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Detailed Solutions

**ESE-2023  
Mains Test Series**

**E & T Engineering  
Test No : 8**

**Section A : Digital Circuits + Analog Circuits**

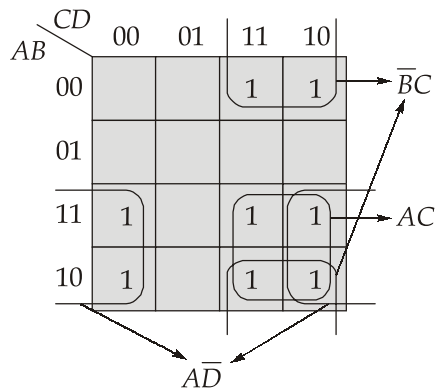
**Q.1 (a) Solution:**

We have,

$$F(A, B, C, D) = \Pi(0, 1, 4, 5, 6, 7, 9, 13).$$

$\therefore$

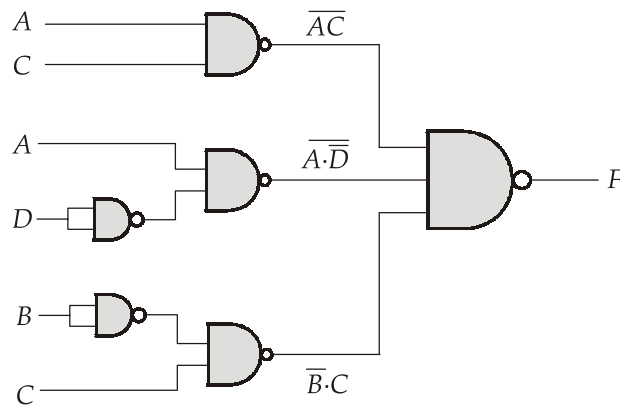
$$F(A, B, C, D) = \Sigma(2, 3, 8, 10, 11, 12, 14, 15)$$



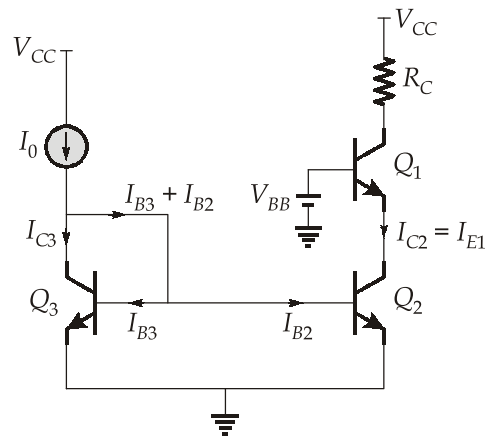
$$F = AC + A\bar{D} + \bar{B}C$$

Implementation using NAND gate:

$$\begin{aligned} F &= \overline{\overline{AC + A\bar{D} + \bar{B}C}} \\ &= \overline{\overline{AC} \cdot \overline{A\bar{D}} \cdot \overline{\bar{B}C}} \end{aligned}$$



Q.1 (b) Solution:



Applying KCL at collector of  $Q_3$ , we get

$$I_{C3} + I_{B3} + I_{B2} = I_0$$

$$I_0 = I_{C3} + 2I_{B3} \quad (\because \text{All transistors are identical, } I_{B3} = I_{B2})$$

$$I_0 = I_{C3} \left[ 1 + \frac{2}{\beta} \right] = I_{C3} \left[ \frac{\beta + 2}{\beta} \right]$$

Also,

$$I_{C3} = I_{C2} = I_{E1} \quad (\because V_{BE3} = V_{BE2})$$

$\therefore$

$$I_0 = I_{E1} \left[ \frac{\beta + 2}{\beta} \right]$$

But,

$$I_{E1} = \frac{(1 + \beta)I_{C1}}{\beta}$$

$$I_0 = \frac{(1 + \beta)(\beta + 2)I_{C1}}{\beta^2}$$

$$I_{C1} = \frac{I_0 \beta^2}{(1 + \beta)(2 + \beta)} \quad \dots(i)$$

But, we know,

$$\text{Stability factor } S'' = \frac{\partial I_{C1}}{\partial \beta} \quad (\text{For collector current of } Q_1)$$

$\therefore$  Differentiating equation (i) w.r.t.  $\beta$

$$\begin{aligned} S'' = \frac{\partial I_{C1}}{\partial \beta} &= I_0 \left[ \frac{(1+\beta)(2+\beta)2\beta - \beta^2[2\beta+3]}{(1+\beta)^2(2+\beta)^2} \right] \\ &= I_0 \left[ \frac{2\beta^2 + 6\beta + 4 - 2\beta^2 - 3\beta}{(1+\beta)^2(2+\beta)^2} \right] \beta \\ S'' &= I_0 \left[ \frac{(3\beta+4)\beta}{(1+\beta)^2(2+\beta)^2} \right] \end{aligned}$$

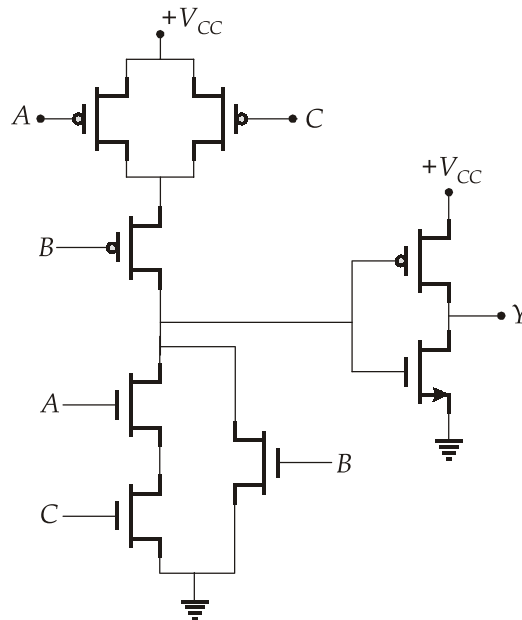
For  $\beta = 100$  and  $I_0 = 1 \text{ mA}$

$$S'' = 1 \times 10^{-3} \left[ \frac{100[300+4]}{(101)^2 \times (102)^2} \right]$$

$$S'' = 2.865 \times 10^{-7}$$

**Q.1 (c) Solution:**

(i)  $Y = B + AC$



(ii)  $Y = A + \bar{B}C$



$$\text{or} \quad \frac{V_0(j\omega)}{V_i(j\omega)} = \frac{1}{(j\omega r_f C + 1)}$$

$$\therefore \quad \text{Phase shift} = -\tan^{-1}(\omega r_f C)$$

(ii) Given,  $C = 25 \text{ nF}$ ,  $f = 250 \text{ kHz}$

and  $\text{Phase shift} = -30^\circ$

$$\therefore \quad -30^\circ = -\tan^{-1}(\omega r_f C)$$

$$\frac{1}{\sqrt{3}} = \omega r_f C$$

$$r_f = \frac{1}{\sqrt{3}\omega C} = \frac{1}{\sqrt{3} \times 2\pi \times 250 \times 10^3 \times 25 \times 10^{-9}} = 14.7 \, \Omega$$

Also,

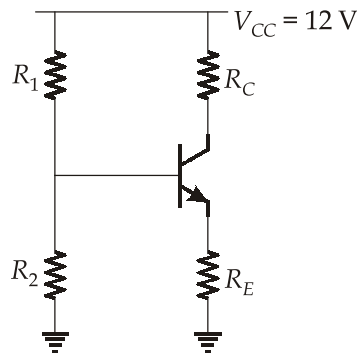
$$r_f = \frac{V_T}{I_{DC}}$$

$$14.7 = \frac{25 \times 10^{-3}}{I_{DC}}$$

$$I_{DC} = 1.7 \text{ mA}$$

### Q.1 (e) Solution:

Self Bias circuit,



Drawing the simplified self bias circuit,

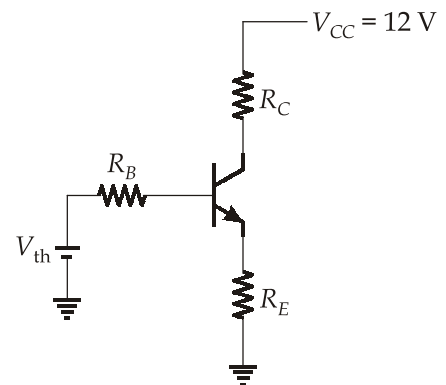
$$R_B = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{th} = \frac{V_{CC} R_2}{R_1 + R_2}$$

$$\frac{R_B}{V_{th}} = \frac{R_1 R_2}{V_{CC} R_2}$$

$$R_1 = \frac{V_{CC} R_B}{V_{th}} \quad \dots(i)$$

Now,



Applying KVL in collector emitter loop, we get,

$$V_{CE} - V_{CC} + I_C R_C + I_E R_E = 0 \quad (\text{Assuming } I_E \cong I_C)$$

$$V_{CE} = V_{CC} - I_C R_C - I_C R_E$$

$$V_{CE} = V_{CC} - I_C [R_C + R_E]$$

$$5 = 12 - I_C [R_C + R_E]$$

$$R_C + R_E = 3.5 \text{ k}\Omega$$

Let,

$$R_E = 1 \text{ k}\Omega$$

(Assuming  $R_C > 2R_E$ )

$\therefore$

$$R_C = 2.5 \text{ k}\Omega$$

Now, according to the condition of stability of  $I_C$  in self bias,

$$(1 + \beta)R_E = 10 R_B$$

$$(1 + 120) \times 1 = 10 R_B$$

$$R_B = 12.1 \text{ k}\Omega$$

Applying KVL in base-emitter loop, we get,

$$-V_{th} + R_B I_B + V_{BE} + I_E R_E = 0$$

$$V_{th} = (R_B + R_E)I_B + I_C R_E + V_{BE}$$

$$[\because I_E = I_B + I_C]$$

$$= 13.1 \times \frac{2}{120} + 2 \times 1 + 0.7$$

$$V_{th} = 2.9183 \text{ V}$$

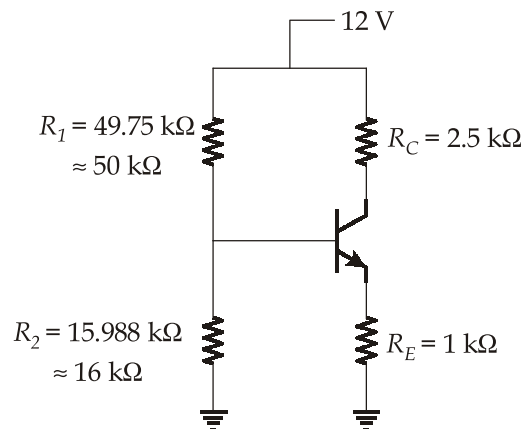
Now, from equation (i),

$$R_1 = \frac{12 \times 12.1}{2.9183}$$

$$R_1 = 49.75 \text{ k}\Omega \approx 50 \text{ k}\Omega$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2} = 12.1 \text{ k}\Omega$$

$$R_2 = 15.988 \text{ k}\Omega \approx 16 \text{ k}\Omega$$



**Q.2 (a) Solution:**

(i) Assuming transistor is in saturation,

$$\therefore I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) [V_{GS} - V_T]^2 \quad \dots(i)$$

Given:  $V_G = 1.8 \text{ V}, \quad \mu_n C_{ox} \left( \frac{W}{L} \right) = 2 \text{ mA/V}^2$

$$V_S = 0.5 I_D, \quad V_T = 1 \text{ V}$$

$$\therefore V_{GS} = 1.8 - 0.5 I_D$$

and  $I_D = \frac{1.8 - V_{GS}}{0.5} = 2[1.8 - V_{GS}]$

$$I_D = 3.6 - 2V_{GS} \quad \dots(ii)$$

Now, from equation (i) and (ii)

$$3.6 - 2V_{GS} = \frac{1}{2} \times 2 [V_{GS} - 1]^2$$

$$3.6 - 2V_{GS} = V_{GS}^2 + 1 - 2V_{GS}$$

$$V_{GS}^2 = 2.6$$

$$V_{GS} = 1.6 \text{ Volt}$$

Hence, from equation (ii),

$$I_D = 0.4 \text{ mA}$$

$$V_S = 0.2 \text{ V}$$

Now,

$$V_D = 3.3 - I_D R_D$$

$$= 3.3 - 0.4 \times 10$$

$$V_D = -0.7 \text{ V} \quad \text{i.e., } V_D < V_T$$

 $V_D$  can't be negative. It means conditions for saturation are not being satisfied. $\therefore$  Transistor is in triode region.

$$\therefore I_D = \mu_n C_{ox} \left( \frac{W}{L} \right) \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$I_D = 2 \left[ V_{GS} - V_T - \frac{V_{DS}}{2} \right] V_{DS} \quad \dots(iii)$$

But

$$V_D = 3.3 - 10I_D, \quad V_S = 0.5I_D$$

$$I_D = 2 \left[ V_G - V_S - V_T - \frac{V_D}{2} + \frac{V_S}{2} \right] [V_{DS}]$$

$$\begin{aligned}
 &= 2 \left[ 1.8 - 0.5V_S - 1 - \frac{V_D}{2} \right] [3.3 - 10.5I_D] \\
 &= 2[0.8 - 0.5V_S - 0.5V_D][3.3 - 10.5I_D] \\
 &= (1.6 - V_S - V_D)(3.3 - 10.5I_D) \\
 &= (1.6 - 0.5I_D - 3.3 + 10I_D)(3.3 - 10.5I_D) \\
 &= (9.5I_D - 1.7)(3.3 - 10.5I_D)
 \end{aligned}$$

$$I_D = 31.35I_D - 99.75I_D^2 - 5.61 + 17.85I_D$$

$$99.75I_D^2 - 48.2I_D + 5.61 = 0$$

After solving we get,

$$I_D = 0.28778 \text{ mA}, 0.1954 \text{ mA}$$

**Case (I):**

$$I_D = 0.1954 \text{ mA}$$

$$V_S = 0.5 \times I_D = 0.0977 \text{ V}$$

$$V_D = 3.3 - 10I_D = 1.346 \text{ V}$$

$$V_{DS} = V_D - V_S = 1.2483 \text{ V}$$

$$(V_{GS} - V_T) = (1.8 - 0.0977 - 1) = 0.7023 \text{ V}$$

$V_{DS} > V_{GS} - V_T$  which is not valid, as transistor is in triode region.

**Case (2):**

$$I_D = 0.28778 \text{ mA}$$

$$V_S = 0.5 \times 0.28778 = 0.14389 \text{ V}$$

$$V_D = 3.3 - 10I_D = 0.4222 \text{ V}$$

$$V_{DS} = V_D - V_S = 0.27831 \text{ V}$$

$$(V_{GS} - V_T) = V_G - V_S - V_T = 1.8 - 0.14389 - 1 = 0.65611 \text{ V}$$

As  $V_{DS} < (V_{GS} - V_T)$   $\therefore I_D = 0.28778 \text{ mA}$  is valid value which confirm that transistor is in triode region.

$\therefore$  Drain voltage  $V_D = 0.4222 \text{ V}$

- (ii) **Transconductance:** Transconductance is the ratio of change in drain current ( $\partial I_D$ ) to change in the gate to source voltage ( $\partial V_{GS}$ ) at a constant drain to source voltage ( $V_{DS} = \text{constant}$ )

$$g_m = \frac{\partial I_D}{\partial V_{GS}} \text{ at constant } V_{DS}$$

This value is maximum at  $V_{GS} = 0$ . This is denoted by  $g_{m0}$ .

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{2I_{DSS}}{V_{GS(\text{off})}} \left[ 1 - \frac{V_{GS}}{V_{GS(\text{off})}} \right]$$

where,  $g_{m0} = \frac{2I_{DSS}}{V_{GS(off)}}$

$\therefore g_m = g_{m0} \left[ 1 - \frac{V_{GS}}{V_{GS(off)}} \right]$

**Dynamic Output Resistance :** This is the ratio of change in drain to source voltage ( $\partial V_{DS}$ ) to the change in drain current ( $\partial I_D$ ) at a constant gate to source voltage ( $V_{GS} = \text{constant}$ ). It is denoted as  $r_d$ .

$$r_d = \frac{\partial V_{DS}}{\partial I_D} \text{ at constant } V_{GS}$$

**Amplification factor:** It is defined as the ratio of change in drain voltage ( $\partial V_{DS}$ ) to change in gate voltage ( $\partial V_{GS}$ ) at a constant drain current ( $I_D = \text{constant}$ )

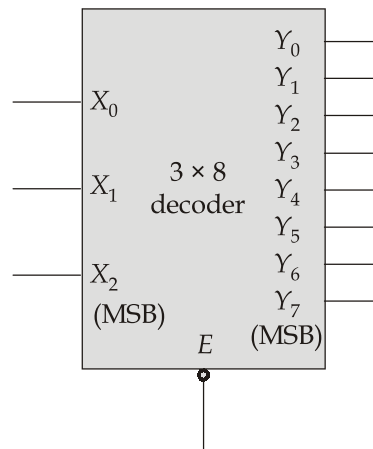
$$\mu = \frac{\partial V_{DS}}{\partial V_{GS}} \text{ at constant } I_D$$

There is a relation between transconductance ( $g_m$ ), dynamic output resistance ( $r_d$ ) and amplification factor ( $\mu$ ) given by

$$\mu = \frac{\partial V_{DS}}{\partial V_{GS}} = \frac{\partial V_{DS}}{\partial I_D} \times \frac{\partial I_D}{\partial V_{GS}}$$

$$\mu = r_d \times g_m$$

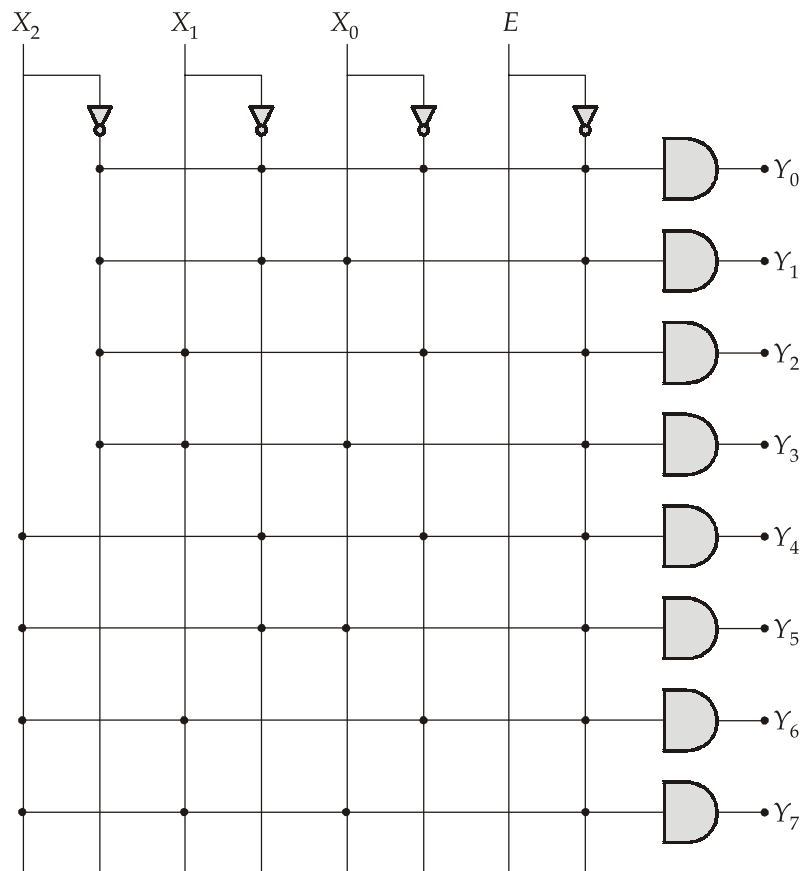
**Q.2 (b) Solution:**



⇒ Truth table:

$E$	$X_2$	$X_1$	$X_0$	$Y_7$	$Y_6$	$Y_5$	$Y_4$	$Y_3$	$Y_2$	$Y_1$	$Y_0$
1	X	X	X	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	0	1	0	0
0	0	1	1	0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	1	0	0	0	0
0	1	0	1	0	0	1	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0	0	0
0	1	1	1	1	0	0	0	0	0	0	0

Logic diagram:



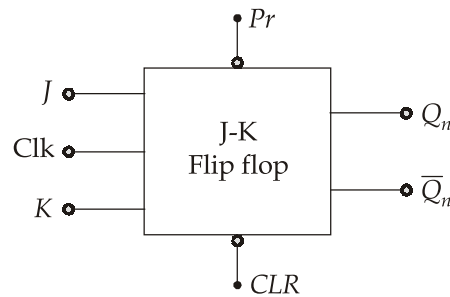
**Q.2 (c) Solution:**

**(i) Truth table of J-K flip flop:**

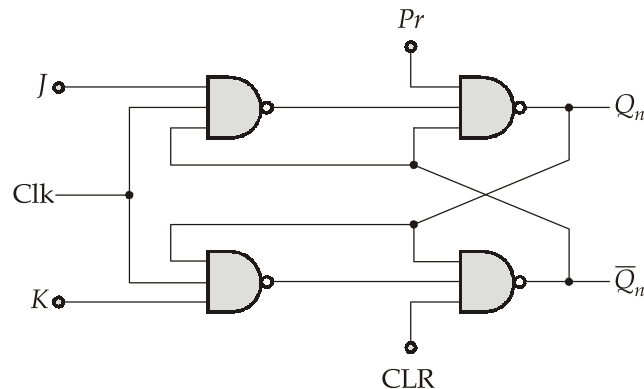
$J$	$K$	$Q_{n+1}$
0	0	$Q_n$
0	1	0
1	0	1
1	1	$\bar{Q}_n$

$$Q_{n+1} = J\bar{Q}_n + \bar{K} \cdot Q_n$$

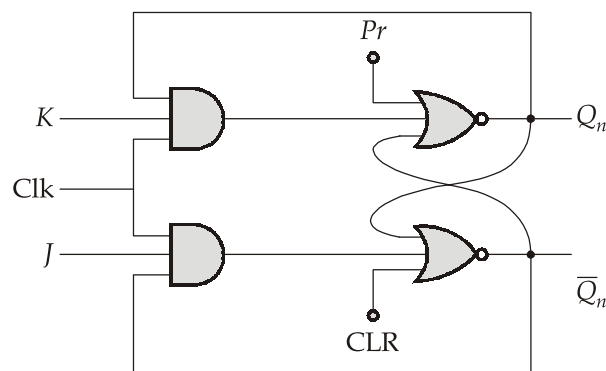
**Logic Symbol:**



**JK flip-flop using NAND latch:**



**JK flip-flop using NOR latch:**



(ii) SR flip flop into JK flip flop:

J	K	$Q_{n+1}$
0	0	$Q_n$
0	1	0
1	0	1
1	1	$\bar{Q}_n$

$Q_n$	$Q_{n+1}$	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

J	K	$Q_n$	$Q_{n+1}$	S	R
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	1	X	0
1	1	0	1	1	0
1	1	1	0	0	1

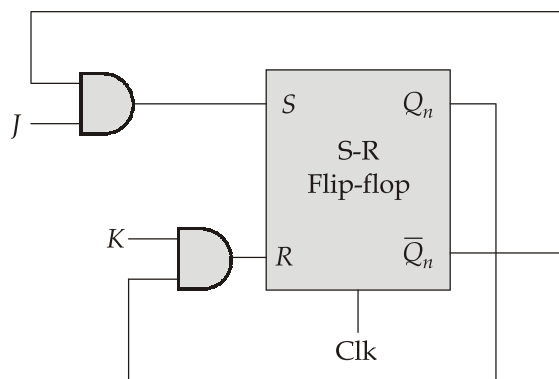
K-Map:

$J \backslash KQ_n$	00	01	11	10
0		X		
1	1	X		1

$$S = J\bar{Q}_n$$

$J \backslash KQ_n$	00	01	11	10
0	X		1	X
1			1	

$$R = KQ_n$$



Race around condition:

- When level triggered clock is given to JK flip flop, then for  $J = K = 1$ , the output of flip flop may toggle between 0 and 1 for given clock pulse and hence, at the

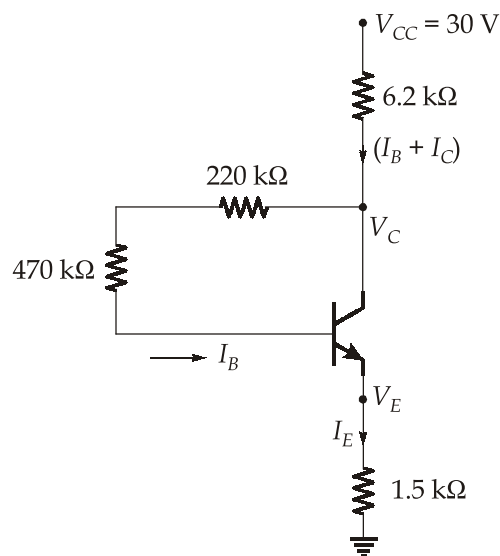


end of the clock pulse, the value of output is uncertain. This situation is known as Race-around condition.

- To avoid Race-around condition, we use Master-slave configuration in J-K flip-flop.

### Q.3 (a) Solution:

- (i) To find the biasing parameters, we carry out the DC analysis. For DC analysis, all the capacitors acts as open circuit. Therefore, the simplified DC circuit is shown below.



Now, applying KVL to the emitter base loop, we have

$$V_{CC} = (6.2)(I_B + I_C) + (470 + 220) I_B + V_{BE} + (I_B + I_C)1.5$$

$$30 = (6.2 + 1.5)(1 + \beta)I_B + 690I_B + 0.7$$

$$I_B = \frac{30 - 0.7}{[7.7(1 + 100) + 690]}$$

$$I_B = 19.96 \mu\text{A} \cong 20 \mu\text{A}$$

1.  $I_C = \beta I_B = 100 \times 20 = 2 \text{ mA}$

2.  $I_E = (1 + \beta)I_B = 101 \times 20 = 2.02 \text{ mA}$

$$V_E = I_E \times 1.5 = 2.02 \times 1.5 = 3.03 \text{ Volts}$$

3. Applying KVL to the collector circuit, we get

$$-V_{CC} + 6.2(I_B + I_C) + V_{CE} + V_E = 0$$

$$V_{CE} = V_{CC} - V_E - 6.2[2.02]$$

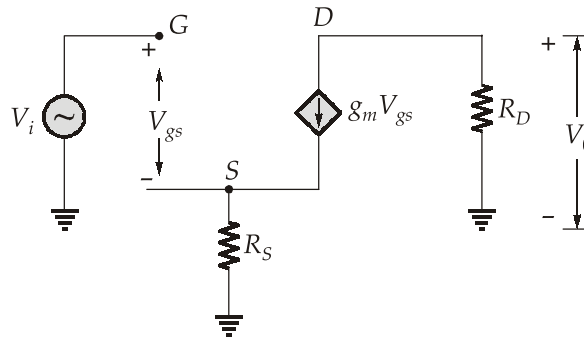
$$= 30 - 3.03 - 12.524$$

$$V_{CE} = 14.446 \text{ Volt}$$

$$\begin{aligned}
 4. \quad V_C &= V_{CC} - 6.2 \times (I_B + I_C) \\
 &= 30 - 6.2[2.02] \\
 V_C &= 17.476 \text{ Volt}
 \end{aligned}$$

(ii) Given,  $I_D = 1 \text{ mA}$ ,  $g_m = 1 \text{ mA/V}$ ,  $r_0 = \infty$

Drawing AC equivalent circuit,



From the output circuit,

$$V_0 = -g_m V_{gs} R_D \quad \dots(i)$$

From input side,

$$\begin{aligned}
 -V_i + V_{gs} + R_s g_m V_{gs} &= 0 \\
 V_i &= V_{gs} [1 + g_m R_s] \quad \dots(ii)
 \end{aligned}$$

$$\therefore \text{Midband gain } \frac{V_0}{V_i} = A_v = \frac{-g_m V_{gs} R_D}{V_{gs} [1 + g_m R_s]}$$

$$A_v = \frac{-g_m R_D}{1 + g_m R_s}$$

After substituting given values,

$$\begin{aligned}
 A_v &= \frac{-1 \times 10^{-3} \times 10 \times 10^3}{1 + 10^{-3} \times 6 \times 10^3} \\
 A_v &= -1.43 \text{ V/V}
 \end{aligned}$$

### Q.3 (b) Solution:

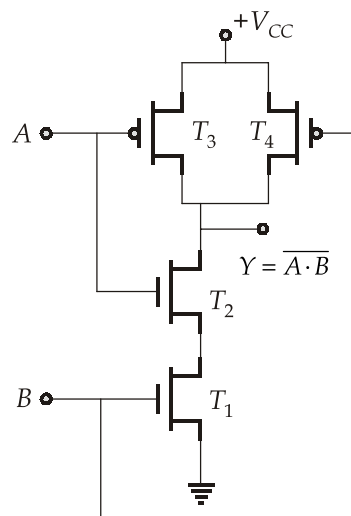
#### CMOS logic:

- A complementary MOSFET (CMOS) is obtained by connecting a p-channel and a n-channel MOSFET in series, with drains tied together and the output is taken at the common drain.
- In a CMOS, p-channel and n-channel MOS devices are fabricated on the same chip, which makes its fabrication more complicated and reduces the packing density.

Because of negligible small power consumption, CMOS is ideally suited for battery operated system.

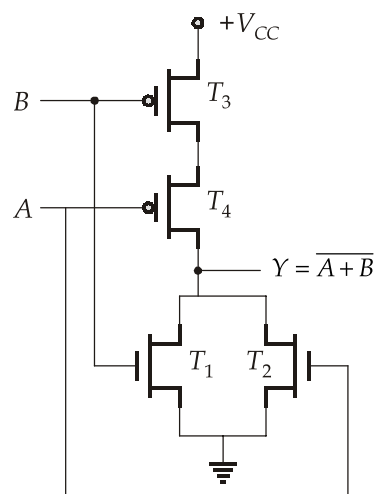
- Speed of conventional CMOS is limited by substrate capacitances and hence, to reduce the effect of these substrate capacitance, silicon on sapphire (SOS) technology is used.
- CMOS has become the most popular in MSI and LSI areas and is the most dominant logic for the fabrication of VLSI devices.

(i) 2-input CMOS NAND Gate



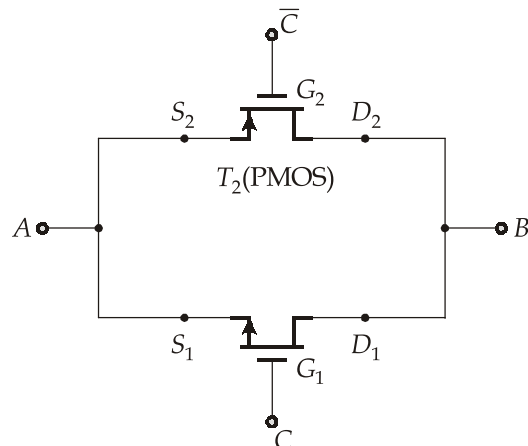
A	B	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	Y
0	0	OFF	OFF	ON	ON	$V_{DD}$
0	$V_{DD}$	ON	OFF	ON	OFF	$V_{DD}$
$V_{DD}$	0	OFF	ON	OFF	ON	$V_{DD}$
$V_{DD}$	$V_{DD}$	ON	ON	OFF	OFF	0

(ii) 2-input CMOS NOR gate



A	B	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	Y
0	0	OFF	OFF	ON	ON	$V_{DD}$
0	$V_{DD}$	ON	OFF	OFF	ON	0
$V_{DD}$	0	OFF	ON	ON	OFF	0
$V_{DD}$	$V_{DD}$	ON	ON	OFF	OFF	0

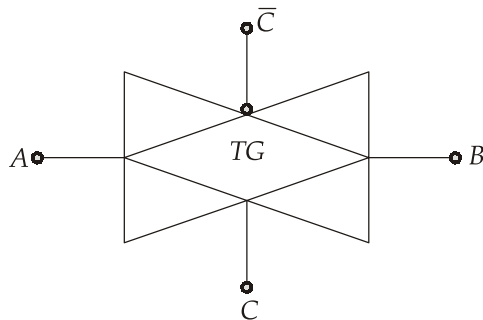
(iii) A CMOS transmission Gate



CMOS Transmission Gate operates as a bidirectional switch between the nodes  $A$  and  $B$  which is controlled by signal  $C$ .

1. When the control signal is logic high, both the N-MOS and P-MOS transistors are turned on and provide a low resistance path between the nodes  $A$  and  $B$ .
2. When the control signal is logic low, both the transistors are OFF and the path between the nodes  $A$  and  $B$  will be open circuit. This condition is also called the high impedance state.

CMOS transmission gate symbol



**Note:**   $\Rightarrow$  PMOS logic;   $\Rightarrow$  NMOS logic

### Q.3 (c) Solution:

- (i) Excitation table for S-R flip flop:

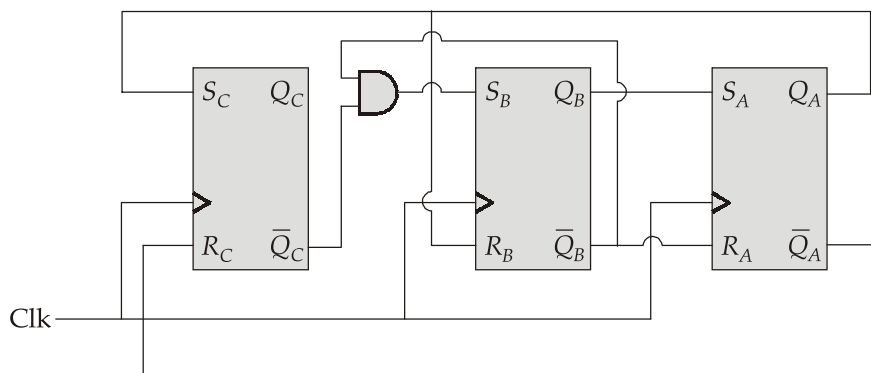
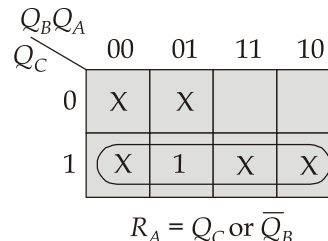
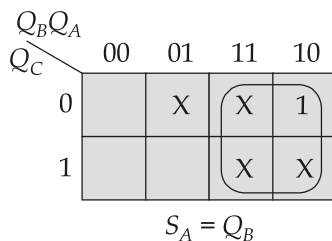
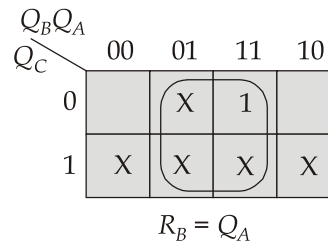
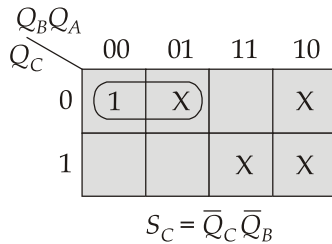
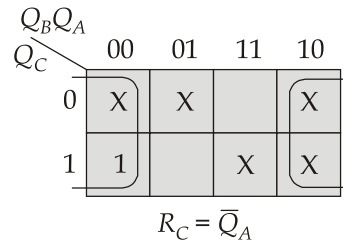
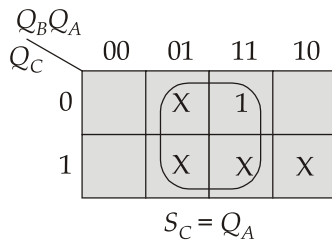
$Q_n$	$Q_{n+1}$	$S$	$R$
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

The state table for the counter can be written as below:

$Q_C$	$Q_B$	$Q_A$	$Q_C^+$	$Q_B^+$	$Q_A^+$	$S_C$	$R_C$	$S_B$	$R_B$	$S_A$	$R_A$
0	0	0	0	1	0	0	X	1	0	0	X
0	1	0	0	1	1	0	X	X	0	1	0
0	1	1	1	0	1	1	0	0	1	X	0
1	0	1	1	0	0	X	0	0	X	0	1
1	0	0	0	0	0	0	1	0	X	0	X

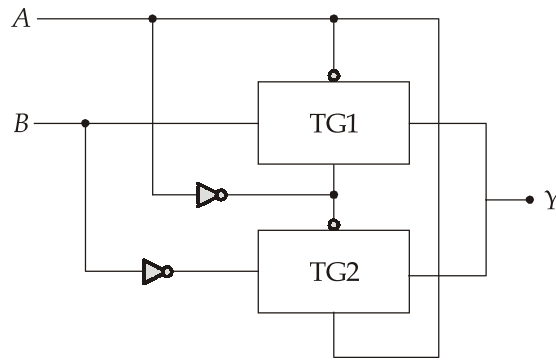
Unused states are: 001, 110, 111

## K-Map



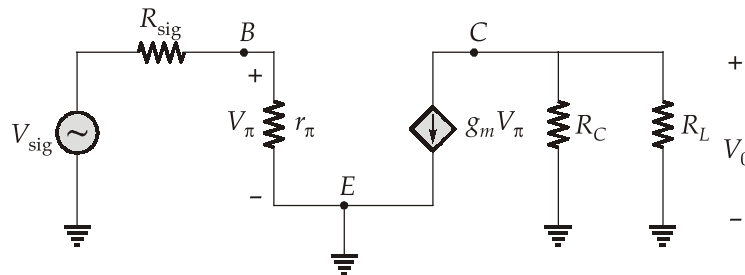
- (ii) In the below circuit, when  $A = 0$ , TG1 is closed and passes the logic B to the output. When  $A = 1$ , TG2 is closed and the complement of logic B is passed to the output. The truth table of the logic circuit can be drawn as below which represents the XOR Gate.

A	B	TG1	TG2	O/P(Y)
0	0	Close	Open	0
0	1	Close	Open	1
1	0	Open	Close	1
1	1	Open	Close	0

**Q.4 (a) Solution:**

- (i) At the mid-band frequency, the capacitances  $C_E$  and  $C_C$  acts as short-circuit.

**Drawing AC equivalent model:**



$A_V$  = Transistor midband voltage gain

$$A_V = \frac{V_0}{V_{be}} = \frac{V_0}{V_\pi}$$

From the circuit,

$$V_0 = -g_m V_\pi (R_C \parallel R_L)$$

$\therefore$

$$A_V = \frac{V_0}{V_\pi} = \frac{-g_m R_C R_L}{(R_C + R_L)} \quad \dots(i)$$

$$A_{vs} = \frac{V_0}{V_{sig}}$$

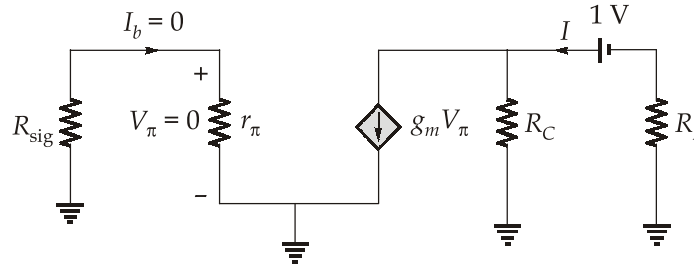
$$V_\pi = \frac{V_{sig} r_\pi}{(R_{sig} + r_\pi)} \quad \text{(using voltage division rule)}$$

$$A_{vs} = \frac{V_0}{V_{sig}} = \frac{V_0 \times r_\pi}{V_\pi (R_{sig} + r_\pi)}$$

$$A_{vs} = \frac{-g_m V_\pi (R_C \parallel R_L) r_\pi}{V_\pi (R_{sig} + r_\pi)}$$

$$A_{vs} = \frac{-g_m (R_C \parallel R_L) r_\pi}{(R_{sig} + r_\pi)} \quad \dots(ii)$$

(ii) **Case 1:** Thevenin resistance across  $C_C$  is calculated as below:



$$\therefore V_{\pi} = 0, g_m V_{\pi} = 0$$

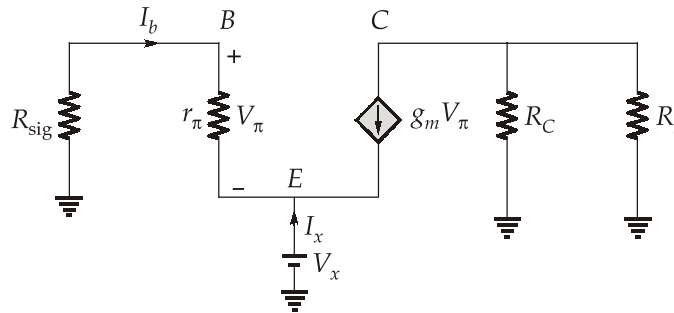
$$\therefore I = \frac{1}{(R_C + R_L)}$$

$$\therefore R_{th} = \frac{1}{I} = (R_C + R_L)$$

$$\therefore f_{CC} = \frac{1}{2\pi R_{th} C_C}$$

$$f_{CC} = \frac{1}{2\pi C_C (R_C + R_L)}$$

**Case 2:** Thevenin resistance across  $C_E$  is calculated as below:



Applying KCL at emitter,

$$-(I_x) = I_b + g_m V_{\pi} = I_b + \beta I_b = I_b(1 + \beta)$$

Applying KVL in base emitter loop,

$$R_{sig} \cdot I_b + r_{\pi} I_b + V_x = 0$$

$$V_x = -I_b [R_{sig} + r_{\pi}]$$

$$\therefore R_{th} = \frac{V_x}{I_x} = \frac{I_b (R_{sig} + r_{\pi})}{I_b (1 + \beta)}$$

$$R_{th} = \frac{r_{\pi} + R_{sig}}{(1 + \beta)}$$

$$\therefore f_{CE} = \frac{1}{2\pi R_{th} \cdot C_E}$$

$$f_{CE} = \frac{[1 + \beta]}{2\pi C_E (r_\pi + R_{sig})}$$

$$f_{CE} = \frac{(1 + \beta)}{2\pi C_E (r_\pi + R_{sig})}$$

(iii) Given  $V_T = 26 \text{ mV}$ ,  $R_C = R_L = R_{sig} = 10 \text{ k}\Omega$ ,  $\beta = 100$

$$g_m = \frac{I_C}{V_T} = \frac{I_E}{(1 + \beta)} \times \beta \times \frac{1}{V_T}$$

$$= \frac{1 \text{ mA} \times 100}{101} \times \frac{1}{26 \text{ mV}}$$

$$g_m = 0.038 \text{ S}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.038} = 2626 \text{ }\Omega$$

From equation (i),

$$A_V = \frac{V_0}{V_\pi} = \frac{-g_m R_C R_L}{(R_C + R_L)}$$

$$= -0.038 \times 5 \text{ k}\Omega$$

$$A_V = -190 \text{ V/V}$$

From equation (ii),

$$A_{VS} = \frac{-g_m (R_C \parallel R_L) r_\pi}{(R_{sig} + r_\pi)}$$

$$= \frac{-0.038 [5 \times 10^3] \times 2626}{[10 \times 10^3 + 2626]}$$

$$A_{VS} = -39.52 \text{ V/V}$$

(iv) Lower 3 dB frequency is given as 100 Hz. From the expressions of break frequencies caused by  $C_C$  and  $C_E$  obtained in part(ii), we can see that the lower frequency is due to  $C_C$ . Hence,

$$f_{L1} = \frac{1 \times (1 + \beta)}{2\pi C_E (r_\pi + R_{sig})} = \frac{101}{2\pi C_E (2626 + 10^4)} = 100$$

$$C_E = 12.73 \text{ }\mu\text{F}$$

Also,

$$f_{L2} = 10 \times 100 = 1000 \text{ Hz}$$

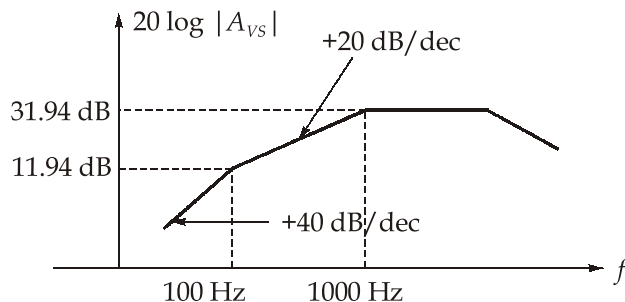
$$f_{L2} = \frac{1}{2\pi C_C (R_C + R_L)} = 1000 \text{ Hz}$$



$$\therefore R_C = R_L = 10 \text{ k}\Omega$$

$$\therefore C_C = 7.958 \text{ nF}$$

(v) The Bode magnitude plot can be drawn as below:



At 1000 Hz,

$$\text{Midband gain } A_{VS} = 20 \log_{10} |A_{VS}| = 20 \log_{10} |39.52|$$

$$[A_{VS}] = 31.94 \text{ dB}$$

At frequency 100 Hz, gain is 20 dB below:

$$\therefore [A_{VS}] = 11.94 \text{ dB}$$

**Q.4 (b) Solution:**

**Truth Table:**

Gray Code				Binary Code			
$X_3$	$X_2$	$X_1$	$X_0$	$Y_3$	$Y_2$	$Y_1$	$Y_0$
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
1	0	0	0	1	1	1	1
1	0	0	1	1	1	1	0
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	0	1	0	1	1
1	1	1	1	1	0	1	0

K-map

For  $Y_3$ :

		$X_1X_0$			
		00	01	11	10
$X_3X_2$	00				
	01				
	11	1	1	1	1
	10	1	1	1	1

$Y_3 = X_3$

For  $Y_2$ :

		$X_1X_0$			
		00	01	11	10
$X_3X_2$	00				
	01	1	1	1	1
	11				
	10	1	1	1	1

$Y_2 = \bar{X}_3X_2 + X_3\bar{X}_2$   
 $= X_3 \oplus X_2$

For  $Y_1$ :

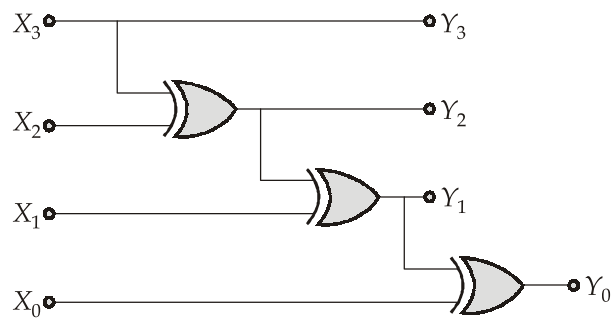
		$X_1X_0$			
		00	01	11	10
$X_3X_2$	00			1	1
	01	1	1		
	11			1	1
	10	1	1		

$Y_1 = \bar{X}_3\bar{X}_2X_1 + \bar{X}_3X_2\bar{X}_1 + X_3X_2X_1 + X_3\bar{X}_2\bar{X}_1$   
 $= (X_3 \odot X_2)X_1 + \bar{X}_1(X_3 \oplus X_2)$   
 $= X_3 \oplus X_2 \oplus X_1$   
 $= Y_2 \oplus X_1$

For  $Y_0$ :

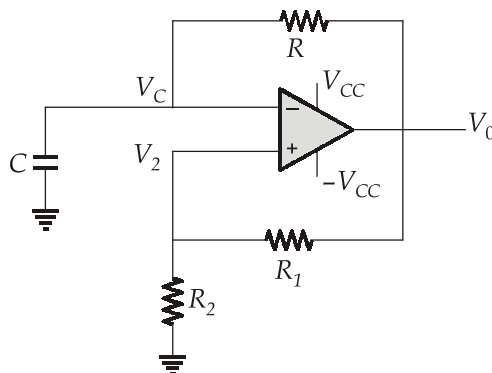
		$X_1X_0$			
		00	01	11	10
$X_3X_2$	00		1		1
	01	1		1	
	11		1		1
	10	1		1	

$Y_0 = \bar{X}_3[X_2 \oplus X_1 \oplus X_0] + X_3[X_1 \odot X_2 \odot X_3]$   
 $Y_0 = X_3 \oplus X_2 \oplus X_1 \oplus X_0$   
 $Y_0 = Y_1 \oplus X_0$

 $\Rightarrow$  Combinational Circuit:

**Q.4 (c) Solution:**

Astable multivibrator using operational amplifier is constructed as shown in the figure below:



- Output has two quasi stable states i.e.,  $+V_{\text{sat}}$  and  $-V_{\text{sat}}$ .
- Output waveform resembles square wave. It is also called square wave generator or free running oscillator.
- Because of positive feedback, the output is saturated to  $+V_{\text{sat}}$  or  $-V_{\text{sat}}$ .

Operation of the Astable Multivibrator:

$$V_2 = \frac{R_2}{R_1 + R_2} V_0 = \beta V_0$$

If  $V_0 = +V_{\text{sat}}$

$$V_2 = +\beta V_{\text{sat}}$$

If  $V_0 = -V_{\text{sat}}$

$$V_2 = -\beta V_{\text{sat}}$$

The circuit compares capacitor voltage,  $V_C$  with  $V_2$

**Case 1:** If  $V_0 = +V_{\text{sat}}$

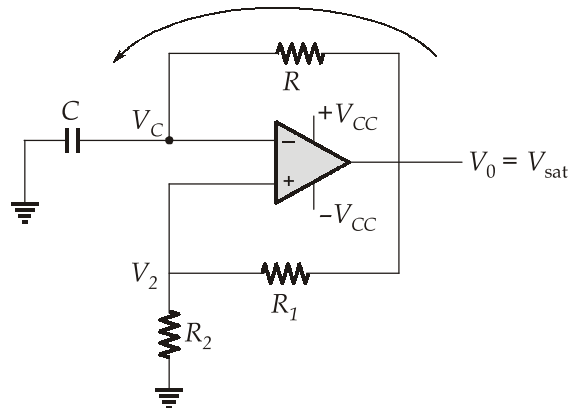
$$V_2 = +\beta V_{\text{sat}}$$

$$V_d = V_2 - V_C$$

Capacitor charges through resistance  $R$  and  $V_C$  continuously increases.

$V_C$  continuous to increase till  $+\beta V_{\text{sat}}$ .

When  $V_C$  becomes slightly greater than  $+\beta V_{\text{sat}}$ ,  $V_d$  becomes negative and then  $V_0$  changes to  $-V_{\text{sat}}$ .



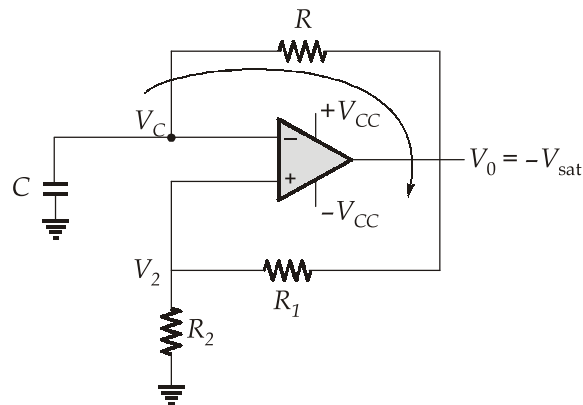
**Case 2: If**

$$V_0 = -V_{\text{sat}}$$

$$V_2 = -\beta V_{\text{sat}}$$

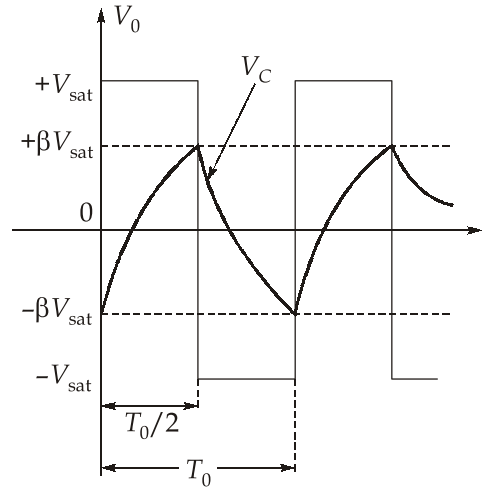
$$V_d = V_2 - V_C$$

Capacitor stops charging and start discharging through  $R$ .  $V_C$  gradually decreases and continue to decrease till  $-\beta V_{\text{sat}}$ .



When  $V_C$  becomes slightly less than  $-\beta V_{\text{sat}}$ ,  $V_d$  becomes positive and  $V_0$  changes to  $+V_{\text{sat}}$ . The capacitor again starts charging upto  $+\beta V_{\text{sat}}$  and the cycle repeats generating a square wave at the output. As capacitor charges and discharges through same resistance, the charging and discharging interval will be same and output waveform has 50% duty cycle.

Calculation of  $T_0$ :



The voltage across the capacitor,

$$V_c(t) = (V_i - V_f)e^{-t/RC} + V_f$$

$$V_c(0) = V_i = -\beta V_{sat}$$

$$V_c(\infty) = V_f = +V_{sat}$$

$$V_c(t) = (-\beta V_{sat} - V_{sat})e^{-t/RC} + V_{sat}$$

$$V_c(t) = V_{sat} - V_{sat}(1 + \beta)e^{-t/RC}$$

$$V_c(t) = V_{sat}[1 - (1 + \beta)e^{-t/RC}]$$

At  $t = T_0/2$ ,  $V_c(t) = +\beta V_{sat}$

$$+\beta V_{sat} = V_{sat} \left[ 1 - (1 + \beta)e^{-T_0/RC \times 2} \right]$$

$$(1 - \beta) = (1 + \beta)e^{-T_0/2RC}$$

$$e^{T_0/2RC} = \frac{(1 + \beta)}{(1 - \beta)}$$

$$T_0 = 2RC \ln \left[ \frac{(1 + \beta)}{(1 - \beta)} \right]$$

where  $\beta$  = feedback factor,  $\beta = \frac{R_2}{R_1 + R_2}$

If  $R_1 = R_2 = R$  then  $\beta = 1/2$ .

$\therefore$

$$T_0 = 2RC \ln \left[ \frac{1 + 1/2}{1 - 1/2} \right]$$

$$T_0 = 2RC \ln 3$$

**Section B : Advanced Electronics-1 + Electronic Measurements and Instrumentation-1  
Electromagnetics-2 + Basic Electrical Engineering-2**

**Q.5 (a) Solution:**

$$\alpha = \frac{2}{\eta_o} \sqrt{\frac{\pi f \mu / \sigma}{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{2}{\eta_o} \sqrt{\frac{\pi \mu}{\sigma}} \cdot \sqrt{\frac{f}{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\alpha = K \sqrt{\frac{f}{1 - \left(\frac{f_c}{f}\right)^2}} = K \sqrt{\frac{f^3}{f^2 - f_c^2}}$$

$\therefore$  We have to find the maximum value, thus we will have to differentiate  $\alpha$  with respect to frequency ' $f$ '

$$\therefore \alpha = K \cdot \frac{f^{3/2}}{\sqrt{f^2 - f_c^2}}$$

$$\therefore \frac{d\alpha}{df} = K \frac{\left(\frac{3}{2} f^{1/2}\right)(\sqrt{f^2 - f_c^2}) - f^{3/2} \left(\frac{1}{2}\right)(2f)(f^2 - f_c^2)^{-1/2}}{f^2 - f_c^2}$$

Now, for maxima  $\frac{d\alpha}{df} = 0$ .

$$\Rightarrow K \frac{\frac{3}{2} \cdot f^{1/2}(f^2 - f_c^2) - f^{5/2}}{(f^2 - f_c^2)^{3/2}} = 0$$

$$\Rightarrow \frac{3}{2} f^{1/2}(f^2 - f_c^2) - f^{5/2} = 0$$

$$\Rightarrow \frac{3}{2} f^{5/2} - \frac{3}{2} f_c^2 \cdot f^{1/2} - f^{5/2} = 0$$

$$\frac{3}{2} - \frac{3}{2} f_c^2 \cdot f^{-2} - 1 = 0$$

$$-\frac{3}{2} f_c^2 f^{-2} = -\frac{1}{2}$$

$$f^2 = 3f_c^2$$

$$f = \sqrt{3} f_c$$

Hence, the value of  $\alpha$  will be maximum at  $f = \sqrt{3} f_c$ .

**Q.5 (b) Solution:**

When the two resistances are connected in parallel, the resultant resistance is

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{200 \times 150}{200 + 150} = 85.71 \, \Omega$$

$$\frac{\partial R}{\partial R_1} = \frac{(R_1 + R_2)R_2 - R_1 R_2}{(R_1 + R_2)^2} = \frac{R_2^2}{(R_1 + R_2)^2} = \frac{(150)^2}{(350)^2} = \frac{9}{49}$$

$$\frac{\partial R}{\partial R_1} = \frac{9}{49}$$

$$\frac{\partial R}{\partial R_2} = \frac{(R_1 + R_2)R_1 - R_1 R_2}{(R_1 + R_2)^2} = \frac{R_1^2}{(R_1 + R_2)^2} = \frac{(200)^2}{(350)^2}$$

$$\frac{\partial R}{\partial R_2} = \frac{16}{49}$$

Hence, uncertainty in total resistance is,

$$\omega_R = \pm \sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 \cdot \omega_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 \cdot \omega_{R_2}^2}$$

$$\therefore \omega_{R1} = 0.2 \, \Omega, \omega_{R2} = 0.04 \, \Omega$$

$$\therefore \omega_R = \pm \sqrt{\left(\frac{9}{49}\right)^2 \cdot (0.2)^2 + \left(\frac{16}{49}\right)^2 \cdot (0.04)^2}$$

$$\omega_R = 0.0389 \, \Omega$$

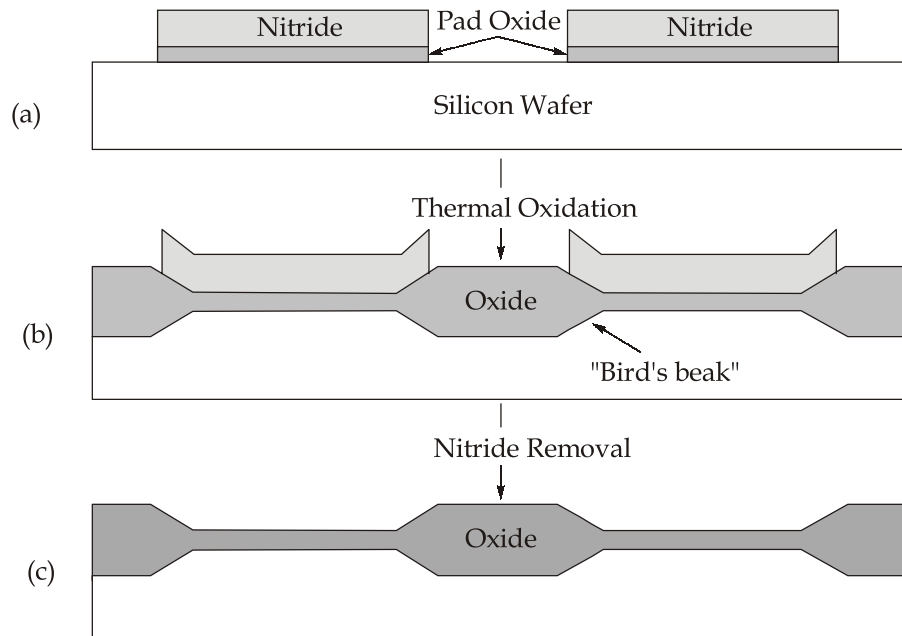
$\therefore$  The total resistance can be given as

$$R = 85.71 \pm 0.0389 \, \Omega$$

**Q.5 (c) Solution:**

Local Oxidation of Silicon (LOCOS) is the traditional isolation technique. At first a very thin silicon oxide layer is grown on the wafer, the so-called pad oxide. Then a layer of silicon nitride is deposited which is used as an oxide barrier. The pattern transfer is performed by photolithography. After lithography, the pattern is etched into the nitride. The result is the nitride mask as shown in figure (a), which defines the active areas for the oxidation process. The next step is the main part of the LOCOS process, the growth of the thermal oxide. The result is the formation of very thick oxide between two regions as shown in figure (b). After the oxidation process is finished, the last step is the removal of the nitride layer.

The main drawback of this technique is the so-called bird's beak effect and the surface area which is lost to this encroachment. The advantages of LOCOS fabrication are the simple process flow and the high oxide quality, because the whole LOCOS structure is thermally grown.



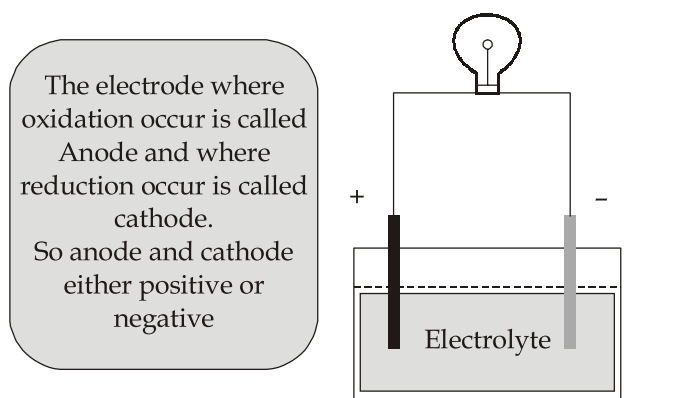
**Process sequence for local oxidation of silicon (LOCOS)**

**Q.5 (d) Solution:**

A battery is an electrochemical device that can store energy in the form of chemical energy. It translates to electric energy when the battery is connected in a circuit due to the flow of electrons because of the specific placement of chemicals. It was invented by Alessandro Volta, whereas Gaston Plante invented the rechargeable battery.

The battery consists of three elements: the negative side, the positive side, and electrolyte (the chemical which reacts with both sides). The electrolyte is used as an electron transportation medium between the anode and cathode.

It works due to electrochemical reactions called oxidation and reduction. In this reaction, electrons flow from one side to another side when the external circuit is connected to the anode and cathode.





**Types of Batteries:**

Based on functionality, there are two types of batteries available in the market.

1. Primary Batteries.
2. Secondary Batteries.

**Primary Batteries:** The batteries made for one-time use only and unable to recharge, are called **primary batteries**. This type of battery is thrown away after use. It is also known as **non-rechargeable batteries**. It's a very simple and convenient source of power for portable devices like a watch, camera, torch, etc.

These batteries are cheap, small, lightweight, and there is no or low maintenance required.

**Some common primary batteries**

1. Alkaline Battery
2. Button Cell Battery

**Secondary Batteries**

The battery which is made for reusable purposes by recharging are called **secondary batteries**. They are also called **rechargeable batteries**. They have the same electrochemical reaction as alkaline batteries, but the electrochemical reaction can be reversed. This type of battery is used for portable devices like mobile phones, laptops, electric vehicles, etc. Also, a rechargeable battery is used with an inverter which stores power to supply our household devices.

**Some common secondary batteries**

1. Lead-Acid Batteries
2. Nickel Cadmium Batteries
3. Lithium Ion Batteries

**Lithium-ion batteries**

Lithium-ion batteries have anode made of graphite and cathode made of lithium metal oxide. The lithium salt as an organic solvent is used as an electrolyte. When the battery is connected to the circuit or load, lithium-ion migrates from the negative electrode to the positive electrode.

**Construction:** The lithium metal oxide is coated on aluminum foil which is the positive electrode. The graphite is coated on copper foil which is the negative electrode. Both foils are rolled in a cylindrical shape with a separator between them. The separator is soaked with electrolyte material which generally is lithium salt as an organic solvent. The outer metal casing is negative, and the top cap is the positive terminal. Both are separated by a gasket, which is made of insulating material.

Lithium-ion batteries are used in mobiles, laptops, and many portable devices. It is also used in the military and aerospace due to its lightweight nature. It has a higher energy density and low self-discharge compared to other types of batteries. It is also available in various sizes. Its single-cell voltage is higher. These have a significant risk of explosion when it is short-circuited or externally damaged.

### Q.5 (e) Solution:

Let  $R_1$  and  $L_1$  be the effective resistance and inductance of the specimen respectively.

$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2 + \frac{1}{j\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = \frac{1}{j\omega C_4}$$

At balance,

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1) \frac{1}{j\omega C_4} = R_3 \left( R_2 + \frac{1}{j\omega C_2} \right)$$

$$L_1 = R_2 R_3 C_4 = 834 \times 100 \times 0.1 \times 10^{-6} \text{ H} = 8.34 \text{ mH}$$

and

$$R_1 = \frac{R_3 C_4}{C_2} = \frac{100 \times 0.1}{0.12} = 83.33 \Omega$$

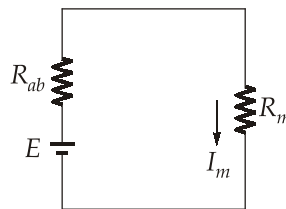
Reactance of specimen at 2 kHz

$$X_1 = 2\pi \times 2 \times 1000 \times 8.34 \times 10^{-3} = 104.8 \Omega$$

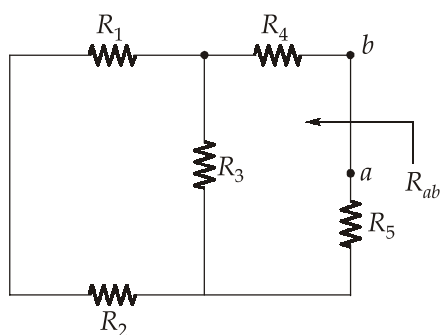
$$\begin{aligned} \therefore \text{Impedance of specimen, } Z_1 &= \sqrt{R_1^2 + X_1^2} \\ &= \sqrt{(83.33)^2 + (104.8)^2} = 133.89 \Omega \end{aligned}$$

### Q.6 (a) Solution:

The given circuit can be reduced into



To calculate the resistance  $R_{ab}$ , short circuiting all the sources,



where,  $R_{ab}$  is resistance seen across  $ab$ .

$$R_{ab} = [(R_1 + R_2) \parallel R_3] + R_4 + R_5$$

$$\therefore R_{ab} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} + R_4 + R_5$$

Let  $I_{mo}$  be the current in the circuit without meter,

$$I_{mo} = \frac{E}{R_{ab}}$$

Let  $I_m$  be the current in the circuit with meter,

$$I_m = \frac{E}{R_{ab} + R_m}$$

Error due to meter loading,

$$\epsilon = \frac{I_{mo} - I_m}{I_{mo}} = \frac{\frac{E}{R_{ab}} - \frac{E}{R_{ab} + R_m}}{\frac{E}{R_{ab}}}$$

$$\epsilon = \frac{R_m}{R_{ab} + R_m}$$

(i) Given,  $R_1 = R_2 = R_3 = R_4 = R_5 = 100 \Omega$ ;  $R_m = 10 \Omega$

$$\therefore R_{ab} = \frac{(100 + 100)100}{300} + 100 + 100$$

$$\therefore R_{ab} \approx 267 \Omega$$

$$\therefore \text{error, } \epsilon = \frac{10}{267 + 10} = 0.036 = 3.6\%$$

(ii) Given,

$$R_1 = R_2 = R_3 = R_4 = R_5 = 200; R_m = 1 \Omega$$

$$\therefore R_{ab} = \frac{(200 + 200)200}{600} + 200 + 200 \simeq 534$$

$$\text{error, } \epsilon = \frac{1}{1 + 534} = 0.00186 = 0.186\%$$

Q.6 (b) Solution:

(i)

$$U_{\max} = 1$$

$$\begin{aligned} U_{\text{avg}} &= \frac{P_{\text{rad}}}{4\pi} = \frac{\int U d\Omega}{4\pi} \\ &= \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \sin^2(2\theta) \cdot \sin(\theta) d\theta \cdot d\phi \\ &= \frac{1}{4\pi} (2\pi) \int_0^{\pi} (2 \sin \theta \cdot \cos \theta)^2 d(-\cos \theta) \\ &= 2 \int_0^{\pi} (\cos^4 \theta - \cos^2 \theta) d(\cos \theta) \\ &= 2 \left[ \frac{\cos^5 \theta}{5} - \frac{\cos^3 \theta}{3} \right]_0^{\pi} = 2 \left[ -\frac{2}{5} + \frac{2}{3} \right] = \frac{8}{15} \end{aligned}$$

$$U_{\text{avg}} = 0.533$$

$$D = \frac{U_{\max}}{U_{\text{avg}}} = 1.875$$

(ii)

$$U_{\max} = 4$$

$$\begin{aligned} U_{\text{avg}} &= \frac{1}{4\pi} \int U \cdot d\Omega = \frac{4}{4\pi} \iint \frac{1}{\sin^2 \theta} \sin \theta \cdot d\theta \cdot d\phi \\ &= \frac{1}{\pi} \int_0^{\pi} d\phi \int_{\pi/3}^{\pi/2} \frac{d(-\cos \theta)}{1 - \cos^2 \theta} = \int_{1/2}^0 \frac{dv}{v^2 - 1} = \frac{1}{2} \ln \left[ \frac{1-v}{1+v} \right]_{0.5}^0 \\ &= \frac{1}{2} \left[ \ln 1 - \ln \left[ \frac{0.5}{1.5} \right] \right] = \frac{1}{2} \ln 3 \end{aligned}$$

$$U_{\text{avg}} = 0.5493$$

$$D = \frac{U_{\max}}{U_{\text{avg}}} = \frac{4}{0.5493} = 7.28$$

(iii)

$$U_{\max} = 2$$

$$\begin{aligned} U_{\text{avg}} &= \frac{1}{4\pi} \int U \cdot d\Omega \\ &= \frac{1}{4\pi} \iint 2 \sin^2 \theta \cdot \sin^2 \phi \sin \theta d\theta d\phi \\ &= \frac{1}{2\pi} \cdot \frac{\pi}{2} \left( \frac{\cos^3 \theta}{3} - \cos \theta \right) \Bigg|_0^{\pi} = \frac{1}{4} \left[ -\frac{2}{3} + 2 \right] = \frac{1}{3} \end{aligned}$$

$$U_{\text{avg}} = 0.333$$

$$D = \frac{U_{\text{max}}}{U_{\text{avg}}} = 6$$

**Q.6 (c) Solution:**

- (i) From the given state diagram and its truth table, only the state *A* has an output of 1, so state *A*'s encoding must be 00 and output  $Y = 1$  regardless of the input because it is a Moore machine. From the state *A*, when  $\text{IN} = 1$ , the machine moves to state *B*. According to the truth table, from state 00 (which is state *A*) when  $\text{IN} = 1$ , it moves to state 01 (state *B*). From state *B*, when  $\text{IN} = 0$  it goes to state *D* so it is state 10. Similarly, from state *B*, when  $\text{IN} = 1$  machine move back to state *A* which we know to be 00. The encoding for state *C* is 11. Looking at the state 10 (state *D*), when  $\text{IN} = 1$  the corresponding transition in the state diagram shows that it goes back to state *D*. Since the encoding of state *D* is 10, the  $S'_0$  entry is 0. Because the output associated with the current state *D* regardless of the input is 0, so the missing output entry is 0. Finally from state 11 (state *C*), when  $\text{IN} = 1$ , the state transits to *A* (state 00).

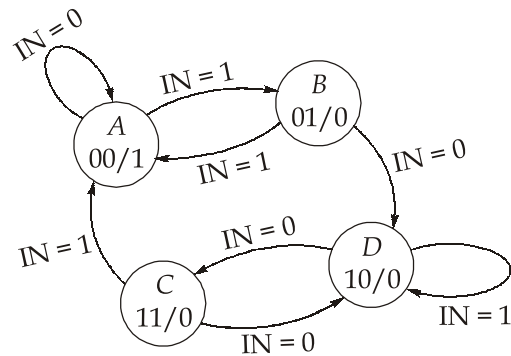
So, the remaining missing  $S'_0$  value is 0.

The complete truth table:

$S_1$	$S_0$	IN	$S'_1$	$S'_0$	Y
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	0	0	0
1	0	0	1	1	0
1	0	1	1	0	0
1	1	0	1	0	0
1	1	1	0	0	0

- (ii) In a Moore machine, equivalent states have the same output, and the same input transitions.

The state transition diagram:



To reduce the above state diagram,

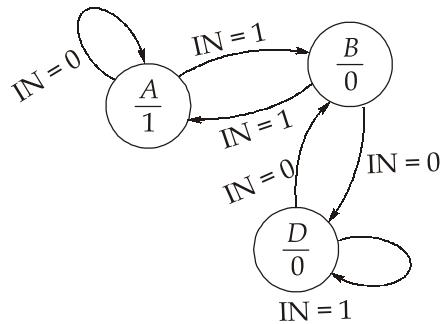
Present State	Output Y	Next State	
		IN = 0	IN = 1
A	1	A	B
B	0	D	A
D	0	C	D
C	0	D	A

For states B and C, output is same and the next states corresponding to a given input are also same. Hence, state B and C are said to be equivalent and the state C can be replaced with state B. The resultant state diagram will be

Present State	Output Y	Next State	
		IN = 0	IN = 1
A	1	A	B
B	0	D	A
D	0	B	D
B	0	D	A

Present State	Output Y	Next State	
		IN = 0	IN = 1
A	1	A	B
B	0	D	A
D	0	B	D

∴ The reduced state diagram is



**Q.7 (a) Solution:**

(i) Weight of water available is

$$\begin{aligned}
 W &= \text{Volume of water} \times \text{density} \\
 &= 5 \times 10^6 \times 1000 \quad (\because \text{Mass of } 1 \text{ m}^3 \text{ of water is } 1000 \text{ kg}) \\
 &= 5 \times 10^9 \text{ kg} \\
 W &= 5 \times 10^9 \times 9.81 \text{ Newton} = 4.905 \times 10^{10} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 \text{Electrical energy available} &= W \times H \times \eta_{\text{overall}} \\
 &= 5 \times 10^9 \times 9.81 \times 200 \times 0.75 \\
 &= 7.3575 \times 10^{12} \text{ Watt-sec} \\
 &= \frac{7.3575 \times 10^{12}}{3600 \times 1000} \text{ kWh} \\
 &= 2.044 \times 10^6 \text{ kWh}
 \end{aligned}$$

(ii) Weight of water available

$$\begin{aligned}
 W &= 94 \times 1000 = 94000 \text{ kg/sec} \\
 \text{Water head, } H &= 39 \text{ m} \\
 \text{Work done/sec} &= W \times H = 94000 \times 39 \times 9.81 \text{ W} \\
 &= 35963.46 \times 10^3 \text{ W} \\
 &= 35963.46 \text{ kW}
 \end{aligned}$$

This is gross plant capacity.

1. Firm capacity = Plant efficiency  $\times$  Gross plant capacity
 
$$\begin{aligned}
 &= 0.8 \times 35963.46 \text{ kW} \\
 &= 28770.77 \text{ kW}
 \end{aligned}$$
2. Yearly Gross output = Firm capacity  $\times$  Hours in a year
 
$$\begin{aligned}
 &= 28770.77 \times 10^3 \times 8760 \\
 &= 252.03 \times 10^6 \text{ kWh}
 \end{aligned}$$

**Q.7 (b) Solution:****(i) Galvanometer used as ammeter :**

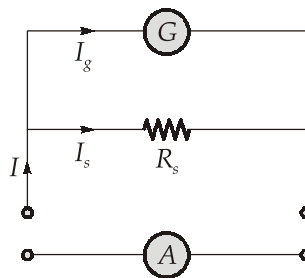
Since galvanometer is a very sensitive instrument, therefore it cannot measure heavy currents. In order to convert a galvanometer into an ammeter, a very low resistance known as shunt resistance is connected in parallel to galvanometer. Value of shunt is so adjusted that most of the current passes through the shunt. In this way a galvanometer is converted into ammeter and can measure heavy currents without fully deflected.

Let resistance of galvanometer =  $R_g$  and it gives full scale deflection when current  $I_g$  is passed through it.

Then,

$$V_g = I_g R_g$$

Let a shunt of resistance  $R_s$  is connected in parallel to galvanometer. If total current through the circuit is  $I$ .



Then current through shunt,  $I_s = (I - I_g)$

Potential difference across the shunt :

$$V_s = I_s R_s \text{ or } V_s = (I - I_g) R_s$$

But,

$$V_s = V_g$$

$$(I - I_g) R_s = I_g R_g$$

$$R_s = \frac{I_g}{I - I_g} R_g$$

$$\Rightarrow R_s = \frac{R_g}{(m - 1)} ; \quad \text{where } m = \frac{I}{I_g}$$

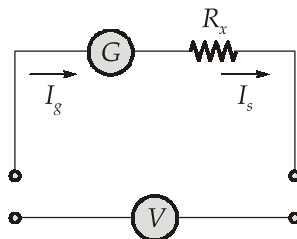
**Galvanometer used as voltmeter:**

Since galvanometer is a very sensitive instrument, therefore it can not measure high potential difference. In order to convert a galvanometer into voltmeter, a very high resistance known as series resistance is connected in series with the galvanometer.



Let, resistance of galvanometer =  $R_g$

and consider a resistance  $R_x$  (high) is connected in series to it. Then combined resistance =  $(R_g + R_x)$



If potential between the points to be measured =  $V$  and if galvanometer gives full scale deflection when current " $I_g$ " passes through it.

Then,

$$I_g = \frac{V - V_g}{R_x} = \frac{V_g}{R_g}$$

$$R_x = \frac{R_g(V - V_g)}{V_g}$$

$$= R_g(m - 1); \quad \text{where } m = \frac{V}{V_g}$$

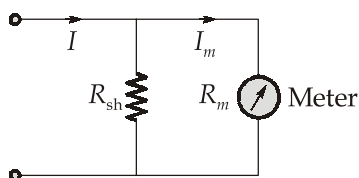
(ii) Full scale deflection current,  $I_m = 1 \text{ mA}$

Required range is 0 - 10 mA

$$\therefore I = 10 \text{ mA}$$

Multiplying power,

$$m = \frac{I}{I_m} = 10$$



The range of an ammeter may be extended by adding a shunt resistance

Shunt resistance,

$$R_{sh} = \frac{R_m}{m - 1}$$

Given that,

$$R_m = 5 \Omega$$

$$\therefore R_{sh} = \frac{5}{10 - 1} = 0.55 \Omega$$

By using a shunt resistance of  $0.55 \Omega$ , we can extend the range of meter to 10 mA.

## Q.7 (c) Solution:

Step 1: Simplification of Boolean function using K-map.

$F_1$ :

$AB \backslash CD$	00	01	11	10
00	1	1	1	1
01	1		1	
11	1		1	
10	1		1	

$$F_1 = \bar{C}\bar{D} + CD + \bar{A}\bar{B}$$

$F_2$ :

$AB \backslash CD$	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

$$F_2 = A\bar{C} + \bar{A}C$$

$F_3$ :

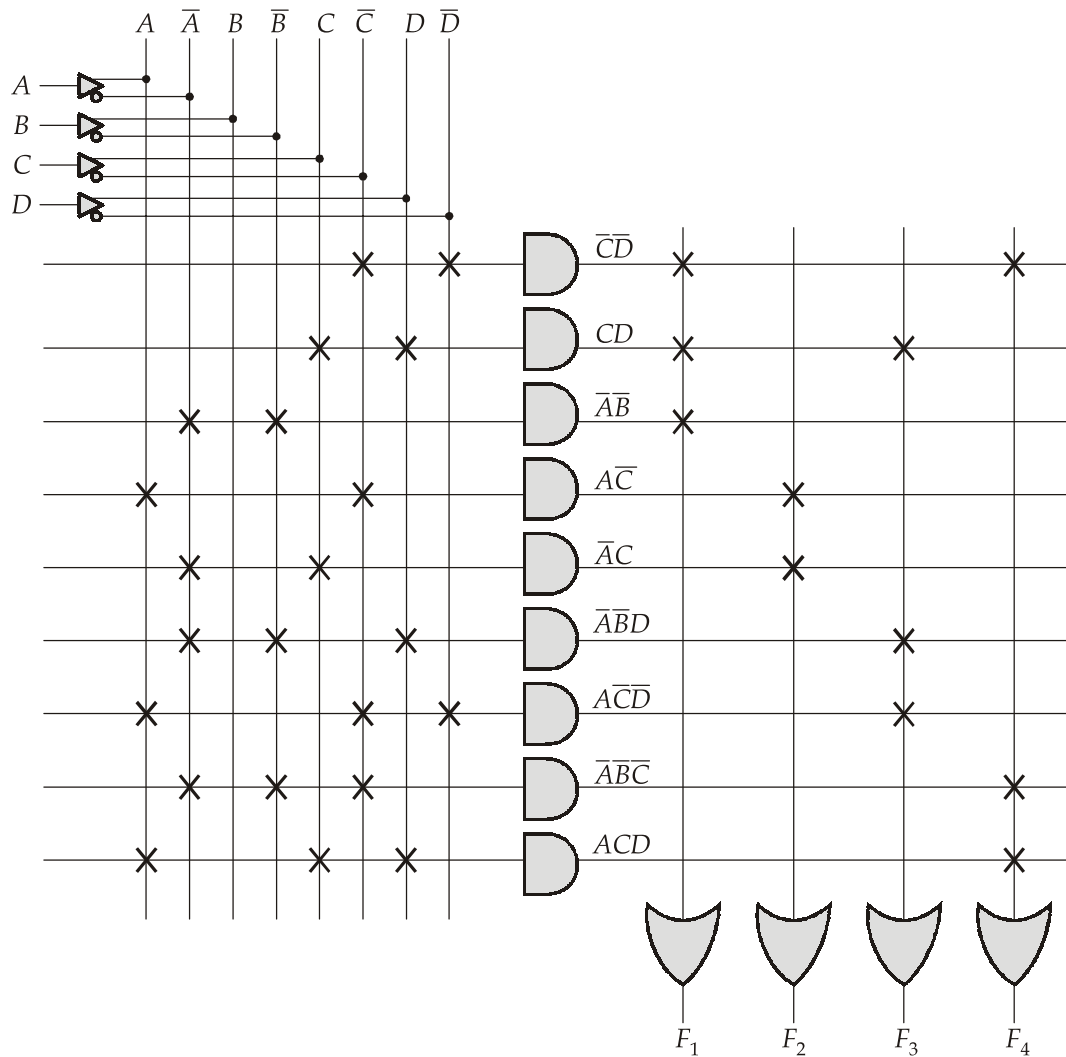
$AB \backslash CD$	00	01	11	10
00		1	1	
01			1	
11	1		1	
10	1		1	

$$F_3 = \bar{A}\bar{B}D + CD + A\bar{C}\bar{D}$$

$F_4$ :

$AB \backslash CD$	00	01	11	10
00	1	1		
01	1			
11	1		1	
10	1		1	

$$F_4 = \bar{C}\bar{D} + \bar{A}\bar{B}\bar{C} + ACD$$

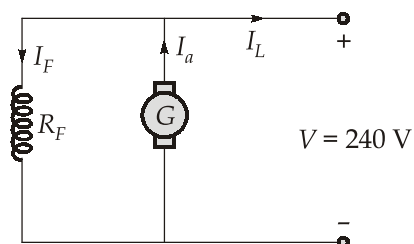
**Step 2: PLA implementation****Q.8 (a) Solution:**

Field resistance ( $R_F$ ) = 100  $\Omega$

Armature resistance ( $R_a$ ) = 0.3  $\Omega$

Output power to load ( $P_0$ ) = 30 kW

Terminal voltage ( $V$ ) = 240 V

**(i) Running as a generator delivering 30 kW:**

$$I_L = \frac{P_0}{V} = \frac{30 \times 10^3}{240} = 125 \text{ Amp}$$

$$I_F = \frac{V}{R_F} = \frac{240}{100} = 2.4 \text{ Amp}$$

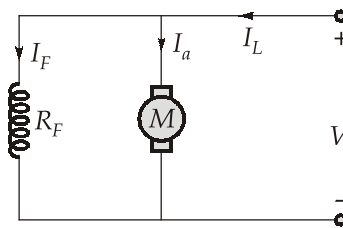
$$I_a = I_L + I_F = 125 + 2.4 = 127.4 \text{ Amp}$$

Now,

$$\begin{aligned} E_g &= I_a \cdot R_a + V \\ &= (127.4)(0.3) + 240 \\ &= 278.22 \text{ Volt} \end{aligned}$$

$$\begin{aligned} \therefore \text{Armature power developed } (P_a) &= E_g \cdot I_a \\ &= 278.22 \times 127.4 \\ &= 35.44 \text{ kW} \end{aligned}$$

(ii) Running as a motor taking 30 kW input:



$$I_F = \frac{V}{R_F} = \frac{240}{100} = 2.4 \text{ Amp}$$

$$P_0 = V \cdot I_L$$

$$I_L = \frac{30 \times 10^3}{240} = 125 \text{ Amp}$$

Now,

$$\begin{aligned} I_a &= I_L - I_F \\ &= 125 - 2.4 \\ &= 122.6 \text{ Amp} \end{aligned}$$

$$E_b = V - I_a \cdot R_a = 240 - (122.6)(0.3) = 203.22 \text{ Volt}$$

$$\begin{aligned} \therefore \text{Armature Power Developed } (P_a) &= I_a \cdot E_b \\ &= 122.6 \times 203.22 \\ &= 24.92 \text{ kW} \end{aligned}$$

**Q.8 (b) Solution:**

(i) Since the transmission line is one-quarter wavelength long thus,

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{50 - j50} = 25 + j25 \text{ Ohms}$$

Now, to calculate the power dissipated by the load, we must calculate

$$V_{in} = V_s \cdot \frac{Z_{in}}{Z_s + Z_{in}} = 10 \left[ \frac{25 + j25}{50 + 25 + j25} \right] = 4 + j2 \text{ V}$$

As the line is lossless, thus the power dissipated by load will be

$$\begin{aligned} P_{in} &= P_L = \frac{1}{2} \operatorname{Re} \{ V_{in} I_{in}^* \} = \frac{1}{2} \operatorname{Re} \left\{ \frac{V_{in} \cdot V_{in}^*}{Z_{in}^*} \right\} \\ &= \frac{1}{2} \operatorname{Re} \left\{ \frac{(4 + j2)(4 - j2)}{25 - j25} \right\} = 0.2 \text{ W} \end{aligned}$$

- (ii) The phasor voltage at any point in the line is given by the sum of forward and backward wave.

$$V_o(s) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

where,

$$V_0^- = \Gamma_L V_0^+$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j50 - 50}{50 - j50 + 50} = 0.2 - j0.4$$

Now,  $z = 0$  is located at the load, thus for  $z = l = -\frac{\lambda}{4}$ ,  $\beta l = \frac{\pi}{2}$ .

$$\begin{aligned} \therefore V_{in} &= V_s(-l) = V_0^+ [e^{j\beta l} + \Gamma_L e^{-j\beta l}] \\ &= V_0^+ [j + (0.2 - j0.4)(-j)] = V_0^+ (-0.4 + j0.8) \end{aligned}$$

$$\text{Thus, } V_0^+ = \frac{(4 + j2)}{(-0.4 + j0.8)}$$

Now, the voltage across the load will be

$$\begin{aligned} V_L &= V_0^+ (1 + \Gamma_L) = \frac{(4 + j2)}{(-0.4 + j0.8)} \times (1.2 - j0.4) \\ &= -2 - j6 \text{ V} \end{aligned}$$

### Q.8 (c) Solution:

$$(i) \text{ Terminal voltage per phase } (V_t) = \frac{240}{\sqrt{3}} = 138.56 \text{ Volt}$$

$$\text{Rated armature current } (I_a) = \frac{15 \times 1000}{\sqrt{3} \times 240} = 36.08 \text{ Amp.}$$

$$\text{We know; } E_F = \sqrt{(V_t \cos \phi + I_a R_a)^2 + (V_t \sin \phi + I_a X_s)^2}$$

$$\therefore R_a = 0, \cos \phi = 1 \text{ (unity p.f.)}$$

$$E_F = \sqrt{V_t^2 + (I_a X_s)^2}$$

$$E_F = \sqrt{(138.56)^2 + (36.08 \times 2)^2} \dots X_s = 2 \Omega/\text{phase}$$

$$= 156.22 \text{ Volt}$$

$$\begin{aligned} \therefore \text{Voltage regulation} &= \frac{E_F - V_t}{V_t} \\ &= \frac{156.22 - 138.56}{138.56} = 0.1275 = 12.75\% \end{aligned}$$

(ii) 1. Absolute error =  $1.3 - 1.2 = +0.1 \text{ k}\Omega$

or  $= 1.1 - 1.2 = -0.1 \text{ k}\Omega$

$\therefore$  absolute error =  $\pm 0.1 \text{ k}\Omega$

Largest possible resistance at  $40^\circ\text{C}$ :

$$R = 1.2 + 0.1 = 1.3 \text{ k}\Omega$$

Now, resistance change per  $^\circ\text{C}$ :

$$\begin{aligned} 400 \text{ ppm per degree} &= \frac{1.3 \times 10^3}{10^6} \times 400 \\ &= 0.52 \Omega/^\circ\text{C} \end{aligned}$$

Here, temperature increase  $\Rightarrow T_2 - T_1 = 100 - 40 = 60^\circ\text{C}$

$$\begin{aligned} \text{Total increase in resistance } (\Delta R) &= 0.52 \times 60 \\ &= 31.2 \Omega \end{aligned}$$

Now, the maximum resistance at  $100^\circ\text{C}$ :

$$\begin{aligned} R_T &= R + \Delta R \\ &= 1300 + 31.2 \\ &= 1331.2 \Omega \end{aligned}$$

$$\begin{aligned} 2. \quad \% \text{ tolerance} &= \frac{\text{Absolute error}}{\text{Specified value}} \times 100 \\ &= \frac{0.1}{1.2} \times 100 = 8.33\% \end{aligned}$$

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