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Detailed Solutions

ESE-2024 Mains Test Series

Mechanical Engineering Test No : 1

Section A : Thermodynamics [All Topics]

Section B : Strength of Materials and Mechanics [All Topics]

Section : A

1. (a)

Given : T_1 = 720°C = 993 K, T_2 = 160°C = 433 K, C_{pg} = 1.088 kJ/kgK, C_{pw} = 4.27 kJ/kgK,

$\dot{m}_g$ = 1450 kg/min, $\dot{m}_w$ = 2000 kg/min, T_3 = 42°C = 315 K, T_0 = 27°C = 300 K

For flow of fluids across a heat exchanger, applying energy balance on the heat exchanger

$$\dot{m}_g c_{pg}(T_1 - T_2) = \dot{m}_w c_{pw}(T_4 - T_3)$$

or $$1450 \times 1.088(720 - 160) = 2000 \times 4.27 \times (T_4 - 42)$$

$\therefore$ $$T_4 = 145.45°C$$

Now, the unavailable energy removed from the gas

$$\dot{Q}_g = T_0(s_1 - s_2) = T_0 \dot{m}_g c_{pg} \ln\left(\frac{T_1}{T_2}\right)$$

$$= 300 \times \frac{1450}{60} \times 1.088 \times \ln\left(\frac{993}{433}\right)$$

$$\dot{Q}_g = 6546.98 \text{ kW}$$

The unavailable energy added to water

$$\dot{Q}_w = T_0(s_4 - s_3) = T_0 \dot{m}_w c_{pw} \ln\left(\frac{T_4}{T_3}\right)$$

$$= 300 \times \frac{2000}{60} \times 4.27 \times \ln\left(\frac{418.45}{315}\right)$$

$$\dot{Q}_w = 12126.15 \text{ kW}$$

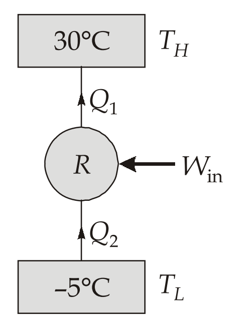
∴ Loss in available energy $= \dot{Q}_w - \dot{Q}_g$

$= 12126.15 - 6546.98$

$= 5579.17$ kW **Ans.**

1. (b)

Given : $T_H = 30°C + 273 = 303$ K; $T_L = -5°C + 273 = 268$ K



$(COP)_{act} = 0.25(COP)_{rev}$

Refrigeration effect, $Q_2 = 500 \times 30 \times 365$

$Q_2 = 5475$ MJ

Now, $$(COP)_{rev} = \frac{T_L}{T_H - T_L} = \frac{268}{303 - 268} = 7.657$$

Actual COP of the refrigerator

$(COP)_{act} = 0.25 \times 7.657 = 1.914$

Further, $$(COP)_{act} = \frac{Q_2}{W_{in}}$$

or $$W_{in} = \frac{5475 \times 10^6}{1.914} = 2860501.56 \text{ kJ}$$

Now, 1 kWh = 3600 kJ

∴ Power consumed by the refrigerator

$P_{in} = 794.58$ kWh

∴ The yearly bill of the refrigerator

$= P_{in} \times$ Unit cost

$= 794.58 \times 2.5$

$=$ ₹1986.45 **Ans.**

1. (c)

Given : $V_1 = 0.15$ m^3; $P_1 = 1$ bar; $T_1 = 80°C = 80 + 273 = 353$ K and $V_2 = 0.02$ m^3; $P_2 = 12$ bar

Considering the compression process is a quasistatic polytropic process, thus

$$P_1V_1^n = P_1V_2^n$$

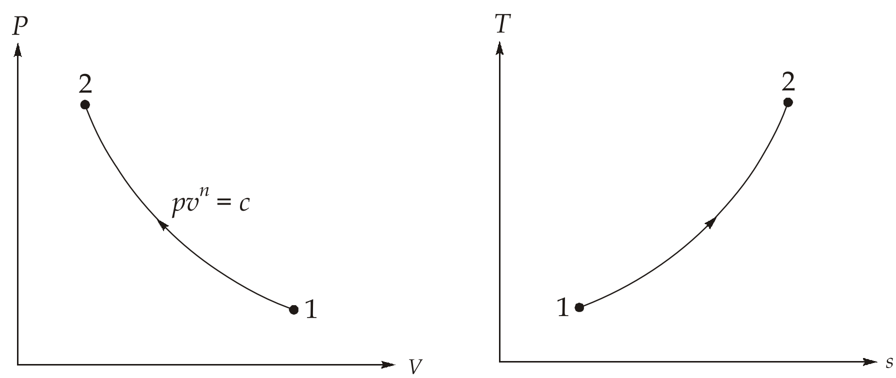
or
$$n = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln\left(\frac{1}{12}\right)}{\ln\left(\frac{0.02}{0.15}\right)}$$

$\therefore$ $$n = 1.233$$

$\therefore$ Temperature after compression,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 353\times(12)^{\frac{0.233}{1.233}}$$

$$T_2 = 564.55 \text{ K}$$



Mass of air in the cylinder, $m = \frac{P_1V_1}{RT_1} = \frac{100\times0.15}{0.287\times353} = 0.148$ kg

Now, change of internal energy during compression,

$$\Delta U = mC_v(T_2 - T_1)$$

$$\Delta U = 0.148 \times 0.717(564.55 - 353)$$

$$\Delta U = 22.44 \text{ kJ}$$ Ans.

Work transfer, $W = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{1200\times0.02 - 100\times0.15}{1-1.233} = -38.62$ kJ

$\therefore$ Heat transfer during compression,

$$Q = \Delta U + W$$

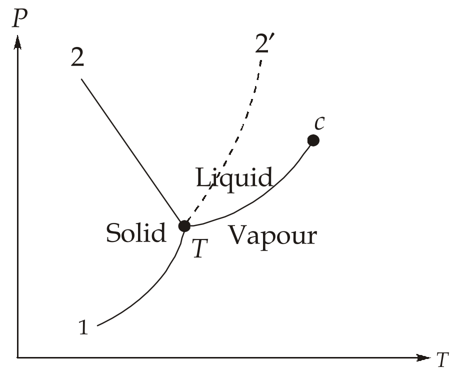
Heat transferred, $Q = 22.44 - 38.62$

$= -16.18$ kJ

or $Q = 16.18$ kJ (Rejected) Ans.

1. (d)

The phase diagram for water is shown in figure below, in which the regions of solid, liquid and vapour are indicated.



The single phase regions are separated from each other by saturation lines. For example, the solid and liquid regions are separated by the fusion curve 2T. Along the fusion curve both solid and liquid phases coexist in equilibrium. It can be observed that the slope of the fusion curve is negative.

i.e. $$\left(\frac{\partial P}{\partial T}\right) < 0$$

which shows that the melting point of ice decreases with increasing pressure. The sublimation curve 1T separates the solid region from the vapour region, while the vaporization curve CT separates the liquid region from the vapour region. Along the sublimation curve the solid and vapour phase coexist in equilibrium while the liquid and vapour phases coexist in equilibrium along the vaporization curve. The phase diagram shown in figure represents the phase behaviour of substances (like water, bismuth and antimony) which contract on melting, that is, the specific volume of the liquid is smaller than the specific volume of the solid. The state T represents the triple point where all the three phases-solid, liquid and vapour-coexist in equilibrium. The state C represents the critical point where the properties of liquid and vapour are identical and they cannot be distinguished from one another. For all other substances the slope of the fusion curve will be positive (as shown by 2′T line on the diagram) which indicates that the melting point increases with increasing pressure.

1. (e)

Consider two bodies A and B. Heat is removed from the body A to bring it to T_2.

Heat removed per kg from the body A,

$$q_L = C_p(T_1 - T_2)$$

Heat discharged per kg by refrigeration to the body B

$$q_H = q + w = C_p(T - T_1)$$

where T becomes the temperature of the body B after heat addition

Work input to the refrigerator,

$$w = C_p(T - T_1) - C_p(T_1 - T_2)$$

$$= C_p(T - 2T_1 + T_2) \quad \text{...(i)}$$

The decrease of entropy of the body A

$$\Delta s_A = \int_{T_1}^{T_2} C_p\left(\frac{dT}{T}\right) = C_p \ln\left(\frac{T_2}{T_1}\right)$$

The entropy increase of the body B

$$\Delta s_B = \int_{T_1}^{T_2} C_p\left(\frac{dT}{T}\right) = C_p \ln\left(\frac{T}{T_1}\right)$$

Net entropy change of the reversible refrigeration cycle = 0

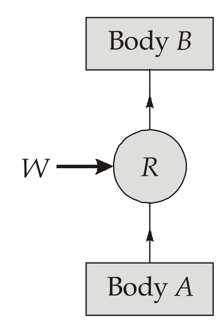
Thus,
$$\oint \frac{\delta q}{T} = \Delta s_A + \Delta s_B = 0$$

$\therefore$
$$C_p \ln\left(\frac{T_2}{T_1}\right) + C_p \ln\left(\frac{T}{T_1}\right) = 0$$

or
$$\ln\left(\frac{T_2 T}{T_1^2}\right) = 0$$

or
$$\left(\frac{T_2 T}{T_1^2}\right) = 1$$

or
$$T = \frac{T_1^2}{T_2}$$



Substituting in equation (i),

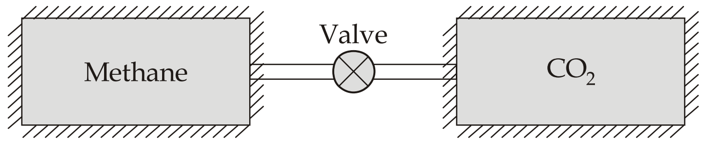
$$W = C_p\left[\frac{T_1^2}{T_2} + T_2 - 2T_1\right]$$

Proved.

2. (a)

Let methane and CO_2 are represented by suffix 1 and 2, respectively

Given : $m_1 = 8$ kg; $P_1 = 1.5$ bar; $T_1 = 27°C = 300$ K; $m_2 = 5$ kg; $P_2 = 4$ bar; $T_2 = 60°C = 333K$; $C_{p_1} = 2.1$ kJ/kgK; $C_{p_2} = 0.85$ kJ/kgK



Now, $$C_{v_1} = C_{p_1} - R_1$$

$$C_{v_1} = 2.1 - \frac{8.314}{16} = 1.58 \text{ kJ/kgK}$$

$$C_{v_2} = C_{p_2} - R_2$$

$$C_{v_2} = 0.85 - \frac{8.314}{44} = 0.661 \text{ kJ/kgK}$$

When the gases are mixed adiabatically, since there is no work or heat transfer in accordance with first law, the total energy does not change. This implies

Final internal energy of mixture = $U_1 + U_2$

∴ $$m_1C_{v_1}T + m_2C_{v_2}T = m_1C_{v_1}(300) + m_2C_{v_2}(333)$$

or $$T = \frac{8 \times 1.58 \times 300 + 5 \times 0.661 \times 333}{8 \times 1.58 + 0.661 \times 5}$$

$$T = 306.84 \text{ K}$$ Ans.

Now, volumes of the two tanks

$$V_1 = \frac{m_1R_1T_1}{P_1} = 8 \times \frac{8.314}{16} \times \frac{300}{1.5 \times 100} = 8.314 \text{ m}^3$$

$$V_2 = \frac{m_2R_2T_2}{P_2} = 5 \times \frac{8.314}{44} \times \frac{333}{4 \times 100} = 0.786 \text{ m}^3$$

The total volume of the mixture,

$$V = V_1 + V_2$$
$$= 8.314 + 0.786 = 9.1 \text{ m}^3$$

Partial pressure of methane in the mixture is

$$P_m = \frac{m_1R_1T}{V} = 8 \times \frac{8.314 \times 306.84}{9.1 \times 16} = 1.4 \text{ bar}$$

Similarly, the partial pressure of CO_2 is

$$P_c = \frac{m_2R_2T}{V} = 5 \times \frac{8.314 \times 306.84}{9.1 \times 44} = 0.318 \text{ bar}$$

∴ The total pressure of the mixture is

$$P = P_m + P_c = 1.4 + 0.318$$

$$P = 1.718 \text{ bar}$$ Ans.

Now, entropy change due to mixing

$$\Delta s = \Delta s_{methane} + \Delta s_{CO_2}$$

$$= m_1\left[C_{p_1}\ln\frac{T}{T_1} - R_1\ln\frac{P_m}{P_1}\right] + m_2\left[C_{p_2}\ln\frac{T}{T_2} - R_2\ln\frac{P_c}{P_2}\right]$$

$\therefore$ $$\Delta s = 8\left[2.1\ln\frac{306.84}{300} - \frac{8.314}{16}\ln\frac{1.4}{1.5}\right] + 5\left[0.85\ln\frac{306.84}{333} - \frac{8.314}{44}\ln\frac{0.318}{4}\right]$$

$\therefore$ $$\Delta s = 0.665 + 2.044 \text{ kJ/k}$$

or $$\Delta s = 2.709 \text{ kJ/k}$$

$\therefore$ Irreversibility, $I = T_0\Delta s$

$$= 300 \times 2.709$$

$$= 812.7 \text{ kJ}$$ Ans.

2. (b)

From steam tables, we have

Inlet condition : $P_1 = 4$ MPa; $T_1 = 500°C$;

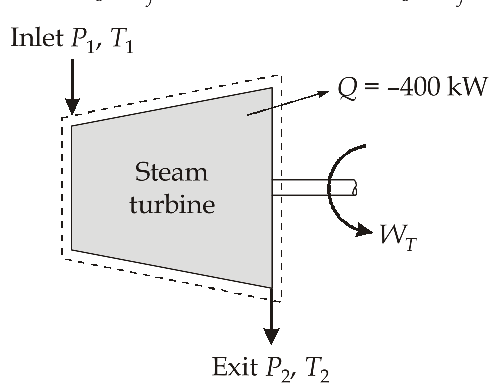
$h_1 = 3446$ kJ/kg; $s_1 = 7.0922$ kJ/kg

Exit condition : $P_2 = 0.3$ MPa; $T_2 = 170°C$;

$h_2 = 2803.7$ kJ/kg; $s_2 = 7.1773$ kJ/kg

Dead state : $P_0 = 100$ kPa; $T_0 = 25°C$;

$h_0 = h_f = 104.92$ kJ/kg; $s_0 = s_f = 0.3672$ kJ/kgK



The actual power output from steam turbine

$$\dot{Q} - \dot{W} = \dot{m}(\Delta h + \Delta ke + \Delta Pe) = \dot{m}(\Delta h)$$

or $$-400 - \dot{W} = 8(2803.7 - 3446)$$

$\therefore \qquad \dot{W}_{act} = 4738.4 \text{ kW}$ Ans. (i)

Reversible work done by the turbine

$$\dot{W}_{rev} = \dot{m}((h_1 - h_2) - T_0(s_1 - s_2))$$
$$= 8[(3446 - 2803.7) - 298(7.0922 - 7.1773)]$$
$$\dot{W}_{rev} = 5341.278 \text{ kW}$$ Ans. (ii)

The second law efficiency,

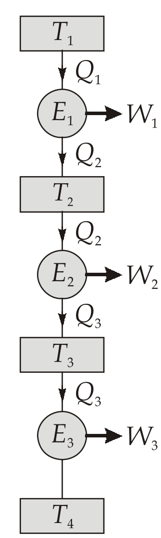
$$\eta_{II} = \frac{\dot{W}_{act}}{\dot{W}_{rev}} = \frac{4738.4}{5341.278}$$
$$= 0.8871 \text{ or } 88.71\%$$ Ans. (iii)

Availability of steam at the inlet condition

$$\psi_1 = (h_1 - h_0) - T_0(s_1 - s_0)$$
$$= (3446 - 104.92) - 298(7.0922 - 0.3672)$$
$$\psi_1 = 1337.03 \text{ kJ/kg}$$ Ans. (iv)

2. (c)

Refer figure,



$$T_1 = 1000 \text{ K}; T_4 = 500 \text{ K}$$
$$W_1 : W_2 : W_3 = 7 : 5 : 4$$

For Carnot engine 'E_1'

$$\eta_1 = \frac{T_1 - T_2}{T_1} = \frac{W_1}{Q_1}$$

or
$$\eta_1 = \frac{1000 - T_2}{1000} = \frac{W_1}{Q_2 + W_1} \qquad \text{...(i)}$$

or
$$(1000 - T_2) \times (Q_2 + W_1) = 1000W_1$$
$$(1000 - T_2)Q_2 = W_1T_2$$

$\therefore$
$$Q_2 = \frac{W_1T_2}{1000 - T_2} \qquad \text{...(ii)}$$

For Carnot engine 'E_2'

$$\eta_2 = \frac{T_2 - T_3}{T_2} = \frac{W_2}{Q_2} \qquad \text{...(iii)}$$

or
$$W_2 = \frac{Q_2(T_2 - T_3)}{T_2}$$

Using the value of Q_2 from equation (ii), we get

$$W_2 = \frac{W_1 T_2 (T_2 - T_3)}{(1000 - T_2) T_2}$$

or
$$\frac{W_2}{W_1} = \frac{T_2 - T_3}{1000 - T_2}$$

or
$$\frac{5}{7} = \frac{T_2 - T_3}{1000 - T_2} \qquad \left(\because \frac{W_2}{W_1} = \frac{5}{7}\right)$$

$$5000 - 5T_2 = 7T_2 - 7T_3$$

or
$$T_2 = \frac{5000 + 7T_3}{12} \quad \text{...(iv)}$$

Again, $Q_2 = Q_3 + W_2$

Substituting in equation (iii), we get

$$\frac{W_2}{W_2 + Q_3} = \frac{T_2 - T_3}{T_2}$$

or
$$W_2 T_2 = W_2 (T_2 - T_3) + Q_3 (T_2 - T_3)$$

or
$$W_2 = \frac{Q_3 (T_2 - T_3)}{T_3} \quad \text{...(v)}$$

Further, the efficiency of third engine 'E_3'

$$\eta_3 = \frac{T_3 - T_4}{T_3} = \frac{W_3}{Q_3}$$

or
$$W_3 = \frac{Q_3 (T_3 - T_4)}{T_3}$$

$\because$
$$\frac{W_2}{W_3} = \frac{5}{4}$$

$\Rightarrow$
$$4W_2 = 5W_3$$

or
$$4 \times \frac{Q_3 (T_2 - T_3)}{T_3} = 5 \times \frac{Q_3 (T_3 - T_4)}{T_3}$$

Now, $4T_2 - 4T_3 = 5T_3 - 5T_4$

or $4T_2 + 5T_4 = 9T_3$

or
$$\frac{4(5000 + 7T_3)}{12} + 5T_4 = 9T_3$$

$$5000 + 7T_3 + 15T_4 = 27T_3$$

$\therefore$ $$5000 + 15 \times 500 = 20T_3$$

$\therefore$ $$T_3 = 625 \text{ K}$$ Ans.

Now, from equation (iv)

$$T_2 = \frac{5000 + 7 \times 625}{12} = 781.25 \text{ K}$$ Ans.

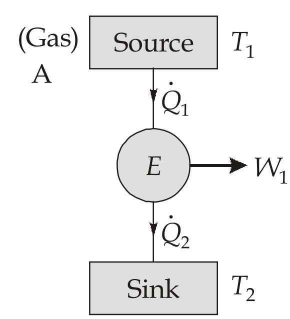
3. (a)

Maximum power can be obtained by employing a reversible engine. We know that for a reversible engine

$$\Delta s_{source} + \Delta s_{sink} = 0$$

The ambient atmosphere is at constant temperature. However, the source temperature continuously decreases till 300 K, while energy is extracted as heat.

If the gases A and B are used as separate sources, then for the heat engine which uses gas A.



$$\Delta s_A + \Delta s_{sink} = 0$$

or $$mc_p \ln\frac{T_2}{T_1} + \frac{\dot{Q}_2}{T_2} = 0$$

or $$1.5 \times 1.005 \ln\frac{300}{1200} + \frac{\dot{Q}_2}{300} = 0$$

$$\dot{Q}_2 = 626.95 \text{ kW}$$

Energy absorbed from gas A

$$\dot{Q}_1 = \dot{m}c_p(T_1 - T_2) = 1.5 \times 1.005(1200 - 300)$$

$$\dot{Q}_1 = 1356.75 \text{ kW}$$

$\therefore$ Power delivered by the engine which uses gas A

$$W_A = \dot{Q}_1 - \dot{Q}_2 = 1356.75 - 626.95$$

$$W_A = 729.8 \text{ kW}$$

Similarly for the engine which uses gas B, we get

$$\Delta s_B + \Delta s_{sink} = 0$$

$$\dot{m}c_p \ln\frac{T_2}{T_1} + \frac{\dot{Q}_2}{T_2} = 0$$

$$2 \times 1.005 \ln \frac{300}{900} + \frac{\dot{Q}_2}{300} = 0$$

$$\therefore \quad \dot{Q}_2 = 662.46 \text{ kW}$$

Energy absorbed from gas '*B*'

$$\dot{Q}_1 = \dot{m}c_p(T_1 - T_2)$$
$$= 2 \times 1.005\,(900 - 300)$$
$$= 1206 \text{ kW}$$

∴ Power delivered by the engine which uses gas *B*,

$$W_2 = \dot{Q}_1 - Q_2$$
$$= 1206 - 662.46$$
$$= 543.54 \text{ kW}$$

Therefore, the maximum power that can be obtained when the gases *A* and *B* are separately used as sources.

$$W_{max} = W_1 + W_2$$
$$= 729.8 + 543.54 = 1273.34 \text{ kW}$$

3. (b)

From steam tables, for saturated liquid, we have

At 80°C, P_1 = 0.047414 MPa; v_1 = 0.00102905 m^3/kg; u_1 = 334.96 kJ/kg

At 60°C, h_e = 251.18 kJ/kg

At 40°C, P_2 = 0.0073849 MPa, v_2 = 0.00100789 m^3/kg; u_2 = 167.52 kJ/kg Ans. (i)

$$\text{Initial volume } V_1 = m_1 v_1$$
$$= 30 \times 0.00102905 = 0.03087 \text{ m}^3$$

$$\text{Final volume, } V_2 = m_2 v_2$$
$$= 20 \times 0.00100789 = 0.02015 \text{ m}^3$$

$$\therefore \quad \text{Movement of piston, } \Delta x = \frac{V_1 - V_2}{A} = \frac{0.03087 - 0.02015}{500 \times 10^{-4}}$$

$$\Delta x = 0.2144 \text{ m}$$

$$\text{Change in force, } \Delta f = (P_1 - P_2) \times A$$
$$= (0.047414 - 0.0073849) \times 10^3 \times 500 \times 10^{-4}$$
$$\Delta f = 2 \text{ kN}$$

$$\therefore \quad \text{Spring constant, } k = \frac{\Delta f}{\Delta x} = \frac{2}{0.2144} = 9.328 \text{ kN/m}$$ Ans. (iii)

Now, for unsteady state, energy equation considering the control volume as volume of cylinder,

$$m_2u_2 - m_1u_1 = Q + W - \dot{m}_e h_e$$

Here, $$W = \frac{1}{2}(P_1 + P_2)(V_1 - V_2)$$

$$= \frac{1}{2}(0.047414 + 0.0073849) \times 10^3 \times (0.03087 - 0.02015)$$

$$= 0.294 \text{ kJ}$$

Substituting, we get

$$20 \times 167.52 - 30 \times 334.96 = Q + 0.294 - 10 \times 251.18$$

$$-6698.4 = Q - 2511.506$$

$$Q = -4186.89 \text{ kJ}$$

$\therefore$ $$Q = 4186.89 \text{ kJ (Rejected)}$$ Ans. (ii)

3. (c)

Given : $V = 3.5$ m^3; $P_1 = 200$ kPa; $T_1 = 27°C = 300$ K; $T_R = 1800$ K; $T_2 = 900$ K;
$P_0 = 1$ bar = 100 kPa; $T_0 = 17°C = 290$ K

Assumptions:

(i) A non-flow process

(ii) Specific heat at constant volume is 0.717 kJ/kgK

(iii) The gas constant of air as 0.287 kJ/kgK

(iv) $\Delta KE = 0$ and $\Delta PE = 0$

The initial availability of air in a closed system

$$\phi_1 = m[(u_1 - u_0) + P_0(v_1 - v_0) - T_0(s_1 - s_0)]$$

Here, $$m = \frac{P_1V}{RT_1} = \frac{200 \times 3.5}{0.287 \times 300} = 8.13 \text{ kg}$$

$$v_0 = \frac{RT_0}{P_0} = \frac{0.287 \times 290}{100} = 0.8323 \text{ m}^3/\text{kg}$$

and $$v_1 = \frac{V}{m} = \frac{3.5}{8.13} = 0.43 \text{ m}^3/\text{kg}$$

$$u_1 - u_0 = C_v(T_1 - T_0) = 0.717(300 - 290)$$

$$= 7.17 \text{ kJ/kg}$$

$$s_1 - s_0 = C_v \ln\left(\frac{T_1}{T_0}\right) + R\ln\left(\frac{v_1}{v_0}\right)$$

$$s_1 - s_0 = 0.717 \ln\left(\frac{300}{290}\right) + 0.287 \ln\left(\frac{0.43}{0.8323}\right)$$

$$s_1 - s_0 = -0.165 \text{ kJ/kgK}$$

∴ Initial availability, $\phi_1 = 8.13[7.17 + 100(0.43 - 0.8323) - 290(-0.165)]$

$$\phi_1 = 120.24 \text{ kJ}$$ Ans.

The final availability of air, $\phi_2 = m[(u_2 - u_0) + p_0(v_2 - v_0) - T_0(s_2 - s_0)]$

For, $v = c$

$$P_2 = P_1 \times \frac{T_2}{T_1} = 200 \times \frac{900}{300} = 600 \text{ kPa}$$

Now, $u_2 - u_0 = C_v(T_2 - T_0)$

$$u_2 - u_0 = 0.717(900 - 290) = 437.37 \text{ kJ/kg}$$

$$s_2 - s_0 = C_v \ln\left(\frac{T_2}{T_0}\right) + R \ln\left(\frac{v_2}{v_0}\right)$$

$$= 0.717 \ln\left(\frac{900}{290}\right) + 0.287 \ln\left(\frac{0.43}{0.8323}\right)$$

$$= 0.622 \text{ kJ/kgK} \qquad (\because \quad v_1 = v_2)$$

Also $P_0(v_2 - v_0) = 100(0.43 - 0.8323) = -40.23$ kJ/kg

∴ $\phi_2 = 8.13[437.37 - 40.23 - 290 \times 0.622]$

$$\phi_2 = 1762.25 \text{ kJ}$$ Ans.

The air receives heat from a constant-temperature reservoir at 1800 K, thus maximum useful work.

$$W_{max} = \phi_1 - \phi_2 + Q_R\left(1 - \frac{T_0}{T_R}\right)$$

Here, $Q_R = m(u_2 - u_1)$

$$= 8.13 \times 0.717 \times (900 - 300)$$

$$= 3497.526$$

∴ $W_{max} = 120.24 - 1762.25 + 3497.526\left(1 - \frac{290}{1800}\right)$

$$W_{max} = 1292.025 \text{ kJ}$$

The irreversibility of the closed system

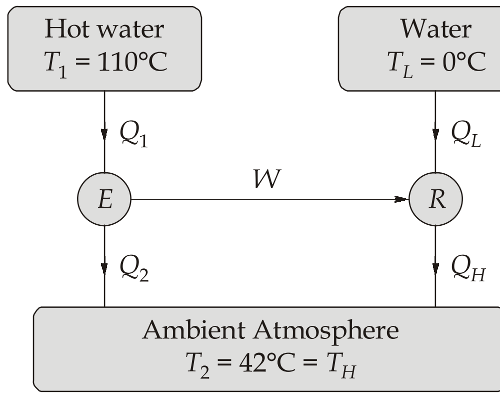
$$I = W_{max} - W_{act}$$

For a constant volume process, $W_{act} = 0$

∴ $I = W_{max} = 1292.025$ kJ

4. (a)

A schematic of the heat engine and the refrigerator is shown below:



Now, $$(\text{COP})_{\text{ref}} = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L}$$

or $$\frac{1500 \times 333.43}{3600 \times W} = \frac{273}{315 - 273} = 6.5$$

∴ $$W = \frac{1500 \times 333.43}{3600 \times 6.5}$$

$$W = 21.37 \text{ kW}$$ Ans. (i)

$$\text{Efficiency of engine, } \eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{315}{383}$$

∴ $$\eta = 0.1775$$

Also, $$\frac{Q_L}{Q_1} = \frac{Q_L / W}{Q_1 / W} = \frac{(COP)_{ref}}{1/\eta} = 6.5 \times 0.1775$$

∴ $$\frac{Q_L}{Q_1} = 1.153$$ Ans. (ii)

Now, $$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \text{ or } Q_2 = Q_1 \times \frac{T_2}{T_1}$$

∴ $$Q_2 = Q_1 \times \frac{315}{383} = 0.822 Q_1$$

Also, $$Q_H = Q_L \times \frac{T_H}{T_L} = Q_L \times \frac{315}{273}$$

∴ $$Q_H = 1.1538 Q_L$$

Now, rate of energy rejection to the ambient atmosphere = $Q_2 + Q_H$

or $$Q_2 + Q_H = 0.822Q_1 + 1.1538Q_L$$

$$= \left(0.822 + 1.1538\frac{Q_L}{Q_1}\right)Q_1$$

$$= (0.822 + 1.1538 \times 1.153) \times \frac{W}{\eta}$$

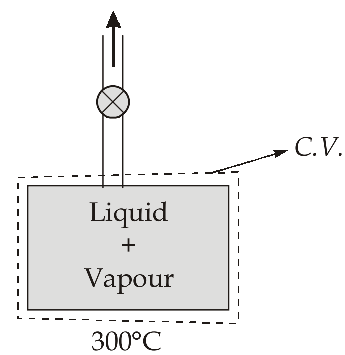
$$= (0.822 + 1.1538 \times 1.153) \times \frac{21.37}{0.1775}$$

$$Q_2 + Q_H = 259.128 \text{ kW}$$

4. (b)

Given : $V = 0.95$ m^3; $x = 0.8$; $T_{sat} = 300$°C

By mass balance,



$$\frac{dm_{cv}}{dt} = -\dot{m}_e$$

By energy balance,

$$\frac{du_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Combining the mass and energy rate balances results in

$$\frac{du_{cv}}{dt} = \dot{Q}_{cv} - \frac{h_e dm_{cv}}{dt}$$

or $$\Delta u_{cv} = Q_{cv} + h_e \Delta m_{cv}$$

or $$Q_{cv} = (m_2u_2 - m_1u_1) - h_e(m_2 - m_1)$$

where m_1 and m_2 denote, respectively, the initial and final amounts of mass within the tank.

From steam table, we have

At 300°C, $v_f = 0.00140423$ m^3/kg, $v_g = 0.021660$ m^3/kg

$u_f = 1332.9$ kJ/kg, $u_g = 2563.6$ kJ/kg

$h_f = 1345$ kJ/kg, $h_g = 2749.6$ kJ/kg

Now, $$u_1 = u_f + x_1(u_g - v_f)$$

$$= 1332.9 + 0.8(2563.6 - 1332.9)$$

$$= 2317.46 \text{ kJ/kg}$$

Also, $$v_1 = v_f + x_1(v_g - v_f)$$

$$= 0.00140423 + 0.8(0.02166 - 0.00140423)$$
$$= 0.0176 \text{ m}^3/\text{kg}$$

∴ The mass initially contained in the tank,

$$m_1 = \frac{V}{v_1} = \frac{0.95}{0.0176} = 53.977 \text{ kg}$$

∴ The final state of the mass in the tank is saturated vapour at 300°C

∴ $$u_2 = u_g = 2563.6 \text{ kJ/kg}, v_2 = v_g = 0.021660 \text{ m}^3/\text{kg}$$

∴ Mass contained within the tank at the end of the process is

$$m_2 = \frac{V}{v_2} = \frac{0.95}{0.02166} = 43.859 \text{ kg}$$

Also, $$h_e = h_g = 2749.6 \text{ kJ/kg}$$

Substituting the values into the expression for the heat transfer gives

$$Q_{cv} = (43.859 \times 2563.6) - (53.977 \times 2317.46) - 2749.6(43.859 - 53.977)$$
$$Q_{cv} = 15167.84 \text{ kJ}$$ Ans.

4. (c)

Writing the SFEE and mass balance for the mixing valve as:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

and $$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

From steam table, for saturated liquid, we have

$$h_1 = 167.53 \text{ kJ/kg}, s_1 = 0.5724 \text{ kJ/kgK}$$
$$h_2 = 377.04 \text{ kJ/kg}, s_2 = 1.1929 \text{ kJ/kgK}$$
$$h_3 = 251.18 \text{ kJ/kg}, s_3 = 0.83129 \text{ kJ/kgK}$$

Substituting these values and the required value of $\dot{m}_3$ = 1.5 kg/min, we get

$$\dot{m}_1 \times 167.53 + (1.5 - \dot{m}_1) \times 377.04 = 1.5 \times 251.18$$

∴ $$\dot{m}_1 = \frac{1.5 \times 251.18 - 1.5 \times 377.04}{167.53 - 377.04} = 0.9 \text{ kg/min}$$

and $$\dot{m}_2 = 1.5 - 0.9 = 0.6 \text{ kg/min}$$

The entropy generation rate in the mixing valve is

$$\dot{s}_{gen} = \dot{m}_3 \dot{s}_3 - \dot{m}_1 \dot{s}_1 - \dot{m}_2 \dot{s}_2$$

$$\dot{s}_{gen} = 1.5 \times 0.83129 - 0.9 \times 0.5724 - 0.6 \times 1.1929$$

$$\dot{s}_{gen} = 0.016 \text{ kJ/minK} = 0.267 \text{ W/K}$$

The entropy generation rate in the electric heater is

$$\dot{s}_{heater} = \dot{m}_2(s_2 - s_1)$$

$$= 0.6(1.1929 - 0.5724) \quad (\because \; s_1 = s_0)$$

$$= 0.3723 \text{ kJ/min.K}$$

$$= 6.205 \text{ W/K}$$

$\therefore$ Total entropy generation rate = 6.205 + 0.267

$$= 6.472 \text{ W/K}$$ Ans.

Section : B

5. (a)

Given : $D_0 = 60$ mm; $D_i = 0.4 \times D_0 = 0.4 \times 60 = 24$ mm

Allowable tensile stress, $S_{yt} = 260$ MPa

$$\text{Allowable shear stress, } \tau_{per} = \frac{0.5 \times S_{yt}}{\text{FOS}} = \frac{0.5 \times 260}{2} = 65 \text{ MPa}$$

Maximum allowable equivalent torque,

$$T_e = \frac{\pi}{16} \times \frac{(D_o^4 - D_i^4)}{D_o} \times \tau_{per}$$

$$T_e = \frac{\pi}{16} \times \frac{(60^4 - 24^4)}{60 \times 1000} \times 65$$

$$T_e = 2686.17 \text{ Nm}$$

Applied torque, $T' = T$ Nm

Applied bending moment, $M' = 0.5T$ Nm

Equivalent torque according to Tresca's failure theory,

$$T_e = \sqrt{(T')^2 + (M')^2}$$

$$= \sqrt{T^2 + (0.5T)^2} = \frac{\sqrt{5}}{2}T$$

$$\Rightarrow \quad T_e = \frac{\sqrt{5}}{2}T$$

$$\Rightarrow \quad \frac{\sqrt{5}}{2}T = 2686.17$$

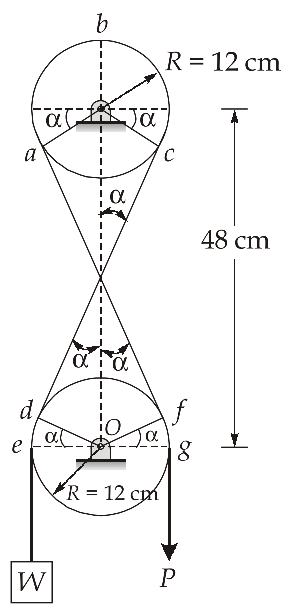
$\Rightarrow \quad T = \dfrac{2}{\sqrt{5}} \times 2686.17$

$\Rightarrow \quad T = 2402.58 \text{ Nm}$

So, the maximum permissible value of $T = 2402.58$ Nm

5. (b)

Refer to figure below,



Total angle of contact, θ = Angle *abc* + Angle *deo* + Angle *ogf*

$$\theta = (\pi + 2\alpha) + \alpha + \alpha = \pi + 4\alpha$$

From figure, $\sin\alpha = \dfrac{12}{24}$

$\Rightarrow \quad \alpha = \sin^{-1}\left(\dfrac{12}{24}\right) = 30^\circ = \dfrac{\pi}{6}$

$\therefore \quad \theta = \pi + 4 \times \dfrac{\pi}{6} = \dfrac{5\pi}{3}$

Also, given, $\mu = \dfrac{7}{8\pi}$

$\therefore \quad \mu\theta = \dfrac{5\pi}{3} \times \dfrac{7}{8\pi} = \dfrac{35}{24} = 1.4583$

$$e^{\mu\theta} = e^{1.4583} = 4.2988$$

(i) To lower the load W, $\dfrac{W}{P_{min}} = e^{\mu\theta} = 4.2988$

$$\Rightarrow \quad P_{min} = \frac{7}{4.2988} = 1.6283 \text{ kN} \quad \text{Ans.}$$

(ii) To raise the load, W, $\frac{P_{max}}{W} = e^{\mu\theta} = 4.2988$

$$\Rightarrow \quad P_{max} = 7 \times 4.2988$$

$$P_{max} = 30.0916 \text{ kN} \quad \text{Ans.}$$

5. (c)

Given : $K_A = K_B = K$; $E_B = 1.02\, E_A$

We know that, $$E = \frac{9KG}{3K+G}$$

$$\Rightarrow \quad 3EK + EG = 9\, KG$$

$$\Rightarrow \quad K(9G - 3E) = EG$$

$$\Rightarrow \quad K = \frac{EG}{3(3G-E)}$$

As given in question, $K_A = K_B$

$$\Rightarrow \quad \frac{E_A G_A}{3(3G_A - E_A)} = \frac{E_B G_B}{3(3G_B - E_B)}$$

$$\Rightarrow \quad E_A G_A(3G_B - E_B) = E_B G_B(3G_A - E_A)$$

$$\Rightarrow \quad 3E_A G_A G_B - E_A G_A E_B = 3E_B G_A G_B - E_A E_B G_B$$

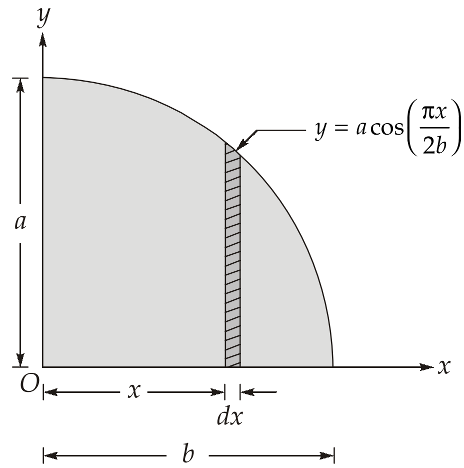
$$\Rightarrow \quad G_B[3E_A \cdot G_A - 3E_B \cdot G_A + E_A E_B] = E_A E_B G_A$$

$$G_B = \frac{E_A E_B G_A}{3E_A G_A - 3E_B G_A + E_A E_B}$$

$$G_B = \frac{1.02\, E_A E_A G_A}{3E_A G_A - 3 \times 1.02 \times E_A G_A + 1.02 \times E_A E_A}$$

$$G_B = \frac{1.02 \times E_A \times G_A}{1.02 E_A - 0.06 G_A} \quad \text{Ans.}$$

5. (d)



Consider an elementry strip of height y and thickness dx at a distance x from oy-axis.

$$\text{Area, } A = \int_0^b y\,dx = \int_0^b a\cos\left(\frac{\pi x}{2b}\right)dx$$

$$= a\left(\frac{2b}{\pi}\right)\left[\sin\left(\frac{\pi x}{2b}\right)\right]_0^b$$

$$A = \frac{2ab}{\pi}$$

First moment of area about oy-axis,

$$M_y = \int_0^b y \cdot x\,dx = \int_0^b a\cos\left(\frac{\pi x}{2b}\right) x\,dx$$

$$= a\left[\frac{2b}{\pi} x \cdot \sin\left(\frac{\pi x}{2b}\right) - \int_0^b \frac{2b}{\pi}\sin\left(\frac{\pi x}{2b}\right)dx\right]_0^b$$

$$= a\left[\frac{2b}{\pi} b \cdot \sin\left(\frac{\pi}{2}\right)\right] + a\left[\frac{4b^2}{\pi^2}\cos\left(\frac{\pi x}{2b}\right)\right]_0^b$$

$$= \frac{2ab^2}{\pi} + \frac{4ab^2}{\pi^2}[0-1] = \frac{2ab^2}{\pi} - \frac{4ab^2}{\pi^2}$$

$$\therefore \quad \bar{x} = \frac{M_y}{A} = \frac{\left[\frac{2ab^2}{\pi} - \frac{4ab^2}{\pi^2}\right]}{\frac{2ab}{\pi}} = b - \frac{2b}{\pi}$$

$$\bar{x} = \left(\frac{\pi - 2}{\pi}\right)b$$

First moment of area about *ox*-axis,

$$M_x = \int_0^b y \cdot dx \cdot \frac{y}{2} = \frac{a^2}{2}\int_0^b \cos^2\left(\frac{\pi x}{2b}\right)dx$$

$$= \frac{a^2}{2}\int_0^b \frac{1}{2}\left\{\cos\left(\frac{\pi x}{b}\right) + 1\right\}dx$$

$$M_x = \frac{a^2}{4}\left[\frac{b}{\pi}\sin\left(\frac{\pi x}{b}\right) + x\right]_0^b = \frac{a^2 b}{4}$$

$$\therefore \quad \bar{y} = \frac{M_x}{A} = \frac{a^2 b}{4} \times \frac{\pi}{2ab} = \frac{\pi a}{8}$$

$$\therefore \quad \bar{y} = \frac{\pi a}{8}$$ Ans.

5. (e)

Given : L = 945 mm; D = 420 mm; t = 10 mm; p = 6 MPa; E = 100 GPa; μ = 0.3; K_{oil} = 2600 MPa

The internal volume of cylinder, $V = \frac{\pi D^2 L}{4} = \frac{\pi \times 420^2 \times 945}{4}$ = 130924303 mm^3

Under the action of internal pressure, the cylinder expands and the oil is compressed.

∴ The volume of oil pumped, δV = Expansion of cylinder (δV_1) + Compression of oil (δV_2)

Expansion of cylinder, $\delta V_1 = \frac{pD}{4tE}(5 - 4\mu) \times V$

$$= \frac{6 \times 420 \times (5 - 4 \times 0.3)}{4 \times 10 \times 100 \times 10^3} \times 130924303 = 313432.78 \text{ mm}^3$$

Compression of oil, $\delta V_2 = \frac{p \cdot V}{K_{oil}} = \frac{6 \times 130924303}{2600} = 302133 \text{ mm}^3$

∴ Volume of oil pumped, $\delta V = \delta V_1 + \delta V_2$

= 313432.78 + 302133

= 615565.787 mm^3

= 615.56 cm^3

6. (a)(i)

Given : D = 87.5 cm = 875 mm; t = 6 mm

$$\delta V = 64 \text{ cm}^3 = 64000 \text{ mm}^3;\ E = 205 \text{ GPa};\ \mu = 0.32$$

Volume of spherical shell, $V = \frac{\pi}{6}D^3 = \frac{\pi}{6}\times 875^3$

$$= 350770273.5 \text{ mm}^3$$

Volumetric strain of spherical shell,

$$\in_V = \frac{\delta V}{V} = \frac{64000}{350770273.5} = 1.82456\times 10^{-4}$$

Also, $$\in_V = \frac{3P.D.}{4tE}(1-\mu)$$

$$\Rightarrow \quad \frac{3P.D.}{4tE}(1-\mu) = 1.82456 \times 10^{-4}$$

$$\Rightarrow \quad \frac{3\times P\times 875}{4\times 6\times 205\times 10^3}(1-0.32) = 1.82456 \times 10^{-4}$$

$$\Rightarrow \quad P = 0.5 \text{ MPa}$$

6. (a)(ii)

Given : σ_1 = 50 MPa; σ_2 = –80 MPa; σ_3 = 0; μ = 0.25; $\sigma_{yc} = \sigma_{yt}$ = 210 MPa

(1) Maximum shear stress theory

$$\sigma_1 - \sigma_2 = \frac{\sigma_{yt}}{N}$$

$$\Rightarrow \quad 50 - (-80) = \frac{210}{N}$$

$$\Rightarrow \quad N = \frac{210}{130} = 1.615$$ Ans.

(2) Maximum strain energy theory

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2) = \left(\frac{\sigma_{yt}}{N}\right)^2$$

$$\Rightarrow \quad 50^2 + (-80)^2 - 2\times 0.25\times[50\times(-80)] = \left(\frac{210}{N}\right)^2$$

$$\Rightarrow \quad N = \frac{210}{\sqrt{50^2 + 80^2 + 2\times 0.25\times 50\times 80}}$$

$$\therefore \quad N = 2.011$$ Ans.

(3) Maximum shear strain energy theory

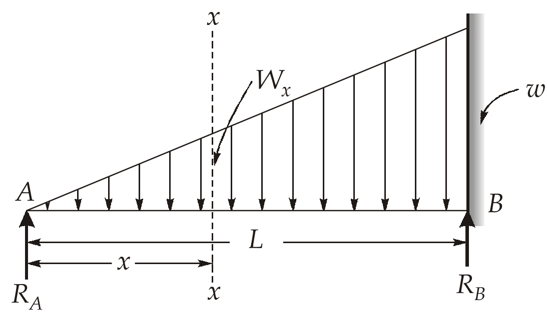
$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \left(\frac{\sigma_{yt}}{N}\right)^2$$

$$\Rightarrow \quad 50^2 + (-80)^2 - \{50 \times (-80)\} = \left(\frac{210}{N}\right)^2$$

$$\Rightarrow \quad N = \frac{210}{\sqrt{50^2 + 80^2 + 50 \times 80}}$$

$$\therefore \quad N = 1.849$$ Ans.

6. (b)



Let the reaction at the propped end A be R_A.

Rate of loading at 'x' distance is given by, $w_x = \frac{w \cdot x}{L}$

Load upto x due to uniformly varying load, (UVL),

$$W_x = w \cdot \frac{x}{L} \cdot \frac{x}{2} = \frac{wx^2}{2L}$$

Bending moment at section x-x due to UVL = $\frac{wx^2}{2L} \times \frac{x}{3} = \frac{wx^3}{6L}$

Bending moment at section x-x, $M_x = R_A x - \frac{wx^3}{6L}$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} = R_A \cdot x - \frac{wx^3}{6L} \quad \text{...(i)}$$

On integrating the above equation, we get

$$\Rightarrow \quad EI\frac{dy}{dx} = \frac{R_A \cdot x^2}{2} - \frac{wx^4}{24L} + C_1 \quad \text{...(ii)}$$

Again integrating equation (ii), we get

$$EIy = \frac{R_A \cdot x^3}{6} - \frac{wx^5}{120L} + C_1 x + C_2 \quad \text{...(iii)}$$

Boundary condition, at $x = L$, $\frac{dy}{dx} = 0$

$$\Rightarrow \quad \frac{R_A L^2}{2} - \frac{wL^4}{24L} + C_1 = 0$$

$$\Rightarrow \quad C_1 = \frac{wL^3}{24} - \frac{R_A L^2}{2} \quad \text{...(iv)}$$

From equation (iii) and (iv), we get

$$EIy = \frac{R_A x^3}{6} - \frac{wx^5}{120L} + \frac{wL^3 x}{24} - \frac{R_A L^2 x}{2} + C_2 \quad \text{...(v)}$$

Boundary condition, $x = 0$, $y = 0$

$$\Rightarrow \quad C_2 = 0$$

$$\therefore \quad EIy = \frac{R_A x^3}{6} - \frac{wx^5}{120L} + \frac{wL^3 x}{24} - \frac{R_A L^2 x}{2} \quad \text{...(vi)}$$

Also, boundary condition, $x = L$, $y = 0$

$$\Rightarrow \quad \frac{R_A L^3}{6} - \frac{wL^4}{120} + \frac{wL^4}{24} - \frac{R_A L^3}{2} = 0$$

$$\Rightarrow \quad R_A \left(\frac{L^3}{2} - \frac{L^3}{6} \right) = \frac{wL^4}{24} - \frac{wL^4}{120}$$

$$\Rightarrow \quad R_A = \frac{wL}{10} \quad \text{Ans.}$$

$$\Rightarrow \quad EIy = \frac{wLx^3}{10 \times 6} - \frac{wx^5}{120L} + \frac{wL^3 x}{24} - \frac{wL^3 x}{2 \times 10}$$

$$\Rightarrow \quad y = \frac{1}{EI} \left[\frac{wLx^3}{60} - \frac{wx^5}{120L} - \frac{wL^3 x}{120} \right] \quad \text{Ans.}$$

Slope at propped end, (i.e. $x = 0$)

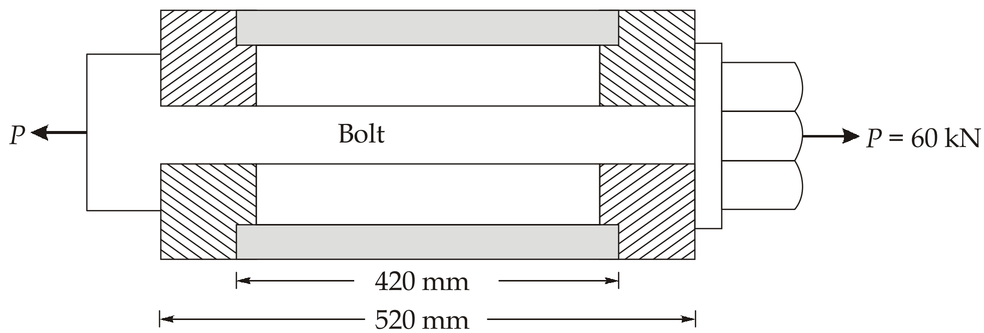
$$\Rightarrow \quad EI \frac{dy}{dx} = C_1 \quad \text{[From equation (ii)]}$$

$$\text{or,} \quad \theta = \frac{1}{EI} \left[\frac{wL^3}{24} - \frac{wL^3}{20} \right]$$

$$\Rightarrow \qquad \theta = \frac{wL^3}{120EI} \text{ (Downward)} \qquad \text{Ans.}$$

6. (c)

Let the suffix B for the bolt and suffix T for the tube.



(a) Effect of tightening the nut : Due to tightening of the nut, tensile stress will be induced in the bolt and compressive stress will be induced in the tube.

For statical equilibrium,

$$\sigma_B A_B = \sigma_T A_T$$

$$\Rightarrow \qquad \sigma_B = \sigma_T \times \left(\frac{500}{700}\right) = \frac{5}{7}\sigma_T \qquad \text{...(i)}$$

Also, $\quad \Delta_B + \Delta_T = \text{Movement of the nut}$

$$\Rightarrow \qquad \frac{\sigma_B \, l_B}{E} + \frac{\sigma_T \cdot l_T}{E} = \frac{1}{5} \times 2.5 = 0.5 \text{ mm}$$

$$\Rightarrow \qquad \sigma_B \times 520 + \sigma_T \times 420 = 0.5 \times 2 \times 10^5$$

$$\Rightarrow \qquad \sigma_B \times 520 + \sigma_T \times 420 = 1 \times 10^5 \qquad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\Rightarrow \qquad \frac{5}{7} \times 520 \times \sigma_T + \sigma_T \times 420 = 1 \times 10^5$$

$$\Rightarrow \qquad \sigma_T = 126.35 \text{ MPa (Compressive)}$$

Hence, $\quad \sigma_B = 90.25 \text{ MPa (Tensile)}$

(b) Effect of external load : Let σ'_T and σ'_B be the additional stresses, both tensile, due to external tensile load of 60 kN.

$$\therefore \qquad \sigma'_B A_B + \sigma'_T A_T = 60000 \qquad \text{...(iii)}$$

Also, from compatibility equation,

$$\frac{\sigma'_B L_B}{E} = \frac{\sigma'_T L_T}{E}$$

$$\Rightarrow \quad \sigma'_B \times 520 = \sigma'_T \times 420$$

$$\Rightarrow \quad \sigma'_B = \frac{42}{52} \times \sigma'_T \quad \text{...(iv)}$$

From equation (iii) and (iv), we get

$$\frac{42}{52} \times \sigma'_T \times 700 + \sigma'_T \times 500 = 60000$$

$$\sigma'_T = 56.32 \text{ MPa (Tensile)}$$

$$\sigma'_B = \frac{42}{52} \times 56.32 = 45.49 \text{ MPa (Tensile)}$$

(c) Final stresses:

Total stresses in bolt = 90.25 + 45.49 = 135.74 MPa (Tensile)

and Total stresses in tube = 126.35 – 56.32 = 70.03 MPa (Compression)

7. (a)

Given : $P = 150$ kW; $l = 4.5$ m; $N = 120$ rpm; $\theta_{per} = 1.8°$; $\tau_{per} = 45$ MPa; E = 200 GPa; $\mu = 0.25$

$$G = \frac{E}{2(1+\mu)} = \frac{200}{2\times(1+0.25)} = 80$$

$$\text{Torque, } T = \frac{P \times 60 \times 10^6}{2\pi N} = \frac{150 \times 60 \times 10^6}{2\pi \times 120} = 11936620.72 \text{ N.mm}$$

For stiffness:

$$\theta = \frac{TL}{GJ} = \frac{11936620.72 \times 4500}{80 \times 10^3 \times \frac{\pi}{32} \times \left(D_o^4 - D_i^4\right)}$$

$$\Rightarrow \quad D_o^4 - D_i^4 = \frac{11936620.72 \times 32 \times 180 \times 4500}{80 \times 10^3 \times \pi \times 1.8 \times \pi}$$

$$\Rightarrow \quad D_o^4 - D_i^4 = 217697857.2 \quad \text{...(i)}$$

For strength:

$$(\tau_s)_{max} = \tau_{per}$$

$$\Rightarrow \quad \frac{T}{Z_p} = \tau_{per}$$

$$\Rightarrow \quad \frac{11936620.72}{\frac{\pi}{32} \times \left(D_o^4 - D_i^4\right)} \times \left(\frac{D_o}{2}\right) = 45$$

$$\Rightarrow \quad \frac{60792710.12 \times D_o}{\left(D_o^4 - D_i^4\right)} = 45$$

From equation (i), we get

$$\Rightarrow \quad \frac{60792710.18 \times D_o}{217697857.2} = 45$$

$$\Rightarrow \quad D_o = 161.14 \text{ mm}$$

Now from equation (i), we get

$$(161.14)^4 - D_i^4 = 217697857.2$$

$$\Rightarrow \quad D_i = 146.18 \text{ mm}$$

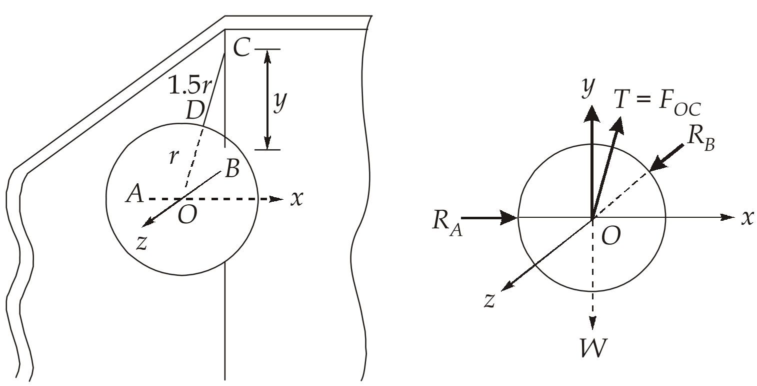
Hence, $\quad D_o = 162 \text{ mm and } D_i = 147 \text{ mm}$

7. (b)

Given : $r = 20$ cm = 0.2 m; $l = 30$ cm = 0.3 m = 1.5 × r; W = 500 N

Selecting the centre of sphere as origin and the coordinates system as shown in figure below, the coordinates of the point of contact with walls are,

$A(-r, 0, 0)$, $B(0, 0, -r)$ and $C(-r, y, -r)$



Now, $\quad OC = CD + r = 1.5r + r = 2.5r$

Also, $\quad OC^2 = (-r)^2 + (y)^2 + (-r)^2$

$$\Rightarrow \quad (2.5r)^2 = 2r^2 + y^2$$

or, $\quad y = 2.0615r$

$\therefore$ The coordinates of C are $(-r, 2.0615r, -r)$ i.e. (–0.2 m, 0.4123 m, –0.2 m)

From FBD of the sphere, which is in equilibrium under the action of reaction $R_A = F_{AO}$ and $R_B = F_{BO}$

from Walls, tension $T = F_{OC}$ in wire and self weight W.

$$\vec{r}_{AO} = r \cdot \hat{i} + O\hat{j} + O\hat{k} = r\hat{i}$$

$$\vec{r}_{AO} = \hat{i} + O\cdot\hat{j} + O\,\hat{k}$$

$\therefore$ $$\vec{F}_{AO} = F_{AO}\left(\hat{i} + O\cdot\hat{j} + O\,\hat{k}\right)$$

Also, $$\vec{r}_{BO} = O\cdot\hat{i} + O\cdot\hat{j} + r\cdot\hat{k} \text{ and } \vec{F}_{BO} = F_{BO}\left(O\hat{i} + O\hat{j} + \hat{k}\right)$$

Now, $$\vec{r}_{OC} = -r\,\hat{i} + 2.0615r\cdot\hat{j} - r\cdot\hat{k}$$

$$|\vec{r}_{OC}| = \sqrt{(-r)^2 + (2.0615\cdot r)^2 + (-r)^2}$$

$$|\vec{r}_{OC}| = 2.5r$$

$$\vec{r}_{OC} = -0.4\hat{i} + 0.825\hat{j} - 0.4\hat{k}$$

$$\vec{F}_{OC} = F_{OC}\left(-0.4\hat{i} + 0.825\hat{j} - 0.4\hat{k}\right)$$

and $$\vec{W} = -W\hat{j}$$

$\therefore$ The equilibrium equation is,

$$\vec{F}_{AO} + \vec{F}_{BO} + \vec{F}_{OC} + \vec{W} = 0$$

i.e. $$\left(F_{AO} + 0 - 0.4F_{OC}\right)\hat{i} + \left(0 + 0 + 0.825F_{OC} - W\right)\hat{j} + \left(0 + F_{BO} - 0.4F_{OC}\right)\hat{k} = 0$$

$\Rightarrow$ $$F_{AO} - 0.4F_{OC} = 0$$

$\Rightarrow$ $$F_{AO} = 0.4F_{OC} \quad \text{...(i)}$$

$\Rightarrow$ $$0.825 \times F_{OC} - W = 0$$

$\Rightarrow$ $$F_{OC} = \frac{W}{0.825} = 1.2121 \times W$$

$\Rightarrow$ $$F_{OC} = 1.2121 \times 500 = 606.05 \text{ N}$$

$\Rightarrow$ $$T = 606.05 \text{ N}$$ **Ans.**

From equation (i), we get

$$F_{AO} = 0.4 \times F_{OC} = 0.4 \times 606.05 = 242.42 \text{ N}$$

$\Rightarrow$ $$R_A = F_{AO} = 242.42 \text{ N}$$ **Ans.**

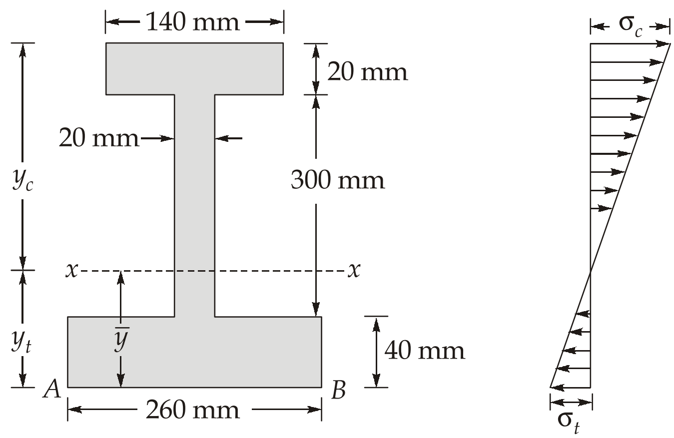
Also, $$F_{BO} - 0.4 \times F_{OC} = 0$$

$\Rightarrow$ $$F_{BO} = 0.4 \times F_{OC} = 0.4 \times 606.05 = 242.42 \text{ N}$$

$\Rightarrow$ $$R_B = F_{BO} = 242.42 \text{ N}$$ **Ans.**

7. (c)

Refer figure,



$$\text{Total area} = 140 \times 20 + 300 \times 20 + 260 \times 40$$
$$= 19200 \text{ mm}^2$$
$$\bar{y} = \frac{(140 \times 20 \times 350) + (300 \times 20 \times 190) + (260 \times 40 \times 20)}{19200}$$
$$\bar{y} = 121.25 \text{ mm}$$
$$I_{AB} = \left(\frac{1}{3} \times 240 \times 40^3\right) + \left(\frac{1}{3} \times 20 \times 340^3\right) + \left(\frac{1}{12} \times 140 \times 20^3 + 140 \times 20 \times 350^2\right)$$

or
$$I_{AB} = 610.24 \times 10^6$$
$$I_{xx} = I_{AB} - A\bar{y}^2 = 610.24 \times 10^6 - 19200 \times 121.25^2$$
$$= 327.97 \times 10^6 \text{ mm}^4$$
$$y_t = \bar{y} = 121.25 \text{ mm}$$
$$y_c = 360 - \bar{y} = 360 - 121.25 = 238.75 \text{ mm}$$

But,
$$\frac{\sigma_c}{y_c} = \frac{\sigma_t}{y_t}$$

⇒
$$\sigma_c = \sigma_t \times \frac{y_c}{y_t} = \sigma_t \times \frac{238.75}{121.25}$$

⇒
$$\sigma_c = 1.9691\,\sigma_t$$

When σ_t reaches 35 MPa, $\sigma_c = 1.9691 \times 35 = 68.92$ MPa, which is lower than the permissible limit. Should σ_c be allowed to go upto 95 MPa, σ_t will evidently be more than 35 MPa, which is not permissible. Hence, the allowable tensile stress will be the criterion for the distribution is shown in above figure.

Hence, $$Z_t = \frac{I_{xx}}{y_t} = \frac{327.97 \times 10^6}{121.25} = 2704907.22 \text{ mm}^3$$

$\therefore$ $$M_R = \sigma_t Z_t = 35 \times 2704907.22 = 94671752.58 \text{ N-mm}$$

Let w be the permissible load on the beam (in kN/m).

$\therefore$ $$M = \frac{wL^2}{8} = \frac{w \times 5^2}{8} = 3.125 \times w \text{ kNm}$$

$$= 3.125 \times w \times 10^6 \text{ Nmm}$$

Equating M_R and M, we get

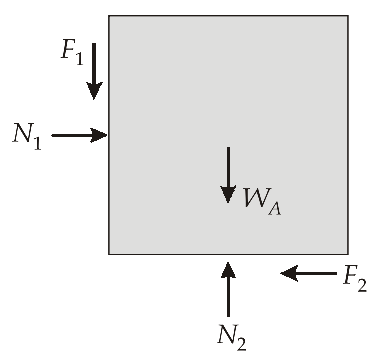
$$3.125 \times w \times 10^6 = 94671752.58$$

$\Rightarrow$ $$w = 30.29 \text{ kN/m}$$

8. (a) (i)

Given : W_A = 840 N; α = 8°; μ = 0.25

FBD 'A':



Applying equilibrium equation on block A, we get

$\Sigma F_x = 0$

$\Rightarrow$ $$N_1 - F_2 = 0$$

$\Rightarrow$ $$N_1 = F_2 = \mu N_2$$

$\Rightarrow$ $$N_1 = 0.25 N_2 \quad \text{...(i)}$$

$\Sigma F_y = 0$

$\Rightarrow$ $$N_2 - W_A - F_1 = 0$$

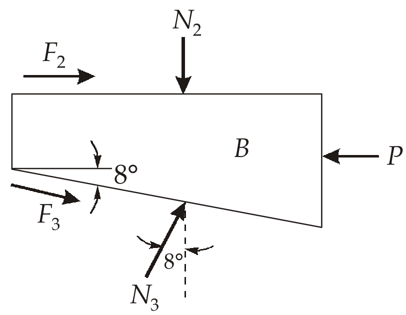
$\Rightarrow$ $$N_2 - W_A - \mu N_1 = 0$$

$\Rightarrow$ $$N_2 - 840 - 0.25 \times (0.25 \times N_2) = 0 \quad \text{(From equation (i))}$$

$\Rightarrow$ $$N_2 \times (1 - 0.25^2) = 840$$

$\Rightarrow$ $$N_2 = \frac{840}{(1 - 0.25^2)} = 896 \text{ N}$$

FBD 'B':



Applying the equations of equilibrium on wedge B, we get

$\Sigma F_y = 0$

$\Rightarrow \quad N_3\cos 8° - N_2 - F_3\sin 8° = 0$

$\Rightarrow \quad N_3\cos 8° - \mu N_3\sin 8° = N_2$

$\Rightarrow \quad N_3 \times (\cos 8° - \mu N_3\sin 8°) = N_2$

$$\Rightarrow \quad N_3 = \frac{896}{\cos 8° - 0.25 \times \sin 8°} = 937.75 \text{ N}$$

Also,

$\Sigma F_x = 0$

$\Rightarrow \quad F_3\cos 8° + F_2 - P + N_3\sin 8° = 0$

$$\Rightarrow \quad P = F_3\cos 8° + F_2 + N_3\sin 8°$$

$$= \mu N_3\cos 8° + \mu N_2 + N_3\sin 8°$$

$$= (0.25 \times \cos 8° + \sin 8°) \times 937.75 + 0.25 \times 896$$

$\Rightarrow \quad P = 586.67$ N **Ans.**

Therefore a force of 586.67 N will be required to raised the block A.

8. (a) (ii)

Given : Mass of vehicle (m) = 680 kg; Initial velocity (u) = 34 m/s; F = 255 N; t = 180s

(1) Velocity of vehicle when the force acts in the direction of motion

$$\text{Acceleration of the vehicle, } a = \frac{F}{m} = \frac{255}{680} = 0.375 \text{ m/s}^2$$

$\therefore$ Velocity of the vehicle after 180 seconds,

$$v_1 = u + at = 34 + 180 \times 0.375 = 101.5 \text{ m/s}$$

On the application of force in the direction of motion, velocity of vehicle increases from 34 m/s to 101.5 m/s i.e. (vehicle speed up).

(2) Velocity of the vehicle when the force acts in the opposite direction of motion, (i.e. a = –0.375 m/s²)

$$v_2 = u + at = 34 + (-0.375) \times 180$$

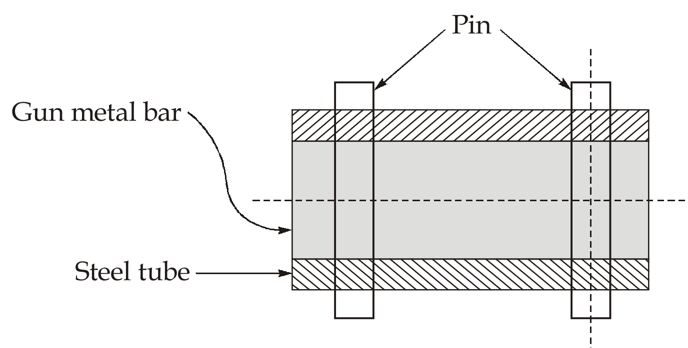
$$v_2 = -33.5 \text{ m/s}$$

Minus sign means that the vehicle is moving in the reverse direction or in other words opposite to the direction in which the vehicle was moving before the force was made to act.

8. (b)

Given : $D_g = 30$ mm; $D_{os} = 45$ mm; $D_{is} = 30$ mm; $D_p = 12$ mm; $\Delta t = 42°C$

Refer to figure,



The thermal expansion coefficient of gun metal bar is more than that of steel tube, so there will be tensile stress in the steel tube and compression stress in the gun metal bar.

Cross-section area of gun metal bar,

$$A_g = \frac{\pi}{4} \times 30^2 = 706.86 \text{ mm}^2$$

Cross-section area of steel tube,

$$A_s = \frac{\pi}{4} \times \left(45^2 - 30^2\right) = 883.57 \text{ mm}^2$$

From statics, $\sigma_g \cdot A_g = \sigma_s \cdot A_s$

$$\Rightarrow \quad \sigma_g = \sigma_s \times \frac{883.57}{706.86}$$

$$\Rightarrow \quad \sigma_g = 1.25 \times \sigma_s \quad \text{...(i)}$$

From compatibility equation,

$$\frac{\sigma_g L}{E_g} + \frac{\sigma_s L}{E_s} = (\alpha_g - \alpha_s) \times \Delta t \times L$$

$$\Rightarrow \quad \frac{\sigma_g}{0.91\times10^5}+\frac{\sigma_s}{2\times10^5} = (20-12)\times10^{-6}\times 42$$

From equation (i), we get

$$\Rightarrow \quad \frac{1.25\times\sigma_s}{0.91}+\frac{\sigma_s}{2} = 8\times10^{-6}\times10^5\times42$$

$$\Rightarrow \quad \sigma_s = 17.93 \text{ MPa}$$ Ans.

Hence, $\sigma_g = 1.25\times17.93$ Ans.

$= 22.42$ MPa

$\therefore$ The force between steel tube and gun metal bar.

$$F = \sigma_g\times A_g = 22.42\times706.86 = 15847.8 \text{ N}$$

Since, the pin is fitted transversely, and passes through the tube and the rod, it will be double shear.

$\therefore$ Shear stress in pin, $\tau_{pin} = \dfrac{F}{2A_P} = \dfrac{15847.8}{2\times\frac{\pi}{4}\times12^2} = 70.062$ MPa Ans.

8. (c)

Force equilibrium in vertical direction,

$\Sigma F_v = 0$

$$\Rightarrow \quad R_A + R_B = w(a+L) \quad \text{...(i)}$$

Taking moment about A, $\Sigma M_A = 0$

$$\Rightarrow \quad \frac{w(L+a)^2}{2} = R_B\times L$$

$$\Rightarrow \quad R_B = \frac{w(L+a)^2}{2L}$$

From equation (i), we get

$$R_A = w(a+L)-R_B = w(L+a)-\frac{w(L+a)^2}{2L} = \frac{w(a+L)(L-a)}{2L}$$

$$\Rightarrow \quad R_A = \frac{w(a+L)(L-a)}{2L}$$

From the above reaction force at B, R_B will be always acting upward (i.e. for $L < a$, $L = a$ and $L > a$)

But, if $L > a$, R_A will act upwards

If $L = a$, $R_A = 0$

If $L < a$, R_A will act downwards

Now, for $w = 4$ kN/m, $L = 8$ m and $a = 4$ m

$$R_A = \frac{w(a+L)(L-a)}{2L} = \frac{4(4+8)(8-4)}{2\times 8} = 12 \text{ kN} (\uparrow)$$

$$R_B = \frac{w(L+a)^2}{2L} = \frac{4(8+4)^2}{2\times 8} = 36 \text{ kN}(\uparrow)$$

S.F.D.: for AB, $F_x = R_A - wx = 12 - 4 \times x$

At $x = 0$, $F_A = 12$ kN

At $x = L = 8$ m, F_B (left) $= 12 - 4 \times 8 = -20$ kN

$$\text{S.F. is zero at } x = \frac{12}{4} = 3 \text{ m}$$

For BC, $F_x = 12 - 4x + 36 = 48 - 4x$

At $x = 8$ m, F_B (right) $= 48 - 4 \times 8 = 16$ kN

At $x = 12$ m, $F_C = 48 - 4 \times 12 = 0$

B.M.D.: for AB, $M_x = R_A x - \frac{w\times x^2}{2} = 12x - \frac{4x^2}{2} = 12x - 2x^2$

At $x = 0$, $M_A = 0$

At $x = L = 8$ m, $M_B = 12 \times 8 - 2 \times 8^2 = -32$ kNm

For maximum B.M., $\frac{dM_x}{dx} = 0$

$\Rightarrow$ $12 - 2 \times 2 \times x = 0$

$\Rightarrow$ $x = 3$ m

Hence, the B.M. is maximum where S.F. is zero.

$$M_{max} = 12 \times 3 - 2 \times 3^2 = 18 \text{ kNm}$$

For B.M. to be zero in AB, we have $12x - 2x^2 = 0$

which gives, $x = 6$ m

For BC, $M_x = R_A \cdot x - \frac{w\times x^2}{2} + R_B(x-8)$

$$M_x = 12x - \frac{4x^2}{2} + 36(x-8)$$

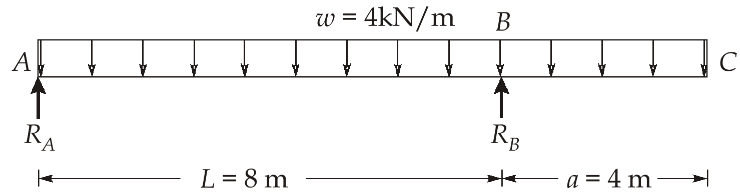
$$= 12x - 2x^2 + 36x - 288$$

$$= 48x - 2x^2 - 288$$

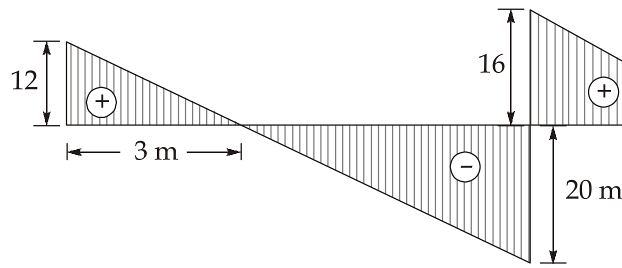
At $x = L = 8$ m, $M_B = 48 \times 8 - 2 \times 8^2 - 288 = -32$ kNm

At $x = L + a = 12$ m, $M_C = 48 \times 12 - 2 \times 12^2 - 288 = 0$

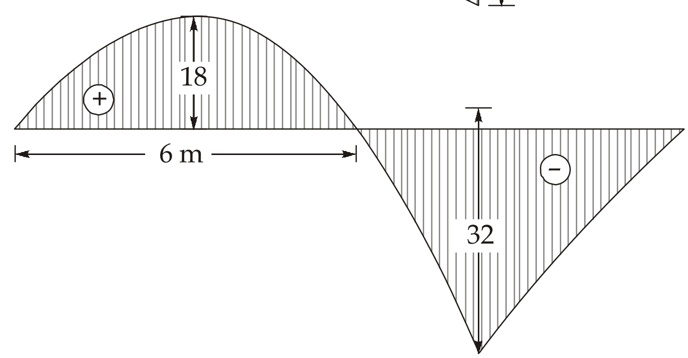
Loading diagram :



S.F.D.



B.M.D.



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