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Detailed Solutions

ESE-2023 Mains Test Series

Electrical Engineering Test No : 6

Section A : Power Electronics & Drives + Engineering Mathematics

Q.1 (a) Solution:

(i) The average terminal voltage is

$$V_t = \frac{2 \times 265}{\pi}\cos(30°)$$

$$V_t = 146.10 \text{ V}$$

The back emf,

$$E_b = V_t - I_a R_a$$

$$= 146.10 - 40 \times 0.25$$

$$E_b = 136.1 \text{ V}$$

Hence, speed of motor

$$N = \frac{E_b}{K_a\phi} = \frac{136.1}{0.15} = 907.33 \text{ rpm}$$

(ii) For motor torque

$$K_a\phi = 0.15 \text{ V/rpm}$$

$$= \frac{0.15 \times 60}{2\pi} \text{ V-sec/rad}$$

$$\text{Torque} = \frac{0.15 \times 60}{2\pi} \times 40 = 57.29 \text{ N-m}$$

(iii) Power to the motor

$$P = (i_a)^2_{\text{rms}} R_a + E_b \cdot I_a$$

$\because$ i_a is ripple free so $\quad (i_a)_{\text{rms}} = (I_a)_{\text{avg}} = I_a$

$$P = 40^2 \times 0.25 + 136.1 \times 40$$

$$P = 5844 \text{ Watts}$$

Q.1 (b) Solution:

(i) Assume the inductor current is continuous and the minimum inductor current,

$$I_{\min} = \frac{V_s}{(1-D)^2 R} - \frac{V_s D}{2Lf}$$

$$= \frac{20}{(1-0.6)^2 \times 50} - \frac{20 \times (0.6)}{2 \times 100 \times 10^{-6} \times 15 \times 10^3} = -1.5 \text{ A}$$

Negative inductor current is not possible, indicating discontinuous current.

(ii) Output voltage of boost converter,

$$V_0 = \frac{V_s}{2}\left[1 + \sqrt{1 + \frac{2D^2 R}{Lf}}\right]$$

$$= \frac{20}{2}\left[1 + \sqrt{1 + \frac{2(0.6)^2(50)}{100 \times 10^{-6} \times 15 \times 10^3}}\right] = 60 \text{ V}$$

(iii) Maximum inductor current in discontinuous mode,

$$I_{L(\max)} = \frac{V_s D}{Lf} = \frac{20 \times 0.6}{100 \times 10^{-6} \times 15 \times 10^3} = 8 \text{ A}$$

Q.1 (c) Solution:

The characteristic equation

$$|A - \lambda I| = \begin{vmatrix} 5-\lambda & -6 & -6 \\ -1 & 4-\lambda & 2 \\ 3 & -6 & -4-\lambda \end{vmatrix} = 0$$

$\Rightarrow (5-\lambda)[(4-\lambda)(-4-\lambda) + 12] - (-6)[-1 \times (-4-\lambda) - 6] - 6[6 - 3(4-\lambda)] = 0$

$\Rightarrow (5-\lambda)(-16 - 4\lambda + 4\lambda + \lambda^2 + 12) + 6(\lambda + 4 - 6) - 6(6 + 3\lambda - 12) = 0$

$\Rightarrow \quad (5-\lambda)(\lambda^2 - 4) + 6\lambda - 12 - 18\lambda + 36 = 0$

$\Rightarrow \quad 5\lambda^2 - 20 - \lambda^3 + 4\lambda + 6\lambda - 12 - 18\lambda + 36 = 0$

$\Rightarrow \quad -\lambda^3 + 5\lambda^2 - 8\lambda + 4 = 0$

$$\Rightarrow \quad \lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

$$\lambda = 1, 2, 2$$

So, eigen values are 1, 2, 2.

Eigen vector X_1 for $\lambda = 1$

$$\begin{bmatrix} 4 & -6 & -6 \\ -1 & 3 & 2 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$4x_1 - 6x_2 - 6x_3 = 0$$

$$-x_1 + 3x_2 + 2x_3 = 0$$

$$\frac{x_1}{6} = \frac{x_2}{-2} = \frac{x_3}{6} = \lambda$$

$$X_1 = \begin{bmatrix} 6\lambda \\ -2\lambda \\ 6\lambda \end{bmatrix} = C\begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$$

Eigen vector X_2, X_3. For $\lambda = 2$

$$\begin{bmatrix} 3 & -6 & -6 \\ -1 & 2 & 2 \\ 3 & -6 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$3x_1 - 6x_2 - 6x_3 = 0$$

$$-x_1 + 2x_2 + 2x_3 = 0$$

Now, let $x_3 = K_1$, $x_2 = K_2$, $x_1 = 2K_2 + 2K_1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2K_1 + 2K_2 \\ K_2 \\ K_1 \end{bmatrix} = K_1\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + K_2\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Hence, eigen vectors are $\begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

Q.1 (d) Solution:

Let $\quad f(x, y, z) = x^2 + y^2 + z^2$

and $\quad \phi(x, y, z) = 3x + 2y^2 + z - 6$

By applying Lagrange's method of multipliers,

$$F(x, y, z) = f(x, y, z) + \lambda\phi(x, y, z)$$
$$= x^2 + y^2 + z^2 + \lambda(3x + 2y + z - 6)$$

Stationary points are :

$$\frac{\partial F}{\partial x} = 0 \Rightarrow 2x + 3\lambda = 0 \Rightarrow x = -\frac{3\lambda}{2}$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow 2y + 2\lambda = 0 \Rightarrow y = -\lambda$$

$$\frac{\partial F}{\partial z} = 0 \Rightarrow 2z + \lambda = 0 \Rightarrow z = -\frac{\lambda}{2}$$

Put the value of x, y, z in $\phi(x, y, z)$

$$3\left(-\frac{3\lambda}{2}\right) + 2(-\lambda) + \left(-\frac{\lambda}{2}\right) = 6$$

$$\lambda = -\frac{6}{7}$$

Substitute into stationary point

$$x = -\frac{3\lambda}{2} = -\frac{3}{2}\left(-\frac{6}{7}\right) = \frac{9}{7}$$

$$y = -\lambda = -\left(-\frac{6}{7}\right) = \frac{6}{7}$$

$$z = -\frac{\lambda}{2} = -\frac{1}{2}\left(-\frac{6}{7}\right) = \frac{3}{7}$$

$$\therefore \quad (x, y, z) = \left(\frac{9}{7}, \frac{6}{7}, \frac{3}{7}\right)$$

Now, minimum value of function $(x^2 + y^2 + z^2)$

$$= \left(\frac{9}{7}\right)^2 + \left(\frac{6}{7}\right)^2 + \left(\frac{3}{7}\right)^2$$

$$= \frac{18}{7} = 2.571$$

Q.1 (e) Solution:

Since $f(x)$ is probability density function and total probability is unity.

$$\int_0^6 f(x)dx = 1$$

$$\int_0^1 Kxdx + \int_2^5 4Kdx + \int_5^6 (5K - Kx)dx = 1$$

$$K\left(\frac{x^2}{2}\right)_0^1 + 4K(x)_2^5 + \left(5Kx - \frac{Kx^2}{2}\right)_5^6 = 1$$

$$\frac{K}{2} + 12K + (30K - 18K - 25K + 12.5K) = 1$$

$$12K = 1 \Rightarrow K = \frac{1}{12}$$

$$\text{Mean of } X \text{ or } E(X) = \int_0^6 xf(x)dx$$

$$= \int_0^1 Kx^2dx + \int_2^5 4Kxdx + \int_5^6 (5Kx - Kx^2)dx$$

$$= \left(\frac{Kx^3}{3}\right)_0^1 + \left(\frac{4Kx^2}{2}\right)_2^5 + \left(\frac{5Kx^2}{2} - \frac{Kx^3}{3}\right)_5^6$$

$$= \frac{K}{3} + 42K + 90K - 72K - 62.5K + 41.67K$$

$$= 39.5K = 39.5 \times \frac{1}{12} = 3.291$$

$$E(X^2) = \int_0^6 x^2 f(x)dx$$

$$= \int_0^1 Kx^3dx + \int_2^5 4Kx^2dx + \int_5^6 (5Kx^2 - Kx^3)dx$$

$$= K\left(\frac{x^4}{4}\right)_0^1 + 4K\left(\frac{x^3}{3}\right)_2^5 + \left(\frac{5Kx^3}{3} - \frac{Kx^4}{4}\right)_5^6$$

$$= \frac{K}{4} + 156K + 360K - 324K - \frac{625}{3}K + \frac{625}{4}K$$

$$= \frac{841}{6}K = \frac{841}{6} \times \frac{1}{12} = 11.68$$

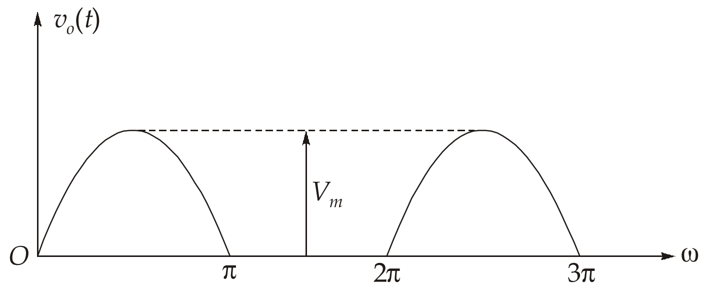
$$\text{Variance} = E(X^2) - [E(X)]^2$$

$$= 11.68 - (3.291)^2$$

$$= 0.8493$$

Q.2 (a) Solution:

(i) Output waveform of half-wave diode rectifier



$\therefore$ a_o = dc component of output voltage,

$$= \frac{1}{2\pi}\int_0^{2\pi} v_o(\omega t)\cdot d(\omega t)$$

$$= \left[\frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t\cdot d(\omega t) + 0\right]$$

$$= \frac{V_m}{\pi} \quad \text{...(i)}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} v_o(\omega t)\cdot \cos n\omega t\cdot d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\cdot \cos n\omega t\cdot d(\omega t)$$

Now, $\sin(\omega t + n\omega t) = \sin \omega t \cos n\omega t + \cos \omega t.\sin n\omega t$

and $\sin(\omega t - n\omega t) = \sin \omega t \cos n\omega t - \cos \omega t.\sin n\omega t$

Adding, we get

$$\sin \omega t \cos n\omega t = \frac{1}{2}[\sin(1+n)\omega t + \sin(1-n)\omega t]$$

$$\therefore \qquad a_n = \frac{V_m}{2\pi}\int_0^{\pi}\{\sin(1+n)\omega t + \sin(1-n)\omega t\}\cdot d(\omega t)$$

$$= \frac{V_m}{2\pi}\left[\frac{1-\cos(1+n)\pi}{1+n} + \frac{1-\cos(1-n)\pi}{1-n}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{1-n+1+n}{1-n^2} - \frac{\cos\pi\cos n\pi - n\cos\pi\cos n\pi + \cos\pi\cos n\pi + n\cos\pi\cos n\pi}{1-n^2}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{2}{1-n^2} + \frac{2\cos n\pi}{1-n^2}\right]$$

$$= \frac{V_m}{\pi}\left[\frac{1+\cos n\pi}{1-n^2}\right] = \frac{V_m}{\pi}\left[\frac{1+(-1)^n}{1-n^2}\right], \text{ as } \cos n\pi = (-1)^n.$$

For $n = 1$, a_1 is indeterminate.

For n odd, $a_n = 0$ and for n even,

$$a_n = -\frac{2V_m}{\pi(n^2-1)}$$

$$a_1 = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\cos\omega t\cdot d(\omega t)$$

$$= \frac{V_m}{2\pi}\int_0^{\pi}\sin 2\omega t\cdot d(\omega t) = 0$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} V_m\sin\omega t\cdot\sin n\omega t\cdot d(\omega t)$$

From trigonometry,

$$\sin\omega t\sin n\omega t = \frac{1}{2}[\cos(1-n)\omega t - \cos(1+n)\omega t]$$

$$\therefore \qquad b_n = \frac{V_m}{2\pi}\int_0^{\pi}\{\cos(1-n)\omega t - \cos(1+n)\omega t\}d(\omega t)$$

$$= \frac{V_m}{2\pi}\left[\left|\frac{\sin(1-n)\omega t}{1-n} - \frac{\sin(1+n)\omega t}{1+n}\right|_0^{\pi}\right] = 0$$

The above expression is indeterminate for $n = 1$ for b_1.

$$\therefore \quad b_1 = \frac{1}{\pi}\int_0^{\pi} V_m \sin^2 \omega t d(\omega t) = \frac{V_m}{2}$$

$$\therefore \quad v_o(t) = a_o + \sum_{n=1}^{1} b_n \sin n\omega t + \sum_{n=2,4,6}^{\infty} a_n \cos n\omega t$$

$$= \frac{V_m}{\pi} + \frac{V_m}{2}\sin \omega t - \frac{2V_m}{\pi}\sum_{n=2,4,6}^{\infty}\frac{1}{n^2-1}\cos n\omega t$$

$$= \frac{V_m}{\pi} + \frac{V_m}{2}\sin \omega t - \frac{2V_m}{3\pi}\cos 2\omega t - \frac{2V_m}{15\pi}\cos 4\omega t - \frac{2V_m}{35\pi}\cos 6\omega t$$

$$\Rightarrow \quad b_1 = \frac{V_m}{2\pi}\left[\frac{\pi}{2}\right] \Rightarrow b_1 = \frac{V_m}{2}$$

$$\Rightarrow \quad \hat{V}_{o1} = \frac{V_m}{2}$$

$$V_{o,\text{avg}} = \frac{V_m}{\pi} = \frac{220 \times \sqrt{2}}{\pi} = 99.035 \text{ V}$$

$$I_{o,\text{avg}} = \frac{99.035}{20} = 4.952 \text{ A}$$

$$\text{Current ripple current} = \frac{I_h}{I_{o,\text{avg}}}$$

$$0.04 = \frac{I_{o1}}{4.952}$$

$$I_{o1} = 0.04 \times 4.954 = 0.19808 \text{ A}$$

$$\hat{V}_{o1} = \frac{V_m}{2} = \frac{220\sqrt{2}}{2 \times \sqrt{2}} = 110 \text{ V}$$

$$I_{o1} = \frac{\hat{V}_{o1}}{|Z_1|}$$

$$0.19817 = \frac{110}{\sqrt{(20)^2 + (2\pi \times 60 \times L)^2}}$$

$$400 + (120\pi L)^2 = (555.331)^2$$

$$(120\pi)L = 555.331$$

$$L = 1.472 \text{ H}$$

(ii) Power circuit diagram of a three phase full converter reveals that source resistance r_s will leads to a voltage drop of $2I_o r_s$. Two thyristors, one from positive and other from negative group will conduct together, therefore, these will be a constant thyristor voltage drop of $2V_T$ and source reactance leads to overlap and it will lead to drop of $6fL_sI_o$.

Hence, mean output voltage of 3-phase full converter is given by

$$E = \frac{3V_{mL}}{\pi}\cos\alpha - 2I_o r_s - 2V_T - 6fL_sI_o$$

As converter is working in inversion mode

$$\alpha = 180° - 45° = 135°$$

The relation between load emf and generator can be written as

$$\frac{3V_{mL}}{\pi}\cos\alpha = -E + 2I_o r_s + 2V_T + 6fL_sI_o$$

$$\frac{3\sqrt{2}\times 400}{\pi}\cos 135° = -E + 2\times 50\times 0.04 + 2\times 1 + \frac{3\times 0.5\times 50}{\pi}$$

$$-381.972 = -E + 4 + 2 + 23.873 \qquad \left\{\begin{aligned} &\omega L = 0.5\ \Omega/\text{Ph} \\ &\frac{3\omega L I_o}{\pi} = 6fLI_o \end{aligned}\right\}$$

$$E = 411.845 \text{ V}$$

Hence, mean generator voltage = 411.845 V

Q.2 (b) Solution:

(i) Let $f(z) = u + iv$ be analytic function.

$$u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$$

Differentiate the function w.r.t. 'x'

$$\frac{\partial f(z)}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

For analytic function, $$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial F(z)}{\partial x} = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$= 3x^2 - 3y^2 + 6x - i(-6xy - 6y)$$

Put $x = z$ and $y = 0$ in above, we get

$$\frac{\partial f(z)}{\partial z} = 3z^2 + 6z$$

$$\int \partial f(z) = \int (3z^2 + 6z)\partial z$$

$$F(z) = z^3 + 3z^2 + C$$

Put $z = x + iy$

$$f(z) = (x + iy)^3 + 3(x + iy)^2 + C$$

$$= x^3 - iy^3 + 3ixy(x + iy) + 3(x^2 - y^2 + 2ixy) + C$$

$$= x^3 - iy^3 + 3ix^2y - 3xy^2 + 3x^2 - 3y^2 + i6xy \quad \text{(Considering } C = 0)$$

$$= x^3 - 3xy^2 + 3x^2 - 3y^2 + i(3x^2y + 6xy - y^3)$$

Also, $f(z) = u(x, y) + iV(x, y)$

Hence, $V(x, y) = 3x^2y + 6xy - y^3$

(ii) $r = \sqrt{2}(1+\cos\theta)$; $r = (\sqrt{2}+1)$

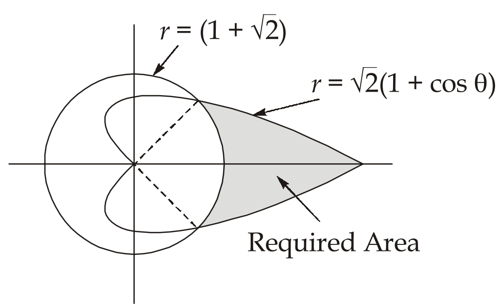
Intersection point is given by

$$\sqrt{2}(1+\cos\theta) = 1+\sqrt{2}$$

$$\sqrt{2}+\sqrt{2}\cos\theta = 1+\sqrt{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4} \text{ or } \left(-\frac{\pi}{4}\right)$$



$$\text{Required area} = \int_{\theta=-\frac{\pi}{4}}^{\theta=\frac{\pi}{4}} \int_{r=(1+\sqrt{2})}^{r=\sqrt{2}(1+\cos\theta)} r\, d\theta\, dr$$

$$= \int_{\theta=-\frac{\pi}{4}}^{\theta=\frac{\pi}{4}} \frac{1}{2}\left(r^2\right)_{1+\sqrt{2}}^{\sqrt{2}(1+\cos\theta)} d\theta$$

$$= \frac{1}{2}\int_{\theta=-\frac{\pi}{4}}^{\theta=\frac{\pi}{4}} \left[2(1+\cos\theta)^2 - \left(1+\sqrt{2}\right)^2\right] d\theta$$

$$= \frac{1}{2}\int_{\theta=-\frac{\pi}{4}}^{\theta=\frac{\pi}{4}} \left[2(1+\cos^2\theta + 2\cos\theta) - \left(1+2+2\sqrt{2}\right)\right] d\theta$$

$$= \frac{1}{2}\int_{\theta=-\frac{\pi}{4}}^{\theta=\frac{\pi}{4}} \left(2\cos^2\theta + 4\cos\theta - 1 - 2\sqrt{2}\right) d\theta$$

$$= \frac{1}{2}\int_{\theta=-\frac{\pi}{4}}^{\theta=\frac{\pi}{4}} \left(1+\cos 2\theta + 4\cos\theta - 1 - 2\sqrt{2}\right) d\theta$$

$$= \frac{1}{2}\int_{\theta=-\frac{\pi}{4}}^{\theta=\frac{\pi}{4}} \left(\cos 2\theta + 4\cos\theta - 2\sqrt{2}\right) d\theta$$

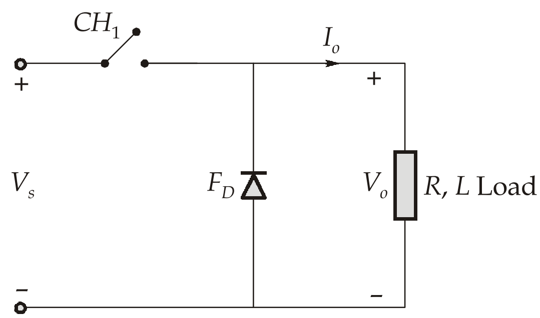
$$= \frac{1}{2}\left(\frac{\sin 2\theta}{2} + 4\sin\theta - 2\sqrt{2}\theta\right)_{\theta=-\frac{\pi}{4}}^{\theta=\frac{\pi}{4}}$$

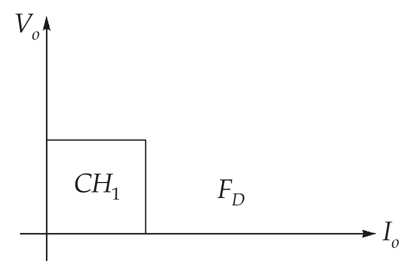
$$= \frac{1}{2}\left[\frac{1-(-1)}{2} + 4\times\frac{2}{\sqrt{2}} - 2\sqrt{2}\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right)\right]$$

$$= \frac{1}{2}\left(1 + 4\sqrt{2} - 2\sqrt{2}\frac{\pi}{2}\right)$$

$= 1.107.$

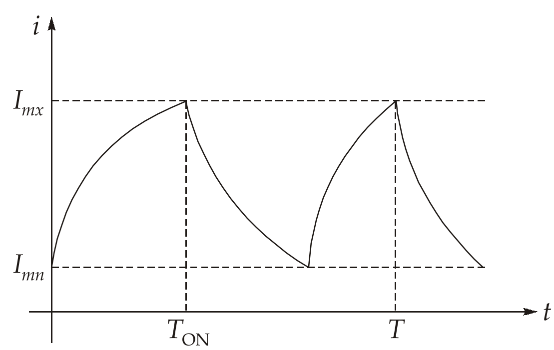
Q.2 (c) Solution:





When CH_1 is ON $(0 \le t \le T_{ON})$

$$V_s = iR + L\frac{di}{dt}$$



$$\frac{V_s}{R} - i = \left(\frac{L}{R}\right)\frac{di}{dt}$$

$$\int_0^t \frac{R}{L}dt = \int_{I_{mn}}^{i} -\frac{di}{i - \frac{V_s}{R}}$$

$$\frac{Rt}{L} = \left[-\ln\left(i - \frac{V_s}{R}\right)\right]_{I_{mn}}^{i}$$

$$-\frac{Rt}{L} = \ln\frac{i - \frac{V_s}{R}}{I_{mn} - \frac{V_s}{R}}$$

$$i - \frac{V_s}{R} = \left(I_{mn} - \frac{V_s}{R}\right)e^{-\frac{Rt}{L}}$$

$$i = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right) + I_{mn}e^{-\frac{Rt}{L}} \quad ...(i)$$

When CH_1 is OFF ($t_{ON} \le t \le T$)

$$0 = iR + L\frac{di}{dt}$$

$$-\frac{R}{L}\int_{t_{ON}}^{T} dt = \int_{I_{mx}}^{I_{mn}} \frac{di}{i}$$

$$-\frac{R}{L}[T - t_{ON}] = [\ln i]_{I_{mx}}^{I_{mn}}$$

$$\ln\frac{I_{mn}}{I_{mx}} = -\frac{R}{L}T_{OFF}$$

$$I_{mn} = I_{mx}e^{-\frac{R}{L}T_{OFF}} \quad ...(ii)$$

From eqn. (i), at $t = t_{ON}$, $i = I_{mx}$

and also put the value of I_{mn} from eqn. (ii) to eqn. (i)

$$I_{mx} = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}T_{ON}}\right) + I_{mx}e^{-\frac{R}{L}T_{OFF}}e^{-\frac{R}{L}T_{ON}}$$

$$I_{mx}\left[1 - e^{-\frac{R}{L}T}\right] = \frac{V_s}{R}\left[1 - e^{-\frac{R}{L}T_{ON}}\right]$$

$$I_{mx} = \frac{\frac{V_s}{R}\left[1 - e^{-\frac{R}{L}T_{ON}}\right]}{1 - e^{-\frac{R}{L}T}}$$

Now on putting all the values,

$$I_{mx} = \frac{\frac{100}{1}\left[1 - e^{-\frac{1}{2\times10^{-3}}\times3\times10^{-3}}\right]}{1 - e^{-\frac{1}{2\times10^{-3}}\times5\times10^{-3}}}$$

$$= 84.634 \text{ A}$$

$$I_{mn} = I_{mx} e^{-\frac{R}{L} T_{OFF}}$$

$$= 84.634 e^{-\frac{2\times10^{-3}}{2\times10^{-3}}}$$

$$= 31.135 \text{ A}$$

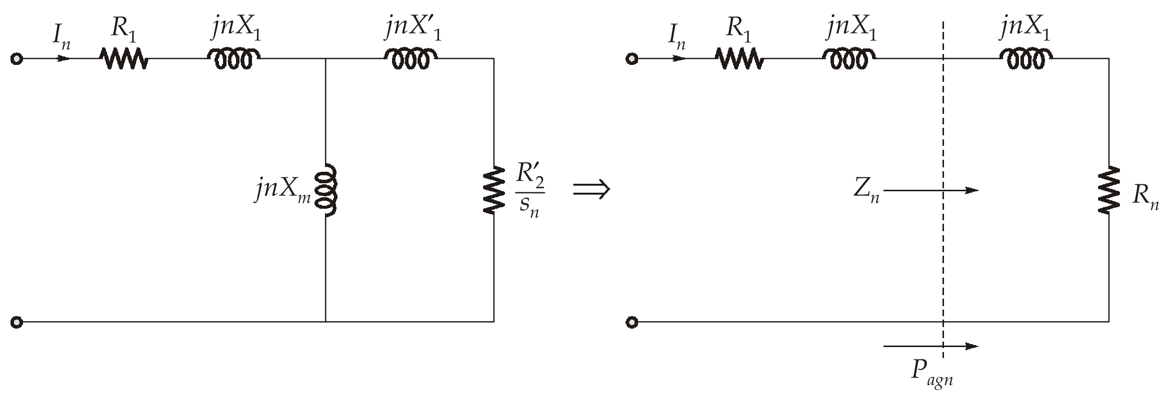
Hence, maximum load current,

$$I_{mx} = 84.634 \text{ A}$$

Minimum load current, $I_{mn} = 31.135 \text{ A}$

Q.3 (a) Solution:

(i) Equivalent circuit at n^{th} harmonic



(ii)

$$N_{s1} = \frac{120 \times f_1}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$s_1 = \frac{1800 - 1740}{1800} = 0.0333$$

$$\frac{R'_2}{s_1} = \frac{0.5}{0.0333} = 15\ \Omega$$

$$X_m = 35\ \Omega$$

$$X'_2 = 1\ \Omega$$

$$Z_1 = \frac{j35(15 + j1)}{15 + j36} = 12.08 + j6 = R_1 + jX_1$$

Fundamental air gap power :

$$P_{ag1} = 3 \times I_1^2 \times R_1$$

$$= 3 \times \left(\frac{1.1 \times 10}{\sqrt{2}}\right)^2 \times 12.08$$

$$P_{ag1} = 2192.52 \text{ W}$$

$$T_1 = \frac{P_{ag1}}{\omega_s} = \frac{2192.52}{60\pi} = 11.63 \text{ N-m}$$

(iii) $n = 5$. This is a negative sequence harmonic.

$$N_{s5} = \frac{-120 \times 5 \times 60}{4} = -9000 \text{ rpm}$$

$$s_5 = \frac{-9000 \times -1740}{-9000} = 1.19$$

$$\frac{R_2'}{s_5} = \frac{0.5}{1.19} = 0.42$$

$$nX_m = 5 \times 35 = 175\ \Omega$$

$$nX_2' = 5 \times 1 = 5\ \Omega$$

$$Z_5 = \frac{j175 \times (0.42 + j5)}{0.42 + j180} = (0.397 + j4.86)\ \Omega$$

$$P_{ag5} = 3 \times \left(\frac{0.22 \times 10}{\sqrt{2}}\right)^2 \times 0.397 = 2.88 \text{ W}$$

$$T_5 = \frac{2.88}{-300\pi} = -3.06 \times 10^{-3} \text{ N-m}$$

For $n = 7$: This is positive sequence harmonic.

$$N_{s7} = \frac{120 \times 7 \times 60}{4} = 12600 \text{ rpm}$$

$$s_7 = \frac{12600 - 1740}{12600} = 0.862$$

$$\frac{R_2'}{s_7} = \frac{0.5}{0.862} = 0.58$$

$$nX_m = 7 \times 35 = 245\ \Omega$$

$$nX_2' = 7 \times 1 = 7\ \Omega$$

$$Z_7 = \frac{j245(0.58 + j7)}{0.580 + j252} = 0.548 + j6.807\ \Omega$$

$$P_{ag7} = 3 \times \left(\frac{0.16 \times 10}{\sqrt{2}}\right)^2 \times 0.548$$

$$P_{ag7} = 2.10432 \text{ W}$$

$$T_7 = \frac{P_{ag}}{\omega_{s7}} = \frac{2.10432}{420\pi} = 1.595 \times 10^{-3} \text{ N-m}$$

Parasitic torques produced by time harmonics are insignificant compared to fundamental torque.

Q.3 (b) Solution:

(i) Expression for output voltage is

$$V_o = \sum_{n=1,3,5...}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$$

$$= \frac{800}{\pi}\sin(2\pi \cdot 50)t + \frac{800}{3\pi}\sin(3\times 2\pi \times 50)t + \frac{800}{5\pi}\sin(5\times 2\pi \times \cdot 50)t$$

$$= 254.64 \sin(314t) + 84.88 \sin(942t) + 50.929 \sin(1570t)$$

Load impedance at frequency nf is

$$Z_n = 20 + j(2\pi n \times 50 \times 15 \times 10^{-3})$$

$$= (20 + j4.71n)$$

$$Z_1 = (20 + j4.71) = 20.547\angle 13.25°$$

$$Z_3 = (20 + j4.71 \times 3) = 24.48\angle 35.24°$$

$$Z_5 = (20 + j4.71 \times 5) = 30.89\angle 49.66°$$

Load current expression

$$I_o = \frac{254.64}{20.547}\sin(314t - 13.25°) + \frac{84.88}{24.48} + \sin(942t - 35.24°)$$

$$+ \frac{50.929}{30.89}\sin(1570t - 49.66°)$$

$$= 12.393 \sin(314t - 13.25°) + 3.467 \sin(942t - 35.24°) + 1.648 \sin(1570t - 49.66°)$$

(ii)

$$I_{o,\text{rms}} = \sqrt{\left(\frac{12.393}{\sqrt{2}}\right)^2 + \left(\frac{3.467}{\sqrt{2}}\right)^2 + \left(\frac{1.648}{\sqrt{2}}\right)^2}$$

$$= 9.173 \text{ A}$$

Power absorbed by load is $I^2_{o,\text{rms}}R$

$$= (9.173)^2 \times 20 \text{ W}$$

$$= 1683.22 \text{ W}$$

(iii) Average current in the DC source is obtained by

$$V_{DC}I_{DC} = \text{Power consumed by the load}$$

$$I_{DC} = \frac{1683.22}{200} = 8.416 \text{ A}$$

Q.3 (c) Solution:

We know the fourier series of $f(x)$ as

$$f(x) = a_o + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x) \quad \left(\because \omega_n = \frac{2\pi}{2} = \pi\right)$$

where,

$$a_o = \int_0^2 f(x)dx$$

$$= \int_0^1 \pi x dx + \int_1^2 \pi(1-x)dx$$

$$= \left(\frac{\pi x^2}{2}\right)_0^1 + \left(\pi\left(x - \frac{x^2}{2}\right)\right]_1^2$$

$$= \frac{\pi}{2} + \pi\left(0 - \frac{1}{2}\right] = 0$$

$$a_n = \frac{2}{2}\int f(x)\cos n\pi x\, dx$$

$$= \int_0^1 \pi x \cos n\pi x dx + \int_1^2 \pi(1-x)\cos n\pi x dx$$

$$= \left(\frac{\pi x \sin n\pi x}{n\pi}\right)_0^1 - \int_0^1 \frac{\pi \sin n\pi x}{n\pi}dx + \left(\pi(1-x)\frac{\sin n\pi x}{n\pi}\right)_1^2 + \int_1^2 \frac{\pi \sin n\pi x}{n\pi}dx$$

$$= 0 - \frac{1}{n^2\pi}[-\cos n\pi x]_0^1 + 0 + \frac{1}{n^2\pi}[-\cos n\pi x]_1^2$$

$$= -\frac{1}{n^2\pi}\left[-(-1)^n + 1\right] + \frac{1}{n^2\pi}\left[-1 + (-1)^n\right]$$

$$= \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$= -\frac{4}{n^2\pi}, \text{ for } n = 1, 3, 5$$

$$b_n = \frac{2}{2}\int_0^2 f(x)\sin n\pi x\,dx$$

$$= \int_0^1 \pi x \sin n\pi x\,dx + \int_1^2 \pi(1-x)\sin n\pi x\,dx$$

$$= \left[-\pi x\frac{\cos n\pi x}{n\pi}\right]_0^1 + \int_0^1 \pi\frac{\cos n\pi x}{n\pi}dx + \left[-\pi(1-x)\frac{\cos n\pi x}{n\pi}\right]_1^2$$

$$- \int_1^2 \pi\frac{\cos n\pi x}{n\pi}dx$$

$$= \frac{-(-1)^n}{n} + \left[\frac{\sin n\pi x}{n^2\pi}\right]_0^1 + \frac{1}{n} - \left[\frac{\sin n\pi x}{n^2\pi}\right]_1^2$$

$$= \frac{1}{n}\left[1-(-1)^n\right]$$

$$= \frac{2}{n}, \text{ for n} = 1, 3, 5, 7$$

Fourier series of $f(x)$ is

$$f(x) = 0 - \frac{4}{\pi}\left[\frac{\cos\pi x}{1^2} + \frac{\cos^3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \ldots..\right]$$

$$+ 2\left[\frac{\sin\pi x}{1} + \frac{\sin 3\pi x}{3} + \frac{\sin 5\pi x}{5} + \ldots..\right]$$

Now put $x = 1$ in above expression

$$f(1) = 0 - \frac{4}{\pi}\left[\frac{-1}{1^2} + \frac{(-1)}{3^2} + \frac{(-1)}{5^2} + \ldots..\right] + 2\times 0$$

$$\pi = \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots..\right]$$

$$\Rightarrow \quad \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.. = \frac{\pi^2}{4}$$

Q.4 (a) Solution:

(i) (a) Input kVA at full load

$$S = \frac{\text{Output power}}{\text{Efficiency} \times \text{Power factor}}$$

$$= \frac{100 \times 0.746}{0.8 \times 0.85}$$

$$S = 109.70 \text{ kVA}$$

Input line current I_L :

$$I_L = \frac{109.71 \times 10^3}{\sqrt{3} \times 460} = 137.7 \text{ A}$$

Motor phase current I_P :

$$I_P = \frac{I_L}{\sqrt{3}} = \frac{137.7}{\sqrt{3}} = 79.5 \text{ A}$$

Thyristor rms current $I_{T(\text{rms})}$:

$$I_{T(\text{rms})} = \frac{I_P}{\sqrt{2}} = \frac{79.5}{\sqrt{2}} = 56.22 \text{ A}$$

(b) Peak voltage across thyristor :

$$V_{T(\text{peak})} = \sqrt{2} \times 460 = 650.54 \text{ V}$$

Power factor of motor cos ϕ :

$$\phi = \cos^{-1}(0.85) = 31.8°$$

Control range of firing angle

$$\phi < \alpha < 180° \Rightarrow 31.80° < \alpha < 180°$$

(ii) For $\alpha < 30°$,

$$V_o = \frac{3V_{mL}}{2\pi}\cos\alpha$$

$$V_{o(\max)} = \frac{3V_{mL}}{2\pi}$$

Required average output voltage

$$= \frac{1}{2}\left(\frac{3V_{mL}}{2\pi}\right)$$

$$\cos\alpha = \frac{1}{2}$$

$$\alpha = 60° > 30°$$

For $\alpha \geq 30°$

$$V_o = \frac{3V_{mP}}{2\pi}[1 + \cos(\alpha + 30°)]$$

$$\frac{3V_{mP}}{2\pi}[1 + \cos(\alpha + 30°)] = \frac{1}{2} \cdot \frac{3\sqrt{3}V_{mP}}{2\pi}$$

$$1 + \cos(\alpha + 30°) = \frac{\sqrt{3}}{2}$$

$$\cos(\alpha + 30°) = -0.13397$$

$$\alpha + 30° = 97.69°$$

$$\alpha = 67.69°$$

Firing angle, $\alpha = 67.7°$

$$\text{Average load current} = \frac{V_o}{R} = \frac{1}{2R} \times \frac{3V_{mL}}{2\pi}$$

$$= \frac{3 \times 400\sqrt{2}}{2 \times 20 \times 2 \times \pi} \text{A}$$

$$= 6.752 \text{ A}$$

RMS output voltage

$$V_{or} = \frac{V_{mL}}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6} - \alpha\right) + \frac{1}{2}\sin\left(2\alpha + \frac{\pi}{3}\right)\right]^{1/2}$$

$$= \frac{400\sqrt{2}}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6} - 0.3761\pi\right) + \frac{1}{2}\sin(2 \times 67.7° + 60°)\right]^{1/2}$$

$$= 182.202 \text{ V}$$

RMS current, $I_{or} = \frac{V_{or}}{R} = \frac{182.202}{20} = 9.11 \text{ A}$

Q.4 (b) Solution:

(i) $$\frac{dy}{dx} = \frac{y^2 - x^2}{x^2 + y^2} \Rightarrow f(x, y) = \frac{y^2 - x^2}{x^2 + y^2}$$

$x_o = 0; y_o = 1$

$$K_1 = hf(x_o, y_o) = 0.2 \times \left[\frac{1^2 - 0^2}{1^2 + 0^2}\right] = 0.2$$

$$K_2 = hf\left[x_o + \frac{h}{2}, y_o + \frac{K_1}{2}\right]$$

$$= hf(0.1, 1.1) = 0.2 \times \left[\frac{1.1^2 - 0.1^2}{1.1^2 + 0.1^2}\right] = 0.1967$$

$$K_3 = hf\left[x_o + \frac{h}{2},\ y_o + \frac{K_2}{2}\right]$$

$$= hf(0.1, 1.098) = 0.2 \times \left[\frac{1.098^2 - 0.1^2}{1.098^2 + 0.1^2}\right]$$

$$= 0.196709$$

$$K_4 = h.f(x_o + h, y_o + K_3)$$

$$= hf(0.2, 1.196709)$$

$$= 0.2\left[\frac{(1.196709)^2 - 0.2^2}{(1.196709)^2 + 0.2^2}\right] = 0.1891$$

$$K = \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$$

$$= \frac{0.2 + 0.1967 \times 2 + 2 \times 0.196711 + 0.1891}{6}$$

$$= 0.195986$$

$$y_1 = y_0 + K = 1 + 0.195986$$

$$= 1.195986$$

(ii) Let $u = xy + z; \quad V = x^2 + y^2 - z^2$

and $u = \phi(V)$

$$xy + z = \phi(x^2 + y^2 - z^2)$$

$$z = \phi(x^2 + y^2 - z^2) - xy$$

Differentiating z w.r.t. x

$$\frac{\partial z}{\partial x} = \phi'(x^2 + y^2 - z^2)\left(2x - 2z\frac{\partial z}{\partial x}\right) - y$$

$$P = \phi'(x^2 + y^2 - z^2)(2x - 2zP) - y$$

$$2(x - zP)\phi'(x^2 + y^2 - z^2) = P + y \quad ...(i)$$

Again, differentiating z w.r.t. y,

$$\frac{\partial z}{\partial y} = \phi'(x^2 + y^2 - z^2)\left(2y - 2z\frac{\partial z}{\partial y}\right) - x$$

$$q = \phi'(x^2 + y^2 - z^2)(2y - 2zq) - x$$

$$2(y - zq)\phi'(x^2 + y^2 - z^2) = q + x \quad ...(ii)$$

Divide eqn. (i) and (ii)

$$\frac{P + y}{q + x} = \frac{x - Pz}{y - qz}$$

$$\Rightarrow \quad Py - Pqz + y^2 - qyz = xq - Pqz + x^2 - Pxz$$

$$\Rightarrow \quad P(y + xz) - q(yz + x) = x^2 - y^2$$

So, partial differential equation is

$$(y + xz)\frac{\partial z}{\partial x} - (x + yz)\frac{\partial z}{\partial y} = x^2 - y^2$$

Q.4 (c) Solution:

Given :

$$f(D)y = 4x^2e^{3x}\cos 2x$$

$$f(D) = D^3 - 9D^2 + 27D - 27$$

The auxiliary equation is

$$D^3 - 9D^2 + 27D - 27 = 0$$

$$(D - 3)^3 = 0$$

$$D = 3, 3, 3$$

$$\text{CF} = (C_1 + C_2x + C_3x^2)e^{3x}$$

$$\text{PI} = \frac{4x^2e^{3x}\cos 2x}{(D - 3)^3}$$

$$= 4e^{3x}\frac{1}{(D + 3 - 3)^3}x^2\cos 2x$$

$$= 4e^{3x}\frac{1}{D^3}x^2\cos 2x$$

Let

$$I = \frac{1}{D^3}x^2\cos 2x = \frac{1}{D^3}x^2\ (\text{Real } e^{i2x})$$

$$RP = \text{Real Part}$$

$$= \text{RP of } \frac{1}{D^3}(x^2 e^{i2x})$$

$$= \text{RP of } e^{i2x} \frac{x^2}{(D+i2)^3}$$

$$= \text{RP of } e^{i2x} \frac{x^2}{(i2)^3}\left(1+\frac{D}{2i}\right)^{-3}$$

$$\left\{\text{As } (1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + ...\right\}$$

$$= \text{RP of } \frac{e^{i2x}}{-i8}x^2\left[1 - \frac{3D}{2i} + \frac{3\times 4}{2}\left(\frac{D}{2i}\right)^2 - .....\right]$$

$$= \text{RP of } \frac{i}{8}e^{i2x}x^2\left[1 + i\frac{3}{2}D + 6\frac{D^2}{-4} - .....\right]$$

$$= \text{RP of } \frac{i}{8}e^{i2x}\left[x^2 + i\frac{3}{2}\cdot 2x - \frac{3}{2}\times 2\right]$$

$$= \text{RP of } \frac{i}{8}e^{i2x}\left[x^2 - 3 + i3x\right]$$

$$= \text{RP of } \frac{i}{8}(\cos 2x + i\sin 2x)(x^2 - 3 + i3x)$$

$$I = -\frac{1}{8}\left[(x^2-3)\sin 2x + 3x\cos 2x\right]$$

$$\text{P.I.} = -\frac{4e^{3x}}{8}[(x^2-3)\sin 2x + 3x\cos 2x]$$

$$= -\frac{e^{3x}}{2}[(x^2-3)\sin 2x + 3x\cos 2x]$$

$$Y = \text{C.F.} + \text{P.I.}$$

$$= (C_1 + C_2x + C_2x^2)e^{3x} - \frac{e^{3x}}{2}[(x^2-3)\sin 2x + 3x\cos 2x]$$

Section B : Basic Electronics Engineering-1 + Analog Electronics-1 + Electrical Materials-1 + Electrical Machines-2

Q.5 (a) Solution:

Number of poles,

$$P = \frac{120f}{N_s} = \frac{120 \times 50}{125} = 48 \text{ poles}$$

Average flux per pole is given as,

$$\phi_{avg} = \frac{2B_m}{\pi} \times \frac{\pi DL}{P}$$

$$= \frac{2 \times 1.1}{\pi} \times \frac{\pi \times 6.1 \times 1.2}{48}$$

$$\phi_{avg} = 0.335 \text{ Wb/pole}$$

Now, distribution for 1-ϕ,

$$m = \text{slot per pole per phase}$$

$$m = \frac{\text{Slot}}{\text{Phase} \times \text{Pole}} = \frac{576}{48} = 12$$

$$\beta = \frac{180° \times \text{Pole}}{\text{Slots}} = \frac{180° \times 48}{572} = 15°$$

Now,

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin(90°)}{12\sin(7.5°)} = 0.638$$

Total number of series turns,

$$N = \frac{576 \times 2}{2} = 576 \text{ turns}$$

Therefore, emf induced is,

$$E = 4.44\phi_m f K_d N_{ph}$$

$$= 4.44 \times 0.335 \times 50 \times 0.638 \times 576$$

$$E = 27330 \text{ Volts}$$

Now for 3-ϕ,

$$m = \frac{\text{Number of slots}}{\text{Pole} \times \text{Phase}}$$

$$m = \frac{576}{48 \times 3} = 4$$

$$\beta = \frac{180° \times 48}{576} = 15°$$

$$K_d = \frac{\sin\left(\frac{4 \times 15°}{4}\right)}{4\sin(7.5°)} = 0.9576$$

$$\text{Emf, } E = 4.44 \times 0.335 \times 0.9576 \times 50 \times \frac{576}{3}$$

$$E = 13673.60 \text{ Volts}$$

Q.5 (b) Solution:

(i) The two sources of magnetic moments for electrons are : (a) Spin; (b) Orbital motion.

(a) **Spin :** Spin is an intrinsic property of electrons and is a form of angular momentum. It results in a magnetic moment because a spin charged particles generates magnetic field. Electrons can have either spin up or spin down, which results in a magnetic moment.

(b) **Orbital Motion :** Electrons in an atom also have orbital motion, which results in a magnetic moment which an electron moves around the nucleus, it generates a magnetic field. The direction of magnetic moment depends on the direction of the electron's motion and orientation of its orbit.

(ii) Not all electrons have a net magnetic moment only electrons that have an unpaired spin or an unpaired orbital momentum have a net magnetic moment. Electrons with paired orbital angular momentum cancel each other out and do not produce a net magnetic moment.

(iii) Not all atoms have a net magnetic moment. Atoms with an even number of electrons with paired spin or paired orbital angular momentum have a net magnetic moment of zero because the magnetic moments of the paired electrons cancel each other out. Atoms with an odd number of electrons or with unpaired spins or orbital angular momentum have a net magnetic moment. However, this net magnetic moment may be very small and in some cases, it may be difficult to measure.

Q.5 (c) Solution:

The equation of diode current is

$$I_D = I_o\left[e^{\frac{V_D}{\eta V_T}} - 1\right] \cong I_o e^{\frac{V_D}{\eta V_T}}$$

Substituting the respective values,

$$0.6 \text{ mA} = I_o e^{\frac{400}{\eta \times 26}} \quad ...(1)$$

$$20 \text{ mA} = I_o e^{\frac{500}{\eta \times 26}}$$

Dividing eqn. (2) by (1)

$$\frac{20}{0.6} = \frac{I_o e^{\frac{500}{26\eta}}}{I_o e^{\frac{400}{26\eta}}}$$

$$\frac{100}{3} = e^{\frac{500-400}{26\eta}}$$

$$\eta = \frac{100}{26 \times \ln\left(\frac{100}{3}\right)}$$

$$\eta = 1.097$$

Current at 450 mV

$$\frac{I\ (450 \text{ mV})}{I\ (400 \text{ mV})} = \frac{I_o e^{\frac{450}{26\eta}}}{I_o e^{\frac{400}{26\eta}}}$$

$$I\ (450 \text{ mV}) = 0.6 \times 10^{-3} e^{\frac{50}{26 \times 1.097}}$$

$$= 3.463 \times 10^{-3} \text{ A}$$

$$I\ (450 \text{ mV}) = 3.463 \text{ mA}$$

Q.5 (d) Solution:

Given : $I_{DSS} = 3$ mA, $V_{GS(off)} = -6$ V, $g_{mo} = 5$ mS

(i) The maximum transconductance g_{mo} occurs at $V_{GS} = 0$ V.

The transconductance,

$$g_m = g_{mo}\left[1 - \frac{V_{GS}}{V_{GS(\text{off})}}\right]$$

$$g_m = 5 \times 10^{-3}\left[1 - \frac{-4}{-6}\right]$$

$$g_m = \frac{5}{3}\text{mS} = 1.667 \text{ mS}$$

(ii) The drain current,

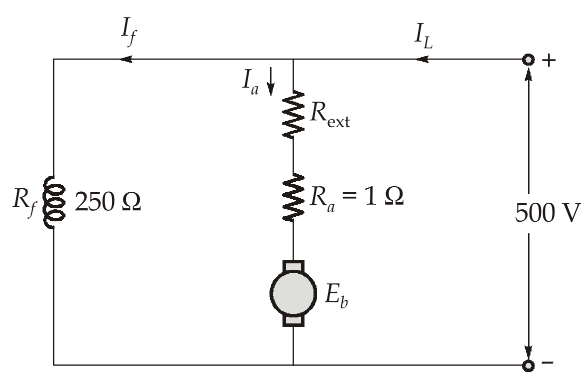
$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_{GS(\text{off})}}\right]^2$$

$$= 3 \times 10^{-3}\left[1 - \frac{-4}{-6}\right]^2$$

$$= 3 \times 10^{-3} \times \frac{1}{9}$$

$$I_D = 0.33 \text{ mA}$$

Q.5 (e) Solution:



When $R_{ext} = 0$,

$$I_L = 42 \text{ A}, \quad I_{a1} = 40 \text{ A}$$

Back emf,
$$E_{b1} = V_t - I_a R_a$$
$$= 500 - 40 \times 1$$
$$E_{b1} = 460 \text{ V}$$

Now, at the instant braking, armature current should not exceed 150% of rated armature current

$$I_{a2} = 1.5 \times I_{a1} = 1.5 \times 40 = 60 \text{ A}$$

Plugging of motor involves the reversing the supply terminals. Therefore, armature current at the instant of braking

$$I_{a2} = \frac{E_{a1} + V_t}{R_a + R_{ext}}$$

$$I_{a2} = \frac{500 + 460}{1 + R_{ext}}$$

$$1 + R_{ext} = \frac{960}{60} = 16\ \Omega$$

$$R_{ext} = 16 - 1 = 15\ \text{A}$$

Plugging torque, $$T_P = \frac{460 \times 60}{130} = 212.308\ \text{N-m}$$

Load torque, $$T_L = \frac{460 \times 40}{130} = 141.53\ \text{N-m}$$

Therefore, braking torque,

$$T_b = T_P + T_L = 353.838\ \text{N-m}$$

Q.6 (a) Solution:

Given : $V_{rms} = 10$ V, $f_o = 50$ Hz, $R_L = 1.1$ kΩ, $C = 50$ μF

Peak value of voltage across load,

$$V_m = 10\sqrt{2} = 14.14\ \text{Volts}$$

We know,

DC voltage, $$V_{DC} = V_m - \frac{I_{DC}}{4 f_o C}$$

But, $$V_{DC} = I_{DC}.R_L$$

$$V_{DC} + \frac{V_{DC}}{4 R_L f_o C} = V_m$$

$$V_{DC} = \frac{V_m}{\left[1 + \frac{1}{4 f_o R_L C}\right]}$$

$$= \frac{14.14}{\left[1 + \frac{1}{4 \times 50 \times 1.1 \times 10^3 \times 50 \times 10^{-6}}\right]}$$

$$V_{DC} = 12.962\ \text{V}$$

Voltage regulation,

$$\% \text{ V.R.} = \frac{V_m - V_{DC}}{V_{DC}} \times 100\%$$

$$= \frac{14.14 - 12.962}{12.962} \times 100\%$$

$$= 9.09\%$$

Ripple output voltage,

$$V_r = \frac{I_{DC}}{2f_o C}$$

$$V_r = \frac{V_{DC}}{2f_o C R_L}$$

$$V_r = \frac{12.962}{2 \times 50 \times 1.1 \times 10^3 \times 50 \times 10^{-6}}$$

$$V_r = 2.357 \text{ Volts}$$

Ripple ac output voltage,

$$V_{ac,rms} = \frac{V_r}{2\sqrt{3}} = \frac{2.357}{2\sqrt{3}}$$

$$V_{ac,rms} = 0.68 \text{ Volt}$$

Q.6 (b) Solution:

(i) Speed of generator, $\omega_s = \frac{4\pi f}{P} = \frac{4\pi \times 50}{6} = \frac{100}{3}\pi$ rad/sec

(ii) At 0.8 leading pf :

$$\vec{V}_t = \vec{E} - j\vec{I}_a X_s$$

$$= \frac{480}{\sqrt{3}} - j1 \times 60\angle 36.86°$$

$$\vec{V}_t = 316.77\angle -8.71° \text{ Volts (L-N)}$$

$$\vec{V}_t = 548.67\angle -8.71° \text{ Volts (L-L)}$$

At 0.8 lagging pf :

$$\vec{V}_t = \vec{E} - j\vec{I}_a X_s$$

$$= \frac{480}{\sqrt{3}} - j1 \times 60\angle -36.86$$

$$\vec{V}_t = 245.87\angle -11.26° \text{ Volts (L-N)}$$

or $$\vec{V}_t = \sqrt{3} \times 245.87\angle -11.26° = 425.857\angle -11.26° \text{ Volts (L-L)}$$

(iii) Developed power in generator,

$$\vec{S}_{\text{dev}} = \sqrt{3} \cdot \vec{E} \cdot \vec{I}_a^*$$

$$= \sqrt{3} \times 425.857\angle -11.26 \times 60\angle 36.86$$

$$\vec{S}_{\text{dev}} = (39.91 + j19.12) \text{ kVA}$$

Output power of generator,

$$P_{\text{out}} = P_{\text{dev}} - \text{Losses}$$

$$= 39.91 - 1.5 - 1 = 37.41 \text{ kW}$$

Therefore, efficiency of generator is given as

$$\eta = \frac{P_{\text{out}}}{P_{i/p}} \times 100\% \quad (\because \text{ Cu loss neglected. So } P_{\text{dev}} = P_{i/p})$$

$$\eta = \frac{37.41}{39.91} \times 100 = 93.736\%$$

(iv) Voltage regulation of generator at 50 A and 0.8 leading power factor is,

$$\% \text{ V.R.} = \frac{|\vec{E}| - |\vec{V}_t|}{|\vec{V}_t|} \times 100$$

Now, $$\vec{V}_t = \vec{E}_f - j\vec{I}_a X_s$$

$$= \frac{480}{\sqrt{3}} - j50\angle 36.86$$

$$\vec{V}_t = 309.71\angle -7.42° \text{ Volts (L-N)}$$

Now, $$\% \text{ V.R.} = \frac{\frac{480}{\sqrt{3}} - 309.71}{309.71} \times 100$$

$$\% \text{ V.R.} = -10.52\%$$

Q.6 (c) Solution:

Given : $N_A = 1 \times 10^{16}/\text{cm}^3,\ N_D = 1 \times 10^{15}/\text{cm}^3$

$n_i = 1 \times 10^{10}/\text{cm}^3,\ V_R = 5\text{ V}$

Built-in potential, $V_D = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$

$$V_o = 0.0258 \ln\left(\frac{1\times10^{15}\times1\times10^{16}}{(1\times10^{10})^2}\right)$$

$$V_o = 0.653$$

$$W_{\text{dep}} = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_o+V_R)}$$

$$= \sqrt{\frac{2\times11.7\times8.854}{1.6\times10^{-19}}\left(\frac{1}{10^{15}}+\frac{1}{10^{16}}\right)(0.653+5)\times10^{-14}}$$

$$= 2.838 \times 10^{-4}\text{ cm}$$

$$W_{\text{dep}} = 283.8\ \mu\text{m}$$

But $W_{\text{dep}} = x_n + x_p$

$$283.8 = x_n + x_p \quad ...(1)$$

But $\frac{x_p}{x_n} = \frac{N_D}{N_A}$

$$\frac{x_p}{x_n} = \frac{10^{15}}{10^{16}}$$

$$x_n = 10x_p \quad ...(2)$$

From eqn. (1) and (2),

$$x_p + 10x_p = 283.8$$

$$x_p = 25.8\ \mu\text{m}$$

$$x_n = 258\ \mu\text{m}$$

Charge stored

$$q_j = qN_Dx_nA = qN_Ax_pA$$

$$= 1.6 \times 10^{-19} \times 10^{16} \times 25.8 \times 400 \times 10^{-8}$$

$$q_j = 1.6512 \times 10^{-7}\text{ C}$$

Junction capacitance,

$$C_j = \frac{\varepsilon A}{W_{dep}} = \frac{11.7 \times 8.854 \times 10^{-14} \times 400 \times 10^{-8}}{2.838 \times 10^{-4}}$$

$$C_j = 1.46 \times 10^{-14} \text{ F}$$

Q.7 (a) Solution:

(i) (a) Magnetic susceptibility (χ) is defined as the ratio of induced magnetization (M) to the applied magnetic field (H). Hence,

$$\chi = \frac{M}{H} = \frac{3.2 \times 10^5}{50} = 6400$$

(b) The permeability (μ) of the material is given by relation

$$B = \mu H$$

$$\mu = \frac{B}{H} = \frac{\mu_o(1+\chi)H}{H}$$

$$\mu = \mu_o(1 + X) = (6400 + 1) \times 4\pi \times 10^{-7}$$

$$\mu = 8.043 \times 10^{-3} \text{ H/m}$$

(c) Magnetic field density,

$$B = \mu_o(1+\chi)H$$

$$= 4\pi \times 10^{-7} \times 6401 \times 50$$

$$B = 0.4022 \text{ T}$$

(d) Based on the high value of magnetic susceptibility, we can suggest that the material is displaying ferromagnetism. This is because of the ferromagnetic materials have a high magnetic susceptibility due to the alignment of magnetic moment in the material.

(ii) The information can be stored magnetically through the use of magnetic field to represent binary digit (bits) of the data. The magnetic medium, such as a hard disk or magnetic tape, contains a surface coated with a magnetic materials that can be magnetized in different directions.

The direction of magnetization of a small area on the surface, called magnetic domain, represents a bit data. The magnetization direction can be either up or down, representing 1 or 0 respectively. The magnetization of each domain is set by magnetic head that creates a magnetic field that aligns the magnetic domains in a particular direction.

To read the stored information, a magnetic head detects the magnetization direction of each domain as the storage medium spins past it. The magnetic head senses the changes in the magnetic field as it moves over each domain and converts these changes into electrical that can be interpreted as 1s and 0s by computer.

The storage capacity of magnetic storage medium can be increased by reducing the size of the magnetic domains, allowing more bits to be stored per unit area. Magnetic storage technology has been widely used in computer systems due to its high capacity, low cost and reliability.

Q.7 (b) Solution:

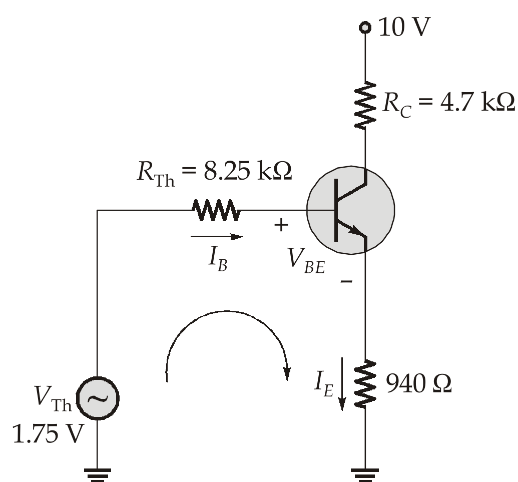
(i) DC Analysis : For DC source, all the capacitor will be open circuited.

Thevenin equivalent circuit,

$$V_{Th} = \frac{10 \times 10}{10 + 47} = 1.75 \text{ V}$$

$$R_{Th} = 10 \parallel 47 = \frac{10 \times 47}{10 + 47} = 8.246 \text{ k}\Omega$$

Thevenin equivalent circuit,



Apply kVL in Base-Emitter loop

$$-1.75 + 8.25 I_B + V_{BE} + \frac{(940)}{1000} \times (1 + \beta_{dc}) I_B = 0$$

$$I_B = \frac{1.75 - 0.7}{8.246 + 0.94 \times 151} \text{mA}$$

$$I_B = 6.99 \, \mu\text{A}$$

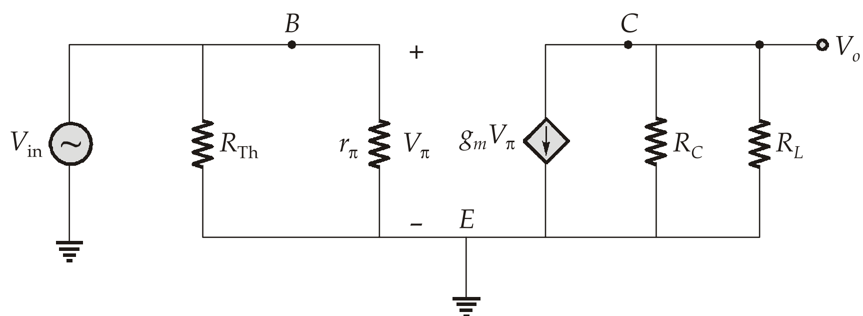
$$I_C = \beta_{dc} I_B = 150 \times 6.99 \ \mu A$$
$$I_C = 1.05 \text{ mA}$$

Small signal parameters :

$$r_\pi = \frac{V_T}{I_B} = \frac{26 \times 10^{-3}}{6.99 \times 10^{-3}} \text{k}\Omega = 3.72 \text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{1.05}{26} = 40.38 \text{ m℧}$$

Small signal model in ac analysis



$$V_o = -g_m(R_C \| R_L)V_\pi$$
$$= -40.38(4.7 \| 47)V_\pi$$
$$= -172.533 V_\pi$$

Voltage gain, $A_v = -172.533 \times 1$

$$A_v = -172.533$$

Q.7 (c) Solution:

(i) As generator :

Load current, $I_{L1} = \dfrac{50 \times 10^3}{250} = 200 \text{ A}$

Armature current, $I_{a1} = I_{L1} + \dfrac{250}{50} = 205 \text{ A}$

Generated emf, $E_1 = V_t + I_{a1}R_a + \text{Voltage drop in the brushes}$

$$= 250 + 205 \times 0.02 + 2$$

$$E_1 = 256.10 \text{ Volts}$$

Speed of generator, $N_1 = 400 \text{ rpm}$

Now as motor :

Input current, $I_{L2} = \frac{P_L}{V} = \frac{50 \times 10^3}{250} = 200$ A

Armature current, $I_{a2} = 200 - \frac{250}{50} = 195$ A

Back emf of motor, $E_2 = V_t - I_{a2}R_a$ – brush drop

$E_2 = 250 - 195 \times 0.02 - 2$

$= 244.10$ Volts

As we know, $N \propto \frac{E_b}{\phi}$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_1}{\phi_2}$$

But excitation remain unchanged.

Therefore, $\phi_1 = \phi_2$

Hence, speed of motor,

$$N_2 = \frac{E_2}{E_1} \times N_1$$

$$= \frac{244.1}{256.1} \times 400 = 381.25 \text{ rpm}$$

$N_2 = 381.25$ rpm

(ii) Given : 28 slots, 2 pole, 16 turns/coil

Length, $l = 20$ cm

Radius, $r = \frac{l}{2} = 10$ cm

As the winding is double layered, therefore,

Number of coils = Number of slots

Total number of turns = 28 × 16 = 448 turns

Total number of conductors,

$$Z = 2 \times 448 = 896$$

Effective area per pole :

$$A = \frac{\pi DL}{P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

$$= \frac{\pi \times 0.2 \times 0.2}{2} \times 0.75$$

$$A = 0.047 \text{ m}^2$$

$$\text{Average flux/pole} = B_{avg} \times A$$

$$\phi_{avg} = 1.08 \times 0.047 = 0.0509 \text{ Wb/pole}$$

Now, induced emf in machine is given as

$$E = \frac{ZNP\phi}{60\ A} \qquad [\because \text{ for lap winding, } A = P]$$

$$= \frac{896 \times 1750 \times 0.0509}{60}$$

$$E = 1330.247 \text{ Volts}$$

Q.8 (a) Solution:

(i) In dc series motor,

$$\text{Power} = E_a I_a$$

Given that, $\text{Power} = \text{Constant}$

$\therefore$ $E_a I_a = \text{Constant}$

$\Rightarrow$ $E_{a1} I_{a1} = E_{a2} I_{a2}$ ...(1)

Let rated value of speed, $N_1 = 1$ pu

Voltage, $E_{a1} = 1$ pu

Current, $I_{a1} = 1$ pu

We know that,

$$T \propto \phi I_a \propto I_a^2 \qquad (\because \phi \propto I_a \text{ for sereis motor}) \quad ...(2)$$

Given : $\text{Power} = \text{Constant}$

$\Rightarrow$ $T\omega = \text{Constant}$

$\Rightarrow$ $TN = \text{Constant}$

$\Rightarrow$ $T \propto \frac{1}{N}$ ...(3)

From eqn. (1) and (2)

$$I_a^2 \propto \frac{1}{N}$$

$\Rightarrow$ $$I_{a2} = \sqrt{\frac{N_1}{N_2}} \times I_{a1}$$

$$\Rightarrow \qquad I_{a2} = \sqrt{\frac{1}{0.25}} \times 1 = 2 \text{ pu}$$

From eqn. (1),

$$E_{a2} = \frac{E_{a1} \cdot I_{a1}}{I_{a2}} = \frac{1 \times 1}{2} = 0.5 \text{ pu}$$

Therefore, supply voltage should be reduced to 50%.

(ii) For active and reactive power, we need generated emf,

$$E' = \sqrt{(V_t \cos\phi + I_a R_a)^2 + (V_t \sin\phi + I_a X_q)^2}$$

$$= \sqrt{(440 \times 0.85)^2 + (231.784 + \frac{1000}{\sqrt{3}} \times 0.075)^2}$$

$$E' = 464.271 \text{ V}$$

$$\tan\psi = \frac{V_t \sin\phi + I_a X_q}{V_t \cos\phi + I_a R_a} = 0.7355$$

$$\psi = 36.335°$$

As we know,

$$\psi = \delta + \phi$$

$$\delta = 36.335° - 31.78 = 4.555$$

Now,

$$I_d = I_a \sin\psi = \frac{1000}{\sqrt{3}} \times \sin(36.335°)$$

$$I_d = 342.083 \text{ A}$$

Now, generated emf

$$|E_f| = E' + I_d(X_d - X_q)$$

$$= 464.271 + 342.083(0.025) = 472.823 \text{ V}$$

$$\vec{E}_f = 472.823 \text{ V}$$

Now, active developed power in generator,

$$P_{dev} = \left[\frac{E_f V_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right] \times 3$$

$$= \left[\frac{472.823 \times 440}{0.1}\sin(4.555°) + \frac{440^2}{2}\left(\frac{1}{0.075} - \frac{1}{0.1}\right) \times \sin(2 \times 4.555°)\right] \times 3$$

$$P_{\text{dev}} = 648.92 \text{ kW}$$

$$Q_{\text{dev}} = \left[\frac{VE_f}{X_d}\cos\delta - \frac{V^2}{2X_dX_q}[(X_d + X_q) - (X_d - X_q)\cos 2\delta]\right] \times 3$$

$$Q_{\text{dev}} = \left[\frac{472.823 \times 440}{0.1}\cos 4.555 - \frac{440^2}{2 \times 0.1 \times 0.075}[0.175 - 0.025\cos(2 \times 4.555)]\right] \times 3$$

$$Q_{\text{dev}} = 401.34 \text{ kVAr}$$

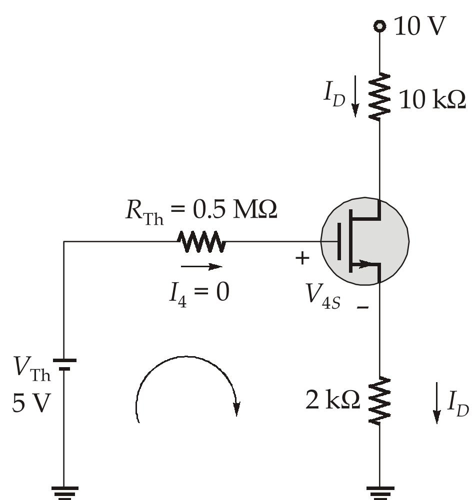
Q.8 (b) Solution:

DC Analysis : Thevenin equivalent circuit across gate terminal,

$$V_G = V_{\text{Th}} = \frac{1}{1+1} \times 10 = \frac{10}{2} = 5 \text{ V}$$

$$R_{\text{Th}} = 1 \,||\, 1 = \frac{1 \times 1}{1+1} = 0.5 \text{ M}\Omega$$

Thevenin equivalent circuit



Apply KVL in Gate-source loop

$$V_{\text{Th}} = I_G R_{\text{Th}} + V_{GS} + 2000 I_D = 0$$

$$I_D = \frac{5 - V_{GS}}{2} \text{mA} \qquad \text{...(1)}$$

But

$$I_D = k_n(V_{GS} - V_{\text{TN}})^2$$

$$I_D = 200 \times 10^{-3}(V_{GS} - 1)^2 \qquad \text{...(2)}$$

From eqn. (1) and (2)

$$\frac{5 - V_{GS}}{2} = 0.2(V_{GS} - 1)^2$$

$$5 - V_{GS} = 0.4(V_{GS}^2 - 2V_{GS} + 1)$$

$$0.4V_{GS}^2 + 0.2V_{GS} - 4.6 = 0$$

$$V_{GS} = 3.15, -3.65$$

$$V_{GS} = 3.15$$

$$I_D = \frac{5 - 3.15}{2000} = 0.925 \text{ mA}$$

$$g_m = 2k_n(V_{GS} - V_{TN})$$

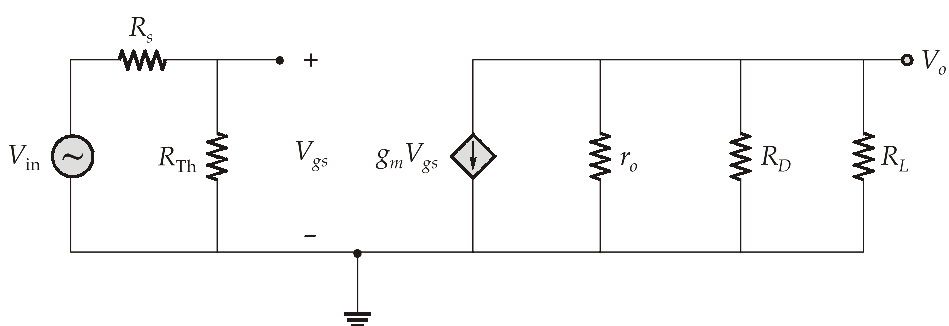
$$= 2 \times 200(3.15 - 1) \times 10^{-6}$$

$$g_m = 0.86 \text{ m℧}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 0.925} \text{k}\Omega$$

$$r_o = 21.62 \text{ k}\Omega$$

Small signal equivalent,



$$V_o = -g_m V_{gs}(r_o \,||\, R_D \,||\, R_L)$$

$$V_{gs} = V_{in} \times \frac{500}{500 + 50}$$

$$V_{gs} = 0.909V_{in}$$

$$V_o = -0.86 \times 0.909 \times (10 \,||\, 60 \,||\, 21.62)V_{in}$$

$$\frac{V_o}{V_{in}} = -4.798$$

$$A_v = -4.798$$

Q.8 (c) Solution:

(i) The conductivity of semi-conducting materials depends on the number of free electrons in the conduction band and also the number of holes in valence band according to equation

$$\sigma = ne\mu_e + pe\mu_h$$

where p = number of holes per cubic meter

n = number of electrons per cubic meters

Thermal energy associated with lattice vibrations can promote electron excitations in which free electrons and holes are created. Additional charge carriers may be generated as consequence of photon-induced electron transitions in which light is absorbed; the attendant increase in conductivity is called photo-conductivity.

Thus, when a specimen of a photoconductive materials is illuminated, the conductivity increases.

This phenomenon is used in photographic light meters. A photoinduced current is measured and its magnitude is a direct function of the intensity of the incident light radiation or the rate at which the photons of light strike photoconductive material. Of course, visible light radiation must induce electronic transitions in the photoconductive material. Cadmium sulfide is commonly used in light meters.

Sunlight may be directly converted into electrical energy in solar cells, which also employ semiconductors. The operation of these devices is, in a sense, the reverse of that for light-emitting diode. A p-n junction is used in which photoexcited electrons and holes are drawn away from the junction, in opposite direction, and become a part of external current.

(ii) We know,

Absolute temperature, $T = T + 273$

$= 1273 \text{ K}$

Let the number of atomic sites/m^3 of copper be N, Atomic weight of Cu be A_{Cu}, density ρ, and Avogadro number, N_A.

Then, $$N = \frac{N_A \rho}{A_{Cu}}$$

$$N = \frac{(6.023 \times 10^{23}) \times 8.4 \times 10^6}{63.5} = 7.96 \times 10^{28} \text{ atoms/m}^3$$

Thus, the number of vacancies at 1000°C are

$$N_V = N\exp\left[\frac{-Q_V}{kT}\right]$$

$$= 7.96 \times 10^{28} \times \exp\left[\frac{-0.9}{8.62 \times 10^{-5} \times 1273}\right]$$

$$N_V = 2.184 \times 10^{25} \text{ atoms/m}^3$$

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