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ESE 2023

Main Exam Detailed Solutions

Electrical Engineering

PAPER-II

EXAM DATE : 25-06-2022 | 02:00 PM to 05:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/error/typo or want to contest the solution given by us, kindly send your suggested answer(s) with detailed explanation(s) at: **info@madeeasy.in**

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ANALYSIS

Electrical Engineering Paper-II
ESE 2023 Main Examination

Sl.	Subjects	Marks
1.	Analog and Digital Electronics	72
2.	Power Systems	84
3.	Systems & Signal Processing	72
4.	Control Systems	84
5.	Electrical Machines	84
6.	Power Electronics	84
		Total 480

Scroll down for detailed solutions

SECTION : A

Q.1 (a) Draw memory read machine cycle of 8085 microprocessor and explain.

[12 marks : 2023]

Solution:

Microprocessor 8085 machien cycles.

1. Opcode fetch; 2. Memory read; 3. Memory write; 4. I/O Read; 5. I/O Write

Required machine cycle timing diagram → Memory Read

Memory Read indicates to read/access data from memory into microprocessor.

e.g., MVI A, 90 H; 2B instruction to move 90H into accumualtor.

Machine cycles → Fetch and memory read.

Assuming the instruction at 4000H, 90H would be at 4001H.

Control signals : $\overline{RD}$ → 0 (active) in T2 and T3 states.

Status signals : S_1 = 1; S_0 = 0

$IO/\overline{M}$ = 0; Memory operation

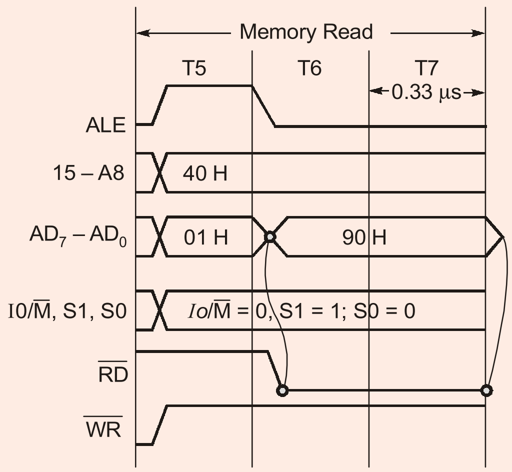
XX ⇒ Opcode of instruction

	Memory
4000H	XX
4001H	90H

Considering fetch as 4 T-states.

Memory Read starts from T5 and till T7.

If f_{CLK} = 3 MHz, $T = \frac{1}{f_{CLK}} = \frac{1}{3\times10^6} = 0.33\ \mu s$



In T6 and T7, $\overline{RD}$ = 0, i.e., active.

Data is accessed into Reg-A in T7 as per example.

After opcode is decoded in T4 of opcode fetch machine cycle, the microprocessor interpretes the instruction as 2B and knows that fetch and memory read cycles are required.

Memory Read Cycle :

T5 : When ALE → 1, all 16 address lines carry the address 4001H as per consideration.

$\therefore A_{15} - A_8$ = 40H $\{AD_7 - AD_0$ = 01 H.$\}$

Status lines, $\frac{IO}{\overline{M}} = 0$; $S_1 = 1$ and $S_0 = 0$ for memory read.

$\overline{RD} = 1$; Inactive as there is not data bus available.

$\overline{WR} = 1$; Inactive.

T_6 → ALE = 0, $A_{15} - A_8$ → Higher by of address, i.e., 40H but $AD_7 - AD_0$ → Data Bus.

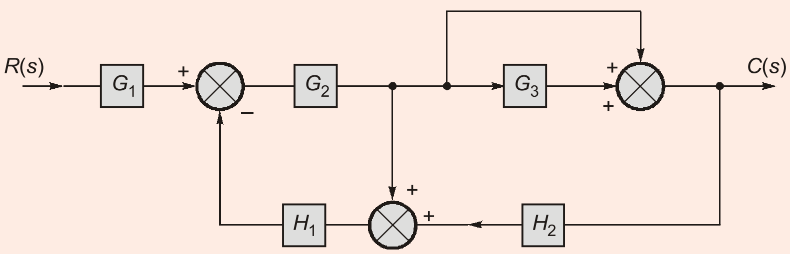
As data bus is available, $\overline{RD} = 0$, Active (or) $\overline{MEMR} = 0$; Active, i.e., Memory Read Control Signal : active.

Data from memory location is accessed onto data bus of memory chip.

T_7 → Data from memory chip data bus is accessed into microprocessor register A.

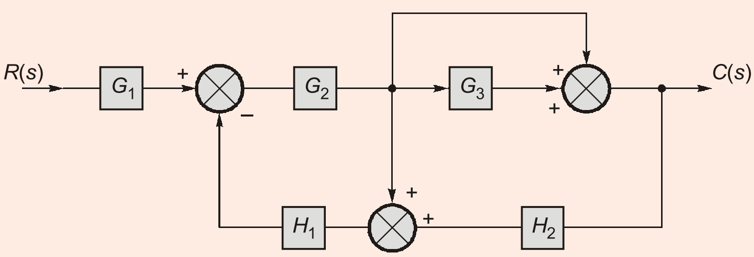
End of Solution

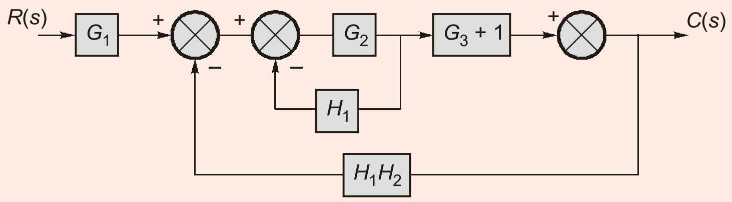
Q.1 (b) Reduce the block diagram shown below, using block diagram reduction technique and find the transfer function $\frac{C(s)}{R(s)}$.

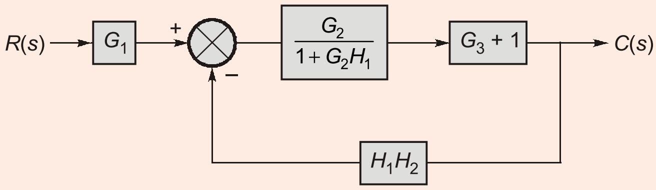


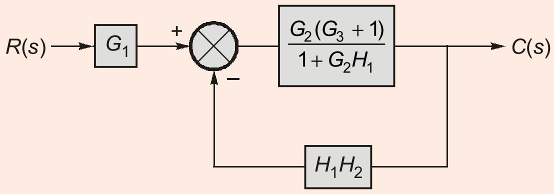
[12 marks : 2023]

Solution:









$$\frac{C(s)}{R(s)} = \frac{G_1G_2(G_3+1)}{1+G_2H_1+G_2H_1H_2(G_3+1)}$$

End of Solution

Q.1 (c) The maximum efficiency of a 500 KVA, 3300/500 V, 50 Hz single-phase transformer is 97% and occurs at $\frac{3}{4}$ full load, unity power factor. If the impedance is 10%, find the voltage regulation at full load, power factor 0.8 leading.

[12 marks : 2023]

Solution:

$$0.97 = \frac{0.75\times 500\times 1}{0.75\times 500\times 1+2P_i}$$

$$363.75 + 1.94P_i = 375$$

$$\Rightarrow \quad P_i = 5.7989 \simeq 5.8 \text{ kW}$$

According to maximum, $\eta \quad x^2P_{Cu} = P_i$

$$0.75^2(P_{Cu}) = 5.7989$$

$$\therefore \quad P_{Cu} = \frac{5.7989}{0.75^2} = 10.309 \text{ kW}$$

$$\%R = \% \text{ Cu loss}$$

$$\Rightarrow \quad \frac{10.309}{500}\times 100 = 0.0206$$

$$\%R = 2.06\%$$

$$\%Z = 10\%$$

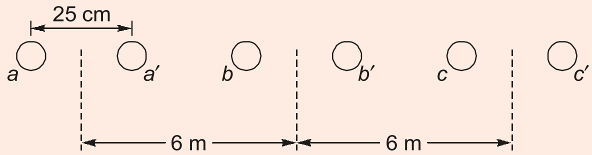
$$\therefore \quad \%X = \sqrt{10^2-2.06^2} = 9.7855\%$$

V.R. (FL) 0.8 p.f. lead $\Rightarrow$ $\%R \cos\phi - \% \times \sin\phi$

$$= 2.06 \times 0.8 - 9.7855 \times 0.6$$
$$= 1.648 - 5.87131 = -4.22\%$$

End of Solution

Q.1 (d) Calculate the inductance and capacitance of the single-circuit, two-bundle conductor, 200 km long line as shown below. The diameter of each conductor is 5 cm.



[12 marks : 2023]

Solution:

For bundle conductors,

$$L = 2\times10^{-7}\ln\frac{D_m}{D_s} \text{ H/m}$$

$$C = \frac{2\pi\in_o}{\ln\frac{D_m}{D_s}} \text{ F/m}$$

Here, $D_m = 3\sqrt{d_1 d_2 d_3} = 3\sqrt{6\times6\times12} = 7.56$ m

The diameter, $D = 5$ cm

$$r = \frac{D}{2} = 2.5 \text{ cm}$$

$$D_s \text{ for inductance} = \sqrt[4]{DaaDaa'Da'aDa'a'}$$
$$Daa = Da'a' = 0.7788 \times 2.5 = 1.947$$
$$Da'a = Daa' = 25$$
$$D_s = \sqrt[4]{1.947\times25\times25\times1.947} = 6.976 \text{ cm}$$
$$L = 2\times10^{-7}\ln\frac{756}{6.976} = 9.37\times10^{-7} \text{ H/m}$$

For 200 km length, $L = 200 \times 10^3 \times 9.37 \times 10^{-7} = 0.1874$ H

$$D_s \text{ for capacitor} = \sqrt[4]{DaaDaa'Da'aDa'a'}$$

Here, $Daa = Da'a' = 2.5$ cm

$$Daa' = Da'a = 25 \text{ cm}$$
$$D_s = \sqrt[4]{2.5\times2.5\times2.5\times2.5} = 7.9 \text{ cm}$$
$$C = \frac{2\pi\in_o}{\ln\frac{D_m}{n_s}} = \frac{2\pi\times8.854\times10^{-12}}{\ln\frac{756}{7.9}} = 12.2\times10^{-12} \text{ F/m}$$

For 200 km, $C = 200 \times 10^3 \times 12.2 \times 10^{-12} = 2.44$ μF

End of Solution

Q.1 (e) Explain the concept of Pulse Width Modulation. How is it used in the reduction of harmonics in a single-phase full bridge. Inverter?

[12 marks : 2023]

Solution:

PWM (Pulse Width Modulation) is an internal voltage control of inserter. In this method, a fixed dc input voltage is given to the inverter and a controlled output voltage is obtained by, adjusting the on and off periods of the inverter component.

In pulse width modulation, the width of the pulse is adjusted to reduce the harmonic. In general, the RMS value of the amplitude of harmonics voltage of a single phase full bridge inverter is given by

$$E_{Ln} = \frac{4E_{dc}}{\sqrt{2}\, n\pi} \times \sin nP$$

where P is the width of pulse, E_{dc} is the supply dc voltage

If the n^{th} harmonic is to be eliminated,

$$E_{Ln} = 0$$

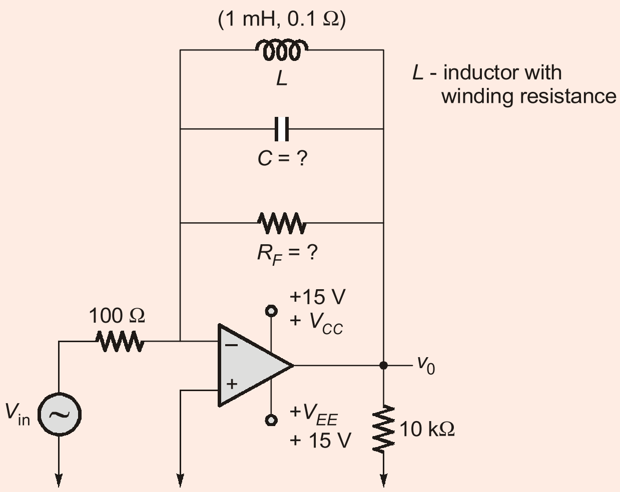
$$\therefore \quad \frac{4E_{dc}}{\sqrt{2}\, n\pi} \times \sin nP = 0$$

$$nP = 2\pi$$

$$P = \frac{2\pi}{n}$$

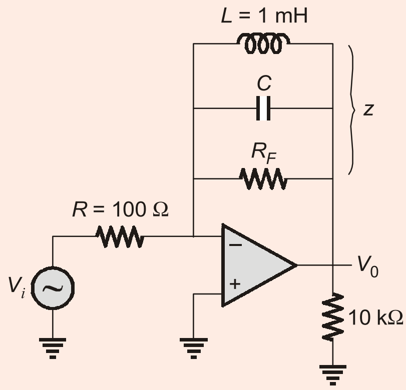
End of Solution

Q.2 (a) For the circuit given below, find the value of the components. Gain is 5 at a frequency of 32 kHz.



[20 marks : 2023]

Solution:



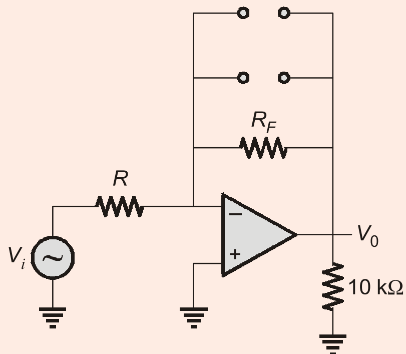
Assume resonant frequency of parallel RLC network as 32 kHz

$$f_o \cong \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{4\pi^2 f_o^2 L}$$

$$C = 24.736 \text{ nF}$$

At $f = f_o$, L and C become open circuit.



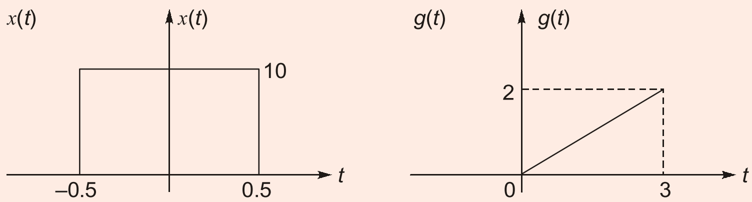
$$\frac{V_o}{V_i} = \frac{-R_F}{R}$$

$$\left|\frac{V_o}{V_i}\right| = \frac{R_F}{R} = 5$$

$$R_F = 5 \times R = 500 \ \Omega$$

End of Solution

Q.2 (b) Find $x(t) * g(t)$, using graphical convolution.



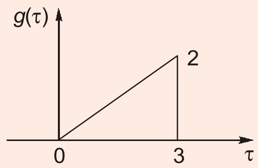
[20 marks : 2023]

Solution:

Given, $y(t) = x(t) * g(t)$

So, $y(t) = \int_{-\infty}^{\infty} g(\tau) \cdot x(t - \tau) d\tau$

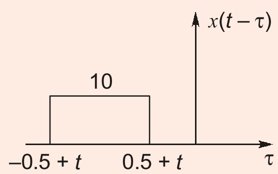
Case-I:



When $0.5 + t < 0$

$\Rightarrow$ $t < -0.5$

then, $y(t) = 0$



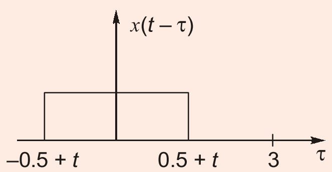
Case-II:

When $-0.5 + t < 0$ but $0.5 + t > 0$

i.e. $-0.5 < t < 0.5$

then, $y(t) = \int_{0}^{0.5+t} t_0 \cdot \frac{2}{3} \tau \, d\tau = \frac{20}{3}\left[\frac{\tau^2}{2}\right]_0^{0.5+t}$

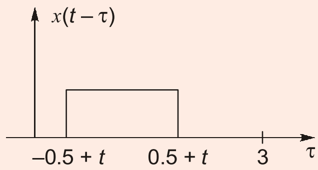
$= \frac{10}{3}(0.5 + t)^2$



Case-III:

When $-0.5 + t > 0$ but $0.5 + t < 3$

i.e $0.5 < t < 2.5$



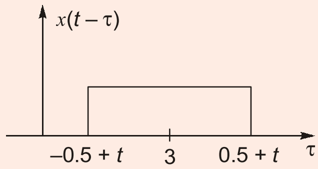
$$y(t) = \int_{-0.5+t}^{0.5+t} 10 \cdot \frac{2}{3}\tau\, d\tau = \frac{20}{3}\left[\frac{\tau^2}{2}\right]_{-0.5+t}^{0.5+t}$$

$$= \frac{10}{3}\left[(0.5+t)^2 - (-0.5+t)^2\right]$$

$$= \frac{20}{3}t$$

Case-IV:

When –0.5 + t < 3 but 0.5 + t > 3



i.e. 2.5 < t < 3.5

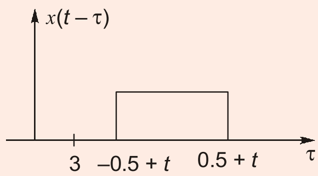
then,
$$y(t) = \int_{-0.5+t}^{3} 10 \cdot \frac{2}{3}\tau\, d\tau = \frac{20}{3}\left[\frac{\tau^2}{2}\right]_{-0.5+t}^{3}$$

$$= \frac{10}{3}\left[9 - (-0.5+t)^2\right] = \frac{10}{3}\left[8.75 - t^2 + t\right]$$

Case-V:

When –0.5 + t > 3,

i.e. t > 3.5



then
$$y(t) = 0$$

Thus,
$$y(t) = \begin{cases} 0, & \text{for } t < -0.5 \\ \dfrac{10}{3}(0.5+t)^2, & \text{for } -0.5 < t < 0.5 \\ \dfrac{20t}{3}, & \text{for } 0.5 < t < 2.5 \\ \dfrac{10}{3}[8.75 - t^2 + t], & \text{for } 2.5 < t < 3.5 \\ 0, & \text{for } t > 3.5 \end{cases}$$

End of Solution

Q.2 (c) Design a PD controller for a unity feedback system whose open loop transfer function.

$$G(s)H(s) = \frac{10}{(s+1)(s+4)}$$

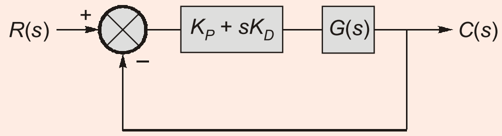
will have poles at $s = -4 \pm j4$.

[20 marks : 2023]

Solution:

Given OLTF is :

$$G(s)H(s) = \frac{10}{(s+1)(s+4)}$$



$$\frac{C(s)}{R(s)} = \frac{(K_P + sK_D)\cdot 10}{(s+1)(s+4)+(K_P+sK_D)\cdot 10}$$

Characteristic equation is,

$$s^2 + 5s + 4 + 10sK_D + 10K_P = 0$$

$$s^2 + s(5 + 10K_D) + 4 + 10K_P = 0$$

Poles will be :

$$s = \frac{-(5+10K_D) \pm \sqrt{(5+10K_D)^2 - 4(4+10K_P)}}{2}$$

Since given, $s = -4 \pm 4j$

On comparing,

$$\frac{-(5+10K_D)}{2} = -4$$

$$K_D = 0.3$$

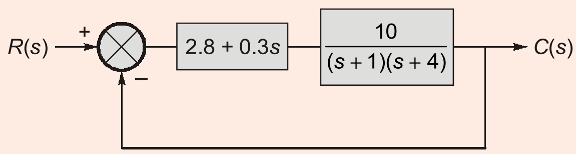
$$\frac{\sqrt{(4(4+10K_P)-64)}}{2} = j4$$

$$4(4 + 10K_P) - 64 = 64$$

$$4 + 10K_P = \frac{128}{4}$$

$$K_P = 2.8$$

Required system is



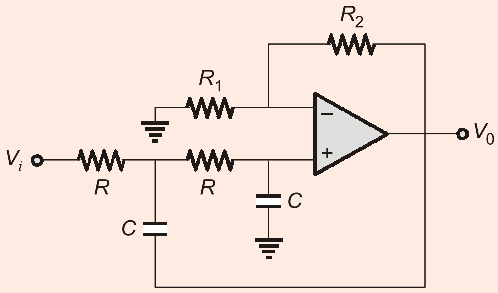
End of Solution

Q.3 (a) Design a second order low pass filter using Op-Amp with feedback gain 1.586. High cut-off frequency is 10 kHz. Assume capacitor 0.1 μF and R_1 = 10 kΩ (resistor connected between input source to input terminal of Op-Amp). Draw the circuit diagram and plot the frequency response.

[20 marks : 2023]

Solution:

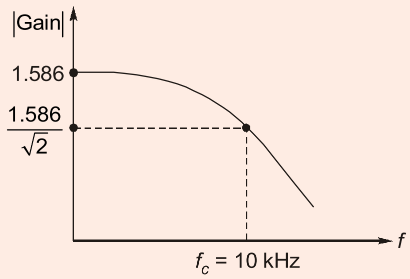
Second order low pass filter



Given :

$$C = 0.1\ \mu F, \quad R_1 = 10\ k\Omega$$
$$f_C = 10\ kHz, \quad A_{max} = 1.586$$
$$f_C = \frac{1}{2\pi RC}$$
$$\Rightarrow \quad R = \frac{1}{2\pi C f_C} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 10 \times 10^3}$$
$$R = 159.15\ \Omega$$
$$A_{max} = 1 + \frac{R_2}{R_1} = 1.586$$
$$\Rightarrow \quad R_2 = 0.586 \times R_1$$
$$R_2 = 5.86\ k\Omega$$

Frequency Response :



End of Solution

Q.3 (b) A dc motor is mechanically connected to a constant torque load. When the armature is connected to a 120 volt dc supply, it draws an armature current of 10 amperes and runs at 1800 rpm. The armature resistance is R_a = 0.1 Ω. Accidentally, the field circuit breaks and the flux drops to the residual flux, which is only 5% of the original flux.

(i) Determine the value of the armature current immediately after the field circuit breaks (i.e. before the speed has had time to change from 1800 rpm).

(ii) Determine the hypothetical final speed of the motor after the field circuit breaks.

Neglect the inductance of the armature circuit.

[20 marks : 2023]

Solution:

$$V = 120\text{ V},\ I_a = 10\text{ A},\ R_a = 0.1\ \Omega,\ N = 1800$$
$$\phi_2 = 0.05\ \phi_1,\ (\phi \text{ reduced by } 95\%)$$

(i) I_a when field circuit breaks immediately before speed change from 1800 rpm.

$$E_b = V - I_aR_a$$
$$= 120 - 10(0.1) = 119\text{ V}$$

$E_b \propto \phi N$, when $\phi \downarrow 95\%$ N : Constant

E_b also drops to 95% instantly.

(OR) If flux is 5% of original, E_b also same.

$\therefore$ $$E_{b2} = 0.05E_{b1} = 5.95\text{ V}$$

Suddenly, $$I_{a2} = \frac{V - E_b}{R_a} = \frac{120 - 5.95}{0.1} = 1140.5\text{ A}$$

(ii) Final speed (Hypothetically)

Given constant torque : $T \propto \phi I_a$

To maintain torque constant I_{a2} should be :

$$\frac{T_2}{T_1} = \frac{\phi_2}{\phi_1}\cdot\frac{I_{a2}}{I_{a1}}$$

$\Rightarrow$ $$1 = \frac{0.05\phi_1}{\phi_1}\cdot\frac{I_{a2}}{10}$$

$\Rightarrow$ $$I_{a2} = 200\text{ A}$$

For $I_{a2} = 200$ A, $$E_{b2} = V - I_{a2}R_a$$
$$E_{b2} = 120 - 200(0.1) = 100\text{ V}$$
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$\Rightarrow$ $$\frac{N_2}{1800} = \frac{100}{119} \times \frac{\phi_1}{0.05\phi_1}$$
$$N_2 = 30252.1\text{ rpm}$$

End of Solution

Q.3 (c) A 250 km long, three-phase, 50 Hz, transmission line has the following line constants :

$$A = D = 0.9\angle 1°,\quad B = 120\angle 72°,\quad C = 0.001\angle 90°\ \Omega$$

The sending end voltage is 230 kV.

Find:

(i) Line charging current

(ii) Maximum active power that can be transferred at 220 kV and also the corresponding reactive power.

[20 marks : 2023]

Solution:

$$A = D = 0.9\angle 1^\circ$$
$$B = 120\angle 72^\circ$$
$$C = 0.001\angle 90^\circ$$
$$V_s = 230 \text{ kV}, \; l = 250 \text{ km}$$

(i) Line charging current, $I_c = VY$

In main lines Y is C parameter

$$I_c = \frac{230\times 10^3}{\sqrt{3}}\times 0.001 = 132.8 \text{ Amp}$$

(ii)
$$P_{R,max} = \frac{V_S V_R}{B} - \left|\frac{A}{B}\right|\left|V_R^2\right|\cos(\theta-\alpha)$$

$$V_S|_{Ph} = \frac{230}{\sqrt{3}} = 132.8 \text{ kV}$$

$$V_R|_{Ph} = \frac{220}{\sqrt{3}} = 127 \text{ kV}$$

$$P_{R,max} = \frac{132.8\times 127}{120} - \frac{0.9}{120}(127)^2\cos(72-1)$$
$$= 101.163 \text{ MW/ph}$$

For 3-ϕ, $P_{R,max} = 303.5$ MW

$$\text{R.P. at } P_{R,max} = -\left(\frac{A}{B}\right)V_R^2\sin(\theta-\alpha)$$
$$= -\frac{0.9}{120}(127)^2\sin(72-1) = -114.38 \text{ MVAR/ph}$$

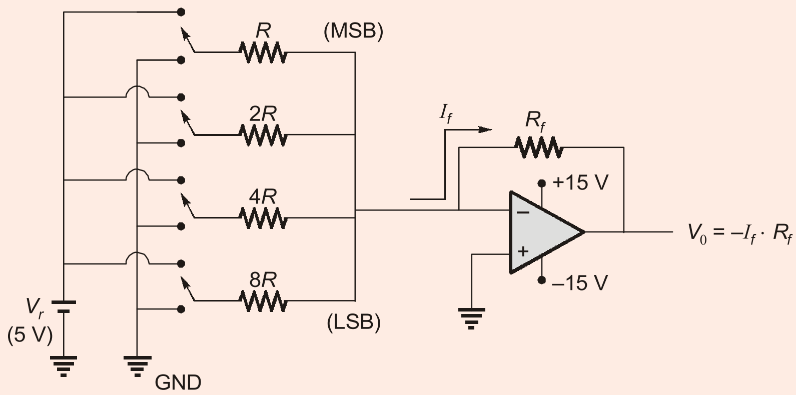
For 3-ϕ = 343.13 MVAR to be injected.

End of Solution

Q.4 (a) Draw a 4-bit digital to analog (D-A) converter circuit diagram using Op-Amp and binary weighted resistors. Derive the output voltage equation to get bidirectional signal output. Assume digital input 5 V and bias power supply are ±15 V.

[20 marks : 2023]

Solution:



Output voltage,

$$V_o = -I_f R_f$$

$$V_o = -R_f[I_3 + I_2 + I_1 + I_0]$$

$$= -R_f\left[\frac{V_r}{R} + \frac{V_r}{2R} + \frac{V_r}{4R} + \frac{V_r}{8R}\right]$$

$$= -R_f\left(\frac{V_r}{R}\right)\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}\right]$$

$$= -\frac{V_r}{R} \cdot R_f\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}\right]$$

$$= \left|\frac{V_r}{2^3}\right| \cdot \frac{R_f}{R} \underbrace{[2^3 b_3 + 2^2 b_2 + 2^1 b_1 + 2^0 b_0]}_{\text{Decimal equivalent of binary data}}$$

V_o = (Resolution).(Gain).(Decimal equivalent of binary data)

$$V_o = \frac{5}{2^3} \cdot \frac{R_f}{R}[2^3 b_3 + 2^2 b_2 + 2^1 b_1 + 2^0 b_0] \quad [\because V_r = 5 \text{ V}]$$

End of Solution

Q.4 (b) A unity feedback control system has

$$G(s) = \frac{10 * K}{s\left(\frac{s}{2} + 1\right)(s + 10)}$$

(i) Find gain and phase margin for $K = 1$.

(ii) If a phase-lag element with transfer of $\frac{(1+2s)}{(1+5s)}$ is added in the forward path, find the new value of K to keep the same gain margin.

[20 marks : 2023]

Solution:

(i) Given OLTF is

$$G(s) = \frac{10K}{s\left(\frac{s}{2}+1\right)(s+10)}$$

For $K = 1$ (given)

$$G(s) = \frac{20}{s(s+2)(s+10)}$$

For ω_{pc}, imaginary would be zero on negative real axis.

$$G(-j\omega_{pc}) = \frac{20}{-j\omega_{pc}^3 - 12\omega_{pc}^2 + 20j\omega_{pc}}$$

$$= \frac{20}{-12\omega_{pc}^2 + j(20\omega_{pc} - \omega_{pc}^3)}$$

Putting imaginary $[G(j\omega_{pc})] = 0$

$$\omega_{pc}(20 - \omega_{pc}^2) = 0$$

$$\omega_{pc} = 0, \sqrt{20} \text{ rad/sec}$$

Now, $$|G(j\omega_{pc})| = \left|\frac{20}{12\times 20}\right|$$

$$\text{G.M.} = \left|\frac{1}{G(j\omega_{pc})}\right|$$

$$\text{G.M.} = 12$$

$$\text{G.M. (dB)} = 20 \log 12$$

$$\text{G.M.} = 21.538 \text{ dB}$$

For ω_{gc} (gain cross over frequency)

$$|G(j\omega_{gc})| = 1$$

$$\left|\frac{20}{\omega\sqrt{\omega^2+4}\,\sqrt{\omega^2+100}}\right| = 1$$

$$400 = \omega^2(\omega^2 + 4)(\omega^2 + 100)$$

$$\omega^6 + 104\omega^4 + 400\omega^2 = 400$$

Assume : $\omega^2 = x$

$$x^3 + 104x^2 + 400x - 400 = 0$$

On solving,

$$x = -99.958, 0.822, -4.86$$

Valid, $$x = 0.822$$

$$\omega^2 = 0.822$$

$$\omega_{gc} = 0.906 \text{ rad/sec}$$

$$\angle G(\omega)\Big|_{\omega_s=\omega_{gc}} = -90° - \tan^{-1}\frac{\omega_{gc}}{2} - \tan^{-1}\frac{\omega_{gc}}{10}$$

$$= -90° - \tan^{-1}\frac{0.906}{2} - \tan^{-1}\frac{0.906}{10}$$

$$\angle G(j\omega_{pc}) = -119.56$$

$$\text{P.M.} = 180° + \angle G(\omega_{gc})$$

$$= 180° + (-119.56)$$

$$\text{P.M.} = 60.434°$$

(ii) OLTF after adding phase lag element

$$G'(s) = \frac{(1+2s)}{(1+5s)} \times \frac{20k}{s(s+2)(s+10)}$$

$$G'(s) = \frac{20k(1+2s)}{s(1+5s)(s+2)(s+10)} \quad ...(1)$$

∵ Gain margin is same as previous one.

So, G.M. = 21.538 (dB)

or G.M. = 12

For calculating ω_{PC},

$$\angle G'(j\omega) = -180°$$

$$-90° + \tan^{-1}2\omega - \tan^{-1}5\omega - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) = -180°$$

$$\Rightarrow \tan^{-1}\left(\frac{2\omega - 5\omega}{1+2\omega\times 5\omega}\right) - \tan^{-1}\left(\frac{\frac{\omega}{2}+\frac{\omega}{10}}{1-\frac{\omega}{2}\times\frac{\omega}{10}}\right) = -90°$$

$$\Rightarrow \tan^{-1}\left(\frac{-3\omega}{1+10\omega^2}\right) - \tan^{-1}\left(\frac{12\omega}{20-\omega^2}\right) = -90°$$

$$\Rightarrow \frac{\frac{3\omega}{1+10\omega^2}+\frac{12\omega}{20-\omega^2}}{1-\left(\frac{3\omega}{1+10\omega^2}\right)\left(\frac{12\omega}{20-\omega^2}\right)} = \tan 90° = \frac{1}{0}$$

$$\Rightarrow \left(\frac{3\omega}{1+10\omega^2}\right)\left(\frac{12\omega}{20-\omega^2}\right) = 1$$

$$\Rightarrow 36\omega^2 = (1 + 10\omega^2)(20 - \omega^2)$$

$$\Rightarrow 36\omega^2 = 20 - \omega^2 + 200\omega^2 - 10\omega^4$$

$$\Rightarrow 10\omega^4 - 163\omega^2 - 20 = 0$$

$$\Rightarrow \omega^2 = 16.42$$

$$\Rightarrow \omega = 4.05$$

$$\therefore \omega_{pc} = 4.05$$

$$\text{G.M.} = \frac{1}{|G'|_{\omega=\omega_{pc}}}$$

$$12 = \left|\frac{j\omega_{pc}(1+5j\omega_{pc})(j\omega_{pc}+2)(j\omega_{pc}+10)}{20k(1+2j\omega_{pc})}\right|$$

$$12 = \frac{1}{20k}\left|\frac{\omega_{pc}\sqrt{1+25\omega_{pc}^2}\;\sqrt{4+\omega_{pc}^2}\;\sqrt{100+\omega_{pc}^2}}{\sqrt{1+4\omega_{pc}^2}}\right|$$

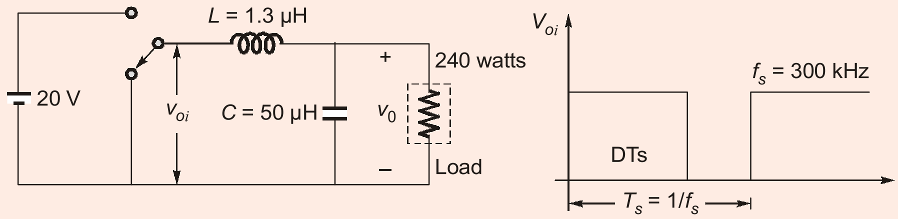
$$12 = \frac{490.3}{20k}$$

$$k = 2.04$$

∴ At $k = 2.04$ gain margin is same as part (i).

End of Solution

Q.4 (c) The equivalent circuit and its associated voltage waveform for a switched mode DC power supply is shown below,



(i) Assuming a pure dc V_0 = 15 V at the output across a load of 240 watts, calculate and draw the waveforms of voltage and current associated with the filter inductor 'L' and current through 'C'. Let switch duty ratio D = 0.75 in this condition.

(ii) Estimate the peak-to-peak ripple in the voltage across capacitor.

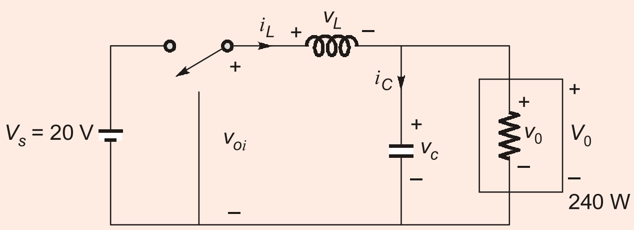
(iii) Calculate the harmonic voltage of v_{oi}.

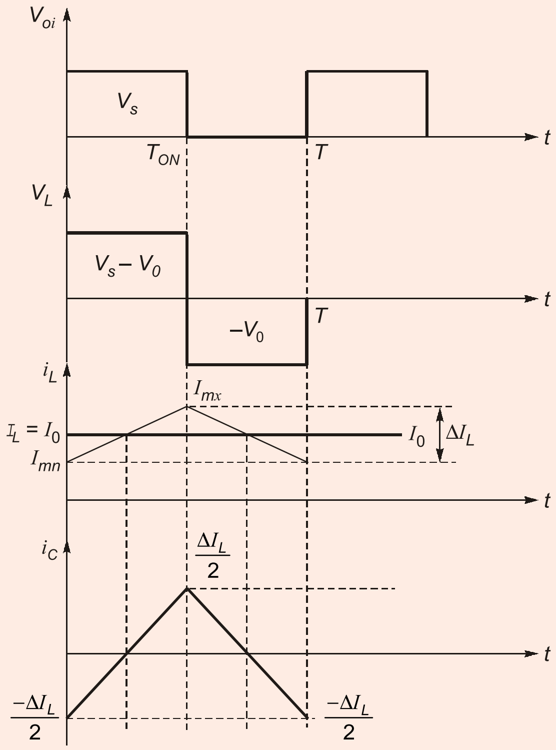
(iv) Calculate the attenuation in decibels of ripple voltage in v_{oi} at harmonic frequency

[20 marks : 2023]

Solution:

(i)





If $V_o = DV_s$, then it is 0.75 × 20, either continuous V_o = 15 V (given) or at the boundary.

$\therefore \quad V_o = 15\text{ V} = DV_s$ (given)

$$I_{OB} = \frac{\alpha(1-\alpha)V_s}{2fL} = \frac{0.75(1-0.75)\times 20}{2\times 300.10^3\times(1.3\times 10^{-6})}$$

$$I_{OB} = \frac{3.75}{0.78} = 4.8\text{ A}$$

$$P_o = 240\text{ W}$$

$$V_oI_o = 240$$

$$I_o = \frac{240}{15} = 16\text{ A}$$

$\because \quad I_o = 16\text{ A} = I_{OB}$

Hence, i_L is continuous.

(ii)
$$\Delta V_o = \Delta V_c = \frac{\alpha(1-\alpha)V_s}{8f^2LC}$$

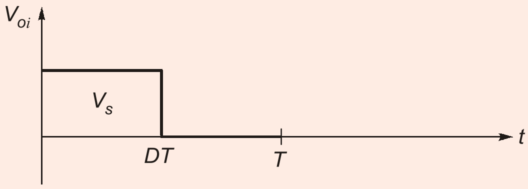
$$= \frac{0.75(1-0.75)\times 20}{8\times(300.10^3)^2\times(1.3\times 10^{-6})\times(50.10^{-6})}$$

$$= \frac{3.75}{1560.10^{-2}}$$

$$= 0.24\text{ V}$$

$$(V_o)_{\text{ripple,harmonic rms}} = \frac{0.24/2}{\sqrt{3}} = \frac{0.12}{\sqrt{3}} = 0.06928$$

(iii)



$$V_{oi,\text{Avg}} = DV_s = 0.75 \times 20 = 15 \text{ V}$$

$$V_{oi,\text{rms}} = \sqrt{D} \cdot V_s = \sqrt{0.75} \times 20 = 17.32 \text{ V}$$

$$V_{oi,\text{harmonic voltage}} = \sqrt{V_{oi,\text{rms}}^2 - V_{oi,\text{avg}}^2}$$

$$= \sqrt{17.32^2 - 15^2}$$

$$= 8.66 \text{ V}$$

(iv) Attenuation in decibels of ripples voltage (V_{oi})

$$= 20\log_{10}\left[\frac{(V_{oi})_{\text{Harmonic rms}}}{(V_o)_{\text{Harmonic rms}}}\right]$$

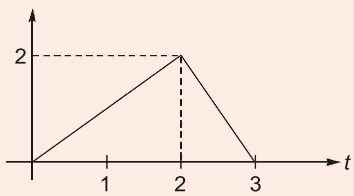
$$= 20\log_{10}\left(\frac{8.66}{0.06928}\right)$$

$$= 41.9 \text{ dB}$$

End of Solution

SECTION : B

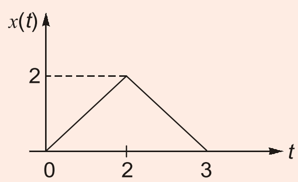
Q.5 (a) Find the Laplace transform of the signal given below.



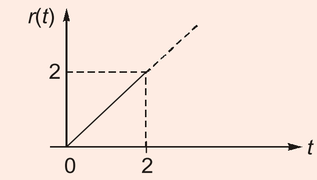
[12 marks : 2023]

Solution:

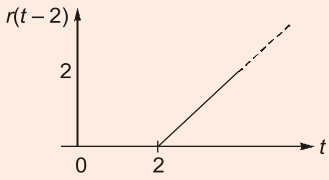
Given wave form is



Step-I:



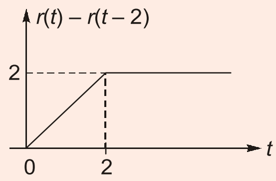
Step-II:



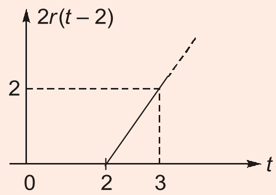
Step-III:

Waveform, (i) - (ii),

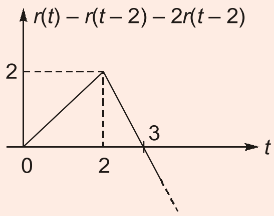
$r(t) - r(t-2)$



Step-IV:

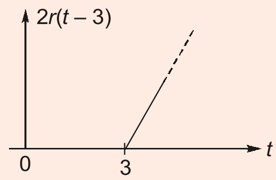


Step-V:



Waveform, (iii) - (iv),

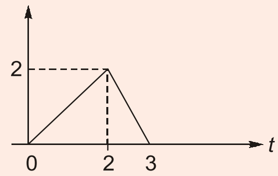
Step-VI:



Step-VII:

Wave form (v) + (vi)

$$x(t) = [r(t) - r(t-2) - 2r(t-2)] + 2r(t-3)$$



Thus, $$x(t) = r(t) - 3r(t-2) + 2r(t-3)$$

Now, $$r(t) \rightleftharpoons \frac{1}{s^2}$$

$$-3\,r(t-2) \rightleftharpoons \frac{-3}{s^2}e^{-2s} \quad \text{...by time-shifting property}$$

$$2\,r(t-3) \rightleftharpoons \frac{2}{s^2}e^{-3s} \quad \text{...by time shifting property}$$

Therefore Laplace transform of $x(t)$ is

$$X(s) = \frac{1}{s^2} - \frac{3}{s^2}e^{-2s} + \frac{2}{s^2}e^{-3s}$$

$$= \frac{1}{s^2}\left[1 - 3e^{-2s} + 2e^{-3s}\right]$$

End of Solution

Q.5 (b) Find the time response, initial value and final value of the given function,

$$F(s) = \frac{12(s+1)}{s(s+2)^2(s+3)}$$

[12 marks : 2023]

Solution:

Given function is :

$$F(s) = \frac{12(s+1)}{s(s+2)^2(s+3)}$$

For residue of $s = 0$

$$A = 1$$

For residue of $s = -2$.

Put $s = -2$ in $F(s)$

$$B = \frac{12 \times -1}{-2 \times 1} = 6$$

For residue of $s = -3$

$$C = \frac{12 \times -2}{-3 \times 1} = 8$$

For residue of $s = -2$, apply partial fraction

$$\frac{12(s+1)}{s(s+2)^2(s+3)} = \frac{A}{s}+\frac{B}{(s+2)^2}+\frac{C}{(s+3)^2}+\frac{D}{s+2}$$

$$\frac{12(s+1)}{s(s+2)^2(s+3)} = \frac{1}{s}+\frac{6}{(s+2)^2}+\frac{8}{(s+3)^2}+\frac{D}{(s+2)}$$

$$= \frac{(s+2)^2(s+3)+6(s+3)s+8s(s+2)^2+D[s(s+2)(s+3)]}{s(s+3)(s+2)^2}$$

On comparing,

$$16 + 18 + 32 + 6D = 12$$

$$D = -9$$

$$F(s) = \frac{1}{s}+\frac{6}{(s+2)^2}+\frac{8}{(s+3)}-\frac{9}{(s+2)}$$

On taking inverse laplace transform

$$f(t) = 1 + 6te^{-2t} + 8e^{-3t} - 9e^{-2t}$$

By applying initial value theorem

$$f(t)\big|_{t=0} = \lim_{s\to\infty} sF(s)$$

$$= \lim_{s\to\infty}\frac{12s(s+1)}{s(s+2)^2(s+3)}$$

$$\lim_{s\to 0} sF(s) = 0$$

By applying final value theorem

$$f(t)\big|_{t\to\infty} = \lim_{s\to 0} sF(s)$$

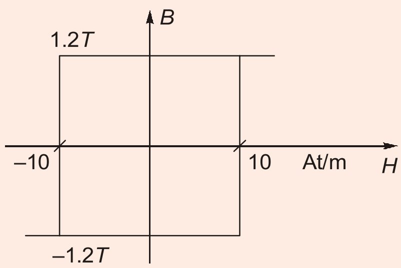
$$= \lim_{s\to 0}\frac{s\cdot 12(s+1)}{s(s+2)^2(s+3)}$$

$$\lim_{s\to 0} sF(s) = 1$$

End of Solution

Q.5 (c) A toroidal core of mean length 15 cm and cross-sectional area 10 cm^2 has a uniformly distributed winding of 300 turns.

The B-H characteristic of the core can be assumed to be of rectangular form, as shown in the figure below. The coil is connected to a 100 V, 400 Hz supply. Determine the hysteresis loss in the core.



[12 marks : 2023]

Solution:

$$\text{Hysteresis energy density} = \text{Area under B-H curve}$$
$$\text{Hysteresis energy density} = 2.4 \times 20 = 48 \text{ J/m}^3$$
$$\text{Volume of toroidal core} = \text{Area} \times \text{Mean length}$$
$$= 10 \times 10^{-4} \times 15 \times 10^{-2}$$
$$= 150 \times 10^{-6} \text{ m}^3$$

Now, Hysteresis energy loss, E = (Hysteresis energy density) × (Volume of toroidal core)

$$E = 48 \times 150 \times 10^{-6}$$
$$= 7.2 \text{ mJ}$$

$$\text{Hysteresis power loss, } P = \frac{E}{T}$$
$$= E \times f = 7.2 \times 10^{-3} \times 400$$
$$P = 2.88 \text{ Watts}$$

End of Solution

Q.5 (d) The incremental fuel cost for a generating plant having two units are

$$IC_1 = 20 + 0.1P_1 \quad ₹/\text{MWhr}$$
$$IC_2 = 15 + 0.012P_2 \quad ₹/\text{MWhr}$$

If the total demand P_D = 200 MW, determine the division of load between the units for the most economical operation.

[12 marks : 2023]

Solution:

$$I_{C1} = 0.1P_1 + 20 \text{ Rs/MW hr}$$
$$I_{C2} = 0.12P_2 + 15 \text{ Rs/MW hr}$$

For the total load as 200 MW, give ELD

In general, $P_G = P_1 + P_2 = P_D + P_L$

Here there are no losses, so

$$P_G = P_1 + P_2 = P_D$$

i.e., $P_1 + P_2 = 200$ ...(1)

For ELD losses, $I_{C1} = I_{C2}$

$$0.1P_1 + 20 = 0.12P_2 + 15$$
$$0.1P_1 - 0.12P_2 = -5$$
$$10P_1 - 12P_2 = -500 \quad \text{...(2)}$$

By solving (1) and (2)

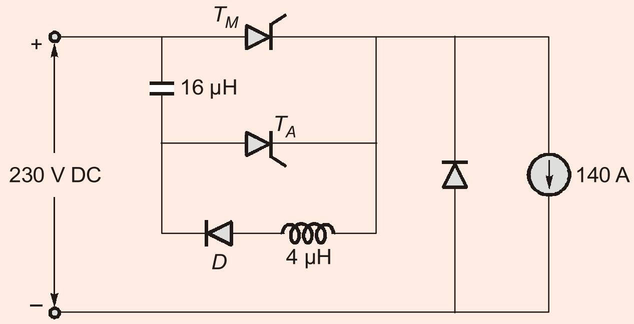
$$P_1 = 86.34 \text{ MW}$$
$$P_2 = 113.64 \text{ MW}$$

End of Solution

Q.5 (e) For a Class-D Commutation circuit shown below, calculate

(i) peak current through Main and Auxiliary thyristor

(ii) turn-off time(s) for Main and Auxiliary thyristors

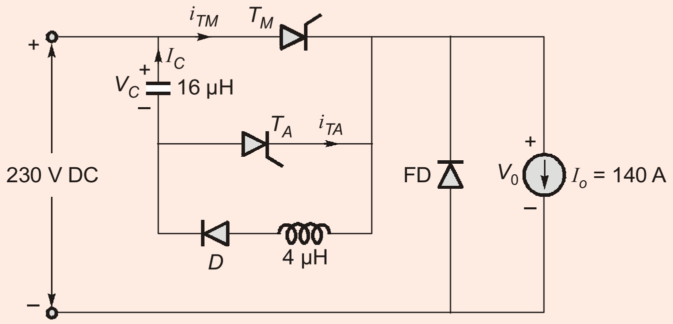


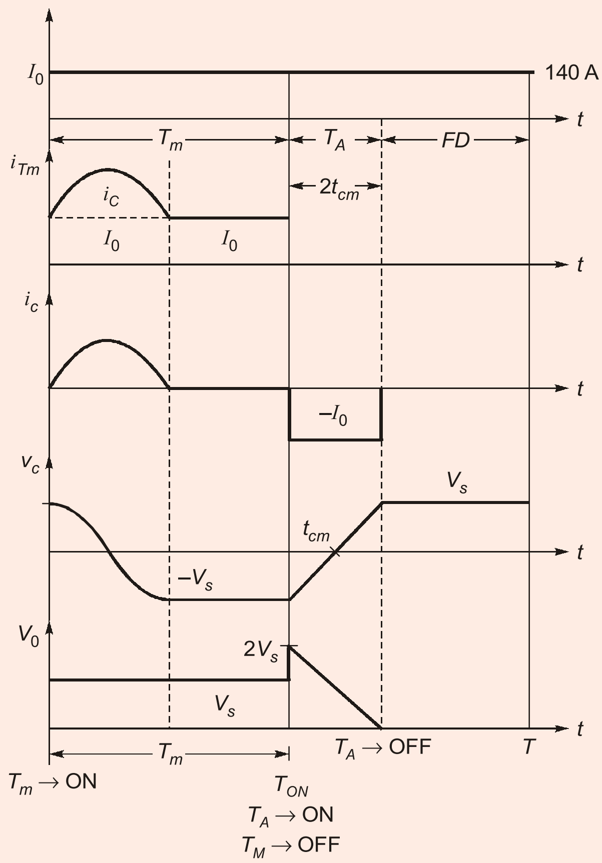
where T_M is main thyristor and T_A is Auxiliary thyristor.

[12 marks : 2023]

Solution:

Class D - Commutation :





(i)

$$(i_{TM})_{Peak} = I_o + I_P$$

$$= I_o + V_s\sqrt{\frac{C}{L}}$$

$$= 140 + 230\sqrt{\frac{16\ \mu F}{4\ \mu H}}$$

$$= 140 + (230 \times 2)$$

$$= 600\ A$$

$$(i_{TA})_{Peak} = I_o = 140\ A$$

(ii) Circuit turn OFF time of Main Thyristor

$$t_{cm} = \frac{C}{I_o}V_s$$

$$= \frac{16.10^{-6}}{140} \times 230$$

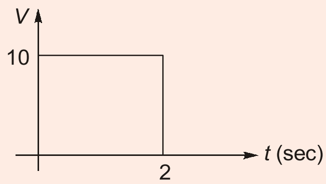
$$t_{cm} = 26.28\ \mu S$$

Circuit turn OFF time of T_A

$$t_{CA} = \frac{\pi}{2}\sqrt{LC} = \frac{\pi}{2}\sqrt{4.10^{-6} \times 16.10^{-6}}$$
$$= 12.56\ \mu S$$

End of Solution

Q.6 (a) Determine the Fourier transform of a pulse shown below.

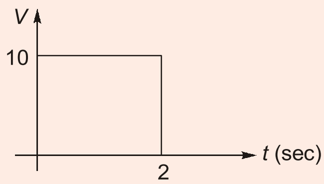


Find the magnitude at $\omega = 2\pi$.

[20 marks : 2023]

Solution:

Given pulse is



The fourier transform of $V(t)$ is given by,

$$V(\omega) = \int_{-\infty}^{\infty} v(t)\cdot e^{-j\omega t}dt = \int_0^2 10\cdot e^{-j\omega t}dt$$
$$= 10\cdot\left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^2 = 10\cdot\left[\frac{e^{-j20}}{-j\omega}\right]$$
$$= \frac{-10}{j\omega}\left[e^{-j2\omega} - 1\right] = \frac{10}{j\omega}\left[1 - e^{-j2\omega}\right]$$

At $\omega = 2\pi$,

$$V(2\pi) = \frac{10}{j2\pi}\left[1 - e^{-j4\pi}\right]$$
$$= \frac{10}{j2\pi}[1-1] \qquad \left[\because e^{-j4\pi} = 1\right]$$
$$= 0$$

Thus magnitude of $V(\omega)$ at $\omega = 2\pi$ is 'zero'.

End of Solution

Q.6 (b) For a single machine infinite bus shown below, if δ_c is the critical clearing angle for a three-phase short circuit 'F', prove that the clearing time 't_c' of the circuit breaker CB must satisfy the following:

$$t_c \le \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_i}}$$

where P_i is the mechanical power input,

δ_0 is the initial power angle,

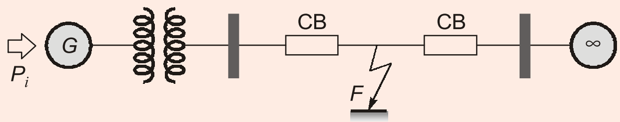
f is the frequency and

H is the machine inertia constant and is given by $H = \frac{\pi f}{G} J \left(\frac{2}{P}\right)^2 \omega_e \times 10^{-6}$

J is moment of inertia of rotor (kg-m^2),

ω_c is synchronous speed in electrical rad/sec

G is three-phase MVA rating (base) of machine

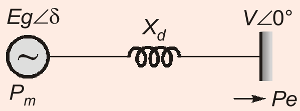


Also express the relation of δ_{cr} with δ_o, δ_{cr} is the critical clearing angle and corresponding critical clearing time.

[20 marks : 2023]

Solution:

For SMIB system



$$M\frac{d^2\delta}{dt} = P_a = P_m - P_e$$

Here, $$M = \frac{GH}{\pi f}$$

$$H = M\cdot\frac{\pi f}{G} = \frac{\pi f}{G} J\left(\frac{2}{P}\right)^2 \omega_c \times 10^{-6}$$

The above equation can be solved for δ as

$$\delta = \delta_o + \frac{P_a}{2M} t^2$$

if $\delta \to \delta_{cr}$, $t \to t_{cr}$, $P_a = P_m - P_e$

$$\delta_{cr} = \delta_o + \frac{(P_m - P_e)}{2M} t_{cr}^2$$

$$t_{cr} = \left[\frac{(\delta_{cr} - \delta_o)2M}{(P_m - P_e)}\right]^{1/2}$$

As 3-ϕ fault is occured at point F, P_e = 0.0.

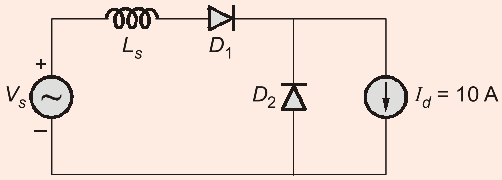
$$t_{cr} = \left[\frac{(\delta_{cr} - \delta_o)}{P_m} \cdot 2\frac{GH}{\pi f}\right]^{1/2}$$

As 3-ϕ fault accelerates the machine more, the clearance line should be less than this,

i.e., $t_c < \left[\frac{(\delta_{cr} - \delta_o)}{P_m} \cdot \frac{2GH}{\pi f}\right]^{1/2}$. (Proved).

End of Solution

Q.6 (c) A half-wave uncontrolled rectifier circuit is fed from ac source with source inductance 'L_s'. It is driving a dc load at a constant current I_d as shown in figure below,



Calculate average output voltage V_d, average power Pdf commutation overlap angle μ and plot the waveform of source current i_s , if

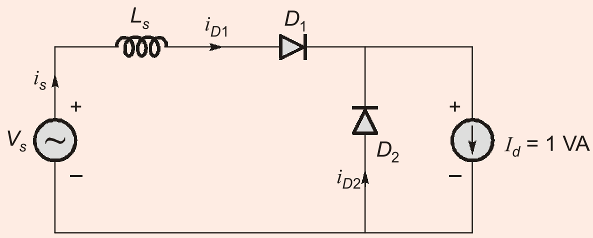
(i) V_s = 310 sin(314t) and L_s = 0.

(ii) V_s = 310 sin(314t) and L_s = 5 mH.

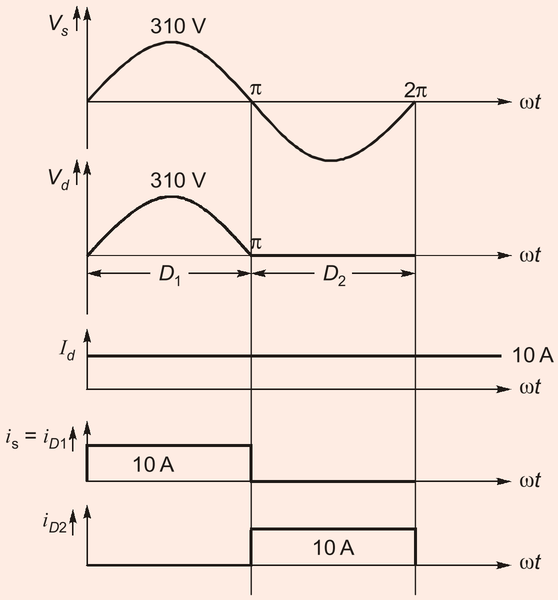
(iii) V_s is a square wave of 310 V and 50 Hz with a source inductance L_s = 5 mH.

[20 marks : 2023]

Solution:



(i) $V_s = 310 \sin(314t)$ and $L_s = 0$



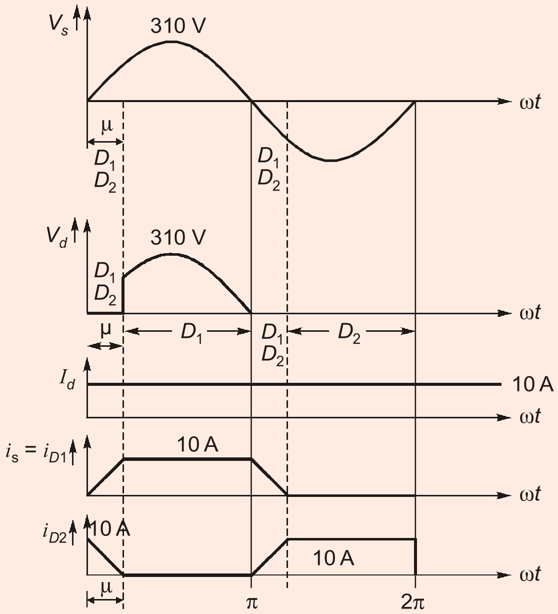
$$V_{d,avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{\pi} = \frac{310}{\pi} = 98.67 \text{ V}$$

$$P_d = V_d \cdot I_d = 98.676 \times 10$$
$$= 986.76 \text{ W}$$

Commutation angle, $\mu = 0$

$\because$ $L_s = 0$

(ii) $V_s = 310\sin(314t)$ and $L_s = 5$ mH



$$V_d = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \cdot d(\omega t)$$

$$V_d = \frac{V_m}{2\pi}[1+\cos\mu] = \frac{310}{2\pi}[1+\cos(18.31)]$$

$$= 96.178 \text{ V}$$

$\therefore$

$$P_d = V_d I_d$$

$$= 96.178 \times 10$$

$$= 961.78 \text{ W}$$

During overlap angle,

$$V_s = V_L$$

$$V_m \sin\omega t = L_s \frac{di_s}{dt}$$

$$di_s = \frac{V_m \sin\omega t}{L_s} dt$$

$$V_m \sin\omega t\, dt = L_s\, di_s$$

$$\int_0^{\mu} V_m \sin\omega t d(\omega t) = \omega L_s \int_0^{I_d} di_s$$

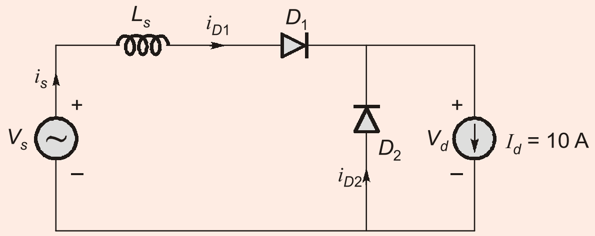
$$V_m[1 - \cos\mu] = \omega L_s I_d$$

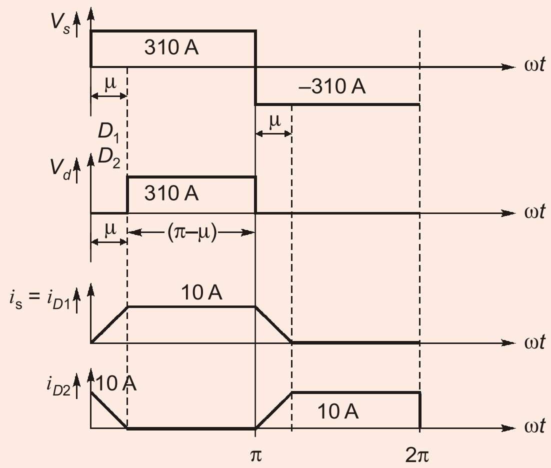
$$1 - \cos\mu = \frac{\omega L_s I_d}{V_m}$$

$$\cos\mu = 1 - \frac{314 \times 5.10^{-3} \times 10}{310}$$

$$\mu = 18.31°$$

(iii)





During overlap period : $D_1 D_2 \rightarrow$ ON

$$\therefore \quad V_s = L_s \frac{di_s}{dt}$$

$$\omega = 2\pi.50$$

$$\omega = 100\pi$$

$$V_s.dt = L_s di_s$$

$$\int_0^{\mu} V_s d(\omega t) = \omega L_s \int_0^{I_d} di_s$$

$$V_s \mu = \omega L_s I_d$$

$$\mu = \frac{\omega L_s I_d}{V_s} = \frac{100\pi \times 5.10^{-3} \times 10}{310}$$

$$\mu = 0.0506 \text{ rad}$$

$$\mu = 2.9^\circ$$

$$V_d = V_s\left(\frac{\pi - \mu}{2\pi}\right) = V_s\left(\frac{180^\circ - \mu^\circ}{360^\circ}\right)$$

$$= 310\left(\frac{180^\circ - 2.9^\circ}{360^\circ}\right) = 152.5 \text{ V}$$

$$P_d = V_d I_d = 152.5 \times 10 = 1525 \text{ W}$$

End of Solution

Q.7 (a) A certain system is described by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-2 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 3\end{bmatrix}u(t);\quad y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Determine the transformation matrix [P] so that if [x] = [P][Z]; the state matrics [$\tilde{A}$],[$\tilde{B}$],[$\tilde{C}$] and [$\tilde{D}$] describing the dynamics of [Z] are in control canonical form.

[20 marks : 2023]

Solution:

Given :

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix}=\begin{bmatrix}-2 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}+\begin{bmatrix}1\\ 3\end{bmatrix}u(t)$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}$$

$$A=\begin{bmatrix}-2 & 1\\ -2 & 0\end{bmatrix},\ B=\begin{bmatrix}1\\ 3\end{bmatrix},\ C=\begin{bmatrix}1 & 0\end{bmatrix},\ D=0$$

The characteristic equation of A is

$$|sI-A|=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}-2 & 1\\ -2 & 0\end{bmatrix}$$

$$|sI-A|=\begin{bmatrix}s+2 & -1\\ 2 & s\end{bmatrix}$$

$$|sI-A|=s(s+2)+2$$

$$|sI-A|=s^2+2s+2 \qquad ...(1)$$

Thus, the coefficients of the characteristics equation are $a_0 = 2$, $a_1 = 2$.

$$M=\begin{bmatrix}a_1 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix}$$

The controllability matrix is

$$s=\begin{bmatrix}B & AB\end{bmatrix}=\begin{bmatrix}1 & 1\\ 3 & -2\end{bmatrix}$$

$$AB=\begin{bmatrix}-2 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}1\\ 3\end{bmatrix}=\begin{bmatrix}1\\ -2\end{bmatrix}$$

Transformation matrix,

$$P=SM=\begin{bmatrix}1 & 1\\ 3 & -2\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix}$$

$$P=\begin{bmatrix}3 & 1\\ 4 & 3\end{bmatrix}$$

such that $[X] = [P]\ [Z]$

Thus, the control cannonical form is given by

$$[\bar{A}] = P^{-1}AP = \frac{1}{9-4}\begin{bmatrix} 3 & -1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -6 & -2 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 0 & 5 \\ -10 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$$

$$[\bar{B}] = P^{-1}B = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[\bar{C}] = CP = [1 \quad 0]\begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$$

$$= [3 \quad 1]$$

$$[\bar{D}] = D = 0$$

End of Solution

Q.7 (b) A 3-phase, 440 V, 50 Hz, four pole wound rotor induction motor develops full load torque at a slip of 0.04 (i.e. 4%) when the slip rings are short circuited. The maximum torque it can develop is 2.5 per unit. The stator leakage impedance is negligible. The rotor resistance measured between two slip rings is 0.5 Ω.

(i) Determine the speed of the motor at maximum torque. Derive the formula used.

(ii) Determine the starting torque in per unit. (Full load torque is one per unit torque).

(iii) Determine the value of resistance to be added to each phase of the rotor circuit so that maximum torque is developed at the starting condition.

(iv) Determine the speed at full-load torque with the added rotor resistance of part (iii).

[20 marks : 2023]

Solution:

440 V, 4-pole, 50 Hz, N_s = 1500 rpm, S_F = 0.04, T_{max} = 2.5 pu

∴ $T_{max} = 2.5T_f$

$$R_2 = \frac{0.5}{2} = 0.25\ \Omega \text{ per phase}$$

(i) Speed of motor at max torque

Slip at which max torque occurs :

Torque relation neglecting stator impedance :

$$T_1 = \frac{3 \times 60}{2\pi N_s} \cdot \frac{SE_2^2 R_2}{R_2^2 + (SX_2)^2}$$

For a constant V and F

$$T \propto \frac{SR_2}{R_2^2 + (SX_2)^2}$$

Let $T = \frac{1}{Y}$

where $$Y \propto \frac{R_2^2 + (SX_2)^2}{SR_2}$$

If T to be maximum, Y must be minimum.

According to maximum-minimum theorem,

$$\frac{dy}{ds} = 0$$

$$Y \propto \frac{R_2}{S} + \frac{SX_2^2}{R_2}$$

$$\frac{dy}{ds} = 0 \left| \frac{-R_2}{S^2} + \frac{X_2^2}{R_2} \right. = 0$$

$$R_2^2 = (SX_2)^2 \text{ or } R_2 = SX_2$$

or $$S_{mT} = \frac{R_2}{X_2}$$

The above slip is the slip at which maximum torque occurs.

For given question : $S_f = 0.04$

$$\frac{T_{max}}{T_f} = 2.5 \text{ or } \frac{T_f}{T_{max}} = \frac{1}{2.5}$$

$$\frac{T_f}{T_{max}} = \frac{2S_f S_m}{S_m^2 + S_f^2} = \frac{1}{2.5}$$

$$\frac{2 \times 0.04 \times S_m}{S_m^2 + (0.04)^2} = \frac{1}{2.5}$$

$$0.2S_m = S_m^2 + 0.0016$$

$$S_m^2 - 0.2S_m + 0.0016 = 0$$

By solving, $S_m = 0.191$

∴ Speed at which Max torque occurs

$$N_m = N_s(1 - S_m) = 1500(1 - 0.191)$$

$$N_m = 1213.5 \text{ rpm}$$

(ii) $$\frac{T_{st}}{T_f} = ?; \quad T_{st} \text{ in p.u.} = ?$$

$$\frac{T_{st}}{T_{max}} = \frac{2S_m}{S_m^2+1}$$

$$= \frac{2\times 0.191}{0.191^2+1}$$

$$= 0.3685$$

$$\frac{\frac{T_{st}}{T_{max}}}{\frac{T_f}{T_{max}}} = \frac{T_{st}}{T_f} = \frac{0.3685}{\frac{1}{2.5}} = 0.921 \text{ p.u.}$$

(iii) $$S_m = \frac{R_2}{X_2} = 0.191$$

$\therefore$ $$X_2 = \frac{R_2}{0.191} = \frac{0.25}{0.191} = 1.3089\ \Omega$$

For maximum T_{st} :

$$R_2 + R_{ext} = X_2$$

$\therefore$ $$R_{ext} = X_2 - R_2 = 1.3089 - 0.25$$

$$= 1.0589\ \Omega$$

(iv) If 1.0589 Ω is added to rotor then total R_2 = 1.3089 Ω

$$T_f \propto \frac{S}{R_2}$$ For same full load 'T'

$$S \propto R_2$$

$\therefore$ $$\frac{S_2}{S_1} = \frac{R_2 + R_{ext}}{R_2}$$

$$\frac{S_2}{0.04} = \frac{1.3089}{0.25}$$

$$S_2 = \frac{1.3089}{0.25}\times 0.04 = 0.2094$$

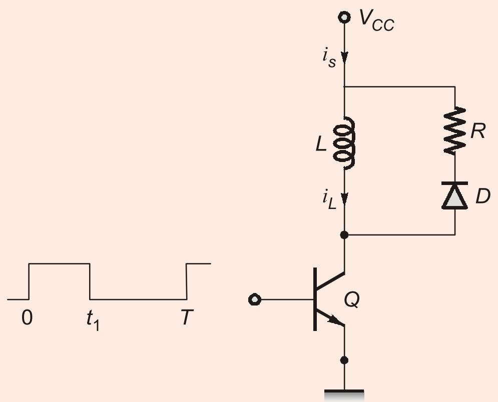
$\therefore$ $$N_2 = N_{S2}(1 - S_2)$$

$$= 1500(1 - 0.2094)$$

$$N_2 = 1185.9 \text{ rpm}$$

End of Solution

Q.7 (c) For the figure shown below, the transistor 'Q' is excited by a pulse of duration 't_1' with a periodicity of $\frac{1}{T}$.



(i) Draw the current waveforms of 'i_s' and 'i_L'.

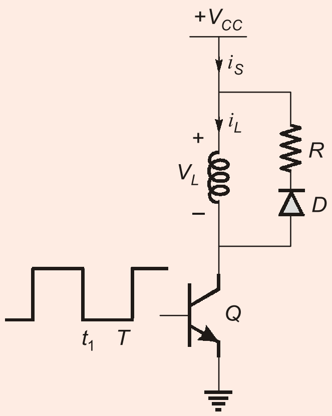
(ii) Expression for absorbed average power by resistor '*R*' in the circuit.

Assume $\frac{L}{R}$ ratio to be too small in comparison to '*T*'.

(iii) Expression for $i_L(t)$, the current through inductor '*L*'.

[20 marks : 2023]

Solution:



(I) $0 \le t \le t_1$:

$Q \to$ ON :

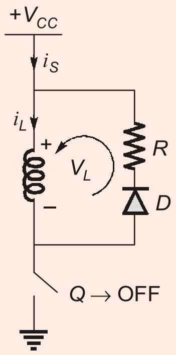
$\therefore$
$$V_L = V_{CC}$$
$$L \cdot \frac{di_L}{dt} = V_{CC}$$
$$\int di_L = \frac{V_{CC}}{L} \int dt$$
$$i_L = \frac{V_{CC}}{L} t + K$$

At $t = 0$, $\quad i_L = 0 \quad \therefore K = 0$

(i)
$$i_S = i_L = \frac{V_{CC}}{L}\cdot t$$
$$I_{mx} = \frac{V_{CC}}{L}\cdot t_1$$

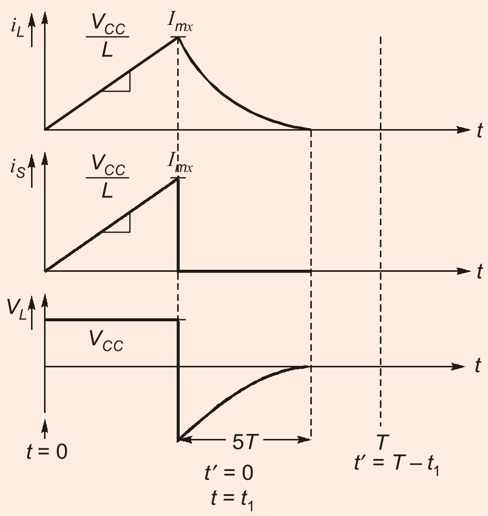
(II) $t_1 \le t \le T : [0 \le t' \le (T - t_1)]$

$Q \rightarrow$ OFF :



$\because$
$$i_s = 0,\ \tau = \frac{L}{R}$$
$$i_L = K_i e^{-t'/\tau}$$

(i)
$$i_L = I_{mx}\cdot e^{-\frac{t'}{\tau}} = \frac{V_{CC}}{L}\cdot t_1 e^{\frac{-t'}{\tau}}$$
$$v_L = L\cdot\frac{di_L}{dt} = L\cdot\frac{d}{dt}\left(I_{mx}\cdot e^{-\frac{t'}{\tau}}\right)$$
$$v_L = L\cdot I_{mx} - \frac{1}{\tau}e^{\frac{-t'}{\tau}}$$
$$v_L = \frac{-LI_{mx}}{\tau}\cdot e^{\frac{-t'}{\tau}}$$



At $t = t_1$, energy stored in inductance

$$E = \frac{1}{2}LI_{mx}^2$$

$\therefore$ During the interval : $t_1 \le t \le T; \quad 0 \le t' \le (T - t_1)$

$\frac{1}{2}LI_m^2 \Rightarrow$ Energy dissipated in resistor

$\therefore$
$$P_{avg} = \frac{1}{T} \times \text{Energy dissipated in resistor}$$
$$= \frac{1}{T} \times \frac{1}{2}LI_{mx}^2$$
$$P_{avg} = \frac{1}{T}\cdot\frac{1}{2}\cdot L\left(\frac{V_{CC}}{L}t_1\right)^2$$

(ii)
$$P_{avg} = \frac{V_{CC}^2 t_1^2}{2T \cdot L}$$

End of Solution

Q.8 (a) For a causal system $H(z) = \frac{z}{z - 0.5}$, find the zero state response to input

$$x(n) = \left(\frac{1}{4}\right)^n u(n) + 5(3)^n u[-(n+1)].$$

[20 marks : 2023]

Solution:

For causal LTI-system:

$$H(z) = \frac{z}{z - 0.5}, \quad |z| > 0.5$$

System input:

$$x(n) = \left(\frac{1}{4}\right)^n u(n) + 5(3)^n u[-(n+1)]$$

Now,
$$\left(\frac{1}{4}\right)^n u(n) \rightleftharpoons \frac{1}{1 - \frac{1}{4}z^{-1}}, \qquad |z| > \frac{1}{4}$$

$$-(3)^n u(-n-1) \rightleftharpoons \frac{1}{1 - 3z^{-1}}, \qquad |z| < 3$$

$$5(3)^n u(-n-1) \rightleftharpoons \frac{-5}{1 - 3z^{-1}}, \qquad |z| < 3$$

Thus,
$$x(n) \rightleftharpoons X(z) = \frac{1}{1 - \frac{1}{4}z^{-1}} - \frac{5}{1 - 3z^{-1}}, \qquad \frac{1}{4} < |z| < 3$$

For zero-state response:
Initial conditions are zero,
Hence, we can write

$$Y(z) = H(z)\cdot X(z)$$

$$= \frac{1}{1-0.5z^{-1}}\cdot\left[\frac{1}{1-\frac{1}{4}z^{-1}} - \frac{5}{1-3z^{-1}}\right], \qquad 0.5<|z|<3$$

$$\Rightarrow \quad Y(z) = \frac{1}{(1-0.5z^{-1})\left(1-\frac{1}{4}z^{-1}\right)} - \frac{5}{(1-0.5z^{-1})(1-3z^{-1})}$$

$$= \frac{2}{1-0.5z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}} - \left[\frac{-1}{1-0.5z^{-1}} + \frac{6}{1-3z^{-1}}\right]$$

$$= \frac{3}{1-0.5z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}} - \frac{6}{1-3z^{-1}}, \qquad 0.5<|z|<3$$

By applying inverse ZT,

$$y(n) = 3(0.5)^n u(n) - \left(\frac{1}{4}\right)^n u(n) + 6(3)^n u(-n-1)$$

End of Solution

Q.8 (b) Two identical 250 KVA, 230/460 volt transformers are connected in open delta to supply a balanced 3-phase star connected load at 460 volt and at a power factor of 0.8 lagging.

Answer the following:

(i) Draw the phasor diagram of the open-delta condition.

(ii) Find the maximum secondary line current without overloading the transformers.

(iii) Find the real power delivered by each transformer and the total real power delivered.

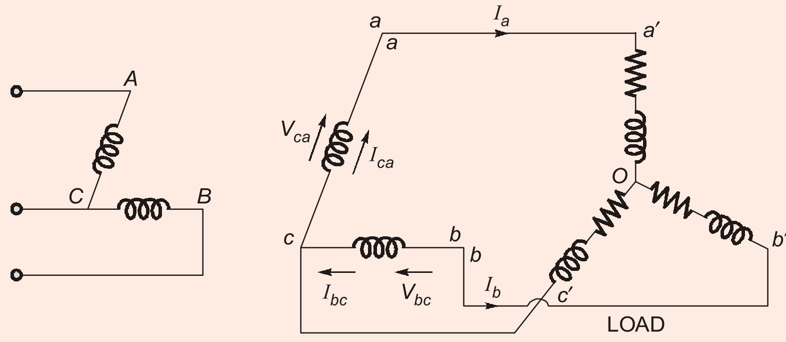
(iv) Find the primary line currents.

(v) If a similar transformer is now added to complete the Δ, find the percentage increase in real power that can be supplied. Assume that the load voltage and power factor remain unchanged at 460 volt and 0.8 lagging, respectively.

[20 marks : 2023]

Solution:

Two 250 kVA, 230/460 V, V-V



To draw phasor diagram :

Applying KVL at loop : *Caa′oC′C*

KVL at loop : *bCC′ob′b*

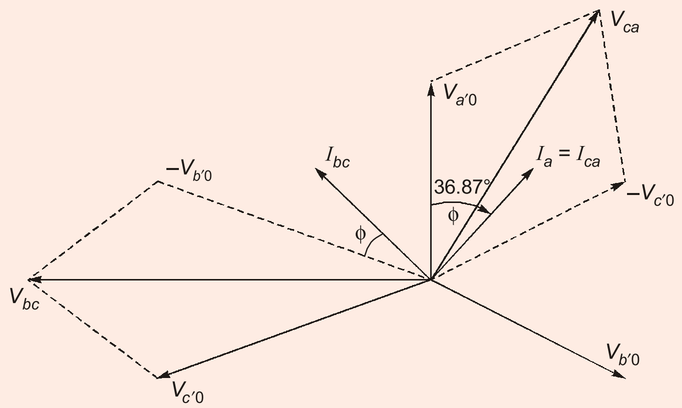
KCL at node *a* and KCL at node *b*

$$V_{ca} = V_{a'o} + V_{oc'} = V_{a'o} + (-V_{c'o})$$

$$V_{bc} = V_{c'o} + V_{ob'} = V_{c'o} + (-V_{b'o})$$

KCL at *a*, $I_{ca} = I_a$

KCL at *b*, $I_{bc} = -I_b$



KCL at *b*, $I_{bc} = -I_b$

I_{ca} laggs V_{ca} by an angle of 6.87°.

∴ p.f. is cos (6.87) lagg 0.9928 lagg.

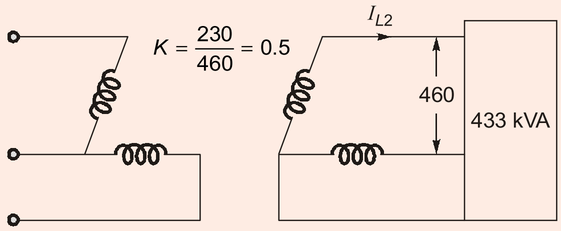
I_{bc} laggs V_{bc} by an angle of 66.87°.

∴ P.f. is cos(66.87) lags = 0.3928 lagg.

(ii) V-V Correction Capacity is :

$$2 \times 250 \times 0.866 = 433 \text{ kVA}$$

∴ Load on V-V must be 433 kVA (not to overload)



$$\sqrt{3}V_{L2}I_{L2} = 433 \text{ kVA}$$

$$\sqrt{3}\times 460\times I_{L2} = 433 \times 10^3$$

$$\therefore \quad I_{L2} = 543.478 \text{ A} = I_{ph2}$$

$$I_{ph1} = \frac{I_{ph2}}{K} = 1086.956 \text{ A}$$

$$I_{ph1} = I_{L1} = 1086.956 \text{ A}$$

Maximum sec. line current without overloading is 543.478 A.

(iii) Real power delivered by each T/F

T/F 1 : $460 \times 543.478\angle 6.87°$

$$= 248204.9 + j29904.239$$

T/F 2 : $460 \times 543.478\angle 66.87°$

$$= 98204.623 + j227903.875$$

Total real power supplied is

$$248204.9 + 98204.523 = 346409.523$$

or 346.409 kW

(iv) Primary line current is done in (ii).

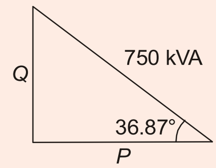
$$I_{L1} = I_{ph1} = 1086.956 \text{ A}$$

(v) If another 250 kVA T/F is added.

Total capacity is $3 \times 250 = 750$ kVA

Load can be 750 kVA, at 0.8 pf lagg.

Real power is : $\cos\phi = \frac{P}{S}$



$$0.8 = \frac{P}{750}$$

$$\therefore \quad P = 600 \text{ kW}$$

V-V supplying 346.409 kW.

Δ-Δ supplying 600 kW.

$$\% \text{ Increase} = \frac{600 - 346.409}{346.409}\times 100 = 73.2\%$$

End of Solution

Q.8 (c) The positive, negative and zero sequence reactances of a 25 MVA, 13.2 kV synchronous generator are 0.3 pu, 0.2 pu and 0.1 pu respectively. The generator is star connected and neutral is solidly grounded. When it is unloaded, find the fault current and line-line voltages when a fault of

(i) Line-line occurs,

(ii) Double line to ground occurs.

[20 marks : 2023]

Solution:

$Z_1 = j0.3$ pu; $Z_2 = j0.2$ pu; $Z_0 = j0.1$ pu

$G = 25$ MVA; $V = 13.2$ kV

$Y =$ Current generation, Solid grounded neutral

$$I_b = \frac{25\times 10^3}{\sqrt{3}\times 13.2} = 1.09(10)^3 = 1093.5 \text{ A}$$

(i) L-L form

$$I_f = -j\sqrt{3}I_{a1}$$

$$I_{a1} = \frac{E_a}{Z_1+Z_2} = \frac{1.0}{j0.3+j0.2} = -j2.0$$

$$I_f = -j\sqrt{3}(-j2.0) \times 1093.5 = 3787.88 \text{ A}$$

(ii)

$$I_f = 3I_{ao}$$

$$I_{ao} = \frac{I_o Z_1 - E_a}{Z_o}$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 \parallel Z_0}$$

$$I_{a1} = \frac{1.0}{j0.3 + j0.2 \parallel j0.1} = -j2.727$$

$$I_{a0} = \frac{(-j2.727)(j0.3)-1.0}{j0.1} = j1.818$$

$$I_f = 3I_{a0} = 3 \times 1.818 \times 1093.5$$

$$= 5964 \text{ Amp}$$

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