \text{(for 2\% criterion)}$$

$$t_s = \frac{3}{\xi\omega_n} = \frac{3}{0.331 \times 16.97} = 0.534 \text{ sec} \quad \text{(for 5\% criterion)}$$

The bandwidth $\omega_b = \omega_n \left[1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}\right]^{1/2}$

$$= 16.97\left[1 - 2 \times (0.331)^2 + \sqrt{2 - 4 \times (0.331)^2 + 4 \times (0.331)^4}\right]^{1/2}$$

$$= 24.295 \text{ rad/sec}$$

The result is

(a) For $M_r = 1.6$, $\omega_r = 15$ rad/sec, $k = 288.23$ and $b = 11.245$.

(b)For $k = 288.23$, $b = 11.245$, $t_s = 0.712$s and B.W = 24.295 rad/sec

Q.6 (a) Solution:

We have,

Open loop transfer function, $G(s) = \dfrac{2K + 5}{s(s - (2 + K))}$

Thus,

Characteristic equation of the system,

$$1 + G(s)H(s) = 0 \qquad \because H(s) = 1$$

$$1 + \frac{2K + 5}{s(s - (2 + K))} = 0$$

$$s^2 - (2 + K)s + 2K + 5 = 0 \qquad \text{...(i)}$$

(i) Using Routh-Hurwitz criterion,

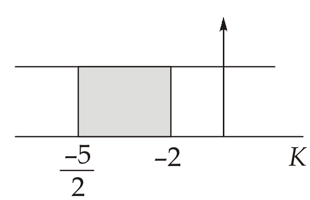
s^2	1	$2K + 5$
s^1	$-(2 + K)$	
s^0	$2K + 5$	

For stable system

$-(K + 2) > 0$ and $2K + 5 > 0$

$K + 2 < 0$ and $2K > -5$

$K < -2$ and $K > \dfrac{-5}{2}$



Therefore, the system is stable for $-\dfrac{5}{2} < K < -2$

(ii) In characteristic equation (i), put $s = z - 1$ (for analysis of poles in reference to $s + 1 = 0$ line)

then

$(z - 1)^2 - (K + 2)(z - 1) + (2K + 5) = 0$

On solving, we get

$$z^2 + z(-4 - K) + 3K + 8 = 0$$

Using Routh-Hurwitz criterion,

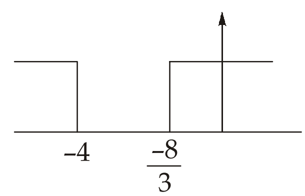
$$\begin{array}{lll} z^2 & 1 & 3K+8 \\ z^1 & -4-K & \\ z^0 & 3K+8 & \end{array}$$

For all poles to lie to the left of $s + 1 = 0$ line,

$-4 - K > 0$ and $3K + 8 > 0$

$-4 > K$ and $3K > -8$

$$K > \frac{-8}{3}$$



Hence, for no value of 'K', both the poles lies to the left of $s + 1 = 0$ line.

(iii) Using Routh array obtained in part (b),

$$\begin{array}{lll} z^2 & 1 & 3K+8 \\ z^1 & -4-K & \\ z^0 & 3K+8 & \end{array}$$

Now, possibilities for one pole in left and one in right side of $s + 1 = 0$ line is listed below:

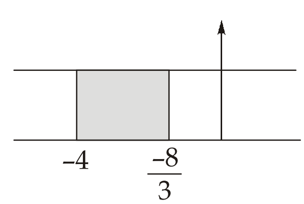
Case-I			**Case-II**	
z^2	+		z^2	+
z^1	–	or	z^1	+
z^0	–		z^0	–

For Case-I:

$-4 - K < 0$ and $3K + 8 < 0$

$-K < 4$ and $3K < -8$

$K > -4$ and $K < -\frac{8}{3}$



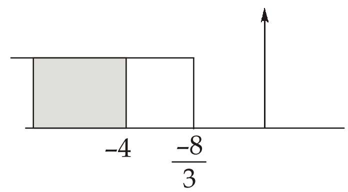
Hence, $-4 < K < -\frac{8}{3}$ ...(ii)

For Case-II:

$-4 - K > 0$ and $3K + 8 < 0$

$-4 > K$ and $3K < -8$

$-4 > K$ and $K < -\frac{8}{3}$



Hence, $K < -4$ ...(iii)

From equation (ii) and (iii), we can conclude that for one pole of the characteristic equation to be present in the left of $s + 1 = 0$ line $-4 < K < \frac{-8}{3}$ or $K < -4$.

(iv) According to question,

Poles are present at $s_1 = -0.125 + 0.7i$ and at

$s_2 = -0.125 - 0.7i$

Thus, characteristic equation must satisfy at s_1 and s_2.

On putting s_1 in the characteristic equation, we get

$(-0.125 + 0.7i)^2 - (2 + K)(-0.125 + 0.7i) + 2K + 5 = 0$

$(-0.47 - 0.175i) + 0.25 - 1.4i + 0.125K - 0.7Ki + 2K + 5 = 0$

$4.78 - 1.575i + 2.125K - 0.7Ki = 0$

$(4.78 + 2.125K) - 1.575i - 0.7Ki = 0$

equating real part to zero or equating imaginary part to zero

$$4.78 + 2.125K = 0 \qquad -1.575i - 0.7Ki = 0$$

$$2.125K = -4.78 \qquad -0.7K = 1.575$$

$$K = -2.25 \qquad K = -2.25$$

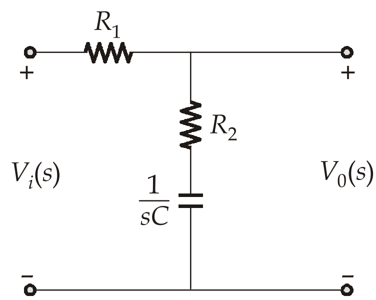
Hence for $K = -2.25$, pole is at $s = -0.125 + 0.7i$.

Since s_1 and s_2 are complex conjugate poles. Therefore we get that for $K = -2.25$; poles are at $s_1 = -0.125 + 0.7i$ and at $s_2 = -0.125 - 0.7i$.

Q.6 (b) Solution:

(i) Lag compensator:

The lag compensator is an electrical network which produces a sinusoidal output having the phase lag when a sinusoidal input is applied. The lag compensator circuit in the 's' domain is shown below:



The transfer function of this lag compensator is

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{\alpha}\left(\frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}}\right)$$

where, $\tau = R_2C$ and $\alpha = \dfrac{R_1 + R_2}{R_2}$

From the above equation, α is always greater than one.

From the transfer function, we can conclude the lag compensator has one pole at $s = -\dfrac{1}{\alpha\tau}$. This means, the pole will be nearer to origin in the pole-zero configuration of the lag compensator.

Substitute, $s = j\omega$ the transfer function.

$$\frac{V_0(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha}\left(\frac{j\omega+\frac{1}{\tau}}{j\omega+\frac{1}{\alpha\tau}}\right)$$

Phase angle, $\phi = \tan^{-1}\omega\tau - \tan^{-1}\alpha\omega\tau$

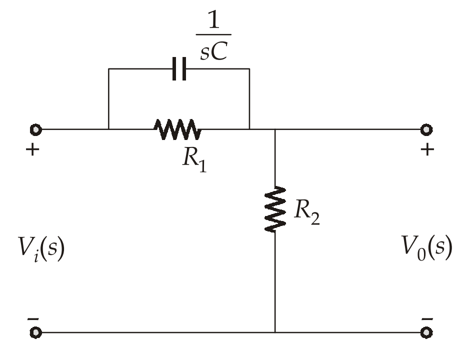
Since the phase of the output sinusoidal signal is equal to the sum of the phase angles of input sinusoidal signal and the transfer function.

So, in order to produce the phase lag at the output of this compensator, the phase angle of the transfer function should be negative.

This will happen when $\alpha > 1$.

(ii) Lead compensator:

The lead compensator is an electrical network which produces a sinusoidal output having the phase lead when a sinusoidal input is applied. The lead compensator circuit in '*s*' domain is shown below:



Here, the capacitor is parallel to the resistor R, and the output is measured across resistor R_2.

The transfer function of this lead compensator is

$$\frac{V_0(s)}{V_i(s)} = \beta\left(\frac{s\tau+1}{\beta s\tau+1}\right)$$

where, $\tau = RC$; $\beta = \dfrac{R_2}{R_1 + R_2} < 1$

From the transfer function, we conclude that the lead compensator has pole at $s = -1/\beta\tau$ and zero at $s = -1/\tau$. It has a dominant zero i.e. zero is nearer to origin than the pole.

Substitute, $s = j\omega$ in the transfer function

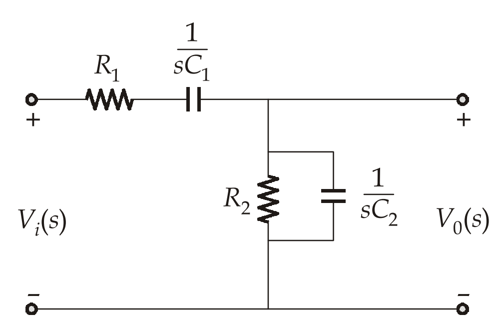
$$\frac{V_0(j\omega)}{V_i(j\omega)} = \beta\left(\frac{j\omega\tau+1}{\beta j\omega\tau+1}\right)$$

$$\text{Phase angle, } \phi = \tan^{-1}\omega\tau - \tan^{-1}\beta\omega\tau$$

In order to produce the phase lead at the output of this compensator, the phase angle of the transfer function should be positive. This will happen when $0 < \beta < 1$.

(iii) Lead-lag compensator:

Lead-lag compensator is an electrical network which produces phase lead at one frequency region and phase lag at other frequency region. It is the combination of both the lead and lag compensators. The lead-lag compensator circuit in the s domain is shown below:



This circuit like both the compensators are cascaded. So the transfer function of this circuit will be product of transfer functions of the lag and lead compensators.

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{\alpha}\left[\frac{\left(s+\frac{1}{\tau_2}\right)}{\left(s+\frac{1}{\alpha\tau_2}\right)}\right]\frac{(s\tau_1+1)}{(\beta s\tau_1+1)}$$

where, $\alpha\beta = 1$

$$\frac{V_0(s)}{V_i(s)} = \frac{\left(s+\frac{1}{\tau_1}\right)}{\left(s+\frac{1}{\beta\tau_1}\right)}\frac{\left(s+\frac{1}{\tau_2}\right)}{\left(s+\frac{1}{\alpha\tau_2}\right)}$$

where, $\tau_1 = R_1C_1$; $\tau_2 = R_2C_2$

Q.6 (c) Solution:

(i) The solution of the homogeneous state equation is given by

$$x(t) = \phi(t)X(0)$$

$\phi(t)$, the state transition matrix is computed as follows.

$$[sI - A] = \left[s\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\right] = \begin{bmatrix}s-1 & 0\\-1 & s-1\end{bmatrix}$$

$\therefore$

$$[sI - A]^{-1} = \frac{\begin{bmatrix}s-1 & 1\\0 & s-1\end{bmatrix}^T}{s^2-2s+1}$$

$$= \frac{\begin{bmatrix}s-1 & 0\\1 & s-1\end{bmatrix}}{(s-1)^2} = \phi(s), \text{ the resolvent matrix}$$

$$\phi(t) = L^{-1}[sI-A]^{-1} = L^{-1}\begin{bmatrix}\frac{1}{s-1} & 0\\ \frac{1}{(s-1)^2} & \frac{1}{s-1}\end{bmatrix}$$

$$= \begin{bmatrix}e^t & 0\\te^t & e^t\end{bmatrix}$$

The solution of the homogeneous state equation is given by

$$x(t) = \phi(t)x(0) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

This can also be obtained by finding $X(s)$ and taking its inverse Laplace transform,

$$X(s) = \phi(s)x(0) = \frac{\begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{(s-1)^2} = \frac{\begin{bmatrix} s-1 \\ 1 \end{bmatrix}}{(s-1)^2} = \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{(s-1)^2} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = L^{-1}[X(s)] = L^{-1}\begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{(s-1)^2} \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

(ii) The solution of the nonhomogeneous equation called the state transition equation is

$$x(t) = \phi(t)x(0) + \int_0^t \phi(t-\tau)Bu(\tau)d\tau$$

$$= \phi(t)\left[x(0) + \int_0^t \phi(-\tau)Bu(\tau)d\tau\right]$$

as $\phi(t-\tau) = e^{A(t-\tau)} = e^{At}.e^{-A\tau} = \phi(t)\cdot\phi(-\tau)$

The input is a unit-step function, therefore $u(t) = 1$

$$x(t) = \phi(t)\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} e^{-\tau} & 0 \\ -\tau e^{-\tau} & e^{-\tau} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau\right]$$

$$= \phi(t)\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} 0 \\ e^{-\tau} \end{bmatrix} d\tau\right] = \phi(t)\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -e^{-\tau} \end{bmatrix}_0^t\right]$$

$$= \phi(t)\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -e^{-t}+1 \end{bmatrix}\right] = \phi(t)\begin{bmatrix} 1 \\ 1-e^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 1-e^{-t} \end{bmatrix} = \begin{bmatrix} e^t \\ te^t + e^t - 1 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} e^t \\ te^t + e^t - 1 \end{bmatrix}$$

Q.7 (a) Solution:

We have,

open loop transfer function, $G(s)H(s) = \dfrac{K}{s(s+2)}$

(i) Closed loop transfer function,

$$\frac{V_0(s)}{V_{in}(s)} = \frac{\left(\dfrac{K}{s(s+2)}\right)}{1+\dfrac{K}{s(s+2)}}$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{K}{s(s+2)+K}$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{K}{s^2+2s+K} \quad \text{...(i)}$$

(ii) We have characteristic equation as;

$$1 + G(s)H(s) = 0$$

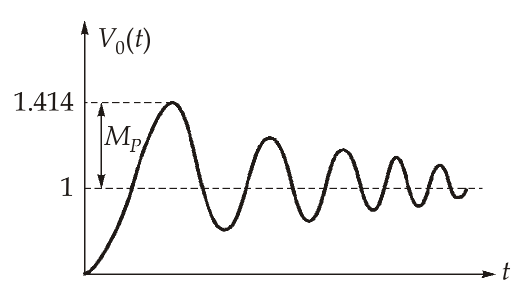
$$s^2 + 2s + K = 0$$

On comparison with standard 2nd order characteristic equation,

$$2\xi\omega_n = 2 \text{ and } \omega_n = \sqrt{K}$$

$$\xi = \frac{1}{\sqrt{K}}$$

From figure (ii) we get,



$$\text{Peak overshoot; } \%M_p = \left(\frac{1.414-1}{1}\right)\times 100$$

$$\%M_p = 41.4\%$$

We know that,

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = \frac{41.4}{100}$$

$$\frac{-\pi\xi}{\sqrt{1-\xi^2}} = -0.88$$

On squaring both side, we get

$$\frac{\pi^2\xi^2}{1-\xi^2} = 0.77$$

$$\pi^2\xi^2 = 0.77 - 0.77\xi^2$$

$$\xi^2 = 0.07$$

$$\xi = 0.26 = \frac{1}{\sqrt{K}}$$

$$\sqrt{K} = 3.85$$

$$K = 14.82$$

Hence, for $M_p = 41.4\%$, $K_{min} = 14.82$

(iii) Here, $K = 2K_{min}$

$$K = 29.64$$

Put K in equation (i)

$$\frac{V_0(s)}{V_{in}(s)} = \frac{29.64}{s^2 + 2s + 29.64}$$

We get, $2\xi\omega_n = 2$ $\qquad \omega_n = \sqrt{29.64}$

$$\xi = \frac{1}{\omega_n} = 0.18 \qquad \omega_n = 5.44 \text{ rad/sec}$$

$\because$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d = 5.44\sqrt{1-(0.18)^2}$$

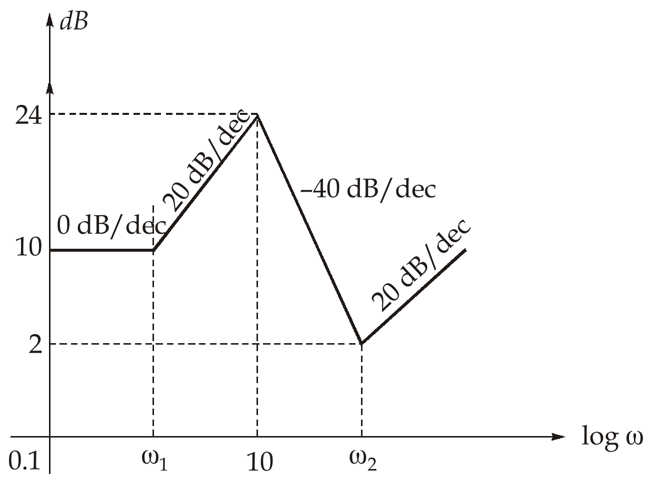
$$\omega_d = 5.35 \text{ rad/sec}$$

$\therefore$

$$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{5.35} = 1.17 \text{ sec}$$

Q.7 (b) Solution:

(i)



At frequency ω_1, the slope of the line changes from 0 dB/dec to 20 dB/dec, hence the transfer function contains the term $G_1(s)$ givens as:

$$G_1(s) = \left(1 + \frac{s}{\omega_1}\right)$$

where ω_1 can be calculated as below:

$$20 = \frac{24 - 10}{\log \frac{10}{\omega_1}}$$

$$\log \frac{10}{\omega_1} = \frac{14}{20}$$

$$\frac{10}{\omega_1} = 10^{0.7}$$

$$\omega_1 = \frac{10}{10^{0.7}} = 1.995 \cong 2 \text{ rad/sec}$$

At the corner frequency of 10 rad/sec, the slope of the line again changes to –40 dB/dec from 20 dB/dec. Hence, the transfer function contains the following term:

$$G_2(s) = \frac{1}{\left(1 + \frac{s}{10}\right)^3}$$

Now, at corner frequency of ω_2, the slope of line changes to 20 dB/dec from –40 dB/dec, therefore, transfer function contains the following term:

$$G_3(s) = \left(1+\frac{s}{\omega_2}\right)^3$$

where, ω_2 is calculated as below:

$$-40 \text{ dB/dec} = \frac{2-24}{\log\frac{\omega_2}{10}}$$

$$\log\frac{\omega_2}{10} = \frac{22}{40}$$

$$\omega_2 = 10^{0.55} \times 10 = 35.48 \text{ rad/sec}$$

Therefore, overall transfer function becomes

$$T(s) = KG_1(s)\,G_2(s)\,G_3(s)$$

$$T(s) = \frac{K\left(1+\frac{s}{2}\right)\left(1+\frac{s}{35.48}\right)^3}{\left(1+\frac{s}{10}\right)^3}$$

We have, $20 \log K = 10$

$$K = 10^{0.5} = 3.162$$

$$T(s) = \frac{3.162(s+2)(s+35.48)^3 \times 10^3}{2\times(35.48)^3(s+10)^3}$$

$\therefore$
$$T(s) = \frac{0.035(s+2)(s+35.48)^3}{(s+10)^3}$$

(ii) Given,
$$G(s) = \frac{1+5s}{s^2(1+s)(1+2s)}$$

$$G(j\omega) = \frac{1+5j\omega}{-\omega^2(1+j\omega)(1+2j\omega)}$$

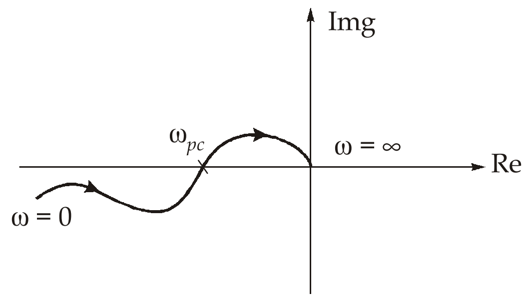
$$|G(j\omega)| = \frac{\sqrt{1+25\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

$$\angle G(j\omega) = \tan^{-1}5\omega - 180° - \tan^{-1}\omega - \tan^{-1}2\omega$$

So, at $\omega = 0$
$$|G(j\omega)| = \infty$$
$$\angle G(j\omega) = -180°$$

at $\omega = \infty$
$$|G(j\omega)| = 0$$
$$\angle G(j\omega) = -270°$$

The polar plot can be drawn as below:



Note:

The frequency at which the polar plot cuts the horizontal axis (i.e., 180° line) is none other than phase crossover frequency.

$\because$ at ω_{pc}, $\angle G(j\omega) = -180°$

$\therefore$ $-180° + \tan^{-1} 5\,\omega_{pc} - \tan^{-1}\omega_{pc} - \tan^{-1} 2\omega_{pc} = -180°$

$$\tan^{-1} 5\omega_{pc} - \tan^{-1}\frac{3\omega_{pc}}{1-2\omega_{pc}^2} = 0$$

$$5\omega_{pc} = \frac{3\omega_{pc}}{1-2\omega_{pc}}$$

$$1-2\omega_{pc}^2 = \frac{3}{5}$$

$$2\omega_{pc}^2 = 1-\frac{3}{5} = \frac{2}{5}$$

$$\omega_{pc} = \frac{1}{\sqrt{5}} = 0.45\text{ rad/sec}$$

$$\text{G.M} = \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$$

$$= \frac{1}{\dfrac{\sqrt{1+25\omega_{pc}^2}}{\omega_{pc}^2\sqrt{1+\omega_{pc}^2}\sqrt{1+4\omega_{pc}^2}}}$$

$$= \frac{(0.45)^2\sqrt{1+(0.45)^2}\sqrt{1+4\times(0.45)^2}}{\sqrt{1+25(0.45)^2}} = 0.121$$

In decibels, $$\text{G.M.} = 20\log\frac{1}{|G(j\omega)|}\bigg|_{\omega=\omega_{pc}}$$

$$= 20\log(0.121) = -18.344\text{ dB}$$

Q.7 (c) Solution:

- The open-loop transfer function of the uncompensated system is,

$$G_f(s) = \frac{10}{s(s+1)}$$

Finding the phase margin of uncompensated system :

- The gain crossover frequency of uncompensated system is,

$$\frac{10}{\omega_{gc}\sqrt{1+\omega_{gc}^2}} = 1$$

$$\omega_{gc}^4 + \omega_{gc}^2 - 100 = 0$$

$$\omega_{gc}^2 = 9.51 \quad \text{(taking only positive value)}$$

$$\omega_{gc} = 3.1 \text{ rad/sec}$$

- The phase of the system at $\omega = \omega_{gc}$ is,

$$\phi_{gc} = -90° - \tan^{-1}(\omega_{gc}) = -90° - \tan^{-1}(3.1) = -162°$$

- The phase margin of the uncompensated system is,

$$(PM)_{\text{uncompensated}} = 180° - 162° = 18°$$

To design the required compensator:

$$(PM)_{\text{uncompensated}} = 18°$$

$$(PM)_{\text{overall}} \geq 43°$$

- So, the compensator to be designed should be a lead-compensator and the phase lead to be provided at new gain crossover frequency is,

$$\phi_m = 43° - 18° + \varepsilon$$

ε = margin of safety and it can be taken as 5° for this problem, as the gain of the uncompensated system is rolling-off with a slope of –40 dB/decade nearer to its gain crossover frequency.

So, $$\phi_m = 43° - 18° + 5° = 30°$$

- The general form of the transfer function of lead compensator to be designed can be given as,

$$G_c(s) = \frac{K_c(1+s\tau)}{(1+s\alpha\tau)} \quad \text{...(i)}$$

- The value of K_c can be determined by using desired K_v as follows:

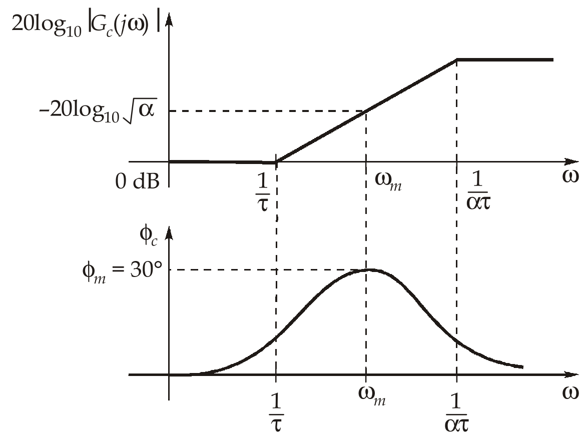
$$K_v = \lim_{s\to 0} sG_f(s)G_c(s) = \lim_{s\to 0}\frac{10K_c(1+s\tau)s}{s(s+1)(1+s\alpha\tau)}$$

$$10K_c = 10 \Rightarrow K_c = 1$$

- The constant "α" can be given as,

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1-0.5}{1+0.5} = \frac{1}{3}$$

- The magnitude and phase bode plots of the lead compensator block can be given as follows:



$$\omega_m = \sqrt{\left(\frac{1}{\tau}\right)\left(\frac{1}{\alpha\tau}\right)} = \frac{1}{\sqrt{\alpha}\,\tau}$$

$$\tau = \frac{1}{\omega_m\sqrt{\alpha}}$$

- The value of ω_m should be selected at the new gain crossover frequency. So, at $\omega = \omega_m$ the gain of overall open-loop transfer function should be unity and it can be determined as follows:

$$|G_f(j\omega)||G_c(j\omega)| = 1$$

$$|G_c(j\omega)|_{\omega=\omega_m} = \frac{1}{\sqrt{\alpha}}$$

So,

$$|G_f(j\omega)|\Big|_{\omega=\omega_m} = \sqrt{\alpha}$$

$$\frac{10}{\omega_m\sqrt{(1+\omega_m^2)}} = \sqrt{\alpha} = \frac{1}{\sqrt{3}}$$

$$\omega_m^4 + \omega_m^2 - 300 = 0$$

$$\omega_m^2 = 16.83 \quad \text{(considering only positive value)}$$

$$\omega_m = 4.1 \text{ rad/sec}$$

$$\tau = \frac{1}{\omega_m\sqrt{\alpha}} = 0.42$$

$$\alpha\tau = 0.14$$

- By substituting the value of τ, $\alpha\tau$ and K_c in equation (i), we get the transfer function of the desired compensator, which is as follows:

$$G_c(s) = \frac{(1+0.42s)}{(1+0.14s)}$$

- The open-loop transfer function of the overall system (or) compensated system is,

$$G(s) = G_f(s)\,G_c(s) = \frac{10(1+0.42s)}{s(1+s)(1+0.14s)}$$

- The gain crossover frequency of the overall system is $\omega_{gc} = \omega_m = 4.1$ rad/sec
- The phase margin of the overall system can be calculated as,

$$\phi_{gc} = -90° - \tan^{-1}(\omega_{gc}) - \tan^{-1}(0.14\,\omega_{gc}) + \tan^{-1}(0.42\omega_{gc})$$

$$= -136.3°$$

$$(\text{PM})_{\text{compensated}} = 180° - 136.3° = 43.7°$$

- The phase margin of uncompensated system is 18°. So, the designed compensator improved the phase margin of the system by 25.7°.

Q.8 (a) Solution:

(i)
- open-loop transfer function, $G(s)H(s) = \dfrac{K}{s(s+2)(s+5)}$.
- The open-loop poles are located at $s = 0; -2; -5$. Hence, $p = 3$.
- There are no open-loop zero's. Hence, $z = 0$.
- The root locus on the real axis is in between

$$s = 0 \text{ to } -2 \text{ and}$$

$$s = -5 \text{ to } -\infty$$

- Angle of asymptotes,

$$\alpha_n = \frac{(2n+1)180°}{p\text{-}z};\ n = 0, 1, \ldots p\text{-}z\text{-}1$$

$$\alpha_n = \frac{(2n+1)180°}{3-0}$$

$$\alpha_n = (2n+1)60°$$

$$\alpha_0 = 60°$$

$$\alpha_1 = 180°$$

$$\alpha_2 = 300°$$

- Point of intersection of asymptotes and real axis,

$$\text{Centroid} = \frac{\Sigma\text{real part of poles} - \Sigma\text{real part of zeros}}{p\text{-}z}$$

$$\text{Centroid} = \frac{(-0-2-5)}{3-0}$$

$$\text{Centroid} = \frac{-7}{3} = -2.33$$

- Break away points, $\frac{dK}{ds} = 0$

The characteristic equation is $1 + G(s)H(s) = 0$

$$s(s+2)(s+5) + K = 0$$

$$K = -s(s+2)(s+5) = -s(s^2+7s+10)$$

$$-\frac{dK}{ds} = s(2s+7) + (s^2+7s+10)$$

$$= 3s^2 + 14s + 10$$

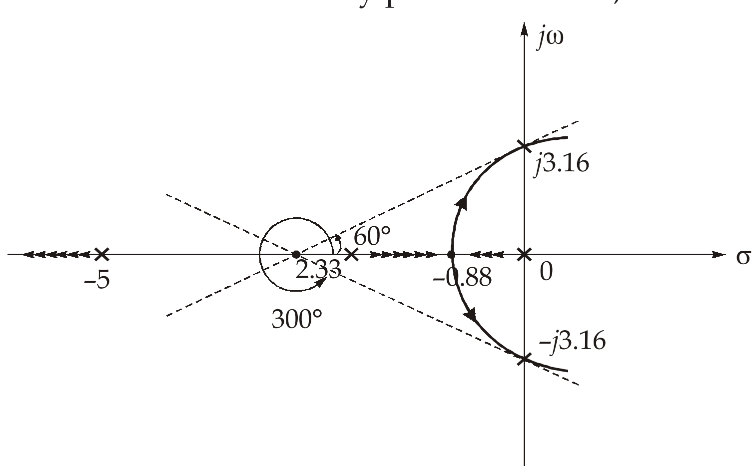
$$\frac{dK}{ds} = 0$$

$$3s^2 + 14s + 10 = 0$$

$$s = -0.88; -3.79$$

Since, root locus is only present between (–2, 0) and (–5, –∞).

Hence, –3.79 is not a valid break away point. Therefore, s = –3.79 is discarded.



Intersection of root locus with imaginary Axis

The characteristic equation

$$1 + G(s)H(s) = 0$$

$$s(s + 2)(s + 5) + K = 0$$

$$s^3 + 7s^2 + 10s + K = 0$$

From Routh-Hurtwiz criterion,

s^3	1	10
s^2	7	K
s^1	$\frac{70 - K}{7}$	
s^0	K	

For the system to be on the verge of stability

$$\frac{70 - K}{K} = 0 \text{ i.e., } K = 70$$

The auxiliary equation gives the location of poles on the imaginary axis,

$$7s^2 + 70 = 0$$

$$s = \pm j3.16$$

(ii) We have,

$$G(s) = \frac{K}{s(s+2)(s+5)}$$

characteristic equation, $1 + G(s)H(s) = 0$

$$s(s + 2)(s + 5) + K = 0$$

$$s^3 + 7s^2 + 10s + K = 0 \quad ...(i)$$

Now, suppose a characteristic equation of a 3rd order system having one pole at $s = -a$

$$(s + a)(s^2 + 2\xi\omega_n s + \omega_n^2) = 0$$

On solving we get it as,

$$s^3 + s^2(2\xi\omega_n + a) + s\left(\omega_n^2 + 2\xi a\omega_n\right) + a\omega_n^2 = 0 \quad ...(ii)$$

On comparing (i) and (ii), we get

$$2\xi\omega_n + a = 7; \quad \omega_n^2 + 2\xi a\omega_n = 10; \quad a\omega_n^2 = K$$

according to question, we have $\xi = 0.5$

$$\omega_n + a = 7; \quad \omega_n^2 + a\omega_n = 10; \ a\omega_n^2 = K$$

$$a = 7 - \omega_n$$

$$\omega_n^2 + (7-\omega_n)\omega_n = 10$$

$$\omega_n^2 + 7\omega_n - \omega_n^2 = 10$$

$$\omega_n = 1.43 \text{ rad/sec}$$

Since, $\omega_n + a = 7$

$$a = 7 - \omega_n = 7 - 1.43$$

$$a = 5.57$$

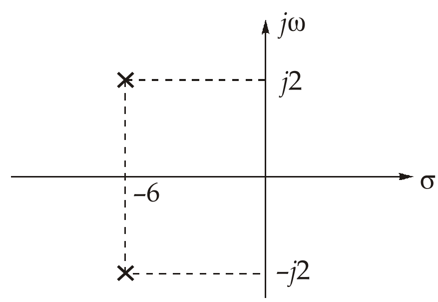
$\therefore$ $a\omega_n^2 = K$

$$(5.57)(1.43)^2 = K$$

$$K = 11.40$$

Q.8 (b) Solution:

(i) We have,



The poles of a second order system are given by $s = -\xi\omega_n \pm j\omega_d$

Thus, $-\xi\omega_n = -6$ and $\omega_d = \omega_n\sqrt{1-\xi^2} = 2$ ...(ii)

$$\xi\omega_n = 6 \quad \text{...(i)}$$

On solving (i) and (ii), we get

$$\omega_n = 6.32 \text{ rad/sec} \quad \text{and} \quad \xi = 0.95$$

1. Transfer function for the standard 2nd order system is given as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{(6.32)^2}{s^2 + 2(6.32)(0.95)s + (6.32)^2}$$

$$\frac{C(s)}{R(s)} = \frac{40}{s^2 + 12s + 40}$$

2. Settling time $= \frac{4}{\xi\omega_n}$ sec (for 2% tolerance band)

$$= \frac{4}{6} = 0.66 \text{ sec}$$

3. Percentage peak overshoot;

$$\%M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$$

$$= e^{\frac{-\pi(0.95)}{\sqrt{1-(0.95)^2}}} \times 100\%$$

$$= 7.06 \times 10^{-3}\%$$

As ξ is close to 1; M_p is close to 0.

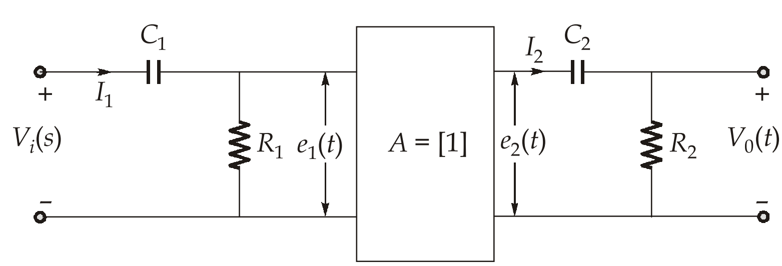
4. Rise time:

$$t_r = \frac{\pi - \cos^{-1}(\xi)}{\omega_d} \text{sec} = \frac{\pi - \cos^{-1}(0.95)}{2} \text{sec} = 1.41 \text{ sec}$$

5. Delay time:

$$t_s = \frac{1 + 0.7\xi}{\omega_n} = \frac{1 + 0.7(0.95)}{6.32} = 0.26 \text{ sec}$$

(ii) Redrawing the circuit diagram as shown in figure below and applying Kirchoff's laws, we get



$$V_i(s) = I_1(s)R_1 + \frac{I_1(s)}{sC_1}; \quad E_1(s) = I_1(s)R_1$$

As amplifier of unity voltage gain is used, it implies $E_1(s) = E_2(s)$

and $$E_2(s) = I_2(s)R_2 + \frac{I_2(s)}{sC_2}; \quad V_0(s) = I_2(s)R_2$$

Transfer function, $$\frac{V_0(s)}{V_i(s)} = \frac{I_2(s)R_2}{I_1(s)R_1 + \frac{I_1(s)}{sC_1}} = \left(\frac{I_2(s)}{I_1(s)}\right) \times \frac{R_2}{\left(R_1 + \frac{1}{sC_1}\right)}$$

where, $$I_2(s) = \frac{E_2(s)}{R_2 + \frac{1}{sC_2}} \quad \text{and} \quad I_1(s) = \frac{E_1(s)}{R_1}$$

Then, $$\frac{V_0(s)}{V_i(s)} = \left[\frac{E_2(s)/\left(R_2 + \frac{1}{sC_2}\right)}{E_1(s)/R_1}\right] \times \frac{R_2}{\left(R_1 + \frac{1}{sC_1}\right)}$$

$\because$ $E_1(s) = E_2(s)$ then, after substituting in the above equation, we get

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{\left(1 + \frac{1}{sR_2C_2}\right)\left(1 + \frac{1}{sR_1C_1}\right)}$$

Substituting the value of R_1, R_2, C_1 and C_2, we get

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{\left(1 + \frac{2}{s}\right)\left(1 + \frac{1}{s}\right)} = \frac{s^2}{(s+2)(s+1)}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{s^2}{(s^2 + 3s + 2)}$$

For unit step input, $$V_i(s) = \frac{1}{s}$$

$$V_0(s) = \frac{s^2}{(s^3 + 3s + 2)} \times \frac{1}{s}$$

$$V_0(s) = \frac{s}{(s^2 + 3s + 2)} = \frac{2}{(s+2)} - \frac{1}{(s+1)}$$

$$V_0(t) = L^{-1}(V_0(s))$$

$$V_0(t) = (2e^{-2t} - e^{-t})u(t)$$

Q.8 (c) Solution:

The given open loop system has no poles in right half of s-plane i.e., $P = 0$. So for closed loop system to be stable, the Nyquist plot must not encircle the $(-1 + j0)$ point of the $q(s)$ plane.

$$G(j\omega)H(j\omega) = \frac{K(j\omega+4)}{(j\omega)^2(j\omega+2)} = \frac{K(j\omega+4)(2-j\omega)}{-\omega^2(2+j\omega)(2-j\omega)}$$

$$= \frac{-K(8+\omega^2)}{\omega^2(4+\omega^2)} + \frac{j2K\omega}{\omega^2(4+\omega^2)}$$

Along the segment (C_1) of the Nyquist contour on the $j\omega$-axis, $j\omega$ varies from $-j\infty$ to $+j\infty$.

At $\omega = -\infty$, $G(j\omega)H(j\omega) = -0 - j0$.

At $\omega = 0^-$, $G(j\omega)H(j\omega) = -\infty - j\infty$

At $\omega = 0^+$, $G(j\omega)H(j\omega) = -\infty + j\infty$

At $\omega = +\infty$, $G(j\omega)H(j\omega) = -0 + j0$

So, we get points to draw an approximate Nyquist plot. The semicircular indent around the plot at the origin of the Nyquist contour represented by

$$s = \underset{\text{limit}\to 0}{\in e^{j\theta}} \quad (\theta \text{ varying from } -90° \text{ to } 0° \text{ to } 90°)$$

is mapped into

$$\lim_{\in\to 0} \frac{K(\in e^{j\theta}+4)}{(\in e^{j\theta})(\in^{j\theta}+1)} = \infty e^{-j2\theta}$$

$$= \infty\angle 180° \to \infty\angle 0° \to \infty\angle -180°$$

It is an infinite circular arc with clockwise direction. The infinite semicircular arc of Nyquist contour (C_2) is

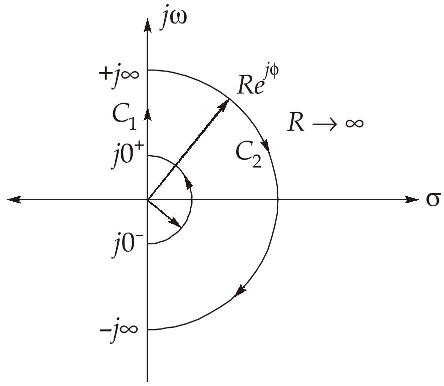
$$s = \lim_{R\to\infty} \text{Re}^{j\phi} \quad (\phi \text{ varying from } +90° + 0° \text{ to } -90°)$$

is mapped into

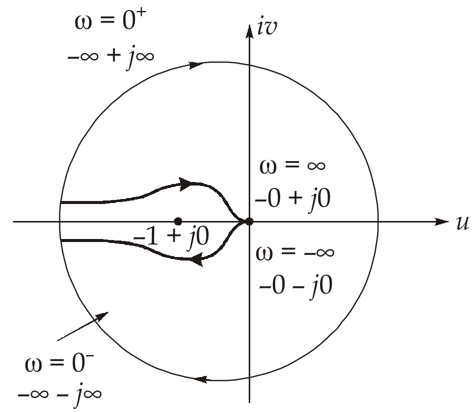
$$\lim_{R\to\infty} \frac{K(\text{Re}^{j\phi}+4)}{(\text{Re}^{j\phi})^2(\text{Re}^{j\phi}+1)} = 0e^{-2j\phi}$$

$$= \angle -180° \to \angle 0° \to \angle 180°$$

The map turns around the origin from $\angle -180° \to \angle 0° \to \angle 180°$ as shown in figure below,



(i) Nyquist contour



(ii) Nyquist plot

The point of intersection of Nyquist plot on real axis is obtained by setting imaginary part to zero.

$$\frac{2K\omega}{\omega^2(4+\omega^2)} = 0$$

$$\omega = 0$$

The value of real part at the frequency is obtained by substituting this value of ω in the real part of $G(j\omega)H(j\omega)$ i.e.,

$$\frac{-K(8+\omega^2)}{\omega^2(4+\omega^2)} = -\infty$$

Based on the above information, it can be concluded that system is unstable as the Nyquist plot encircles the $(-1 + j0)$ point twice in clockwise direction.

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