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Detailed Solutions

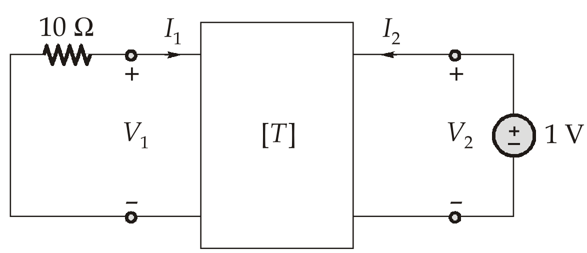
ESE-2024 Mains Test Series

Electrical Engineering Test No : 1

Section A : Electrical Circuits

Q.1 (a) Solution:

Calculating Z_{th} using the circuit in figure,



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Substituting the given *ABCD* parameters,

We obtain,

$$V_1 = 4V_2 - 20I_2 \quad ...(i)$$
$$I_1 = 0.1V_2 - 2I_2 \quad ...(ii)$$

At the input port, $V_1 = -10I_1$

Substituting this into equation (i) gives,

$$-10I_1 = 4V_2 - 20I_2$$
$$I_1 = -0.4V_2 + 2I_2 \quad ...(iii)$$

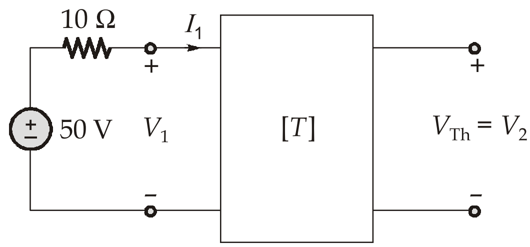
Setting the right hand sides of equations (ii) and (iii) equal,

$$0.1V_2 - 2I_2 = -0.4V_2 + 2I_2$$

$$0.5V_2 = 4I_2$$

Hence, $$Z_{Th} = \frac{V_2}{I_2} = \frac{4}{0.5} = 8\ \Omega$$

To find V_{th}, we use the circuit shown in figure below,



At the output port $I_2 = 0$ and at the input port

$$V_1 = 50 - 10I_1$$

Substituting these into equation (i),

$$50 - 10I_1 = 4V_2 \quad \text{...(iv)}$$

$$I_1 = 0.1V_2 \quad \text{...(v)}$$

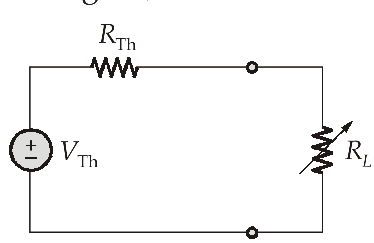
Substituting equation (v) into equation (iv),

$$50 - V_2 = 4V_2$$

$$V_2 = 10$$

Thus, $$V_{Th} = V_2 = 10\ \text{V}$$

The equivalent circuit shown in figure,



For maximum power transfer,

$$R_L = Z_{Th} = 8\ \Omega$$

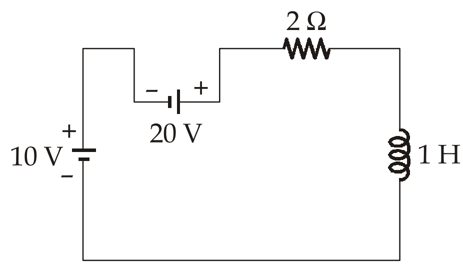
The maximum power is

$$P = I^2R_L = \left(\frac{V_{Th}}{2R_L}\right)^2 R_L = \frac{V_{Th}^2}{4R_L}$$

$$= \frac{100}{4\times 8} = 3.125 \text{ W}$$

Q.1 (b) Solution:

After opening the switch, by using KVL in the loop,



The circuit equation can be written as

$$2i(t) + \frac{d\,i(t)}{dt} = 30 \quad \text{...(i)}$$

By applying Laplace transform to the above differential equation (i), we get

$$2\,I(s) + [sI(s) - i(0^-)] = \frac{30}{s} \quad \text{...(ii)}$$

Given, $i(0^-) = 1$ at the instant of opening the switch,

The equation (ii) can written as,

$$2I(s) + sI(s) = \frac{30}{s} + 1$$

$$I(s) = \frac{s+30}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$A = \left[s\times\frac{(s+30)}{s(s+2)}\right]_{s=0} = \frac{30}{2} = 15$$

and

$$B = \left[(s+2)\times\frac{(s+30)}{s(s+2)}\right]_{s=-2} = -14$$

∴

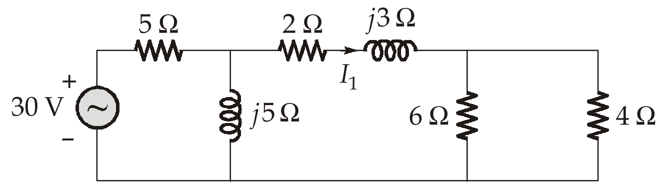
$$I(s) = \frac{15}{s} - \frac{14}{s+2}$$

$$I(s) \rightleftharpoons i(t)$$

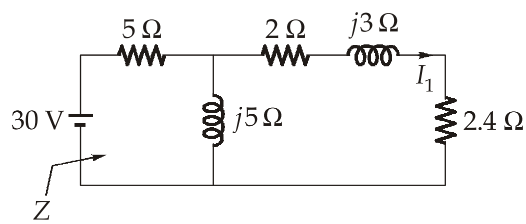
$$i(t) = (15 - 14e^{-2t})u(t)$$

Q.1 (c) Solution:

When the 30 V source is acting alone, let the current through the branch $(2 + j3)\Omega$ be I_1 .



Circuit can be redrawn as



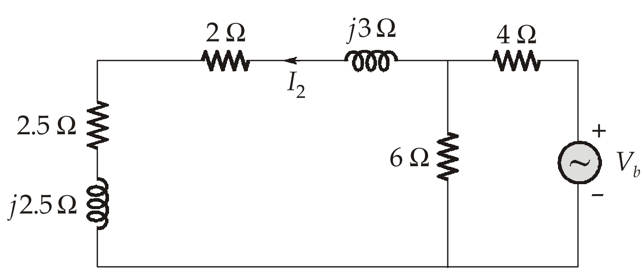
$$\text{Impedance,} \quad Z = 5 + \frac{j5 \times (4.4 + j3)}{4.4 + j8} = \left(\frac{7 + j62}{4.4 + j8}\right)\Omega$$

$$\therefore \quad I = \frac{30}{Z} = \frac{30(4.4 + j8)}{7 + j62}$$

$$\therefore \quad I_1 = I \times \frac{j5}{4.4 + j8}$$

$$= \frac{30(4.4 + j8)}{7 + j62} \times \left(\frac{j5}{4.4 + j8}\right) = \frac{j150}{7 + j62}\text{A}$$

When the V_b source is acting alone, let the current through the branch $(2 + j3)\Omega$ be I_2



$$\text{Impedance, } Z = 4 + \frac{6 \times (4.5 + j5.5)}{10.5 + j5.5} = \frac{69 + j55}{10.5 + j5.5}\Omega$$

$$\therefore \quad I' = \frac{V_b}{Z} = \frac{V_b(10.5 + j5.5)}{69 + j55}$$

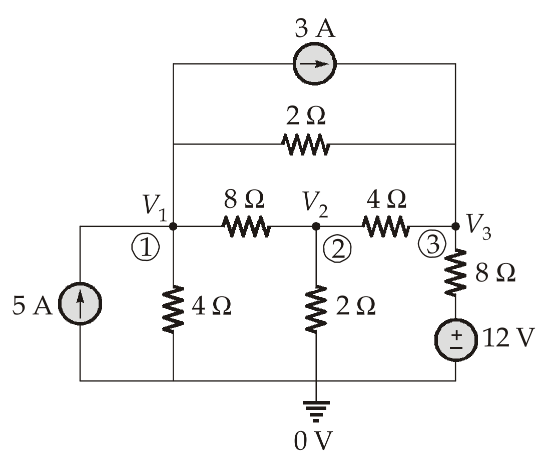
$\therefore$
$$I_2 = I' \times \frac{6}{10.5 + j5.5}$$
$$= \frac{V_b(10.5 + j5.5)}{69 + j55} \times \frac{6}{10.5 + j5.5}$$
$$= \frac{6V_b}{69 + j55} \text{A}$$

Current through the branch $(2 + j3)\Omega$ will be zero

If $I_1 = I_2$ ...(superposition theorem)

$$\frac{j150}{7 + j62} = \frac{6V_b}{69 + j55}$$
$$V_b = (25 + j25)\text{V}$$
$$= 35.35\angle 45° \text{ V}$$

Q.1 (d) Solution:



Applying KCL at node (1)

$$\frac{V_1}{4} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{2} + 3 = 5$$
$$2V_1 + V_1 - V_2 + 4V_1 - 4V_3 = 16$$
$$7V_1 - V_2 - 4V_3 = 16 \quad ...(1)$$

Applying KCL at node (2)

$$\frac{V_2 - V_1}{8} + \frac{V_2}{2} + \frac{V_2 - V_3}{4} = 0$$
$$V_2 - V_1 + 4V_2 + 2V_2 - 2V_3 = 0$$
$$-V_1 + 7V_2 - 2V_3 = 0 \quad ...(2)$$

Applying KCL at node (3)

$$\frac{V_3 - 12}{8} + \frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{2} = 3$$

$$V_3 - 12 + 2V_3 - 2V_2 + 4V_3 - 4V_1 = 24$$

$$-4V_1 - 2V_2 + 7V_3 = 36 \quad \text{...(3)}$$

$$\begin{bmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 36 \end{bmatrix}$$

Determinant of conductance matrix

$$\Delta = \begin{vmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{vmatrix} \begin{array}{l} \Rightarrow \Delta = 7(49-4) + 1(-7-8) - 4(2+28) \\ \Rightarrow \Delta = 7 \times 45 + 1 \times (-15) - 4 \times 30 \\ \Rightarrow \Delta = 180 \end{array}$$

$$\Delta_1 = \begin{vmatrix} 16 & -1 & -4 \\ 0 & 7 & -2 \\ 36 & -2 & 7 \end{vmatrix} \begin{array}{l} \Rightarrow \Delta_1 = 16(49-4) + 1(0+72) - 4(-252) \\ \Rightarrow \Delta_1 = 1800 \end{array}$$

$$\Delta_2 = \begin{vmatrix} 7 & 16 & -4 \\ -1 & 0 & -2 \\ -4 & 36 & 7 \end{vmatrix} \begin{array}{l} \Rightarrow \Delta_2 = 7(72) - 16(-7-8) - 4(-36) \\ \Rightarrow \Delta_2 = 888 \end{array}$$

$$\Delta_3 = \begin{vmatrix} 7 & -1 & 16 \\ -1 & 7 & 0 \\ -4 & -2 & 36 \end{vmatrix} \begin{array}{l} \Rightarrow \Delta_3 = 7(252-0) + 1(-36) + 16(30) \\ \Rightarrow \Delta_3 = 2208 \end{array}$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{1800}{180} = 10 \text{ V}; V_2 = \frac{\Delta_2}{\Delta} = \frac{888}{180} = 4.933 \text{ V};$$

$$V_3 = \frac{\Delta_3}{\Delta} = 12.267$$

Alternatively,

On solving equation (1), (2), (3)

$$V_1 = 10\text{V}$$
$$V_2 = 4.933\text{V}$$
$$V_3 = 12.26\text{V}$$

Q.1 (e) Solution:

$$\frac{V_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j3} + 5\angle 90 = 0$$

$$V_1\left[\frac{1}{3}+\frac{1}{-j5}+\frac{1}{j3}\right]-V_2\left[\frac{1}{-j5}+\frac{1}{j3}\right] = -j5$$

$$(0.33 - j0.133)V_1 + V_2(j0.133) = -j5$$

$$V_1 = \frac{-j5 - j0.133V_2}{0.33 - j0.133} = 5.26 - j13.02 - V_2[-0.139 + j0.346]$$

$$V_1 = 5.26 - j13.02 + (0.139 - j0.346)V_2$$

Apply KCL at node (2)

$$\frac{V_2 - V_1}{-j5}+\frac{V_2 - V_1}{j3}+\frac{V_2}{6} = 10\angle 0$$

$$-V_1\left[\frac{1}{-j5}+\frac{1}{j3}\right]+V_2\left[\frac{1}{-j5}+\frac{1}{j3}+\frac{1}{6}\right] = 10$$

$$j0.133V_1 + (0.166 - j0.133)V_2 = 10$$

$$j0.133[5.26 - j13.02 + (0.139 - j0.346)\ V_2] + (0.166 - j0.133)V_2 = 10$$

$$j0.7 + 1.73 + j0.0184V_2 + 0.046V_2 + 0.166V_2 - j0.133V_2 = 10$$

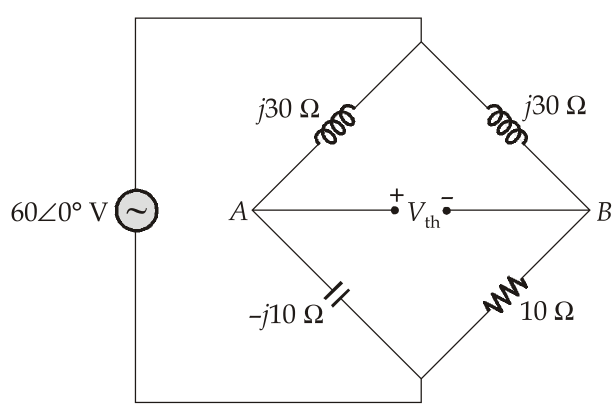
$$(0.212 - j0.1146)V_2 = -j0.7 - 1.73 + 10$$

$$V_2 = 31.569 + j13.76$$

$$V_2 = 34.44\angle 23.56 \text{ V}$$

Q.2 (a) Solution:

For Thevenin's equivalent voltage across AB, i.e., V_{th},



$$V_{th} = V_{AB} = V_A - V_B$$

By using voltage division rule

The node voltage, $$V_A = \frac{-j10}{(j30 - j10)}(60\angle 0°) = -30 \text{ V}$$

The node voltage, $V_B = \left(\dfrac{10}{10+j30}\right)\times(60\angle 0°)$

$$= \frac{600}{(10+j30)}\times\frac{(10-j30)}{(10-j30)}$$

$$V_B = \frac{600}{1000}(10-j30)$$

$$= \frac{3}{5}\times 10 - j\frac{3}{5}\times 30 = (6 - j18)\text{ V}$$

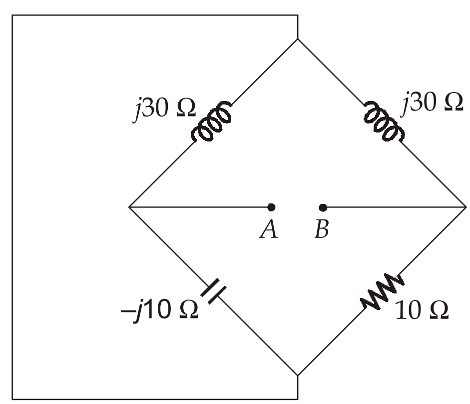
∴ Thevenin's voltage, $V_{th} = V_A - V_B$

$$= -30 - (6 - j18)$$

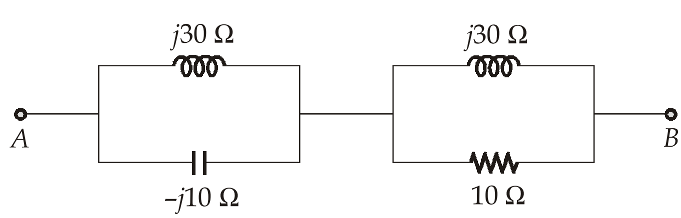
$$= (-36 + j18)\text{ V}$$

∴ $V_{th} = 40.24\angle 153.43°\text{ V}$

For calculating Thevenin's impedance across AB terminal i.e. Z_{th} the independent voltage source is replaced by short circuit as below,



By simplying the above network



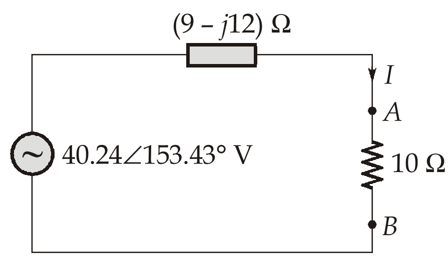
$$Z_{Th} = Z_{AB} = (j30 \,||\, -j10) + (j30 \,||\, 10)$$

$$= \frac{j30(-j10)}{j20} + \frac{j30\times 10}{10+j30}$$

$$= -j15\ \Omega + 9 + j3\ \Omega$$

$$= (9 - j12)\Omega$$
$$= 15\angle{-53.13°}\ \Omega$$

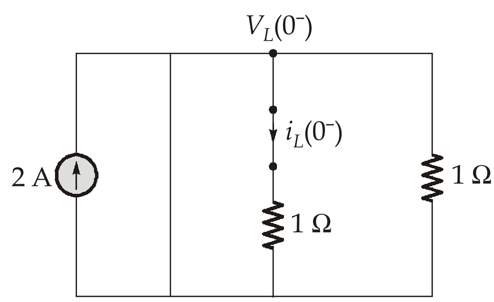
The Thevenin's equivalent network is



The current,
$$I = \frac{V_{th}}{Z_{th} + 10} = \frac{40.24\angle 153.43°}{9 - j12 + 10}$$
$$= 1.79\angle{-174.29}\ \text{A}$$

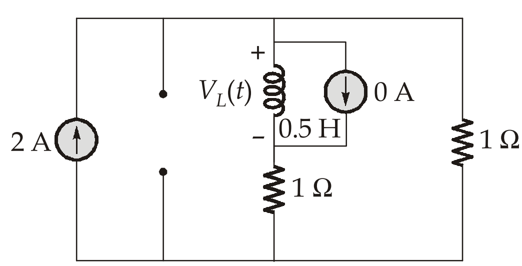
Q.2 (b) (i) Solution:

At $t = 0^-$, the switch is closed and steady state condition is reached.

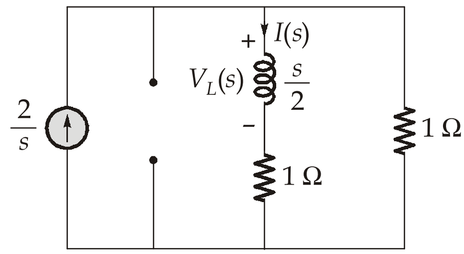


$\therefore$
$$i_L(0^-) = 0 = i_L(0^+)$$
$$V_L(0^-) = 0$$

for $t > 0$; the transformed network is,



By using Laplace transform approach,



$$I(s) = \frac{\frac{2}{s}\times 1}{1+1+\frac{s}{2}} = \frac{\frac{2}{s}}{\frac{s+4}{2}} = \frac{4}{s(s+4)}$$

$$V_L(s) = \frac{s}{2}I(s)$$

$$= \frac{s}{2}\frac{4}{s(s+4)}$$

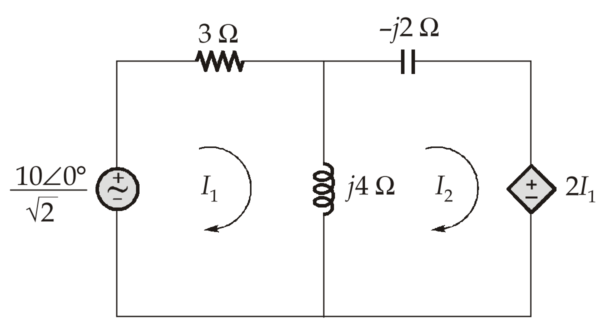
$$V_L(s) = \frac{2}{s+4} \quad \Rightarrow \quad v_L(t) = 2e^{-4t};\ t > 0$$

Q.2 (b) (ii) Solution:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} = \frac{1}{1000\times 500\times 10^{-6}} = 2\Omega$$

$$X_L = \omega L = 1000 \times 4 \times 10^{-3} = 4\Omega$$

Applying KVL in mesh 1:



$$3I_1 + j4(I_1 - I_2) = \frac{10}{\sqrt{2}}\angle 0°$$

$$(3 + j4)I_1 - j4I_2 = \frac{10}{\sqrt{2}}\angle 0° \qquad ...(1)$$

Applying KVL in mesh 2

$$j4(I_2 - I_1) - j2I_2 + 2I_1 = 0$$

$$(2 - j4)I_1 + j2I_2 = 0 \quad \text{...(2)}$$

From eqn. (1) and eqn. (2), we obtain matrix equation

$$\begin{bmatrix} 3+j4 & -j4 \\ 2-j4 & j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{10}{\sqrt{2}}\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3+j4 & -j4 \\ 2-j4 & j2 \end{vmatrix}$$

$$= (3 + j4)(j2) + j4(2 - j4)$$

$$= j6 - 8 + j8 + 16 = 8 + j14$$

$$\Delta_1 = \begin{vmatrix} \frac{10}{\sqrt{2}} & -j4 \\ 0 & j2 \end{vmatrix} = j14.14$$

$$\Delta_2 = \begin{vmatrix} 3+j4 & \frac{10}{\sqrt{2}} \\ 2-j4 & 0 \end{vmatrix} = -14.14 + j28.28$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{j14.14}{8+j14} = 0.87693\angle 29.7^\circ$$

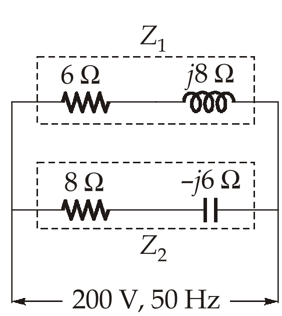
$$I_1 = 1.24\cos(1000t + 29.7^\circ)\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-14.14 + j28.28}{8+j14} = 1.9609\angle 56.3^\circ\text{A}$$

$$I_2 = 2.773\cos(1000t + 56.3^\circ)\text{A}$$

Q.2 (c) Solution:

Given circuit,



(i) The impedance, $Z_1 = 6 + j8 = 10\angle 53.13^\circ\ \Omega$

The impedance, $Z_2 = 8 - j6 = 10\angle -36.87^\circ\ \Omega$

The admittance, $Y_1 = \frac{1}{Z_1} = \frac{1}{10\angle 53.13^\circ} = 0.1\angle -53.13^\circ\ \mho$

The admittance, $Y_2 = \frac{1}{Z_2} = \frac{1}{10\angle -36.87^\circ} = 0.1\angle 36.87^\circ\ \mho$

$$Y = Y_1 + Y_2$$
$$= 0.1\angle -53.13^\circ + 0.1\angle 36.87^\circ$$
$$= 0.14\angle -8.13^\circ\ \mho$$
$$= (0.14 - j0.02)\ \mho$$

Total admittance $Y = 0.14 - j0.02\ \mho$

Total conductance $G = 0.14\ \mho$

Total susceptance $B_L = 0.02\ \mho$

(ii) We have, source voltage, $V = 200\angle 0^\circ$ V

Current, $I = V \cdot Y$

$$= (200\angle 0^\circ)(0.14\angle -8.13^\circ) = 28\angle -8.13^\circ \text{ A}$$

Total power factor (p.f) $= \cos\phi$

$$= \cos(8.13^\circ) = 0.989 \text{ (lagging)}$$

(iii) Since the current lags behind voltage, the circuit is inductive in nature. In order to make the total p.f unity, a pure capacitor is connected in parallel and hence, imaginary part of required admittance (Y_{req}) becomes zero.

i.e., $Y_{req} = 0.14 - j0.02 + j0.02 = 0.14\ \mho$

$$B_{C_{req}} = 0.02 = 2\pi f C_{req}$$

$$C_{req} = \frac{0.02}{2\pi \times 50} = 63.66\ \mu\text{F}$$

Alternatively:

$\therefore$ $\frac{1}{X_C} = 0.02$

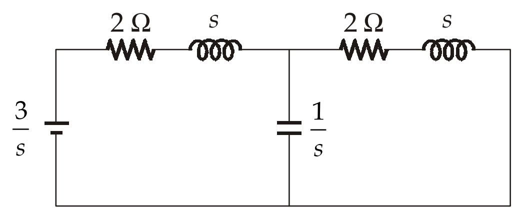
$$X_C = 50\ \Omega$$

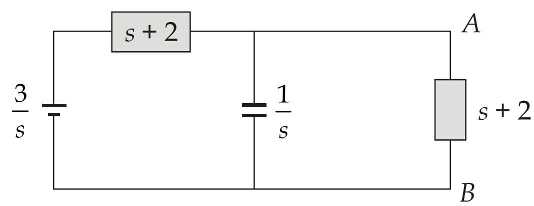
$\therefore$ The required capacitance,

$$C = \frac{1}{2\pi \times 50 \times 50} = 63.66\ \mu\text{F}$$

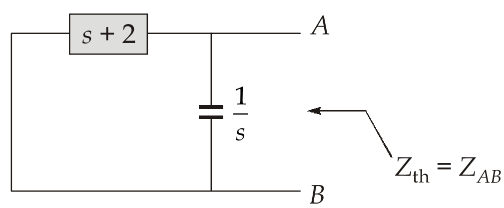
Q.3 (a) Solution:

The network for $t > 0$ in Laplace domain,



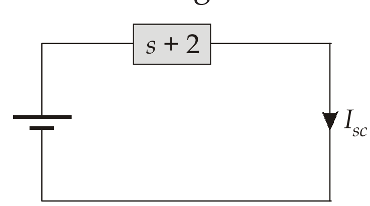


To find current in L_2, we have to find Norton's equivalent circuit across the terminal A and B. The impedance between terminals A and B is calculated deactivating all the sources (voltage source by S.C. and current source by O.C) as shown below.



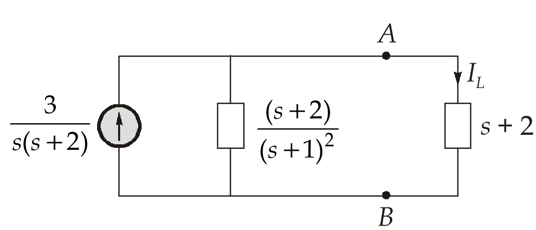
$$Z_{th} = Z_{AB} = \frac{(s+2)\times\frac{1}{s}}{s+2+\frac{1}{s}} = \frac{(s+2)}{s^2+2s+1} = \frac{(s+2)}{(s+1)^2}$$

Short circuit current flowing from A to B is given as



$$I_{SC} = \frac{3/s}{s+2} = \frac{3}{s(s+2)}$$

The Norton's equivalent circuit,



$$I_L = \frac{3}{s(s+2)} \times \frac{(s+2)}{(s+1)^2} \times \frac{1}{\frac{(s+2)}{(s+1)^2} + (s+2)}$$

$$= \frac{3}{s(s+2)(s^2+2s+2)}$$

By Partial fraction expansion,

$$I_L = \frac{3}{s(s+2)(s^2+2s+2)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+1+j1} + \frac{K_3^*}{s+1-j1}$$

Where,

$$K_1 = \left.\frac{3}{(s+2)(s^2+2s+2)}\right|_{s=0} = \frac{3}{4}$$

$$K_2 = \left.\frac{3}{s(s^2+2s+2)}\right|_{s=-2} = \frac{-3}{4}$$

$$K_3 = \left.\frac{3}{s(s+2)(s+1-j1)}\right|_{s=-1-j1} = j\frac{3}{4}$$

$$K_3^* = -j\frac{3}{4}$$

$\therefore$

$$I_L = \frac{\frac{3}{4}}{s} - \frac{\frac{3}{4}}{s+2} + \frac{j\frac{3}{4}}{s+1+j1} - \frac{j\frac{3}{4}}{s+1-j1}$$

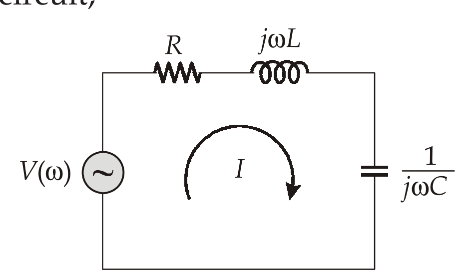
$$= \frac{\frac{3}{4}}{s} - \frac{\frac{3}{4}}{s+2} - \frac{\frac{3}{2}}{(s+1)^2+1}$$

Taking inverse Laplace transform we get the required current as

$$i(t) = \frac{3}{4} - \frac{3}{4}e^{-2t} - \frac{3}{2}e^{-t}\sin t$$

Q.3 (b) Solution:

Figure shows a series RLC circuit,



By KVL
$$I = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$|I| = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At half power frequency,

$$I = \frac{I_0}{\sqrt{2}}$$

$$I_0 = \frac{V}{R}$$

$$\frac{V}{\sqrt{2}R} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

from here
$$\omega L - \frac{1}{\omega C} = \pm R$$

At lower half power frequency,

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad (\because \omega_1 \text{ is positive})$$

At upper half power frequency,

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad (\because \omega_2 \text{ is positive})$$

$$\omega_1\omega_2 = \left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

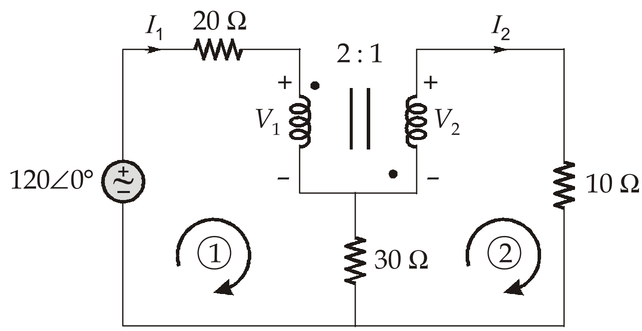
$$= \frac{1}{LC} = \omega_0^2$$

$$\omega_1 \omega_2 = \omega_0^2$$

$$\Rightarrow \qquad \omega_0 = \sqrt{\omega_1 \omega_2}$$

Thus, resonant frequency ω_0 of a series RLC circuit is geometrical mean of ω_1, ω_2.

Q.3 (c) Solution:



For mesh-1,

$$-120 + (20 + 30)I_1 - 30I_2 + V_1 = 0$$

or $$50I_1 - 30I_2 + V_1 = 120 \qquad \text{...(i)}$$

For mesh-2,

$$-V_2 + (10 + 30)I_2 - 30I_1 = 0$$

$$-30I_1 + 40I_2 - V_2 = 0 \qquad \text{...(ii)}$$

At the transformer terminals,

$$V_2 = -\frac{1}{2}V_1 \qquad \text{...(iii)}$$

$$I_2 = -2I_1 \qquad \text{...(iv)}$$

Substitute V_1 and I_1 in terms of V_2 and I_2

$$-55I_2 - 2V_2 = 120 \qquad \text{...(v)}$$

$$15I_2 + 40I_2 - V_2 = 0$$

$$V_2 = 55I_2 \qquad \text{...(vi)}$$

Substituting equation (vi) in equation (v),

$$-165I_2 = 120$$

$$I_2 = \frac{-120}{165} = -0.7272 \text{ A}$$

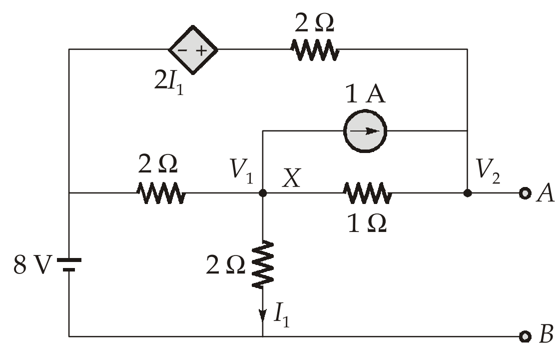
The power absorbed by the 10 Ω resistor is

$$P = (-0.7272)^2 \times 10$$
$$= 5.3 \text{ W}$$

Q.4 (a) Solution:

To calculate the Thevenin's voltage across *A*-*B*, the given circuit is redrawn after opening the resistance across the terminals *A*-*B* as shown in figure below.

Let the voltage at node *X* be V_1 and that at terminal *A* be V_2



Using nodal analysis at node *X*, we obtain

$$\frac{8-V_1}{2} = \frac{V_1-V_2}{1}+\frac{V_1}{2}+1$$

$$2V_1 - V_2 = 3 \qquad \text{...(i)}$$

Again using nodal analysis at terminal *A*, we obtain

$$\frac{8+2I_1-V_2}{2}+1+\frac{V_1-V_2}{1} = 0$$

$$3V_1 - 3V_2 = -10 \qquad \left(\because I_1 = \frac{V_1}{2}\right) \qquad \text{...(ii)}$$

Solving the above equation (i) and (ii), we get

$$V_1 = \frac{19}{3}\text{V}$$

and $$V_2 = \frac{29}{3}\text{V}$$

∴ The open circuit voltage,

$$V_{OC} = V_2 = \frac{29}{3}\text{V} = 9.67 \text{ V}$$

Thevenin's voltage, $$V_{th} = 9.67 \text{ V}$$

After shorting the terminals

KCL at Node (X):

$$\frac{V_1 - 8}{2} + 1 + \frac{V_1}{2} + \frac{V_1}{1} = 0$$

$$V_1 - 8 + 2 + V_1 + 2V_1 = 0$$

$$4V_1 = 6$$

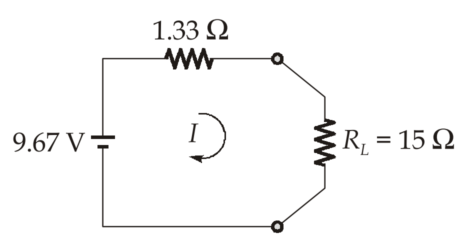
$$V_1 = \frac{3}{2}V$$

$$I_{sc} = 1 + \frac{V_1}{1} + \frac{8 + 2I_1}{2}$$

$$I_{sc} = V_1 + \frac{8 + V_1}{2} + 1$$

$$I_{sc} = 7.25\text{A}$$

$$Z_{\text{Th}} = \frac{V_{OC}}{I_{sc}} = 1.33\Omega$$



$\therefore$ $$\text{Current, } I = \frac{9.67}{1.33 + 15} = 0.592 \text{ A}$$

To get the maximum power transfer through R_L, as per maximum power transfer theorem,

$$R_L = R_{\text{th}} = 1.33\ \Omega$$

$\therefore$ $$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(9.67)^2}{4 \times 1.33} = 17.57 \text{ W}$$

Q.4 (b) Solution:

To find h-parameter, we consider two cases,

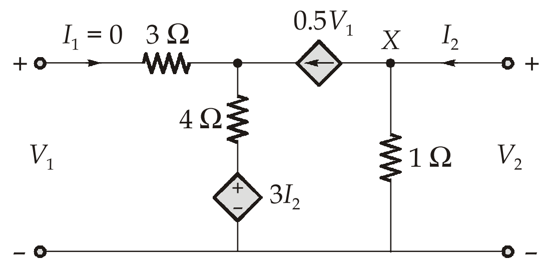
When $I_1 = 0$

Here, no current will flow through the 3 Ω resistance by KVL at the left mesh, we get

$$V_1 = 4 \times 0.5\ V_1 + 3I_2$$

$$= 2V_1 + 3I_2$$

$$V_1 = -3I_2$$



Also, by KCL at node (X), we get

$$I_2 = \frac{V_2}{1} + 0.5V_1 = V_2 + 0.5V_1$$

$$= V_2 + 0.5 \times (-3I_2)$$

$\Rightarrow$ $$2.5I_2 = V_2$$

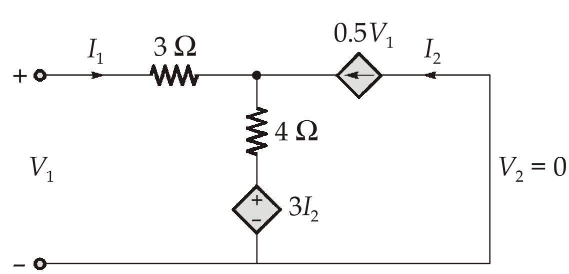
$\therefore$ $$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{2.5} = 0.4\,\mho$$

$\therefore$ $$V_1 = -3I_2 = -3\times\left(\frac{V_2}{2.5}\right) = -1.2V_2$$

$\therefore$ $$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = -1.2$$

Then $$V_2 = 0$$

Here, port-2 is short circuited. The 1 Ω resistance becomes redundant. The modified circuit is shown in figure,



$\therefore$ $$I_2 = 0.5\,V_1$$

$$= 0.5 \times [3I_1 + 4I_1 + 4I_2 + 3I_2]$$

$$= 3.5I_1 + 3.5I_2$$

$$2.5I_2 = -3.5I_1$$

$\therefore$ $$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{3.5}{2.5} = -1.4$$

Also, $V_1 = 3I_1 + 4I_1 + 4I_2 + 3I_2 = 7I_1 + 7I_2$

$= 7I_1 + 7 \times (-1.4I_1)$

$= -2.8\ I_1$

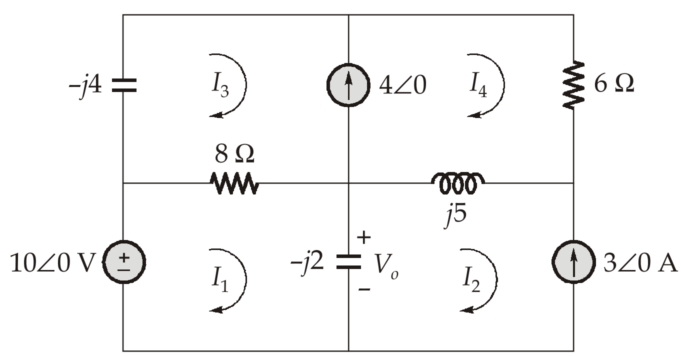
$$\therefore \quad h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = -2.8\ \Omega$$

Therefore, the h-parameter of the network are given as

$$[h] = \begin{bmatrix} -2.8\,\Omega & -1.2 \\ -1.4 & 0.4\,℧ \end{bmatrix}$$

Q.4 (c) Solution:

As shown in figure, meshes 3 and 4 form a supermesh due to the current source between the meshes.



For mesh 1, KVL gives

$$-10 + (8 - j2)I_1 - (-j2)I_2 - 8I_3 = 0$$

$$(8 - j2)I_1 + j2I_2 - 8I_3 = 10 \quad ...(1)$$

$$I_2 = -3 \quad ...(2)$$

For the super mesh,

$$(8 - j4)I_3 - 8I_1 + (6 + j5)I_4 - j5I_2 = 0 \quad ...(3)$$

Due to the current source between meshes 3 and 4

$$-I_3 + I_4 = 4 \quad ...(4)$$

Substitute eqn. (2) in eqn. (1) and eqn. (3)

$$(8 - j2)I_1 - 8I_3 = 10 + j6 \quad ...(5)$$

$$-8I_1 + (8 - j4)I_3 + (6 + j5)I_4 = -j15 \quad ...(6)$$

Substitute $I_4 = 4 + I_3$ in eqn. (6)

$$-8I_1 + (8 - j4)I_3 + (6 + j5)(4 + I_3) = -j15$$

$$-8I_1 + (14 + j1)I_3 = -24 - j35 \quad ...(7)$$

From eqn. (5) and eqn. (7), we obtain matrix equation.

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & (14 + j1) \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j1 \end{vmatrix}$$

$$= (8 - j2)(14 + j1) - 64$$

$$= 112 + j8 - j28 + 2 - 64$$

$$= (50 - j20)$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j1 \end{vmatrix}$$

$$= 140 + j10 + j84 - 6 - 192 - j280$$

$$= -58 - j186$$

Current I_1 is obtained as

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\angle 274.5 \text{ A}$$

The required voltage V_o is

$$V_o = -j2(I_1 - I_2) = -j2(3.618\angle 274.5 + 3)$$

$$V_o = 9.756\angle 222.32° \text{ V}$$

Section B : Control System

Q.5 (a) Solution:

From the given bode magnitude plot, we can write the open loop transfer function as,

$$G(s)H(s) = \frac{K\left(\frac{s}{\omega_2} + 1\right)}{s\left(\frac{s}{\omega_1} + 1\right)\left(\frac{s}{\omega_3} + 1\right)}$$

The slope between the corner frequency ω_1 rad/s and 10 rad/s is –40 dB/decade, therefore, we can write

$$-40 = \frac{0 - 24.1}{\log 10 - \log \omega_1}$$

$$\log\left(\frac{10}{\omega_1}\right) = \frac{24.1}{40}$$

Therefore, corner frequency,

$$\omega_1 = 2.5 \text{ rad/sec}$$

The slope between 10 rad/s and the corner frequency ω_2 rad/s is –40 dB/decade, therefore, we can write,

$$-40 = \frac{-15.92 - 0}{\log \omega_2 - \log 10}$$

$$\log\left(\frac{\omega_2}{10}\right) = \frac{-15.92}{-40}$$

Therefore, corner frequency,

$$\omega_2 = 25 \text{ rad/sec}$$

The slope between the corner frequencies ω_2 rad/sec and ω_3 rad/sec is –20 dB/decade, therefore, we can write,

$$-20 = \frac{-23.52 + 15.92}{\log \omega_3 - \log 25}$$

$$\log\left(\frac{\omega_3}{25}\right) = \frac{-7.6}{-20}$$

∴ Therefore, corner frequency,

$$\omega_3 \simeq 60 \text{ rad/sec}$$

For the initial slope line, the magnitude,

$$M = -20 \times \log \omega + 20 \log K$$

At $\omega_1 = 2.5$ rad/sec, $$M = 24.1 \text{ dB}$$

$$24.1 = -20 \times \log(2.5) + 20 \log K$$

∴ $$K = 40.08 \simeq 40$$

∴ $$G(s)H(s) = \frac{40\left(\frac{s}{25}+1\right)}{s\left(\frac{s}{2.5}+1\right)\left(\frac{s}{60}+1\right)}$$

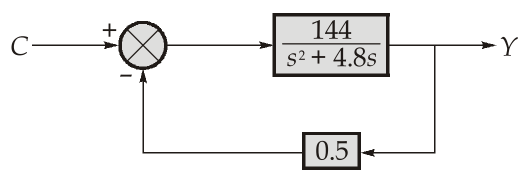
∴ $$G(s)H(s) = \frac{240(s+25)}{s(s+2.5)(s+60)}$$

Q.5 (b) Solution:

Given :
$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144E$$
$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144(C - 0.5y)$$

Taking Laplace on both sides,
$$s^2Y(s) + 4.8sY(s) = 144[C(s) - 0.5Y(s)]$$
$$\frac{Y(s)}{C(s)} = \frac{144}{s^2 + 4.8s + 72} \quad ...(1)$$

Block diagram of the system is shown in figure below :



Comparing $\frac{Y(s)}{C(s)}$ from eqn. (1) to standard 2nd order equation
$$\omega_n = \sqrt{72} = 8.48 \text{ rad/sec}$$
$$2\xi\omega_n = 4.8 \Rightarrow \xi = \frac{4.8}{2\times\omega_n} = \frac{4.8}{2\times 8.48}$$
$$\xi = 0.283$$

Damped frequency of oscillation
$$\omega_d = \omega_n\sqrt{1-\xi^2}$$
$$= 8.48\sqrt{1-0.283^2}$$
$$\omega_d = 8.13 \text{ rad/sec}$$

Q.5 (c) Solution:

To improve transient response, PD controller can be used.

Controller transfer function $= K_P + K_D s$

Forward path transfer function,
$$G(s)H(s) = \frac{28.8(K_P + sK_D)}{s(1+0.2s)}$$

Characteristic equation :

$$1 + G(s)H(s) = 0$$

$$s(1 + 0.2s) + 28.8(K_P + sK_D) = 0$$

$$0.2s^2 + s + s28.8K_D + 28.8K_P = 0$$

$$0.2s^2 + (28.8K_D + 1)s + 28.8K_P = 0$$

$$s^2 + (144K_D + 5)s + 144K_P = 0$$

Comparing above equation with standard 2nd order equation :

$$\omega_n = \sqrt{144K_P} = 12\sqrt{K_P} \quad ...(1)$$

and $$2\xi\omega_n = 144K_D + 5 \quad ...(2)$$

Given there is no overshoot and settling time is minimum.

$$\therefore \quad \xi = 1$$

From eqn. (2),

$$2\omega_n = 144K_D + 5 \quad ...(3)$$

From eqn. (1),

$$\omega_n = 12\sqrt{K_P} \quad \text{(Given : } K_P = 5)$$

$$\omega_n = 12\sqrt{5}$$

$$\omega_n = 26.83 \text{ rad/sec}$$

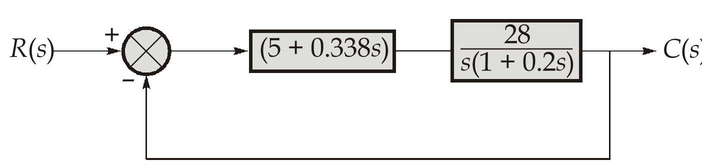
From eqn. (3),

$$2\omega_n = 144K_D + 5$$

$$\frac{2 \times 26.83 - 5}{144} = K_D$$

$$K_D = 0.338$$

$$\text{Controller transfer function} = 5 + 0.338s$$



Q.5 (d) Solution:

Given : $$G(s)H(s) = \frac{K/T}{s\left(s + \frac{1}{T}\right)}$$

Closed loop transfer function $= \dfrac{K/T}{s^2 + \dfrac{s}{T} + \dfrac{K}{T}}$

Comparing characteristic equation with the standard form : $s^2 + 2\xi\omega_n s + \omega_n^2$

$$\omega_n = \sqrt{\frac{K}{T}}$$

and $$2\xi\omega_n = \frac{1}{T}$$

$$\xi = \frac{1}{2T\omega_n} = \frac{1}{2T\sqrt{\frac{K}{T}}} = \frac{1}{2\sqrt{KT}} \quad ...(1)$$

Now, Peak overshoot : $$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \quad ...(2)$$

At $M_{P1} = 80\%(0.8) \Rightarrow \xi = \xi_1$

At $M_{P2} = 50\%(0.5) \Rightarrow \xi = \xi_2$

From eqn. (2),

$$M_{P1} = e^{-\xi_1\pi/\sqrt{1-\xi_1^2}} = 0.8$$

On solving, $\xi_1 = 0.0708$

and $$M_{P2} = e^{-\xi_2\pi/\sqrt{1-\xi_2^2}} = 0.5$$

On solving, $\xi_2 = 0.2154$

At $\xi_1 = 0.0708$, $K = K_1$

and $\xi_2 = 0.2154$, $K = K_2$

From eqn. (1)

$$\frac{\xi_1}{\xi_2} = \sqrt{\frac{K_2}{K_1}}$$

$$\sqrt{K_2} = \frac{\xi_1}{\xi_2}\sqrt{K_1}$$

$$K_2 = \frac{\xi_1^2}{\xi_2^2}K_1 = \frac{(0.0708)^2}{(0.2154)^2} \times K_1$$

$$K_2 = \frac{1}{9.25}K_1$$

So, value of K should be reduced by 9.25 times.

Q.5 (e) Solution:

Given: open loop transfer function,

$$G(s) = \frac{K}{2s(1+0.1s)(1+s)}$$

Put $s = j\omega$,

$$G(j\omega) = \frac{K}{2j\omega(1+j0.1\omega)(1+j\omega)}$$

We know that,

The gain margin, $$\text{G.M} = \left.\frac{1}{|G(j\omega)|}\right|_{\omega=\omega_{pc}}$$

where, ω_{pc} is the frequency at which phase of the system is –180°.

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(0.1\,\omega) - \tan^{-1}(\omega)$$

At $\omega = \omega_{pc}$; $$\angle G(j\omega_{pc}) = -180^\circ$$

$$-180^\circ = -90^\circ - \tan^{-1}(0.1\,\omega_{pc}) - \tan^{-1}(\omega_{pc})$$

$$-90^\circ = -\tan^{-1}\left(\frac{0.1\omega_{pc}+\omega_{pc}}{1-0.1\omega_{pc}^2}\right)$$

$$1-0.1\omega_{pc}^2 = 0$$

$\therefore$ $$\omega_{pc} = \sqrt{10} = 3.162 \text{ rad/sec}$$

Magnitude of the system,

$$|G(j\omega)| = \frac{K}{2\omega\sqrt{\omega^2+1}\sqrt{1+(0.1\omega)^2}}$$

At phase crossover frequency ω_{pc},

$$|G(j\omega_{pc})| = \frac{K}{2\omega_{pc}\sqrt{1+\omega_{pc}^2}\sqrt{1+(0.1\omega_{pc})^2}}$$

$$|G(j\omega_{pc})| = \frac{K}{2\times 3.162\sqrt{(3.162)^2+1}\sqrt{1+(0.1\times 3.162)^2}}$$

$$= \frac{K}{22}$$

Gain margin, $$\text{G.M} = \left.\frac{1}{|G(j\omega)|}\right|_{\omega=\omega_{pc}} = \frac{22}{K}$$

Given, Gain margin = 14 dB

i.e., 20 log (G.M) = 14

$$\therefore \quad G.M = 10^{14/20} = 5.012$$

$$5.012 = \frac{22}{K}$$

$$K = \frac{22}{5.012}$$

$$\therefore \quad K = 4.389$$

Q.6 (a) Solution:

Given : Open loop transfer function = $e^{-Ts}G_1(s)$

where $G_1(s)$ is minimum phase system.

Given magnitude plot belongs to $G_1(s)$ as magnitude plot of e^{-Ts} is a straight line on 0 dB axis.

∴ From the Bode plot

$$G_1(s) = \frac{K}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$$

and also, $20 \log_{10} K - 20 \log_{10} \omega = 40$

Given : $20 \log_{10} K - 20 \log_{10} (0.05) = 40$

After solving, we get $K = 5$

$$\therefore \quad G_1(s) = \frac{5}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$$

Therefore, the open loop transfer function

$$G(s)H(s) = \frac{5 \cdot e^{-Ts}}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$$

$$G(s)H(s) = \frac{100e^{-Ts}}{s(s+2)(s+10)}$$

Now, finding gain crossover frequency to calculate phase margin

$$|G(s)H(s)|_{\omega=\omega_{gc}} = 1$$

$$\frac{100 \times 1}{\omega_{gc}\sqrt{4+\omega_{gc}^2}\ \sqrt{100+\omega_{gc}^2}} = 1$$

On solving, $\omega_{gc} = 2.8$ rad/sec

$$PM = 180 + \angle G(s)H(s)|_{\omega=\omega_{gc}}$$

$$= 180 + \left[-90° - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{gc}}{10}\right) - \frac{\omega_{gc} \times T \times 180°}{\pi}\right]$$

$$= 180 + \left[-90° - \tan^{-1}\left(\frac{2.8}{2}\right) - \tan^{-1}\left(\frac{2.8}{10}\right) - \frac{2.8 \times 180° \times T}{\pi}\right]$$

$$PM = 180 + [-160.1 - 160.428T]$$

Given : Phase margin $= -18.19°$

$$-18.19° = 180 - 160.1 - 160.428T$$

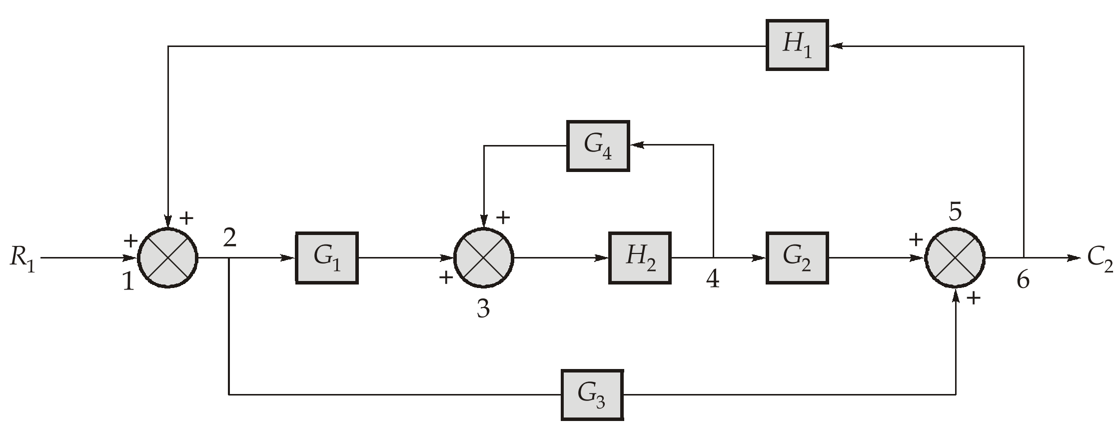
$$-18.19 - 19.9 = -160.428T$$

$$T \approx 237.42 \text{ ms}$$

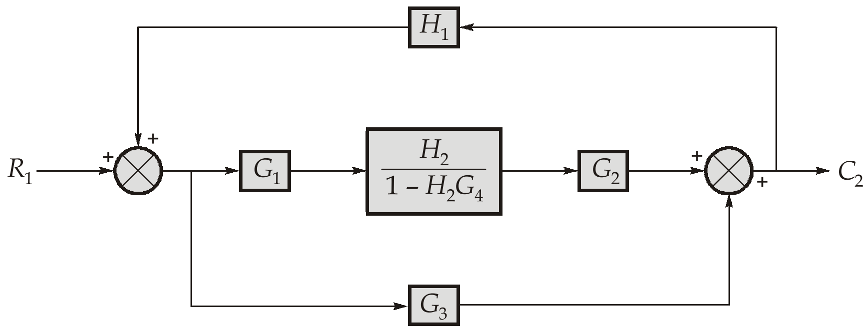
Q.6 (b) (i) Solution:

Assume $R_2 = 0$ and $C_1 = 0$

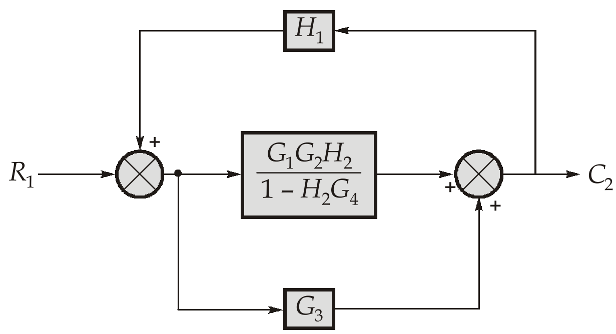
Rearranging, we get

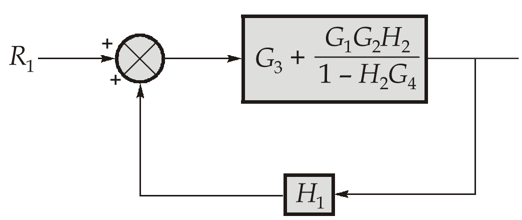


Eliminating feedback path between point 3 and point 4



Further simplifying





$$\frac{C_2}{R_1} = \frac{G_3 + \dfrac{G_1G_2H_2}{1 - H_2G_4}}{1 - \left(G_3 + \dfrac{G_1G_2H_2}{1 - H_2G_4}\right)H_1}$$

$$\frac{C_2}{R_1} = \frac{G_3 - H_2G_3G_4 + G_1G_2H_2}{(1 - H_2G_4 - G_3H_1 + H_1H_2G_4G_3 - G_1G_2H_1H_2)}$$

Q.6 (b) (ii) Solution:

The transfer function of the mechanical system is

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Fs + K} \qquad ...(1)$$

and $$F(s) = \frac{10}{s}$$

So, $$X(s) = \frac{10}{s(Ms^2 + Fs + K)} \quad ...(2)$$

The steady state value of 'x' is

$$\lim_{s \to 0} sX(s) = \frac{10}{K} = 0.02$$

$$K = \frac{10}{0.02} = 500$$

Given : $$M_p = \frac{0.00193}{0.02} \times 100 = 9.65\%$$

So, $$\frac{9.65}{100} = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\xi = 0.597$$

Peak time, $$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 3$$

On solving, $$\omega_n = 1.31 \text{ rad/sec}$$

From eqn. (1), on comparing with standard 2nd order equation,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = \frac{K}{M}$$

$$M = \frac{K}{\omega_n^2} = \frac{500}{1.31^2} = 291.358 Kgs$$

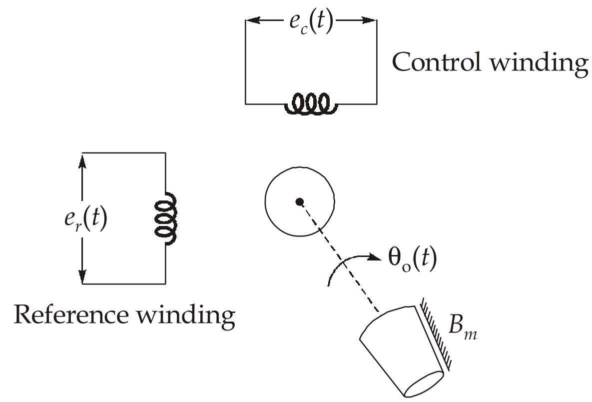
and $$\frac{F}{M} = 2\xi\omega_n$$

$$F = 2 \times 0.6 \times 1.31 \times 291.81 = 458.0015 \text{ N/m/sec.}$$

Q.6 (c) Solution:

An AC servomotor is essentially two phase induction motor made up of two stator coils and a high resistance rotor.

The two stator coils are called the control winding and reference winding. Control signal of variable voltage and polarity is applied to the control winding and, reference winding is supplied with a fixed signal that is phase shifted by 90° relative to the control signal. Schematic diagram of such a construction is shown below :

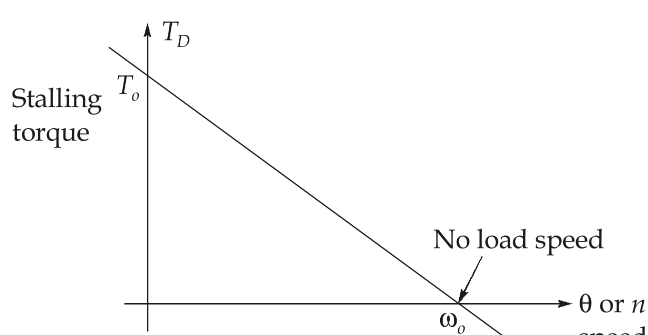


Working :

Let

$$e_r(t) = \text{Reference field voltage}$$
$$J_m = \text{Moment of inertia}$$
$$e_c(t) = \text{Control field voltage}$$
$$B_m = \text{Friction}$$
$$T_D = \text{Torque developed}$$
$$n = \text{Speed}$$

As the control voltage $e_c(t)$ is varied, the torque T_D proportional to $e_r(t)$, $e_c(t)$ and the sine of angle between them and speed n vary. Torque speed curves for varying values of control voltage are shown in figure below :



Linearised Characteristics

Calculation of Transfer Function : The developed torque T_D is a function of speed and control voltage from the linearized torque-speed characteristics. The equation which relates the torque of the motor and the speed is given by :

$$T_D(t) = m\omega_m(t) + Ke_c(t) \quad ...(1)$$

where,

(a) m = slope of the linearized characteristic

$$= \frac{-T_o}{\omega_o}$$

(b) $$T_o = \text{stalling torque at speed equal to zero}$$
$$= Ke_c(t)$$

Note : Stalling torque is proportional to $e_c(t)$

or $$K = \frac{T_o}{e_c(t)} \text{ in Nm/Volt}$$

(c) $$\omega_m = \frac{d\theta_m(t)}{dt}$$

Substituting the value in eqn. (1) and we get

$$T_D(t) = \frac{md\theta_m(t)}{dt} + Ke_c(t) \quad ...(2)$$

In Laplace form, $$T_D(s) = ms\theta_m(s) + KE_c(s) \quad ...(3)$$

Writing torque developed in terms of moment of inertia and friction :

$$T_D(t) = \frac{J_m d^2\theta_m(t)}{dt^2} + \frac{B_m d\theta_m}{dt}$$

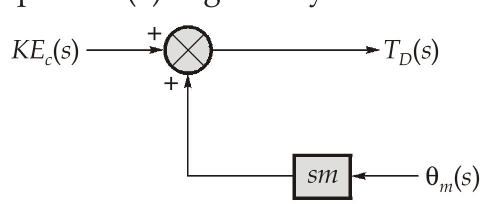
In Laplace form

$$T_D(s) = s^2 J_m \theta_m(s) + sB_m \theta_m(s) \quad ...(4)$$

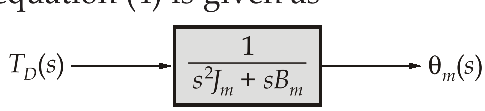
On comparing eqn. (3) and eqn. (4)

$$ms\theta_m(s) + KE_c(s) = s^2 J_m \theta_m(s) + sB_m \theta_m(s)$$

Block diagram of equation (2) is given by



Block diagram of equation (4) is given as

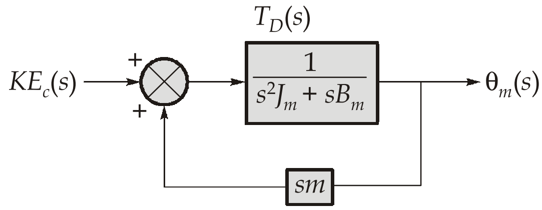


$$KE_c(s) = \theta_m(s)[s^2 J_m + sB_m - ms]$$

Transfer function :

$$\frac{\theta_m(s)}{E_c(s)} = \frac{K}{s[sJ_m + (B_m - m)]} \quad ...(5)$$

Combining the two block diagram, we get



Solving numerical :

Converting no load speed = 2904 rpm to rad/sec,

We get
$$\omega_o = \frac{2904 \times 2\pi}{60} = 303.9 = 304 \text{ rad/sec}$$

$$m = \frac{-T_o}{\omega_o} = \frac{-0.166}{304} = -5.5 \times 10^{-4}$$

$$K = \frac{T_o}{e_c} = \frac{0.166}{115} = 1.44 \times 10^{-3} \text{ Nm/Volt}$$

Substituting values in eqn. (6), we get

$$\frac{\theta_m(s)}{E_c(s)} = \frac{1.44 \times 10^{-3}}{s(s \times 1 \times 10^{-5} + 5.5 \times 10^{-4})} \quad \text{(Given : } B_m = 0)$$

$$= \frac{0.262 \times 10^{-3}}{s \times 10^{-4}(0.18 \times 10^{-1} s + 1)} = \frac{2.62}{s(1 + 0.018s)}$$

Q.7 (a) (i) Solution:

The characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+a)}{s(s+1)(s+2)(s+5)} = 0$$

$$s(s+1)(s+2)(s+5) + Ks + Ka = 0$$

$$s^4 + 8s^3 + 17s^2 + 10s + Ks + Ka = 0$$

$$s^4 + 8s^3 + 17s^2 + s(10 + K) + Ka = 0$$

The Routh's array is :

s^4	1	17	Ka
s^3	8	$10+K$	
s^2	$\dfrac{126-K}{8}$	Ka	
s^1	$\dfrac{\left(\dfrac{126-K}{8}\right)(10+K)-8Ka}{\dfrac{126-K}{8}}$		
s^0	Ka		

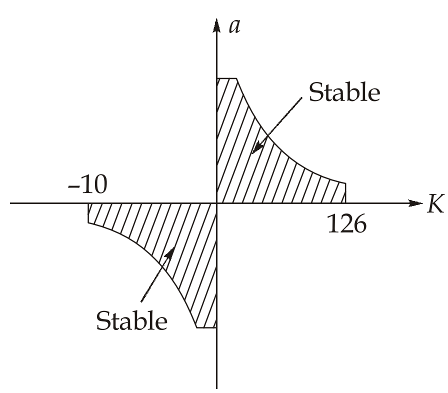
The condition of stability are :

$$\frac{126-K}{8} > 0 \quad \Rightarrow \quad K < 126$$

and $$(K+10)(126-K) - 64Ka > 0$$

and $$Ka > 0$$

Figure below depicts the shaded area satisfying the above three conditions and where the system is stable.



Q.7 (a) (ii) Solution:

$$\text{Poles} = 0, 0, -2 \text{ and Zeros} = -1$$

Since, the order of the characteristic equation is three, the number of root-locus are three. The three root loci will originate from three poles. Out of the three, one root locus path will terminate at the zero $s = -1$ and the other two root loci will terminate at infinity.

Asymptotes, $$a_1 = \pm\frac{180°}{3-1} = \pm 90°$$

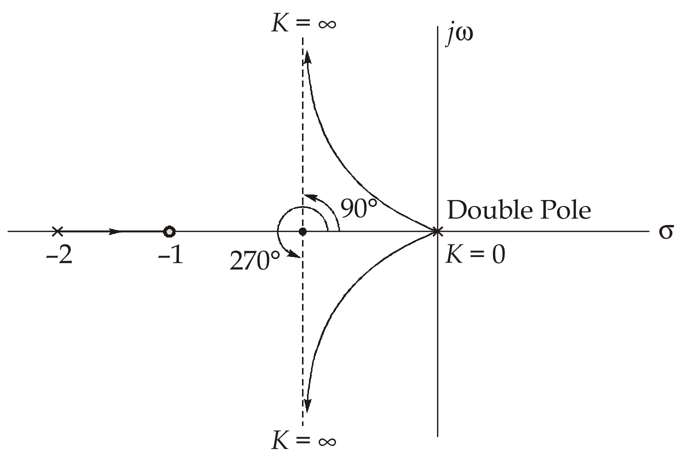
$$a_3 = \pm\frac{180°\times 3}{3-1} = \pm 270°$$

Intersection of the asymptotes with real axis :

$$s = \frac{\sum \text{real part of pole} - \sum \text{real part of zero}}{P-Z}$$

$$= \frac{-2-(-1)}{2} = -0.5$$

Root locus is shown in figure below :



Q.7 (b) Solution:

Given, $$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

The sinusoidal transfer function is

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+j2\omega)}$$

Rationalizing, $$G(j\omega) = \frac{(1-j\omega)(1-j2\omega)}{-\omega^2(1+j\omega)(1-j\omega)(1+j2\omega)(1-j2\omega)}$$

$$= \frac{(1-2\omega^2)-j3\omega}{-\omega^2(1+\omega^2)(1+4\omega^2)}$$

$$= -\frac{(1-2\omega^2)}{\omega^2(1+\omega^2)(1+4\omega^2)} + j\frac{3}{\omega(1+\omega^2)(1+4\omega^2)}$$

When $\omega = 0$,

$$G(j0) = -\infty + j\infty$$

When $\omega = \infty$,

$$G(j\infty) = 0 + j0$$

The polar plot does not cross the real axis. It crosses the imaginary axis at a frequency given by the solution of

$$\frac{-(1-2\omega^2)}{\omega^2(1+\omega^2)(1+4\omega^2)} = 0$$

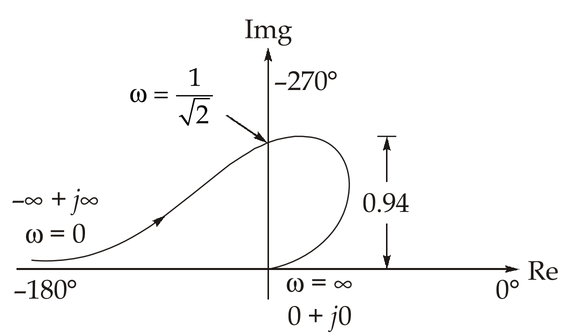
$$1 - 2\omega^2 = 0$$

$$\therefore \qquad \omega = \frac{1}{\sqrt{2}}$$

The value of $G(j\omega)$ at the frequency $\left(\omega = \frac{1}{\sqrt{2}}\right)$ will be

$$j\frac{3}{\omega(1+\omega^2)(1+4\omega^2)}\Bigg|_{\omega=\frac{1}{\sqrt{2}}} = j\cdot\frac{3}{\frac{1}{\sqrt{2}}\left(1+\frac{1}{2}\right)\left(1+\frac{4}{2}\right)} = j0.94$$

Based on the above information, an approximate polar plot is drawn as shown in figure below,



Q.7 (c) Solution:

$$T(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^3 + 6s^2 + 11s + 6} = \frac{1}{(s+1)(s+2)(s+3)}$$

On doing partial fraction,

$$T(s) = \frac{\frac{1}{2}}{(s+1)} + \frac{-1}{(s+2)} + \frac{\frac{1}{2}}{(s+3)}$$

Hence, $$Y(s) = \frac{\frac{1}{2}}{(s+1)}U(s) + \frac{-1}{(s+2)}U(s) + \frac{\frac{1}{2}}{(s+3)}U(s)$$

We define, $$X_1(s) = \frac{\frac{1}{2}}{(s+1)}U(s)$$

$$X_2(s) = \frac{-1}{(s+2)}U(s)$$

$$X_3(s) = \frac{\frac{1}{2}}{(s+3)}U(s)$$

Taking inverse laplace of above three equations, we get

$$\dot{x}_1 = -x_1 + \frac{1}{2}u$$

$$\dot{x}_2 = -2x_2 - u$$

$$\dot{x}_3 = -3x_3 + \frac{1}{2}u$$

Since, $$Y(s) = X_1(s) + X_2(s) + X_3(s)$$

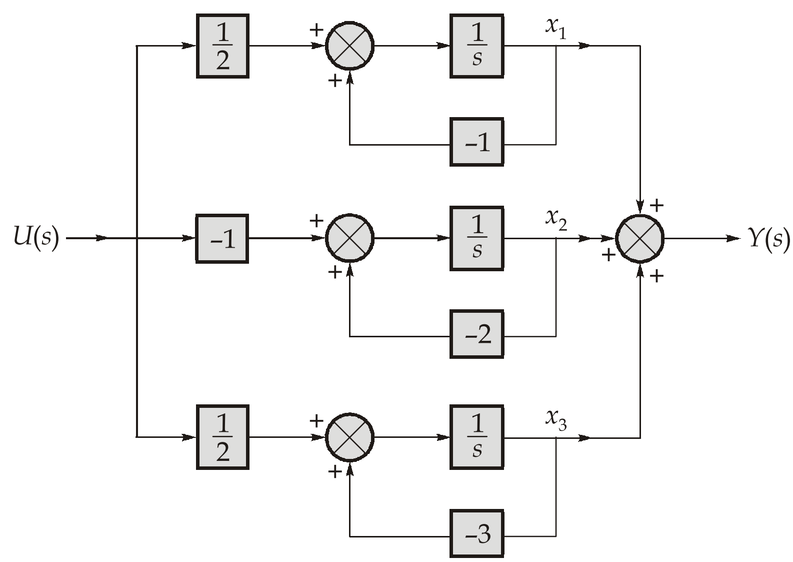
We get $$y = x_1 + x_2 + x_3$$

In terms of vector matrix notation,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Block diagram representation :



Q.8 (a) Solution:

The open-loop poles are located at $s_1 = 0$,

$$s_2 = -2$$

and
$$s_3, s_4 = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 2}}{2} = -1 \pm j1$$

As $Z = 0$ and $P = 4$

The angles of asymptotes are 45°, 135°, 225° and 315°,

The asymptotes intersect on the real axis at

$$x = \frac{\Sigma P - \Sigma Z}{P - Z} = \frac{(0 - 2 - 1 + j1 - 1 - j1)}{4 - 0} = -1$$

The characteristic equation is given by

$$= 1 + \frac{K}{s(s+2)(s^2 + 2s + 2)}$$

$$s(s + 2)(s^2 + 2s + 2) + K = 0$$

$$K = -(s^4 + 4s^3 + 6s^2 + 4s)$$

$$\frac{dK}{ds} = -(4s^3 + 12s^2 + 12s + 4)$$

The breakaway points are determined by solving $\left(\frac{dK}{ds}\right) = 0$

$$4s^3 + 12s^2 + 12s + 4 = 0$$

$$(s+1)^3 = 0$$

The solution of the above equation is $s = -1$

Thus the breakaway point is located $s = -1$

The characteristic equation is $s^4 + 4s^3 + 6s^2 + 4s + K = 0$

Applying Routh-Hurwitz criterion to the characteristic equation,

s^4	1	6	K
s^3	4	4	
s^2	5	K	
s^1	$\frac{20-4K}{5}$		
s^0	K		

The value of K for marginal stability is determined below:

$$\frac{20-4K}{5} = 0$$

$\therefore$ $K = 5$ for marginal stability

The auxiliary equation is

$$5s^2 + K = 0$$

$\therefore$

$$5s^2 + 5 = 0$$

$$s^2 = -1$$

$\therefore$ $s = \pm j1$ are the intersection points of the root locus with the imaginary axis,

The angles of departure from the complex poles are determined below,

$$\phi_{d(-1+j1)} = 180° - (90° + 45° + 135°) = -90°$$

$$\phi_{d(-1-j1)} = 180° - (-90° + 45° + 135°) = +90°$$

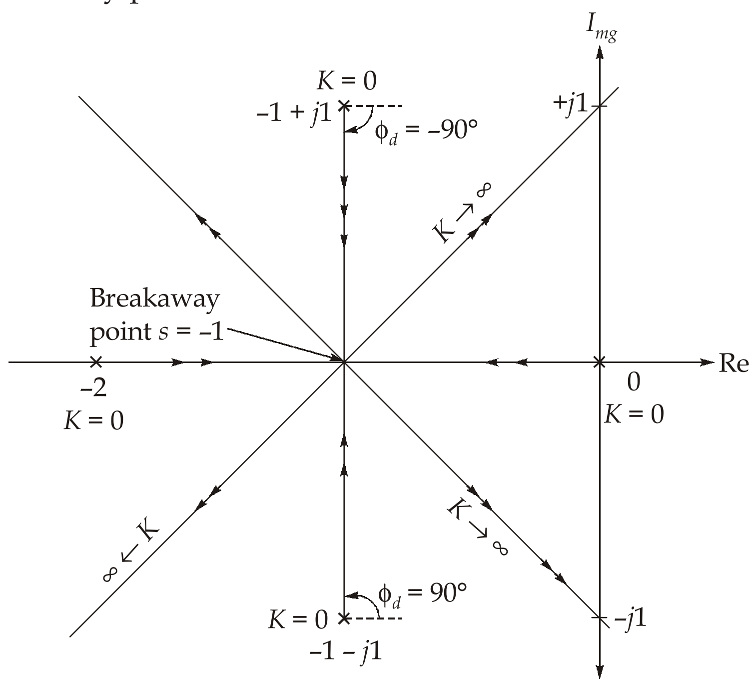
The value of K at the breakaway i.e. $s = -1$, is determined by applying root locus magnitude condition to the open-loop transfer function,

$$G(s) = \frac{K}{[s(s+2)(s^2+2s+2)]}$$

i.e.,

$$1 = \left|\frac{K}{(-1)(-1+2)[(-1)^2+2(-1)+2]}\right| = \left|\frac{K}{-1}\right|$$

$\therefore$ $K = 1$ at the breakaway point.



Q.8 (b) Solution:

$$G(s)H(s) = \frac{K(1+2s)}{s(1+s)(1+s+s^2)}$$

$$G(j\omega)H(j\omega) = \frac{K(1+j2\omega)}{j\omega(1+j\omega)(1-\omega^2+j\omega)} \quad ...(1)$$

$$M = \frac{K\sqrt{1+4\omega^2}}{\omega\sqrt{1+\omega^2}\ \sqrt{(1-\omega^2)^2+\omega^2}} \quad ...(2)$$

$$\phi = -90^\circ + \tan^{-1}(2\omega) - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) \quad ...(3)$$

At $\omega = 0$ $\Rightarrow$ $M = \infty$ and $\phi = -90^\circ$

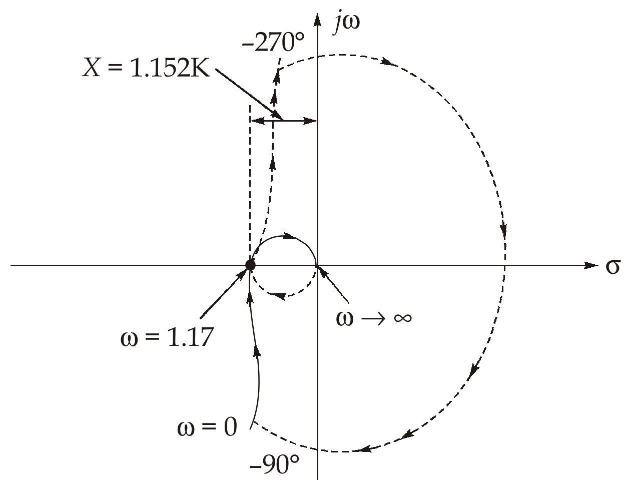
At $\omega = \infty$ $\Rightarrow$ $M = 0$ and $\phi = -270^\circ$

Put $s = Re^{j\theta}$ where $R \to 0$

and $\theta = \frac{-\pi}{2}$ to $\frac{+\pi}{2}$

$$G(s)H(s) = \frac{1}{Re^{j\theta}} = \infty \cdot e^{-j\theta} \qquad \left\{\begin{matrix} \theta = \frac{-\pi}{2} \text{ to } \frac{+\pi}{2} \\ -\theta = \frac{+\pi}{2} \text{ to } \frac{-\pi}{2} \end{matrix}\right\}$$

Nyquist plot drawn as below :



From eqn. (1), separating real and imaginary part and making imaginary part = 0.

$$1 + 2\omega^2 - 2\omega^4 = 0$$

$$\omega = 1.17$$

$$M|_{\omega=1.17} = X = \frac{K\sqrt{1+4\times 1.17^2}}{1.17\sqrt{1+1.17^2}\ \sqrt{(1-1.17^2)^2+1.17^2}}$$

$$X = 1.152K$$

For system to be stable

$$1.152K < 1 \quad \Rightarrow \quad \left\{\begin{matrix} N = 0 \\ N = P - Z \;\Rightarrow\; Z = 0 \text{ (stable)} \end{matrix}\right\}$$

$$K < 0.87$$

$$\text{For gain margin} = 3 \text{ dB}$$

$$20\log_{10}\frac{1}{X} = 3$$

$$20\log_{10}\frac{1}{1.152K} = 3$$

$$K = \frac{1}{1.152\times 1.41}$$

$$K = 0.614$$

For $K = 0.614$ $\Rightarrow$ Gain crossover frequency (ω_{gc})

$$|M|_{\omega=\omega_{gc}} = 1$$

$$\frac{0.614\sqrt{1+4\omega_{gc}^2}}{\omega_{gc}\sqrt{1+\omega_{gc}^2}\,\sqrt{(1-\omega_{gc}^2)^2+\omega_{gc}^2}} = 1$$

On solving, $\omega_{gc} \approx 0.98$

Now phase margin,

$$PM = 180 + \phi|_{\omega=\omega_{gc}}$$

$$= 180 + \left[-90 + \tan^{-1}(2\times 0.98) - \tan^{-1}(0.98) - \tan^{-1}\left(\frac{0.98}{1-0.98^2}\right)\right]$$

$$PM = 20.86°$$

Q.8 (c) Solution:

(i) Condition for the desired response

1. system must be controllable.
2. system must be observable.

The state transition matrices,

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{and } C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

For controllability

$$|Q_c| \neq 0$$

We know that, $Q_c = \begin{bmatrix} B & AB \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$|Q_c| = \begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix}$$

$\therefore$ $|Q| = 0 - 3 \neq 0$ so, system is controllable.

For observability, $|Q_0| \neq 0$

We know that, $Q_0 = \begin{bmatrix} C \\ CA \end{bmatrix}$

$$CA = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \end{bmatrix}$$

$$|Q_0| = \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix}$$

$\therefore \quad |Q_0| = 3 - 0 \neq 0$ so, the system is observable.

So, the desired response is possible.

(ii) Given: Settling time, $T_s = 0.5$ sec, damping frequency $\omega_d = 6$ rad/sec

We know that,
$$T_s = \frac{4}{\xi\omega_n} \Rightarrow \xi\omega_n = \frac{4}{0.5} = 8$$

We know that,
$$\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{\omega_n^2 - (\omega_n\xi)^2}$$
$$6 = \sqrt{\omega_n^2 - 8^2} \Rightarrow \omega_n = 10 \text{ rad/sec}$$

$\therefore$
$$\xi = \frac{8}{\omega_n} = \frac{8}{10} = 0.8$$

The second order characteristic equation,
$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$
$$s^2 + 16s + 100 = 0 \quad \text{...(i)}$$

Let the observer gain matrix be,
$$K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$$

The desired response is given by,
$$\dot{x} = (A - KC)x + Bu$$

Characteristic equation,
$$q(s) = |sI - (A - KC)| = 0$$
$$(A - KC) = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix}$$
$$(A - KC) = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} K_1 & 0 \\ K_2 & 0 \end{bmatrix} = \begin{bmatrix} 2-K_1 & 3 \\ -1-K_2 & 4 \end{bmatrix}$$
$$[sI - (A - KC)] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2-K_1 & 3 \\ -1-K_2 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} s-2+K_1 & -3 \\ 1+K_2 & s-4 \end{bmatrix}$$

Characteristic equation,
$$q(s) = |sI - (A-KC)| = \begin{vmatrix} s-2+K_1 & -3 \\ 1+K_2 & s-4 \end{vmatrix}$$

$$(s - 2 + K_1)(s - 4) + 3(1 + K_2) = 0$$

$$s^2 - 4s + (-2 + K_1)s + 8 - 4K_1 + 3 + 3K_2 = 0$$

$$s^2 + (-6 + K_1)s + (11 - 4K_1 + 3K_2) = 0 \quad \text{...(ii)}$$

On comparing equation (i) and (ii),

$$-6 + K_1 = 16$$

$$\therefore \quad K_1 = 16 + 6 = 22$$

$$11 - 4K_1 + 3K_2 = 100$$

$$11 - 4 \times 22 + 3K_2 = 100$$

$$3K_2 = 100 - 11 + 88 = 177$$

$$\therefore \quad K_2 = 59$$

The observer gain matrix,

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 22 \\ 59 \end{bmatrix}$$

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