Detailed Solutions

ESE-2023 Mains Test Series

Electrical Engineering Test No : 8

Section A : Electromagnetic Theory + Digital Electronics + Communication Systems

Q.1 (a) Solution:

(i)
$$(1100000)_2 = (341)_X$$

Converting into decimal

$$1 \times 2^6 + 1 \times 2^5 + 0 = 3X^2 + 4X + 1$$
$$96 = 3X^2 + 4X + 1$$
$$3X^2 + 4X - 95 = 0$$

On solving,
$$X = 5, -\frac{19}{3}$$
$$X = 5$$

(ii)
$$(504)_6 = (X)_{12}$$

Converting to decimal,

$$(504)_6 = (5 \times 6^2 + 0 + 4 \times 6^0)_{10}$$
$$= (184)_{10}$$

Converting to base 12

12	184	R
12	15	4
12	1	3
	0	1

$$(504)_6 = (134)_{12}$$

$$X = 134$$

(iii) $$(10101.110)_2 = (X)_4$$

Converting to decimal

$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3}$

$= (21.75)$

Converting it to base 4

4	21	R
4	5	1
4	1	1
	0	1

$$0.75 \times 4 = 3.00$$

$$(21)_{10} = (111)_4$$

$$(0.75)_{10} = (0.3)_4$$

$$(10101.110)_2 = (111.3)_4$$

$$X = 111.3$$

Q.1 (b) Solution:

(i) Sensitivity : The sensitivity of a radio receiver may be defined as its ability to amplify weak signals. It is generally, defined in terms of the voltage which must be applied at receiver input terminals to provide a standard output power measured at the output terminals.

The sensitivity is generally quoted in terms of signal power required to produce a minimum acceptable output signal with a minimum acceptable output noise level.

(ii) Selectivity : The selectivity of a receiver may be defined as the ability to reject unwanted signals. It also expresses the attenuation that the receiver offers to signal at frequencies adjacent to the one to which it is tuned.

(iii) Fidelity : The fidelity is the ability of a receiver to reproduce all the modulating frequencies equally. The fidelity basically depends on the frequency response of the AF amplifier.

High fidelity is essential in order to reproduce a good quality signal, i.e., without introducing any distortion.

(iv) Double Spotting : When a receiver picks up the same short wave station at two nearby points on the receiver dial, the double spotting phenomenon takes place.

The main cause for double spotting is poor front-end selectivity, i.e., inadequate image-frequency rejection.

The adverse effect of double spotting is that a weak station may be marked by the reception of a nearby strong station at the spurious point on the receiver dial.

Q.1 (c) Solution:

Given : $$E = \frac{2r}{(r^2 + a^2)^2}\hat{a}_r \text{ V/m}$$

We know the potential,

$$V(r) = -\int E \cdot dL$$

$$dL = dr\,\hat{a}_r$$

$$V(r) = -\int \frac{2r}{(r^2 + a^2)^2} dr$$

$$V(r) = \frac{1}{(r^2 + a^2)} + C$$

(i) $V = 0$ at infinity, i.e.,

$$V(\infty) = \frac{1}{\infty} + C = 0$$

$$C = 0$$

$$V(r) = \frac{1}{r^2 + a^2}$$

(ii) $V = 0$ at $r = 0$

$$V(0) = \frac{1}{0 + a^2} + C = 0$$

$$C = \frac{-1}{a^2}$$

$$V(r) = \frac{1}{r^2 + a^2} + \left(\frac{-1}{a^2}\right)$$

$$V(r) = \frac{1}{r^2 + a^2} - \frac{1}{a^2} = \frac{-r^2}{(r^2 + a^2)a^2}$$

$$V(r) = \frac{-r^2}{a^2(r^2 + a^2)}$$

(iii) $V = 100$ at $r = a$

$$V(a) = \frac{1}{a^2 + a^2} + C = 100$$

$$C = 100 - \frac{1}{2a^2}$$

$$V(r) = \frac{1}{r^2 + a^2} + 100 - \frac{1}{2a^2}$$

$$V(r) = \frac{a^2 - r^2}{2a^2(r^2 + a^2)} + 100$$

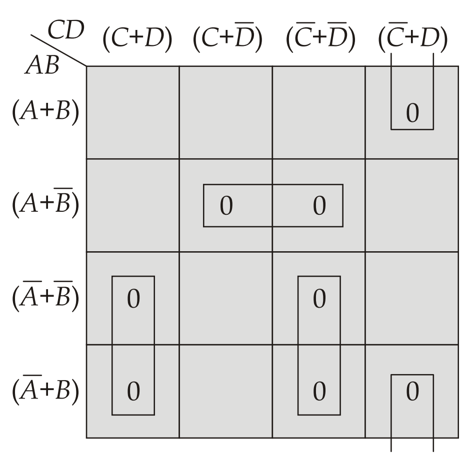
Q.1 (d) Solution:

Given : $F(A, B, C, D) = \Sigma m(0, 1, 3, 4, 6, 9, 13, 14)$

For POS expression :

$$F(A, B, C, D) = \pi M(2, 5, 7, 8, 10, 11, 12, 15)$$

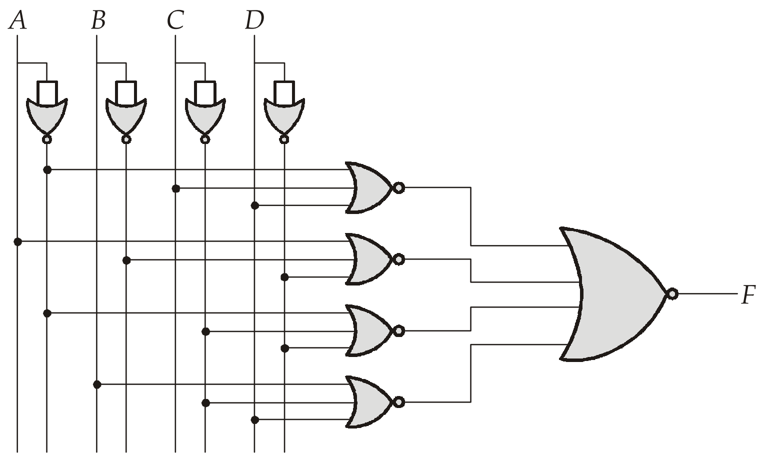
K-Map :



$$F(A, B, C, D) = (\bar{A} + C + D)(A + \bar{B} + \bar{D})(\bar{A} + \bar{C} + \bar{D})(B + \bar{C} + D)$$

$$\overline{F(A,B,C,D)} = \overline{(\bar{A} + C + D)(A + \bar{B} + \bar{D})(\bar{A} + \bar{C} + \bar{D})(B + \bar{C} + D)}$$

$$F(A, B, C, D) = \overline{\overline{(\bar{A} + C + D)} + \overline{(A + \bar{B} + \bar{D})} + \overline{(\bar{A} + \bar{C} + \bar{D})} + \overline{(B + \bar{C} + D)}}$$



Q.1 (e) Solution:

Given : $SN_qR \geq 44$ dB

We know,

$$SN_qR\ (\text{dB}) = 1.76 + 6.02n$$

$$1.76 + 6.02n \geq 44$$

$$n \geq \frac{44.24}{6.02}$$

$$n \geq 7.017$$

But n must be an integer.

$$n \simeq 8$$

Number of required levels,

$$L = 2^n = 2^8 = 256$$

The corresponding signal-to-quantizing noise,

$$SN_qR\ (\text{dB}) = 1.76 + 6.02n$$

$$= 1.76 + 6.02 \times 8 = 49.92$$

$$SN_qR\ (\text{dB}) = 49.92 \text{ dB}$$

Q.2 (a) Solution:

(i) Given : $H = 10^5 \rho^2 \hat{a}_\phi$ A/m

$$\rho = 5 \text{ mm}, V = 0.1 \text{ V}, L = 20 \text{ m}$$

From Maxwell's equation,

$$J = \nabla \times H$$

$$J = \frac{1}{\rho}\begin{bmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & \rho 10^5\rho^2 & 0 \end{bmatrix}$$

$$= \frac{1}{\rho}\left[0 + 0 + a_z\left(\frac{\partial}{\partial\rho}(10^5\rho^3)\right) - 0\right]$$

$$= \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}(10^5\rho^3)a_z$$

$$J = \frac{3\times 10^5\rho^2}{\rho}\hat{a}_z = 3\times 10^5\rho\hat{a}_z$$

Electric field,

$$\vec{E} = \frac{V}{L} = \frac{0.1}{20} = 5 \times 10^{-3} \text{ V/m}$$

Also, $$\vec{J} = \sigma\vec{E}$$

$$\sigma = \frac{\vec{J}}{\vec{E}}$$

$$\sigma = \frac{3\times 10^5\rho}{5\times 10^{-3}}$$

$$\sigma = \frac{3\times 10^5(5\times 10^{-3})}{5\times 10^{-3}}$$

$$\sigma = 3 \times 10^5 \text{ S/m}$$

(ii) The current in the wire,

$$I = \int_s \vec{J}\cdot ds$$

$$ds = \rho d\rho d\phi \hat{a}_z$$

$$I = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{5\text{mm}} 3\times 10^5\rho\cdot\rho d\rho d\phi$$

$$= 3\times 10^5\left[\frac{\rho^3}{3}\right]_0^{5\text{mm}} \times [\phi]_0^{2\pi}$$

$$I = 1 \times 10^5 \left[\rho^3\right]_0^{5\text{mm}} \times [\phi]_0^{2\pi}$$

$$= 1 \times 10^5[(5 \times 10^{-3})^3 - 0] \times 2\pi$$

$$I = 0.0785 \text{ A}$$

Resistance, $$R = \frac{V}{I}$$

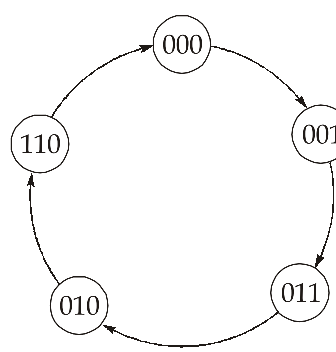
$$R = \frac{0.1}{0.0785} = 1.273\ \Omega$$

$$R = 1.273\ \Omega$$

Q.2 (b) Solution:

For Mod-5 gray up counter, we require 3 J-K flip flops.

State diagram for Mod-5 gray counter.



Excitation Table :

Present State			Next State			J-K Flip Flop					
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	0	1	0	×	0	×	1	×
0	0	1	0	1	1	0	×	1	×	×	0
0	1	1	0	1	0	0	×	×	0	×	1
0	1	0	1	1	0	1	×	×	0	0	×
1	1	0	0	0	0	×	1	×	1	0	×

Remaining states are taken as don't care.

K-Map for J_2 :

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	0	0	0	1
1	×	×	×	×

$J_2 = Q_1\overline{Q}_0$

K-Map for K_2 :

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	×	×	×	×
1	×	×	×	1

$K_2 = 1$

K-Map for J_1 :

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	0	1	×	×
1	×	×	×	×

$J_1 = Q_0$

K-Map for K_1 :

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	×	×	0	0
1	×	×	×	1

$K_1 = Q_2$

K-Map for J_0 :

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	1	×	×	0
1	×	×	×	0

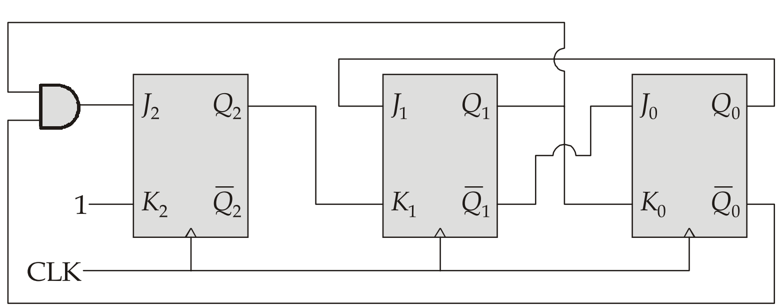
$J_0 = \overline{Q}_1$

K-Map for K_0 :

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	×	0	1	×
1	×	×	×	×

$K_0 = Q_1$

Circuit :



Q.2 (c) Solution:

(i) From figure, the signals v_1 and v_2 are

$$v_1 = k[x(t) + A\cos\omega_c t]$$

$$v_2 = [x(t) - A\cos\omega_c t]$$

Output,

$$v_{out} = av_1^2 - bv_2^2$$

$$v_{out} = a\{k[x(t) + A\cos\omega_c t]\}^2 - b[x(t) - A\cos\omega_c t]^2$$

$$= ak^2[x^2(t) + 2Ax(t)\cos\omega_c t + A^2\cos^2\omega_c t] - b[x^2(t) - 2Ax(t)\cos\omega_c t + A^2\cos^2\omega_c t]$$

$$= (ak^2 - b)[x^2(t) + A^2\cos^2\omega_c t] + 2A(ak^2 + b)x(t)\cos\omega_c t$$

To yield the suppressed carrier DSB signal coefficient of $x^2(t)$ and $A^2\cos^2\omega_c t$ should be zero.

i.e.,

$$ak^2 - b = 0$$

$$k = \sqrt{\frac{b}{a}}$$

(ii) We know,

Total current,

$$I_T = I_C\sqrt{1 + \frac{\mu^2}{2}}$$

When $\mu = 0\%$, $I_T = I_C$

When $\mu = 65\%$, $\mu = 0.65$

$$I_T = I_C\sqrt{1 + \frac{(0.65)^2}{2}}$$

$$I_T = 1.10057 I_C$$

% increase in antenna current

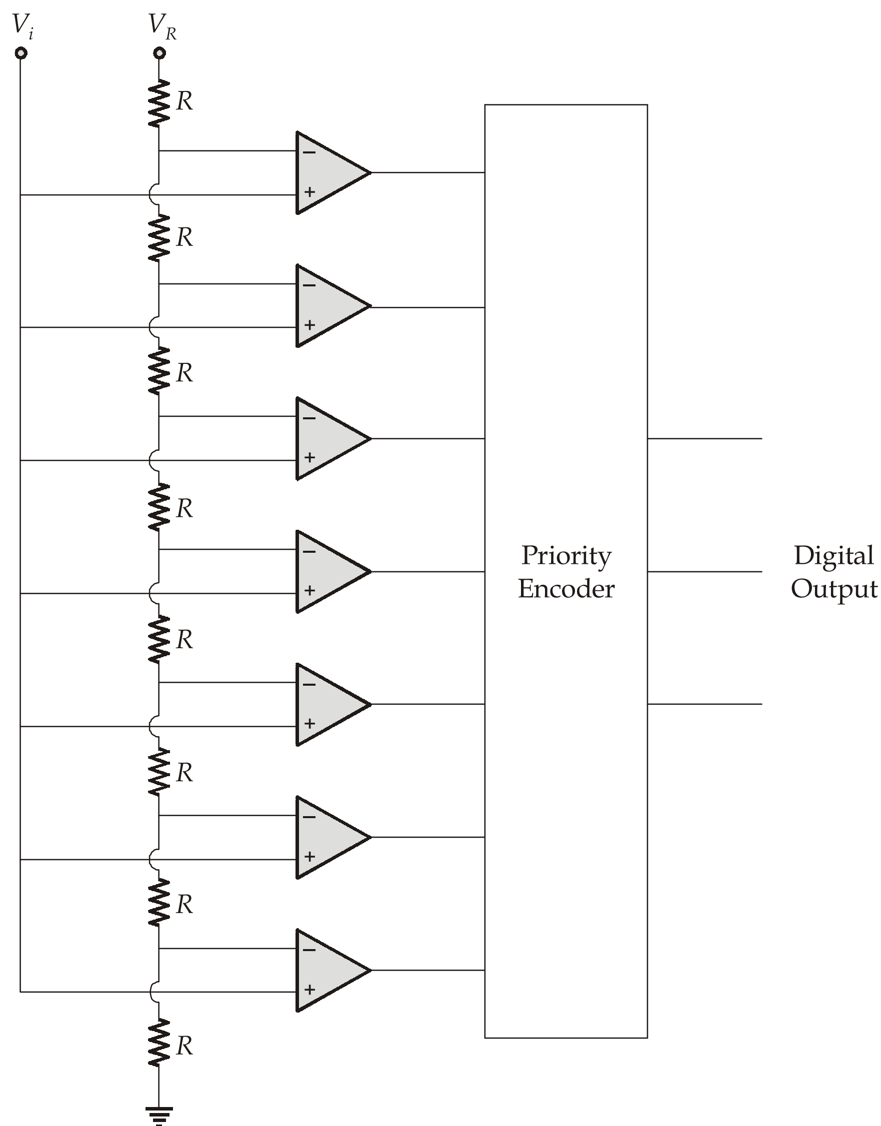
$$= \frac{1.10057 I_C - I_C}{I_C} \times 100\%$$

$$= (1.10057 - 1) \times 100$$

$$= 10.057\%$$

Q.3 (a) Solution:

The circuit of a 3-bit flash type ADC is shown below :



The 3-bit flash type ADC consists of a voltage divider network having 7 comparator and a priority encoder.

The working of 3-bit flash type ADC is as follows :

- The voltage divider network contains 8 equal resistors. A reference voltage V_R is applied across that entire network with respect to the ground. The voltage drop across each resistor from bottom-to-top with respect to ground will be the integer multiple of $\frac{V_R}{8}$.

- The external input voltage V_i is applied to the non-inverting terminals of all comparators. The voltage drop across each resistor from bottom-to-top is applied to the inverting terminal of the comparator.
- At a time, all the comparators compare the external input voltage with the voltage drops present at the respective other input terminals. That means the comparison operations take place by each comparator parallely.
- The output of the comparator will be '1' as long as V_i is greater than the voltage drop present at the respective other input terminal. Similarly, the output of comparator will be '0', when V_i is less than or equal to the voltage drop present at the respective other input terminal.
- All the outputs of comparators are connected as the inputs of priority encoder. This priority encoder produces a binary code (digital output), which is corresponding to the high priority input that has '1'.
- Therefore, the output of priority encoder is nothing but the binary equivalent (digital output) of external analog input voltage V_i.

Advantages :

1. It is the fastest type of ADC because the conversion is performed simultaneously through a set of comparators.
2. The construction is simple and easier.

Disadvantages :

1. It is not suitable for higher number of bits.
2. To convert the analog input voltage into digital signal of n-bit output, $2^n - 1$ comparators are required. Hence, making it more complex for higher number of bits.
3. It is costliest.

Q.3 (b) Solution:

(i) Given :

$$A = 0.12\text{ m}^2,\ d = 80\ \mu\text{m}$$

$$V = 12\text{ V and } W_E = 1\ \mu\text{J}$$

We know,

$$W_E = \frac{1}{2}CV^2$$

$$C = \frac{2\times1\times10^{-6}}{(12)^2}$$

$$C = 1.389\times10^{-8}\text{ F}$$

But, $$C = \frac{\in A}{d} = \frac{\in_o \in_r A}{Q}$$

$$1.389 \times 10^{-8} = \frac{8.854 \times 10^{-12} \times 0.12 \times \in_r}{80 \times 10^{-6}}$$

$$\in_r = 1.046$$

(ii) Given : $u_E = 100 \text{ J/m}^3, V = 200, d = 45 \text{ µm}$

We know, $$u_E = \frac{1}{2} \in E^2 = \frac{1}{2} \in_o \in_r \left(\frac{V}{d}\right)^2$$

$$100 = \frac{1}{2} \times 8.854 \times 10^{-12} \times \in_r \left(\frac{200}{45 \times 10^{-6}}\right)^2$$

$$\in_r = 1.1436$$

(iii) Given : $E = 200 \text{ kV/m}, \rho_s = 20 \text{ µc/m}^2$

We know, $$E = \frac{\rho_s}{\in} = \frac{D}{\in} = \frac{\rho_s}{\in_o \in_r}$$

$$\in_r = \frac{\rho_s}{\in_o E}$$

$$= \frac{20 \times 10^{-6}}{8.854 \times 10^{-12} \times 200 \times 10^3}$$

$$= 11.294$$

$$\in_r = 11.294$$

Q.3 (c) Solution:

(i) Given : Message signal $x(t) = \cos(6000\pi t)$

Sampling rate of PCM, $f_s = 8 \text{ kHz}$

Number of quantization level,

$$L = 64$$

Number of bits, $n = \log_2(L)$

$$n = \log_2(64) = 6$$

Signaling rate for PCM,

$$(R_s)_{PCM} = nf_s$$

$$= 6 \times 8 \times 10^3$$

$$= 48 \text{ kHz}$$

For delta modulation,

Step size, $\Delta = 31.25 \text{ mV}$

Message signal frequency,

$$f_m = 3000 \text{ Hz}$$

Amplitude of message signal,

$$A_m = 1 \text{ V}$$

To avoid slope overload,

$$\frac{\Delta}{T_s} \geq 2\pi f_m A$$

$$f_s \geq \frac{2\pi f_m A}{\Delta}$$

$$f_s \geq \frac{2\pi \times 3 \times 10^3 \times 1}{31.25 \times 10^{-3}}$$

$$f_s \geq 603.186 \text{ kHz}$$

Signaling rate for DM $\geq$ 603.18 kHz.

Comment : To transmit the same voice signal the Delta Modulation (DM) needs a very large signaling rate as compared to PCM.

(ii) Advantages :

1. Since, the delta modulation transmits only one bit for one sample, therefore, the signaling rate and transmission channel bandwidth is quite small as compared to PCM.
2. The transmitter and receiver implementation is very much simple for delta modulation. There is no analog-to-digital converter required in delta modulation.

Disadvantages :

The delta modulation has two major drawbacks :

1. Slope-overload distortion
2. Granular noise

Q.4 (a) Solution:

(i) Given : $N = 12, f_m = 25 \text{ kHz}$, Control bits, $a = 6$

Nyquist rate, $\text{NR} = 2f_m = 2 \times 25$

$$\text{NR} = 50 \text{ kHz}$$

Sampling frequency, $f_s = 3\text{ NR}$

$$f_s = 150\text{ kHz}$$

$$[Q_e]_{max} = 1\% A_m$$

But $$[Q_e] = \frac{\Delta}{2} = \frac{1}{2}\left[\frac{2A_m}{2^n}\right]$$

$$\frac{A_m}{2^n} \le \frac{A_m}{100}$$

$$2^n \ge 100$$

$$n = 7$$

Bit rate, $R_b = (nN + a)f_s$

$$= (7 \times 12 + 6) \times 150 \times 10^3$$

$$R_b = 13500 \times 10^3\text{ bps}$$

$$R_b = 13.5\text{ Mbps}$$

(ii) Given : $A_m = 5\text{ V}, f_m = 2\text{ kHz}, k_f = 40\text{ Hz/Volt}$

(a) Frequency deviation,

$$\Delta f = K_f A_m$$

$$= 40 \times 5$$

$$\Delta f = 200\text{ Hz}$$

(b) Modulation index,

$$\beta = \frac{\Delta f}{f_m} = \frac{200}{2 \times 10^3} = \frac{1}{10} = 0.1$$

$$\beta = 0.1$$

(c) Here, $\beta < 1$. Hence, it is a narrow-band FM.

Bandwidth of NBFM $= 2f_m$

$$= 2 \times 2 \times 10^3$$

$$BW = 4\text{ kHz}$$

Q.4 (b) Solution:

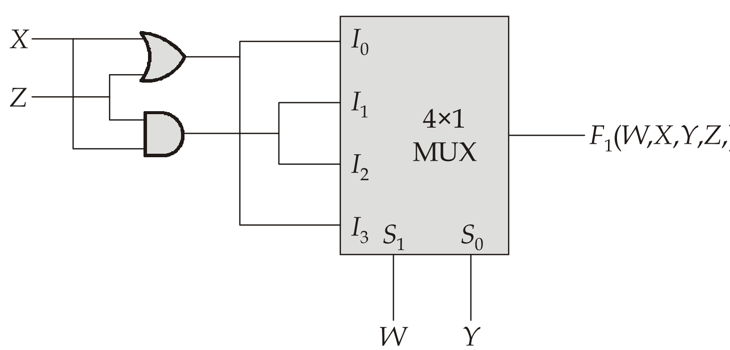
(i) $$F_1(W, X, Y, Z) = XZ + \overline{W}X\overline{Y} + \overline{W}\overline{Y}Z + WXY + WYZ$$

K-map :

$WX \backslash YZ$	00	01	11	10
00		1		
01	1	1	1	
11		1	1	1
10			1	

$$F_1(W, X, Y, Z) = \Sigma m(1, 4, 5, 7, 11, 13, 14, 15)$$

	I_0 $\overline{W}\overline{Y}$	I_1 $\overline{W}Y$	I_2 $W\overline{Y}$	I_3 WY
$\overline{X}\overline{Z}$	0	2	8	10
$\overline{X}Z$	①	3	9	⑪
$X\overline{Z}$	④	6	12	⑭
XZ	⑤	⑦	⑬	⑮
	$(X + Z)$	XZ	XZ	$X + Z$



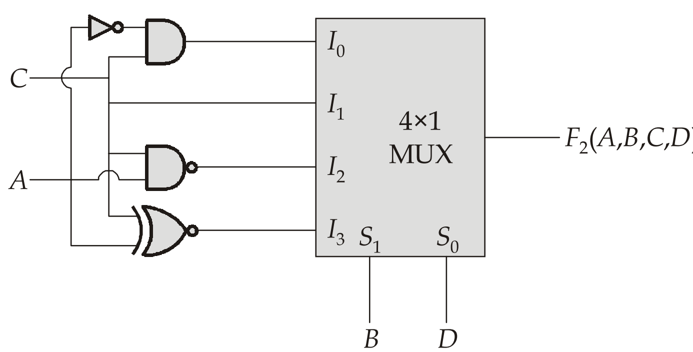
(ii) $$F_2(A, B, C, D) = \bar{A}B\bar{C} + B\bar{C}\bar{D} + ACD + \bar{A}\bar{B}C + \bar{A}C\bar{D}$$

K-map :

AB \ CD	00	01	11	10
00			1	1
01	1	1		1
11	1		1	
10			1	

$$F_2(A, B, C, D) = \Sigma m(2, 3, 4, 5, 6, 11, 12, 15)$$

	I_0 $\bar{B}\bar{D}$	I_1 $\bar{B}D$	I_2 $B\bar{D}$	I_3 BD
$\bar{A}\bar{C}$	0	1	(4)	(5)
$\bar{A}C$	(2)	(3)	(6)	7
$A\bar{C}$	8	9	(12)	13
AC	10	(11)	14	(15)
	$\bar{A}C$	C	$\bar{A}+\bar{C}$ $=\overline{AC}$	$A \odot C$



Q.4 (c) Solution:

Given : $$\vec{E}_1 = -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z \text{ V/m}$$

(i) The tangential components and the normal components of the electric field in region 1.

$$\vec{E}_{1t} = -30\hat{a}_x + 50\hat{a}_y \text{ V/m}$$

$$\vec{E}_{1n} = 70\hat{a}_z \text{ V/m}$$

Apply boundary conditions,

$$\vec{E}_{2t} = \vec{E}_{1t}$$

$$\vec{E}_{2t} = -30\hat{a}_x + 50\hat{a}_y$$

$$\vec{D}_{2t} = \epsilon_2 E_{2t}$$

$$\vec{D}_{2t} = 8.854 \times 10^{-12} \times 4[-30\hat{a}_x + 50\hat{a}_y]$$

$$\vec{D}_{2t} = [-1.062\hat{a}_x + 1.771\hat{a}_y] \times 10^{-9} \text{ C/m}^2$$

Also,

$$\vec{D}_{2n} = \vec{D}_{1n}$$

$$\epsilon_2 \vec{E}_{2n} = \epsilon_1 \vec{E}_{1n}$$

$$\vec{E}_{2n} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \vec{E}_{1n}$$

$$= \frac{2.5}{4} \times 70\hat{a}_z$$

$$\vec{E}_{2n} = 43.75\hat{a}_z$$

$$\vec{D}_{2n} = \epsilon_2 \vec{E}_{2n}$$

$$\vec{D}_{2n} = 8.854 \times 10^{-12} \times 4 \times 43.75\hat{a}_z$$

$$\vec{D}_{2n} = [1.55\hat{a}_z] \times 10^{-9} \text{C/m}^2$$

Electric flux density,

$$\vec{D}_2 = \vec{D}_{2t} + \vec{D}_{2n}$$

$$\vec{D}_2 = [-1.062\hat{a}_x + 1.771\hat{a}_y + 1.55\hat{a}_z] \text{ nC/m}^2$$

(ii) Polarization,

$$\vec{P}_2 = \epsilon_o \chi_{e2} \vec{E}_2$$

$$= \frac{\epsilon_o}{\epsilon_{r2}} \cdot \epsilon_{r2} \chi_{e2} \vec{E}_2$$

$$\vec{P}_2 = \frac{\chi_{e2}\vec{D}_2}{\epsilon_{r2}} = \frac{(\epsilon_{r2} - 1)}{\epsilon_{r2}} \cdot \vec{D}_2$$

$$\vec{P}_2 = \left(\frac{4-1}{4}\right) \cdot [-1.062\hat{a}_x + 1.771\hat{a}_y + 1.55\hat{a}_z] \text{ nC/m}^2$$

$$\vec{P}_2 = (-0.7965\hat{a}_x + 1.328\hat{a}_y + 1.1625\hat{a}_z) \text{ nC/m}^2$$

(iii) The angle between E_1 and normal

$$\overrightarrow{E_1 . a_z} = |E_1| \cos\theta_n$$

$$\cos\theta_n = \frac{\vec{E}_1 \cdot \hat{a}_z}{|E_1|}$$

$$= \frac{70}{\sqrt{(-30)^2 + (50)^2 + (70)^2}}$$

$$\cos\theta_n = 0.7683$$

$$\theta_n = 39.794°$$

Section B : Computer Fundamentals - 1 + Electrical and Electronic Measurements - 1 Power Electronics & Drives - 2 + Engineering Mathematics - 2

Q.5 (a) Solution:

Assume that inductor current is continuous.

When the inductor current is continuous, $I_{L(\text{avg})} > \frac{\Delta I_L}{2}$.

When switch is ON, then

$$V_L = V_{\text{in}} - V_o$$

$$L\frac{di_L}{dt} = V_{\text{in}} - V_o$$

$$\int_{I_{L\min}}^{I_{L\max}} di_L = \frac{1}{L}\int_0^{DT} (V_{\text{in}} - V_o)dt$$

$$\Delta I_L = \frac{1 \cdot D(1-D)V_{\text{in}}}{Lf} = \frac{0.5 \times 0.5 \times 100}{0.5 \times 10}$$

$$\Delta I_L = 5 \text{ A}$$

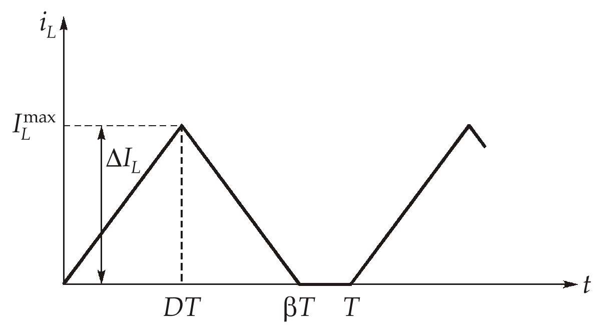
Also, we know that average capacitor current is zero, therefore,

$$I_{L(avg)} = I_o$$
$$I_{L(avg)} = 2 \text{ A}$$

But $$I_{L(avg)} < \frac{\Delta I_L}{2}$$

Therefore, converter is operating in discontinuous conduction mode.

Inductor Current Waveform :



$\therefore$ $$I_L^{max} = \left(\frac{V_{in} - V_o}{L}\right) \cdot DT \quad ...(1)$$

For $DT < t < \beta T$: $$i_L(t) = \frac{-V_o(t - DT)}{L} + I_L^{max}$$

At $t = \beta T$, $$i_L(t) = 0$$

$\therefore$ $$I_L^{max} = \frac{V_o(\beta T - DT)}{L} \quad ...(2)$$

Equating eqn. (1) and (2)

$$\left(\frac{V_{in} - V_o}{L}\right) DT = \frac{V_o(\beta T - DT)}{L}$$

$\therefore$ $$V_o = V_{in}\left(\frac{D}{\beta}\right) \quad ...(3)$$

where DCM represents Discontinuous Conduction Mode

$\because \beta < 1 \Rightarrow$ $$V_o^{DCM} > V_o^{CCM}$$

and CCM represents Continuous Conduction Mode

(i) The average inductor current $I_{L(avg)}$ is given by

$$I_{L(avg)} = \frac{1}{T}\int_0^T i_L dt = \frac{1}{T}\int_0^{\beta T} i_L dt$$

$$I_{L(avg)} = \frac{1}{T} \times \frac{1}{2} \beta T \cdot I_L^{max} \quad ...(4)$$

From eqn. (1), (3), (4)

$$2 = 0.5 \times \beta \times I_L^{max} = 0.5 \times \beta \times 100 \times \left(1 - \frac{0.5}{\beta}\right) \times \frac{0.5}{0.5 \times 10}$$

$$\Rightarrow \quad \beta = 0.9$$

$$\therefore \quad V_o = 100 \times \frac{0.5}{0.9} = 55.56 \text{ Volts}$$

(ii) The ON period of the switch S = DT = 0.5 msec. For discontinuous conduction, the diode conducts for βT-DT. Therefore, the diode (D) conducts for 0.4 msec.

Q.5 (b) Solution:

IEEE standard for floating-point arithmetic is a technical standard for floating-point computation.

The standard addressed many problems found in the diverse floating point implementations that made them difficult to use reliably and reduce their portability.

IEEE standard 754 floating point is the most common representation today for real numbers on computers, including Intel-based PCs, MACs and most common unix platforms.

There are several ways to represent floating point numbers IEEE 754 is the most efficient in the most cases. IEEE 754 has 3 basic components.

1. **The sign of Mantissa :** Here, 0 represents a positive number while 1 represents a negative number.
2. **The Biased Exponent :** The exponent field needs to represent both positive and negative exponents. A bias is added to the actual exponent in order to get the stored exponent.
3. **The Normalised Mantissa :** The Mantissa is a part of number in scientific notation or a floating point number, consisting of its significant digits. Here, we have only two digits, i.e., 0 and 1. So, a normalised Mantissa is one with only one to the left of the decimal.

IEEE 754 numbers are divided into two based on the three components. Single precision and double precision.

Single precision IEEE 754 floating-point standard :

32-bits

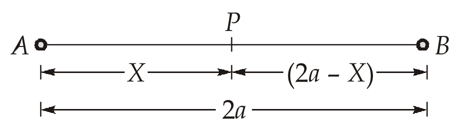
Sign	Exponent	Mantissa
1-bit	8-bits	23-bits

Double Precision : IEEE 754 floating-point integer

64-bits

Sign	Exponent	Mantissa
1-bit	11-bits	52-bits

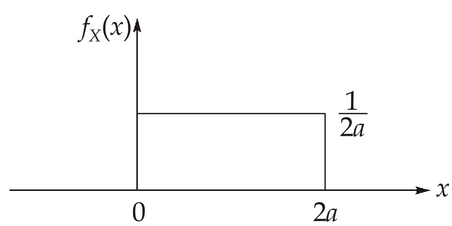
Q.5 (c) Solution:



Let $AP = X \Rightarrow PB = (2a - X)$

Since all positions of the point P are equally likely, ($X = AP$) is uniformly distributed over $(0, 2a)$.

Pdf of X :



Now, $$P\left[AP \times PB > \frac{a^2}{2}\right] = P\left[X(2a - X) > \frac{a^2}{2}\right]$$

$$= P[2X^2 - 4aX + a^2 < 0]$$

Now, factors of $2X^2 - 4aX + a^2$ are : $X - \left(1 - \frac{1}{\sqrt{2}}\right)a$ and $X - \left(1 + \frac{1}{\sqrt{2}}\right)a$.

Therefore, $$P[AB \times PB > a^2] = P[2X^2 - 4aX + a^2 < 0]$$

$$= P\left[\left\{X - \left(1 - \frac{1}{\sqrt{2}}\right)a\right\} \cdot \left\{X - \left(1 + \frac{1}{\sqrt{2}}\right)a\right\} < 0\right]$$

$$= P\left[\left(1 - \frac{1}{\sqrt{2}}\right) \cdot a < X < \left(1 + \frac{1}{\sqrt{2}}\right)a\right]$$

Now, $$P\left[AP \times PB > \frac{a^2}{2}\right] = \int_{\left(1-\frac{1}{\sqrt{2}}\right)a}^{\left(1+\frac{1}{\sqrt{2}}\right)a} f_X(x)\cdot dx \qquad \left[\because P(a < X < b) = \int_a^b f_X(x)\cdot dx\right]$$

Therefore, $$P\left[AP \times PB > \frac{a^2}{2}\right] = \frac{1}{2a} x\Bigg|_{\left(1-\frac{1}{\sqrt{2}}\right)a}^{\left(1+\frac{1}{\sqrt{2}}\right)a} = \frac{\sqrt{2}a}{2a} = \frac{1}{\sqrt{2}}$$

$$P\left[AP \times PB > \frac{a^2}{2}\right] = \frac{1}{\sqrt{2}}$$

Q.5 (d) Solution:

$$v(t) = At \text{ where } A \text{ is slope of ramp.}$$

Slope, $$A = \frac{100-0}{2-0} = 50$$

∴ $$v(t) = 50t;\ 0 \le t \le T$$

∴ $$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T v^2(t)\cdot dt\right]^{1/2}$$

$$= \left[\frac{1}{2}\int_0^T 50t^2 \cdot dt\right]^{1/2}$$

$$= \left[\frac{2500t^3}{6}\Bigg|_0^2\right]^{1/2}$$

$$= \sqrt{\frac{2500 \times 8}{6}} = 57.735 \text{ Volts}$$

While $$V_{av} = \frac{1}{T}\int_0^T v(t)\cdot dt$$

$$= \frac{1}{2}\int_0^2 50t \cdot dt = \frac{1}{2} \times \frac{50t^2}{2}\Bigg|_0^2$$

$$V_{av} = 50 \text{ Volts}$$

$\therefore$ Form factor, $K_f = \frac{V_{rms}}{V_{av}} = \frac{57.73}{50} = 1.1547$

For sine wave, $K_f = 1.11$

Hence, the meter will read less by a factor.

$$\frac{K_f(\text{sine})}{K_f(\text{sawtooth})} = \frac{1.11}{1.1547} = 0.9613$$

$$\therefore \quad \%\text{ error} = \frac{\text{Measured} - \text{True}}{\text{True}} \times 100\%$$

$$= \frac{0.9613 - 1}{1} \times 100\%$$

$$\%\text{ error} = -3.87\%$$

Q.5 (e) Solution:

(i) For maximum field current,

$$\alpha_f = 0°$$

Now, $V_f = \frac{3V_{ml}}{\pi} = \frac{3 \times 208\sqrt{2}}{\pi} = 280.90$ Volts

Field current, $I_f = \frac{V_f}{R_f} = \frac{280.90}{145} = 1.9372$ A

Now, we know that

$$\omega = \frac{2\pi N}{60}$$

$$\Rightarrow \quad \omega = \frac{2\pi \times 900}{60}$$

$$\Rightarrow \quad \omega = 94.24 \text{ rad/s}$$

and Armature current, $I_a = \frac{T_a}{K_v I_f} = \frac{116}{1.2 \times 1.9372} = 49.90$ A

Now, $E_b = K_v I_f . w = 1.2 \times 1.9372 \times 94.24 = 219.09$ V

and $V_a = E_b + I_a R_a = 219.09 + 49.90 \times 0.25 = 231.568$ V

$$V_a = 231.568 = \frac{3V_{ml}}{\pi} \cos\alpha_a$$

$$231.568 = \frac{3 \times 208\sqrt{2}}{\pi} \cos\alpha_a \Rightarrow \alpha_a = 34.474°$$

(ii) $\alpha_a = 0$ and

$$V_a = \frac{3 \times 208\sqrt{2}}{\pi} = 280.90 \text{ V}$$

$$E_b = 280.90 - 49.90 \times 0.25 = 268.423 \text{ V}$$

And the speed,

$$\omega = \frac{E_g}{K_v I_f} = \frac{268.423}{1.2 \times 1.9372} = 115.47 \text{ rad/s or } 1102.657 \text{ rpm}$$

(iii)

$$\omega = \frac{1800\pi}{30} = 188.5 \text{ rad/sec}$$

$$E_b = 268.423 \text{ V} = 1.2 \times 188.5 \times I_f$$

$\Rightarrow$

$$I_f = \frac{268.423}{1.2 \times 188.50} = 1.1867 \text{ A}$$

$$V_f = I_f R_f = 172.066 \text{ Volts}$$

Now,

$$V_f = \frac{3V_{ml}}{\pi} \cos\alpha$$

$$172.066 = \frac{3 \times \sqrt{2} \times 208}{\pi} \cos\alpha_f$$

$$\alpha_f = \cos^{-1}\left(\frac{171.50\pi}{3 \times 208\sqrt{2}}\right)$$

$$\alpha_f = 52.225°$$

Q.6 (a) Solution:

(i) Given equation is

$$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \sin(\ln x)$$

$$(x^2D^2 - 4xD + 2)y = \sin(\ln x) \qquad ...(1)$$

Put $x = e^z \Rightarrow$

$$z = \ln x$$

$$x.Dx = D' \qquad ...(2)$$

$$x^2D^2 = D'(D' - 1) \qquad ...(3)$$

Substituting (2) and (3) in eqn. (1), we get

$$[D'(D' - 1) + (4D' + 2)]y = \sin z \qquad ...(4)$$

The auxiliary equation is

$$m^2 + 3m + 2 = 0$$
$$(m+1)(m+2) = 0$$
$$m = -1, -2$$

Therefore, C.F., $y_{CF} = Ae^{-z} + Be^{-2z}$

Now, calculation of P.I.

$$y_{PI} = \frac{1}{D'^2 + 3D' + 2}\sin z = \frac{1}{-1 + 3D' + 2}\sin z$$
$$= \frac{1}{3D'+1}\sin z = \frac{3D'-1}{9D'^2 - 1}\sin z$$
$$= \frac{(3D'-1)\sin z}{9(-1)-1} \quad (\because D'^2 \to -1)$$
$$= \frac{3D'(\sin z) - \sin z}{-10} = \frac{3\cos z - \sin z}{-10}$$

Now, the solution of equation (4),

$$Y = Y_{CF} + Y_{PI}$$
$$Y = Ae^{-z} + Be^{-2z} + \frac{3\cos z - \sin z}{-10}$$

Substitute, $z = \ln x$

$$x = e^z$$
$$Y = Ae^{-\ln x} + Be^{-2\ln x} - \frac{3\cos(\ln x) - \sin(\ln x)}{10}$$
$$Y = \frac{A}{x} + \frac{B}{x^2} - \frac{3\cos(\ln x) - \sin(\ln x)}{10}$$

(ii) The poles of $f(z) = \dfrac{z-3}{z^2 + 2z + 5}$

are given by $z^2 + 2z + 5 = 0$

$$z = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

(a) Here only one pole $z = 1 + 2i$ lies inside the circle $C : |z + 1 - i| = z$. Therefore, $f(z)$ is analytic within C except at this pole.

$$\therefore \quad \text{Res}\, f(-1+2i) = \lim_{z \to -1+2i} [\{z - (-1+2i)\} f(z)]$$

$$= \lim_{z \to -1+2i} \frac{(z+1-2i)(z-3)}{z^2+2z+5}$$

$$\lim_{z \to -1+2i} \frac{z-3}{z+1+2i} = \frac{-4+2i}{4i} = i + \frac{1}{2}$$

Hence, by residue theorem,

$$\oint_C f(z)dz = 2\pi i \times \text{sum of residues} = \pi(-2+i)$$

(b) Here only the pole $z = -1 - 2i$ lies inside the circle $C : |z + 1 + i| = 2$.

Therefore, $f(z)$ is analytic within C except at this pole.

$$\therefore \quad \text{Res } f(-1-2i) = \lim_{z \to -1-2i} \frac{(z+1+2i)(z-3)}{z^2+2z+5}$$

$$= \lim_{z \to -1-2i} \frac{z-3}{z+1-2i} = \frac{-4-2i}{-4i} = \frac{1}{2} - i$$

Hence, by residue theorem,

$$\oint_C f(z) \cdot dz = 2\pi i \times \text{Sum of residue}$$

$$= 2\pi i \times \left(\frac{1}{2} - i\right) = \pi(2+i)$$

Q.6 (b) Solution:

(i) By voltage divider rule, the true voltage across R_b.

$$V = \frac{R_b}{R_a + R_b} \times 25$$

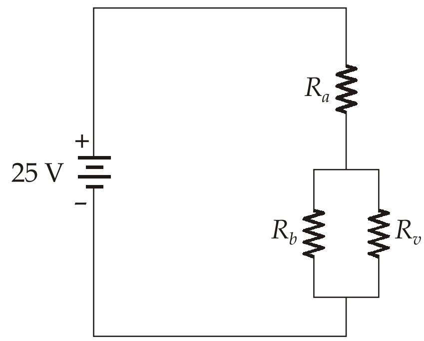
$$= \frac{1}{1+5} \times 25 = 4.167 \text{ V}$$

(ii) Consider first voltmeter with $S = 1 \text{ k}\Omega/\text{V}$

$$\therefore \quad R_v = S \times V_{range} = 1 \times 5$$

$$= 5 \text{ k}\Omega$$

Thus, circuit becomes



$$R_{eq} = R_b \parallel R_v$$

$$= \frac{1 \times 5}{6} = 0.833 \text{ k}\Omega$$

Hence, the voltage reading is

$$V_1 = \frac{R_{eq}}{R_{eq} + R_a} \times 25$$

$$= \frac{0.833}{(0.833 + 5)} \times 25$$

$$V_1 = 3.571 \text{ Volt}$$

(iii) Consider the second voltmeter with $S = 20$ kΩ/V

$$R_V = S \times V_{range}$$

$$R_V = 20 \times 5 = 100 \text{ k}\Omega$$

$\therefore$ $$R_{eq} = R_b \parallel R_v = \frac{100 \times 1}{101} = 0.99 \text{ k}\Omega$$

Hence, the voltage reading is

$$V_2 = \frac{R_{eq}}{R_a + R_{eq}} \times 25$$

$$= \frac{0.99}{0.99 + 5} \times 25$$

$$V_2 = 4.132 \text{ Volt}$$

(iv) The percentage error can be calculated as :

$$\% \text{ error in voltmeter-1} = \frac{\text{Measured} - \text{True}}{\text{True}} \times 100$$

$$= \frac{3.571 - 4.167}{4.167} \times 100$$

$$= -14.30\%$$

$$\text{\% error in voltmeter-2} = \frac{4.132 - 4.167}{4.167} \times 100$$

$$= -0.84\%$$

(v) Percentage accuracy can be obtained as :

% accuracy for voltmeter-1 = (100 – 14.3)% = 85.7%

% accuracy for voltmeter-2 = (100 – 0.84)% = 99.16%

Thus, voltmeter-2 is 99.16% accurate while voltmeter-1 is 85.7% accurate.

Q.6 (c) Solution:

As this voltage waveform has quarter wave symmetry,

$$a_n = 0, a_o = 0$$

$$b_n = \frac{4V_s}{\pi}\left[\int_0^{\alpha_1} \sin(n\omega t)d(\omega t) - \int_{\alpha_1}^{\alpha_2} \sin(n\omega t)d(\omega t) + \int_{\alpha_2}^{\pi/2} \sin(n\omega t)d(\omega t)\right]$$

$$= \frac{4V_s}{\pi}\left[\frac{(1-\cos n\alpha_1)}{n} - \frac{(\cos n\alpha_1 - \cos n\alpha_2)}{n} + \frac{\cos n\alpha_2}{n}\right]$$

$$b_n = \frac{4V_s}{n\pi}[1 - 2\cos n\alpha_1 + 2\cos n\alpha_2]$$

Therefore,

$$b_1 = \frac{4 \times 200}{\pi}[1 - 2\cos(23.62°) + 2\cos(33.30°)]$$

$$b_1 = 213.692 \text{ V}$$

$$b_3 = 0; \quad b_5 = 0$$

$$b_7 = \frac{4 \times 200}{7\pi}[1 - 2\cos(7 \times 23.62°) + 2\cos(7 \times 33.30°)]$$

$$b_7 = 63.08 \text{ V}$$

$$b_9 = \frac{4 \times 200}{9\pi}[1 - 2\cos(9 \times 23.62°) + 2\cos(9 \times 33.30°)]$$

$$b_9 = 104.015 \text{ Volts}$$

$$b_{11} = \frac{4 \times 200}{11\pi}[1 - 2\cos(11 \times 23.62°) + 2\cos(11 \times 33.30°)]$$

$$b_{11} = 77.35 \text{ Volts}$$

Therefore, output voltage expression upto 11th harmonic is

$$v_o(t) = 213.70 \sin(\omega_o t) + 63.08 \sin(7\omega_o t) + 104.015 \sin(9\omega_o t) + 77.35 \sin(11\omega_o t) \text{ Volts}$$

Output current expression can be given as

$$\vec{Z}_1 = R + j\omega_o L = 10 + j314.15 \times 20 \times 10^{-3} = 11.81\angle 32.14° \ \Omega$$

$$\vec{Z}_7 = 10 + j14\pi = 45.10\angle 77.20° \ \Omega$$

$$\vec{Z}_9 = 10 + j18\pi = 57.42\angle 80° \ \Omega$$

$$\vec{Z}_{11} = 10 + j22\pi = 69.83\angle 81.76° \ \Omega$$

$$i_o(t) = \frac{213.70}{11.81}\sin(\omega_o t - 32.14°) + \frac{63.08}{45.10}\sin(7\omega_o t - 77.80°) + \frac{104.015}{57.42}\sin(9\omega_o t - 80°) + \frac{77.35}{69.83}\sin(11\omega_o t - 81.76°)$$

$$i_o(t) = 18.09 \sin(\omega_o t - 32.14°) + 1.40 \sin(7\omega_o t) + 1.81 \sin(9\omega_o t - 80°) + 1.107 \sin(11\omega_o t - 81.76°) \text{ A}$$

$$I_{o(\text{rms})} = \frac{\sqrt{18.09^2 + 1.40^2 + 1.81^2 + 1.107^2}}{\sqrt{2}}$$

$$I_{o(\text{rms})} = 12.917 \text{ A}$$

Power delivered to load, $P_L = I^2_{o(\text{rms})} \times R$

$$P_L = (12.917)^2 \times 10$$

$$P_L = 1668.489 \text{ kW}$$

$\Rightarrow$ Distortion factor, $g = \frac{I_{o1}}{I_{or}} = \frac{12.791}{12.917} = 0.99$

As $I_{o1} = \frac{18.09}{\sqrt{2}}$

Total harmonic distortion in output current is,

$$\text{T.H.D.}_I = \sqrt{\frac{1}{g^2} - 1} = \sqrt{\left(\frac{1}{0.99}\right)^2 - 1}$$

$$\text{T.H.D.}_I = 0.1424 \text{ or } 14.24\%$$

Q.7 (a) Solution:

(i) Capacity of main memory = 256 MB

Capacity of cache memory = 1 MB

Block size = 128 bytes

A set contain 8 blocks.

Since the address space of processor is 256 MB.

The processor will generate address of 28-bits to access a byte (word).

The number of blocks contained by main memory = $\frac{256 \text{ MB}}{128 \text{ B}} = 2^{21}$

Therefore, number of bits required to specify one block in main memory = 21.

Since the block size is 128 bytes.

The number of bits required access each word (byte) = 7.

For associative, the address format :

Tag	Word
21	7

The number of blocks contained by cache memory = $\frac{1 \text{ MB}}{128 \text{ B}} = 2^{13}$.

Therefore, the number of bits required to specify one block in cache memory = 13.

For direct cache, the address format is

Tag	Block	Word
8	13	7

(Block + Word = Index)

(ii) Cache access time, t_c = 50 nsec

Main memory access time, t_m = 500 nsec

Probability of read, p_r = 0.8

Hit ratio for read access, h_r = 0.9

Hit ratio for write access, h_w = 1 ...(by default 1)

Writing scheme : Write through

(a) Considering only memory read cycle

The average access time

$$t_{av,r} = h_r \times t_c + (1 - h_r) \times t_m$$
$$= 0.9 \times 50 + (1 - 0.9) \times 500$$
$$= 95 \text{ nsec}$$

(b) For both read and write cycle,

$$\text{Average access time} = p_r \times t_{av,r} + (1 - p_r) \times t_m$$

In write-through method, access time for write cycle will be main memory access time.

$$= 0.8 \times 95 + (1 - 0.8) \times 500$$
$$= 176 \text{ nsec}$$

Q.7 (b) Solution:

(i) Synchronous speed, $\omega_{sm} = \dfrac{4\pi f}{P} = 50\pi$ rad/sec

and let $K = \dfrac{f}{f_s} = \dfrac{f}{50}$

Breakdown torque can be given as, $T_{\max}$:

$$T_{\max} = \frac{3}{2K\omega_{sm}} \times \frac{K^2V^2}{R_s + \sqrt{R_s^2 + K^2(X_s + X_r')^2}}$$

$$= \frac{3}{2\omega_{sm}} \times \frac{V^2}{\left(\frac{R_s}{K}\right) + \sqrt{\left(\frac{R_s}{K}\right)^2 + (X_s + X_r')^2}}$$

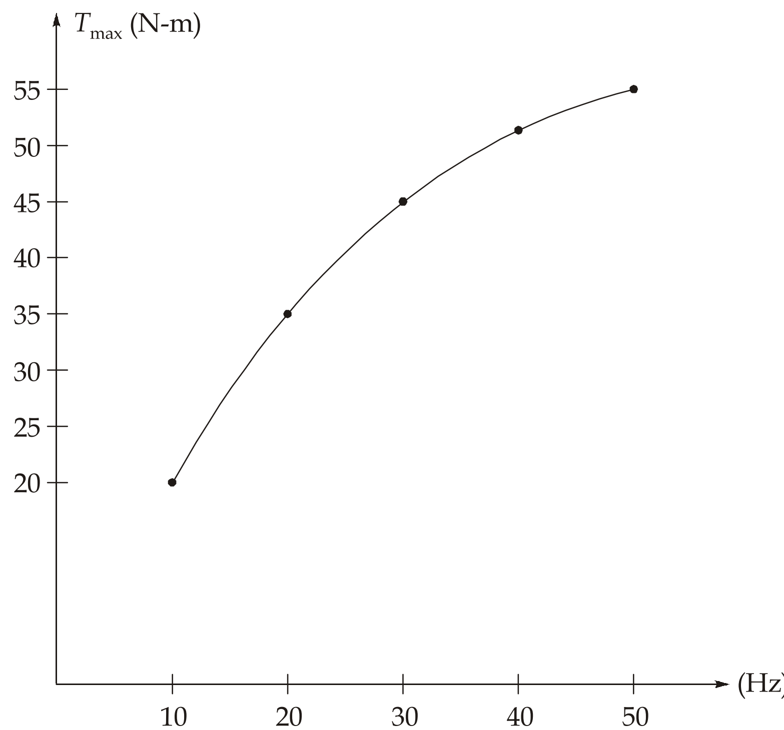
On substituting the respective values

$$= \frac{3}{100\pi} \times \frac{\left(\frac{400}{\sqrt{3}}\right)^2}{\left(\frac{2}{K}\right) + \sqrt{\left(\frac{2}{K}\right)^2 + 49}} \quad \text{...(1)}$$

Now, putting different values of f in between the range 10 to 50 Hz in eqn. (1)

K	1	0.8	0.6	0.4	0.2
f (Hz)	50	40	30	20	10
$T_{\max}$ (N-m)	54.88	51.27	45.94	37.44	22.93

Plot between T_{max} and f:



(ii) Since the minimum frequency available is 10 Hz, so motor will have to started at 10 Hz. So starting torque is

$$T_{st} = \frac{3}{\omega_{sm}} \times \left[\frac{V_m^2 \times R_r'}{(R_s + R_r')^2 + (X_s + X_r')^2}\right] \quad \text{...(1)}$$

$$T_{st} = \frac{3}{50\pi} \times \frac{\left(\frac{400}{\sqrt{3}}\right)^2 \times 3}{(5)^2 + (7)^2} \quad \text{...(at 50 Hz)}$$

$$T_{st} = 41.29 \text{ N-m}$$

Starting current at 50 Hz,

$$I_{st} = \frac{\frac{400}{\sqrt{3}}}{\sqrt{5^2 + 7^2}} = 26.84 \text{ A}$$

With variable frequency central and constant (V/f) ratio. Starting torque at frequency k times the rated

$$T'_{st} = \frac{3}{\omega_{sm}} \times \left[\frac{V_{ph}^2 \times \left(\frac{R'_r}{k}\right)}{\left(\frac{R_s + R'_r}{k}\right)^2 + (X_s + X'_r)^2}\right]$$

For $f = 10$ Hz, $k = 0.2$

$$T'_{st} = \frac{3}{50\pi} \times \frac{\left(\frac{400}{\sqrt{3}}\right)^2 \times \left(\frac{3}{0.2}\right)}{\left(\frac{5}{0.2}\right)^2 + (7)^2} = 22.67 \text{ N-m}$$

Starting current, $$I'_{st} = \frac{\frac{400}{\sqrt{3}}}{\sqrt{\left(\frac{5}{0.2}\right)^2 + 7^2}} = 8.895 \text{ A}$$

Now, $$\frac{T'_{st}(10 \text{ Hz})}{T_{st}(50 \text{ Hz})} = \frac{22.67}{41.29} = 0.549$$

and $$\frac{I'_{st}(10 \text{ Hz})}{I_{st}(50 \text{ Hz})} = \frac{8.895}{26.84} = 0.3314$$

Therefore, as compared to start at frequency, the ratio of (Torque/Current) has increased from 1.54 to 2.55.

Q.7 (c) Solution:

(i) The deflection torque varies as square of the current.

$\therefore$ $$T_d = k_d I^2$$

(a) Spring controlled

$$T_c = k\theta$$

$$T_c = T_d, \text{ i.e., } k\theta = k_d I^2$$

$\therefore$ $$\theta = \frac{k_d}{k} I^2 = k_1 I^2$$

$$90° = k_1 \times (5)^2 \Rightarrow k_1 = 3.6° \text{ per ampere}$$

$\therefore$ $$\theta = k_1 I^2 = 3.6 \times (10)^2 = 360°$$ ...for $I = 10$ A

(b) Gravity controlled

$$T_c = k_g \sin\theta$$

$$T_c = T_d \Rightarrow k_g \sin\theta = k_d I^2$$

$$\sin\theta = \frac{k_d}{k_g} I^2 = k_2 I^2$$

$$\sin(90°) = k_2 \times 5^2 \Rightarrow k_2 = \frac{1}{25}$$

$$\therefore \quad \sin\theta = k_2 I^2 = \frac{1}{25}\times(10)^2 = 4$$

$$\theta = \sin^{-1}(4)$$

But this is mathematically undefined. Thus, for $I = 10$ A, with gravity control, the instrument can not achieve steady state and may get damaged.

(ii) Power consumed by load,

$$P_T = VI\cos\phi = 100 \times 10 \times 0.2 = 200 \text{ W}$$

$$\cos\phi = 0.2 \Rightarrow \phi = \cos^{-1}(0.2) = 78.46°$$

$$R_p = 3000\ \Omega,\ L = 30 \text{ mH},\ X_L = 2\pi \times 50 \times 30 \times 10^{-3} = 9.42\ \Omega$$

$$\beta = \tan^{-1}\left(\frac{X_L}{R_P}\right) = \tan^{-1}\left(\frac{9.42}{3000}\right) = 0.00313 \text{ rad}$$

Consider the pressure coil connected on load side :

$$\text{Actual wattmeter reading} = (1 + \tan\phi \,.\, \tan\beta)P_T$$

$$= (1 + \tan(78.46°) \,.\, \tan(0.00313 \text{ rad})) \times 200$$

$$= 203.06 \text{ Watt}$$

$$\text{Power loss in pressure coil} = \frac{V^2}{R_P} = \frac{(100)^2}{3000} = 3.33 \text{ W}$$

$$\text{Total wattmeter reading} = 203.06 + 3.33 = 206.39 \text{ W}$$

$$\%\text{ error} = \frac{P_W - P_T}{P_T}\times 100\%$$

$$= \frac{206.39 - 200}{200}\times 100\%$$

$$= 3.196\%$$

Now consider that pressure coil is connected on supply side :

$$\text{Total power} = P_T + I^2R_C$$

$$= 200 + 10^2 \times 0.1 = 210 \text{ W}$$

Impedance of load, $Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$

Resistance of load, $R_L = Z\cos\phi = 10 \times 0.2 = 2\ \Omega$

Reactance of load, $X_L = Z\sin\phi = 10 \times 0.979 = 9.798\ \Omega$

The current acts as a load,

$$\text{Total resistance} = 2 + 0.1 = 2.1\ \Omega$$

$$\text{Total reactance} = 9.798\ \Omega$$

Total impedance of current coil

$$= \sqrt{(2.1)^2 + (9.798)^2} = 10.02\ \Omega$$

$$\text{Total power factor of load} = \frac{R_T}{Z_T} = \frac{2.1}{10.02} = 0.2096$$

$$\phi = \cos^{-1}(0.2096) = 77.90°$$

$$\tan\phi = 4.665$$

$$\text{Reading of wattmeter, } P_w = (1 + \tan\phi \, . \, \tan\beta) \times P_T$$

$$= (1 + 4.665 \times 3.13 \times 10^{-3}) \times 210 = 213.06\ \text{W}$$

$$\%\ \text{error} = \frac{213.06 - 200}{200} \times 100$$

$$\%\ \text{error} = 6.53\%$$

Q.8 (a) Solution:

(i) (a)

$$\omega_o = \frac{1}{\sqrt{L_r C_r}} = \frac{1}{\sqrt{10^{-6} \times 0.047 \times 10^{-6}}}$$

$$\omega_o = 4.61 \times 10^6\ \text{rad/sec}$$

$$z_o = \sqrt{\frac{L_r}{C_r}} = \sqrt{\frac{10^{-6}}{0.047 \times 10^{-6}}} = 4.61\ \Omega$$

At $t = t_1$, $V_x = 0$

So diode D_s turns ON.

$$t_1 = \frac{V_s C_r}{I_o} = \frac{20 \times 0.047 \times 10^{-6}}{5} = 0.188\ \mu\text{sec}$$

$$t_2 = \frac{1}{\omega_o}\left[\sin^{-1}\left(\frac{V_s}{I_o Z_o}\right) + \pi\right]$$

$$= \frac{1}{4.61 \times 10^6}\left[\sin^{-1}\left(\frac{20}{5 \times 4.61}\right) + \pi\right]$$

$$t_2 = 0.909\ \mu\text{sec}$$

$$T = t_1 + t_2 = 1.10\ \mu\text{sec}$$

At $t = t_3$, diode D_1 turns ON.

$$t_3 = \left(\frac{L_r \cdot I_o}{V_s}\right)(1 - \cos\omega_o(t_2 - t_1)) + t_2$$

$$= \left(\frac{10^{-6} \times 5}{20}\right) \times [1 - \cos(4.61 \times 10^{6}(0.909 - 0.188) \times 10^{-6})] + 1.10$$

$$t_3 = 1.596 \text{ μsec}$$

So, proper switching frequency,

$$V_o = V_s\left[1 - f_s\left(t_3 - \frac{t_1}{2}\right)\right]$$

$$10 = 20\left[1 - f_s\left(1.47 - \frac{0.188}{2}\right) \times 10^{-6}\right]$$

$$f_s = 363.37 \text{ kHz}$$

(b) Peak inverse voltage across D_s is the same as peak capacitor voltage,

$$V_{Ds,\text{peak}} = V_{C,\text{peak}} = V_D + I_o Z_o$$

$$= 20 + 5 \times 4.61$$

$$V_{Ds,\text{peak}} = 43.05 \text{ V}$$

(ii) Given : $V_{dc} = 200$ V, $I_o = 15$ A

$L = 1.20$ mH, $C = 10$ μF, $f = 200$ Hz

Circuit turn-off time, $t_c = \dfrac{CV_{dc}}{I_o}$

$$t_c = \frac{10 \times 10^{-6} \times 200}{15} = 133.33 \text{ μsec}$$

$$\therefore \quad \alpha_{min} = \frac{t_1}{T} = \frac{\pi\sqrt{LC}}{T} = \pi f\sqrt{LC}$$

$$\alpha_{min} = \pi \times 200\sqrt{10^{-5} \times 1.2 \times 10^{-3}}$$

$$\alpha_{min} = 68.8 \times 10^{-3}$$

$$V_{o(min)} = \alpha_{min} V_{dc} + \frac{2V_{dc} \times 2t_c}{2T}$$

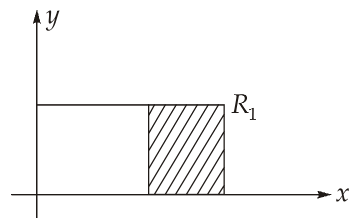
$$= 200 \times 68.8 \times 10^{-3} + \frac{4 \times 200 \times 133.33 \times 10^{-6} \times 200}{2}$$

$$V_{o(min)} = 24.426 \text{ Volts}$$

Q.8 (b) Solution:

Here, the rectangle defined by $0 \le x \le 2$, $0 \le y \le 1$ is the range space R.

(i)
$$P(X > 1) = \underset{\substack{R_1 \\ (x>1)}}{\int\int} f_{XY}(x,y) \cdot dxdy$$



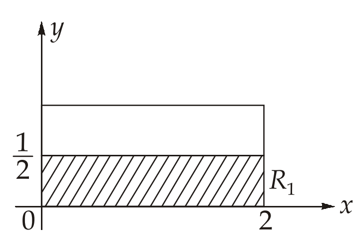
$$= \int_0^1 \int_1^2 \left(xy^2 + \frac{x^2}{8} \right) dxdy = \int_0^1 \left(\frac{x^2y^2}{2} + \frac{x^3}{24} \Bigg|_1^2 \right) dy$$

$$= \int_0^1 \left(2y^2 - \frac{y^2}{2} + \frac{8}{24} - \frac{1}{24} \right) dy$$

$$= \int_0^1 \left(\frac{3}{2}y^2 + \frac{7}{24} \right) dy = \frac{y^3}{2} + \frac{7y}{24} \Bigg|_0^1$$

$$= \frac{1}{2} + \frac{7}{24} = \frac{19}{24}$$

(ii)
$$P\left(Y < \frac{1}{2}\right) = \underset{\substack{R_1 \\ \left(Y<\frac{1}{2}\right)}}{\int\int} \left(xy^2 + \frac{x^2}{8} \right) dxdy$$



$$= \int_0^{1/2} \int_0^2 \left(xy^2 + \frac{x^2}{8} \right) dxdy$$

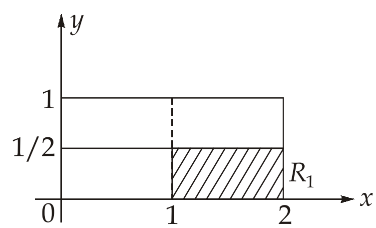
$$= \int_0^{1/2} \left(\frac{x^2y^2}{2} + \frac{x^3}{24} \Big|_0^2 \right) dy$$

$$= \int_0^{1/2} \left(2y^2 + \frac{1}{3} \right) dy$$

$$= \frac{2y^3}{3} + \frac{1}{3} y \Big|^{1/2}$$

$$P\left(y < \frac{1}{2} \right) = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$$

(iii) $$P\left(X > 1, Y < \frac{1}{2} \right) = \iint_{R_1} \left(xy^2 + \frac{x^2}{8} \right) dxdy$$



$$= \int_0^{1/2} \int_1^2 \left(xy^2 + \frac{x^2}{8} \right) dxdy$$

$$= \int_0^{1/2} \left[\frac{x^2y^2}{2} + \frac{x^3}{24} \right]_1^2 dy$$

$$= \int_0^{1/2} \left[2y^2 - \frac{y^2}{2} + \frac{7}{24} \right] dy$$

$$= \frac{y^3}{2} + \frac{7}{24} y \Big|_0^{1/2}$$

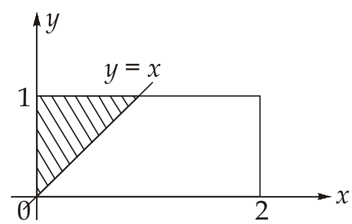
$$= \frac{1}{16} + \frac{7}{48}$$

$$= \frac{10}{48} = \frac{5}{24}$$

(iv)

$$P(X < Y) = \iint_{R_1} \left(xy^2 + \frac{x^2}{8} \right) dxdy$$

$$= \int_0^1 \int_0^y \left(xy^2 + \frac{x^2}{8} \right) dxdy$$



$$= \int_0^1 \left. \frac{x^2y^2}{2} + \frac{x^3}{24} \right|_0^y dy$$

$$= \int_0^1 \left(\frac{y^4}{2} + \frac{y^3}{24} \right) dy$$

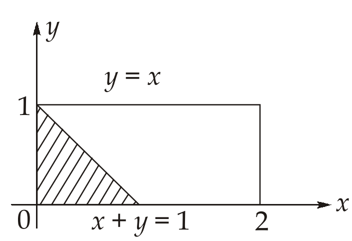
$$= \left. \frac{y^5}{10} + \frac{y^4}{96} \right|_0^1$$

$$= \frac{1}{10} + \frac{1}{32}$$

$$P(X < Y) = \frac{106}{960} = \frac{53}{480}$$

(v)

$$P(X + Y \leq 1) = \iint_{R} \left(xy^2 + \frac{x^2}{8} \right) dxdy$$



$$P(X + Y \leq 1) = \int_0^1 \int_1^{1-y} \left(xy^2 + \frac{x^2}{8} \right) dxdy$$

$$= \int_0^1 \left. \frac{x^2y^2}{2} + \frac{x^3}{24} \right|_0^{1-y} dy$$

$$= \int_0^1 \left[\frac{(1-y)^2 \cdot y^2}{2} + \frac{(1-y)^3}{24} \right] dy$$

$$P(X + Y \le 1) = \frac{13}{480}$$

Q.8 (c) Solution:

(i) Given : $P_1 = 5000 \text{ W}, P_2 = -1000 \text{ W}$

(a)
$$\cos\phi = \cos\left[\tan^{-1}\left(\frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2}\right)\right]$$

$$= \cos\left[\tan^{-1}\left(\frac{\sqrt{3} \times 6000}{4000}\right)\right] = \cos(68.948)$$

$$\cos\phi = 0.3592 \text{ lag}$$

(b) Now capacitance is connected at the source which does not affect the active power consumption.

$\therefore$
$$P = 5000 - 1000 = 4000 \text{ W}$$

Before connecting capacitor :

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$|I_L| = \frac{4000}{\sqrt{3} \times 440 \times 0.3592} = 14.612 \text{ A}$$

For delta,
$$V_{ph} = V_L = 440 \text{ V and } I_L = \sqrt{3} I_{ph}$$

$$|I_{ph}| = 8.4362 \text{ A}$$

$\therefore$
$$|Z_{ph}| = \frac{V_{ph}}{I_{ph}} = \frac{440}{8.4362} = 52.1561\ \Omega,\ \phi = 68.948°$$

$$R_{ph} = Z_{ph} \cos\phi = 18.7352\ \Omega$$

$$X_{ph} = Z_{ph} \sin\phi = 48.6749\ \Omega \text{ (inductive)}$$

After connecting capacitor,

$$P_1 = P = 4000 \text{ W}, P_2 = 0 \text{ W}$$

But,
$$P_2 = V_L I_L \cos(30 + \phi) = 0$$

$$30 + \phi = 90° \Rightarrow \phi = 60°$$

$$\cos\phi = \cos 60° = 0.5 \text{ lagging}$$

Due to pure capacitor, R_{ph} remains same.

$$\therefore \quad R_{ph} = 18.7352\ \Omega$$

$$\tan\phi = \frac{X'_{ph}}{R_{ph}} \Rightarrow X'_{ph} = \text{New reactance}$$

$$\tan 60° = \frac{X'_{ph}}{18.7352}$$

$$X'_{ph} = 32.4503\ \Omega$$

$$X_{C,ph} = X_{ph} - X'_{ph} = 48.6749 - 32.4503 = 16.2246\ \Omega$$

$$X_{C,ph} = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi \times 50 \times 16.2246}$$

$$C = 196.1899\ \mu F$$

(ii) Given : Number of bits = 32 bits

Page size = 4 kB

Page table entry size = 4 bytes

Size of page table = No. of entries in page table × Page table entry size

No. of entries in page table = Process size / Page size

Process size = Number of address bits

Thus, Process size = $2^{32} \times 2^2$ bytes = 2^{34}

Number of entries in page table

$$= \frac{2^{34}}{4 \times 2^{10}} = 2^{24}$$

Size of page table = $2^{24} \times 4 = 2^{26}$ bytes

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