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Detailed Solutions

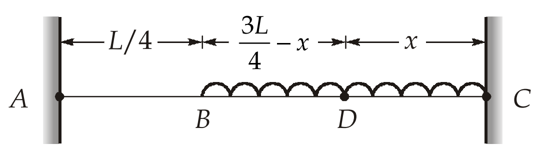
ESE-2023 Mains Test Series

Civil Engineering Test No : 7

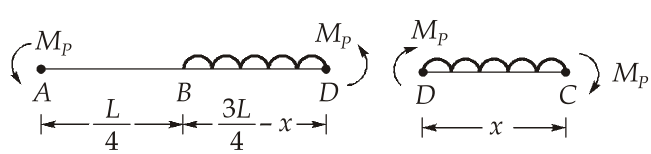
Section A : Design of Steel Structure + Hydrology

Q.1 (a) Solution:

For the fixed ended beam shown in figure, the possible location of plastic hinges will be at A and C, and at point D at a distance x from support C.



Consider the free body diagram of portion AD and DC



Now, $\Sigma M_A = 0$

$$2M_P = w\left(\frac{3L}{4} - x\right)\left(\frac{L}{4} + \frac{\frac{3L}{4} - x}{2}\right) \quad ...(i)$$

Also, $\Sigma M_C = 0$

$$\Rightarrow \quad 2M_P = wx \cdot \frac{x}{2} \quad ...(ii)$$

From equation (i) and (ii), we get

$$w\left(\frac{3L}{4}-x\right) \times \left[\frac{L}{4}+\frac{3L}{8}-\frac{x}{2}\right] = \frac{wx^2}{2}$$

$$\Rightarrow \qquad \left(\frac{3L}{4}-x\right)\left(\frac{5L}{8}-\frac{x}{2}\right) = \frac{x^2}{2}$$

Solving, we get

$$x = \frac{15}{32}L$$

Thus, plastic hinges will be formed at *A*, *C* and at *D*, where $CD = \frac{15}{32}L$

Putting the value of *x* obtained in equation (ii),

$$2M_p = \frac{w}{2}\left(\frac{15}{32}L\right)^2$$

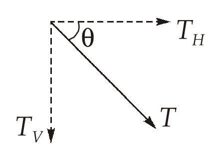
$$\Rightarrow \qquad w = \frac{4M_P}{\left(\frac{15L}{32}\right)^2} = \frac{4096}{225}\frac{M_P}{L^2} = 18.204\frac{M_P}{L^2}$$

Q.1 (b) Solution:

For Fe410 grade steel, $f_u = 410$ MPa

For bolts of grade 4.6, $f_{ub} = 400$ MPa

$f_{yb} = 240$ MPa



For 16 mm diameter bolts,

Net tensile stress area of bolt,

$$A_{nb} = 0.78 \times \frac{\pi}{4} \times 16^2 = 156.83 \text{ mm}^2$$

Nominal shank area, $A_{sb} = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$

Diameter of bolt hole, $d_h = 18$ mm

Factored tensile force, $T = 200$ kN

Horizontal component, $T_H = T\cos\theta = 200 \times \frac{4}{5} = 160$ kN

Vertical component, $T_V = T\sin\theta = 200 \times \frac{3}{5} = 120$ kN

The horizontal component will cause tension in bolts while the vertical component will cause shear in bolts.

Strength of the bolt in single shear,

$$V_{dsb} = A_{nb} \times \frac{f_{ub}}{\sqrt{3}\gamma_{mb}}$$

$$= 156.83 \times \frac{400 \times 10^{-3}}{\sqrt{3} \times 1.25} = 28.97 \text{ kN}$$

Shear force in each bolt, $V_{sb} = \frac{T_V}{8} = \frac{120}{8} = 15 \text{ kN}$

Strength of the bolt in tension,

$$T_{db} = \frac{T_{nb}}{\gamma_{mb}}$$

and $T_{nb} = 0.9 f_{ub} A_{nb} \not> f_{yb} \frac{\gamma_{mb}}{\gamma_{m0}} A_{sb}$

$\Rightarrow$ $T_{nb} = 0.9 \times 400 \times 156.83 \times 10^{-3} \not> 240 \times \frac{1.25}{1.10} \times 201.06 \times 10^{-3}$

$$= 56.46 \text{ kN} \not> 54.83 \text{ kN}$$

$\therefore$ $T_{nb} = 54.83 \text{ kN}$

$\therefore$ $T_{db} = \frac{T_{nb}}{\gamma_{mb}} = \frac{54.83}{1.25} = 43.86 \text{ kN}$

Tensile force in each bolt, $T_b = \frac{T_H}{8} = \frac{160}{8} = 20 \text{ kN}$

Check : $\left(\frac{V_{sb}}{V_{dsb}}\right)^2 + \left(\frac{T_b}{T_{bd}}\right)^2 \leq 1$

$\Rightarrow$ $\left(\frac{15}{28.97}\right)^2 + \left(\frac{20}{43.86}\right)^2 \leq 1$

$\Rightarrow$ $0.476 \leq 1$

which is all right, and the joint at section 1-1 is safe.

The member is composed of double angle section with thickness of each leg as 8 mm. The angles are placed on the opposite side of the web of Tee-bracket. The 12 mm bolts will be in double shear and bearing.

Strength of the bolt in double shear,

$$V_{dsb} = 2 \times A_{nb} \times \frac{f_{ub}}{\sqrt{3}\,\gamma_{mb}}$$

$$= 2 \times 0.78 \times \frac{\pi}{4} \times (12)^2 \times \frac{400}{\sqrt{3} \times 1.25} \times 10^{-3}$$

(Assume both shear planes lie at the threaded zone)

$$= 32.6 \text{ kN}$$

Strength of the bolt in bearing,

$$V_{dpb} = 2.5 K_b dt \frac{f_u}{\gamma_{mb}}$$

Assume a minimum pitch = $2.5d = 2.5 \times 12 = 30$ mm

and a minimum end distance = $1.5\, d_0 = 1.5 \times 13 = 19.5 \approx 20$ mm

$$K_b = \min \begin{cases} \dfrac{e}{3d_0} = \dfrac{20}{3 \times 13} = 0.51 \\ \dfrac{p}{3d_0} - 0.25 = \dfrac{30}{3 \times 13} - 0.25 = 0.52 \\ \dfrac{f_{ub}}{f_u} = \dfrac{400}{410} = 0.98 \\ 1 \end{cases}$$

$\Rightarrow$ $K_b = 0.51$

$t = \min\{8 + 8, 10\} = 10$ mm

$$V_{dpb} = 2.5 \times 0.51 \times 12 \times 10 \times \frac{410}{1.25} \times 10^{-3} = 50.18 \text{ kN}$$

Hence, strength of 12 mm bolt = min {32.6, 50.18} = 32.6 kN

$$\text{Number of bolts required} = \frac{200}{32.6} = 6.13 \approx 7$$

Thus, provide 7 nos. 12 mm diameter bolts for making the connection at a pitch of 30 mm c/c.

Q.1 (c) Solution:

For Fe410 grade of steel, $f_u = 410$ MPa

For site weld, partial safety factor for material,

$$\gamma_{mw} = 1.5$$

Maximum size of weld = 7.4 – 1.5 = 5.9 mm

Minimum size of weld = 5 mm (For thickness between 10 mm and 20 mm)

Let us provide 5 mm size filet weld,

Effective throat thickness, $t_t = KS = 0.7 \times 5 = 3.5$ mm

Design strength of weld, $P_{dw} = l_w t_t \dfrac{f_u}{\sqrt{3}\gamma_{mw}}$

$$\text{Strength of weld per mm length} = 1 \times 3.5 \times \frac{410}{\sqrt{3} \times 1.5} = 552.33 \text{ N/mm}$$

$$\text{Length of the weld required, } l_w = \frac{1000 \times 10^3}{552.33} = 1810.51 \text{ mm} \approx 1811 \text{ mm}$$

Because of the restriction of 360 mm overlap, the length of the weld that can be provided in the usual way is

$$= 2 \times 360 + 350 = 1070 \text{ mm} < 1811 \text{ mm}$$

Hence, let us provide slot welds.

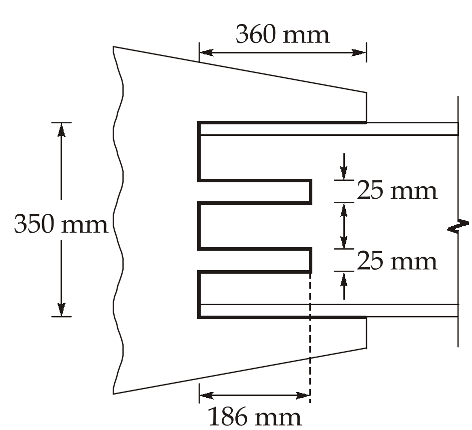
Provide width of slot = 25 mm ($3t = 3 \times 7.4 = 22.2$ mm or 25 mm whichever is greater)

Let us provide two slots and let the length of slot be l_1

$\therefore$ $1811 = 2 \times 360 + 350 + 4l_1$

$\Rightarrow$ $l_1 = 185.25 \simeq 186$ mm

Provide 186 mm × 25 mm slots, two in numbers as shown below:



Q.1 (d) Solution:

(i) 1. Probable maximum flood: The extreme flood that is physically possible in a region as a result of severemost combinations, including rare combinations of metrological and hydrological factors. The PMF is used in situations where the failure of the structure would result in loss of life and catastrophic damage and as such complete security from potential floods in sought. On the other hand, SPF is often used where the failure of a structure would cause less severe damages. Typically, the SPF is about 40% to 60% of the PMF for the same discharge basin.

2. **Standard project flood:** The flood that would result from a severe combination of meteorological and hydrological factors that are reasonably applicable to the region. Extremely rare combinations of factors are excluded.

3. **Spillway design flood:** Design flood used for the specific purpose of designing the spillway of a storage structure. This term is frequently used to denote the maximum discharge that can be passed in a hydraulic structure without any damage or serious threat to the stability of the structure.

(ii) If a rainfall is applied to an impervious surface on the catchments, it will finally reach at a rate equal to the rate of rainfall. Initially, only a certain amount of water will reach the outlet, but after some time, the water will reach the outlet from the entire area and in that case, the runoff rate would become equal to the rainfall rate. The time required to reach this conditions is time of concentration (T_C).

But in practice, impervious surface does not occur, so the peak runoff rate is,

$$Q_P = K.\, p\, .\, A \qquad \text{for } t > T_c$$

where, Q_p = peak flood

k = Coefficient of runoff

P_c = Mean rainfall intensity for duration equal to T_C.

A = Catchment area (hectare)

If p(cm) of rainfall has fallen in T-hours during an individual storm. The mean rainfall intensity will be given as,

$$p = \frac{P}{T}$$

The intensity p' of the same storm during a small time interval (hr),

$$p' = \frac{P}{T}\left(\frac{T+1}{t+1}\right) \qquad (t = \text{duration of intensity } p')$$

If t = 1 hr, than p' is called one hour rainfall and given as,

$$p_0 = \frac{P}{T}\left(\frac{T+1}{t+1}\right) = \frac{P}{T}\left(\frac{T+1}{2}\right)$$

and, p_c is calculated as,

$$p_c = p_0\left(\frac{2}{1+T_c}\right)$$

T_c is estimated by Kirpich equation

$$T_c = 0.01947 L^{0.77} S^{-0.385}$$

where, T_c = Time concentration (minutes)

L = Maximum length of travel of water (m)

S = Slope of catchment

$$S = \frac{\Delta H}{L}$$

ΔH = Difference between elevation of most remote point on the catchment and outlet.

If non-homogeneous catchment have component subareas distributed in a complex manner, in such case a weighted equivalent run off coefficient K_c is used.

$$K_c = \frac{\sum_{i=1}^{N} K_i A_i}{A}$$

Q.1 (e) Solution:

For given data

Area of ring, $A = \frac{\pi}{4} \times 35^2 = 962.1 \text{ cm}^2$

Time (Minute)	Increase in time (Δt) (Minute)	Commulative volume (V) (cm^3)	Commulative infiltration (V/A) (cm)	Incremental infiltration (cm)	Incremental Rate (cm/hr)
0	0	0	0		
2	2	278	0.289	0.289	8.67
5	3	658	0.684	0.395	7.90
10	5	1173	1.219	0.535	6.42
20	10	1924	2.00	0.781	4.686
30	10	2500	2.598	0.598	3.588
60	30	3345	3.477	0.879	1.758
90	30	3875	4.028	0.551	1.102
150	60	4595	4.776	0.748	0.748
210	60	5315	5.524	0.748	0.748

(i) Minimum infiltration capacity= 0.748 cm/hr

(ii) Average infiltration for first 10 minutes = $\frac{1.219}{10} \times 60 = 7.314$ cm/hr

(iii) Average infiltration for first 30 minutes = $\frac{2.598}{30} \times 60 = 5.196$ cm/hr

Q.2 (a) Solution:

For Fe410 grade of steel, $f_u = 410$ MPa

For bolts of grade 4.6, $f_{ub} = 400$ MPa

For ISWB 200:

Gauge, $g = 80$ mm

Flange thickness, $t_f = 9.0$ mm

The factored end reaction is transmitted on the two bracket plates. Thus the load for which the bracket connection is to be designed is,

$$P = \frac{500}{2} = 250 \text{ kN}$$

Let us provide 20 mm diameter bolts of grade 4.6

Hole dia, $d_0 = 20 + 2 = 22$ mm

End distance, $e = 1.5d_0 = 1.5 \times 22 = 33$ mm

Pitch, $p = 2.5 \times 20 = 50$ mm

Strength of bolt in single shear:

$$V_{dsb} = A_{nb} \frac{f_{ub}}{\sqrt{3}\gamma_{mb}}$$

$$= 0.78 \times \frac{\pi}{4} \times 20^2 \times \frac{400}{\sqrt{3} \times 1.25} \times 10^{-3} = 45.27 \text{ kN}$$

Strength of bolt in bearing:

$$V_{dpb} = 2.5K_b dt \frac{f_u}{\gamma_m b}$$

$$K_b = \min\left\{\frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1\right\}$$

$$= \min\left\{\frac{33}{3 \times 22}, \frac{50}{3 \times 22} - 0.25, \frac{400}{410}, 1\right\}$$

$$= \{0.5, 0.51, 0.98, 1\} = 0.5$$

$$\Rightarrow \quad V_{dpb} = 2.5 \times 0.5 \times 20 \times 9 \times \frac{410}{1.25} \times 10^{-3} \quad [t = \min(9, 12)]$$

$$= 73.8 \text{ kN}$$

Hence, strength of bolt,

$$V_{sd} = 45.27 \text{ kN}$$

Let us provide bolts in two vertical rows. Number of bolts required in one row.

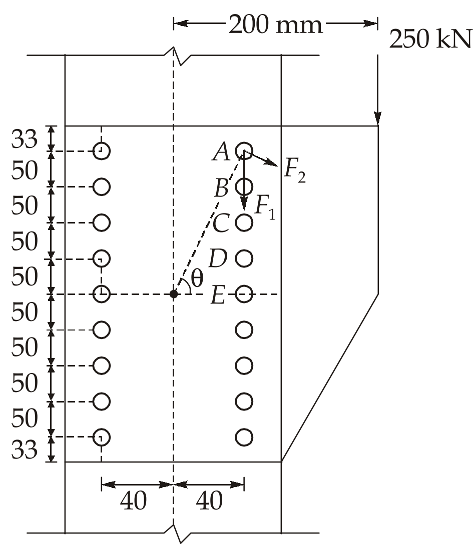
$$n = \sqrt{\frac{6M}{pn'V_{sd}}} = \sqrt{\frac{6 \times 250 \times 200 \times 10^3}{50 \times 2 \times 45.27 \times 10^3}} = 8.14 \approx 9$$

Provide 18 bolts on each bracket plate with 9 bolts in each row.

The critical bolt will be bolt A as shown in figure

Force on critical bolt A:

Direct force, $$F_1 = \frac{P}{n} = \frac{250}{18} = 13.89 \text{ kN}$$



Force due to twisting moment,

$$F_2 = \frac{Per_A}{\Sigma r^2}$$

$$e = 200 \text{ mm}$$

$$r_A = \sqrt{40^2 + 200^2} = 203.96 \text{ mm}$$

$$\Sigma r^2 = 4\left[r_A^2 + r_B^2 + r_C^2 + r_D^2\right] + 2r_E^2$$

$$\Sigma r^2 = 4 \times [40^2 + 200^2 + 40^2 + 150^2 + 40^2 + 100^2 + 40^2 + 50^2] + 2(40^2)$$
$$= 328,800 \text{ mm}^2$$

$$\therefore \quad F_2 = \frac{250 \times 10^3 \times 200 \times 203.96 \times 10^{-3}}{328800} = 31.02 \text{ kN}$$

$$\cos\theta = \frac{40}{203.96} = 0.196$$

Resultant force on the critical rivet,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(13.89)^2 + (31.02)^2 + 2(13.89)(31.02)(0.196)}$$

$$= 36.39 \text{ kN} < 45.26 \text{ kN} \quad \text{(OK)}$$

Hence, provide 18 nos. M20 bolts as shown.

Q.2 (b) Solution:

- **For section A:**

$$y_1 = 4.5 \text{ m},$$
$$B_1 = 10 \text{ m}$$

$\therefore$ Area $(A_1) = y_1 \times B_1 = 4.5 \times 10 = 45 \text{ m}^2$

and Perimeter $(P_1) = B_1 + 2y_1 = 10 + 2 \times 4.5 = 19$ m

Now, hydraulic mean radius,

$$R_1 = \frac{A_1}{P_1} = \frac{45}{19} = 2.37 \text{ m}$$

and conveyance,

$$K_1 = \frac{1}{n} \times A_1 R_1^{2/3}$$

$$= \frac{1}{0.02} \times 2.37^{2/3} \times 45 = 3999.58$$

- **For section B:**

$$y_2 = 3.2 \text{ m},$$
$$B_2 = 10 \text{ m}$$

$\therefore$ Area $(A_2) = y_2 \times B_2 = 3.2 \times 10 = 32 \text{ m}^2$

and Perimeter $(P_2) = 2y_2 + B_2 = 2 \times 3.2 + 10 = 16.4$ m

Now, hydraulic mean radius,

$$R_2 = \frac{A_2}{P_2} = \frac{32}{16.4} = 1.95 \text{ m}$$

and conveyance,

$$K_2 = \frac{1}{n} \times A_2 R_2^{2/3}$$

$$= \frac{1}{0.02} \times 1.95^{2/3} \times 32 = 2497.33$$

Average conveyance is given by,

$$K_{avg.} = \sqrt{K_1 \cdot K_2} = \sqrt{3999.58 \times 2497.32} = 3160.42 \text{ m}$$

1. I^st Iteration:

Assuming, $V_1 = V_2$

$$\therefore \quad h_f = (h_1 - h_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - K\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

$$\Rightarrow \quad h_f = (104.771 - 103.852) = 0.919 \text{ m}$$

Now,

$$Q = K_{avg}\sqrt{\frac{h_f}{L}} = 3160.42\sqrt{\frac{0.919}{5000}} = 42.847 \text{ m}^3/\text{sec}$$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{42.846}{45} = 0.952 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{42.847}{32} = 1.34 \text{ m/sec}$$

2. 2^st Iteration:

Take $K = 0.1$ for gradual contraction

$$\therefore \quad h_f = (h_1 - h_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - K\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

$$= (104.771 - 103.852) + \left(\frac{0.95^2}{2 \times 9.81} - \frac{1.34^2}{2 \times 9.81}\right)$$

$$-0.1\left(\frac{0.95^2}{2 \times 9.81} - \frac{1.34^2}{2 \times 9.81}\right)$$

$$\Rightarrow \quad h_f = 0.842 \text{ m}$$

Now, $$Q = K_{avg}\sqrt{\frac{h_f}{L}} = 3160.42\sqrt{\frac{0.842}{5000}} = 41.012 \text{ m}^3/\text{sec}$$

$\therefore$ $$V_1 = \frac{Q}{A_1} = \frac{41.012}{45} = 0.9114 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{41.012}{32} = 1.282 \text{ m/sec}$$

3. **3rd Iteration:**

$$h_f = (104.771 - 103.852) + \left(\frac{(0.9114)^2}{2\times 9.81} - \frac{(1.282)^2}{2\times 9.81}\right)$$

$$-0.1\times\left(\frac{(0.9114)^2}{2\times 9.81} - \frac{(1.282)^2}{2\times 9.81}\right) = 0.882 \text{ m}$$

$\therefore$ $$Q = K_{avg}\sqrt{\frac{h_f}{L}} = 3160.42\sqrt{\frac{0.882}{5000}} = 41.98 \text{ m}^3/\text{sec}$$

Q.2 (c) Solution:

(i) **ISI Standard Pan:** This pan evaporator is also known as modified class-A pan. It consists of a pan 1220 mm in diameter with 255 mm of depth. The pan is made of copper sheet of 0.9 mm thickness, tinned inside and painted white outside. A fixed point gauge indicates the level of water. The top of pan is covered fully with a hexagonal wire netting of galvanized iron to protect the water in the pan from bird. Further, the presence of a wire mesh makes the water temperature more uniform during day and night. The evaporation from this pan is found to be less by about 14% compared to that from unscreened pan.

(ii) Evaporation pans are not exact models of large reservoir and have the following major drawbacks:

- They differ in the heat storing capacity and heat transfer from sides and bottom. While a pan of 3 m diameter is known to give a value which is about same as from a neighbouring lake, a pan of size 1.0 m diameter indicates about 20% excess evaporation than that of 3 m diameter pan.
- The height of rim in an evaporation pan affects the wind action over the surface.
- The heat-transfer characteristics of the pan material is different from that of reservoir.

In view of the above, the evaporation observed from a pan has to be corrected to get the evaporation from a lake under similar climatic and exposure conditions. Thus a coefficient is introduced as,

$$\text{Lake evaporation} = C_p \times \text{Pan evaporation}$$

Types of Pan	**Average value of C_P**
Class A pan	0.7
ISI pan	0.8
Colorado Sunken pan	0.78
45 GS floating pan	0.8

(iii) Mayer's formula is given as,

$$E_L = K_m(e_w - e_a)\left(1 - \frac{U_9}{16}\right)$$

where,

E_L = Lake evaporation in mm/day

e_w = Saturated vapour pressure = 17.54 mm of Hg

e_a = Actual vapour pressure = Relative humidity × e_w = 0.6 × 17.54 = 10.524 mm of Hg

U_9 = Wind velocity at a height of 9 m above ground

$$= U_1 \times (9)^{1/7}$$

$$= 16 \times (9)^{1/7} = 21.9 \text{ km/h}$$

K_m = Coefficient accounting for various other factors.

For deep lakes, $K_m = 0.36$

$$\therefore \quad E_L = 0.36 \times (17.54 - 10.524) \times \left(1 + \frac{21.9}{16}\right) = 5.98 \text{ mm/day}$$

Now, for ISI pan,

Pan evaporation coefficient,

$$C_p = 0.8$$

∴ Daily evaporation as per pan evaporimeter

$$= \left(\frac{62}{7}\right) \times 0.8 = 7.086 \text{ mm/day}$$

$$\% \text{ error by meyer's formula} = \frac{7.086 - 5.98}{7.086} \times 100 = 15.6\%$$

Now, volume of water evaporated from the lake in 7 days

$$= 7 \times \frac{7.086}{1000} \times 250 \times 10^4 = 124005 \text{ m}^3$$

Q.3 (a) Solution:

Step-1 : Calculation of factored load:

Dead load = 1.5 × 10 = 15 kN/m

Live load = 1.5 × 20 = 30 kN/m

Total factored load, w = 45 kN/m

Step-2 : Calculation of bending moment and shear force:

$$\text{Maximum B.M., } M = \frac{wl^2}{2} = \frac{45 \times 5^2}{2} = 562.5 \text{ kNm}$$

Maximum S.F., $V = wl = 45 \times 5 = 225$ kN

Step-3: Choosing a trial section:

Plastic section modulus required,

$$Z_{\text{pz req}} = \frac{M\gamma_{mo}}{f_y} = \frac{562.5 \times 10^6 \times 1.1}{250} = 2475 \times 10^3 \text{ mm}^4$$

Choose a section of ISMB 550 @ 1.017 kN/m

Overall depth, h = 550 mm

Width of flange, b_f = 190 mm

Thickness of flange, t_f = 19.3 mm

Radius at root, R = 18 mm

Thickness of web, t_w = 11.2 mm

Depth of web, $d = h - 2(t_f + R)$

$= 550 - 2(19.3 + 18) = 475.4$ mm

Moment of inertia, $I_Z = 64893.6 \times 10^4 \text{ mm}^4$

Elastic section modulus, $Z_e = 2359.8 \times 10^3 \text{ mm}^3$

Plastic section modulus, $Z_p = 2711.98 \times 10^3 \text{ mm}^3$

Section classification:

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

Outstand of flange, $$b = \frac{b_f}{2} = \frac{190}{2} = 95 \text{ mm}$$

$$\frac{b}{t_f} = \frac{95}{19.3} = 4.92 < 9.4$$

$$\frac{d}{t_w} = \frac{475.4}{11.2} = 42.45 < 84$$

Hence the section is classified as plastic section.

Step-4 : Calculation of shear capacity of section:

$$V_d = \frac{f_y}{\gamma_{mo} \times \sqrt{3}} \times ht_w = \frac{250}{1.1 \times \sqrt{3}} \times 550 \times 11.2 = 808.3 \text{ kN}$$

and $$V = 225 \text{ kN } (V_d > V) \text{ (OK)}$$

Also $$0.6\, V_d = 485 \text{ kN}$$

As $V < 0.6\, V_d$, it is the case is of low shear

Step-5 : Design capacity of the section:

$$M_d = \frac{\beta_b Z_p f_y}{\gamma_{m0}} = \frac{1 \times 2711.98 \times 10^3 \times 250}{1.1} \text{N.mm} = 616.36 \text{ kNm}$$

$$\leq \frac{1.5 Z_e f_y}{\gamma_{m0}} = \frac{1.5 \times 2359.8 \times 10^3 \times 250}{1.1} \text{N.mm} = 804.48 \text{ kNm}$$

which is all right,

Hence, design bending strength of section

$$= 616.36 \text{ kNm}$$

Step-6: Check for deflection:

$$\delta = \frac{wl^4}{8EI} = \frac{30 \times (5000)^4}{8 \times 2 \times 10^5 \times 64893.6 \times 10^4} = 18.06 \text{ mm}$$

Allowable deflection, $$\delta_{\text{allowable}} = \frac{L}{150} = \frac{5000}{150} = 33.33 \text{ mm}$$

$\delta < \delta_{\text{allowable}}$

which is all right.

Step-7: Check for web buckling:

$$\frac{d}{t_w} = \frac{475.4}{11.2} = 42.45 < 67\ (67\epsilon = 67 \times 1)$$

Hence, shear buckling check of web is not required.

Step-8: Check for web bearing:

Stiff bearing length, b = 100 mm

$$\text{Bearing strength, } F_w = \frac{A_e f_{yw}}{\gamma_{mo}} = (b + n_1)t_w \frac{f_{yw}}{\gamma_{mo}}$$

$$n_1 = 2.5(R + t_f) = 2.5(18 + 19.3) = 93.25 \text{ mm}$$

$$F_w = \frac{(100 + 93.25) \times 11.2 \times 250}{1.1} \text{ N} = 491.91 \text{ kN} > 225 \text{ kN}$$

which is all right.

Q.3 (b) Solution:

Year	**Rainfall (*P*)(cm)**	**Runoff (*R*) (cm)**	P^2	***P.R.***	R^2
1964	90.5	30.1	8190.25	2724.05	906.01
1965	111.0	50.2	12321	5572.2	2520.04
1966	38.7	5.3	1497.69	205.11	28.09
1967	129.5	61.5	16770.25	7964.25	3782.25
1968	145.5	74.8	21170.25	10883.4	5595.04
1969	99.8	39.9	9960.04	3982.02	1592.01
1970	147.6	64.7	21785.76	9549.72	4186.09
1971	50.9	6.5	2590.81	330.85	42.25
1972	120.2	46.1	14448.04	5541.22	2125.21
1973	90.3	36.2	8154.09	3268.86	1310.44
1974	65.2	24.6	4251.04	1603.92	605.16
1975	75.9	20.1	5760.81	1525.59	400.01
	$\Sigma P = 1165.1$	$\Sigma R = 460$	$\Sigma P^2 = 126900.03$	$\Sigma PR = 53151.19$	$\Sigma R^2 = 23096.6$

Now, the equation for straight line regression between rainfall (P) and runoff (R) is,

$$R = aP + b$$

where, $$a = \frac{N(\Sigma PR) - (\Sigma P)(\Sigma R)}{N(\Sigma P^2) - (\Sigma P)^2}$$

and $$b = \frac{\Sigma R - a\Sigma P}{N}$$

Here, $N = 12$

$$\therefore \quad a = \frac{(12 \times 53151.19) - (1165.1 \times 460)}{12 \times 126900.03 - (1165.1)^2} = 0.616$$

and $$b = \frac{460 - 0.616 \times 1165.1}{12} = -21.475$$

∴ Relation becomes, $R = 0.616P - 21.475$

Now, coefficient of correlation,

$$r = \frac{N(\Sigma PR) - (\Sigma P)(\Sigma R)}{\sqrt{\left[N(\Sigma P^2) - (\Sigma P)^2\right]\left[N(\Sigma R^2) - (\Sigma R)^2\right]}}$$

$$= \frac{12 \times 53151.19 - 1165.1 \times 460}{\sqrt{\left[12 \times 126900.03 - (1165.1)^2\right]\left[12 \times 23096.6 - (460)^2\right]}}$$

$$= 0.978$$

Comment: r is very close to 1, therefore the correlation is very good.

Now, runoff at 100 cm annual rainfall is,

$$R = 0.616 \times 100 - 21.475$$
$$= 40.125 \text{ cm}$$

Q.3 (c) Solution:

Let us assume the design compressive stress as 150 MPa

$$\text{Required area of the column} = \frac{900 \times 10^3}{150} = 6000 \text{ mm}^2$$

Four ISA 100 × 100 × 8 mm are used,

$$A_{provoided} = 4 \times 1540 = 6160 \text{ mm}^2 > 6000 \text{ mm}^2 \text{ (OK)}$$

Design compressive stress, $$f_{cd} = \frac{900 \times 10^3}{6160} = 146.10 \text{ N/mm}^2$$

From table, for buckling class C

$$\lambda = 70 + \frac{10}{(152 - 136)} \times (152 - 146.10) = 73.69$$

For laced column, the effective slenderness ratio

$$= 1.05 \times 73.69 = 77.37 < 180$$

Effective length, $l = 0.65 \times 12 \times 10^3$

$$= 7800 \text{ mm}$$

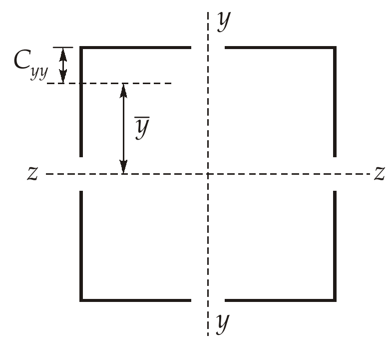
$$r = \frac{7800}{77.37} = 100.81 \text{ mm}$$

Moment of inertia of section required,

$$I = Ar^2 = 6160 \times 100.81^2$$

$$= 62.602 \times 10^6 \text{ mm}^4$$

Moment of inertia required = Moment of inertia provided



$$I = 62.602 \times 10^6 = 4\left[1.45 \times 10^6 + 1540\overline{y}^2\right]$$

$\Rightarrow$ $\overline{y} = 96.03 \text{ mm}$

Spacing of angles, $S = 2 \times (96.03 + 27.6) = 247.26 \text{ mm}$

Therefore, provide, $S = 260 \text{ mm}$

Now, $I_{zz} = I_{yy} = 4 \times 1.45 \times 10^6 + 4 \times 1540\left(\frac{260}{2} - 27.6\right)^2$

$$= 70.39 \times 10^6 \text{ mm}^4$$

$\therefore$ $r = \sqrt{\frac{70.39 \times 10^6}{6160}} = 106.9 \text{ mm}$

$\therefore$ $\frac{L}{r} = \frac{7800}{106.9} = 72.97$

$\therefore$ $f_{cd} = 152 - \frac{(152 - 136)}{(80 - 70)} \times (72.97 - 70) = 147.25 \text{ MPa}$

Capacity of the built up column = $6160 \times 147.25 \times 10^{-3}$

$= 907.1$ kN > 900 kN (O.K.)

Connection system:

Let us provide a double lacing system with the lacing flats inclined at 45°. Both are provided at the centre of the leg of the angle.

Spacing of lacing bars, $a_1 = (260 - 50 - 50) \cot 45° = 160$ mm

$$\frac{a_1}{r_y} = \frac{160}{30.7} = 5.21 < 50 \qquad \text{(OK)}$$

Also should be less than 0.7 × 72.97 (= 51.08), which it is

Shear force, $V = \frac{2.5}{100} \times 900 \times 10^3 = 22500$ N

Transverse shear in each panel $= \frac{V}{N} = \frac{22500}{2} = 11250$ N

As double lacing is provided,

$\therefore$ Compressive force in lacing bar $= \frac{1}{2} \times \frac{V}{N} \text{cosec} 45°$

$$= \frac{1}{2} \times 11250 \times \sqrt{2} = 7954.95 \text{ N} \approx 7955 \text{ N}$$

Section of lacing flat

For 20 mm diameter bolts

Minimum width of flat = 3 × 20 = 60 mm

Length of lacing = (260 – 50 – 50) cosec 45° = 226.27 mm

Minimum thickness of lacing flate= $\frac{1}{60} \times 226.27 = 3.77$ mm

Provide a flat of size 60 × 6 mm

Minimum radius of gyration, $r = \frac{t}{\sqrt{12}} = \frac{6}{\sqrt{12}} = 1.73$ mm

Slenderness ratio, $\frac{L_1}{r} = \frac{0.7(226.27)}{1.73} = 91.55 < 145$

Hence the flat is safe

For buckling class C and $\frac{l_1}{r} = 91.55$,

$$f_{cd} = 121 - \frac{(121-107)}{(100-90)}(91.55-90) = 118.83 \text{ MPa}$$

$$\text{Capacity of lacing bar} = 118.83 \times 60 \times 6$$
$$= 42778.8 \text{ N} > 7955 \text{ N}$$

Hence the lacing bar is safe.

Q.4 (a) (i) Solution:

The data is arranged in descending order and rank is assigned to the recorded events. Then probability of the event being equalled or exceeded is calculated by using Weibull's formula. Return period is then calculated as $1/p$.

m	**Annual rainfall (m)**	**Probability** $= \frac{m}{N+1}$	**Return period** $T = \frac{1}{P}$
1.	143	0.0625	16
2.	134	0.125	8
3.	128	0.1875	5.33
4.	117	0.250	4
5.	112	0.3125	3.2
6.	102	0.375	2.67
7.	100	0.4375	2.29
8.	96	0.500	2
9.	95	0.5625	1.78
10.	90	0.625	1.6
11.	89	0.6875	1.45
12.	87	0.75	1.33
13.	87	0.8125	1.23
14.	85	0.875	1.14
15.	82	0.9375	1.07

$$\text{Probability, } p = \frac{m}{1+N}$$

where, m = rank, N = Number of years of record

1. (*a*) For T = 11 years, the corresponding rainfall magnitude is obtained by interpolation.

$$\therefore \quad p = 143 + \left(\frac{134-143}{8-16}\right) \times (11-16) = 137.375 \text{ cm}$$

(*b*) For $T = 5$ years,

$$p = 134 + \left(\frac{128 - 134}{5.33 - 8}\right) \times (5 - 8) = 127.258 \text{ cm}$$

2. Probability of an annual rainfall of magnitude equal to or exceeding 105 cm

From table:

$$p = 0.3125 + \left(\frac{0.375 - 0.3125}{102 - 112}\right) \times (105 - 112) = 0.356$$

3. For 60% dependable annual rainfall,

$$\text{Probability, } p = 0.6$$

$$\therefore \quad \text{Return period} = \frac{1}{p} = \frac{1}{0.6} = 1.67$$

Now, from table,

$$P = 95 + \left(\frac{90 - 95}{1.6 - 1.78}\right) \times (1.67 - 1.78) = 91.94 \text{ cm}$$

4. Risk for rainfall having return of 6 years:

$$\text{Probability, } p = \frac{1}{T} = \frac{1}{6}$$

$$\text{Risk} = [1 - (1 - p)^n]$$

$$= \left[1 - \left(1 - \frac{1}{6}\right)^{20}\right] = 0.974$$

Q.4 (a) (ii) Solution:

- **Weighing bucket type rain gauge:** Weighing bucket type rain gauge is most common self-recording rain gauge. It consists of a receiver bucket supported by a spring or lever balance or some other weighing mechanism. The movement of bucket due to its increasing weight is transmitted to a pen which traces record or some marking on a clock driven chart. This instrument gives a plot of the accumulated rainfall against the elapsed time i.e. mass curve rainfall is obtained.
- **Tipping bucket type rain gauge:** Tipping bucket type rain gauge is a 30 cm sized circular rainguage adopted by US Weather Bureau. It has 30 cm diameter sharp edged receiver and at the end of receiver, a funnel is provided. A pair of bucket is pivoted under this funnel in such a manner that when one bucket receives 0.25 mm of precipitation, it tips discharging its rainfall into the storage can, brining the other

bucket under the funnel. The tipping actuates an electrically driven pen to force a record on clockwise work driven chart. The water collected in the storage can is measured at regular intervals to provide the total rainfall and also serve as a check. The recordings from the tipping bucket gives data on the intensity of rainfall. The recording of data is done at control room, far away from rain gauge station. This instrument is further suited for digitalizing the output signal.

Q.4 (b) Solution:

(i) Gusset is connected to the 100 mm leg of the angle

Net area of connected leg,

$$A_{nc} = \left(100 - \frac{6}{2} - 18\right) \times 6 = 474 \text{ mm}^2$$

Gross area of outstanding leg,

$$A_{go} = \left(65 - \frac{6}{2}\right) \times 6 = 372 \text{ mm}^2$$

Gross area, $A_g = (100 + 65 - 6) \times 6 = 954 \text{ mm}^2$

1. Strength governed by yielding of gross section,

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}} = \frac{954 \times 250}{1.1} \times 10^{-3} \text{ kN} = 216.82 \text{ kN}$$

2. Strength governed by rupture of critical section

$$T_{dn} = \frac{0.9 f_u A_{nc}}{\gamma_{m1}} + \frac{\beta A_{go} f_y}{\gamma_{m0}}$$

$$\beta = 1.4 - 0.076 \frac{w}{t} \times \frac{f_y}{f_u} \times \frac{b_s}{L_c}$$

$$= 1.4 - 0.076 \times \frac{65}{6} \times \frac{250}{410} \times \frac{(65 + 60 - 6)}{(5 \times 40)} = 1.101$$

$$0.7 \le \beta \le 0.9 \frac{f_u}{f_y} \times \frac{\gamma_{m0}}{\gamma_{m1}}$$

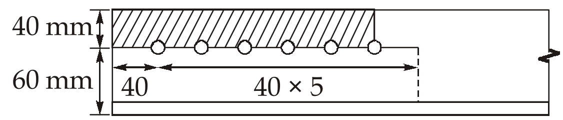
$$\Rightarrow \quad \beta \le 0.9 \times \frac{410}{250} \times \frac{1.1}{1.25}$$

$$\Rightarrow \quad \beta \le 1.299$$

$$\therefore \quad \beta = 1.101$$

$$\therefore \qquad T_{dn} = \frac{0.9 \times 410 \times 474}{1.25} + \frac{1.101 \times 372 \times 250}{1.1} = 233.01 \text{ kN}$$

3. Strength governed by block shear



Gross area in shear, $A_{vg} = (40 + 40 \times 5) \times 6 = 1440 \text{ mm}^2$

Net area in shear, $A_{vn} = (40 + 40 \times 5 - 5.5 \times 18) \times 6 = 846 \text{ mm}^2$

Gross area in tension, $A_{tg} = 40 \times 6 = 240 \text{ mm}^2$

Net area in tension, $A_{tn} = (40 - 0.5 \times 18) \times 6 = 186 \text{ mm}^2$

$$T_{db1} = \frac{A_{vg} f_y}{\sqrt{3}\gamma_{m0}} + \frac{0.9 A_{tn} f_u}{\gamma_{m1}}$$

$$= \left(\frac{1440 \times 250}{\sqrt{3} \times 1.1} + \frac{0.9 \times 186 \times 410}{1.25}\right) \times 10^{-3} = 243.56 \text{ kN}$$

$$T_{db2} = \frac{0.9 A_{vn} f_u}{\sqrt{3}\gamma_{m1}} + \frac{A_{tg} f_y}{\gamma_{m0}}$$

$$= \left(\frac{0.9 \times 846 \times 410}{\sqrt{3} \times 1.25} + \frac{240 \times 250}{1.1}\right) \times 10^{-3} = 198.73 \text{ kN}$$

Hence $T_{db} = 198.73 \text{ kN}$

Thus, the design tensile strength of the angle section

$$= \min \{T_{dg} . T_{dn}, T_{db}\}$$

$$= \min \{217.05, 233.01, 198,73\}$$

$$= 198.73 \text{ kN}$$

(ii) Gusset is connected to the 65 mm leg

$$A_{nc} = \left(65 - \frac{6}{2} - 18\right) \times 6 = 264 \text{ mm}^2$$

$$A_{g0} = \left(100 - \frac{6}{2}\right) \times 6 = 582 \text{ mm}^2$$

$$A_g = 955 \text{ mm}^2$$

1. Strength governed by yielding of gross-section

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}} = \frac{955 \times 250}{1.1} = 217.05 \text{ kN}$$

2. Strength governed by rupture of critical section

$$T_{dn} = \frac{0.9 f_u A_{nc}}{\gamma_{m1}} + \frac{\beta A_{go} f_y}{\gamma_{m0}}$$

$$\beta = 1.4 - 0.076 \times \frac{w}{t} \times \frac{f_y}{f_u} \times \frac{b_s}{L_c}$$

$$= 1.4 - 0.076 \times \frac{100}{6} \times \frac{250}{415} \times \frac{(100+30-6)}{(5 \times 40)}$$

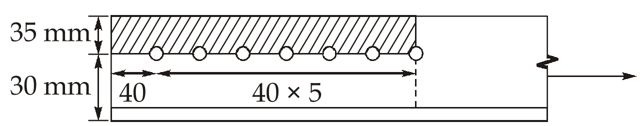
$$= 0.927 > 0.7 \text{ and } < 1.299 \text{ (O.K.)}$$

$\Rightarrow$ $\beta = 0.927$

$\Rightarrow$ $$T_{dn} = \left(\frac{0.9 \times 410 \times 264}{1.25} + \frac{0.927 \times 582 \times 250}{1.1}\right) \times 10^{-3}$$

$$= 200.55 \text{ kN}$$

3. Strength governed by block shear



Gross area in shear, $A_{vg} = (40 + 40 \times 5) \times 6 = 1440 \text{ mm}^2$

Net area in shear, $A_{vn} = (40 + 40 \times 5 - 5.5 \times 18) \times 6 = 846 \text{ mm}^2$

Gross area in tension, $A_{tg} = 35 \times 6 = 210 \text{ mm}^2$

Net area in tension, $A_{tn} = (35 - 0.5 \times 18) \times 6 = 156 \text{ mm}^2$

$$T_{db1} = \frac{A_{vg} f_y}{\sqrt{3}\gamma_{m0}} + \frac{0.9 A_{tn} f_u}{\gamma_{m1}}$$

$$= \left(\frac{1440 \times 250}{\sqrt{3} \times 1.1} + \frac{0.9 \times 156 \times 410}{1.25}\right) \times 10^{-3} = 235 \text{ kN}$$

$$T_{db2} = \frac{0.9 A_{nv} f_u}{\sqrt{3}\gamma_{m1}} + \frac{A_{tg} f_y}{\gamma_{m0}}$$

$$= \left(\frac{0.9 \times 846 \times 410}{\sqrt{3} \times 1.25} + \frac{210 \times 250}{1.1}\right) \times 10^{-3} = 191.91 \text{ kN}$$

Hence, T_{db} = 191.91 kN

Thus, the design tensile strength of the angle

$$= \min\{T_{dg}, T_{dn}, T_{db}\}$$
$$= \min\{216.82, 200.55, 191.91\}$$
$$= 191.91 \text{ kN}$$

Q.4 (c) Solution:

(i) The two basic assumptions for unit hydrograph theory are:

1. **Time invariance:** This first assumption is that the direct runoff response to a given effective rainfall in a catchment is time invariant. This implies that DRH for a given effective rainfall in catchment is always the same irrespective of when it occurs.

2. **Linear response:** The direct-runoff response to the rainfall excess is assumed to be linear. It means that if the excess rainfall of *D* cm occurs in a duration of *T*-hour then the resulting runoff hydrograph will have its ordinates equal to *D*-times the ordinates of a unit hydrograph.

   Use of unit hydrograph: The unit hydrograph establishes a relationship between effective rainfall and direct runoff for a catchment. This relationship is very useful in study of the hydrology of a catchment as

   - in the development of flood hydrograph for extreme rainfall records.
   - in extension of flood-flow records based on rainfall records.
   - in development of flood forecasting and warning system based on rainfall.

(ii) **S-curve method:**

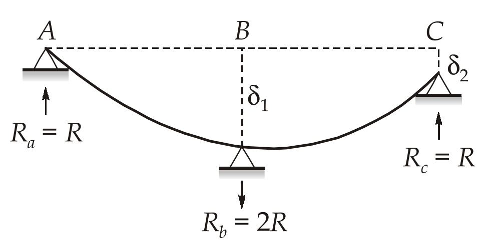
If it is desired to develop a unit hydrograph of duration *mD*, where *m* is a fraction, the method of superposition cannot be used. A different technique known as *S*-curve method is adopted in such cases and this method is applicable for rational values of *m*.

The *S*-curve, also known as *S*-hydrograph, is a hydrograph produced by a continuous effective rainfall at a constant rate for an infinite period. It is a curve obtained by summation of an infinite series of *D*-*h* unit hydrograph spaced *D*-*h* apart.

(1) Time (hrs)	(2) Ordinate of 2 - h UH	(3) S - curve Addtion	(4) S_2 - curve Ordinates	(5) S - curved lagged by 4 hrs	(6) Col. 4 - Col. 5	(7) Col. 6 × $\left(\frac{2}{4}\right)$ = UH unit hydrograph
0	0		0		0	0
2	25	0	25		25	12.5
4	100	25	125	0	125	62.5
6	160	125	285	25	260	130
8	190	285	475	125	350	175
10	170	475	645	285	360	180
12	110	645	755	475	280	140
14	70	755	825	645	180	90
16	30	825	855	755	100	50
18	20	855	875	825	50	25
20	6	875	881	855	26	13
22	0	881	881	875	6	3
24	0	881	881	881	0	0
26	0	881	881	881	0	0

Section B : Structural Analysis-1 + CPM PERT-1 + Flow of fluids, Hydraulic Machines and Hydro Power-2

Q.5 (a) Solution:



Let R_a and R_c be the upward reactions at the end supports A and C respectively

$$M_b = \text{Bending moment at } B.$$

$$\therefore \quad BM \text{ at } B = R_a l = R_c l = M_b$$

$$\Rightarrow \quad R_a = R_c = R\uparrow \text{ (say)}$$

$$\therefore \quad \text{Reaction at } B = R_b = 2R\downarrow$$

Applying the theorem of three moments for the spans AB and BC.

$$M_a l_1 + 2M_b(l_1 + l_2) + M_c l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2} - 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_1 - \delta_2}{l_2}\right)$$

$$\Rightarrow \quad 0 + 2M_b(l + l) + 0 = 0 + 0 - 6EI\left(\frac{\delta_1}{l} + \frac{\delta_1 - \delta_2}{l}\right)$$

$$\Rightarrow \quad 4M_b l = -6EI\left(\frac{2\delta_1 - \delta_2}{l}\right)$$

$$\Rightarrow \quad M_b = \frac{-3EI(2\delta_1 - \delta_2)}{2l^2}$$

The negative sign indicates that M_b is sagging moment.

$$\therefore \quad \text{Bending moment at } B = Rl = \frac{3EI(2\delta_1 - \delta_2)}{2l^2}$$

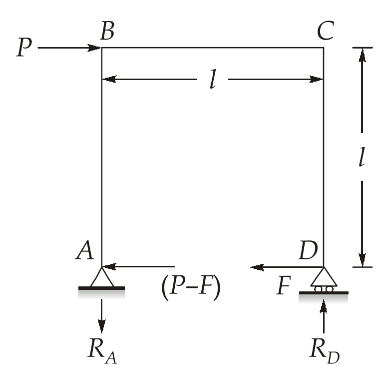
$$\Rightarrow \quad R = \frac{3EI(2\delta_1 - \delta_2)}{2l^3}$$

$$\therefore \quad R_a = R_c = \frac{3EI(2\delta_1 - \delta_2)}{2l^3} \quad \text{(upward)}$$

$$R_b = 2R = \frac{3EI(2\delta_1 - \delta_2)}{l^3} \quad \text{(downward)}$$

Q.5 (b) Solution:

Let F be the horizontal reaction at D and R_A and R_D be the vertical reactions at support A and D respectively.



Taking moments about A:

$$R_D \times l = P \times l$$

$$\Rightarrow \quad R_D = P$$

So, $\quad F = \mu P$

where, $\quad \mu$ = Coefficient of friction

Horizontal reaction at A will be $(P - F)$.

For deflection at D to be zero, $\frac{\partial U}{\partial F} = 0$

Member	Origin at	BM	$\frac{\partial M}{\partial F}$	Limits
AB	*A*	$(P-F)\,y$	$-y$	$0-y$
BC	*C*	$(Px-Fl)$	$-l$	$0-l$
CD	*D*	$-Fy$	$-y$	$0-y$

$$\because \quad \Delta D = 0 \quad \text{or,} \quad \frac{\partial U}{\partial F} = \frac{1}{EI}\int M\frac{\partial M}{\partial F}dS = 0$$

$$\Rightarrow \int_0^l \frac{(P-F)y(-y)dy}{EI} + \int_0^l \frac{(Px-Fl)(-l)dx}{EI} + \int_0^l \frac{(-Fy)(-y)dy}{EI} = 0$$

$$\Rightarrow \left[\frac{(P-F)(-y)^3}{3EI}\right]_0^l + \left[\frac{-Plx^2}{2EI} + \frac{Fl^2x}{EI}\right]_0^l + \left[\frac{Fy^3}{3EI}\right]_0^l = 0$$

$$\Rightarrow \left[\frac{-Pl^3}{3} + \frac{Fl^3}{3}\right]\frac{1}{EI} + \left[\frac{-Pl^3}{2EI} + \frac{Fl^3}{EI}\right] + \left[\frac{Fl^3}{3EI}\right] = 0$$

$$\Rightarrow \frac{-Pl^3}{3} + \frac{Fl^3}{3} - \frac{Pl^3}{2} + Fl^3 + \frac{Fl^3}{3} = 0$$

$$\Rightarrow \quad \frac{-5}{6}Pl^3 + \frac{5}{3}Fl^3 = 0$$

$$\Rightarrow \quad F = \frac{P}{2}$$

But, $\quad F = \mu P$

$\therefore \quad \mu P = 0.5P$

$\Rightarrow \quad \mu = 0.5$

Q.5 (c) Solution:

(i) Clamshells:

- Clamshell is a machine having most of the characteristics of dragline and crane in common. Digging is done like a dragline and once the bucket is filled, it works like a crane.
- If consists of a bucket of two halves which are hinged together at the top.

- The bucket halves can be attached to the shovel crane units or at the boom of dragline.
- It is primarily used for handling loose material such a sand, gravel, coal etc. and for removing materials from cofferdams, pier foundations, sewer manholes etc.
- It is suited for vertically lifting materials from one location to another. The limits of vertical movement depends upon the length of the crane boom.
- Since the shape of the bucket is like a clam fish and has hinged double shell, it is named as CLAMSHELL.

Hoe:

- Hoe is an excavating equipment of the power shovel group.
- Since the digging mechanism resembles to an ordinary garden hoe, it is named as Hoe.
- The machine is placed in operation by settling the boom at desired angle. The dipper is moved to the desired position. The free end of the boom is lowered down by releasing the tension in the hoist cable until the dipper teeth engages the material to be dug. As the cable is pulled in, the dipper is filled up. Then, the boom is raised and swung to the dumping position.
- As the hoe is able to exert greater tooth pressure it is commonly used in quarries which have tough digging-conditions.

(ii) Net mould board capacity $= \frac{3}{1.25} = 2.4\,m^3$

$$\text{Time taken per haul} = (1.08 + 0.49 + 0.3) = 1.87 \text{ min.}$$

$$\text{Trips per hour} = \frac{50}{1.87} = 26.74$$

$$\therefore \quad \text{Output per hour} = 26.74 \times 2.4 = 64.176\ m^3$$

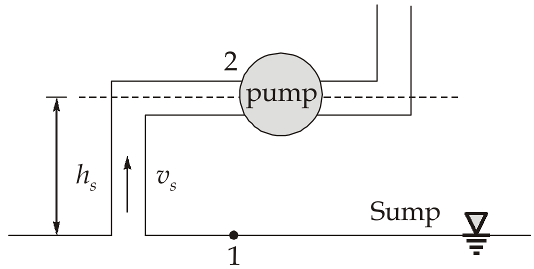
Q.5 (d) Solution:

The net positive suction head (NPSH) is defined as the absolute pressure head at the inlet to the pump minus the vapour pressure head (in absolute units) plus the velocity head.

It is also defined as the total energy available at the eye of impeller above vapour pressure of water in the field.

In other words, NPSH may be defined as the total head required to make the liquid flow through the suction pipe to the pump impeller.

Expression for net positive suction head (NPSH):



$\therefore$ Net energy at position 2 $= \dfrac{p_2}{\rho g} + \dfrac{v_s^2}{2g}$

where, p_2 = absolute pressure

According to definition,

$$\text{NPSH} = \frac{p_2}{\rho g} + \frac{v_s^2}{2g} - \frac{p_v}{\rho g} \quad \text{...(i)}$$

where, p_v = vapour pressure

Applying Bernoulli's equation between 1 and 2,

$$\frac{p_{\text{atm}}}{\rho g} = \frac{p_2}{\rho g} + \frac{v_s^2}{2g} + h_s + \text{losses}$$

$$\Rightarrow \quad \frac{p_{\text{atm}}}{\rho g} = \frac{p_2}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{fs} \quad \text{...(ii)}$$

where, h_{fs} = head loss in suction pipe

From equation (i) and (ii);

$$\text{NPSH} = \frac{p_{\text{atm}}}{\rho g} - h_s - h_{fs} - \frac{p_v}{\rho g}$$

$$\Rightarrow \quad \text{NPSH} = H_a - H_v - h_s - h_{fs}$$

$$\Rightarrow \quad \text{NPSH} = [(H_a - h_s - h_{fs}) - H_v]$$

where, $\dfrac{p_{\text{atm}}}{\rho g} = H_a$ = Atmospheric pressure head

$\dfrac{p_v}{\rho g} = H_v$ = Vapour pressure head

- Thoma's cavitation factor is used to indicate whether cavitation will occur in pumps. The value of Thoma's cavitation factor for pumps is given by the equation as,

$$\sigma = \frac{(H_a - H_v) - h_s - h_{fs}}{H_m}$$

Also, $$\text{NPSH} = (H_a - h_s - h_{fs}) - H_v$$

$\therefore$ $$\sigma = \frac{NPSH}{H_m}$$

where, H_m = Manometric head

If the value of σ calculated is less than the critical value, σ_c then cavitation will occur in the pumps. The value of σ_c depends upon the specific speed of the pump $\left(N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}\right)$.

The following empirical relation is used to determine the value of σ_c.

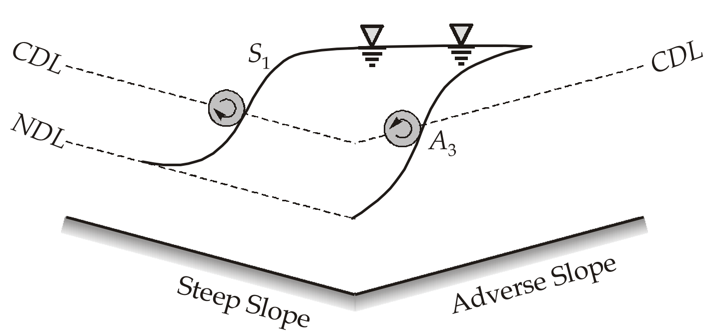
$$\sigma_c = 0.103\left(\frac{N_S}{1000}\right)^{4/3} = 1.03 \times 10^{-5} N_s^{4/3}$$

Q.5 (e) Solution:

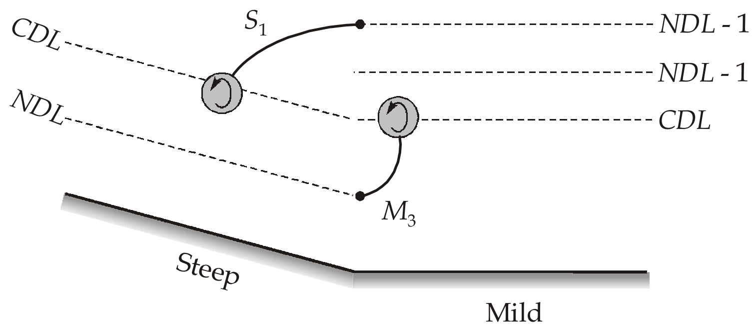
(i) The assumptions in gradually varied flow (GVF) are:

- Pressure distribution is hydrostatic.
- Resistance to flow is given by Manning's or Chezy's equation with the slope taken as slope of energy line.
- Channel bed slope is small.
- Velocity distribution is invariant.
- Resistance coefficients (C & n) are constant with depth.
- Channel is prismatic.
- There is no lateral inflow or outflow from the channel.

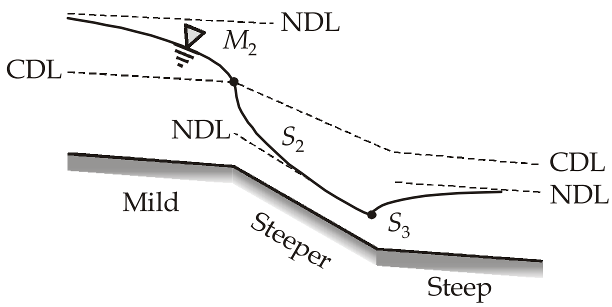
(i) 1.



2.

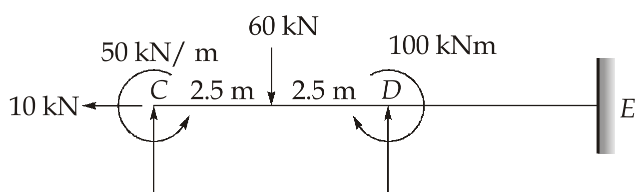


3.



Q.6 (a) Solution:

The beam loading diagram given in question can be resolved into equivalent diagram as shown in figure below:



Fixed end moments:

$$\text{Overhang } ABC,\ \bar{M}_{cb} = M_{cb} = +50\,\text{kNm}$$

$$\text{Span } CD,\ \bar{M}_{cd} = -\frac{Wl}{8} = \frac{-60\times 5}{8} = -37.5 \text{ kNm}$$

$$\bar{M}_{dc} = \frac{Wl}{8} = \frac{60\times 5}{8} = 37.5\,\text{kNm}$$

External moment at D = +100 kNm

Equilibrium condition to be satisfied at D is

$$\bar{M}_{dc} + \bar{M}_{de} = +100 \text{ kNm}$$

Distribution factors:

Joint	Member	Relative stiffness	Total Relative Stiffness	Distribution factors
D	DC	$\frac{3}{4}\cdot\frac{I}{5}=\frac{3I}{20}$	$\frac{7I}{20}$	$\frac{3}{7}$
	DE	$\frac{I}{5}=\frac{4I}{20}$		$\frac{4}{7}$

Moment distribution:

Joints	C	D		E	Explanation
$D.F$	1	3/7	4/7	–	Fixed end moment distribution
FEM	– 37.5	37.5			
Balance	37.5	–16.071	–21.428		
COM		18.75		–10.714	
Balance		–8.0357	–10.714		
COM				– 5.351	
	0	32.143	–32.142	–16.065	
External moment balancing	–50	42.857	57.1428		Externally applied moment distribution
COM		–25		+28.5714	
Balance		10.714	14.285		
COM				7.142	
Final End moment	–50	60.714	39.286	19.642	

Reactions:

B.M just on LHS of D $\Rightarrow V_c \times 5 - 10 \times 5 - 60 \times 2.5 + 60.714 = 0$

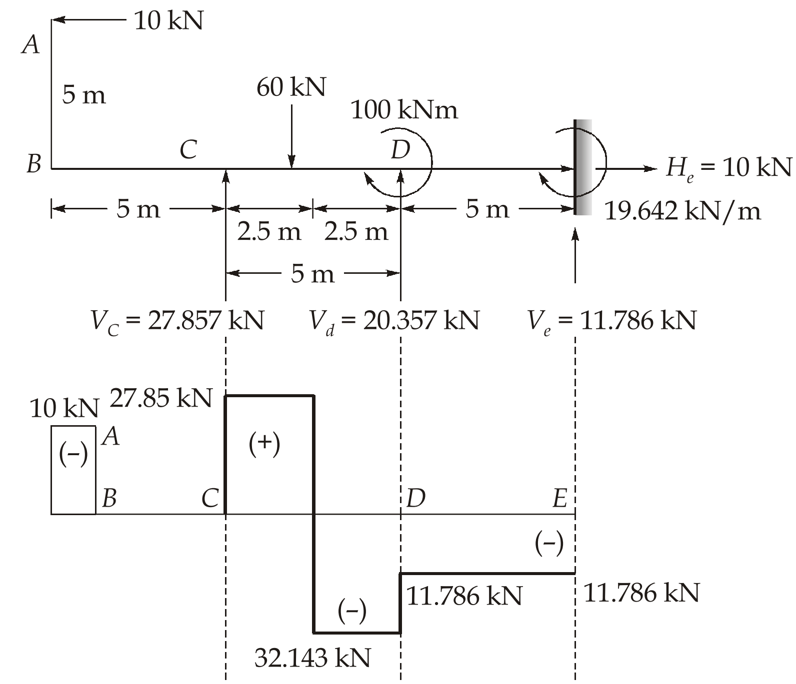
$\Rightarrow$ $V_c = 27.857$ kN

BM just on RHS of D $\Rightarrow V_e \times 5 - 19.642 - 39.286 = 0$

$\Rightarrow$ $V_e = 11.786$ kN

$\therefore$ $V_d = 60 - 27.857 - 11.786 = 20.357$ kN

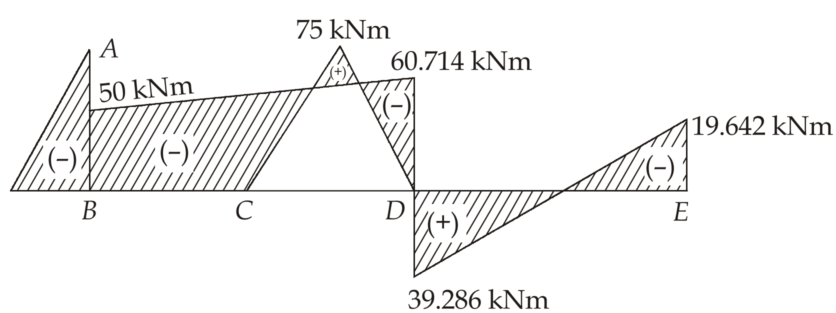
and, $H_e = 10$ kN $\rightarrow$



Shear force diagram

Free bending moment of span $CD = \frac{Wl}{4} = \frac{60 \times 5}{4} = 75\,\text{kNm}$

Sign Convention: Hogging (-ve)
Sagging (+ve)



Bending moment diagram

Q.6 (b) Solution:

Due to 8 kNm moment at C, joint C deflects or sinks. So, final end moments are calculated by adding end moments due to applied loading and end moments due to sinking or sway.

A support is assumed at 'C' which will deflect say by 'δ'.

Since, the reaction at C will be zero, $\theta_{CA} = \theta_{CB} = \theta_C$

Also, $\theta_A = \theta_B = 0$

Slope deflection equations:

$$M_{AC} = \bar{M}_{AC} + \frac{2EI}{l}\left(2\theta_A + \theta c - \frac{3\delta}{l}\right)$$

$$\Rightarrow \quad M_{AC} = 0 + \frac{2EI}{4}\left(\theta_C - \frac{3\delta}{4}\right)$$

Similarly,

$$M_{CA} = \frac{2EI}{4}\left(2\theta_C + 0 - \frac{3\delta}{4}\right)$$

$$M_{CB} = \frac{2E(2I)}{8}\left(2\theta_C + 0 + \frac{3\delta}{8}\right)$$

$$M_{BC} = \frac{2E(2I)}{8}\left(\theta_C + \frac{3\delta}{8}\right)$$

For equilibrium, the sum of moments at C is 8 kNm

$$\therefore \quad M_{CA} + M_{CB} = 8$$

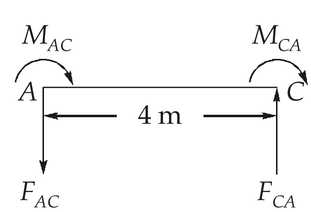
$$\Rightarrow \quad \frac{EI}{2}\left(2\theta_C - \frac{3\delta}{4}\right) + \frac{EI}{2}\left(2\theta_C + \frac{3\delta}{8}\right) = 8$$

$$\Rightarrow \quad EI\theta_C - \frac{3EI\delta}{8} + EI\theta_C + \frac{3EI\delta}{16} = 8$$

$$\Rightarrow \quad 2EI\theta_C - \frac{3EI\delta}{16} = 8$$

$$\Rightarrow \quad 2\theta_C - \frac{3\delta}{16} = \frac{8}{EI} \quad \text{...(i)}$$

Due to moments M_{CA} and M_{AC} acting on ends of beam section AC, reaction force F_{CA} will develop at C.

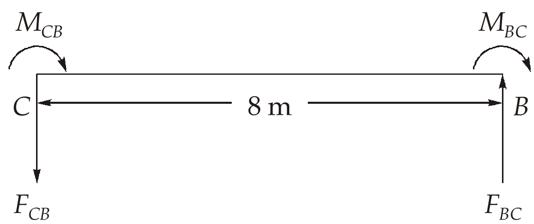


$$\therefore \quad F_{CA} = \frac{M_{CA} + M_{AC}}{l}$$

$$= \frac{\frac{EI}{2}\left(2\theta_C - \frac{3\delta}{4}\right) + \frac{EI}{2}\left(\theta_C - \frac{3\delta}{4}\right)}{4}$$

$\therefore \quad F_{CA} = \dfrac{\dfrac{3EI\theta_C}{2} - \dfrac{6EI\delta}{8}}{4} = \dfrac{3EI\theta_C}{8} - \dfrac{3EI\delta}{16}$

Similarly,



$\therefore \quad F_{CB} = -\left(\dfrac{M_{CB} + M_{BC}}{8}\right)$

$\Rightarrow \quad F_{CB} = \dfrac{-\left[\dfrac{EI}{2}\left(2\theta_C + \dfrac{3\delta}{8}\right) + \dfrac{EI}{2}\left(\theta_C + \dfrac{3\delta}{8}\right)\right]}{8}$

$\Rightarrow \quad F_{CB} = -\left[\dfrac{3EI\theta_C}{16} + \dfrac{3EI\delta}{64}\right]$

$\therefore \quad R_C = F_{CA} + F_{CB} = 0$

$\Rightarrow \quad \dfrac{3EI\theta_C}{8} - \dfrac{3EI\delta}{16} = \dfrac{3EI\theta_C}{16} + \dfrac{3EI\delta}{64}$

$\Rightarrow \quad \dfrac{3EI\theta_C}{16} = \dfrac{15EI\delta}{64}$

$\Rightarrow \quad \theta_C = \dfrac{5\delta}{4}$ ...(ii)

Substituting the value of θ_c in equation (i),

$\Rightarrow \quad 2 \times \left(\dfrac{5\delta}{4}\right) - \dfrac{3\delta}{16} = \dfrac{8}{EI}$

$\Rightarrow \quad \dfrac{37\delta}{16} = \dfrac{8}{EI}$

$\Rightarrow \quad \delta = \dfrac{128}{37EI}$

Hence, $\quad \theta_C = \dfrac{5}{4}\left(\dfrac{128}{37EI}\right) = \dfrac{160}{37EI}$

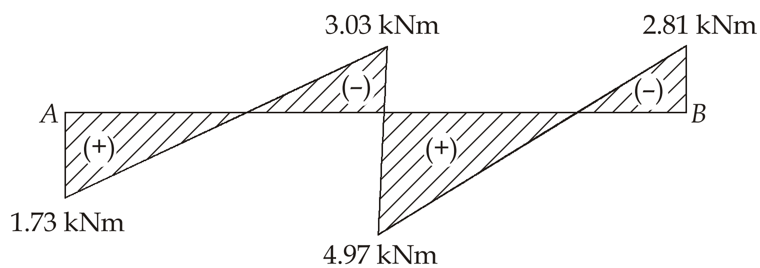
$\therefore$
$$M_{AC} = \frac{EI}{2}\left(\frac{160}{37EI} - \frac{3\times128}{4\times37EI}\right) = \frac{64}{37} = 1.73 \text{ kNm}$$

$$M_{CA} = \frac{EI}{2}\left(2\times\frac{160}{37EI} - \frac{3}{4}\times\frac{128}{37EI}\right) = 3.03 \text{ kNm}$$

$$M_{CB} = \frac{EI}{2}\left(2\times\frac{160}{37EI} + \frac{3}{8}\times\frac{128}{37EI}\right) = 4.97 \text{ kNm}$$

$$M_{BC} = \frac{EI}{2}\left(\frac{160}{37EI} + \frac{3}{8}\times\frac{128}{37EI}\right) = 2.81 \text{ kNm}$$

Bending moment diagram:



Q.6 (c) Solution:

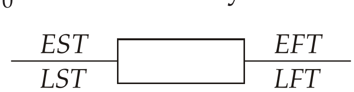
(i) Advantages of A-O-N system are as follows:

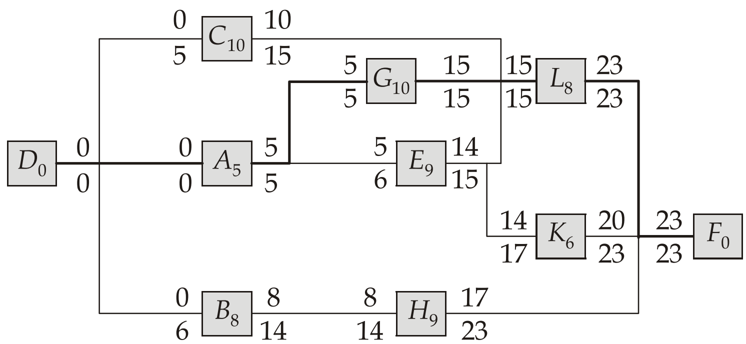
- AON system can be advantageously used in case of project having a large number of overlapping or intermediate activities in the network.
- AON system completely eliminates the use of dummy activities.
- This system is self sufficient as it contains all activity times (EST, LST, EFT, LFT) on the diagram itself.
- Revisions and modifications can be carried out easily without affecting most of the activities.
- Pre-operations and post-operations of the activity under consideration are distinctly visible.
- This system adopts simple notations similar to the engineering flow charts and hence can be easily understood by non-specialist also.

(ii) AON diagram for the given AOA diagram is shown below:

D_0 = Debut activity with zero time to indicate start

F_0 = End activity with zero time to indicate finish





Total float = LST – EST or LFT – EFT

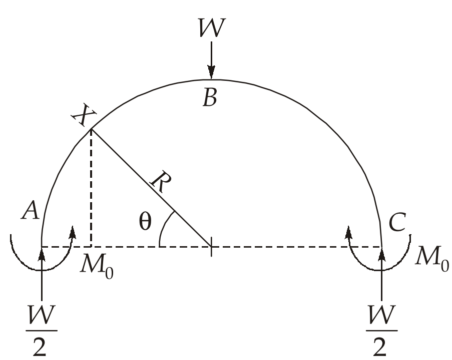
Critical path is A–G–L

Project duration = 23 days

Activity	Duration	EST	EFT	LST	LFT	Total float (F_T)	Remark
D_0	0	0	0	0	0	–	–
A	5	0	5	0	5	0	Critical
B	8	0	8	6	14	6	
C	10	0	10	5	15	5	
E	9	5	14	6	15	1	
G	10	5	15	5	15	0	Critical
H	9	8	17	14	23	6	
K	6	14	20	17	23	3	
L	8	15	23	15	23	0	Critical
F_0	0	23	23	23	23	–	

Q.7 (a) Solution:

Consider the equilibrium of half of the ring ABC.



Let us assume reacting moments M_0 is acting at A and C and upward reaction at A and C be $\dfrac{W}{2}$ each.

Consider any section X whose radius vector makes an angle θ with the horizontal.

The BM at the section X is

$$M = \frac{WR}{2}(1-\cos\theta) - M_0$$

∴ Strain energy stored by the part ABC,

$$U = \int \frac{M^2 dS}{2EI}$$

$$\Rightarrow \quad U = 2\int_0^{\pi/2} \frac{\left[\frac{WR}{2}(1-\cos\theta) - M_0\right]^2 R d\theta}{2EI}$$

M_0 is determined from the condition that the strain energy stored is minimum.

$$\frac{\partial U}{\partial M_0} = 0$$

$$\therefore \quad \frac{\partial U}{\partial M_0} = \int_0^{\pi/2} 2\frac{\left[\frac{WR}{2}(1-\cos\theta) - M_0\right](-1) R d\theta}{EI} = 0$$

$$\Rightarrow \quad \int_0^{\pi/2} \left[\frac{WR}{2}(1-\cos\theta) - M_0\right] d\theta = 0$$

$$\Rightarrow \quad \left[\frac{WR}{2}(\theta - \sin\theta) - M_0\theta\right]_0^{\pi/2} = 0$$

$$\Rightarrow \quad \frac{WR}{2}\left[\frac{\pi}{2} - 1\right] - M_0\frac{\pi}{2} = 0$$

$$\Rightarrow \quad M_0 = \frac{WR}{\pi}\left[\frac{\pi}{2} - 1\right] = \frac{WR}{2\pi}(\pi - 2)$$

∴ BM at any section is

$$M = \frac{WR}{2}(1-\cos\theta) - \frac{WR}{2\pi}(\pi - 2)$$

∴ At $\theta = 0$, $\quad M_{\theta=0} = \frac{-WR}{2\pi}(\pi - 2)$

At $\theta = \frac{\pi}{2}$, $\quad M_{\theta=\pi/2} = \frac{WR}{2} - \frac{WR}{2\pi}(\pi - 2)$

$$\therefore \qquad M_{\theta = \pi/2} = \frac{WR}{\pi}$$

Point of contraflexure:

$$\because \qquad M = 0$$

$$\Rightarrow \qquad \frac{WR}{2}(1-\cos\theta) - \frac{WR}{2\pi}(\pi - 2) = 0$$

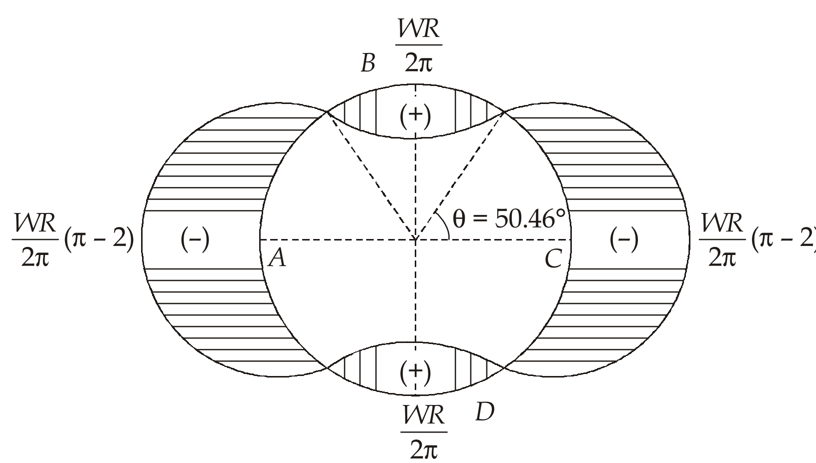
$$\Rightarrow \qquad \frac{WR}{2}(1-\cos\theta) = \frac{WR}{2\pi}(\pi - 2)$$

$$\Rightarrow \qquad 1 - \cos\theta = 1 - \frac{2}{\pi}$$

$$\Rightarrow \qquad \cos\theta = \frac{2}{\pi}$$

$$\therefore \qquad \theta = \cos^{-1}\left(\frac{2}{\pi}\right) = 50.459° \simeq 50.46°$$

Bending moment diagram:



Decrease in length of the vertical diameter:

The decrease in the length of the vertical diameter equals double the vertical deflection at *B*.

$$\therefore \qquad \text{BM at any section, } M = \frac{WR}{2}(1-\cos\theta) - \frac{WR}{2\pi}(\pi - 2)$$

Strain energy stored by the part *ABC*,

$$U = \int \frac{M^2 dS}{2EI}$$

Vertical deflection of B, $\delta = \dfrac{\partial U}{\partial W}$

$$\therefore \qquad \delta = \frac{\partial}{\partial W}\int_0^{\pi/2}\frac{2\left[\frac{WR}{2}(1-\cos\theta)-\frac{WR}{2\pi}(\pi-2)\right]^2 Rd\theta}{2EI}$$

$$\Rightarrow \qquad \delta = \int_0^{\pi/2}\frac{2\left[\frac{WR}{2}(1-\cos\theta)-\frac{WR}{2\pi}(\pi-2)\right]\left[\frac{R}{2}(1-\cos\theta)-\frac{R}{2\pi}(\pi-2)\right]Rd\theta}{EI}$$

$$\Rightarrow \qquad \delta = \int_0^{\pi/2}\frac{2\left[-\frac{WR}{2}\cos\theta+\frac{WR}{\pi}\right]\left[-\frac{R\cos\theta}{2}+\frac{R}{\pi}\right]Rd\theta}{EI}$$

$$\Rightarrow \qquad \delta = \int_0^{\pi/2}\frac{2\left[\frac{WR}{2}(2-\pi\cos\theta)\right]\frac{R}{2\pi}[2-\pi\cos\theta]Rd\theta}{EI}$$

$$\Rightarrow \qquad \delta = \int_0^{\pi/2}\frac{WR^3}{2\pi^2}\frac{\left(4+\pi^2\cos^2\theta-4\pi\cos\theta\right)d\theta}{EI}$$

$$\Rightarrow \qquad \delta = \frac{WR^3}{2\pi^2 EI}\left[4\theta+\pi^2\frac{(\theta-\sin 2\theta)}{2}-4\pi\sin\theta\right]_0^{\pi/2}$$

$$\Rightarrow \qquad \delta = \frac{WR^3}{2\pi^2 EI}\left[4\times\frac{\pi}{2}+\pi^2\left(\frac{\pi}{4}\right)-4\pi\right]$$

$$\Rightarrow \qquad \delta = \frac{WR^3}{2\pi^2 EI}\left[2\pi+\frac{\pi^3}{4}-4\pi\right]$$

$$\Rightarrow \qquad \delta = \frac{WR^3}{2\pi^2 EI}\times\frac{\pi}{4}[\pi^2-8]$$

$$\Rightarrow \qquad \delta = \frac{WR^3}{8\pi EI}[\pi^2-8]$$

Decrease in length of vertical diameter = 2δ

$$= \frac{WR^3}{4\pi EI}[\pi^2-8]$$

Q.7 (b) Solution:

$$\text{Net head, } H = (200 - 90Q^2)$$

$$\therefore \quad \text{Velocity of jet, } V = C_v\sqrt{2gH} = 0.98\sqrt{2\times 9.81\times(200-90Q^2)}$$

Assuming jet diameter to be same as nozzle diameter.

$$\therefore \quad \text{Discharge, } Q = \frac{\pi}{4}(0.1)^2\times 0.98\sqrt{2\times 9.81\times(200-90Q^2)}$$

$$\text{On solving, } Q = 0.4587 \simeq 0.459 \text{ m}^3/\text{sec.}$$

$$\text{Since, power, } P = 1000 \text{ HP}$$

$$= 1000 \times 746 \text{ Watt} \qquad (\because 1 \text{ HP} = 746 \text{ W})$$

$$= 746 \text{ kW}$$

$$\text{Net head, } H = [200 - 90(0.459)^2] = 181.04 \text{ m}$$

$$\therefore \text{ Overall efficiency, } \quad \eta_0 = \frac{P}{\rho QgH}$$

$$\Rightarrow \quad \eta_0 = \frac{746\times 10^3}{10^3\times 0.459\times 9.81\times 181.04}$$

$$\Rightarrow \quad \eta_0 = 0.9151 = 91.51\%$$

(i) When the power produced is reduced to 800 HP by operating the needle in the nozzle, then

$$\text{Power } (P) \propto Q \propto d^2$$

So, the required jet diameter, d is,

$$\therefore \quad d = 10\times\left(\frac{800}{1000}\right)^{1/2}$$

$$\Rightarrow \quad d = 8.944 \text{ cm} \simeq 8.95 \text{ cm}$$

$$\therefore \quad \text{Discharge, } Q = \frac{\pi}{4}(0.0895)^2\times 0.98\sqrt{2\times 9.81\times(200-90Q^2)}$$

On solving for Q;

$$Q = 0.3738 \text{ m}^3/\text{sec} \simeq 0.374 \text{ m}^3/\text{s}$$

(ii) When the power produced is reduced to 800 HP by closing the valve provided in the main then, the jet diameter remains the same as 10 cm, but, there will be additional head loss at the valve.

Let, h_L be the loss of head at the valve,

$\therefore$ Net head, $H = (200 - 90Q^2 - h_L)$

$\therefore$ Discharge, $Q = \frac{\pi}{4}(0.1)^2 \times 0.98\sqrt{2 \times 9.81 \times (200 - 90Q^2 - h_L)}$ ..(i)

Assuming, the efficiency to remain constant,

$$\eta_0 = \frac{P}{\rho QgH}$$

$$\Rightarrow \quad 0.9151 = \frac{800 \times 746}{10^3 \times Q \times 9.81 \times (200 - 90Q^2 - h_L)}$$

$$\Rightarrow \quad (200 - 90Q^2 - h_L) = \frac{66.48}{Q} \quad ...(ii)$$

From equation (i) and (ii)

$$Q = \frac{\pi}{4}(0.1)^2 \times 0.98\sqrt{2 \times 9.81 \times \frac{66.48}{Q}}$$

On solving, we get, $Q = 0.4259 \text{ m}^3/\text{s} \simeq 0.426 \text{ m}^3/\text{s}$

Putting value of Q in equation (ii), we get

$$(200 - 90 \times 0.426^2 - h_L) = \frac{66.48}{0.426}$$

$$\Rightarrow \quad h_L = 27.61 \text{ m}$$

So, additional head loss in case (ii) = 27.61 m

The provision of a valve in the main to reduce the discharge when the power produced is reduced, results in additional head loss and also requires more discharge and hence it is not a satisfactory arrangement.

On the other hand, the provision of a needle in the nozzle and its operation to reduce the discharge when the power produced is reduced, results in negligible head loss and requires less discharge and hence, it is a satisfactory arrangement, which is commonly adopted.

Q.7 (c) Solution:

(i) For a wide rectangular channel, $R \simeq y$

Let, y_n be the normal depth of flow.

According to Manning's formula,

$$Q = \frac{1}{n} \times A \times R^{2/3} \cdot S^{1/2}$$

$$\Rightarrow \qquad 8 = \frac{1}{0.015} \times y_n \times y_n^{2/3} \cdot \left(\frac{1}{1000}\right)^{1/2}$$

$$\Rightarrow \qquad y_n = 2.2259 \text{ m} \simeq 2.226 \text{ m}$$

The critical depth y_c is given by

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{8^2}{9.81}\right)^{1/3}$$

$$\Rightarrow \qquad y_c = 1.869 \text{ m}$$

Since, $y_n > y_c$, the bed slope is mild. Further, the depth of flow at vena-contracta is 0.6 m, which is less than critical depth. In this case, hydraulic jump will form.

The jump will be located at a section where the depth of flow will be conjugate to the normal depth of flow y_n equal to 2.226 m.

For this the depth of flow will increase from 0.6 m at vena-contracta to this conjugate depth along M_3 type surface profile.

Let, y_1 be the depth conjugate to normal depth y_n.

$$\therefore \qquad \frac{y_1}{y_n} = \frac{1}{2}\left[-1 + \sqrt{1 + 8F_r^2}\right]$$

$$\Rightarrow \qquad \frac{y_1}{y_n} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gy_n^3}}\right] \qquad \left[\because F_r^2 = \frac{q^2}{gy^3}\right]$$

$$\Rightarrow \qquad \frac{y_1}{2.226} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8 \times 8^2}{9.81 \times (2.226)^3}}\right]$$

$$\Rightarrow \qquad y_1 = 1.5517 \simeq 1.552 \text{ m}$$

At vena-contracta section, the specific energy is

$$E_1 = 0.6 + \frac{(8/0.6)^2}{2 \times 9.81}$$

$$\therefore \qquad E_1 = 9.661 \text{ m}$$

Similarly, at the section when the depth of flow is 2.226 m, the specific energy is

$$E_2 = 2.226 + \frac{\left(\frac{8}{2.226}\right)^2}{2 \times 9.81}$$

$$\Rightarrow \qquad E_2 = 2.884 \text{ m}$$

$$\therefore \qquad \Delta E = (E_2 - E_1) = (2.884 - 9.661) = -6.777 \text{ m}$$

According to Manning's formula,

At vena contracta section,

$$S_{f1} = \frac{V^2n^2}{R^{4/3}} = \frac{(8/0.6)^2(0.015)^2}{(0.6)^{4/3}} = 0.07904$$

At section where depth of flow is 2.226 m

$$S_{f2} = \frac{V^2n^2}{R^{4/3}} = \frac{(8/2.226)^2(0.015)^2}{(2.226)^{4/3}} = 9.9987\times10^{-4}$$

$$\therefore \quad \bar{S}_f = \frac{S_{f1}+S_{f2}}{2} = \frac{0.07904+9.9987\times10^{-4}}{2}$$

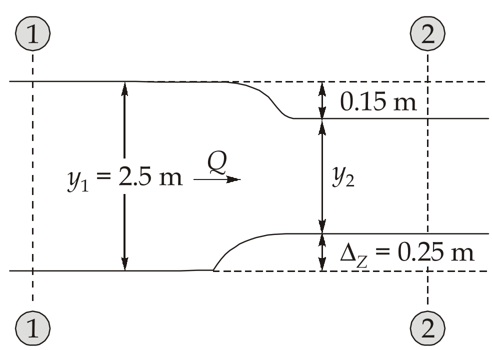
$$\Rightarrow \quad \bar{S}_f = 0.04002$$

The distance between the vena contracta and the section where jump is located is

$$x = \frac{\Delta E}{S_0 - S_f} = \frac{-6.777}{\left(\frac{1}{1000} - 0.04002\right)}$$

$$\therefore \quad x = 173.68 \text{ m}$$

(ii)



The depth of flow at the contracted section is

$$y_2 = (2.5 - 0.15 - 0.25) = 2.1 \text{ m}$$

It Q is the discharge flowing in the channel then, specific energy at the upstream section E_1 will be

$$E_1 = y_1 + \frac{v_1^2}{2g}$$

$$\therefore \quad E_1 = 2.5 + \frac{Q^2}{2g\times(3.5\times2.5)^2} = 2.5 + \frac{Q^2}{1502.156}$$

Also, the specific energy at the contracted section is

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

$$\Rightarrow \quad E_2 = 2.1 + \frac{Q^2}{2g \times (2.5 \times 2.1)^2} = 2.1 + \frac{Q^2}{540.776}$$

Assuming, no loss of energy between two sections.

$$E_1 = E_2 + \Delta Z$$

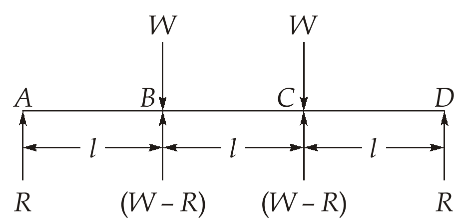
$$\Rightarrow \quad 2.5 + \frac{Q^2}{1502.156} = 2.1 + \frac{Q^2}{540.776} + 0.25$$

$$\Rightarrow \quad Q = 11.258 \text{ m}^3/\text{sec}$$

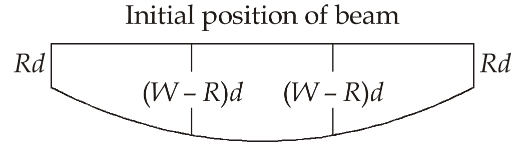
Q.8 (a) Solution:

(i) Let, R tonnes be the reaction at A.

By symmetry reaction R_D will also be R.



Initial position of beam



Final position

$$R_B = R_C = W - R$$

Immersion of floats at A and B will be Rd and that of at B and C will be $(W - R)d$.

Deflection of B relative at A will be

$$(W - R)d - Rd = (W - 2R)d$$

Deflection of B relative to C will be zero. Ends A and C are simply supported so,

$$M_A = M_D = 0$$

Also, $$M_B = M_C$$

(Beam is symmetrical and loading is also symmetrical)

Applying theorem of three moments to spans AB and BC.

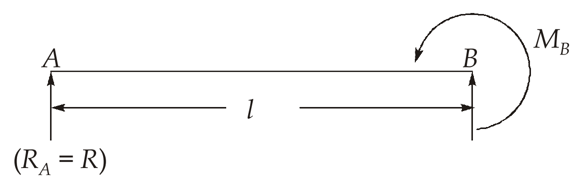
$$M_A \times l + 2M_B(l + l) + M_C \times l = -6\left(\frac{A_1\bar{x}_1}{l^2} + \frac{A_2\bar{x}_2}{l^2}\right) + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

$$\Rightarrow \quad 0 + 2M_B(2l) + M_B \times l = 0 + 6EI\left[\frac{(W-2R)d}{l} + 0\right]$$

$$\Rightarrow \quad 5M_B l = \frac{6EId(W-2R)}{l}$$

$$\Rightarrow \quad M_B = \frac{6EId(W-2R)}{5l^2}$$

Taking moments about B,



$$R \times l = M_B$$

$$\Rightarrow \quad R \times l = \frac{6EId(W-2R)}{5l^2}$$

$$\Rightarrow \quad R \times l + \frac{12EIdR}{5l^2} = \frac{6EIdW}{5l^2}$$

$$\Rightarrow \quad Rl\left(1 + \frac{12EId}{5l^3}\right) = \frac{6EIdW}{5l^2}$$

$$\Rightarrow \quad R = \frac{\dfrac{W \times 6EId}{5l^3}}{\left(1 + \dfrac{12EId}{5l^3}\right)}$$

$$\therefore \quad R_B = W - R = W - W \times \frac{\dfrac{6EId}{5l^3}}{\left(1 + \dfrac{12EId}{5l^3}\right)}$$

$$= \frac{W\left(1 + \dfrac{6EId}{5l^3}\right)}{\left(1 + \dfrac{12EId}{5l^3}\right)}$$

$\therefore$ Portion of load carried by central float is

$$R_B = \frac{W\left(1 + \dfrac{6EId}{5l^3}\right)}{\left(1 + \dfrac{12EId}{5l^3}\right)}$$

(ii) Approximate methods for the analysis of multistorey frames subjected to horizontal forces are:

1. Portal method
2. Cantilever method
3. Factor method.

Assumptions involved in portal method are:

- Point of contraflexure occurs at the middle of all the members of frame.
- Horizontal shear taken by each interior column is double of that taken by external column.

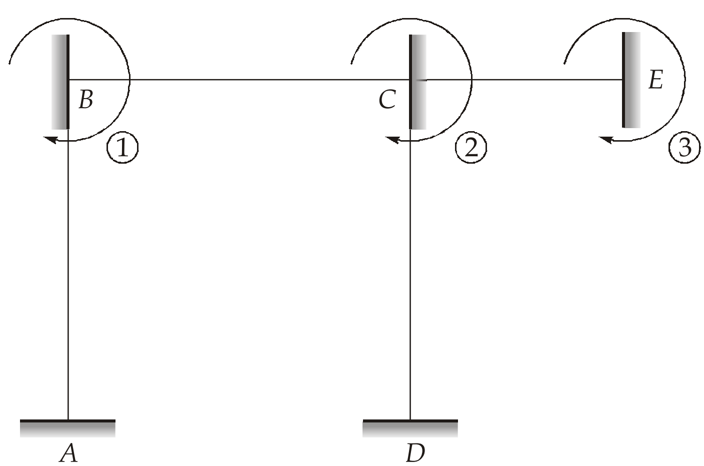
Assumptions involved in cantilever method are:

- There is a point of contraflexure at the centre of each member.
- The intensity of axial stress in each column of a storey is proportional to the horizontal distance of that column from the centre of gravity of all the columns of the storey under consideration.

Factor method is an approximate slope deflection method by which the desired results can be obtained without the knowledge of principle of elasticity upon which it is based.

Q.8 (b) Solution:

The degree of freedom of the framed structure is 3. The three coordinates selected and fully restrained structure are shown in figure:



Fixed end moments:

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-50\times 6^2}{12} = -150\,\text{kNm}$$

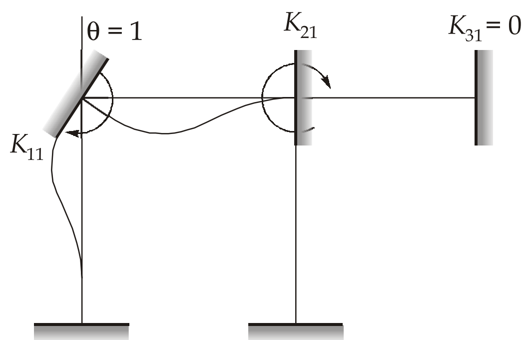
$$M_{FCB} = +\frac{wl^2}{12} = \frac{50 \times 6^2}{12} = 150\,\text{kNm}$$

All other fixed end moments are zero.

$$\therefore \quad [P_L] = \begin{bmatrix} -150 \\ 150 \\ 0 \end{bmatrix} \text{ and } P = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness matrix:

(a) Unit rotation in coordinate direction-1:

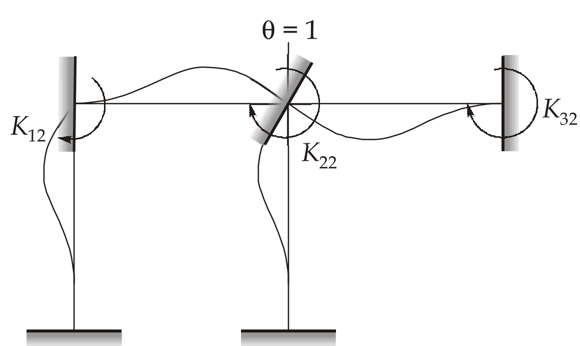


$$k_{11} = \frac{4EI}{6} + \frac{4EI}{6} = \frac{4EI}{3}$$

$$k_{21} = k_{12} = \frac{2EI}{6} = \frac{EI}{3}$$

$$k_{31} = 0$$

(b) Unit rotation in coordinate direction-2:

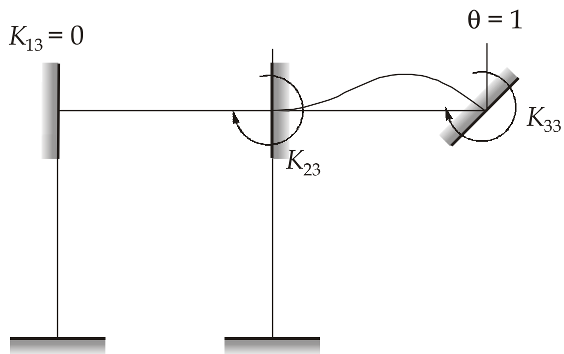


$$k_{12} = \frac{2EI}{6} = \frac{EI}{3}$$

$$k_{22} = \frac{4EI}{6} + \frac{4EI}{6} + \frac{4EI}{4} = \frac{7EI}{3}$$

$$k_{32} = \frac{2EI}{4} = \frac{EI}{2}$$

(c) Unit rotation in coordinate direction-3:



$$k_{13} = 0$$

$$k_{23} = \frac{2EI}{4} = \frac{EI}{2}$$

$$k_{33} = \frac{4EI}{4} = EI$$

Therefore, the stiffness matrix is

$$[k] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} \frac{4EI}{3} & \frac{EI}{3} & 0 \\ \frac{EI}{3} & \frac{7}{3}EI & \frac{EI}{2} \\ 0 & \frac{EI}{2} & EI \end{bmatrix}$$

$$= \frac{EI}{3}\begin{bmatrix} 4 & 1 & 0 \\ 1 & 7 & \frac{3}{2} \\ 0 & \frac{3}{2} & 3 \end{bmatrix}$$

Stiffness matrix equation is:

$$[k][\Delta] = [P - P_L]$$

$$\Rightarrow \quad \frac{EI}{3}\begin{bmatrix} 4 & 1 & 0 \\ 1 & 7 & 3/2 \\ 0 & 3/2 & 3 \end{bmatrix}\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \begin{bmatrix} 0 & - & (-150) \\ 0 & - & (150) \\ 0 & - & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \left(\frac{3}{EI}\right)\left[\frac{1}{72}\right]\begin{bmatrix} 18.75 & -3 & 1.5 \\ -3 & 12 & -6 \\ 1.5 & -6 & 27 \end{bmatrix}\begin{bmatrix} 150 \\ -150 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_E \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 135.9375 \\ -93.75 \\ 46.875 \end{bmatrix}$$

Now using slope-deflection equations:

$$M_{AB} = \frac{2EI}{6}[\theta_B] = \frac{2EI}{6}\left[\frac{135.9375}{EI}\right] = 45.3125\,\text{kNm}$$

$$M_{BA} = \frac{2EI}{6}[2\theta_B + \theta_A] = \frac{2EI}{6}\left[2 \times \frac{135.9375}{EI}\right] = 90.625\,\text{kNm}$$

$$M_{BC} = -150 + \frac{2EI}{6}[2\theta_B + \theta_C]$$

$$= -150 + \frac{2EI}{6}\left[2 \times \frac{135.9375}{EI} + \left(\frac{-93.75}{EI}\right)\right]$$

$$= -90.625\text{ kNm}$$

$$M_{CB} = 150 + \frac{2EI}{6}[2\theta_C + \theta_B]$$

$$= 150 + \frac{2EI}{6}\left[2\left(\frac{-93.75}{EI}\right) + \frac{135.9375}{EI}\right] = 132.8125\text{ kNm}$$

$$M_{CD} = \frac{2EI}{6}[2\theta_C + \theta_D] = \frac{2EI}{6}\left[2\left(\frac{-93.75}{EI}\right) + 0\right] = -62.5\,\text{kNm}$$

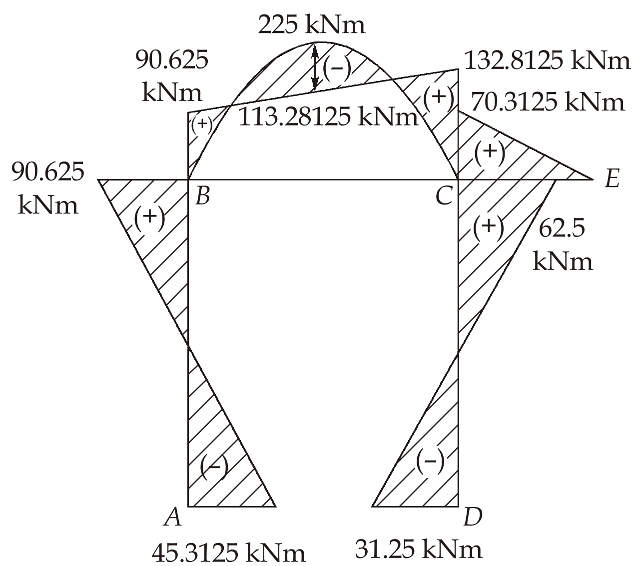
$$M_{DC} = \frac{2EI}{6}[2\theta_D + \theta_C] = \frac{2EI}{6}\left[0 + \left(\frac{-93.75}{EI}\right)\right] = -31.25\,\text{kNm}$$

$$M_{CE} = \frac{2EI}{4}[2\theta_C + \theta_E]$$

$$= \frac{2EI}{4}\left[2\left(\frac{-93.75}{EI}\right) + \frac{46.875}{EI}\right] = -70.3125\,\text{kNm}$$

$$M_{EC} = \frac{2EI}{4}[2\theta_E + \theta_C] = \frac{2EI}{4}\left[2\left(\frac{46.875}{EI}\right) + \left(\frac{-93.75}{EI}\right)\right] = 0\,\text{kNm}$$

Bending moment diagram:



Q.8 (c) Solution:

(i) For centrifugal pump

Outside diameter, $D_2 = 1.2$ m

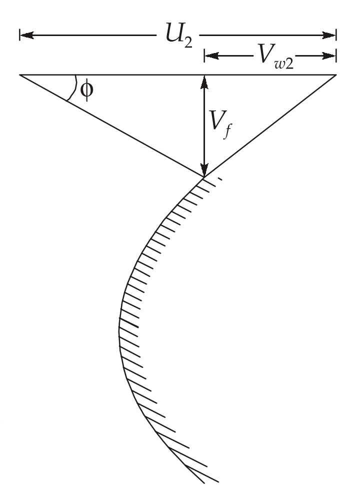
Inside diameter, $D_1 = 60$ cms $= 0.6$ m

Speed, $N = 200$ rpm

Discharge, $Q = 1800$ LPS $= 1.8\ m^3/s$

Manometric head, $H = 6$ m

Velocity of flow, $V_f = 2.4$ m/s



Assuming radial entry at the inlet

Pump velocity at outlet, $U_2 = \frac{\pi D_2 N}{60}$

$$= \frac{\pi \times 1.2 \times 200}{60} = 12.57 \text{ m/s}$$

Using velocity triangle at outlet

$$\tan\phi = \frac{V_f}{U_2 - V_{w_2}}$$

$$\tan(180° - 150°) = \frac{2.4}{12.57 - V_{w_2}}$$

$$V_{w_2} = 8.41 \text{ m/s}$$

Manometric efficiency, $n = \frac{gH_m}{V_{w_2}U_2} = \frac{9.81\times 6}{12.57\times 8.41} = 0.557$ or 55.7%

For minimum starting speed, N_1

$$\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2 = 2gH$$

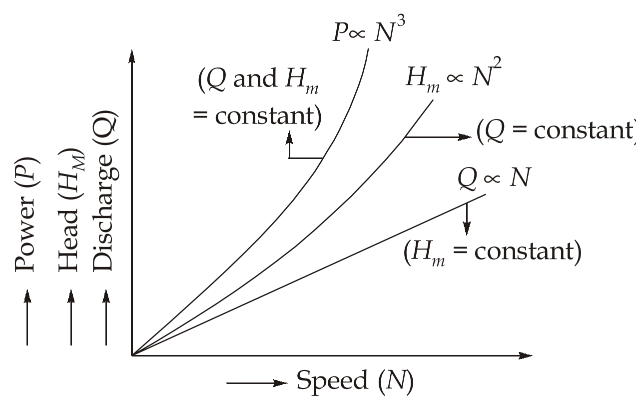
$$\left(\frac{\pi N_1}{60}\right)^2 (1.2^2 - 0.6^2) = 2 \times 9.81 \times 6$$

$$N_1 = 199.4 \text{ rpm}$$

(ii) Characteristic curves of centrifugal pump are necessary to predict the behavior and performance of the pump when the pump is working under different flow rate, head and speed. The following are the important characteristic curves for pumps:

1. Main characteristic curves.
2. Operating characteristic curves.
3. Constant efficiency or Muschel curves.

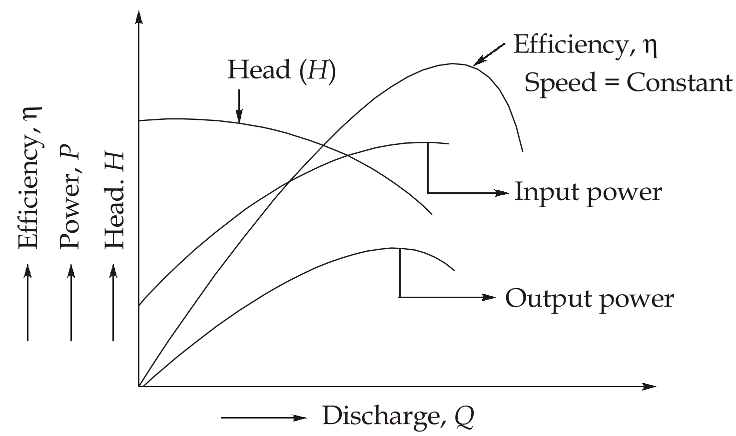
Main Characteristic Curves: The main characteristic curves of a centrifugal pump consists of variation of manometric head, H_m, power and discharge with respect to speed. For plotting curves of manometric head versus speed, discharge is kept constant. Similarly, for plotting curves of discharge versus speed and curves of power versus speed, manometric head and discharge are kept constant respectively.



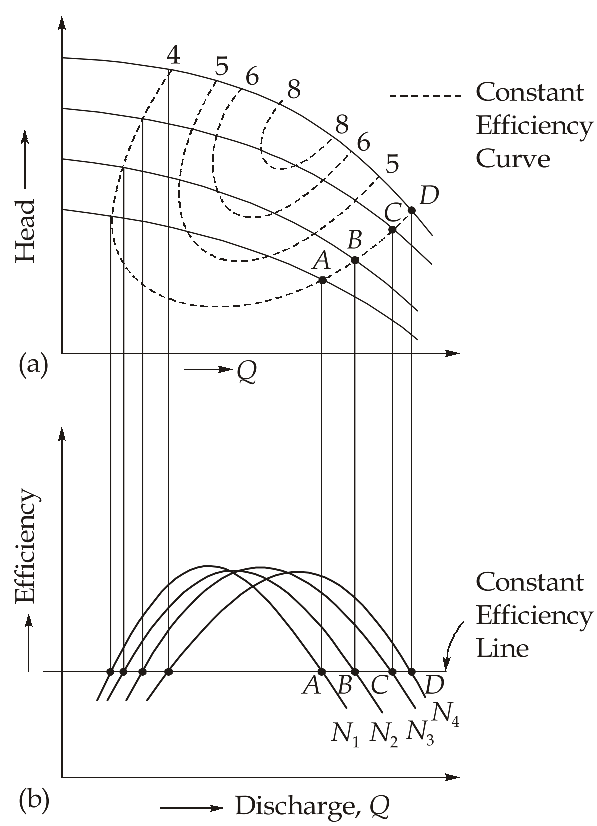
Operating Characteristic Curves: If the speed is kept constant, the variation of manometric head, power and efficiency with respect to discharge gives the operating characteristics of the pump.

The input power curve for pumps shall not pass through the origin as, even at zero discharge some power is needed to overcome mechanical losses. The head curve will have maximum value of head when discharge is zero.

The output power curve will start from origin as at $Q = 0$, output power ($\rho Q g H$) will be zero. The efficiency curve will start from origin as at $Q = 0$, $\eta = 0$.



Constant Efficiency Curve: For obtaining constant efficiency curve, the head versus discharge curves and efficiency versus discharge curves for different speeds are used. By combining these curves ($H \sim Q$ curves and $\eta \sim Q$ curves), constant efficiency curves are obtained.



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